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Article

Lump-Type Solutions, Lump Solutions, and Mixed Rogue Waves for Coupled Nonlinear Generalized Zakharov Equations

Aly R. Seadawy [1,*], Syed T. R. Rizvi [2] and Hanadi Zahed [1]

1 Mathematics Department, Faculty of Science, Taibah University, Al-Madinah Al-Munawarah 41411, Saudi Arabia
2 Department of Mathematics, COMSATS University Islamabad, Lahore Campus, Islamabad 45550, Pakistan
* Correspondence: aabdelalim@taibahu.edu.sa

Abstract: This article studies diverse forms of lump-type solutions for coupled nonlinear generalized Zakharov equations (CNL-GZEs) in plasma physics through an appropriate transformation approach and bilinear equations. By utilizing the positive quadratic assumption in the bilinear equation, the lump-type solutions are derived. Similarly, by employing a single exponential transformation in the bilinear equation, the lump one-soliton solutions are derived. Furthermore, by choosing the double exponential ansatz in the bilinear equation, the lump two-soliton solutions are found. Interaction behaviors are observed and we also establish a few new solutions in various dimensions (3D and contour). Furthermore, we compute rogue-wave solutions and lump periodic solutions by employing proper hyperbolic and trigonometric functions.

Keywords: CNL-GZE; lump-type solitons; rogue wave; appropriate transformation technique

MSC: 35J05; 35J10; 35K05; 35L05

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1. Introduction

The study of partial differential equations (PDEs) occurs in various fields such as theoretical physics, applied mathematics, biological sciences, and engineering sciences. These PDEs play a crucial role in explaining key scientific phenomena. For instance, the Korteweg–de Vries equation governs shallow water wave dynamics near ocean shores and beaches, and the nonlinear Schrödinger's equation governs the propagation of solitons through optical fibers. Some examples of PDEs and their applications can be found in [1–8].

Although the above-mentioned PDEs are scalar, a large number of PDEs are coupled. Some of them are two-coupled PDEs such as the Gear–Grimshaw equation, whereas others are three-coupled PDEs. An example of a three-coupled PDE is the Wu–Zhang equation. These coupled PDEs are also calculated in distinct areas of theoretical physics. In this paper, we will study CNL-GZE used in plasmas.

Lump waves (LWs), as superior nonlinear wave phenomena, have been visualized in various fields. LWs are theoretically viewed as a limited type of soliton and move with higher propagating energy compared to general solitons. Consequently, LWs can be destructive and even catastrophic in certain systems, such as in the ocean and finance. It is important to be able to find and anticipate LWs in practical applications. In recent years, studies on lump solutions have increased, leading to more specialized investigations. Therefore, theoretical investigations of LWs are instrumental in enhancing our understanding and predicting possible extremes in nonlinear systems [9–13].

Finding the lump solutions of PDEs has become a primary focus in recent years. As a result, several mathematical experts have developed important schemes in order to solve PDEs [14–16].

In this article, we consider the CNL-GZE for the complex envelope $u(x,t)$ of the high-frequency wave and the real low-frequency field $v(x,t)$, as follows [17]:

$$\begin{cases} ih_1\psi_t + \psi_{xx} - 2h_2|\phi|^2\psi + 2\psi\phi = 0, \\ \phi_{tt} - \psi_{xx} - (|\phi|^2)_{xx} = 0. \end{cases} \quad (1)$$

where h_1 and h_1 are real constants. The cubic term in Equation (1) represents the nonlinear self-interaction in the high-frequency subsystem, which corresponds to a self-focusing effect in plasma physics.

Several researchers have worked on the stated model. For instance, Wang et al. evaluated periodic wave solutions for GZEs using the extended F-expansion method [17]. Zheng et al. performed a numerical simulation of a GZ system [18]. Bao et al. developed numerical schemes for a GZ system [19]. Bhrawy et al. constructed an efficient Jacobi pseudospectral approximation for a nonlinear complex GZ system [20]. Zhang et al. studied solitary wave solutions through a variational approach [21]. Similarly, Yildirim et al. studied some newly discovered soliton solutions of GZEs by applying He's variational approach [22]. Li et al. computed additional exact solutions of GZEs through the Exp-function method [23]. Buhe et al. studied symmetry reductions, conservation laws, and exact solutions for GZEs [24]. Lin et al. constructed some additional exact solutions for GZEs through the Exp-function method [23]. Wu et al. studied exact solutions for GZEs using a variational approach [25]. However, in this paper, we will explore lump, lump-type, lump one-strip, and lump two-strip solutions for CNL-GZEs through appropriate transformation methods and bilinear equations. We compute the lump solutions by choosing the appropriate polynomial function. In addition, we compute lump-periodic and rogue-wave solutions by using logarithmic transformation.

This article is organized as follows. In Section 2, we form bilinear equations and evaluate lump solutions for the coupled nonlinear generalized Zakharov equations in plasma physics through appropriate transformation approaches. The solutions are presented along with with their corresponding graphs. The mixed solutions of soliton and lump waves are provided in Section 3. We evaluate the lump one-strip and lump two-strip solutions using suitable profiles in Section 3. By employing a trigonometric ansatz in the bilinear equation, we compute lump periodic solutions in Section 4. By utilizing a hyperbolic ansatz in the bilinear equation, we explore rogue-wave solutions in Section 5. Section 6 discusses the results of the obtained solutions, and finally, in Section 7, we present some concluding remarks.

2. Lump Solution

For the lump solutions of Equation (1), we apply the following ansatz: [26–30],

$$\psi(x,t) = \frac{h_3 e^{(ict)} p(x,t)}{q(x,t)}, \quad \phi(x,t) = 2[\ln q(x,t)]_x - c, \quad (2)$$

then, we obtain the bilinear equations,

$$\begin{aligned} 2h_2h_3^2p^3 + 2ch_3pqt^2 + ch_1h_3pq^2 - ih_1h_3q^2 + p_t + ih_1h_3pqq_t - 4h_3pqq_x \\ +2h_3qp_xq_x - 2h_3pq_x^2 - h_3q^2p_{xx} + h_3pqq_{xx} = 0, \end{aligned} \quad (3)$$

and

$$\begin{aligned} h_3^2q^2p_x^2q_t^2q_x - q^2q_{tt}q_x - 4h_3^2pqp_xq_x + 3h_3^3p^2q_x^2 - 2qq_x^3 - 2q^2q_tq_{xt} + q^3q_{xtt} \\ +h_3^2pq^2p_{xx} - h_3^2p^2qq_{xx} + 3q^2q_xq_{xx} - q^3q_{xxx} = 0, \end{aligned} \quad (4)$$

respectively.

Now, to obtain the LP solution, the functions p and q in Equations (3) and (4) are assumed to be [27,28],

$$p = \xi_1^2 + \xi_2^2 + a_2\,, \quad q = \xi_1^2 + \xi_2^2 + a_3, \tag{5}$$

where $\xi_1 = a_0 x + t$, $\xi_2 = a_1 x + t$.

In addition, $a_i (1 \leq i \leq 3)$ are specific constants. Now, by substituting Equation (5) into Equations (3) and (4) and solving the equations obtained from the coefficients of x and t, we obtain:

Set I. The values of unknowns for Equations (3) and (4), respectively, are as follows:

$$\begin{cases} a_0 = \frac{-1+i\sqrt{3}}{2},\ h_1 = -\frac{2\left(h_2 h_3^2 + c\right)}{c},\ a_2 = a_2,\ a_3 = a_3,\ a_0 = a_0. \\ and \\ a_0 = \frac{1-i}{2},\ a_1 = \frac{1+i}{2},\ h_3 = 0,\ a_2 = a_2,\ a_3 = a_3. \end{cases} \tag{6}$$

Then, the values in Equation (6) generate the required solutions for Equations (3) and (4), which are, respectively,

$$\begin{cases} \psi_{1,1} = -\frac{2e^{ict}\left(c+h_2h_3^2\right)\left(a_2+\left(t+\frac{(-1+i\sqrt{3})}{2}x\right)^2+(t+a_1x)^2\right)}{c\left(a_3+\left(t+\frac{(-1+i\sqrt{3})x}{2}\right)^2+(t+a_1x)^2\right)}, \\ and \\ \phi_{1,1} = \frac{2\left(-1+i\sqrt{3}\right)\left(\left(t+\frac{(-1+i\sqrt{3})^2}{2}x\right)+2a_1(t+a_1x)\right)}{a_3+\left(t+\frac{(-1+i\sqrt{3})x}{2}\right)^2+(t+a_1x)^2} - c. \end{cases} \tag{7}$$

and

$$\begin{cases} \psi_{1,2} = \frac{e^{ict}h_1\left(a_2+\left(t+\left(\frac{1-i}{2}\right)x\right)^2+\left(t+\left(\frac{1+i}{2}\right)x\right)^2\right)}{a_3+\left(t+\left(\frac{1-i}{2}\right)x\right)^2+\left(t+\left(\frac{1+i}{2}\right)x\right)^2}, \\ and \\ \phi_{1,2} = \frac{2\left((1-i)\left(t+\left(\frac{1-i}{2}\right)x\right)+(1+i)\left(t+\left(\frac{1+i}{2}\right)x\right)\right)}{a_3+\left(t+\left(\frac{1-i}{2}\right)x\right)^2+\left(t+\left(\frac{1+i}{2}\right)x\right)^2} - c. \end{cases} \tag{8}$$

Set II. The values of the parameters in Equations (3) and (4) are, respectively,

$$\begin{cases} a_0 = \frac{-3+3i}{4}, a_1 = \frac{3+3i}{4}, h_1 = -2, a_2 = 0, a_3 = a_3. \\ and \\ a_0 = 1, a_1 = 1, h_3 = h_3, a_2 = a_2, a_3 = a_3. \end{cases} \tag{9}$$

Then, the values in Equation (9) generate the required solutions for Equations (3) and (4), which are, respectively,

$$\begin{cases} \psi_{2,1} = -\frac{2e^{ict}\left(\left(t-\left(\frac{3-3i}{4}\right)x\right)^2+\left(t+\left(\frac{3+3i}{4}\right)x\right)^2\right)}{a_3+\left(t-\left(\frac{3-3i}{4}\right)x\right)^2+\left(t+\left(\frac{3+3i}{4}\right)x\right)^2}, \\ and \\ \phi_{2,1} = \frac{2\left(\left(\frac{-3+3i}{2}\right)\left(t-\left(\frac{3-3i}{4}\right)x\right)+\left(\frac{3+3i}{2}\right)\left(t+\left(\frac{3+3i}{4}\right)x\right)\right)}{a_3+\left(t-\left(\frac{3-3i}{4}\right)x\right)^2+\left(t+\left(\frac{3+3i}{4}\right)x\right)^2} - c. \end{cases} \tag{10}$$

and

$$\begin{cases} \psi_{2,2} = \frac{e^{ict} h_1 \left(a_2 + 2(t+x)^2\right)}{a_3 + 2(t+x)^2}, \\ and \\ \phi_{2,2} = -c + \frac{8(t+x)}{a_3 + 2(t+x)^2}. \end{cases} \tag{11}$$

3. Mixed Solutions of Soliton and Lump Waves

In this section, we study the interaction of a lump soliton with a single kink wave and the interaction of a lump soliton with double kink waves.

3.1. Lump One-Strip Soliton Interaction Solution

To obtain the lump one-strip solution, we use the transformations given in Equations (3) and (4) [22,27–30]:

$$p = \xi_1^2 + \xi_2^2 + a_2 + b_0 e^{k_1 x + k_2 t}, \quad q = \xi_1^2 + \xi_2^2 + a_3 + b_0 e^{k_1 x + k_2 t}, \tag{12}$$

where $\xi_1 = a_0 x + t$, $\xi_2 = a_1 x + t$, and $a_i (1 \le i \le 3)$, k_1, k_2, and b_0 are any constants. Now, from Equations (12) and (4), we obtain the coefficients of x and t and solve the equations as follows:

Set I. The values of the parameters in Equations (3) and (4) are, respectively,

$$\begin{cases} c = \frac{-18h_2h_3^2 + 8 + 4\sqrt{-9ih_1k_2 - 72h_2h_3^2 + 4}}{9h_1 + 18}, \; k_1 = \frac{2 + \sqrt{-9ih_1k_2 - 72h_2h_3^2 + 4}}{3}, \; a_0 = ia_1, \; h_3 = h_3, \; a_2 = a_2, \\ and \\ a_0 = \frac{1 - 2i\sqrt{5}}{3}, a_1 = \frac{1 + 2i\sqrt{5}}{3}, a_2 = -\frac{69300}{19h_3^4}, a_3 = -\frac{62100}{19h_3^4}, k_1 = -\frac{19}{90}h_3^2, k_2 = -\frac{19}{90}h_3^2. \end{cases} \tag{13}$$

Then, the values in Equation (13) generate the required results for Equations (3) and (4), which are, respectively,

$$\begin{cases} \psi_{3,1} = \dfrac{e^{i\frac{\left(-18h_2h_3^2 + 8 + 4\sqrt{-9ih_1k_2 - 72h_2h_3^2 + 4}\right)t}{9h_1 + 18}} h_1 \left(a_2 + b_0 e^{k_2 t + \frac{\left(2 + \sqrt{-9ih_1k_2 - 72h_2h_3^2 + 4}\right)x}{3}} + (t + ia_1 x)^2 + (t + a_1 x)^2 \right)}{a_3 + b_0 e^{k_2 t + \frac{\left(2 + \sqrt{-9ih_1k_2 - 72h_2h_3^2 + 4}\right)x}{3}} + (t + ia_1 x)^2 + (t + a_1 x)^2}, \\ and \\ \phi_{3,1} = -\dfrac{-18h_2h_3^2 + 8 + 4\sqrt{-9ih_1k_2 - 72h_2h_3^2 + 4}}{9h_1 + 18} + \dfrac{2\left(\frac{1}{3} b_0 e^{k_2 t + \frac{\left(2 + \sqrt{-9ih_1k_2 - 72h_2h_3^2 + 4}\right)x}{3}} \Pi_1 \right)}{a_3 + b_0 e^{k_2 t + \frac{\left(2 + \sqrt{-9ih_1k_2 - 72h_2h_3^2 + 4}\right)x}{3}} + (t + ia_1 x)^2 + (t + a_1 x)^2}, \\ \Pi_1 = \left(2 + \sqrt{-9ih_1k_2 - 72h_2h_3^2 + 4}\right) + 2ia_1(t + ia_1 x) + 2a_1(t + a_1 x). \end{cases} \tag{14}$$

and

$$\begin{cases} \psi_{4,1} = \dfrac{e^{ict} h_1 \left(b_0 e^{\frac{19h_3^2 t}{90} - \frac{19h_3^2 x}{90}} - \frac{69300}{19h_3^4} + \left(t + \frac{(1-2i\sqrt{5})x}{3} \right)^2 + \left(t + \frac{(1+2i\sqrt{5})x}{3} \right)^2 \right)}{b_0 e^{\frac{19h_3^2 t}{90} - \frac{19h_3^2 x}{90}} - \frac{69300}{19h_3^4} + \left(t + \frac{(1-2i\sqrt{5})x}{3} \right)^2 + \left(t + \frac{(1+2i\sqrt{5})x}{3} \right)^2}, \\ and \\ \phi_{4,1} = \dfrac{2\left(-\frac{19}{90} b_0 e^{\frac{19h_3^2 t}{90} - \frac{19h_3^2 x}{90}} h_3^2 + \frac{2(1-2i\sqrt{5})}{3} \left(t + \frac{(1-2i\sqrt{5})x}{3} \right) + \Pi_2 \right)}{b_0 e^{\frac{19h_3^2 t}{90} - \frac{19h_3^2 x}{90}} - \frac{62100}{19h_3^4} + \left(t + \frac{(1-2i\sqrt{5})x}{3} \right)^2 + \left(t + \frac{(1+2i\sqrt{5})x}{3} \right)^2} - c, \\ \Pi_2 = \dfrac{2(1+2i\sqrt{5})}{3} \left(t + \dfrac{(1-2i\sqrt{5})x}{3} \right). \end{cases} \tag{15}$$

Set II. The values of the parameters in Equations (3) and (4) are, respectively,

$$\begin{cases} a_0 = ia_1,\ k_1 = \frac{6h_2 h_3^2 + 3ch_1 + 6c}{4},\ a_3 = a_3,\ h_3 = h_3,\ a_2 = a_2. \\ and \\ a_0 = \frac{-1-2i\sqrt{5}}{3}, a_1 = \frac{-1+2i\sqrt{5}}{3}, a_2 = -\frac{69300}{19h_3^4}, a_3 = -\frac{62100}{19h_3^4}, k_1 = -\frac{19}{90} h_3^2, k_2 = -\frac{19}{90} h_3^2. \end{cases} \tag{16}$$

Then, the values in Equation (16) generate the required results for Equations (3) and (4), which are, respectively,

$$\begin{cases} \psi_{5,1} = \dfrac{e^{ict} h_1 \left(a_2 + b_0 e^{k_2 t + \frac{(6h_2 h_3^2 + 3ch_1 + 6c)x}{4}} + (t + ia_1 x)^2 + (t + a_1 x)^2 \right)}{a_3 + b_0 e^{k_2 t + \frac{(6h_2 h_3^2 + 3ch_1 + 6c)x}{4}} + (t + ia_1 x)^2 + (t + a_1 x)^2}, \\ and \\ \phi_{5,1} = \dfrac{2\left(\frac{1}{4} b_0 e^{k_2 t + \frac{(6h_2 h_3^2 + 3ch_1 + 6c)x}{4}} + (6h_2 h_3^2 + 3ch_1 + 6c) + 2ia_1 (t + ia_1 x)^2 + 2a_1 (t + a_1 x)^2 \right)}{a_3 + b_0 e^{k_2 t + \frac{(6h_2 h_3^2 + 3ch_1 + 6c)x}{4}} + (t + ia_1 x)^2 + (t + a_1 x)^2} - c. \end{cases} \tag{17}$$

and

$$\begin{cases} \psi_{5,2} = \dfrac{e^{ict} h_1 \left(b_0 e^{\frac{19h_3^2 t}{90} - \frac{19h_3^2 x}{90}} - \frac{69300}{19h_3^4} + \left(t + \frac{(-1-2i\sqrt{5})x}{3} \right)^2 + \left(t + \frac{(-1+2i\sqrt{5})x}{3} \right)^2 \right)}{b_0 e^{\frac{19h_3^2 t}{90} - \frac{19h_3^2 x}{90}} - \frac{69300}{19h_3^4} + \left(t + \frac{(-1-2i\sqrt{5})x}{3} \right)^2 + \left(t + \frac{(-1+2i\sqrt{5})x}{3} \right)^2}, \\ and \\ \phi_{5,2} = \dfrac{2\left(-\frac{19}{90} b_0 e^{\frac{19h_3^2 t}{90} - \frac{19h_3^2 x}{90}} h_3^2 + \frac{2(-1-2i\sqrt{5})}{3} \left(t + \frac{(-1-2i\sqrt{5})x}{3} \right) + \Pi_3 \right)}{b_0 e^{\frac{19h_3^2 t}{90} - \frac{19h_3^2 x}{90}} - \frac{62100}{19h_3^4} + \left(t + \frac{(-1-2i\sqrt{5})x}{3} \right)^2 + \left(t + \frac{(-1+2i\sqrt{5})x}{3} \right)^2} - c, \\ \Pi_3 = \dfrac{2(-1+2i\sqrt{5})}{3} \left(t + \dfrac{(-1-2i\sqrt{5})x}{3} \right). \end{cases} \tag{18}$$

3.2. Lump Double-Strip Soliton Interaction Solution

To obtain the lump two-strip solution, we assume the following transformation [22,27–30]:

$$p = \bigwedge_1^2 + \bigwedge_2^2 + a_3 + m_1 e^{k_1 x + k_2 t + k_3} + m_2 e^{k_4 x + k_5 t + k_6}, \quad q = \bigwedge_1^2 + \bigwedge_2^2 + a_4 + m_1 e^{k_1 x + k_2 t + k_3} + m_2 e^{k_4 x + k_5 t + k_6}, \tag{19}$$

where $\bigwedge_1 = a_1x + a_2t$, $\bigwedge_2 = a_1x + a_2t$, and $a_i(1 \leq i \leq 4)$, $k_i(1 \leq i \leq 6)$, m_1, and m_2 are specific real parameters. Now, from Equation (19) and Equation (4), we obtain the coefficients of x, t, and exp and solve these equations as follows:

Set I. When $k_5 = k_4 = a_1 = 0$ for Equation (3) and $k_3 = k_6 = a_1 = 0$ for Equation (4), the values of the parameters are, respectively,

$$\begin{cases} a_4 = -\frac{i\left(9ih_2h_3^2k_1^2-3a_2a_3h_2h_3^2+ia_2^2a_3\right)}{\left(3ih_2h_3^2+a_2\right)a_2},\ k_2 = \frac{2k_1\left(3ih_2h_3^2-2a_2\right)}{3h_2h_3^2},\ m_1 = -\frac{a_2m_2\left(3ih_2h_3^2m_2+2a_2\right)}{-a_2+3ih_2h_3^2}. \\ and \\ a_2 = \frac{\sqrt{6}h_3^2}{60},\ k_1 = -\frac{\sqrt{6}k_4^2-\frac{2}{3}\sqrt{6}k_5^2+3k_4k_5}{5\sqrt{\frac{2}{5}\sqrt{6}k_4k_5-\frac{6}{5}k_4^2-\frac{1}{5}k_5^2}},\ k_2 = \sqrt{\frac{2}{5}\sqrt{6}k_4k_5-\frac{6}{5}k_4^2-\frac{1}{5}k_5^2},\ a_4 = 0. \end{cases} \tag{20}$$

Then, the values in Equation (20) generate the required results for Equations (3) and (4), which are, respectively,

$$\begin{cases} \psi_{5,1} = \frac{e^{ict}h_1\left(a_3+k_6^2+e^{a_2t}m_2-\frac{a_2m_2e^{a_2t}\left(3ih_2h_3^2m_2+2a_2\right)}{-a_2+3ih_2h_3^2}+\left(k_3+\frac{2k_1\left(3ih_2h_3^2-2a_2\right)t}{3h_2h_3^2}+k_1x\right)^2\right)}{2e^{a_2t}-\frac{i\left(9ih_2h_3^2k_1^2-3a_2a_3h_2h_3^2+ia_2^2a_3\right)}{\left(3ih_2h_3^2+a_2\right)a_2}+k_6^2+\left(k_3+\frac{2k_1\left(3ih_2h_3^2-2a_2\right)t}{3h_2h_3^2}+k_1x\right)^2}, \\ \phi_{5,1} = \frac{4k_1\left(k_3+\frac{2k_1\left(3ih_2h_3^2-2a_2\right)t}{3h_2h_3^2}+k_1x\right)}{2e^{a_2t}-\frac{i\left(9ih_2h_3^2k_1^2-3a_2a_3h_2h_3^2+ia_2^2a_3\right)}{\left(3ih_2h_3^2+a_2\right)a_2}+k_6^2+\left(k_3+\frac{2k_1\left(3ih_2h_3^2-2a_2\right)t}{3h_2h_3^2}+k_1x\right)^2} - c. \end{cases} \tag{21}$$

and

$$\begin{cases} \psi_{6,1} = \frac{e^{ict}h_1\left(\Delta_1+\left(\sqrt{\frac{2}{5}\sqrt{6}k_4k_5-\frac{6}{5}k_4^2-\frac{1}{5}k_5^2}\,t-\frac{\left(\sqrt{6}k_4^2-\frac{2}{3}\sqrt{6}k_5^2+3k_4k_5\right)x}{5\sqrt{\frac{2}{5}\sqrt{6}k_4k_5-\frac{6}{5}k_4^2-\frac{1}{5}k_5^2}}\right)^2\right)}{2e^{\frac{h_3^2t}{10\sqrt{6}}}+(k_5t+k_4x)^2+\left(\sqrt{\frac{2}{5}\sqrt{6}k_4k_5-\frac{6}{5}k_4^2-\frac{1}{5}k_5^2}\,t-\frac{\left(\sqrt{6}k_4^2-\frac{2}{3}\sqrt{6}k_5^2+3k_4k_5\right)x}{5\sqrt{\frac{2}{5}\sqrt{6}k_4k_5-\frac{6}{5}k_4^2-\frac{1}{5}k_5^2}}\right)^2}, \\ \Delta_1 = a_3 + e^{\frac{h_3^2t}{10\sqrt{6}}}m_1 + e^{\frac{h_3^2t}{10\sqrt{6}}}m_2 + (k_5t + k_4x)^2. \\ and \\ \phi_{6,1} = \frac{2\left(\Delta_2-\frac{\left(\sqrt{6}k_4^2-\frac{2}{3}\sqrt{6}k_5^2+3k_4k_5\right)\left(\sqrt{\frac{2}{5}\sqrt{6}k_4k_5-\frac{6}{5}k_4^2-\frac{1}{5}k_5^2}\,t-\frac{\left(\sqrt{6}k_4^2-\frac{2}{3}\sqrt{6}k_5^2+3k_4k_5\right)x}{5\sqrt{\frac{2}{5}\sqrt{6}k_4k_5-\frac{6}{5}k_4^2-\frac{1}{5}k_5^2}}\right)^2}{5\sqrt{\frac{2}{5}\sqrt{6}k_4k_5-\frac{6}{5}k_4^2-\frac{1}{5}k_5^2}}\right)}{2e^{\frac{h_3^2t}{10\sqrt{6}}}+(k_5t+k_4x)^2+\left(\sqrt{\frac{2}{5}\sqrt{6}k_4k_5-\frac{6}{5}k_4^2-\frac{1}{5}k_5^2}\,t-\frac{\left(\sqrt{6}k_4^2-\frac{2}{3}\sqrt{6}k_5^2+3k_4k_5\right)x}{5\sqrt{\frac{2}{5}\sqrt{6}k_4k_5-\frac{6}{5}k_4^2-\frac{1}{5}k_5^2}}\right)^2} - c, \\ \Delta_2 = 2k_4(k_5t + k_4x). \end{cases} \tag{22}$$

Set II. When $k_5 = k_4 = a_1 = 0$ for Equation (3) and $k_3 = k_6 = a_1 = 0$ for Equation (4), the values of the parameters are, respectively,

$$\begin{cases} a_2 = \frac{4ih_2h_3^2c(a_3-a_4)}{-10a_3h_2h_3^2+10a_4h_2h_3^2+2a_3c-2a_4c+9k_1^2},\ h_1 = -\frac{-10a_3h_2h_3^2+10a_4h_2h_3^2+2a_3c-2a_4c+9k_1^2}{c(a_3-a_4)}, \\ k_2 = -\frac{\frac{4}{3}ick_1(a_3-a_4)}{-10a_3h_2h_3^2+10a_4h_2h_3^2+2a_3c-2a_4c+9k_1^2},\ m_1 = -m_2 - 4. \\ and \\ m_1 = -\frac{5a_3m_2-4a_4m_2-8a_3+6a_4}{5a_3-4a_4}, k_1 = ik_4, k_2 = ik_5, a_2 = 0. \end{cases} \tag{23}$$

Then, the values in Equation (23) generate the required results for Equations (3) and (4), which are, respectively,

$$\begin{cases} \psi_{7,1} = \dfrac{-\left(e^{ict}D_1\left(a_3+k_6^2+(-m_2-4)e^{\frac{4ih_2h_3^2c(a_3-a_4)t}{-10a_3h_2h_3^2+10a_4h_2h_3^2+2a_3c-2a_4c+9k_1^2}}+m_2e^{\frac{4ih_2h_3^2c(a_3-a_4)t}{-10a_3h_2h_3^2+10a_4h_2h_3^2+2a_3c-2a_4c+9k_1^2}}D_2\right)\right)}{(a_3-a_4)c\left(a_4+k_6^2+2e^{\frac{4ih_2h_3^2c(a_3-a_4)t}{-10a_3h_2h_3^2+10a_4h_2h_3^2+2a_3c-2a_4c+9k_1^2}}D_2\right)}, \\ D_1 = -10a_3h_2h_3^2+10a_4h_2h_3^2+2a_3c-2a_4c+9k_1^2, \\ D_2 = \left(k_3 - \dfrac{4ik_1c(a_3-a_4)t}{3(-10a_3h_2h_3^2+10a_4h_2h_3^2+2a_3c-2a_4c+9k_1^2)} + k_1x\right)^2, \\ and \\ \phi_{7,1} = -c + \dfrac{4k_1\left(k_3-\frac{4ik_1c(a_3-a_4)t}{3(-10a_3h_2h_3^2+10a_4h_2h_3^2+2a_3c-2a_4c+9k_1^2)}+k_1x\right)}{\left(a_4+k_6^2+2e^{\frac{4ih_2h_3^2c(a_3-a_4)t}{-10a_3h_2h_3^2+10a_4h_2h_3^2+2a_3c-2a_4c+9k_1^2}}D_2\right)}. \end{cases} \tag{24}$$

and

$$\begin{cases} \psi_{8,1} = \dfrac{e^{ict}h_1\left(a_3+m_2-\frac{5a_3m_2-4a_4m_2-8a_3+6a_4}{5a_3-4a_4}+(ik_5t+ik_4x)^2+(k_5t+k_4x)^2\right)}{2+a_4+(ik_5t+ik_4x)^2+(k_5t+k_4x)^2}, \\ and \\ \phi_{8,1} = \dfrac{2\left(2ik_4(ik_5t+ik_4x)^2+2k_4(k_5t+k_4x)^2\right)}{2+a_4+(ik_5t+ik_4x)^2+(k_5t+k_4x)^2} - c. \end{cases} \tag{25}$$

4. Lump Periodic Soliton Solution

To compute the LPS solution, we use the following supposition in Equations (3) and (4) [22,27–30]:

$$p = \bigwedge_1^2 + \bigwedge_2^2 + a_2 + a_3\cos(n_1x+t), \quad q = \bigwedge_1^2 + \bigwedge_2^2 + a_4 + a_5\cos(n_1x+t) \tag{26}$$

where $\bigwedge_1 = B_0x + t$, $\bigwedge_2 = B_1x + t$. In addition, $a_i (1 \le i \le 5)$ and n_1 are various parameters to be determined. Now, by substituting Equation (26) into Equations (3) and (4) and then examining the coefficients of x, cos function, and t, we obtain the following:

Set I. The values of the parameters for Equations (3) and (4) are, respectively,

$$\begin{cases} n_1 = -\dfrac{\frac{1}{4}ih_1(a_4-a_5)}{a_4+a_5}, \; a_0 = -a_1, \; c = c, \; a_4 = a_4. \\ and \\ n_1 = -\dfrac{4(a_0^2+a_1^2)}{(a_1+a_0)(3a_0^2+3a_1^2-2)}, \; a_0 = a_0, \; c = c, \; a_4 = a_4, \; a_3 = a_3. \end{cases} \tag{27}$$

Then, the values in Equation (27) generate the required results for Equations (3) and (4), which are, respectively,

$$\begin{cases} \psi_{9,1} = \dfrac{e^{ict}h_1\left(a_2+(t+a_0x)^2+(t+a_1x)^2+a_4\cos\left(t-\frac{i(a_4-a_5)h_1x}{4(a_4+a_5)}\right)\right)}{\left(a_3+(t+a_0x)^2+(t+a_1x)^2+a_5\cos\left(t-\frac{i(a_4-a_5)h_1x}{4(a_4+a_5)}\right)\right)}, \\ and \\ \phi_{9,1} = -c + \dfrac{2\left(2a_0(t+a_0x)+2a_1(t+a_1x)+\frac{i(a_4-a_5)h_1a_5 sin\left(t-\frac{i(a_4-a_5)h_1x}{4(a_4+a_5)}\right)}{4(a_4+a_5)}\right)}{\left(a_3+(t+a_0x)^2+(t+a_1x)^2+a_5\cos\left(t-\frac{i(a_4-a_5)h_1x}{4(a_4+a_5)}\right)\right)}. \end{cases} \tag{28}$$

and

$$\begin{cases} \psi_{10,1} = \dfrac{e^{ict} h_1 \left(a_2 + (t+a_0 x)^2 + (t+a_1 x)^2 + a_4 \cos\left(t + \frac{4\left(a_0^2+a_1^2\right)x}{(a_1+a_0)\left(3a_0^2+3a_1^2-2\right)} \right) \right)}{\left(a_3 + (t+a_0 x)^2 + (t+a_1 x)^2 + a_5 \cos\left(t + \frac{4\left(a_0^2+a_1^2\right)x}{(a_1+a_0)\left(3a_0^2+3a_1^2-2\right)} \right) \right)}, \\ and \\ \phi_{10,1} = -c + \dfrac{2\left(2a_0(t+a_0 x) + 2a_1(t+a_1 x) - \frac{4\left(a_0^2+a_1^2\right) a_5 sin\left(t + \frac{4\left(a_0^2+a_1^2\right)x}{(a_1+a_0)\left(3a_0^2+3a_1^2-2\right)} \right)}{(a_1+a_0)\left(3a_0^2+3a_1^2-2\right)} \right)}{\left(a_3 + (t+a_0 x)^2 + (t+a_1 x)^2 + a_5 \cos\left(t + \frac{4\left(a_0^2+a_1^2\right)x}{(a_1+a_0)\left(3a_0^2+3a_1^2-2\right)} \right) \right)}. \end{cases} \tag{29}$$

5. Rogue-Wave Solutions

To compute the LPS solution, we use the following supposition in Equations (3) and (4) [22,27–30]:

$$p = \bigwedge_1^2 + \bigwedge_2^2 + a_2 + a_3 \cosh(n_1 x + t), \quad q = \bigwedge_1^2 + \bigwedge_2^2 + a_4 + a_5 \cosh(n_1 x + t) \tag{30}$$

where $\bigwedge_1 = a_0 x + t$, $\bigwedge_2 = a_1 x + t$. In addition, $a_i (1 \leq i \leq 5)$ and n_1 are various parameters to be determined. Now, by substituting Equation (26) into Equations (3) and (4) and then examining the coefficients of x, cos function, and t, we obtain the following:

Set I. The values of the parameters for Equations (3) and (4), are, respectively,

$$\begin{cases} a_4 = -\frac{a_5(4in_1+h_1)}{(4in_1-h_1)},\ a_0 = -a_1,\ a_2 = a_2,\ a_4 = a_4,\ a_3 = a_3. \\ and \\ a_1 = ia_0,\ a_4 = 0,\ n_1 = 1,\ a_3 = a_3,\ a_5 = a_5. \end{cases} \tag{31}$$

Then, the values in Equation (31) generate the solutions for Equations (3) and (4), which are, respectively,

$$\begin{cases} \psi_{11,1} = \dfrac{e^{ict} h_1 \left(a_2 + (t-a_1 x)^2 + (t+a_1 x)^2 - \frac{a_5(4in_1+h_1)\cosh(t+n_1 x)}{(4in_1-h_1)} \right)}{\left(a_3 + (t-a_1 x)^2 + (t+a_1 x)^2 + a_5 \cosh(t+n_2 x) \right)}, \\ and \\ \phi_{11,1} = \dfrac{2(-2a_1(t-a_1 x) + 2a_1(t+a_1 x) + a_5 n_2 sinh(t+n_2 x))}{\left(a_3 + (t-a_1 x)^2 + (t+a_1 x)^2 + a_5 \cosh(t+n_2 x) \right)} - c. \end{cases} \tag{32}$$

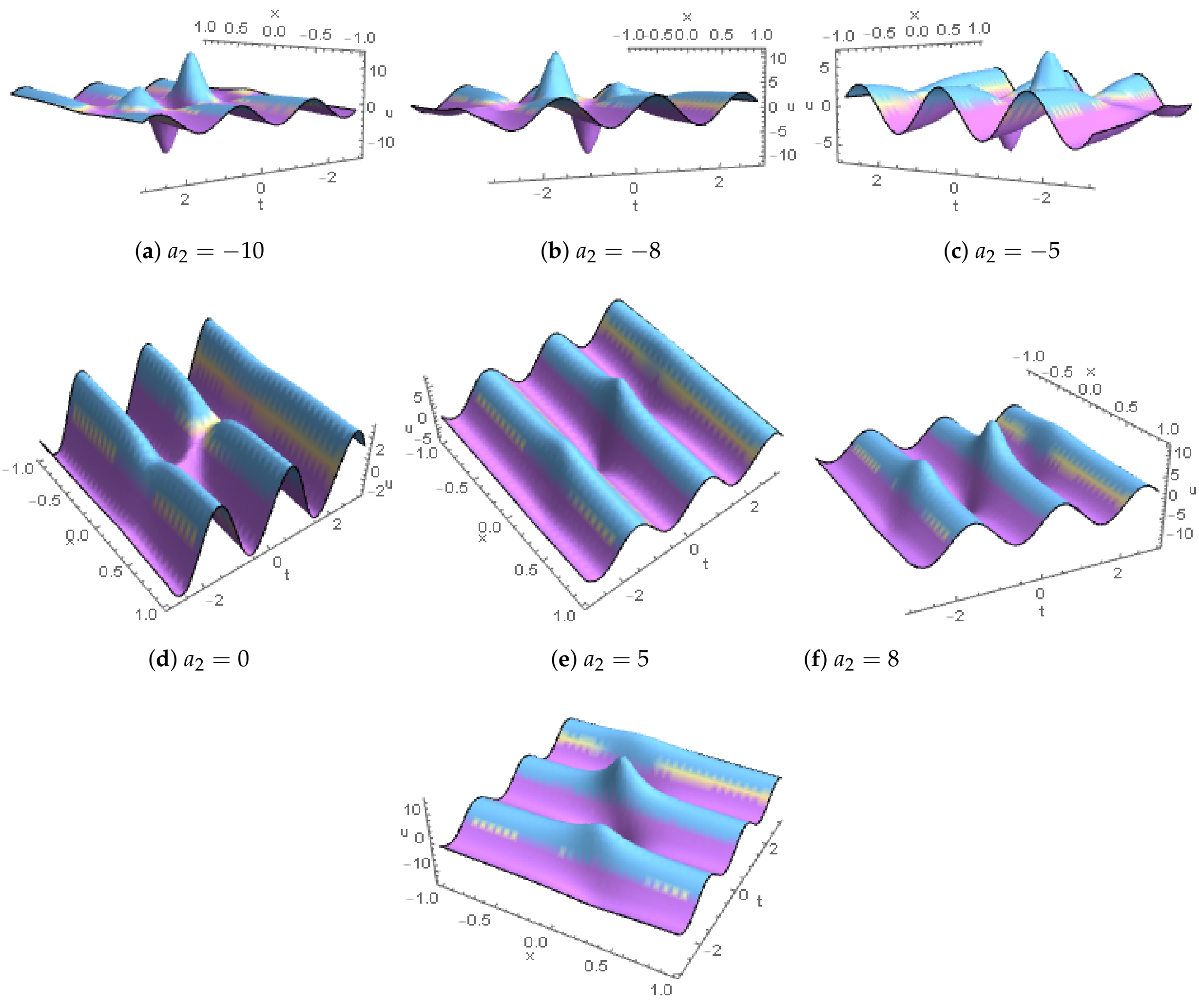
and

$$\begin{cases} \psi_{12,1} = \dfrac{e^{ict} h_1 \left(a_2 + (t+ia_0 x)^2 + (t+a_1 x)^2 \right)}{\left(a_3 + (t+ia_0 x)^2 + (t+a_1 x)^2 + a_5 \cosh(t+n_2 x) \right)}, \\ and \\ \phi_{12,1} = -c + \dfrac{2(2ia_0(t+ia_0 x) + 2a_0(t+a_0 x) + a_5 n_2 \sinh(t+n_2 x))}{\left(a_3 + (t+ia_0 x)^2 + (t+a_1 x)^2 + a_5 \cosh(t+n_2 x) \right)}. \end{cases} \tag{33}$$

6. Results and Discussion

We observed that the solution $\psi_{1,1}(x,t)$ in Equation (7) with $a_1 = 10, h_2 = -2, h_3 = 2$, $a_3 = 2$, and $c = 3$ formed two lump waves (LWs) known as upper-bright and lower-dark LWs, and that the bright and dark LWs were symmetrical about the coordinate plane. As a_2 varied from a minimum to a maximum number, the two LWs rotated counterclockwise. When $a_2 = 0$, the LW disappeared, but at $a_2 = 5$, the LW gradually reappeared (see Figure 1). The contour lump-wave profiles for $\psi_{1,1}(x,t)$ are plotted for $a_1 = 10, h_2 = -2$, $h_3 = 2, a_3 = 2$, and $c = 3$ in Figure 2. The mixed solutions of soliton and lump waves were successfully obtained. Notice that our solution $\phi_{3,1}(x,t)$ in Equation (14) with

$h_1 = 10, b_0 = 10$, and $c = 5$ formed lump one-strip waves (LSWs) known as upper-bright LSWs. The lump one-strip wave profiles for $\phi_{2,1}(x,t)$ are depicted for $h_1 = 10, b_0 = 10$, and $c = 5$ in Figures 3 and 4. The lump double-strip wave profiles for $\phi_{5,1}(x,t)$ are plotted for $k_3 = 4, h_2 = 2, h_1 = 4, h_3 = 3, a_2 = 20, a_3 = 5, k_6 = 2, m_2 = 2$, and $c = 5$ in Figures 3, 5 and 6. By utilizing the assumption of the cosine function in bilinear equations in Equations (3) and (4), we have obtained the lump periodic solutions. We have successfully obtained the lump periodic graphs for $\phi_{9,1}(x,t)$, which are plotted for $a_0 = 10, a_1 = 5, a_2 = 4, a_3 = 2, a_4 = 3, a_5 = 5$, and $h_1 = 20$ in Figure 7. The lump periodic contour graphs for $\phi_{9,1}(x,t)$ are plotted for $a_0 = 10, a_1 = 5, a_2 = 4, a_3 = 2, a_4 = 3, a_5 = 5$, and $h_1 = 20$ in Figure 8. By utilizing the assumption of cosine hyperbolic functions in bilinear equations in Equations (3) and (4), we have obtained the lump periodic solutions. As a_1 varied from -10 to 10, the rogue wave rotated, and its behavior can be seen for $\psi_{11,1}(x,t)$ for $h_1 = 4, a_2 = 3, a_3 = 1.5, a_5 = 5, n_1 = 3, n_2 = 4$, and $c = 5$ in Figure 9.

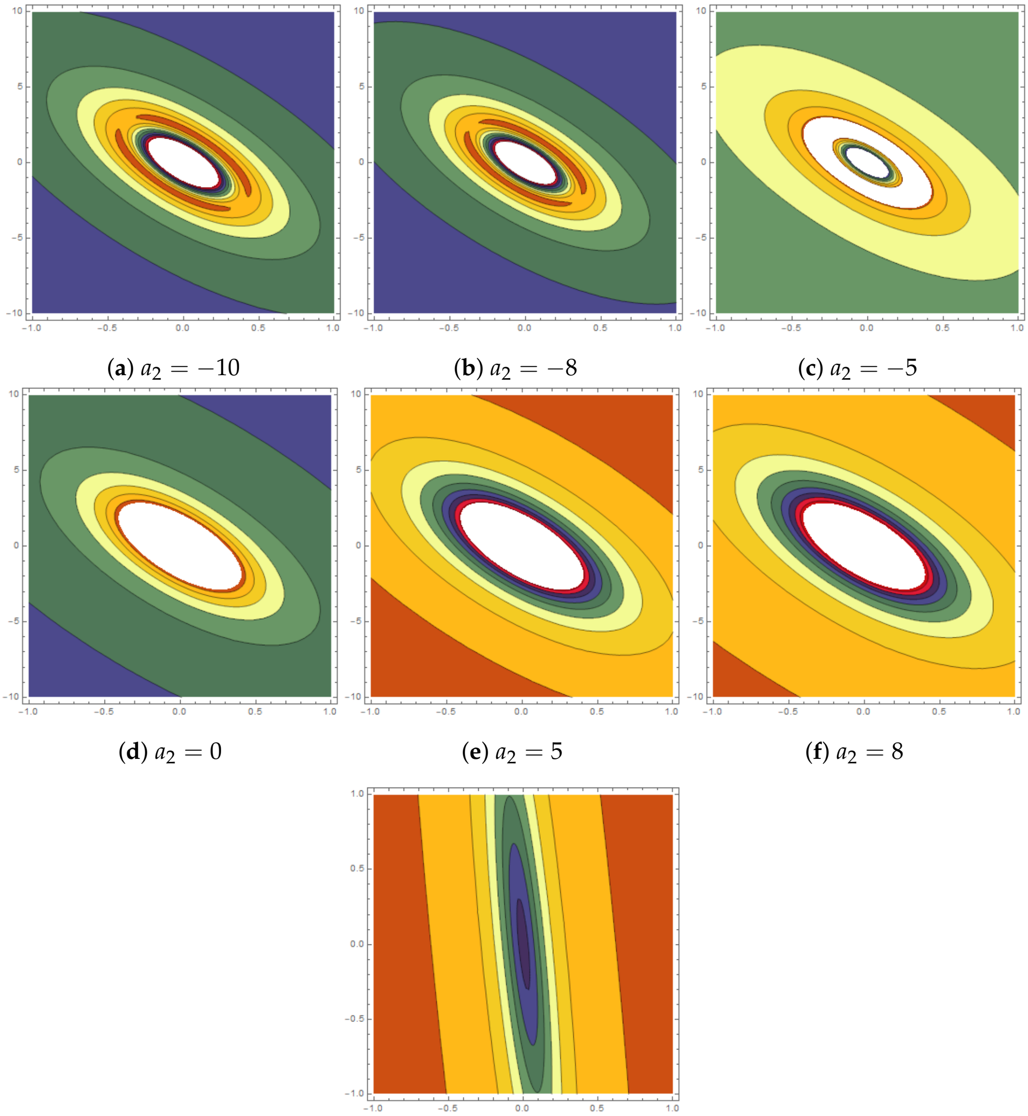


(**a**) $a_2 = -10$ (**b**) $a_2 = -8$ (**c**) $a_2 = -5$

(**d**) $a_2 = 0$ (**e**) $a_2 = 5$ (**f**) $a_2 = 8$

(**g**) $a_2 = 10$

Figure 1. Lump-wave profiles for $\psi_{1,1}(x,t)$ are plotted for $a_1 = 10, h_2 = -2, h_3 = 2, a_3 = 2, c = 3$.

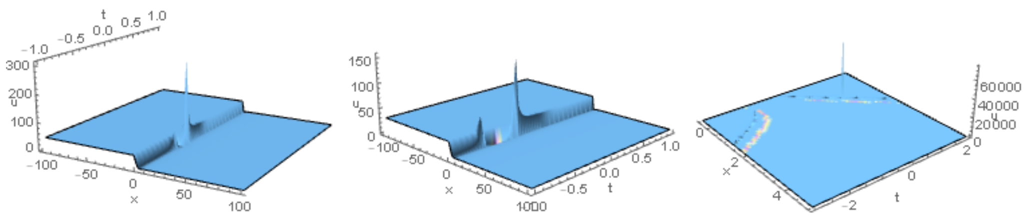


(**a**) $a_2 = -10$ (**b**) $a_2 = -8$ (**c**) $a_2 = -5$

(**d**) $a_2 = 0$ (**e**) $a_2 = 5$ (**f**) $a_2 = 8$

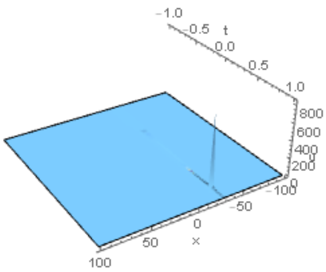
(**g**) $t = 10$

Figure 2. Contour lump-wave profiles for $\psi_{1,1}(x,t)$ are plotted for $a_1 = 10, h_2 = -2, h_3 = 2, a_3 = 2, c = 3$.

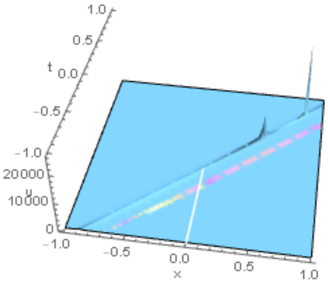


(**a**) $h_3 = -10$ (**b**) $h_3 = -8$ (**c**) $h_3 = -5$

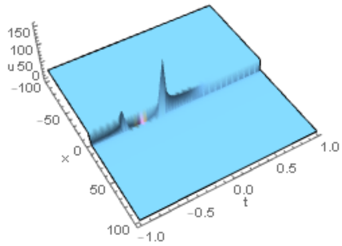
Figure 3. *Cont.*



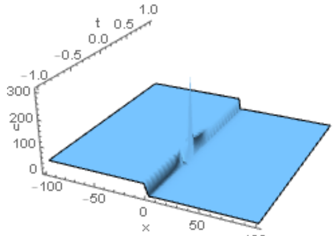
(**d**) $h_3 = -1$



(**e**) $h_3 = 5$

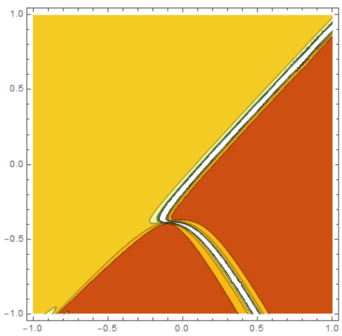


(**f**) $h_3 = 8$

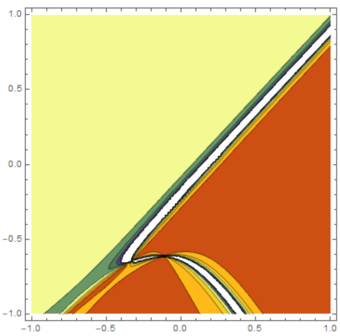


(**g**) $h_3 = 10$

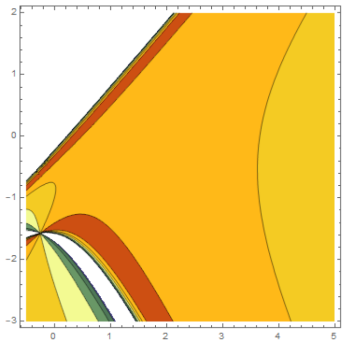
Figure 3. Lump one-strip wave profiles for $\phi_{3,1}(x,t)$ are plotted for $h_1 = 10, b_0 = 10, c = 5$.



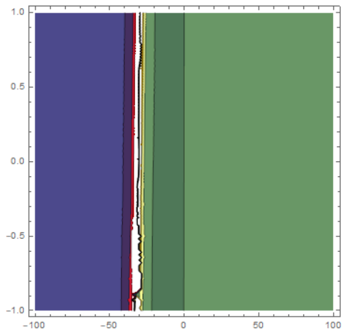
(**a**) $h_3 = -10$



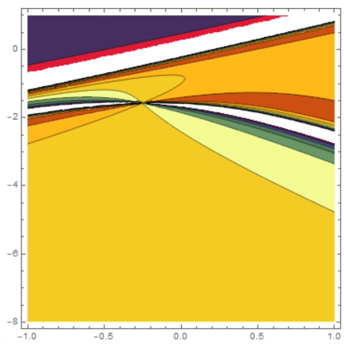
(**b**) $h_3 = -8$



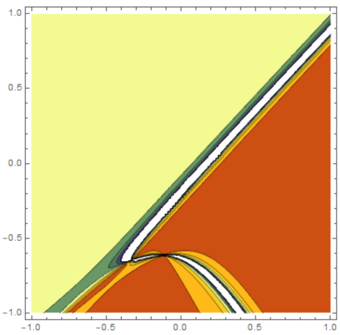
(**c**) $h_3 = -5$



(**d**) $h_3 = -1$

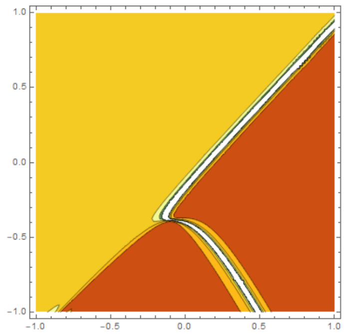


(**e**) $h_3 = 5$



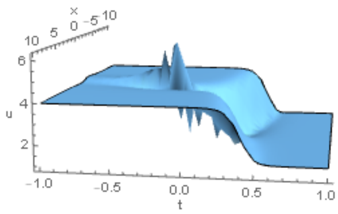
(**f**) $h_3 = 8$

Figure 4. *Cont.*

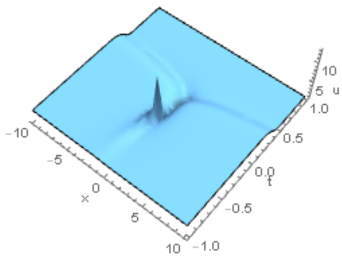


(**g**) $h_3 = 10$

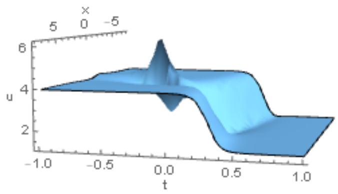
Figure 4. Contour lump one-strip wave profiles for $\phi_{3,1}(x,t)$ are plotted for $h_1 = 10, b_0 = 10, c = 5$.



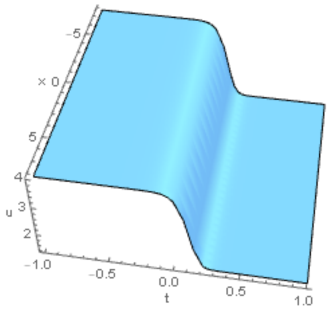
(**a**) $k_1 = -10$



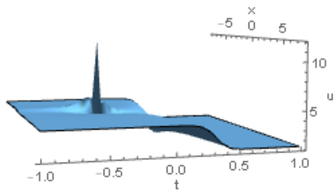
(**b**) $k_1 = -8$



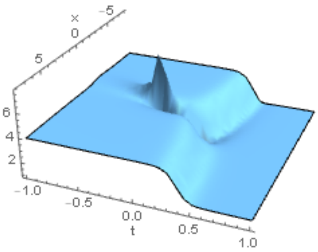
(**c**) $k_1 = -5$



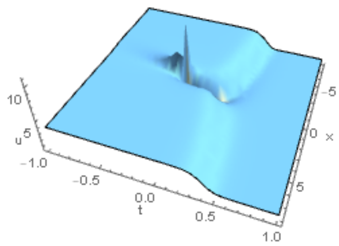
(**d**) $k_1 = -1$



(**e**) $k_1 = 5$

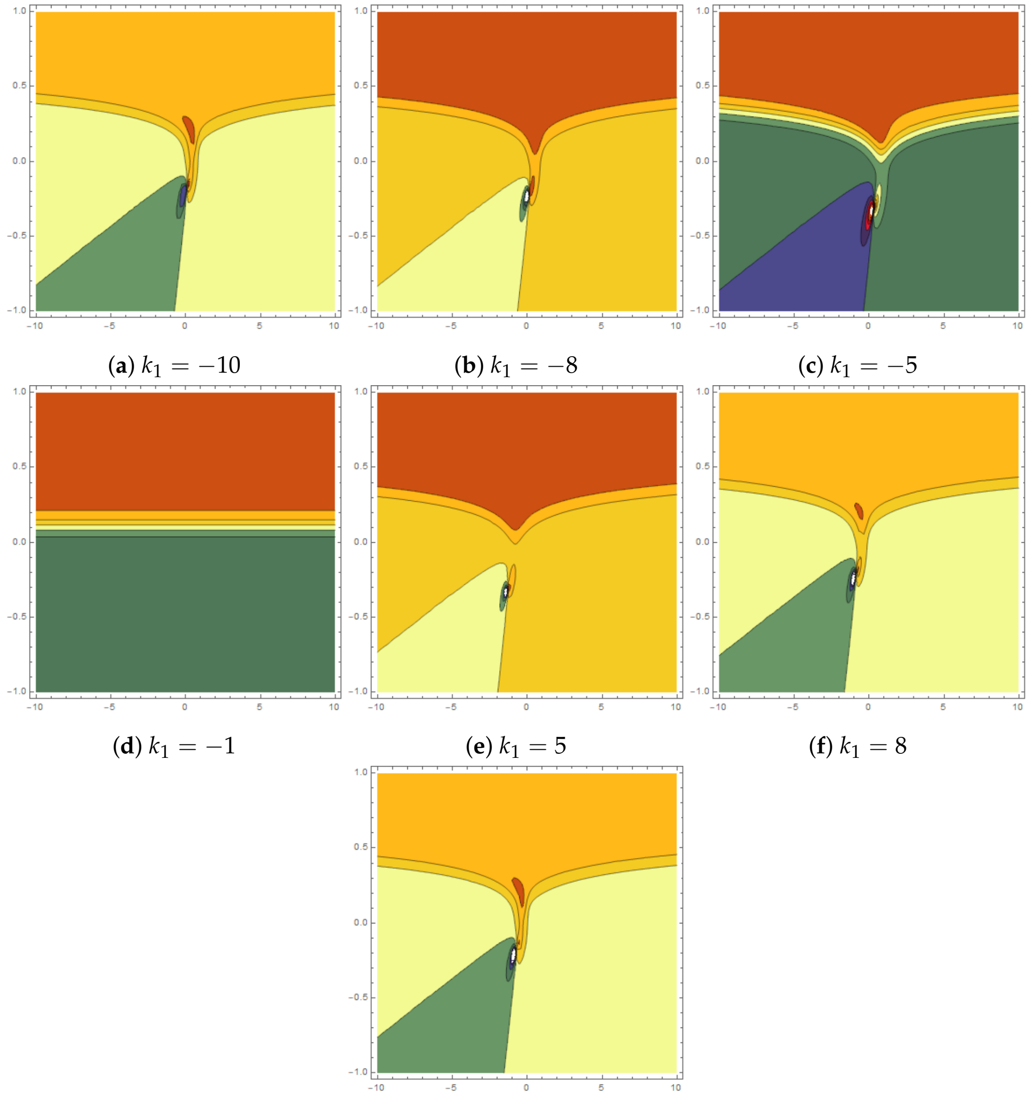


(**f**) $k_1 = 8$



(**g**) $k_1 = 10$

Figure 5. Lump double-strip wave profiles for $\phi_{5,1}(x,t)$ are plotted for $k_3 = 4, h_2 = 2, h_1 = 4, h_3 = 3, a_2 = 20, a_3 = 5, k_6 = 2, m_2 = 2, c = 5$.

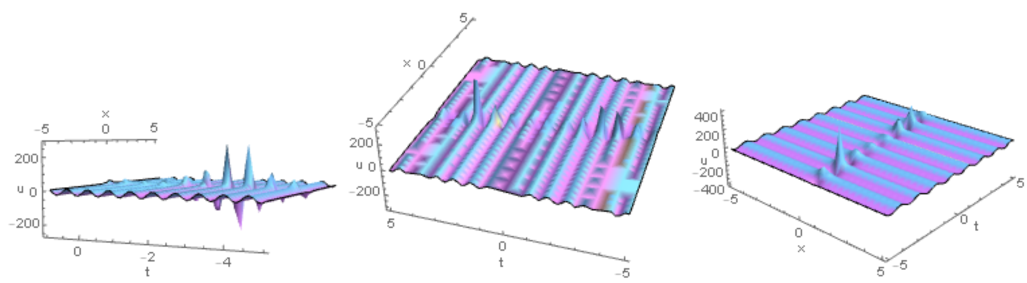


(**a**) $k_1 = -10$ (**b**) $k_1 = -8$ (**c**) $k_1 = -5$

(**d**) $k_1 = -1$ (**e**) $k_1 = 5$ (**f**) $k_1 = 8$

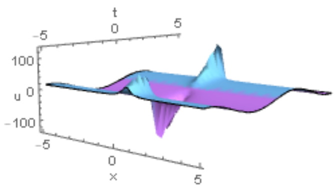
(**g**) $k_1 = 10$

Figure 6. Contour profiles for Figure 5.

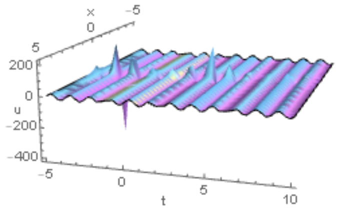


(**a**) $c = -10$ (**b**) $c = -8$ (**c**) $c = -5$

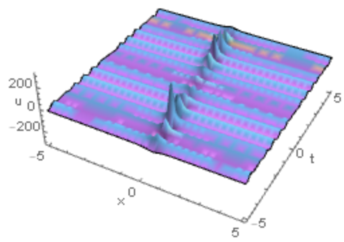
Figure 7. *Cont.*



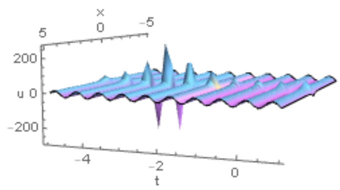
(**d**) $c = -1$



(**e**) $c = 5$

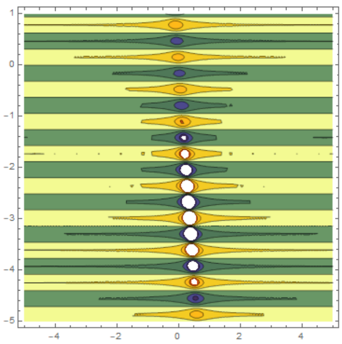


(**f**) $c = 8$

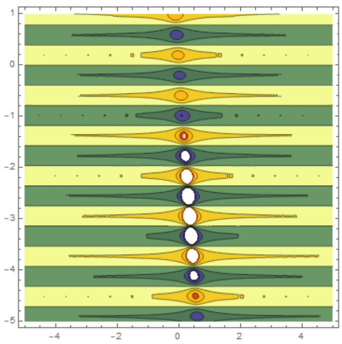


(**g**) $c = 10$

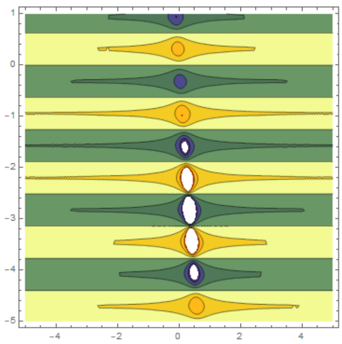
Figure 7. Lump periodic graphs for $\phi_{9,1}(x,t)$ are plotted for $a_0 = 10, a_1 = 5, a_2 = 4, a_3 = 2, a_4 = 3, a_5 = 5, h_1 = 20$.



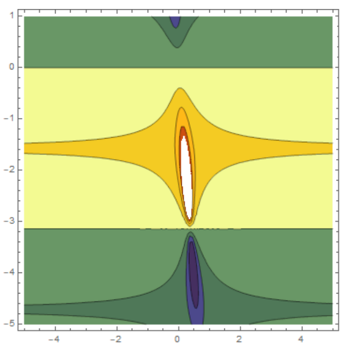
(**a**) $c = -10$



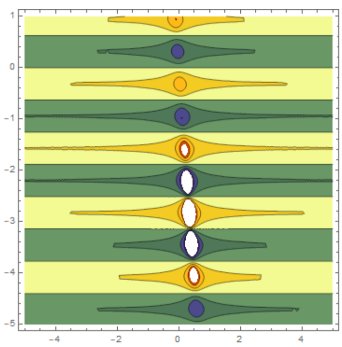
(**b**) $c = -8$



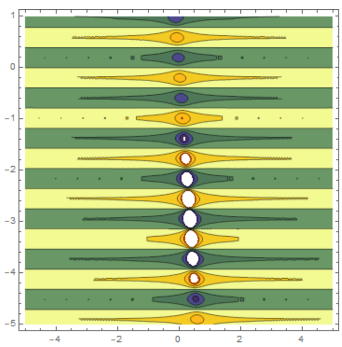
(**c**) $c = -5$



(**d**) $c = -1$

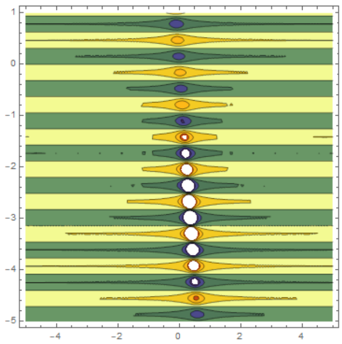


(**e**) $c = 5$



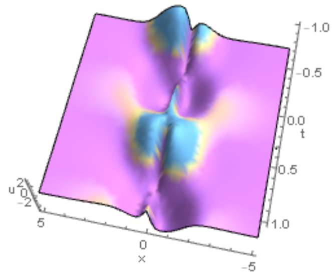
(**f**) $c = 8$

Figure 8. *Cont.*

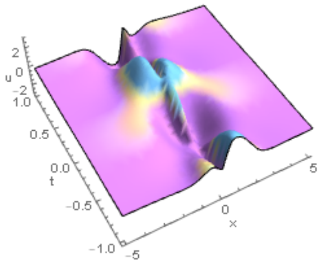


(**g**) $c = 10$

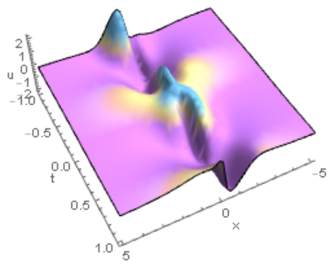
Figure 8. Lump periodic contour graphs for $\phi_{9,1}(x,t)$ are plotted for $a_0 = 10, a_1 = 5, a_2 = 4, a_3 = 2, a_4 = 3, a_5 = 5, h_1 = 20$.



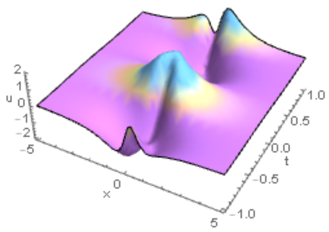
(**a**) $a_1 = -10$



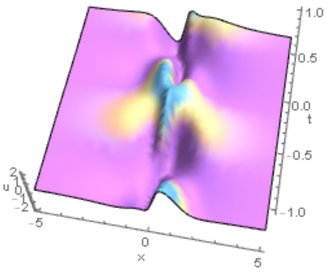
(**b**) $a_1 = -8$



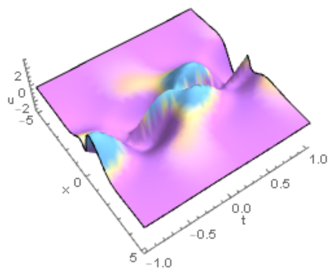
(**c**) $a_1 = -5$



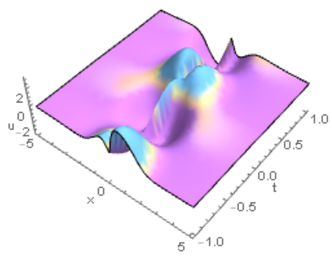
(**d**) $a_1 = -1$



(**e**) $a_1 = 5$



(**f**) $a_1 = 8$



(**g**) $a_1 = 10$

Figure 9. Rogue-wave profiles for $\psi_{11,1}(x,t)$ are plotted for $h_1 = 4, a_2 = 3, a_3 = 1.5, a_5 = 5, n_1 = 3, n_2 = 4, c = 5$.

7. Concluding Remarks

In this paper, we have studied multiple forms of lump solutions for CNL-GZEs in plasma physics using appropriate transformation approaches, bilinear equations, and symbolic computations. By utilizing the positive quadratic assumption in the bilinear equation, we have derived the lump-type solutions. We have evaluated the lump one-soliton solutions through a single exponential function transformation in the bilinear equation. Similarly, we have computed the lump two-soliton solutions using a double exponential function transformation in the bilinear equation. Mixed solutions of lump waves and solitons have been successfully evaluated. Furthermore, we have computed rogue-wave solutions and lump periodic solutions by utilizing appropriate hyperbolic and trigonometric functions. We have identified certain constraint values throughout the derivation of the solutions that must hold for the soliton solution to exist. The presented solutions have valuable uses in plasma physics.

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