

Article

ESN-Observer-Based Adaptive Stabilization Control for Delayed Nonlinear Systems with Unknown Control Gain

Shuxian Lun *, Zhaoyi Lv, Xiaodong Lu * and Ming Li

School of Control Science and Engineering, Bohai University, Jinzhou 121013, China; ll981211@163.com (Z.L.); lm_dq@bhu.edu.cn (M.L.)

* Correspondence: jzlunzi@163.com (S.L.); lxd2211@sina.com (X.L.)

Abstract: This paper investigates the observer-based adaptive stabilization control problem for a class of time-delay nonlinear systems with unknown control gain using an echo state network (ESN). In order to handle unknown functions, a new recurrent neural network (RNN) approximation method called ESN is utilized. It improves accuracy, reduces computing cost, and is simple to train. To address the issue of unknown control gain, the Nussbaum function is used, and the Lyapunov–Krasovskii functionals are used to address the delay term. The backstepping strategy and command filtering methodology are then used to create an adaptive stabilization controller. All of the closed-loop system’s signals are predicted to be confined by the Lyapunov stability theory. Finally, a simulation example is used to demonstrate the effectiveness of the suggested control mechanism.

Keywords: echo state network (ESN); stabilization control; unknown control gain; time-delay nonlinear systems

MSC: 93D05



Citation: Lun, S.; Lv, Z.; Lu, X.; Li, M. ESN-Observer-Based Adaptive Stabilization Control for Delayed Nonlinear Systems with Unknown Control Gain. *Mathematics* **2023**, *11*, 2965. <https://doi.org/10.3390/math11132965>

Academic Editors: Nicholas Christakis and António Lopes

Received: 24 April 2023

Revised: 13 June 2023

Accepted: 24 June 2023

Published: 3 July 2023



Copyright: © 2023 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>).

1. Introduction

In the past decades, the time-delay problem which may seriously affect system performance has become a common feature of various control systems. To solve this issue, a number of schemes have been proposed. As in [1], by using a nominal feedback technique that satisfied the nonlinear growth limit and transformed the delayed system into fully delay-free equivalent systems, predictor-based controllers were developed by Ponomarev for nonlinear systems with scattered input delays. Additionally, the small constant input delay in [2] was addressed using the Pade approximation method. The adding-a-power integrator technique was used in [3] to solve the stability problem of stochastic time-delayed systems, but the problem of determining whether the control of systems with time-delayed was in the deterministic or stochastic case remained unsolved because of the difficulty of developing a suitable Lyapunov–Krasovskii functional. Then, in [4], Lyapunov–Krasovskii functions and backstepping design were used to handle an approximate-based fuzzy logic control problem of nonlinear time-delay systems. However, all of the works listed above are fundamentally predicated on the idea that the state variables in the systems are measurable, and do not take time-delay terms into account.

In practical systems, unmeasurable or partially measurable states are very common situations. For instance, the controlled systems in [5] must be predicated on the availability of state variables, which restricted the applicability of these control strategies in practical projects. So far, a state observer-based adaptive neural network (NN) control technique is an effective method for addressing the state unpredictability issue. The work in [6] presented the problem of the observer-based adaptive neural controller for a class of nonlinear strict-feedback systems. In [7], Hua et al. studied the robust output tracking control problem for a class of time-delayed nonlinear systems and developed an observer-based NN controller using the backstepping strategy. In [8], the adaptive output-feedback

control problem for uncertain nonlinear single-input and single-output (SISO) systems with partially unpredictable states was studied, where a reduced-order observer was designed to estimate unpredictable states. An adaptive output-feedback controller was constructed by using a radial basis function (RBF) NN and incorporating the reduced-order observer into a new backstepping design in [9]. Furthermore, a dynamic output-feedback controller based on NN approximation theory was built to address the issue of dynamic output-feedback control for a class of nonlinear time-delay linked systems in [10]. The regionally stable conclusion from [6] was later extended to nonlinear strict-feedback time-delay systems in [8,9], and then to nonlinear linked systems in [10]. Sufficient conditions for obtaining the consistency of both sides of the considered system are obtained by constructing appropriate Lyapunov–Krasovskiy generalized functions and using the linear matrix inequality (LMI) technique [11]. Based on the explanations above, the primary restriction is that the control direction must be known. However, in practical applications, there are frequently indications of the unknown control gains.

It is challenging to construct a controller for a controlled system when the sign of the control coefficient is unknown because a controlled force acting in the wrong direction could cause the system to deviate from the expected behavior. For an adaptive control design, the researchers presented alternatively shifting the symbol of the control force. The Nussbaum function was invented by Nussbaum (1983), which relied on increasing the controller gain to move the system quickly to the desired state. Nussbaum-type functions were first introduced in parametric adaptive control in [12]. After that, the Nussbaum function was then expanded adaptive neural control of nonlinear time-delay systems in [13], where an adaptive fuzzy output-feedback controller was developed by employing the Nussbaum function to guarantee the uniformly ultimately bounded of all closed-loop system signals. For tracking the control problem of a class of strict-feedback stochastic nonlinear systems with unknown virtual control gain function as the primary characteristic, a new fuzzy adaptive control scheme was put forth in [14]. Razumikhin–Nussbaum lemma was proposed in [15] to solve the adaptive NN control problem for a class of stochastic nonlinear time-delay systems with unknown control gain. With an asymmetric control gain matrix, the control problem of a class of uncertain multi-input multi-output nonlinear systems with unknown control directions and unmeasured states was addressed by an observer-based fuzzy adaptive control technique in [16]. However, in [16], the problem of unknown control gains did not affect the observer design, which is not reasonable. In order to address the issue of unknown control gains for the observer, our research is focused on this topic.

Additionally, designing the controllers by using the backstepping method will lead to the “explosion of complexity” problem. To solve this problem, dynamic surface control (DSC) and command filtering techniques have been introduced. DSC was employed in [17] to get around the difficult math involved in the higher-order nonlinear dynamics. This method used several sliding surfaces and a step-by-step recursive procedure to stabilize the dynamic system. Different from DSC, the command filter is used to replace the traditional DSC, which simplifies the derivation process and the design of the controller. For example, in [18], an adaptive fuzzy tracking control problem for nonlinear systems was investigated based on the fuzzy approximate method and the command-filtering technology. Sun et al. investigated a command filtering-based finite-time adaptive fuzzy tracking control issue in [19] for uncertain nonlinear systems with specified performance. The command filters enable a more practical design where the controller is not differentiated, eliminating the complexity brought on by the differential explosion. Thus, it is essential that the command filtering technique is introduced into the design of the adaptive backstepping controllers.

In a word, it is relevant to investigate the adaptive stabilization control problem of nonlinear time-delay systems based on ESN. In this study, an adaptive stabilization scheme based on the ESN observer is proposed for a class of nonlinear time-delay systems with unknown control gains. The following are the primary contributions of this article as compared to the findings of earlier studies:

- (1) This paper takes advantage of the low computational complexity of ESN without changing the connection weights between the energy reservoirs of the input and hidden layers, simple training and high accuracy to approximate unknown nonlinear function terms. For the learning algorithm of an RBF neural network, the key issue is the reasonable determination of the central parameters of the hidden layer neurons. There are still shortcomings that make the application scope limited. In this work, ESN is applied to adaptive stabilization control of nonlinear time-delay system with unknown control gain.
- (2) Considering the unmeasurable variables in the system, an ESN-based full-dimensional state observer is developed in this paper to estimate them in the system. This method does not assume that all states of the controlled system are measurable. In contrast to [16], the construction of the observer is more difficult due to the consideration of the unknown control gains.
- (3) The Nussbaum function is employed in the system to manage the unknown control gain term. The difference is that, in this paper, the Nussbaum function is applied to handle the estimates of the unknown gain due to the observer. The Lyapunov–Krasovskii functionals and the Nussbaum function are used to provide an adaptive stabilization controller for the time-delay nonlinear system. All of the signals in the closed-loop system are guaranteed to be bounded by the proposed controller.

Design the time-delayed system model, estimate the controlled object with state observer, redesign the system equations on this basis, next design the compensation signal to solve the computational complexity explosion caused by command filtering, use Lyapunov–Krasovskii functionals to solve the time-delayed problem in the process of designing the controller, and finally introduce the Nussbaum function in the actual controller design to deal with the unknown control gain. Finally, the actual controller and the adaptive law are designed.

The remainder of this article is structured as follows. Section 2 gives the problem formulation and preliminary information. Section 3, provides a system stability analysis as well as a new adaptive stabilization control mechanism. Section 4 provides a simulated example. Section 5 contains the conclusion.

2. Preliminaries

2.1. Problem Formulation

Consider the following nonlinear nonstrict time-delay system with unknown control gain

$$\begin{aligned} \dot{x}_i &= x_{i+1} + f_i(x) + b_i(x(t - \tau_i)) \\ \dot{x}_n &= g_n(t)u + f_n(x) + b_n(x(t - \tau_n)) \\ y &= x_1 \end{aligned} \tag{1}$$

where $x = [x_1, \dots, x_n]^T \in R^n$ represents the unmeasurable state vector of the system. The system input is $u \in R$, and $y \in R$ is the system output. $f_i(x)$ and $b_i(x(t - \tau_i))$ are smooth functions that have no known properties. When τ_{\max} is a known constant and τ_i signifies the unknown time-varying delay that satisfies $0 < \tau_i < \tau_{\max}$. $g_n(t) \neq 0$ represents a time-varying unknown bounded control gain function.

Definition 1 ([19]). *If there exists a function satisfying*

$$\limsup_{s \rightarrow \infty} \frac{1}{s} \int_0^s N(\Phi) d\Phi = +\infty \tag{2}$$

$$\liminf_{s \rightarrow \infty} \frac{1}{s} \int_0^s N(\Phi) d\Phi = -\infty \tag{3}$$

then $N(\Phi)$ is a Nussbaum-type function. In this paper, $N(\Phi)$ is chosen as $\Phi^2 \cos(\Phi)$.

Lemma 1 ([20]). Assuming $\Phi(t)$ is a smooth function on $[0, t_f]$, if the following inequality holds

$$\dot{V}_0 \leq -\pi V + \frac{1}{r_n} [g_n(t)N(\Phi) + 1] \Phi \tag{4}$$

where $V_0 \geq 0$ is radially unbounded, and π, r_n are positive parameters, then $\frac{1}{r_n} [g_n(t)N(\Phi) + 1] \Phi$ is bounded on $[0, t_f]$.

2.2. Echo State Network

An ESN is a new type of RNN with the above structural description. In Figure 1, K, N and L are the number of neurons in the input, hidden and output layers, respectively. Large-scale RNNs are used as a dynamic library in the hidden layer, which can also be abbreviated as dynamic reservoir (DR). These leaky-integrator neurons that make up these DRs are triggered by appropriate input and output feedback. When compared to come forward NNs such as an RBFNN, an ESN has the advantage of faster training and does not require changing the connection weights between inputs and energy storage in the hidden layer. The continuous-time dynamics of the ESN used are followed by

$$\dot{\zeta}(x) = (1 - \iota) \tanh(W^{it}\zeta(x) + W^f y + W^i u) + \iota \zeta(x) \tag{5}$$

where the activation function of DR is given as $\zeta(x)$, the energy storage neuron’s leakage rate is given as $\iota > 0$, and the hyperbolic tangent function is indicated by $\tanh(\cdot)$. The input, internal, and feedback connection weight matrices are represented by $W^i, W^{it}, W^f \in R^n$, correspondingly. u is the external input in dimension K , so that the output equation reads in the following

$$y = W^T \zeta(x) \tag{6}$$

and $W \in R^n$ stands for the output weight matrix.

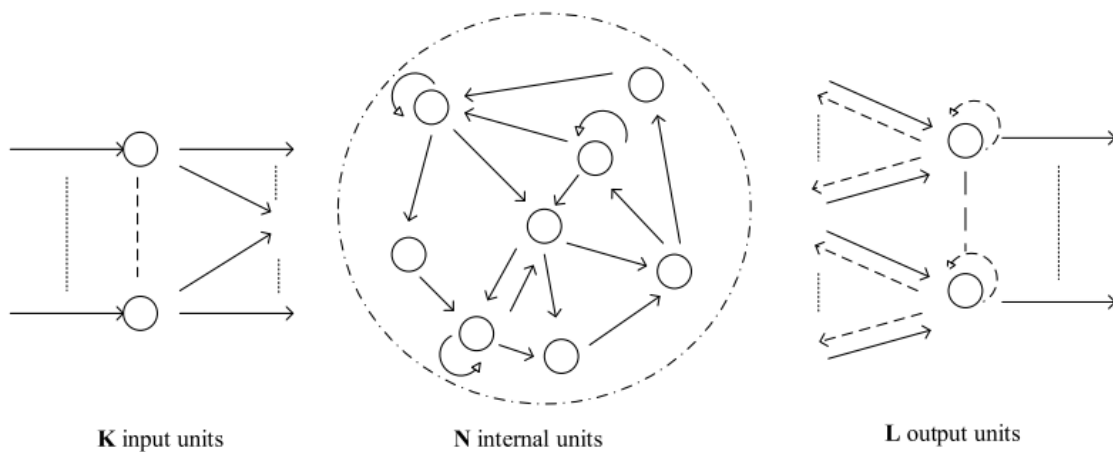


Figure 1. Architecture of ESN.

Through the [21–23], we obtained the method of an ESN for approximating the unknown function. Since any continuous function can be approximated by the results of RNN, this also proves that the existence of an ESN system $y(x)$ in the form of (6) such that the inequality is valid for any continuous function $f(\cdot) : R^n \rightarrow R$ on a large enough compact set $\Omega \in R$

$$\sup_{x \in \Omega} |f(x) - y(x)| \leq \bar{\epsilon} \tag{7}$$

where $\bar{\epsilon} > 0$ can be any arbitrary positive constant, and the function $f(x)$ can be roughly approximated by

$$f(x) = \epsilon + W^T \zeta(x), \forall x \in \Omega \tag{8}$$

where $W = \arg \min_{W \in R^n} \{ \sup_{x \in \Omega} |f(x) - W^T \zeta(x)| \}$ is satisfied by the ideal weight matrix $W \in R^n$, ε denotes the ESN approximation error bounded by $|\varepsilon| \leq \bar{\varepsilon}$, where $\zeta_j(x)$ is selected in the sigmoid form provided by

$$\zeta_j(x) = \frac{p_j}{o_j + a^{-\frac{x}{d_j}}} + c_j, j = 1, 2, \dots, N \tag{9}$$

where p_j, o_j and d_j, c_j are the constant parameters, and $\zeta_j(x)$ is a constant in $0 < \zeta_j(x) < o_m$ with $o_m = \max \left\{ \left| \frac{p_j}{o_j} + c_j, \frac{p_j}{o_j+1} + c_j \right| \right\}$. Since the ideal weight matrix W is typically unknown in practice, \hat{W} , an estimation of W is used and updated by creating adaptive laws online to ensure the asymptotic tracking performance.

Remark 1. Based on the aforementioned descriptions, it is found that the ESN is an alternative functional approximation tool. Despite the fact that the RBFNN is frequently used in articles for approximation, the ESN may be trained with more ease and accuracy than the RBFNN because it does not require altering the weights between both the hidden and input layers.

Lemma 2 ([24]). Let $\zeta(\cdot)$ be a function vector of the ESN. The center of the receptive field and the input variable are $\omega_i = [\omega_{i1}, \dots, \omega_{in}]^T$ and $X_n = [x_1, \dots, x_n]$, respectively. Create a new center ω_i , with the input variable S_k being $\omega_i = [\omega_{i1}, \dots, \omega_{ik}]^T$ and $S_k = [x_1, \dots, x_k]^T$ for an integer, $k < n$. Then, one has

$$\zeta^T(X_n)\zeta(X_n) \leq \zeta^T(S_k)\zeta(S_k) \tag{10}$$

for the same dimension constant, in which ω_i and S_k are indeed the vectors made up of the first k components of ω_i^k and X_n .

Assumption 1 ([25]). The underlying inequality

$$\underline{b}_{ij} \leq \frac{\partial f_i}{\partial x_j} \leq \bar{b}_{ij} \tag{11}$$

is satisfied by the partial derivative $\partial f_i / \partial x_i$, where $1 \leq i \leq n$ and $2 \leq j \leq n$, \underline{b}_{ij} and \bar{b}_{ij} are known constants.

Assumption 2 ([18]). Positive constants d_i and h_i exist for nonlinear functions $f_i(\cdot)$ and $q_i(\cdot)$ of that kind that for $1 \leq i \leq n$

$$\begin{cases} |f_i(x) - f_i(\hat{x})| \leq d_i \|x - \hat{x}\| \\ |q_i(x) - q_i(\hat{x})| \leq h_i \|x - \hat{x}\| \end{cases} \tag{12}$$

where \hat{x} is the estimation of x .

Remark 2. Taking $\hat{x} = 0$ from Assumption 2 results in $|f_i(x)| \leq d_i \|x\|$, proving the existence of strictly rising smooth functions $K_i(a) = d_i a$ with $a \in R$, which serve as the bounding functions for $f_i(\cdot)$. Similar to this, strictly growing smooth functions $k_i(a) = h_i a$ with $a \in R$ serving as the bounding functions of $q_i(\cdot)$ are also possible.

Lemma 3 ([26]). For $\forall (x, y) \in R^n$, the following inequality holds

$$xy \leq \frac{j^p}{p} |x|^p + \frac{1}{qj^q} |y|^q \tag{13}$$

where $j > 0, p > 1, q > 1$ and $(p - 1)(q - 1) = 1$.

Lemma 4 ([19]). For Lemma 1 and state transformation, the following inequality holds

$$\|\hat{x}\| \leq \sum_{i=1}^n |s_i| k_i(\theta_i) \tag{14}$$

where $k_i = 2\left(1 + c_i + \frac{1}{2}\right) + \frac{1}{q_i^2} \hat{\theta}_i^2 v_i^T(S_i) v_i(S_i)$, and c_i, q_i are positive design parameters.

Lemma 5 ([27]). If input noise satisfies $|x_{(i+1)_c} - \alpha_i| \leq \zeta$, where $x_{(i+1)_c}$ is the output of the command filtering, the inequality as following is real for a scalar $o_{i+1} > 0$

$$|\varphi_{i+1} - x_{(i+1)_c}| \leq o_{i+1} \zeta \tag{15}$$

3. Observer-Based Controller Design and Stability Analysis

A backstepping design process for system (1) using the adaptive neural control protocol is presented in this section.

An ESN-based observer is built to estimate the unknown state x_1, \dots, x_n of the system (1) as below

$$\begin{cases} \dot{\hat{x}}_i = \hat{x}_{i+1} - l_i(y - \hat{x}_1) + \hat{W}_i^T \xi_i(x_1, \hat{\zeta}) \\ \dot{\hat{x}}_n = \hat{g}_n(t)u - l_n(y - \hat{x}_1) + \hat{W}_n^T \xi_n(x_1, \hat{\zeta}) \end{cases} \tag{16}$$

where \hat{x}_i is the estimation of $x_i, 1 \leq i \leq n - 1, \hat{\zeta} = [\hat{x}_2, \dots, \hat{x}_n]^T, \hat{W}_i$ is used to estimate the unknown ideal weight vector, and it estimates $W_i. l_i$ is the to-be-designed observation gain parameter. $\hat{g}_n(t)$ is the estimation of $g_n(t). f_i(x_1, \hat{\zeta})$ is an unidentified nonlinear function, thus an ESN $W_i^T \xi_i(x_1, \hat{\zeta})$ is used to model it in a way that, for a given degree of precision $\bar{\varepsilon} > 0$

$$f_i(x_1, \hat{\zeta}) = W_i^T \xi_i(x_1, \hat{\zeta}) + \varepsilon_i(x_1, \hat{\zeta}), |\varepsilon_i| \leq \bar{\varepsilon} \tag{17}$$

with W_i is the ideal right vector, and ε_i is the approximation error function.

The estimating error is defined as $w_i = x_i - \hat{x}_i, \text{ for } 1 \leq i \leq n.$

Using (16) and (17), the error dynamic can be given as

$$\begin{aligned} \dot{w} = & A_0 w + F(x_1, \zeta) - \hat{W}^T \xi(x_1, \hat{\zeta}) + F(x) \\ & + B(x(t - \tau)) - F(x_1, \zeta) \end{aligned} \tag{18}$$

where $\zeta = [x_2, \dots, x_n]^T, w = [w_1, \dots, w_n]^T, \text{ and}$

$$\begin{aligned} F(x_1, \zeta) &= [f_1(x_1, \zeta), \dots, f_n(x_1, \zeta)]^T \\ \hat{W}^T \xi(x_1, \hat{\zeta}) &= [\hat{W}_1^T \xi_1(x_1, \hat{\zeta}), \dots, \hat{W}_n^T \xi_n(x_1, \hat{\zeta})]^T \\ F(x) &= [f_1(x), \dots, f_n(x)]^T \\ B(x(t - \tau)) &= [b_1(x(t - \tau_1)), \dots, b_n(x(t - \tau_n))]^T \\ A_0 &= A - LC, A = \begin{bmatrix} L & I_{n-1} \\ l_n & 0 \end{bmatrix} \\ L &= [l_1, \dots, l_{n-1}]^T. \end{aligned}$$

Define $\tilde{W} = [W_1 - \hat{W}_1, \dots, W_n - \hat{W}_n]^T$, and $F(x_1, \hat{\zeta}) = [f_1(x_1, \hat{\zeta}), \dots, f_n(x_n, \hat{\zeta})]^T$ is to address the impact of the $F(x_1, \zeta)$ on the error dynamic.

Then, (18) can be rewritten as

$$\begin{aligned} \dot{w} = & A_0 w + (F(x_1, \zeta) - F(x_1, \hat{\zeta})) + \varepsilon(x_1, \hat{\zeta}) + F(x) \\ & + \tilde{W}^T \xi(x_1, \hat{\zeta}) + B(x(t - \tau)) - F(x_1, \zeta) \end{aligned} \tag{19}$$

Choose the Lyapunov function candidate V_w as

$$V_w = w^T P w + \sum_{i=1}^n \int_{t-\tau_i}^t \|P\| d_i^2 w_i^2(a) da \tag{20}$$

with P being the to-be-designed positive definite matrix and the constant d_i will be specified later.

Differentiating V_w can get

$$\begin{aligned} \dot{V}_w &= w^T (P A_0 + A_0^T P) w + 2 w^T P (F(x_1, \zeta) - F(x_1, \hat{\zeta})) \\ &\quad + 2 w^T P \varepsilon(x_1, \hat{\zeta}) + 2 w^T P \tilde{W}^T \xi(x_1, \hat{\zeta}) \\ &\quad + 2 w^T P F(x) + 2 w^T P B(x(t - \tau)) + \|P\| \sum_{i=1}^n d_i^2 w_i^2 \\ &\quad - \|P\| \sum_{i=1}^n d_i^2 w_i^2(\tau_i) - 2 w^T P F(x_1, \zeta) \\ &= w^T (P A_0 + A_0^T P) w + 2 w^T P (F(x_1, \zeta) - F(x_1, \hat{\zeta})) \\ &\quad + 2 w^T P (\varepsilon(x_1, \hat{\zeta}) + \tilde{W}^T \xi(x_1, \hat{\zeta})) + 2 w^T P (F(x) \\ &\quad - F(\hat{x})) + 2 w^T P (B(x(t - \tau)) - B(\hat{x}(t - \tau))) \\ &\quad + 2 w^T P F(\hat{x}) + 2 w^T P B(\hat{x}(t - \tau)) + \|P\| \sum_{i=1}^n d_i^2 w_i^2 \\ &\quad - \|P\| \sum_{i=1}^n d_i^2 w_i^2(\tau_i) - 2 w^T P F(x_1, \zeta) \end{aligned} \tag{21}$$

According to differential mid-value theorem, one can get

$$2w^T P (F(x_1 - \zeta) - F(x_1 - \hat{\zeta})) = 2w^T P E w \tag{22}$$

where matrix E is defined as

$$E = \begin{bmatrix} 0 & \frac{\partial f_1}{\partial x_2} & \dots & \frac{\partial f_1}{\partial x_n} \\ \vdots & \vdots & \vdots & \vdots \\ 0 & \frac{\partial f_n}{\partial x_2} & \dots & \frac{\partial f_n}{\partial x_n} \end{bmatrix} \tag{23}$$

From Assumption 2 and Lemma 5, we have

$$\begin{aligned} &2 w^T P \varepsilon(x_1, \hat{\zeta}) + 2 w^T P \tilde{W}^T \xi(x_1, \hat{\zeta}) \\ &\leq \sigma w^T P w + \frac{n \bar{\varepsilon}^2}{\sigma} \|P\|^2 \\ &\quad + \frac{\|P\|}{\sigma} \sum_{i=1}^n a_i \tilde{W}_i^T \tilde{W}_i \end{aligned} \tag{24}$$

$$\begin{aligned} 2w^T P (F(x) - F(\hat{x})) &\leq 2w^T P d_i \|x - \hat{x}\| \\ &\leq w^T P w + \|P\| \sum_{i=1}^n d_i^2 w_i^2 w \end{aligned} \tag{25}$$

$$\begin{aligned} &2w^T P B(x(t - \tau)) - 2w^T P B(\hat{x}(t - \tau)) \\ &\leq 2w^T P h_i \|x(t - \tau) - \hat{x}(t - \tau)\| \\ &\leq w^T P w + \|P\| \sum_{i=1}^n h_i^2 w_i^2(\tau_i) \end{aligned} \tag{26}$$

where a_i and σ are positive design constants.

Invoking (22)–(26) into (21), one can obtain

$$\begin{aligned} \dot{V}_w \leq & w^T \left(PA_0 + A_0^T P + 2P + \sigma P + PE + E^T P \right) w \\ & + \frac{n\bar{\epsilon}^2}{\sigma} \|P\| + \|P\| \sum_{i=1}^n \left(h_i^2 + d_i^2 \right) w^T w \\ & + 2w^T PF(\hat{x}) + 2w^T PB(x(t - \tau)) \\ & + \frac{\|P\|}{\sigma} \sum_{i=1}^n a_i \tilde{W}_i^T \tilde{W}_i - 2w^T PF(x_1, \zeta) \end{aligned} \tag{27}$$

Then, a stabilization control method is proposed for this kind of nonlinear system. The coordinate transformation of z is described by

$$\begin{cases} z_1 = \hat{x}_1 \\ z_i = \hat{x}_i - x_{i_c} \end{cases} \tag{28}$$

where $i = 2, \dots, n$ and x_{i_c} denotes the output of command filtering.

To avoid the increased complexity caused by the backstepping controller design, the command filter is defined as

$$\dot{\varphi}_i \omega_i = \alpha_{i-1} - \varphi_i \tag{29}$$

where φ_i is denoted as the output of the command filter that the virtual control signal α_{i-1} is the corresponding input signal, and ω_i is a positive design parameter. The initial condition is $\varphi_i(0) = \alpha_i(0)$. Since signal noise is ubiquitous, observation and control require real signals, specified frequencies, and sudden changes in interference are excluded or limited as much as possible, so command filters are applied and parameters are set.

To deal with the filtering error z , the error compensation signals η_i for $i = 1, \dots, n$ are designed. Define the compensated errors as

$$\begin{cases} s_i = z_i - \eta_i \\ s_n = z_n - \eta_n \end{cases} \tag{30}$$

Step 1: Together with (16), (28) and (30), one can differentiate s_1 as

$$\dot{s}_1 = \hat{x}_2 - l_1(y - \hat{x}_1) + \hat{W}_1^T \xi_1(x_1, \hat{\zeta}) - \dot{\eta}_1 \tag{31}$$

Choose the Lyapunov–Krasovskii function candidate as

$$V_1 = \frac{1}{2} s_1^2 + m \int_{t-\tau_1}^t s_1^2 k_1^2(\theta_1(a)) da + \frac{1}{2\rho_1} \tilde{\theta}_1^2 \tag{32}$$

where ρ_1 is a design constant, $\tilde{\theta}_1 = \theta_1 - \hat{\theta}_1$ and $\hat{\theta}_1$ is the estimation of θ_1 , m is a positive design parameter that will be designed later.

The derivative forms of V_1 can be presented as

$$\begin{aligned} \dot{V}_1 = & s_1(\hat{x}_2 - l_1 w_1 + \hat{W}_1^T \xi_1(x_1, \hat{\zeta}) - \dot{\eta}_1) + m s_1^2 k_1^2(\theta_1) \\ & - m s_1^2(\tau_1) k_1^2(\theta_1(\tau_1)) - \frac{1}{\rho_1} \tilde{\theta}_1 \dot{\hat{\theta}}_1 \end{aligned} \tag{33}$$

According to Lemma 3, one has

$$-s_1 l_1 w_1 \leq \frac{1}{2} \beta_1 w_1^2 + \frac{1}{2\beta_1} s_1^2 l_1^2 \tag{34}$$

where β_1 is a positive design parameter.

Invoking (34) into (33) yields

$$\begin{aligned} \dot{V}_1 \leq & s_1(\hat{x}_2 + \hat{W}_1^T \zeta_1(x_1, \hat{\zeta}) - \dot{\eta}_1) + \frac{1}{2\beta_1} s_1^2 l_1^2 + m s_1^2 k_1^2(\theta_1) \\ & - m s_1^2(\tau_1) k_1^2(\theta_1(\tau_1)) - \frac{1}{\rho_1} \tilde{\theta}_1 \dot{\hat{\theta}}_1 + \frac{1}{2} \beta_1 w_1^2 \end{aligned} \tag{35}$$

Design the compensation signal η_1 as

$$\dot{\eta}_1 = -c_1 \eta_1 - \frac{1}{2} \eta_1 + \eta_2 + s_2 + x_{2c} - \alpha_1 \tag{36}$$

where c_1 is a positive design parameter.

Substituting (36) into (35) gives

$$\begin{aligned} \dot{V}_1 \leq & s_1 \left(\frac{1}{2\beta_1} s_1 l_1^2 + W_1^T \zeta_1(x_1, \hat{\zeta}) - \tilde{W}_1^T \zeta_1(x_1, \hat{\zeta}) \right) \\ & + s_1 \left(c_1 \eta_1 + \frac{1}{2} \eta_1 + \alpha_1 \right) + m s_1^2 k_1^2(\theta_1) \\ & - m s_1^2(\tau_1) k_1^2(\theta_1(\tau_1)) - \frac{1}{\rho_1} \tilde{\theta}_1 \dot{\hat{\theta}}_1 + \frac{1}{2} \beta_1 w_1^2 \end{aligned} \tag{37}$$

where

$$\bar{f}_1(X_1) = W_1^T \zeta_1(x_1, \hat{\zeta}) + j s_1 K_1^2(\theta_1) + \frac{1}{2\beta_1} s_1 l_1^2 + m s_1 k_1^2(\theta_1)$$

with $X_1 = [x_1, \dots, x_n, \hat{W}_1^T, \hat{\zeta}]^T$. For the unknown $\bar{f}_1(X_1)$, an ESN $W_1^T v_1(X_1)$ is employed to approximate it and such that for given $\bar{\epsilon}_1 > 0$, $\bar{f}_1(X_1) = W_1^T v_1(X_1) + \epsilon_1(X_1)$, with $|\epsilon_1| < \bar{\epsilon}_1$.

Using Lemmas 2 and 3, the following inequality holds

$$\begin{aligned} s_1 \bar{f}_1(X_1) &= s_1 W_1^T v_1(X_1) + s_1 \epsilon_1(X_1) \\ &\leq \frac{1}{2q_1^2} s_1^2 \|W_1\|^2 v_1^T(X_1) v_1(X_1) + \frac{q_1^2}{2} + \frac{1}{2} s_1^2 + \frac{1}{2} \epsilon_1^2 \\ &\leq \frac{1}{2q_1^2} s_1^2 \theta_1 v_1^T(S_1) v_1(S_1) + \frac{q_1^2}{2} + \frac{1}{2} s_1^2 + \frac{1}{2} \bar{\epsilon}_1^2 \end{aligned} \tag{38}$$

where q_1 being a positive constant, $\|W_1\|^2 = \theta_1$, and $S_1 = [x_1]^T$.

From (38), (37) can be rewritten as

$$\begin{aligned} \dot{V}_1 \leq & s_1 \left(-\tilde{W}_1^T \zeta_1(x_1, \hat{\zeta}) + c_1 \eta_1 + \frac{1}{2} \eta_1 + \frac{1}{2} s_1 + \alpha_1 \right) \\ & - j s_1^2 K_1^2(\theta_1) - m s_1^2(\tau_1) k_1^2(\theta_1(\tau_1)) + \frac{1}{2} q_1^2 + \frac{1}{2} \beta_1 w_1^2 \\ & + \frac{1}{2} \bar{\epsilon}_1^2 + \frac{1}{2q_1^2} s_1^2 \hat{\theta}_1 v_1^T(S_1) v_1(S_1) \\ & + \frac{1}{\rho_1} \tilde{\theta}_1 \left(\frac{\rho_1}{2q_1^2} s_1^2 v_1^T(S_1) v_1(S_1) - \dot{\hat{\theta}}_1 \right) \end{aligned} \tag{39}$$

Develop the virtual control signal α_1 and adaptive law $\hat{\theta}_1$ as

$$\alpha_1 = - \left(c_1 + \frac{1}{2} \right) z_1 - \frac{1}{2q_1^2} s_1 \hat{\theta}_1 v_1^T(S_1) v_1(S_1) \tag{40}$$

$$\dot{\hat{\theta}}_1 = \frac{\rho_1}{2q_1^2} s_1^2 v_1^T(S_1) v_1(S_1) - \gamma_1 \hat{\theta}_1 \tag{41}$$

where γ_1 is a positive design parameter.

By substituting (40) and (41) into (39), it follows that

$$\begin{aligned} \dot{V}_1 \leq & -c_1 s_1^2 - j s_1^2 K_1^2(\theta_1) - m s_1^2(\tau_1) k_1^2(\theta_1(\tau_1)) \\ & + \frac{\gamma_1}{\rho_1} \tilde{\theta}_1 \hat{\theta}_1 - s_1 \hat{W}_1^T \zeta_1(x_1, \hat{\zeta}) + \frac{1}{2} q_1^2 + \frac{1}{2} \beta_1 w_1^2 + \frac{1}{2} \bar{\varepsilon}_1^2 \end{aligned} \tag{42}$$

Step $i(2 \leq i \leq n - 1)$: Considering (16), (28), (30) and differentiating s_i with respect to time yields

$$\dot{s}_i = \hat{x}_{i+1} - l_i(y - \hat{x}_1) + \hat{W}_i^T \zeta_i(x_1, \hat{\zeta}) - \dot{x}_{i_c} - \dot{\eta}_i \tag{43}$$

Consider the Lyapunov–Krasovskii function candidate

$$V_i = \frac{1}{2} s_i^2 + m \int_{t-\tau_i}^t s_i^2 k_i^2(\theta_i(a)) da + \frac{1}{2\rho_i} \tilde{\theta}_i^2 \tag{44}$$

where ρ_i is a design constant, $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$ and $\hat{\theta}_i$ is estimation of θ_i .

Invoking (43) and (44), the derivative of V_i is given by

$$\begin{aligned} \dot{V}_i = & s_i(\hat{x}_{i+1} - l_i w_1 + \hat{W}_i^T \zeta(x_1, \hat{\zeta}) - \dot{x}_{i_c} - \dot{\eta}_i) + m s_i^2 k_i^2(\theta_i) \\ & - m s_i^2(\tau_i) k_i^2(\theta_i(\tau_i)) - \frac{1}{\rho_i} \tilde{\theta}_i \dot{\hat{\theta}}_i \end{aligned} \tag{45}$$

Consider the following fact

$$-s_i l_i w_1 \leq \frac{1}{2} \beta_i w_1^2 + \frac{1}{2\beta_i} s_i^2 l_i^2 \tag{46}$$

where β_i is a positive design parameter.

According to (46), it can be obtained

$$\begin{aligned} \dot{V}_i \leq & s_i(\hat{x}_{i+1} + \hat{W}_i^T \zeta_i(x_1, \hat{\zeta}) - \dot{x}_{i_c} - \dot{\eta}_i) + \frac{1}{2\beta_i} s_i^2 l_i^2 \\ & + m s_i^2 k_i^2(\theta_i) - m s_i^2(\tau_i) k_i^2(\theta_i(\tau_i)) - \frac{1}{\rho_i} \tilde{\theta}_i \dot{\hat{\theta}}_i + \frac{1}{2} \beta_i w_1^2 \end{aligned} \tag{47}$$

The compensation signal η_i is designed as

$$\dot{\eta}_i = -c_i \eta_i - \frac{1}{2} \eta_i + \eta_{i+1} - \eta_{i-1} + s_{i+1} + x_{(i+1)_c} - \alpha_i \tag{48}$$

where c_i is a positive design parameter.

Then, (47) can be rewritten in the following form

$$\begin{aligned} \dot{V}_i \leq & s_i(-\dot{x}_{i_c} + \frac{1}{2\beta_i} s_i l_i^2 + W_i^T \zeta_i(x_1, \hat{\zeta}) - \hat{W}_i^T \zeta_i(x_1, \hat{\zeta})) \\ & + s_i(c_i \eta_i + \frac{1}{2} \eta_i + \eta_{i-1} + \alpha_i) - m s_i^2(\tau_i) k_i^2(\theta_i(\tau_i)) \\ & + m s_i^2 k_i^2(\theta_i) - \frac{1}{\rho_i} \tilde{\theta}_i \dot{\hat{\theta}}_i + \frac{1}{2} \beta_i w_1^2 \end{aligned} \tag{49}$$

where

$$\bar{f}_i(X_i) = j s_i K_i^2(\theta_i) + W_i^T \zeta_i(x_1, \hat{\zeta}) + s_i \eta_{i-1} + m s_i k_i^2(\theta_i) + \frac{1}{2\beta_i} s_i l_i^2$$

with $X_i = [x_1, \dots, x_n, \hat{W}_i^T, \hat{\zeta}]^T$. Once again, an ESN $W_i^T v_i(X_i)$ is used to approximate the unknown function $\bar{f}_i(X_i)$ such that for given $\bar{\varepsilon}_i > 0$, $\bar{f}_i(X_i) = W_i^T v_i(X_i) + \varepsilon_i(X_i)$, with $|\varepsilon_i| < \bar{\varepsilon}_i$.

Similar to step 1, one has

$$\begin{aligned} s_i \bar{f}_i(X_i) &\leq \frac{1}{2q_i^2} s_i^2 \|W_i\|^2 v_i^T(X_i) v_i(X_i) + \frac{q_i^2}{2} + \frac{1}{2} s_i^2 + \frac{1}{2} \bar{\varepsilon}_i^2 \\ &\leq \frac{1}{2q_i^2} s_i^2 \theta_i v_i^T(S_i) v_i(S_i) + \frac{q_i^2}{2} + \frac{1}{2} s_i^2 + \frac{1}{2} \bar{\varepsilon}_i^2 \end{aligned} \tag{50}$$

with $\|W_i\|^2 = \theta_i$, and $S_i = [x_1, \dots, x_i]^T$.

It follows from substituting (50) into (49) that

$$\begin{aligned} \dot{V}_i &\leq s_i(-\dot{x}_{ic} + c_i \eta_i + \frac{1}{2} \eta_i + \frac{1}{2} s_i + \alpha_i) - j s_i^2 K_i^2(\theta_i) \\ &\quad - m s_i^2(\tau_i) k_i^2(\theta_i(\tau_i)) + \frac{1}{2} q_i^2 + \frac{1}{2} \beta_i w_1^2 \\ &\quad + \frac{1}{2} \bar{\varepsilon}_i^2 + \frac{1}{2q_i^2} s_i^2 \hat{\theta}_i v_i^T(S_i) v_i(S_i) - s_i \tilde{W}_i^T \zeta_i(x_1, \hat{\zeta}) \\ &\quad + \frac{1}{\rho_i} \tilde{\theta}_i (\frac{\rho_i}{2q_i^2} s_i^2 v_i^T(S_i) v_i(S_i) - \hat{\theta}_i) \end{aligned} \tag{51}$$

Choose the virtual control signal α_i and adaptive law $\hat{\theta}_i$ as

$$\alpha_i = -c_i z_i - \frac{1}{2} z_i - \frac{1}{2q_i^2} s_i \hat{\theta}_i v_i^T(S_i) v_i(S_i) + \dot{x}_{ic} \tag{52}$$

$$\dot{\hat{\theta}}_i = \frac{\rho_i}{2q_i^2} s_i^2 v_i^T(S_i) v_i(S_i) - \gamma_i \hat{\theta}_i \tag{53}$$

where γ_i is a positive design parameter.

According to (52) and (53), one has

$$\begin{aligned} \dot{V}_i &\leq -c_i s_i^2 - j s_i^2 K_i^2(\theta_i) - m s_i^2(\tau_i) k_i^2(\theta_i(\tau_i)) \\ &\quad + \frac{\gamma_i}{\rho_i} \tilde{\theta}_i \hat{\theta}_i + \frac{1}{2} q_i^2 + \frac{1}{2} \beta_i w_1^2 + \frac{1}{2} \bar{\varepsilon}_i^2 - s_i \tilde{W}_i^T \zeta_i(x_1, \hat{\zeta}) \end{aligned} \tag{54}$$

Step n: According to (16), (28), (30), the derivative of s_n is

$$\dot{s}_n = \hat{g}_n(t) u - l_n(y - \hat{x}_1) + \hat{W}_n^T \zeta_n(x_1, \hat{\zeta}) - \dot{x}_{nc} - \dot{\eta}_n \tag{55}$$

Construct the Lyapunov–Krasovskii function candidate as

$$V_n = V_{n-1} + \frac{1}{2} s_n^2 + m \int_{t-\tau_n}^t s_n^2 k_n^2(\theta_n(a)) da + \frac{1}{2\rho_n} \tilde{\theta}_n^2 \tag{56}$$

where ρ_n is a design constant and $\tilde{\theta}_n = \theta_n - \hat{\theta}_n$, $\hat{g}_n = g_n - \tilde{g}_n$. $\hat{\theta}_n$ and \hat{g}_n are estimation of θ_n and g_n .

\dot{V}_n is the time derivative of V_n

$$\begin{aligned} \dot{V}_n &= \dot{V}_{n-1} + m s_n^2 k_n^2(\theta_n) - m s_n^2(\tau_n) k_n^2(\theta_n(\tau_n)) \\ &\quad + s_n((g_n(t) - \tilde{g}_n(t)) u - l_n w_1 + \hat{W}_n^T \zeta_n(x_1, \hat{\zeta}) - \dot{x}_{nc} \\ &\quad - \dot{\eta}_n) - \frac{1}{\rho_n} \tilde{\theta}_n \dot{\hat{\theta}}_n \end{aligned} \tag{57}$$

according to Lemma 3, one can get

$$-s_n l_n w_1 \leq \frac{1}{2} \beta_n w_1^2 + \frac{1}{2\beta_n} s_n^2 l_n^2 \tag{58}$$

where β_n is a positive design parameter.

By taking (58) into consideration, one arrives at

$$\begin{aligned} \dot{V}_n &\leq \dot{V}_{n-1} + \frac{1}{2\beta_n} s_n^2 l_n^2 + m s_n^2 k_n^2(\theta_n) - m s_n^2(\tau_n) k_n^2(\theta_n(\tau_n)) \\ &\quad + s_n((g_n(t) - \tilde{g}_n(t))u + \hat{W}_n^T \xi_n(x_1, \hat{\zeta}) - \dot{x}_{n_c} - \dot{\eta}_n) \\ &\quad - \frac{1}{\rho_n} \tilde{\theta}_n \dot{\hat{\theta}}_n + \frac{1}{2} \beta_n w_1^2 \end{aligned} \tag{59}$$

The compensation signal η_n is proposed as

$$\dot{\eta}_n = -c_n \eta_n - \eta_{n-1} - \frac{\eta_n}{2} \tag{60}$$

where c_n is a positive design parameter.

Substituting (60) into (59) yields

$$\begin{aligned} \dot{V}_n &\leq \dot{V}_{n-1} + s_n \left((g_n(t) - \tilde{g}_n(t))u + c_n \eta_n + \eta_{n-1} + \frac{\eta_n}{2} \right) \\ &\quad + s_n \left(\frac{1}{2\beta_n} s_n l_n^2 + W_n^T \xi_n(x_1, \hat{\zeta}) - \tilde{W}_n^T \xi_n(x_1, \hat{\zeta}) \right) \\ &\quad - s_n \dot{x}_{n_c} + m s_n^2 k_n^2(\theta_n) - m s_n^2(\tau_n) k_n^2(\theta_n(\tau_n)) \\ &\quad - \frac{1}{\rho_n} \tilde{\theta}_n \dot{\hat{\theta}}_n + \frac{1}{2} \beta_n w_1^2 \end{aligned} \tag{61}$$

Let

$$\bar{f}_n(X_n) = m s_n k_n^2(\theta_n) + j s_n K_n^2(\theta_n) + W_n^T \xi_n(x_1, \hat{\zeta}) + g(t)u + \frac{1}{2\beta_n} s_n l_n^2 + s_n \eta_{n-1} - 2 w^T P F(x_1, \zeta)$$

with $X_n = [x_1, \dots, x_n, \hat{W}_n^T, \hat{\zeta}]^T$. Similarly to step i , it follows that $\bar{f}_n(X_n) = W_n^T v_n(X_n) + \varepsilon_n(X_n)$ and $|\varepsilon_n| < \bar{\varepsilon}_n$.

The following inequality holds

$$\begin{aligned} s_n \bar{f}_n(X_n) &\leq \frac{1}{2q_n^2} s_n^2 \|W_n\|^2 v_n^T(X_n) v_n(X_n) \\ &\quad + \frac{q_n^2}{2} + \frac{1}{2} s_n^2 + \frac{1}{2} \bar{\varepsilon}_n^2 \\ &\leq \frac{1}{2q_n^2} s_n^2 \theta_n v_n^T(S_n) v_n(S_n) \\ &\quad + \frac{q_n^2}{2} + \frac{1}{2} s_n^2 + \frac{1}{2} \bar{\varepsilon}_n^2 \end{aligned} \tag{62}$$

with $\|W_n\|^2 = \theta_n$, and $S_n = [x_1, \dots, x_n]^T$.

From (61) and (62), one has

$$\begin{aligned} \dot{V}_n &\leq \dot{V}_{n-1} + s_n(-\tilde{g}_n(t)u + \frac{1}{2} s_n - \dot{x}_{n_c} + c_n \eta_n + \frac{\eta_n}{2}) \\ &\quad + \frac{1}{2q_n^2} s_n^2 \hat{\theta}_n v_n^T(S_n) v_n(S_n) + \frac{1}{2} \beta_n w_1^2 + \frac{1}{2} q_n^2 \\ &\quad + \frac{1}{2} \bar{\varepsilon}_n^2 - j s_n^2 K_n^2(\theta_n) - m s_n^2(\tau_n) k_n^2(\theta_n(\tau_n)) \\ &\quad + \frac{1}{\rho_n} \tilde{\theta}_n \left(\frac{\rho_n}{2q_n^2} s_n^2 v_n^T(S_n) v_n(S_n) - \dot{\hat{\theta}}_n \right) \\ &\quad - s_n \tilde{W}_n^T \xi_n(x_1, \hat{\zeta}) + 2 w^T P F(x_1, \zeta) \end{aligned} \tag{63}$$

Design the actual control signal u and the adaptive law $\hat{\theta}_n$ as

$$u = N(\Phi) \bar{u} \tag{64}$$

$$\bar{u} = -c_n z_n - \frac{1}{2} z_n - \frac{1}{2q_n^2} s_n \hat{\theta}_n v_n^T(S_n) v_n(S_n) + \dot{x}_{n_c} \tag{65}$$

$$\dot{\Phi} = -\bar{u} s_n r_n \tag{66}$$

$$\dot{\hat{\theta}}_n = \frac{\rho_n}{2q_n^2} s_n^2 v_n^T(S_n) v_n(S_n) - \gamma_n \hat{\theta}_n \tag{67}$$

where c_n, r_n are positive parameters to be designed.

Along with (63)–(67), \dot{V}_n can be concluded as

$$\begin{aligned} \dot{V}_n \leq & - \sum_{i=1}^n c_i s_i^2 + \frac{1}{r_n} [\tilde{g}_n(t) N(\Phi) + 1] \dot{\Phi} \\ & - \sum_{i=1}^n j s_i^2 K_i^2(\theta_i) - \sum_{i=1}^n m s_i^2(\tau_i) k_i^2(\theta_i(\tau_i)) \\ & + \sum_{i=1}^n \frac{\gamma_i}{\rho_i} \tilde{\theta}_i \hat{\theta}_i + \sum_{i=1}^n \left(\frac{1}{2} \beta_i w_1^2 + \frac{1}{2} q_i^2 + \frac{1}{2} \tilde{\varepsilon}_i^2 \right) \\ & - s_n \tilde{W}_n^T \tilde{\zeta}_n(x_1, \hat{\zeta}) + 2 w^T P F(x_1, \zeta) \end{aligned} \tag{68}$$

Notice that

$$\tilde{\theta}_i \hat{\theta}_i = \tilde{\theta}_i (\theta_i - \tilde{\theta}_i) \leq \frac{1}{2} \theta_i^2 - \frac{1}{2} \tilde{\theta}_i^2 \tag{69}$$

Consequently, \dot{V}_n is rewritten as

$$\begin{aligned} \dot{V}_n \leq & - \sum_{i=1}^n c_i s_i^2 + \frac{1}{r_n} [\tilde{g}_n(t) N(\Phi) + 1] \dot{\Phi} \\ & - \sum_{i=1}^n j s_i^2 K_i^2(\theta_i) - \sum_{i=1}^n m s_i^2(\tau_i) k_i^2(\theta_i(\tau_i)) \\ & - \sum_{i=1}^n \frac{\gamma_i}{2\rho_i} \tilde{\theta}_i^2 + \sum_{i=1}^n \left(\frac{1}{2} \beta_i w_1^2 + \frac{1}{2} q_i^2 + \frac{1}{2} \tilde{\varepsilon}_i^2 + \frac{\gamma_i}{2\rho_i} \theta_i^2 \right) \\ & - s_n \tilde{W}_n^T \tilde{\zeta}_n(x_1, \hat{\zeta}) + 2 w^T P F(x_1, \zeta) \end{aligned} \tag{70}$$

To develop the adaptive law regarding \hat{W} , the Lyapunov function candidate can be constructed as

$$V_m = \sum_{i=1}^n V_i + \sum_{i=1}^n \frac{1}{2\mu_i} \tilde{W}_i^T \tilde{W}_i$$

with μ_i being a positive constant.

Differentiating V_m and taking (70) into account, we have

$$\begin{aligned} \dot{V}_m \leq & - \sum_{i=1}^n s_i \tilde{W}_i^T \tilde{\zeta}_i - \sum_{i=1}^n c_i s_i^2 - \sum_{i=1}^n j s_i^2 K_i^2(\theta_i) \\ & - \sum_{i=1}^n m s_i^2(\tau_i) k_i^2(\theta_i(\tau_i)) + \frac{1}{r_n} [\tilde{g}_n(t) N(\Phi) + 1] \dot{\Phi} \\ & - \sum_{i=1}^n \frac{\gamma_i}{2\rho_i} \tilde{\theta}_i^2 + \sum_{i=1}^n \left(\frac{1}{2} \beta_i w_1^2 + \frac{1}{2} q_i^2 + \frac{1}{2} \tilde{\varepsilon}_i^2 + \frac{\gamma_i}{2\rho_i} \theta_i^2 \right) \\ & - \sum_{i=1}^n \frac{1}{\mu_i} \tilde{W}_i^T \dot{\tilde{W}}_i + 2 w^T P F(x_1, \zeta) \end{aligned} \tag{71}$$

Choosing

$$\dot{W}_i = -\vartheta_i \hat{W}_i - \mu_i s_i \zeta_i(x_1, \hat{\zeta}) \tag{72}$$

with ϑ_i being a positive constant. Substituting (71) into (70) yields

$$\begin{aligned} \dot{V}_m \leq & - \sum_{i=1}^n c_i s_i^2 - \sum_{i=1}^n j s_i^2 K_i^2(\theta_i) - \sum_{i=1}^n m s_i^2(\tau_i) k_i^2(\theta_i(\tau_i)) \\ & + \frac{1}{r_n} [\tilde{g}_n(t) N(\Phi) + 1] \Phi - \sum_{i=1}^n \frac{\gamma_i}{2\rho_i} \tilde{\theta}_i^2 + \sum_{i=1}^n \frac{\vartheta_i}{\mu_i} \tilde{W}_i^T \hat{W}_i \\ & + \sum_{i=1}^n \left(\frac{1}{2} \beta_i w_1^2 + \frac{1}{2} q_i^2 + \frac{1}{2} \bar{\varepsilon}_i^2 + \frac{\gamma_i}{2\rho_i} \theta_i^2 \right) + 2w^T PF(x_1, \zeta) \end{aligned} \tag{73}$$

Therefore, based on the previous discussion, we construct the Lyapunov function $V_0 = V_w + V_m$ and differentiate it, one can obtain

$$\begin{aligned} \dot{V}_0 \leq & w^T \left(PA_0 + A_0^T P + 2P + \sigma P + PE + E^T P \right) w \\ & + \frac{n\bar{\varepsilon}^2}{\sigma} \|P\| + \|P\| \sum_{i=1}^n \left(h_i^2 + d_i^2 \right) w^T w + 2w^T PF(\hat{x}) \\ & + 2w^T PB(\hat{x}(t - \tau)) - \sum_{i=1}^n c_i s_i^2 - \sum_{i=1}^n j s_i^2 K_i^2(\theta_i) \\ & - \sum_{i=1}^n m s_i^2(\tau_i) k_i^2(\theta_i(\tau_i)) + \frac{1}{r_n} [\tilde{g}_n(t) N(\Phi) + 1] \Phi \\ & - \sum_{i=1}^n \frac{\gamma_i}{2\rho_i} \tilde{\theta}_i^2 + \sum_{i=1}^n \left(\frac{1}{2} \beta_i w_1^2 + \frac{1}{2} q_i^2 + \frac{1}{2} \bar{\varepsilon}_i^2 + \frac{\gamma_i}{2\rho_i} \theta_i^2 \right) \\ & + \frac{\|P\|}{\sigma} \sum_{i=1}^n a_i \tilde{W}_i^T \tilde{W}_i + \sum_{i=1}^n \frac{\vartheta_i}{\mu_i} \tilde{W}_i^T \hat{W}_i + 2w^T PF(x_1, \zeta) \end{aligned} \tag{74}$$

Using Lemmas 3 and 4, one can get

$$\begin{aligned} 2w^T PF(\hat{x}) & \leq \varepsilon_0 w^T w + \frac{1}{\varepsilon_0} F^T(\hat{x}) p^2 F(\hat{x}) \\ & \leq \varepsilon_0 w^T w + \frac{1}{\varepsilon_0} \|P\|^2 \|F(\hat{x})\|^2 \\ & \leq \varepsilon_0 w^T w + j \|\hat{x}\|^2 \\ & \leq \varepsilon_0 w^T w + \sum_{i=1}^n j s_i^2 K_i^2(\theta_i) \end{aligned} \tag{75}$$

$$\begin{aligned} 2w^T PB(\hat{x}(t - \tau)) & \leq \frac{1}{\varepsilon_0} B^T(\hat{x}(t - \tau)) P^2 B(\hat{x}(t - \tau)) \\ & \quad + \varepsilon_0 w^T w \\ & \leq \varepsilon_0 w^T w + \frac{1}{\varepsilon_0} \|P\|^2 \|B(\hat{x}(t - \tau))\|^2 \\ & \leq \varepsilon_0 w^T w + m \|\hat{x}(t - \tau)\|^2 \\ & \leq \varepsilon_0 w^T w + \sum_{i=1}^n m s_i^2(\tau_i) k_i^2(\theta_i(\tau_i)) \end{aligned} \tag{76}$$

with $j = \frac{n}{\varepsilon_0} \|P\|^2 \sum_{i=1}^n d_i^2$, $m = \frac{n}{\varepsilon_0} \|P\|^2 \sum_{i=1}^n h_i^2$, and ε_0 is a positive design parameter.

Then, (74) can be rewritten as

$$\begin{aligned}
 \dot{V}_0 \leq & w^T \left(PA_0 + A_0^T P + 2P + 2\varepsilon_0 + \sigma P + PE + E^T P \right) w \\
 & + \frac{n\varepsilon^2}{\sigma} \|P\| + \|P\| \sum_{i=1}^n \left(h_i^2 + d_i^2 \right) w^T w \\
 & + \frac{\|P\|}{\sigma} \sum_{i=1}^n a_i \tilde{W}_i^T \tilde{W}_i + \frac{1}{r_n} [\tilde{g}_n(t)N(\Phi) + 1] \Phi \\
 & - \sum_{i=1}^n c_i s_i^2 - \sum_{i=1}^n \frac{\gamma_i}{2\rho_i} \tilde{\theta}_i^2 + \sum_{i=1}^n \frac{\vartheta_i}{\mu_i} \tilde{W}_i^T \hat{W}_i \\
 & + \sum_{i=1}^n \left(\frac{1}{2} \beta_i w_1^2 + \frac{1}{2} q_i^2 + \frac{1}{2} \varepsilon_i^2 + \frac{\gamma_i}{2\rho_i} \theta_i^2 \right)
 \end{aligned} \tag{77}$$

Theorem 1. For the system (1) that fulfilling all presumptions construct the observer (16). Assume that ESN can approximate the packaged nonlinear function \bar{f}_i such that the approximation errors are constrained. If a definite positive matrix P exists, then

$$\begin{aligned}
 & PA_0 + A_0^T P + 2P + 2\varepsilon_0 + \sigma P + PE + E^T P \\
 & + \|P\| \sum_{i=1}^n \left(d_i^2 + h_i^2 \right) I < 0
 \end{aligned} \tag{78}$$

where $\varepsilon_0, \sigma, d_i$ and h_i are constants and $I = w^T w$ ensure that the closed-loop system is semi-globally stabilized, and by selecting suitable design parameters, it is possible to make any variable in the closed-loop system is infinitely small.

Proof. Combining (77) and (78) yields

$$\begin{aligned}
 \dot{V}_0 \leq & w^T \left(PA_0 + A_0^T P + 2P + 2\varepsilon_0 + \sigma P + PE + E^T P \right) w \\
 & + \frac{n\varepsilon^2}{\sigma} \|P\| + \|P\| \sum_{i=1}^n \left(h_i^2 + d_i^2 \right) w^T w \\
 & + \frac{\|P\|}{\sigma} \sum_{i=1}^n a_i \tilde{W}_i^T \tilde{W}_i + \frac{1}{r_n} [\tilde{g}_n(t)N(\Phi) + 1] \Phi \\
 & - \sum_{i=1}^n c_i s_i^2 - \sum_{i=1}^n \frac{\gamma_i}{2\rho_i} \tilde{\theta}_i^2 + \sum_{i=1}^n \frac{\vartheta_i}{\mu_i} \tilde{W}_i^T \hat{W}_i \\
 & + \sum_{i=1}^n \left(\frac{1}{2} \beta_i w_1^2 + \frac{1}{2} q_i^2 + \frac{1}{2} \varepsilon_i^2 + \frac{\gamma_i}{2\rho_i} \theta_i^2 \right)
 \end{aligned} \tag{79}$$

Notice that

$$\frac{\vartheta_i}{\mu_i} \tilde{W}_i^T \hat{W}_i \leq -\frac{\vartheta_i}{2\mu_i} \tilde{W}_i^T \tilde{W}_i + \frac{\vartheta_i}{2\mu_i} W_i^T W_i \tag{80}$$

Taking (79) and (80) into account gives

$$\begin{aligned} \dot{V}_0 \leq & w^T \left(PA_0 + A_0^T P + 2P + 2\varepsilon_0 + \sigma P + PE + E^T P \right) w \\ & + \frac{n\varepsilon^2}{\sigma} \|P\| + \|P\| \sum_{i=1}^n \left(h_i^2 + d_i^2 \right) w^T w \\ & + \frac{1}{r_n} [\tilde{g}_n(t)N(\Phi) + 1]\Phi - \sum_{i=1}^n c_i s_i^2 - \sum_{i=1}^n \frac{\gamma_i}{2\rho_i} \tilde{\theta}_i^2 \\ & + \sum_{i=1}^n \left(\frac{1}{2} \beta_i w_1^2 + \frac{1}{2} q_i^2 + \frac{1}{2} \varepsilon_i^2 + \frac{\gamma_i}{2\rho_i} \theta_i^2 + \frac{\vartheta_i}{2\mu_i} W_i^T W_i \right) \\ & + \sum_{i=1}^n \frac{1}{2\mu_i} \left(\frac{2\mu_i a_i}{\sigma} \|p\| - \vartheta_i \right) \tilde{W}_i^T \tilde{W}_i \end{aligned} \tag{81}$$

For a given constant σ , a symmetric and positive matrix variable P must exist in order for the error dynamics to be stable. This matrix inequality is

$$\sigma P + PE + E^T P < 0 \tag{82}$$

This matrix inequality is challenging to resolve directly since E is a function matrix. Another aspect, Assumption 1 suggests that a function $\mu_{ij}(t)$ ($0 \leq \mu_{ij} \leq 1$) exists, such that

$$\frac{\partial f_i}{\partial x_j} = \mu_{ij} b_{ij} + (1 - \mu_{ij}) \bar{b}_{ij}$$

Consequently, matrix E can be written as

$$E = \sum_{1 \leq i \leq n, 2 \leq j \leq n} \left(\mu_{ij} b_{ij} + (1 - \mu_{ij}) \bar{b}_{ij} \right) I_{ij} \tag{83}$$

where I_{ij} is a square matrix of n -order. Furthermore, according to a convex combination theory, the solvability of (81) is related to that of a collection of matrix inequalities with the following type of time-invariant

$$\sigma P + PC + C^T P < 0 \tag{84}$$

where the set $\Omega = \{C | [C]_{ij} \text{ is } b_{ij} \text{ or } \bar{b}_{ij}; 1 \leq i \leq n, 2 \leq j \leq n\}$ is a member of the known matrix C .

It is implied by inequality (78) that there is a constant $\Delta > 0$ such that

$$PA_0 + A_0^T P + 2P + 2\varepsilon_0 I + \|P\| \sum_{i=1}^n \left(h_i^2 + d_i^2 \right) I < -\Delta I \tag{85}$$

which implies that

$$\begin{aligned} & w^T \left(PA_0 + A_0^T P + 2p + \sigma P + PE + E^T P \right) w + 2\varepsilon_0 I \\ & + \|P\| \sum_{i=1}^n \left(h_i^2 + d_i^2 \right) w^T w \\ & \leq -\Delta w^T w \\ & = -\frac{\Delta}{\omega_H(P)} \omega_H(P) w^T w \\ & \leq -\frac{\Delta}{\omega_H(P)} w^T P w \end{aligned} \tag{86}$$

where $\omega_H(P)$ denotes the maximum eigenvalue of matrix P .

Furthermore, choose the design parameters $\lambda_i, \mu_i, a_i, \sigma$ and ϑ_i , so that

$$\sum_{i=1}^n \frac{1}{2\mu_i} \left(\frac{2\mu_i a_i}{\sigma} \|P\| - \vartheta_i \right) \tilde{W}_i^T \tilde{W}_i = Y_0.$$

Define

$$\begin{aligned} \Pi &= \min \left\{ -\frac{\Delta}{\omega_H(P)} w^T w, c_i, \rho_i, \gamma_i, Y_0 \mid 1 \leq i \leq n \right\} \\ \aleph &= \sum_{i=1}^n \left(\frac{1}{2} \beta_i w_1^2 + \frac{1}{2} q_i^2 + \frac{1}{2} \varepsilon_i^2 + \frac{\gamma_i}{2\rho_i} \theta_i^2 + \frac{\vartheta_i}{2\mu_i} W_i^T W_i \right) \end{aligned}$$

Then, it follows from (81) that

$$\dot{V}_0 \leq -\Pi V_0 + \frac{1}{r_n} [\tilde{g}_n(t) N(\Phi) + 1] \Phi + \aleph$$

which implies that all the closed-loop signals are bounded.

It can be seen from $s_i = z_i - \eta_i$ that if the convergence of the η_i is guaranteed, the error s_i is actually stable. In the following, we will prove that η_i is bounded.

Construct the following Lyapunov function

$$V_\eta = \sum_{i=1}^n \frac{1}{2} \eta_i^2 \tag{87}$$

Derivative of V_η gives

$$\begin{aligned} \dot{V}_\eta &= -c_1 \eta_1^2 - \frac{1}{2} \eta_1^2 + \eta_1 (s_2 + x_{2c}) - \alpha_1 \eta_1 + \dots \\ &\quad - c_i \eta_i^2 - \frac{1}{2} \eta_i^2 - \eta_{i-1} \eta_i + \eta_i (s_{i+1} + x_{(i+1)c}) \\ &\quad - \alpha_i \eta_i + \dots - c_n \eta_n^2 - \frac{1}{2} \eta_n^2 - \eta_{n-1} \eta_n \\ &= -\sum_{i=1}^n c_i \eta_i^2 + \sum_{i=1}^n s_{i+1} \eta_i \\ &\quad + \sum_{i=1}^{n-1} (x_{(i+1)c} - \alpha_i - \frac{1}{2}) \eta_i \end{aligned} \tag{88}$$

According to Lemma 5, the following inequality is satisfied with choosing suitable parameter satisfying $o_{i+1} > \frac{1}{2}$

$$\begin{aligned} \dot{V}_\eta &\leq -\sum_{i=1}^n c_i \eta_i^2 + \sum_{i=1}^{n-1} (o_{i+1} - \frac{1}{2} + s_{i+1}) \eta_i \\ &\leq -c_m V_{n+1} + \Theta V_{n+1}^{\frac{1}{2}} \end{aligned} \tag{89}$$

where $c_m = 2 \min c_i, \Theta = \sqrt{2} \min \{ (o_{i+1} - \frac{1}{2} + s_{i+1}) \}$. According to Lemma 1, it is proved that η_i is convergent. Since $s_i = z_i - \eta_i$, the errors s_i are stable.

The proof of theorem is thus completed. \square

4. Simulation Results

One example is presented to illustrate the viability of the suggested method in this section.

Think about the following numerical example of a nonlinear system, where the system dynamics is described as

$$\begin{aligned} \dot{x}_1 &= x_2 - 100x_1x_2^2 + b_1(t - \tau_1) \\ \dot{x}_2 &= (0.01 \cos(t) - 5)u + 17x_1^2x_2 + b_2(t - \tau_2) \\ y &= x_1 \end{aligned}$$

To consider the effect of delays, the time-varying delays are given as $b_1(t - \tau_1) = 5(4t + 10)$ and $b_2(t - \tau_2) = 0.002(9t - 50)$. The observer parameters are taken as $l_1 = -3.5$ and $l_2 = 0.6$.

Choose the initial values and adaptive parameters $x_1 = 0.2, x_2 = 0, \hat{x}_1 = 0.1, \hat{x}_2 = 0.01, \varphi_2 = 0.1, \eta_1 = 0.8, \eta_2 = 0.8, \Phi = 1.5, \hat{\theta}_1 = 0.5, c_1 = 3, c_2 = 0.5, q_1 = 1, q_2 = 8, \gamma_1 = 1, \gamma_2 = 0.5, r_2 = 0.5, \omega_2 = 0.5, \rho_1 = 2, \rho_2 = 1, \hat{g}_2 = 4$.

Figures 2–5 show the simulation results. Figure 2 shows the correlation curves between the state variable x_1 and its corresponding estimate. The reaction of the actual controller u is shown in Figure 3. The paths of the adaptive laws $\hat{\theta}_1, \hat{\theta}_2, \hat{W}_1$ and \hat{W}_2 are depicted in Figures 4 and 5, respectively. According to the different points of the results of this paper and others, such as in the design of the controller while considering the observer, time-delay and unknown control gain brought under the influence of the designed controller and adaptive law is more stable. These figures demonstrate that the simulation results support the effectiveness of the suggested control strategy.

For the observer-based time-delayed system stabilization problem, the initials and estimated values of the system can be observed in the images to reach stability after 5 s, while in this paper it can be clearly seen that the images have stabilized before 5 s.

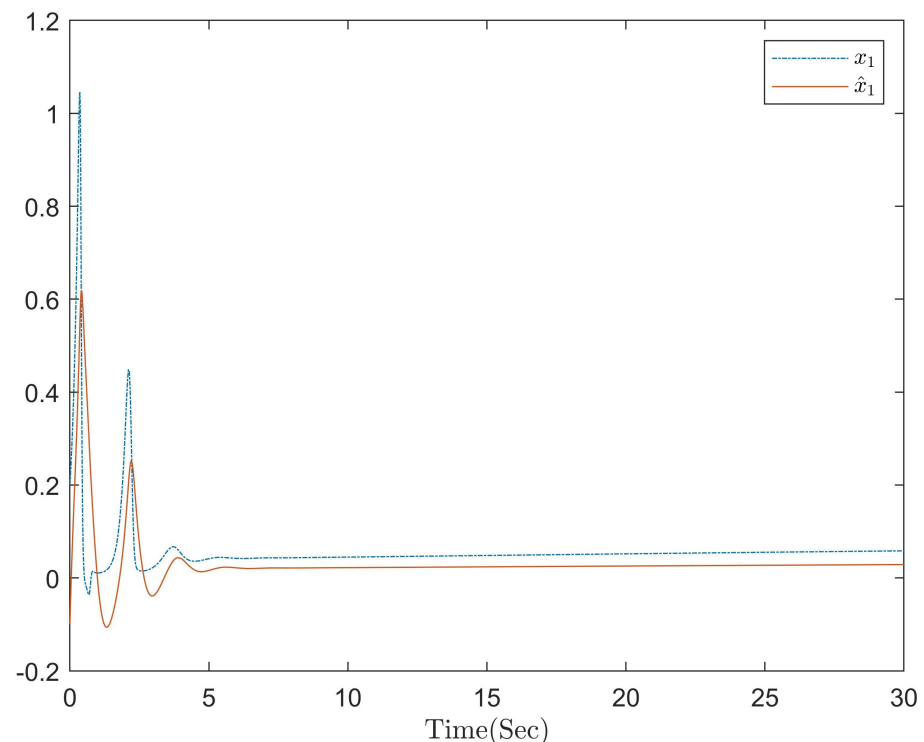


Figure 2. Paths of state variables x_1 and \hat{x}_1 .

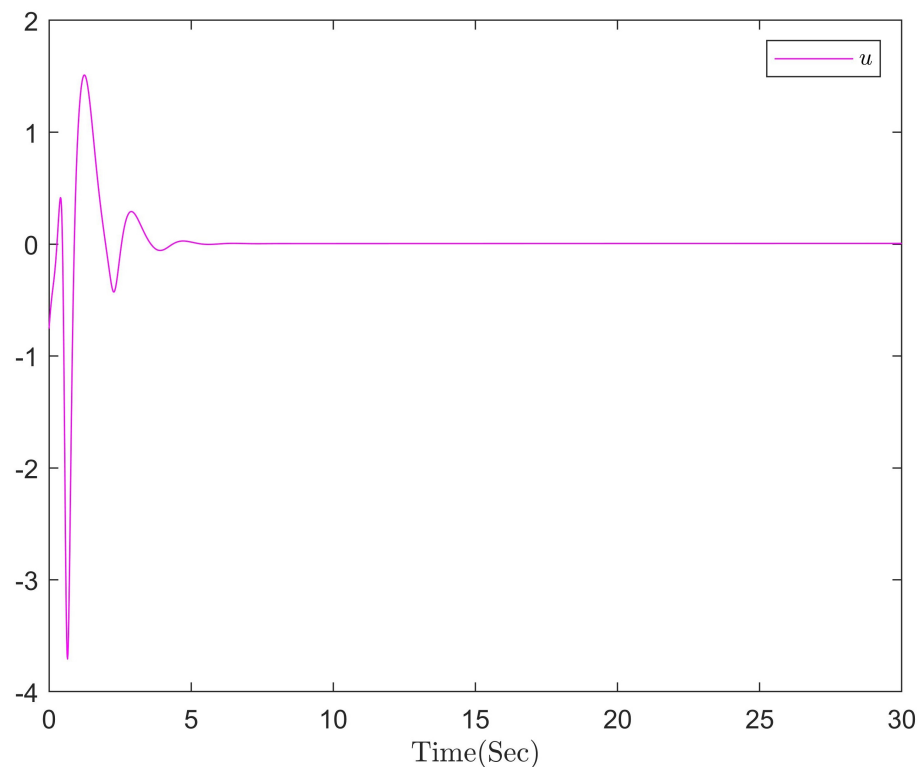


Figure 3. Curve of the controller u .

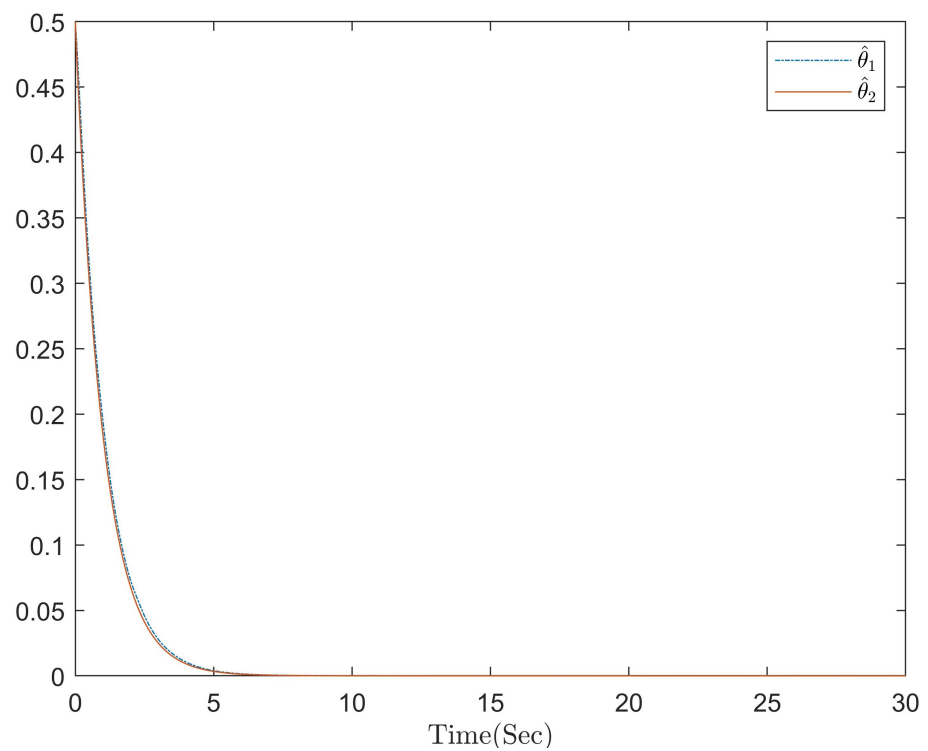


Figure 4. Curves of adaptive parameters $\hat{\theta}_1$ and $\hat{\theta}_2$.

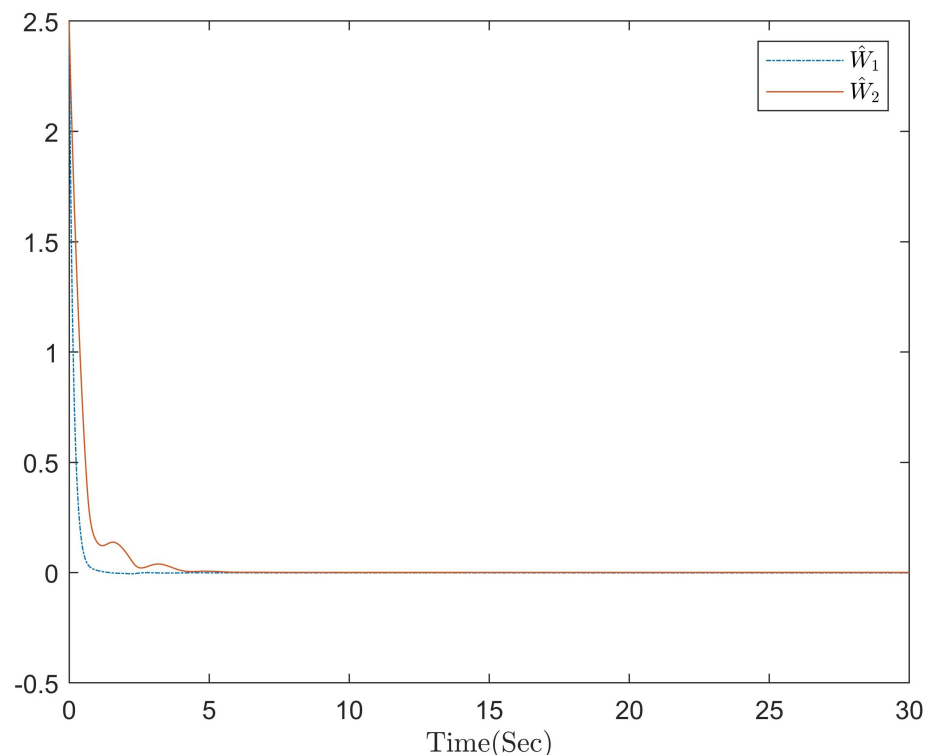


Figure 5. Curves of adaptive parameters \hat{W}_1 and \hat{W}_2 .

5. Conclusions

In this paper, a class of adaptive stabilization control problem for nonlinear time-delay systems with unknown control gain is considered. In order to estimate the unmeasurable state variables, a state observer is established. An adaptive stabilization controller based on an ESN observer is proposed by combining the command filtering technique, ESN, and Lyapunov–Krasovskii functionals. On this basis, the unknown function is approximated by an ESN, and the unknown control gain is addressed with the Nussbaum function. Then, the simulation results prove the effectiveness of the designed scheme.

Author Contributions: Conceptualization, X.L.; Methodology, S.L.; Software, S.L. and Z.L.; Validation, X.L.; Formal analysis, M.L.; Investigation, Z.L. and M.L.; Writing—original draft, Z.L.; Writing—review & editing, S.L. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Data Availability Statement: Not applicable.

Conflicts of Interest: The authors declare no conflict of interest.

Abbreviations

The following abbreviations are used in this paper:

ESN	Echo state network
RBFNN	Radial basis function neural network
RNNs	Recurrent neural network
DSC	Dynamic surface control

References

1. Bekiaris-Liberis, N.; Krstic, M. Stability of predictor-based feedback for nonlinear systems with distributed input delay. *Automatica* **2016**, *70*, 195–203. [[CrossRef](#)]
2. Li, H.; Wang, L.; Du, H.; Boulkroune, A. Adaptive fuzzy backstepping tracking control for strict-feedback systems with input delay. *IEEE Trans. Fuzzy Syst.* **2017**, *25*, 642–652. [[CrossRef](#)]

3. Chen, W.S.; Wu, J.; Jiao, L.C. State-feedback stabilization for a class of stochastic time-delay nonlinear systems. *Int. J. Robust Nonlinear Control*. **2012**, *22*, 1921–1937. [[CrossRef](#)]
4. Yin, S.; Shi, P.; Yang, H.Y. Adaptive tracking control for nonlinear time-varying delay systems with full state constraints and unknown control coefficients. *Automatica* **2018**, *93*, 444–453.
5. Li, F.B.; Shi, P.; Wu, L.G.; Zhang, X. Fuzzy-model-based D-stability and nonfragile control for discrete-time descriptor systems with multiple delays. *IEEE Trans. Fuzzy Syst.* **2014**, *22*, 1019–1025. [[CrossRef](#)]
6. Leu, Y.G.; Lee, T.T.; Wang, W.Y. Observer-based adaptive fuzzy- neural control for unknown nonlinear dynamical systems. *IEEE Trans. Syst. Man Cybern.* **1999**, *29*, 583–591.
7. Hua, C.; Guan, X.P.; Shi, P. Robust Output Feedback Tracking Control for Time-Delay Nonlinear Systems Using Neural Network. *IEEE Trans. Neural Netw.* **2007**, *18*, 495–505. [[CrossRef](#)]
8. Liu, Y.J.; Tong, S.C.; Wang, D.; Li, T.S.; Chen, C.L.P. Adaptive neural output feedback design with reduced-order observer for a class of uncertain nonlinear SISO systems. *IEEE Trans. Neural Netw.* **2011**, *22*, 1328–1334.
9. Wang, T.; Zhang, Y.F.; Qiu, J.B.; Gao, H.J. Adaptive fuzzy backstepping control for a class of nonlinear systems with sampled and delayed measurements. *IEEE Trans. Fuzzy Syst.* **2015**, *23*, 302–312. [[CrossRef](#)]
10. Hua, C.C.; Guan, X.P. Output feedback stabilization for time-delay nonlinear interconnected systems using neural network. *IEEE Trans. Neural Netw.* **2008**, *19*, 673–688.
11. Sakthivel, R.; Manickavalli, S.; Parivallal, A. Observer-based bipartite consensus for uncertain Markovian-jumping multi-agent systems with actuator saturation. *Eur. J. Control.* **2021**, *61*, 13–23. [[CrossRef](#)]
12. Nussbaum, R.D. Some remarks on a conjecture in parameter adaptive control. *Syst. Control. Lett.* **1983**, *3*, 243–246. [[CrossRef](#)]
13. Ge, S.S.; Hong, F.; Lee, T.H. Adaptive neural control of nonlinear time-delay systems with unknown virtual control coefficients. *IEEE Trans. Syst.* **2004**, *34*, 499–516. [[CrossRef](#)]
14. Wang, Y.C.; Zhang, H.G.; Wang, Y.Z. Fuzzy adaptive control of stochastic nonlinear systems with unknown virtual control gain function. *ACTA Autom. Sin.* **2006**, *32*, 170–178.
15. Yu, Z.X.; Li, S.G.; Du, H.B. Razumikhin Nussbaum lemma based adaptive neural control for uncertain stochastic pure-feedback nonlinear systems with time-varying delays. *Int. J. Robust Nonlinear Control* **2013**, *23*, 1214–1239. [[CrossRef](#)]
16. Shi, W.X. Observer-based fuzzy adaptive control for multi-input multi-output nonlinear systems with a nonsymmetric control gain matrix and unknown control direction. *Fuzzy Sets Syst.* **2015**, *263*, 1–26. [[CrossRef](#)]
17. Emran, B.J.; Najjaran, H. Global tracking control of quadrotor based on adaptive dynamic surface control. *Int. J. Dyn. Control* **2021**, *9*, 240–256. [[CrossRef](#)]
18. Yu, J.P.; Shi, P.; Dong, W.J.; Lin, C. Adaptive Fuzzy Control of Nonlinear Systems With Unknown Dead Zones Based on Command Filtering. *IEEE Trans. Fuzzy Syst.* **2018**, *26*, 46–55. [[CrossRef](#)]
19. Sun, W.; Wu, Y.Q.; Sun, Z.Y. Command filter-based finite-time adaptive fuzzy control for uncertain nonlinear systems with prescribed performance. *IEEE Trans. Fuzzy Syst.* **2020**, *28*, 3161–3170. [[CrossRef](#)]
20. Lin, Z.; Liu, Z.; Zhang, Y.; Chen, C.L.P. Command filtered neural control of multi-agent systems with input quantization and unknown control direction. *Neurocomputing* **2021**, *430*, 47–57. [[CrossRef](#)]
21. Xu, D.M.; Lan, J.; Principe, J.C. Direct adaptive control: An echo state network and genetic algorithm approach. In Proceedings of the 2005 IEEE International Joint Conference on Neural Networks, Montreal, QC, Canada, 31 July–4 August 2005.
22. Hart, A.; Hook, J.; Dawes, J. Embedding and approximation theorems for echo state networks. *Neural Netw.* **2020**, *128*, 234–247. [[CrossRef](#)]
23. Chen, Q.; Shi, L.L.; Na, J. Adaptive echo state network control for a class of pure-feedback systems with input and output constraints. *Neurocomputing* **2018**, *274*, 1370–1382. [[CrossRef](#)]
24. Chen, B.; Lin, C.; Liu, X.P.; Liu, K.F. Observer-Based Adaptive Fuzzy Control for a Class of Nonlinear Delayed Systems. *IEEE Trans. Syst.* **2016**, *46*, 27–36. [[CrossRef](#)]
25. Chen, B.; Zhang, H.G.; Liu, X.P.; Lin, C. Neural Observer and Adaptive Neural Control Design for a Class of Nonlinear Systems. *IEEE Trans. Neural Netw. Learn. Syst.* **2018**, *29*, 4261–4271. [[CrossRef](#)]
26. Li, B.M.; Xia, J.W.; Sun, W.; Park, J.H.; Sun, Z.Y. Command filter-based event-triggered adaptive neural network control for uncertain nonlinear time-delay systems. *Int. J. Robust Nonlinear Control*. **2020**, *30*, 6363–6382. [[CrossRef](#)]
27. Levant, A. Higher-order sliding modes, differentiation and output-feedback control. *Int. J. Control* **2010**, *76*, 924–941. [[CrossRef](#)]

Disclaimer/Publisher’s Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.