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Analysis of a State Degradation Model and Preventive Maintenance Strategies for Wind Turbine Generators Based on Stochastic Differential Equations

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Abstract: Preventive maintenance is widely used in wind turbine equipment to ensure their safe and reliable operation, and this mainly includes time-based maintenance (TBM) and condition-based maintenance (CBM). Most wind farms only use TBM as the main maintenance strategy in engineering practice. Although this can meet certain reliability requirements, it cannot fully utilize the characteristics of TBM and CBM. For this, a state model based on the stochastic differential equation (SDE) is established in this paper to describe the spatio-temporal evolution process of the degradation behavior of wind turbine generators, in which the components' failure is represented by a proportional hazards model, the random fluctuation of the state is simulated by the Brownian motion, and the SDE model is solved by a function transformation method. Based on the model, the characteristics of TBM and CBM, and the asymptotic relationship between them, are discussed and analyzed, the necessity and feasibility of their combination are expounded, and a joint maintenance strategy is proposed and analyzed. The results show that the stochastic model can better reflect the real deterioration state of the generator. Moreover, TBM has a fixed maintenance interval, depending on global sample tracks and, only depending on the local sample track, CBM can follow the component state. Finally, the rationality and effectiveness of the proposed model and results are verified by a practical example.

Keywords: stochastic differential equations (SDE); degradation model; condition-based maintenance (CBM); time-based maintenance (TBM); wind turbine

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1. Introduction

In recent years, wind power generation has been widely used due to its advantages of clean, environment-friendly, renewable, and low operation and maintenance costs, and given the increasing demand for energy and the significant enhancement of the awareness of ecological environment protection [1]. Since September 2020, the Chinese government has proposed the "carbon peaking and carbon neutrality goals" policy, has raised the requirement for strengthening the planning and construction of a new energy supply and consumption system, and has made a guarantee in the "Fourteenth Five Year Plan" for the development of national wind power generation in China. This means that, in China, the average annual increase in installed capacity of wind power generation will be no less than 60 GW by 2025, at least 800 GW by 2030, and at least 3 TW by 2060 [2]. This indicates that the wind power industry has encountered a historical opportunity for development, and that wind power generation is poised to play a critical role as the primary driving force in the future energy system of China. Meanwhile, wind farms are typically situated in remote mountainous regions or offshore locations and are distant from urban areas, which inevitably leads to complex and harsh operating environments for wind turbines, due to which, furthermore, wind turbines deteriorate rapidly and fail frequently, making maintenance very inconvenient and costly. Therefore, it is necessary to study the degradation process and maintenance strategy of wind turbine components.

The main maintenance methods for wind turbines include corrective maintenance (CM) and preventive maintenance (PM). CM refers to the maintenance measures taken after the equipment failure, rather than the measures taken before that. CM can maximize the effective life of the equipment, but it requires downtime for maintenance when an unexpected breakdown occurs, which may conduce to greater production losses. Therefore, CM is applicable to non-core components, of which the failure doesn't have a significant impact [3]. And PM is a maintenance activity carried out on equipment to ensure that it is in an acceptable operating state before failure occurs, which is an effective strategy to prevent the sudden occurrence of the malfunctions [4].

There are two main types of PM, TBM and CBM. TBM is a type of periodic maintenance, i.e., maintenance of components such as generators is implemented at fixed intervals. Therefore, TBM is an active maintenance that can be scheduled and planned in advance, which means that maintenance resources (such as materials, spare parts and information) and personnel arrangements and transfers can be adequately prepared in advance in accordance with the maintenance plan [5,6]. CBM, on the other hand, is a real-time dynamic maintenance according to components' state, i.e., its implementation is depended on accurately perceiving the state of components or equipment via a series of condition monitoring and diagnosis systems, such as Supervisory Control and Data Acquisition (SCADA) system. However, uncertainty as to the state or deterioration of components or equipment makes CBM passive maintenance [7,8].

Sarker et al. [9–11] specially proposed a reliability analysis methodology for predicting faults in offshore wind turbines, which incorporates condition monitoring and expert systems. Their research focuses on the development of a detailed TBM strategy for offshore wind turbines, emphasizing the role of reliability in the formulation of TBM intervals. Huang [12] presented a technical research framework for the proportional hazards model (PHM) of wind turbines, which utilizes the operational data obtained from the SCADA system. And they studied key PHM technologies such as data fusion, state evaluation, fault diagnosis and state trend prediction, and applied them to CBM. Tian [13] established an optimization model of CBM for cold storage equipment based on the Markov decision process, and analyzed the optimal CBM strategy of coupling generator state and maintenance cost. Yuan, et al. [14–16] established the state degradation model of equipment under continuous state monitoring and periodic state monitoring based on the life distribution of train on-board equipment, and optimized the CBM strategy with the state threshold as the core under the constraint of component availability and maintenance cost. Chen [17] focused on the study of trouble-free time for various equipment used in urban rail transit, obtained its probability distribution model by a trial error method, and determined the interval of TBM by combining the relationship between reliability characteristics. Based on reliability theory and the PHM model, Luo [18] analyzed the relationship between historical fault data and historical monitoring data of wind turbine components including generator, gearbox, bearing system, etc., and designed the CBM strategy for repairable components of wind turbine. Li et al. [19–23] established a state degradation model of wind turbines based on the Markov state transition process, discussed the relationship between the actual and expected operation curve of wind turbines, and based on system reliability, availability, stability and asymptotic stability as index, analyzed and designed the CBM strategy with inspection frequency as the core and TBM strategy with inspection time as the core. It is not difficult to see that as a traditional maintenance method, TBM has significant advantages that its implementation is proactive and has plans and arrangements established, which makes it still adopted by many wind farms. CBM is a real-time and dynamic process that accurately perceives the deterioration of components such as generators and decreases the demand for CM, thereby reducing the overall cost of maintenance. This has led CBM to rise from being an auxiliary or back-up to an equally important position as TBM in many maintenance occasions, and even in some cases, TBM has become an auxiliary or back-up to CBM.

However, the state degradation models established in most studies are based on traditional ordinary differential equations (ODEs), of which the essence is the average result according to the law of large numbers in probability theory [24]. Although the maintenance strategy based on ODE models can basically meet the maintenance needs of most wind turbines in practice, which is naturally the reason why TBM has always been used extensively, the model still needs to be improved. The complex environment of wind farms, such as those mostly located in high altitude areas, ocean areas and desert areas, where the wind turbine units are widely distributed, results in the operating conditions of each one have certain particularities, and thus the differences of their sample tracks should be taken into consideration. That means, essentially, to provide a more accurate and comprehensive representation of components' degradation, it is essential to consider the interference of various random factors, such as environmental impacts and operational changes, on the state of the component generator. One method to solve this problem is to introduce a stochastic model. Duan et al. [25,26] established a state degradation model for wind turbine generators based on a first-order linear SDE, and analyzed the relationship between CBM, equipment availability, and maintenance costs. Wang et al. [27–29] established the state model of wind turbine gearboxes by a SDE, introduced the Brownian motion to describe the disturbance of random factors to components or equipment, designed CBM and TBM strategies based on reliability analysis, and elaborated the necessity of combining CBM and TBM. There is no doubt that the stochastic model has a further description of the state deterioration behavior for components of wind turbine, and however, neither linear nor non-linear SDE models have been used in these studies to discuss the solution of SDE in more depth. And moreover, the asymptotic relationship between TBM and CBM and the relationship between TBM or CBM and the joint strategy of TBM and CBM are not discussed in detail.

Regarding the above problems, this paper analyzes and describes the spatio-temporal evolution behavior of wind turbine generators based on a non-linear SDE model, and obtains the analytical solution of the model under certain conditions by a function transformation method. Furthermore, based on the stochastic model, the advantages and disadvantages of TBM and CBM, as well as their asymmetric relationship, are analyzed and discussed, and the characteristics of the joint maintenance strategy, which is a maintenance strategy that combines TBM and CBM, are demonstrated by martingale analysis and change in probability measure theory.

Thereupon, the subsequent sections of this paper are organized as follows: Section 2 establishes the state degradation model of a component generator based on the stochastic differential equation, and analyses the characteristics and shortcomings of TBM and CBM, respectively; Section 3 discusses the asymptotic relationship between TBM and CBM and the characteristics of the joint strategy of TBM and CBM based on the established model; and Section 4 takes a component generator of a wind turbine as an example for simulation and verification.

2. SDE Model

Prior to establishing the state degradation model of wind turbine components, including the generator, it is necessary to provide the following definitions and presumptions:

Definition 1. *$X(t) \in \mathbb{R}^{[0,1]}$ $(t \geq 0)$ is a stochastic function which represents the component state with a sample space Ω of the wind turbine at time t, reflecting its health condition, i.e., the component is in a fault operating state when $X(t) = 0$, and the component is in a completely new operating state when $X(t) = 1$.*

Definition 2. *Define a filtered probability space (Ω,$\mathcal{F}$,$\mathbb{R}$,P) where: $\mathcal{F}$ is the σ-algebra on Ω generated by X(t), this consists of some subsets (also called events) of Ω, and $\mathcal{F}_t \subseteq \mathcal{F}$ is the σ-algebra on Ω generated by X(t) up to time t, which contains all the information about the stochastic process {X(t)} up to time t; a filtration $\mathbb{F}$ is a family{$\mathcal{F}_t$} of increasing σ-algebra on measurable space (Ω,$\mathcal{F}$), which specifies how the information about {X(t)} is revealed in time t; and the function P: $\mathcal{F} \rightarrow \mathbb{R}^{[0,1]}$ is the probability measure on (Ω,$\mathcal{F}$), also known as probability.*

Presumption 1 ([30])**.** *The maintenance behavior of components is reasonable and complete, i.e., there are no errors or failures, interruptions, incomplete, etc. On the scale of the full-life cycle of components, maintenance is considered to be implemented and completed immediately, making the equipment "as good as new".*

2.1. Modeling and Model Description

Based on the above definitions and hypotheses, the state degradation model of wind turbine components is established as follows:

$$\mathrm{d}X(t) = \mu(t, X(t))\mathrm{d}t + \sigma(t, X(t))\mathrm{d}B(t) \tag{1}$$

where $\mu(t,X(t)) = -\lambda(t,X(t)) \cdot X(t)$ is the drift coefficient, depending on the failure rate function $\lambda(t,X(t))$ of the component; $\sigma(t,X(t))$ is the diffusion coefficient, characterizing the random disturbance of the component state; and $B(t)$ is Brownian motion.

Equation (1) satisfies the following three conditions:

1. Locally consistent Lipschitz continuity, i.e., for every T and N, there exists a constant C_N depending only on N, such that for all $|x|, |y| \leq N$ and all $0 \leq t \leq T$:

$$|\sigma(t, x) - \sigma(t, y)| + |\mu(t, x) - \mu(t, y)| \leq C_N|x - y| \tag{2}$$

2. Linear growth condition, i.e., for every T, there is a constant C_T depending only on T, such that for all $0 \leq t \leq T$:

$$|\sigma(t, x) + \mu(t, x)| \leq C_T(1 + |x|) \tag{3}$$

3. For initial time, $t_0 \geq 0$, $X(t_0)$ is independent of $\{B(t), t \in [t_0, T]\}$ and the mathematical expectation of $X^2(t_0)$, $\mathrm{E}[X^2(t_0)] < \infty$ [31].

Therefore, Equation (1) provides a unique solution and has integral form:

$$X(t) = X(t_0) + \int_{t_0}^{t} \mu(s, X(s))\mathrm{d}s + \int_{t_0}^{t} \sigma(s, X(s))\mathrm{d}s \tag{4}$$

Then, we can say that $X(t)$ is the solution of the SDE Equation (1) and, according to the definition, $X(t)$ is a diffusion process. Additionally, the drift coefficient and diffusion coefficient in Equation (1) have the following probability meanings:

$$\mu(t, x) = \lim_{h \to 0} \frac{\mathrm{E}[\, X(t + h) - X(t)|X(t) = x]}{h} \tag{5}$$

$$\sigma^2(t, x) = \lim_{h \to 0} \frac{\mathrm{E}\left[\, (X(t + h) - X(t))^2 \middle| X(t) = x\right]}{h} \tag{6}$$

2.1.1. CBM Model

The proportional hazards model (PHM) is often used to describe components' failure in the CBM strategy, whether for the maintenance of repairable components or the replacement of non-repairable components, since it can effectively combine components' failure data with condition-monitoring data and fully consider the state information [32]. According to the PHM model, we have:

$$\lambda(t, X(t)) = \lambda(t, \mathbf{Z}(t)) = \lambda_0(t)\cdot \exp\{\boldsymbol{\gamma}\cdot\mathbf{Z}(t)\} \quad (7)$$

where $\lambda_0(t)$ is the basic failure rate function and its form determines the type of PHM model; $\mathbf{Z}(t)$ represents the real-state information of generators at time t, which consists of p condition-monitoring data (e.g., temperature, stress, deformation, etc.) obtained from a monitoring system such as SCADA, so it is a p-dimensional vector function, i.e., $\mathbf{Z}(t) = [Z_1(t)\ Z_2(t) \cdots\ Z_p(t)]^{\mathrm{T}}$; and $\boldsymbol{\gamma} = [\gamma_1\ \gamma_2 \cdots\ \gamma_p]$ is a p-dimensional constant vector and γ_i represents the weight of the corresponding i-th condition-monitoring data $Z_i(t)$, $i = 1, 2, \ldots, p$.

The basic failure rate function $\lambda_0(t)$ is determined by the specific description model of the failure rate utilized in this research. The Weibull distribution model and the Gamma distribution model are widely used to depict the failure rate. The actual difference between the two is that the Gamma model is generally used when the failure rate is strictly monotonous. When the failure rate fluctuates, the Weibull model is often used. At the same time, they are also different in mathematical treatment, parameter estimation, and numerical simulation. In this paper, the Weibull distribution model is used to represent the basic failure rate function, i.e., the Weibull proportional hazards model (WPHM), and then:

$$\lambda_0(t) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1},\ t \geq 0 \quad (8)$$

where $\beta > 0$ and $\eta > 0$ are the shape and scale parameters of the Weibull distribution W(β,η).

The random disturbance of a generator is independent of the operating time, but its effect is related to the current state of the component. Therefore, the diffusion coefficient $\sigma(t,X(t)) = KX(t)$, represents the fluctuation rate of the generator state.

Then, the state model of CBM is:

$$\mathrm{d}X(t) = -\frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1}\cdot \exp\{\boldsymbol{\gamma}\cdot\mathbf{Z}(t)\}X(t)\mathrm{d}t + KX(t)\mathrm{d}B(t) \quad (9)$$

And Equation (9) is a first-order nonlinear SDE model with non-homogeneous coefficients. It contains the real state information of the component generator represented by the covariate $\mathbf{Z}(t)$, which is the essential feature that distinguishes the CBM strategy from other maintenance strategies. The meaning of the state $X(t)$ in Equation (9) is consistent with that given at the beginning of Section 2. Due to the existence of Brownian motion $B(t)$, the process of components' degradation is random. Furthermore, Equation (9) is an Itō-type SDE, i.e., its solution $X(t)$ is an Itō process [33,34].

2.1.2. TBM Model

TBM has a fixed maintenance interval, for which the sample track of the generator state of TBM can be considered aptotic. Moreover, TBM is the expectation of CBM [28]. Since the mean value of the disturbance is zero, the state model of TBM degrades to an ordinary differential equation (ODE) model, i.e.,

$$\mathrm{d}X(t) = -\lambda^*(t)X(t)\mathrm{d}t \quad (10)$$

where $\lambda^*(t)$ is the average failure rate of all CBM sample tracks, describing the average situation exhibited by many CBM sample tracks.

The state model of TBM described in Equation (10) is a first-order nonlinear ODE model, and its sample path is fixed, which reflects the expectations of many CBM sample paths. In addition, the time required for each sample path of the TBM state to reach the maintenance threshold, as given in Equation (10), is approximately identical. The meaning of the state $X(t)$ therein is consistent with that given at the beginning of Section 2.

2.2. Parameter Estimation

2.2.1. The Drift Coefficient

The parameter estimation of the drift coefficient is mainly to estimate the parameter of the basic fault rate function $\lambda_0(t)$. A set of ordered samples of the random variable Λ of the basic failure rate for wind turbine components, such as generators, is $\lambda_1 \leq \lambda_2 \leq \ldots \leq \lambda_n$, and Λ obeys the Weibull distribution W(β,η) with unknown shape and scale parameters $\beta > 0$, $\eta > 0$. Then, the k-th order moment of Λ is:

$$\mathrm{E}\Lambda^k = M_k = \eta^k \Gamma\left(1 + \frac{k}{\beta}\right) \tag{11}$$

where $\Gamma(\cdot)$ is the Gamma function, $\Gamma(1 + x) = x\Gamma(x)$. Additionally, the calculated sample moment is:

$$S_n(\lambda) = \begin{cases} 0,\ \lambda < \lambda_1 \\ r/n,\ \lambda_r \leq \lambda < \lambda_{r+1} \\ 1,\ \lambda_n \leq \lambda \end{cases},\ r = 1, 2, \ldots, n-1 \tag{12}$$

Furthermore, the function for calculating the sample observed values is:

$$m_k = \int_0^{+\infty} (1 - S_n(u))\mathrm{d}u = \sum_{r=0}^{n-1} \left(1 - \frac{r}{n}\right)^k (\lambda_{r+1} - \lambda_r) \tag{13}$$

where $\lambda_0 = 0$. According to the moment estimation theory, we have:

$$M_k = m_k,\ k = 1, 2, 4 \tag{14}$$

Then, the estimator $\hat{\beta}$ of the shape parameter β can be obtained:

$$\hat{\beta} = \frac{\ln 2}{\ln(M_1 - M_2) - \ln(M_2 - M_4)} \tag{15}$$

and

$$\Gamma\left(1 + \frac{1}{\hat{\beta}}\right)\hat{\eta} = M_1 \tag{16}$$

Solving Equation (16), the estimator $\hat{\eta}$ of the scale parameter η can be obtained.

The drift coefficient parameter estimation has been completed.

2.2.2. The Diffusion Coefficient

It can be seen from Equation (6) that the diffusion coefficient is a characterization of the conditional average second-order moment growth rate of the component state. Since the fluctuation of state is independent of time, the K-value in the diffusion coefficient can be estimated as follows.

In the time interval [0, T], take the step Δt as:

$$\Delta t = \sup\{\Delta t_i = t_i - t_{i-1},\ i = 1, 2, \ \ldots\ , N\} \tag{17}$$

and $0 \leq n\Delta t \leq T$ (n = 1,2, . . . , N). Then:

$$K = \frac{1}{N}\sum_{i=1}^{N} k_i = \frac{1}{N}\sum_{i=1}^{N} \frac{\gamma \cdot \mathbf{Z}(t_i)\Delta t_i - \gamma \cdot \mathbf{Z}(t_{i-1})\Delta t_{i-1}}{\gamma \cdot \mathbf{Z}(t_{i-1})\Delta t} \tag{18}$$

Thus, the diffusion coefficient $\sigma(t,X(t)) = KX(t)$ is obtained.

2.3. Solution of the CBM State Model

In accordance with the definition of the Itō process, it is easy to know that Equation (9) is the differential form of Itō process. Furthermore, its solution is much more complex than traditional ordinary differential equations. It is taken into consideration to solve this SDE by a method of function transformation. Let:

$$\mathrm{d}X(t) = -\frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} \cdot \exp\{\gamma \cdot \mathbf{Z}(t)\} X(t)\mathrm{d}t + KX(t)\mathrm{d}B(t) = \mu(t, X(t))\mathrm{d}t + \sigma(t, X(t))\mathrm{d}B(t) \quad (19)$$

and let a stochastic integral with certainty coefficients and its differential form be:

$$Y(t) = Y(t_0) + \int_{t_0}^{t} \overline{\mu}(s)\mathrm{d}s + \int_{t_0}^{t} \overline{\sigma}(s)\mathrm{d}B(s) \quad (20)$$

$$\mathrm{d}Y(t) = \overline{\mu}(t)\mathrm{d}t + \overline{\sigma}(t)\mathrm{d}B(t) \quad (21)$$

If there exists a transformation $f(u,v)$ and its inverse transformation $g(u,v)$ when u is fixed, such that:

$$Y(t) = f(t, X(t)),\ X(t) = g(t, Y(t)) \quad (22)$$

Then, when $\hat{\mu}(t)$ and $\hat{\sigma}(t)$ are determined, $Y(t)$ is obtained thereupon, and moreover, when f (u,v) or g (u,v) is determined, $X(t)$ is obtained thereupon, which is the solution to the SDE.

In addition, using the Itō formula for $Y(t)$, we have:

$$\mathrm{d}Y(t) = \frac{\partial f(t, X(t))}{\partial t}\mathrm{d}t + \frac{\partial f(t, X(t))}{\partial X(t)}[\mu(t, X(t))\mathrm{d}t + \sigma(t, X(t))\mathrm{d}B(t)] + \frac{1}{2}\frac{\partial^2 f(t, X(t))}{\partial X^2(t)}\sigma^2(t, X(t))\mathrm{d}t \quad (23)$$

Hence, let:

$$\overline{\mu}(t) = \frac{\partial f(t,x)}{\partial t} + \frac{\partial f(t,x)}{\partial x}\mu(t,x) + \frac{1}{2}\frac{\partial^2 f(t,x)}{\partial x^2}\sigma^2(t,x) \quad (24)$$

$$\overline{\sigma}(t) = \frac{\partial f(t,x)}{\partial x}\sigma(t,x) \quad (25)$$

Equations (24) and (25) are hypothesized to be valid.

From Equation (25), we have:

$$\frac{\partial f(t,x)}{\partial x} = \frac{\overline{\sigma}(t)}{\sigma(t,x)} \quad (26)$$

and then

$$\frac{\partial^2 f(t,x)}{\partial x^2} = -\frac{\overline{\sigma}(t)\sigma_2'(t,x)}{\sigma^2(t,x)} \quad (27)$$

$$\frac{\partial^2 f(t,x)}{\partial x \partial t} = \frac{\partial^2 f(t,x)}{\partial t \partial x} = \frac{\overline{\sigma}'(t)\sigma(t,x) - \sigma_1'(t,x)\overline{\sigma}(t)}{\sigma^2(t,x)} \quad (28)$$

and there exists a function $C(u)$ such that:

$$f(u,v) = \overline{\sigma}(u)\left[\int_1^v \frac{1}{\sigma(u,w)}\mathrm{d}w + C(u)\right] \quad (29)$$

Since Equation (24) does not depend on x then, differentiating it with respect to x, we have:

$$0 = \frac{\partial^2 f(t,x)}{\partial t \partial x} + \frac{\partial}{\partial x}\left[\frac{\partial f(t,x)}{\partial x}\mu(t,x) + \frac{1}{2}\frac{\partial^2 f(t,x)}{\partial x^2}\sigma^2(t,x)\right] \quad (30)$$

Substituting Equations (26)–(28) into Equation (30), we have

$$\frac{\mathrm{d}\overline{\sigma}(t)/\mathrm{d}t}{\overline{\sigma}(t)} = \sigma(t,x)\left[\frac{\partial\sigma(t,x)/\partial t}{\sigma^2(t,x)} - \frac{\partial}{\partial x}\left(\frac{\mu(t,x)}{\sigma(t,x)}\right) + \frac{1}{2}\frac{\partial^2\sigma(t,x)}{\partial x^2}\right] \quad (31)$$

Similarly, since Equation (25) does not depend on x, then differentiating Equation (31) with respect to x, we have

$$\frac{\partial}{\partial x}\left(\sigma(t,x)\left[\frac{\partial\sigma(t,x)/\partial t}{\sigma^2(t,x)}-\frac{\partial}{\partial x}\left(\frac{\mu(t,x)}{\sigma(t,x)}\right)+\frac{1}{2}\frac{\partial^2\sigma(t,x)}{\partial x^2}\right]\right)=0 \tag{32}$$

After substituting $\mu(\cdot,\cdot)$ and $\sigma(\cdot,\cdot)$ in the CBM state model Equation (19) into Equation (32) and calculating, we find its value is zero. This means that under the conditions of Equation (19), the previous hypotheses made for Equations (24) and (25) are valid.

Thus, from Equations (24), (29) and (31), it can be obtained:

$$\begin{cases} f(u,v)=\dfrac{C_1}{K}[\ln v+C(v)],\ g(u,v)=\exp\left\{\dfrac{K}{C_1}v-C(u)\right\} \\ \overline{\mu}(t)=\dfrac{C_1}{K}C'(t)-\dfrac{C_1}{K}\dfrac{\beta}{\eta}\left(\dfrac{t}{\eta}\right)^{\beta-1}\cdot\exp\{\boldsymbol{\gamma}\cdot\mathbf{Z}(t)\}-\dfrac{C_1K}{2},\ \overline{\sigma}(t)=C_1 \end{cases} \tag{33}$$

Then according to boundary conditions, from Equations (21), (22) and (33), we have

$$X(t)=X(t_0)\exp\left\{-\left(\frac{t}{\eta}\right)^{\beta}\exp\{\boldsymbol{\gamma}\cdot\mathbf{Z}(t)\}+KB(t)-\frac{K^2}{2}t\right\} \tag{34}$$

The solution to the CBM state model is Equation (34).

3. Preventive Maintenance Strategies Analysis Based on the State Model

3.1. CBM Strategy Analysis

The essence of the CBM is to implement maintenance activities under the guidance of generator state information. This means that there is a preventive maintenance state threshold X_{thr}, i.e., if, at time t:

$$X(\tau)\le X_{\text{thr}} \tag{35}$$

then preventive maintenance shall be scheduled and implemented, and components are repaired "as good as new". Furthermore, τ is defined as follows:

$$\tau=\inf\{t>0: X(t)\le X_{\text{thr}}<1\} \tag{36}$$

where τ represents the time when the process first exceeds the maintenance state threshold, which also represents the maintenance node of CBM, and is a stopping time for process $X(t)$.

Define the transfer density $p(y,t,x,s)$ and the transfer distribution function $P(y,t,x,s)$ of process $X(t)$, i.e.,:

$$P(y,t,x,s)=\mathrm{P}\{X(t)\le y|X(s)=x\}=\int_{-\infty}^{y}p(u,t,x,s)\mathrm{d}u \tag{37}$$

In addition, the transfer density $p(y,t,x,s)$ can be obtained from the Kolmogorov backward equation [35], i.e.,:

$$\frac{\partial p(y,t,x,s)}{\partial s}+L_s p(y,t,x,s)=0 \tag{38}$$

where the differential operator L_s is the generator of the stochastic process, which is defined as:

$$L_s=\frac{1}{2}\sigma^2(s,x)\frac{\partial^2}{\partial x^2}+\mu(s,x)\frac{\partial}{\partial x} \tag{39}$$

Thus, the probability of stopping time τ of process $X(t)$ can be expressed as:

$$\mathrm{P}\{\tau>t\}=1-\mathrm{P}\{\tau\le t\}=1-P(X_{\text{thr}},t,x,s)=1-\int_{-\infty}^{X_{\text{thr}}}p(u,t,x,s)\mathrm{d}u \tag{40}$$

It can be seen that, when the CBM strategy is implemented, the sample paths for the component state of wind turbines are diverse, which makes the maintenance time random and difficult to directly predict. This may lead to inadequate or untimely maintenance preparation such as materials and equipment transfer and personnel arrangements. Therefore, in order to maximize the effectiveness of the CBM strategy, the maintenance department of wind farms is required to respond quickly, arrange reasonably, and maintain high quality in the event of a breakdown under the CBM strategy.

3.2. TBM Strategy Analysis

The interval of TBM reflects the average situation of CBM for a specific component, the equipment, or the entire physical system of a wind turbine, i.e., the time variable concerned by TBM is the mean or expectation of the stopping time corresponding to the critical state of CBM [29]. Additionally, for the operation process, the expectation of the i-th stopping time is:

$$E[\tau_i|\mathcal{F}_{\tau_i}] = \int_0^{+\infty} \tau_i p_{\rm st}(\tau_i)d\tau_i = \int_0^{+\infty}\left(\int_0^{\tau_i} du\right)p_{\rm st}(\tau_i)d\tau_i = \int_0^{+\infty}\int_u^{+\infty} p_{\rm st}(\tau_i)d\tau_i du = \int_0^{+\infty} P\{\tau_i > t|\mathcal{F}_{\tau_i}\}dt \quad (41)$$

where $p_{\rm st}(\cdot)$ is the probability density of the stopping time and $\mathcal{F}_{\tau_i}$ is the σ-algebra of the process $X(t)$ up to time τ_i, which represents all the information of the process up to time τ_i.

Hence, for the same component, the time interval of TBM is defined as the mean of n stopping times of CBM, i.e.,:

$$T_{\rm TBM} = E[E[\tau_i|\mathcal{F}_{\tau_i}]] = \frac{1}{n}\sum_{i=1}^{n} E[\tau_i|\mathcal{F}_{\tau_i}] \quad (42)$$

and

$$\mathcal{F}_{\tau_1} \subseteq \mathcal{F}_{\tau_2} \subseteq \ldots \subseteq \mathcal{F}_{\tau_n} \quad (43)$$

It can be seen that TBM is based on the average of almost all sample tracks of CBM, which indicates that TBM does not simply implement maintenance based on certain time intervals, but constantly corrects maintenance intervals from a large number of sample tracks of CBM. The condition-monitoring technology of CBM is progressively gaining intelligence with the advancement of emerging technologies, including big data and artificial intelligence, which makes the information transmission between CBM and TBM more accurate and timely. Then, TBM is increasingly adaptable to randomly changing operating environments, which also indicates that TBM is gradually and asymptotically approaching CBM.

Furthermore, the asymptotic relationship between TBM and CBM is:

$$\begin{cases} dX_*(t) = \mu(t, X_*(t), Z_*(t))dt + \sigma(t, X_*(t), Z_*(t))dB(t) = -\lambda^*(t)(X_*(t) + Z_*(t))dt + KZ_*(t)dB(t) \\ X_*(T) = X_* \end{cases} \quad (44)$$

where, $X^*(t)$ is the CBM state gradually recovered by TBM; $X^*(T = T_{\rm TBM}) = X^* = X_{\rm thr}$ refers to the terminal state $X_{\rm thr}$ at the terminal time $T_{\rm TBM}$ in a CBM maintenance cycle, which is $\mathcal{F}_T$-measurable; $Z^*(t)$ is a stochastic process unique to the backward process with $X^*(t)$, i.e., $(X^*(t),Z^*(t))$ together is the solution to Equation (44); $\mu(\cdot,\cdot,\cdot)$ and $\sigma(\cdot,\cdot,\cdot)$ are the drift and diffusion coefficients of the corresponding Equation (44), respectively. And Equation (44) can be solved by non-linear Feynman-Kac formula [36].

3.3. The Maintenance Strategy Combined CBM and TBM

For the same component, equipment and even physical system, the essence of CBM is the maintenance state variable, i.e., the monitoring of the system's critical state or threshold value. Meanwhile, the essence of TBM is the maintenance time variable, i.e., the planning and arrangement of maintenance time points.

Under the CBM strategy, therefore, the maintenance behavior can be precisely implemented according to the real-time state of components such as generators. However, at the same time, the unpredictability and randomness of system state inevitably leads to untimely and insufficient arrangements and transfer of personnel or maintenance resources such as materials, spare parts, information, etc. when CBM is implemented, which results in the preventive maintenance delay and incomplete maintenance of equipment as well as increased maintenance costs, and even results in accelerating the deterioration of the component in disguise [27].

Under the TBM strategy, the implementation time of the maintenance behavior is fixed, and the relevant hardware resources and human resources can be reasonably arranged to each TBM time point in strict accordance with the maintenance plan, which can ensure the sufficiency and completeness of the maintenance behavior. However, meanwhile, at each maintenance time point of TBM, on the one hand, the state of the component may not reach the maintenance threshold, and the maintenance behavior at this time would inevitably lead to the waste or excessive redundancy of maintenance resources, resulting in "over-maintenance" behavior. On the other hand, the state of the component may be much lower than the maintenance threshold, and the untimely maintenance may make the component deteriorate more, which would increase the difficulty and cost of maintenance, resulting in "under-maintenance" behavior.

Hence, in order to improve, alleviate or even eliminate the above problems, it is necessary to combine the TBM and CBM strategy.

To simplify the writing, Equations (9) and (10) are expressed as

$$\mathrm{d}X_{\mathrm{C}}(t) = \mu_1(t)X_{\mathrm{C}}(t)\mathrm{d}t + \sigma(t)X_{\mathrm{C}}(t)\mathrm{d}B(t) \tag{45}$$

$$\mathrm{d}X_{\mathrm{T}}(t) = \mu_2(t)X_{\mathrm{T}}(t)\mathrm{d}t \tag{46}$$

where $X_{\mathrm{C}}(t)$ and $X_{\mathrm{T}}(t)$ are the generator state under the CBM and TBM strategy, respectively. Then, the generator state $X_{\mathrm{com}}(t)$ under the joint maintenance strategy satisfies:

$$\begin{aligned} \mathrm{d}X_{\mathrm{com}}(t) &= a_{\mathrm{C}}(t)\mathrm{d}X_{\mathrm{C}}(t) + a_{\mathrm{T}}(t)\mathrm{d}X_{\mathrm{T}}(t) \\ &= a_{\mathrm{C}}(t)X_{\mathrm{C}}(t)[(\mu_1(t) - \mu_2(t))\mathrm{d}t + \sigma(t)\mathrm{d}B(t)] + X_{\mathrm{com}}(t)\mu_2(t)\mathrm{d}t \end{aligned} \tag{47}$$

where $\{a_{\mathrm{C}}(t), a_{\mathrm{T}}(t)\}$ is the joint maintenance strategy. The combination of the TBM strategy and the CBM strategy means that, firstly, through a large number of CBM state data in one maintenance cycle, the maintenance time of TBM is modified, such that the state of the component generator, corresponding to the maintenance time node $T_{\mathrm{TBM}\cdot i}$ of TBM, is close to the state threshold of CBM. Secondly, according to the reliability requirements of the component generator, a time threshold T_{thr} is set for the stopping time τ_i, corresponding to the state threshold of CBM, and τ_i is also the maintenance time node of CBM, such that TBM is implemented instead of CBM when $|\tau_i - T_{\mathrm{TBM}\cdot i}| < T_{\mathrm{thr}}$, otherwise CBM is continued.

Let $\theta(\tau) = [\mu_1(\tau) - \mu_2(\tau)]/\sigma(\tau)$ for the mathematical expectation with respect to probability measure P:

$$\mathrm{E_P}\left[\exp\left\{\frac{1}{2}\int_0^t \theta^2(u)\mathrm{d}u\right\}\right] < \infty \tag{48}$$

There thus exists a unique probability measure Q equivalent to the original probability measure P, which satisfies the change in measure:

$$\frac{\mathrm{dQ}}{\mathrm{dP}}\bigg|\mathcal{F}_t = \exp\left\{-\frac{1}{2}\int_0^T \theta^2(u)\mathrm{d}u - \int_0^T \theta(u)\mathrm{d}B(u)\right\} \tag{49}$$

Then, according to the Girsanov theorem, under the probability measure Q:

$$B^{\mathrm{Q}}(t) = B(t) + \int_0^t \theta(u)\mathrm{d}u \tag{50}$$

is the normal Brownian motion in the filtered probability space $(\Omega, \mathcal{F}_t, \mathbb{F}, \mathrm{P})$. Hence:

$$\begin{aligned} \mathrm{d}X_{\mathrm{com}}(t) &= a_{\mathrm{C}}(t)X_{\mathrm{C}}(t)\sigma(t)[\theta(t)\mathrm{d}t + \mathrm{d}B(t)] + X_{\mathrm{com}}(t)\mu_2(t)\mathrm{d}t \\ &= a_{\mathrm{C}}(t)X_{\mathrm{C}}(t)\sigma(t)\mathrm{d}B^{\mathrm{Q}}(t) + X_{\mathrm{com}}(t)\mu_2(t)\mathrm{d}t \end{aligned} \tag{51}$$

$$X_{\mathrm{com}}^{\mathrm{Q}}(t) = X_{\mathrm{com}}(t)\exp\left\{-\int_0^t \mu_2(u)\mathrm{d}u\right\} \tag{52}$$

Then:

$$\mathrm{d}X_{\mathrm{com}}^{\mathrm{Q}}(t) = a_{\mathrm{C}}(t)X_{\mathrm{C}}(t)\sigma(t)\mathrm{d}B^{\mathrm{Q}}(t) \tag{53}$$

In accordance with Equation (53) and the definition of martingales, $X_{\mathrm{com}}^{\mathrm{Q}}(t)$ is a martingale with respect to probability measure Q, and we have

$$X_{\mathrm{com}}^{\mathrm{Q}}(t) = \mathrm{E}_{\mathrm{Q}}\left[X_{\mathrm{com}}^{\mathrm{Q}}(T)\middle|\mathcal{F}_t\right] \tag{54}$$

From Equation (52), then, under the joint maintenance strategy, the generator state $X_{\mathrm{com}}(t)$ satisfies

$$X_{\mathrm{com}}(t) = \mathrm{E}_{\mathrm{Q}}\left[\exp\left\{-\int_t^T \mu_2(u)\mathrm{d}u\right\}X_{\mathrm{com}}(T)\middle|\mathcal{F}_t\right] \tag{55}$$

The maintenance strategy combining TBM and CBM aims to implement TBM and CBM together, such that near the time point of TBM, the degradation state of the component is also near the maintenance threshold of CBM. This is an ideal situation. More practically, on the one hand, at the time point of TBM, the implementation of the maintenance behavior for TBM is determined by the CBM sensing whether the component state is close to the maintenance threshold. On the other hand, when the component state under CBM is close to or reaches the maintenance threshold, the maintenance should be arranged as far as possible from the nearest time point of TBM, provided that the time difference permits it. The flow chart of the joint maintenance strategy is shown in Figure 1.

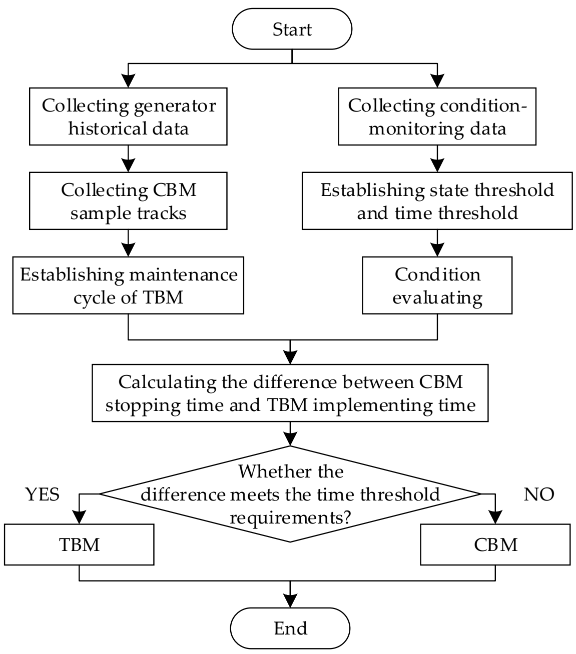

Figure 1. The flow chart of the joint maintenance strategy.

4. Example Analysis

The validity and applicability of the model are verified by employing the component generator of wind turbines as a case study. Part of the life distribution data of generators from 50 wind turbines in a wind farm are shown in Table 1. For the original data of the condition monitoring of generator 1, mainly from the SCADA system, including its main bearing temperature (MBT), main shaft amplitude (MSA), and maximum winding temperature (MWT), 100 data were taken in equal steps, as shown in Table 2. Specifically, the life data in Table 1 are extracted from the historical operation data of the same type of wind turbines in one wind farm. The data in Table 2 are extracted from the historical data of several monitoring indicators of the SCADA system (an operation condition-monitoring and evaluating system) of the same type of wind turbines. These data are all from a wind farm located in Northwest China that cooperated with our Foundation Project.

Table 1. Sample data for life distribution of part generators.

Number	**1**	**2**	**3**	**4**	**5**	**6**	...	**48**	**49**	**50**
Life(h)	7080	8208	7488	11064	5824	5030	...	7756	9331	8065

Table 2. Part of the condition-monitoring data of generator 1.

Number	**Operation Time/h**	**MBT/°C**	**MSA/mm**	**MWT/°C**
1	60	30.3	0.322	39.2
2	135	30.2	0.321	39.2
3	222	30.3	0.324	39.3
4	290	30.5	0.325	39.3
5	360	30.7	0.328	39.4
6	441	30.9	0.327	39.6
7	495	31.4	0.331	40.3
8	571	31.9	0.330	40.8
...	...	...	...	...
97	6801	54.5	2.531	63.0
98	6944	55.2	2.657	64.8
99	7009	56.1	2.864	66.7
100	7080	57.4	3.111	68.8

4.1. Model Analysis of the Example

Based on the parameter estimation method in Section 2.2, using the data in Table 1, we can obtain $\beta = 5.81$, $\eta = 6825$, and $K = 0.015$. Kolmogorov–Smirnov (K-S) test method, as a classical method of goodness-of-fit test, is used to test whether the data in Table 1 obey Weibull distribution [37,38]. The samples with sample size $n = 50$ in Table 1 are sequenced as $x_1, x_2, \ldots, x_n$, and the empirical distribution function (EDF) is:

$$F_n(x) = \begin{cases} 0, \ x < x_1 \\ i/n, \ x_i \leq x < x_{i+1} \\ 1, \ x \geq x_n \end{cases} \tag{56}$$

Then, we have:

$$D = \max_{1 \leq k \leq n} \{ |F_n(x_{k-1}) - F_{\mathrm{W}}(x_k)|, |F_n(x_k) - F_{\mathrm{W}}(x_k)| \} \tag{57}$$

$$F_{\mathrm{W}}(x) = 1 - \exp\left\{ -(x/\eta)^{\beta} \right\} \tag{58}$$

$$D_{\mathrm{m}} = D \cdot \left[\sqrt{n} - (0.021/\alpha) - 0.661 \right] \tag{59}$$

where D is the test statistic, representing the maximum deviation between the EDF and the hypothetical distribution function; $F_n(x_0) = 0$; $F_W(x)$ is the Weibull distribution function; α is the significance level; and D_m is the modified test statistic. Furthermore, according to Equations (56)–(59), $D_m = 0.810$ with $\alpha = 0.01$ is obtained, i.e., at the significance level of 0.01, and it can be considered that the overall distribution of the given data is Weibull distribution with shape and scale parameters $\beta = 5.81$, $\eta = 6825$ [39].

We use the entropy method to calculate the weight of the state vector from the data in Table 2 in order to avoid the state evaluation of the component generator being too complex. The calculation method is shown in Equation (60) [40]:

$$\begin{cases} w_{ij} = Z_{ij} / \sum_{i=1}^{N} Z_{ij} \\ e_j = \frac{1}{\ln N} \sum_{i=1}^{N} w_{ij} \log w_{ij} \\ \gamma_j = \left(1 - e_j\right) \Big/ \sum_{j=1}^{p} \left(1 - e_j\right) \end{cases} \tag{60}$$

where Z_{ij} is the j-th condition-monitoring index at time t_i; w_{ij} is the weight of the j-th condition-monitoring index at time t_i in this index; and e_j is the entropy value of the j-th monitoring index. Thus, we have:

$$\gamma = \begin{bmatrix} \gamma_1 & \gamma_2 & \gamma_3 \end{bmatrix} = \begin{bmatrix} 0.4428 & 0.3147 & 0.2425 \end{bmatrix} \tag{61}$$

$$\gamma \cdot \mathbf{Z}(t) = \begin{bmatrix} \gamma_1 & \gamma_2 & \gamma_3 \end{bmatrix} \cdot \begin{bmatrix} Z_1(t) & Z_2(t) & Z_3(t) \end{bmatrix}^{\mathrm{T}} = \sum_{i=1}^{3} \gamma_i Z_i(t) \tag{62}$$

where $Z_i(t)$ ($i = 1, 2, 3$) are the state-monitoring values of the main bearing temperature, the main shaft amplitude, and the maximum winding temperature at time t.

Therefore, the failure rate function is:

$$\lambda(t, \mathbf{Z}(t)) = \frac{\beta}{\eta} \left(\frac{t}{\eta} \right)^{\beta - 1} \cdot \exp\{\gamma \cdot \mathbf{Z}(t)\} = \frac{5.81}{6825} \left(\frac{t}{6825} \right)^{4.81} \cdot \exp\left\{ \sum_{i=1}^{3} \gamma_i Z_i(t) \right\} \tag{63}$$

Then, the state model under CBM is:

$$\mathrm{d}X(t) = -\frac{5.81}{6825} \left(\frac{t}{6825} \right)^{4.81} \cdot \exp\left\{ \sum_{i=1}^{3} \gamma_i Z_i(t) \right\} X(t)\mathrm{d}t + 0.015 X(t) \mathrm{d}B(t) \tag{64}$$

From the data in Table 1, and the average of sample tracks for CBM state, the state model under TBM is obtained as:

$$\mathrm{d}X(t) = -\frac{6.47}{8102} \left(\frac{t}{8102} \right)^{5.47} X(t)\mathrm{d}t \tag{65}$$

The results of the state model of CBM and TBM in this example can be obtained by using the Euler–Maruyama algorithm to numerically solve the above model, as shown in Figure 2, where the real state is reflected by the condition-monitoring covariates measured from the SCADA system. The meaning of the state $X(t)$ in Figure 2 (similarly hereinafter) is consistent with that determined at the beginning of Section 2, i.e., the state reflects the health condition of the component generator throughout the deterioration process under different maintenance models implemented; when $X(t) = 0$, this means the component is in a fault-operating state, and when $X(t) = 1$, this means the component is in a completely new operating state.

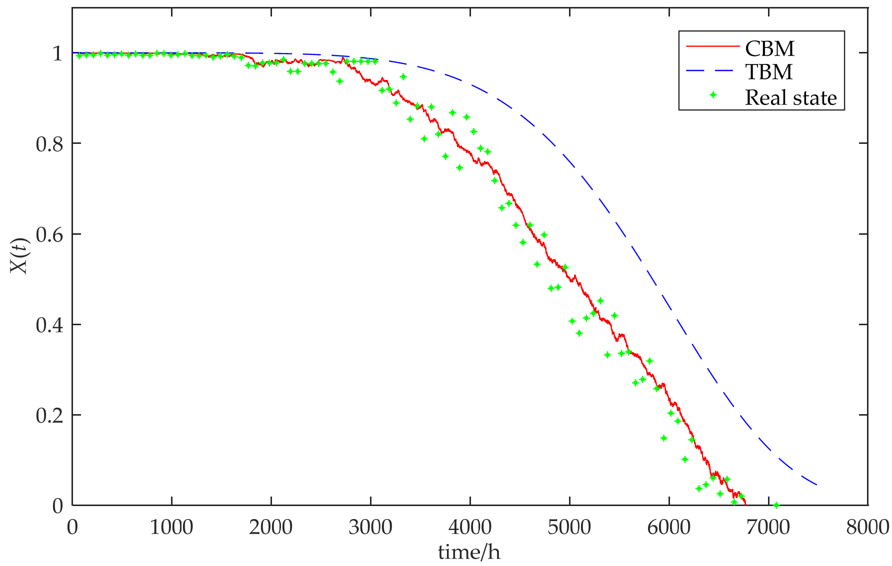

Figure 2. State model of generator 1.

It can be seen from the analysis in Figure 2 that the CBM state has a high degree of fit with the real state of the generator, which means that the description of generator failures based on the PHM model is reasonable, and, furthermore, that it confirms the characteristics of CBM, i.e., it can accurately perceive the state of components such as generators in real time, which enables the CBM model to better predict the component state. Additionally, the curve of the CBM state is not completely smooth, which is caused by the external random disturbance of the generators' state. The ability to characterize this random disturbance is the most essential and distinctive difference between the state model based on the stochastic differential equation and that based on the ordinary differential equation, which is also the advantage of the stochastic model. The CBM state in Figure 2 is only one randomly selected from many sample tracks, while Figure 3 shows the multiple sample tracks of the TBM and CBM states. Therefore, it can be seen that there must be a deviation between the TBM state and a single sample track of the CBM state, and that the TBM state is the average of multiple CBM states.

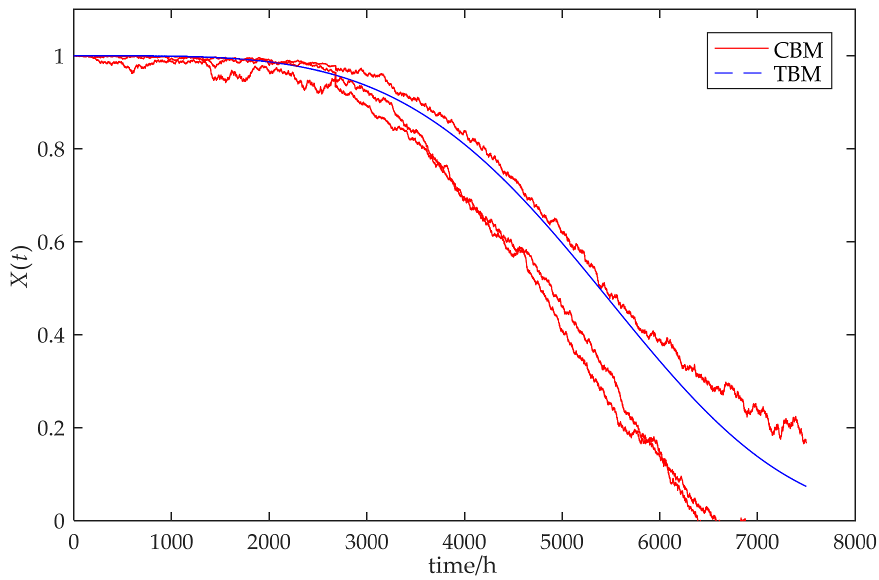

Figure 3. Multiple sample tracks of CBM state.

4.2. Maintenance Strategy Analysis of the Example

The generator deterioration shown in Figure 2 is in a single maintenance cycle. Based on the conditions at the beginning of Section 2 and the data in Section 4.1, we can obtain the full-life-cycle state of the generator under multiple maintenance cycles.

The CBM state of the component generator 1 in the full-life cycle is shown in Figure 4, in which the maintenance threshold is X_{thr} = 0.9 and the time interval for each component state to reach the maintenance threshold is Δt_1 = 2528 h, Δt_2 = 3161 h, Δt_3 = 2880 h, and Δt_4 = 2901 h. It again shows that the maintenance behavior under CBM can be based on and guided by the real-time state of the generator, such that the maintenance can more accurately follow the state, and the real deterioration state of the generator can be perceived. At the same time, the implementation of CBM has great randomness and unpredictability, which inevitably makes maintenance material resources and personnel arrangements and transfer untimely and insufficient.

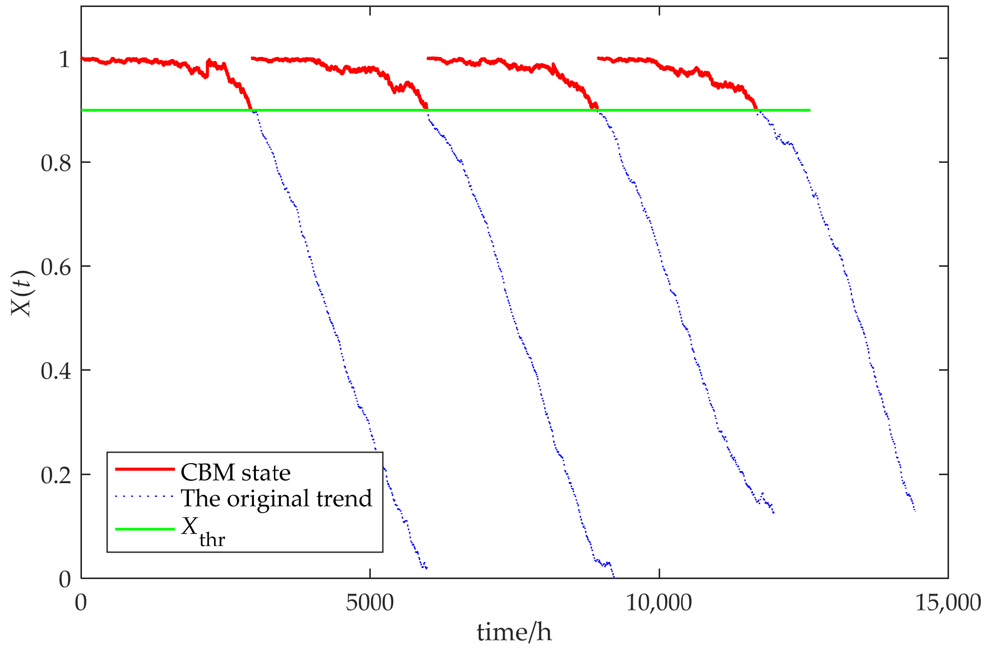

Figure 4. The CBM state for full-life cycle.

The TBM state of generator 1 in the full-life cycle is shown in Figure 5, where $T_{TBM\cdot i}$ (i = 1, 2, 3, 4) is the time point of the TBM implementation, the implementation interval ΔT_{TBM} = 3679 h, and $X(T_{TBM\cdot 1})$ = 0.97, $X(T_{TBM\cdot 2})$ = 0.84, $X(T_{TBM\cdot 3})$ = 0.91, and $X(T_{TBM\cdot 4})$ = 0.94. It is indicated in Figure 5 that the maintenance time of TBM is fixed, which enables the operation and maintenance department of the wind farm to arrange various hardware resources and human resources to each maintenance time point in a reasonable way according to the plan, thus ensuring the sufficiency and completeness of maintenance and reducing the costs to a certain extent. However, from the generator state $X(T_{TBM\cdot i})$ corresponding to each maintenance time $T_{TBM\cdot i}$, it is not difficult to see that there exists, indeed, an "over-maintenance" in TBM, i.e., the state at the maintenance time point is too high in comparison to the maintenance threshold, which results in a waste of maintenance resources. Alternatively, there exists an "under-maintenance" in TBM, i.e., the state at the maintenance time point is too low in comparison to the maintenance threshold, which results in exacerbating the generator deterioration and reducing the life.

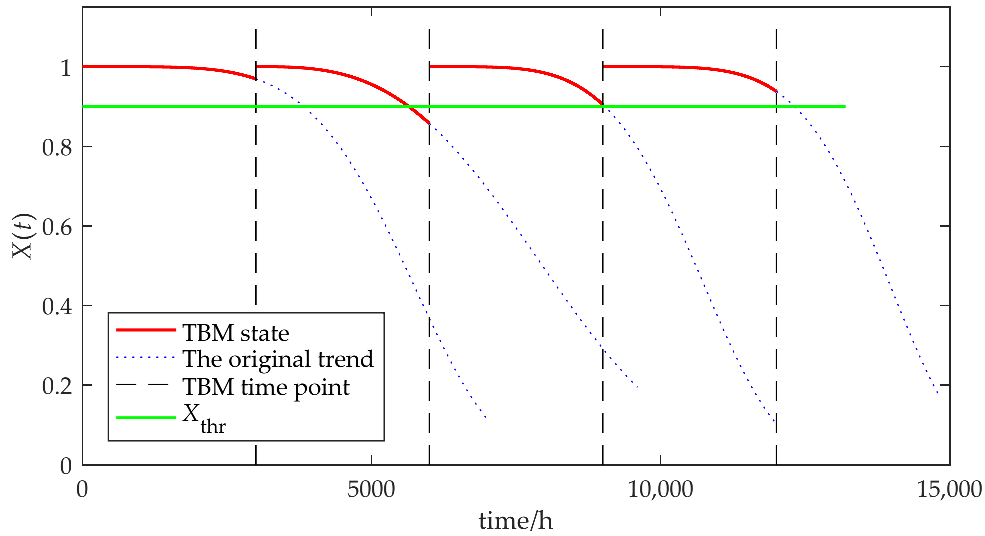

Figure 5. The TBM state for full-life cycle.

Moreover, under the joint strategy, the generator state is shown in Figure 6, where the absolute difference $\Delta T_i = |T_{\text{TBM}\cdot i} - \tau_i|$, between the stopping time τ_i when the state arrives at the maintenance threshold and the TBM time point, is ΔT_1 = 182 h, ΔT_2 = 218 h, ΔT_3 = 832 h, ΔT_4 = 489 h, ΔT_5 = 166 h, and i = 1, 2, 3, 4, 5. The analysis shows that, in the i-th (i = 1, 2, 5) cycle, TBM is implemented when the absolute difference ΔT_i between the CBM stopping time t_i, i.e., the moment when the generator state reaches the maintenance threshold, and the adjacent TBM time $T_{\text{TBM}\cdot I}$, is very small. Furthermore, $\sup\{|X(T_{\text{TBM}\cdot i}) - X_{\text{thr}}|\}$ = 0.025 means that the degree of "over-maintenance" or "under-maintenance" at this time is very small. In the i-th (i = 3, 4) maintenance cycle, CBM is implemented, since $\Delta T_i = |T_{\text{TBM}\cdot i} - \tau_i|$ is large and the deterioration will be exacerbated if the component is still waiting for TBM implementation.

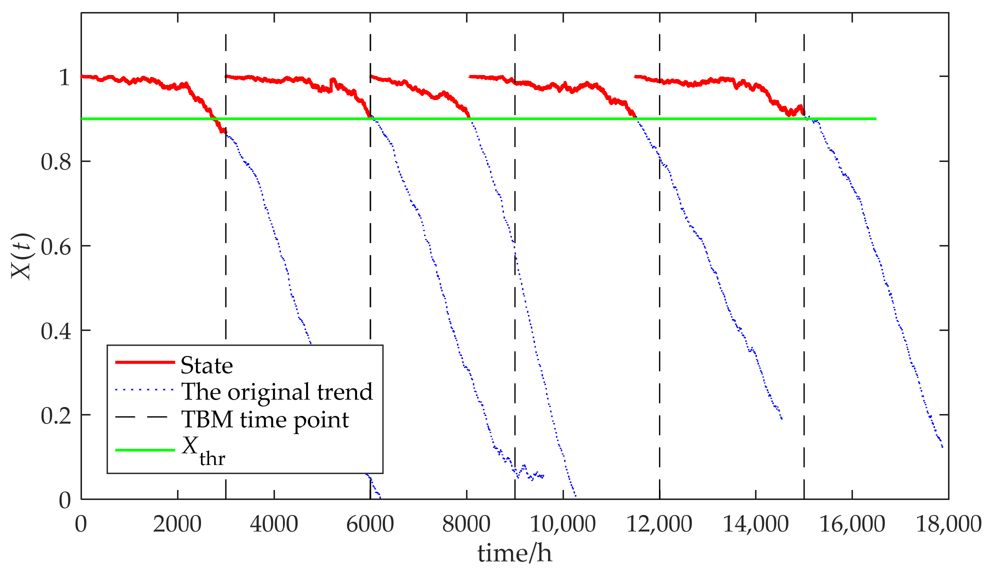

Figure 6. The state of the joint strategy model.

In Figure 7, a comparative analysis of the time information and state information of the generator maintenance behavior under the guidance of three maintenance strategies is shown. It can be seen that the implementation time of TBM is accurate, but the corresponding state is uncertain, CBM can track component state, but maintenance time is random, and the maintenance time of the joint strategy revolves around the time point of the TBM plan, supplemented by the CBM when the state is below the maintenance threshold. This fully demonstrates the characteristics of the joint strategy: to overcome the respective shortcomings of the TBM and CBM strategies and bring their advantages into play. The joint strategy takes into account both the characteristics of the TBM and CBM strategies, such that the maintenance is implemented according to the TBM strategy, when the degree of inappropriate maintenance such as "under-maintenance" and "over-maintenance" is acceptable, and also according to CBM strategy, when the state reaches the state threshold outside of the TBM plans and arrangements.

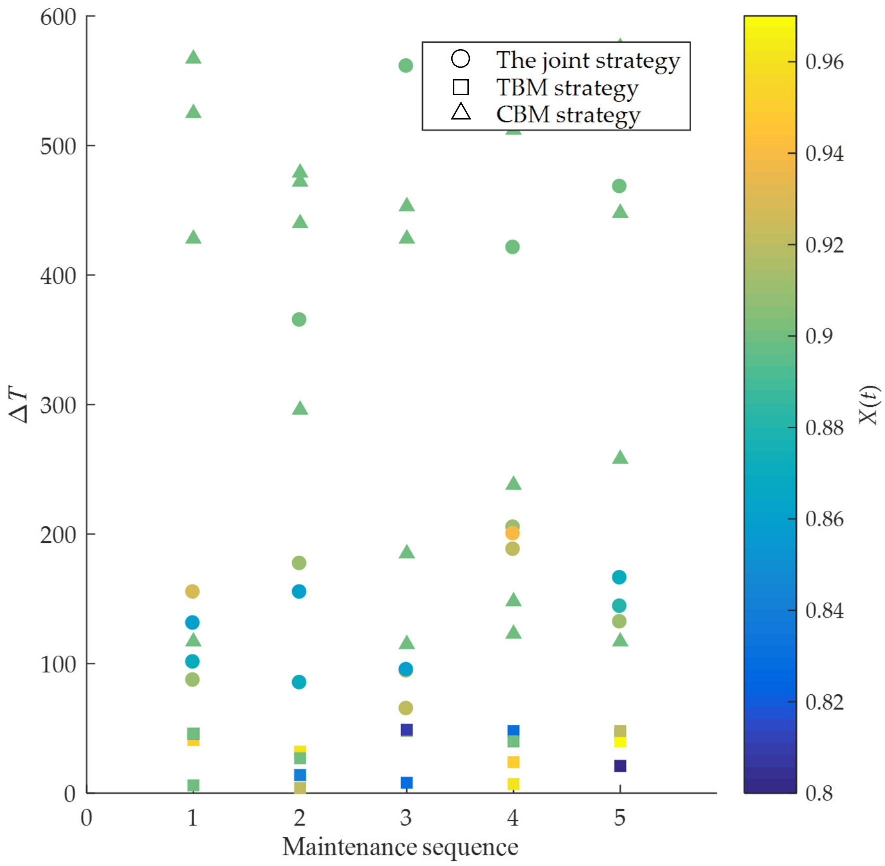

Figure 7. The comparison of three strategies.

The maintenance cost is the main part of the operating expenses (OPEX) of wind turbines, and the maintenance strategy to guide the maintenance behavior shall affect the cost of the component's maintenance. Figure 8 shows the analysis of the generators' maintenance costs and reliability under the implementation of three strategies. Overall, the CBM strategy can maintain high reliability with high costs, while the TBM strategy has a low cost with low reliability, and the joint strategy is between the two. This is because CBM can track the component state to effectively improve reliability, while TBM can prearrange maintenance resources to reduce maintenance costs. For wind farms, maintenance strategies need to be selected according to the specific requirements of relevant indicators such as costs and reliability.

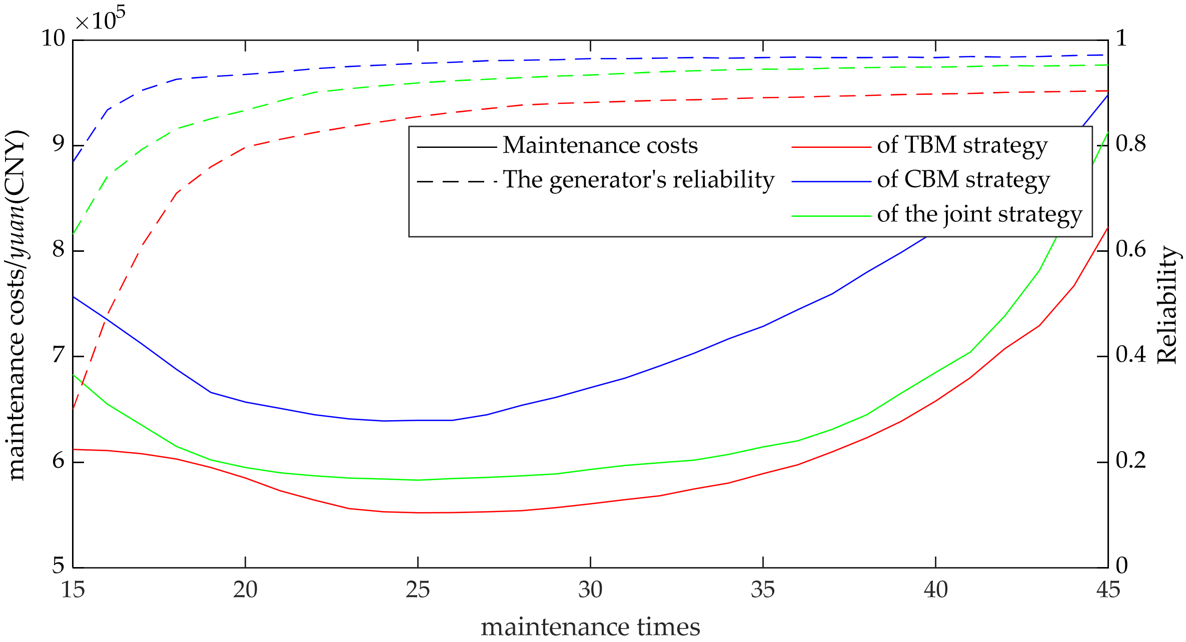

Figure 8. The analysis of generators' maintenance costs and reliability for three strategies.

5. Conclusions

In this paper, a degradation model based on the SDE is established to describe the spatio-temporal evolution behavior of wind turbine generators. In the stochastic model, the failure rate function of the component is represented by Weibull proportional hazards model, the random disturbance is simulated by Brownian motion, and, furthermore, the SDE model is solved by the method of constructing a function transformation. According to the stochastic model established, the strategy of TBM and CBM is compared and analyzed, then the maintenance strategy of combined TBM and CBM is proposed, and the characteristics of the joint strategy are expounded and analyzed by martingale method and change in measure theory. Finally, the model and analysis results are verified by an example of a wind turbine component generator. In the example, the proposed strategies are verified and quantitatively evaluated through an analysis of maintenance time, state evolution, maintenance costs, and the component's reliability. We find that the joint strategy of TBM and CBM is necessary in cases where individual TBM or CBM cannot meet the maintenance requirements of the components. This is mainly based on TBM implementation, which is easy to arrange and plan, but at very random maintenance time points, such as at the time point away from TBM or the time when the state is too far below the maintenance threshold, CBM can well satisfy the maintenance requirements. For the further development of the model, the generalized SDE could be considered to establish a spatio-temporal evolution model that can remember the historical information of components, and, based on this, the corresponding maintenance strategies could be proposed. This could provide some thoughts for the engineering practice of the component maintenance of wind turbines.

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