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Article

SOS Approach for Practical Stabilization of Tempered Fractional-Order Power System

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Abstract: Fractional systems have been widely utilized in various fields, such as mathematics, physics and finance, providing a versatile framework for precise measurements and calculations involving partial quantities. This paper aims to develop a novel polynomial controller for a power system (PS) with fractional-order (FO) dynamics. It begins by studying the practical stability of a general class of tempered fractional-order (TFO) nonlinear systems, with broad applicability and potential for expanding its applications. Afterward, a polynomial controller is designed to guarantee the practical stability of the PS, encompassing the standard constant controller as a specific instance. The design conditions for this controller are resolved using the sum of squares (SOS) approach, a powerful technique for guaranteeing stability and control design. To showcase the practical value of the analytical findings, simulations of the PS are conducted utilizing SOSTOOLS.

Keywords: fractional calculus; Lyapunov theory; numerical solutions; SOSTOOLS; practical stability

MSC: 26A33; 34A08

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1. Introduction

In fractional calculus, non-integer order integrals and differential operators are studied, as well as their applications. Fractional-order models are more accurate and efficient than classical models, and non-integer derivative operators are essential for describing physical events, for example, [1–4]. There has been considerable research on the benefits of fractional calculus in a wide range of fields, including applied mathematics, biology, mechanics, finance, engineering and control theory [5–17].

Stability is one of the most important concepts in control theory. The stability of fractional differential equations (FDEs) has become a hot topic in mathematics and related fields due to the rapid development of fractional calculus. For FDEs, most works on stability are aimed at trivial solutions or equilibrium points [18–23]. On the one hand, Ben Makhlouf et al. [24] introduced the idea of practical stability for FDEs, which is an intrinsic alternative to Lyapunov's original concept of stability, where the origin is not an equilibrium point. In terms of the convergence of solutions to a small ball, practical stability refers to the stability of the systems in a small neighbourhood of the origin [25–27].

On the other hand, due to the growing energy demand, researchers have shown considerable interest in investigating control problems in the field of power systems (PSs). Numerous methodologies have been suggested to generate contemporary outcomes in this particular field. The majority of the studies focus on PSs with integer-order characteristics. For example, a PID controller is applied in [28] for load frequency regulation in PSs. The Takagi Sugeno (TS) method is adopted in [29] to design a fuzzy controller for a single machine infinite bus that is impacted by fault perturbation. It is worth mentioning that all the previously cited results are presented in the form of linear matrix inequalities (LMIs), which can be efficiently addressed through the utilization of the LMI toolbox.

As fractional-order (FO) models have gained significant popularity, numerous researchers have recognized their ability to provide a clearer representation of physical phenomena. For example, Yu et al. in [30] illustrate that the FO model of a PS can accurately present the phenomenon of chaos compared to the integer-order model. However, there have been minimal advancements in the literature regarding the stability and stabilization of fractional-order power systems (FOPSs). For example, the authors of [31] investigate the control design problem of time delay FOPSs.

In 2002, SOSTOOLS v1.00, a tool for polynomial convex optimization, appeared as an alternative to the standard LMI toolbox. The main advantage of this tool compared to the LMI Toolbox is that it allows users to solve polynomial LMI problems, which are quite general and revert to the standard LMI problem when all polynomials are limited to a constant. Recently, the attention of many researchers has been paid to sum of squares (SOS) approach in control applications, e.g., fault tolerant control [32], observer-based control [33] and tracking control [34].

To the best of the authors' knowledge, the utilization of the SOS approach for resolving the stabilization problem of a FOPS remains an unresolved matter.

The following is a list of the advantages and innovations of this paper:

- The TFO derivative is considered, which is more general than the Caputo fractional derivative.
- The practical stability of TFO nonlinear systems is investigated.
- Compared to works [30,31], the proposed method allows us to eliminate a cross-product term that arises in the derivative of the Lyapunov functional.
- Unlike the standard feedback controller, the gain $\mathcal{N}(\xi)$ is not constant; however, it instead follows a polynomial function. Consequently, it allows for greater flexibility in ensuring practical stability.

The subsequent sections of this paper are structured as follows. In Section 2, preliminaries of the TFO and SOS approaches are presented. In Section 3, new results on the practical stability of a TFO nonlinear system is proposed. Based on this result and the SOS approach, a polynomial feedback controller is designed, in Section 4, to guarantee the practical stability of a FOPS.

2. Preliminaries

In this section, some basics results on tempered fractional integrals (FIs) and TFOs are presented.

Definition 1 ([23])**.** *Let $\varpi > 0, \iota \geq 0$ and $u \in C([a,b],\mathbb{R})$. The tempered FI of order ϖ of u is defined by*

$$I_{a^+}^{\varpi,\iota}u(\varsigma) = \frac{1}{\Gamma(\varpi)}\int_a^{\varsigma}(\varsigma-l)^{\varpi-1}\exp\big(-\iota(\varsigma-l)\big)u(l)dl,$$

where $I_{a^+}^{\varpi}$ is the Riemann–Liouville FI of order ϖ.

Definition 2 ([23])**.** *Let $\kappa \in \mathbb{N}, \varsigma-1 < \varpi < \varsigma, \iota \geq 0$ and $u \in AC^{\varsigma}[a,b]$. The TFO of order ϖ of $u(t)$ is defined by*

$$^{C}D_{a^+}^{\varpi,\iota}u(\varsigma) = \frac{\exp(-\iota\varsigma)}{\Gamma(\kappa-\varpi)}\int_a^{\varsigma}(\varsigma-l)^{\kappa-\varpi-1}u_{\iota}^{[\kappa]}(l)dl, \tag{1}$$

where

$$u_{\iota}^{[\kappa]}(\varsigma)=\left[\frac{d}{d\varsigma}\right]^{\kappa}\Big(\exp(\iota\varsigma)u(\varsigma)\Big). \tag{2}$$

For the case when $0<\varpi<1$, then the TFO of order ϖ for an absolutely continuous function u becomes:

$${}^{C}D_{a^+}^{\varpi,\iota}u(\varsigma)=\frac{\exp(-\iota\varsigma)}{\Gamma(1-\varpi)}\int_a^{\varsigma}(\varsigma-l)^{-\varpi}\frac{d}{dl}(\exp(\iota l)u(l))dl. \tag{3}$$

Definition 3 ([35])**.** *The Mittag–Leffler function with two parameters is defined as*

$$E_{d_1,d_2}(l)=\sum_{\lambda=0}^{+\infty}\frac{l^{\lambda}}{\Gamma(\lambda d_1+d_2)},$$

where $d_1>0$, $d_2>0$, $l\in\mathbb{C}$.
When $d_2=1$, we have $E_{d_1}(l)=E_{d_1,1}(l)$.

Lemma 1 ([23])**.** *Let $0<\varpi<1$, $P\in\mathbb{R}^{n\times n}$ be a constant, symmetric, definite positive matrix and $x(\varsigma)\in\mathbb{R}^n$ be an absolutely continuous function, then*

$${}^{C}D_{a^+}^{\varpi,\iota}x^TPx(\varsigma)\leq 2x(\varsigma)^TP\,{}^{C}D_{a^+}^{\varpi,\iota}x(\varsigma). \tag{4}$$

Lemma 2 ([23])**.** *Let $0<\varpi<1$. The solution of the following system*

$${}^{C}D_{\varsigma_0^+}^{\varpi,\iota}u(\varsigma)=ru+c(\varsigma), \tag{5}$$

where $u\in\mathbb{R}^n$ is given by

$$\begin{aligned} u(\varsigma) &= \exp\big(-\iota(\varsigma-\varsigma_0)\big)E_{\varpi}\big(r(\varsigma-\varsigma_0)^{\varpi}\big)u(\varsigma_0) \\ &+ \int_{\varsigma_0}^{\varsigma}(\varsigma-l)^{\varpi-1}E_{\varpi,\varpi}\big(r(\varsigma-l)^{\varpi}\big)\exp\big(-\iota(\varsigma-l)\big)c(l)dl. \end{aligned} \tag{6}$$

Definition 4 ([36])**.** *Consider $v(y)=v(y_1,y_2,\ldots,y_r)$ (in which $y\in\mathbb{R}^r$) as a polynomial. $v(y)$ is an SOS if there exist polynomials $q_1(y),q_2(y),\ldots,$ and $q_g(y)$ such that*

$$v(y)=\sum_{j=1}^{g}q_j^2(y). \tag{7}$$

In the rest, Π_{SOS} denotes the set of SOSs.
It is clear that $v(y)\in\Pi_{SOS}$ implies that $v(y)\geq 0,\ \ \forall y\in\mathbb{R}^r$.

Lemma 3 ([36])**.** *Consider $\mathcal{H}(y)$ as a $l\times l$ symmetric polynomial matrix, a vector $\alpha\in\mathbb{R}^r$ which does not depend on y and a known positive polynomial $\varphi(y)$, then*

$$-\alpha^T\Big(\mathcal{H}(y)+\varphi(y)\Big)\alpha\in\Pi_{SOS} \tag{8}$$

implies that

$$\mathcal{H}(y)<0. \tag{9}$$

Lemma 4 ([37])**.** *The inequality mentioned below holds for any scalar $\epsilon>0$ and matrices K and L with suitable dimensions*

$$K^TL+L^TK\leq\epsilon K^TK+\epsilon^{-1}L^TL. \tag{10}$$

3. Practical Stability of Tempered Fractional-Order Nonlinear Systems

Let us examine the fractional-order system presented below

$$\begin{aligned} {}^{C}D^{\varpi,\iota}_{\varsigma_0}x(\varsigma) &= F(\varsigma,x),\ \varsigma \geq \varsigma_0, \\ x(\varsigma_0) = x_0, \end{aligned} \tag{11}$$

where $\varsigma_0 \in \mathbb{R}_+$ and $F(.,.) \in C\big(\mathbb{R}_+ \times \mathbb{R}^n, \mathbb{R}^n\big)$.

Definition 5. *The system (11) is called generalized practical Mittag-Leffler stable (PMLS) if the following estimation of the solutions of the system (11) is satisfied:*

$$\|x(\varsigma;\varsigma_0,x_0)\| \leq c_1\|x_0\| \exp\big(-c_2(\varsigma-\varsigma_0)\big)\Big(E_\varpi\big(-c_3(\varsigma-\varsigma_0)^\varpi\big)\Big)^{c_4} + r,\ \forall\varsigma \geq \varsigma_0 \geq 0, \tag{12}$$

where $c_i > 0,\ i = 1,\dots,4$ and $r \geq 0$.

Remark 1. *When $r = 0$, system (11) is said to be generalized Mittag-Leffler stable (MLS) (see [23]).*

Theorem 1. *Suppose that there is a C^1 function $V : \mathbb{R}_+ \times \mathbb{R}^n \longrightarrow \mathbb{R}$, such that*

1.

$$\delta_1\|x\|^2 \leq V(\varsigma,x) \leq \delta_2\|x\|^2,\ \forall x \in \mathbb{R}^n,\ \varsigma \geq \varsigma_0. \tag{13}$$

2.

$${}^{C}D^{\varpi,\iota}_{\varsigma_0}V(\varsigma,x(\varsigma;\varsigma_0,x_0)) \leq -\delta_3\|x(\varsigma;\varsigma_0,x_0)\|^2 + \varphi(\varsigma),\ \forall\varsigma \geq \varsigma_0 \geq 0, \tag{14}$$

where $\delta_1,\ \delta_2,\ \delta_3 > 0$ and φ is continuous positive function that satisfies:

$$\sup_{\varsigma\geq 0}\int_0^\varsigma (\varsigma-l)^{\varpi-1}\exp\big(-\iota(\varsigma-l)\big)E_{\alpha,\alpha}\Big(-\frac{\delta_3}{\delta_2}(\varsigma-l)^\varpi\Big)\varphi(l)dl \leq M$$

where $M \geq 0$.
Then, the system (11) is PMLS.

Proof. From inequalities (13) and (14) we get:

$${}^{C}D^{\varpi,\iota}_{\varsigma_0}V(\varsigma,x(\varsigma;\varsigma_0,x_0)) \leq -\frac{\delta_3}{\delta_2}V(\varsigma,x(\varsigma;\varsigma_0,x_0)) + h(\varsigma). \tag{15}$$

Let $h(\varsigma) = {}^{C}D^{\varpi,\iota}_{a^+}V(\varsigma,x(\varsigma;\varsigma_0,x_0)) + \frac{\delta_3}{\delta_2}V(\varsigma,x(\varsigma;\varsigma_0,x_0))$, then we get from Lemma 2

$$\begin{aligned} V(\varsigma,x(\varsigma;\varsigma_0,x_0)) &= \exp\big(-\iota(\varsigma-\varsigma_0)\big)E_\varpi\big(-\frac{\delta_3}{\delta_2}(\varsigma-\varsigma_0)^\varpi\big)V(\varsigma_0,x_0) \\ &\quad + \int_{\varsigma_0}^\varsigma (\varsigma-s)^{\varpi-1}E_{\varpi,\varpi}\big(-\frac{\delta_3}{\delta_2}(\varsigma-s)^\varpi\big)\exp\big(-\iota(\varsigma-s)\big)h(s)ds \\ &\leq \exp\big(-\iota(\varsigma-\varsigma_0)\big)E_\varpi\big(-\frac{\delta_3}{\delta_2}(\varsigma-\varsigma_0)^\varpi\big)V(\varsigma_0,x_0) \\ &\quad + \int_{\varsigma_0}^\varsigma (\varsigma-s)^{\varpi-1}E_{\varpi,\varpi}\big(-\frac{\delta_3}{\delta_2}(\varsigma-s)^\varpi\big)\exp\big(-\iota(\varsigma-s)\big)\varphi(s)ds \\ &\leq \exp\big(-\iota(\varsigma-\varsigma_0)\big)E_\varpi\big(-\frac{\delta_3}{\delta_2}(\varsigma-\varsigma_0)^\varpi\big)V(\varsigma_0,x_0) + M \end{aligned} \tag{16}$$

for every $\varsigma \geq \varsigma_0$.
It follows from inequality (13) that

$$\|x(\varsigma; \varsigma_0, x_0)\| \leq \sqrt{\frac{\delta_2}{\delta_1}} \exp\Big(-\frac{\iota}{2}(\varsigma-\varsigma_0)\Big)\Big(E_{\varpi}\Big(-\frac{\delta_3}{\delta_2}(\varsigma-\varsigma_0)^{\varpi}\Big)\Big)^{\frac{1}{2}}\|x_0\| + \sqrt{M},$$

for every $\varsigma \geq \varsigma_0$.

Thus, the system (11) is PMLS. □

Remark 2. *When $\varphi(\varsigma) = 0$, we get the MLS for system (11) (see [23]).*

4. Practical Stabilization for a Class of Power Systems with Load Disturbance

Now, we consider a power system with load disturbance. Based on the definition of TFO of order ϖ, this system is modeled by the following fractional-order nonlinear model [31]:

$$\begin{cases} {}^{C}D^{\varpi,\iota}_{\varsigma_0}\delta(\varsigma) = \omega(\varsigma) \\ {}^{C}D^{\varpi,\iota}_{\varsigma_0}\omega(\varsigma) = -\eta\sin(\delta(\varsigma)) - \gamma\omega(\varsigma) + \rho + \mu\cos(\beta\varsigma) \end{cases} \tag{17}$$

where

$$\eta = \frac{Q_e}{M}, \quad \gamma = \frac{H}{M}, \quad \rho = \frac{Q_m}{M}, \quad \mu = \frac{Q_e}{M}$$

in which Q_e refers to the electrical power, H refers to the damping coefficient, M refers to the inertia time constant, Q_m refers to the mechanical power and Q_e refers to the disturbance power amplitude.

In the following, to keep it concise, we omit the time variable t.

Using the Taylor expansion, $sin(\delta)$ can be approximated by the following polynomial functions [38,39]:

$$\begin{cases} \sin(\delta) \approx \delta & \text{for small angles} \quad \delta \\ \sin(\delta) \approx \delta - \frac{40}{243}\delta^3 & \text{for} \quad |\delta| < \frac{7\pi}{18} = 70 \\ \sin(\delta) \approx \delta - \frac{40}{243}\delta^3 + \frac{1}{131}\delta^5 & \text{for} \quad |\delta| < \frac{2\pi}{3} = 120 \end{cases} \tag{18}$$

The fifth-order approximation is illustrated in Figure 1.

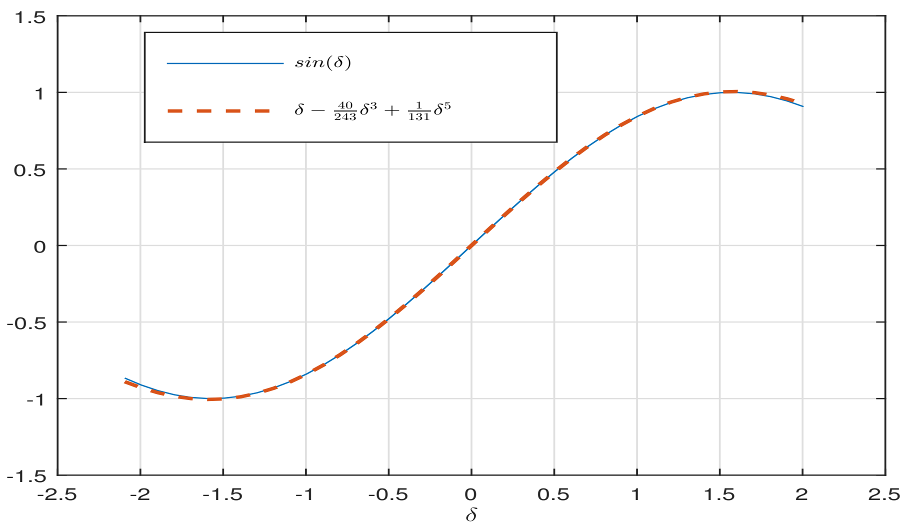

Figure 1. Polynomial approximation to $sin(\delta)$ for $\delta \in [-2\pi/3, 2\pi/3]$.

In this case, the system with feedback gains is given as follows:

$$\begin{cases} {}^{C}D_{\varsigma_0}^{\varpi,\iota}\delta = \omega + \mathcal{N}_{11}(\delta,\omega)\delta + \mathcal{N}_{12}(\delta,\omega)\omega \\ {}^{C}D_{\varsigma_0}^{\varpi,\iota}\omega = -\eta\left(\delta - \frac{40}{243}\delta^3 + \frac{1}{131}\delta^5\right) - \gamma\omega + \rho + \mu\cos(\beta\varsigma) + \mathcal{N}_{21}(\delta,\omega)\delta + \mathcal{N}_{22}(\delta,\omega)\omega \end{cases} \tag{19}$$

Subsequently, the system is expressed in a compact form as follows:

$${}^{C}D_{\varsigma_0}^{\varpi,\iota}\xi = \Big(\mathcal{A}(\xi) + \mathcal{N}(\xi)\Big)\xi + \mathcal{J} \tag{20}$$

where

$$\xi = \begin{bmatrix} \xi_1, & \xi_2 \end{bmatrix}^T = \begin{bmatrix} \delta, & \omega \end{bmatrix}^T, \mathcal{A}(\xi) = \begin{bmatrix} 0 & 1 \\ -\eta\left(1 - \frac{40}{243}\xi_1^2 + \frac{1}{131}\xi_1^4\right) & -\gamma \end{bmatrix},$$
$$\mathcal{N}(\xi) = \begin{bmatrix} \mathcal{N}_{11}(\xi) & \mathcal{N}_{12}(\xi) \\ \mathcal{N}_{21}(\xi) & \mathcal{N}_{22}(\xi) \end{bmatrix}, \mathcal{J} = \begin{bmatrix} 0 \\ \rho + \mu\cos(\beta\varsigma) \end{bmatrix}$$

and $\mathcal{J}$ satisfies $\|\mathcal{J}\| \leq c$, where c is a positive constant.

Theorem 2. *For given positive scalars φ_1, φ_2 and positive polynomial $\varphi_3(\xi)$, the system (20) is PLMS if there exist symmetric matrices $\mathcal{P}$, $\mathcal{R}$ and polynomial matrix $\widehat{\mathcal{N}}(\xi)$ such that the following optimization problem holds:*
Minimize ϵ satisfying the following conditions:

$$\gamma_1^T\Big(\mathcal{P} - \varphi_1 I\Big)\gamma_1 \quad \in \Pi_{SOS}, \tag{21}$$

$$\gamma_1^T\Big(\mathcal{R} - \varphi_2 I\Big)\gamma_1 \quad \in \Pi_{SOS}, \tag{22}$$

$$-\gamma_2^T\Big(\Xi(\xi) + \varphi_3(\xi) I\Big)\gamma_2 \quad \in \Pi_{SOS}, \tag{23}$$

where γ_1 and γ_2 are vectors that are not dependent on ξ,

$$\Xi(\xi) \quad = \quad \begin{bmatrix} \Xi^{11}(y) & \mathcal{P} \\ * & -\epsilon I \end{bmatrix},$$

in which

$$\Xi_{11}(\xi) \quad = \quad \mathcal{P}\mathcal{A}(\xi) + \widehat{\mathcal{N}}(\xi) + \mathcal{A}(\xi) + \widehat{\mathcal{N}}^T(\xi)\mathcal{P} + \mathcal{R}$$

In this case, $\mathcal{N}(\xi) = \mathcal{P}^{-1}\widehat{\mathcal{N}}(\xi)$

Proof. Choose the following polynomial Lyapunov function

$$\mathcal{V}(\xi) = \quad \xi^T\mathcal{P}\xi, \tag{24}$$

Based on Lemma 1, we obtain

$${}^{C}D_{\varsigma_0}^{\varpi,\iota}\mathcal{V}(\xi) \leq \quad 2\,{}^{C}D_{\varsigma_0}^{\varpi,\iota}x^T\mathcal{P}x = \xi^T\Omega(\xi)\xi + \xi^T\mathcal{P}\mathcal{J} + \mathcal{J}^T\mathcal{P}\xi \tag{25}$$

where $\Omega(\xi) = \mathcal{P}\Big(\mathcal{A}(\xi) + \mathcal{N}(\xi)\Big) + \Big(\mathcal{A}(\xi) + \mathcal{N}(\xi)\Big)^T\mathcal{P}$.

By applying Lemma 4, we obtain

$$\xi^T\mathcal{P}\mathcal{J} + \mathcal{J}^T\mathcal{P}\xi \quad \leq \quad \frac{1}{\epsilon}\|\mathcal{P}\xi\|^2 + \epsilon c^2 = \xi^T\Big(\frac{1}{\epsilon}\mathcal{P}\mathcal{P}\Big)\xi + \epsilon c^2 \tag{26}$$

Taking into account the previous inequality, we obtain

$$\dot{\mathcal{V}}(\xi) \leq \quad \xi^T \left(\Omega(\xi) + \tfrac{1}{\epsilon} \mathcal{P}\mathcal{P} \right) \xi + \epsilon c^2 \tag{27}$$

The SOS condition (23) implies that $\Xi(\xi) < 0$ and by applying the Schur complement, we obtain

$$\Omega(\xi) + \frac{1}{\epsilon} \mathcal{P}\mathcal{P} < -\mathcal{R} \tag{28}$$

Then, we obtain

$$\dot{\mathcal{V}}(\xi) \leq -\lambda_{min}(\mathcal{R}) \|\xi\|^2 + \epsilon c^2 \tag{29}$$

It is clear from (22) that $\lambda_{min}(\mathcal{R}) > 0$. Then, according to Theorem 1, the system (20) is PLMS. The proof is complete. □

Remark 3. *In contrast to the conventional feedback controller synthesis for PSs with FO dynamics [30,31], the gain $\mathcal{N}(\xi)$ in this case is not constant but instead varies according to a polynomial function. As a result, this approach offers increased flexibility in achieving practical stability.*

Now, we apply Theorem 2 to PS (17) in which the parameters are selected as in [31]:

$$\eta = 1, \quad \gamma = 0.0052, \quad \rho = 0.03, \quad \beta = 0.026, \quad \mu = 0.5.$$

The polynomial gain of the controller is obtained as follows:

$$\mathcal{N}(\xi) = \begin{bmatrix} \mathcal{N}_{11}(\xi) & \mathcal{N}_{12}(\xi) \\ \mathcal{N}_{21}(\xi) & \mathcal{N}_{22}(\xi) \end{bmatrix}$$

where

$$\begin{aligned}
\mathcal{N}_{11}(\xi) &= -8.123\xi_1^4 - 0.868 \times 10^{-6}\xi_1^3 - 6.819\xi_1^2 + 0.907 \times 10^{-6}\xi_1 - 9.987 \\
\mathcal{N}_{12}(\xi) &= = 0.013\xi_1^4 - 0.162 \times 10^{-4}\xi_1^3 - 0.052\xi_1^2 - 0.264 \times 10^{-7}\xi_1 + 0.392 \times 10^{-2}\xi_1 \\
\mathcal{N}_{21}(\xi) &= = 0.013\xi_1^4 - 0.162 \times 10^{-4}\xi_1^3 - 0.052\xi_1^2 - 0.251 \times 10^{-7}\xi_1 + 0.390 \times 10^{-2}\xi_1 \\
\mathcal{N}_{22}(\xi) &= -8.101\xi_1^4 - 0.972 \times 10^{-5}\xi_1^3 - 6.791\xi_1^2 + 0.126 \times 10^{-6}\xi_1 - 9.967
\end{aligned}$$

The trajectory simulation of the corresponding solution, denoted as $\omega(\varsigma)$ and $\delta(\varsigma)$, for System (17) is presented in Figure 2 by using the software Matlab 7.5.0 (R2007b). To initiate this simulation, the initial conditions are specified as $(\delta(\varsigma_0), \omega(\varsigma_0)) = (0.5, -2)$, where $\varsigma_0 = 0.3$. The simulation employs the parameter values $\varpi = 0.95$ and $\iota = 0.1$. Furthermore, in Figure 3, we demonstrate the trajectory simulation of system (19) using the same parameter values. This simulation serves to illustrate the practical stability of system (19).

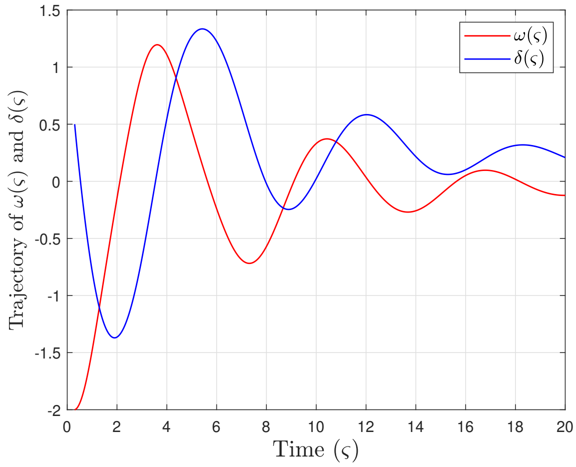

Figure 2. Time evolution of the states $\omega(\varsigma)$ and $\delta(\varsigma)$ of system (17) for $\varsigma \in [0.5, 20]$.

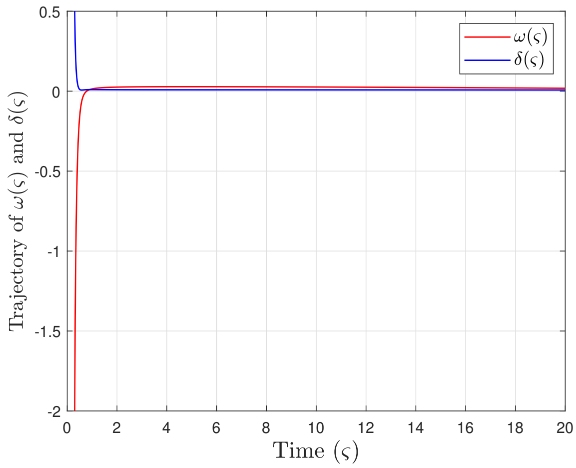

Figure 3. Time evolution of the states $\omega(\varsigma)$ and $\delta(\varsigma)$ of system (19) for $\varsigma \in [0.5, 20]$.

5. Conclusions

In this paper, a polynomial model has been employed to present an SOS approach for the modeling and stabilization of PSs with FO dynamics. First, we have proposed a PMLS stability analysis of a new class of fractional-order systems, specifically employing the TFO model. Building upon this fundamental analysis, the study aims to design a polynomial feedback controller using the SOS approach, ultimately ensuring the practical stabilization of an FOPS. In order to demonstrate the practical significance of the analytical results, simulations on the PS were carried out using SOSTOOLS. By addressing this objective, this study aims to contribute to the advancement of control strategies for fractional-order systems, particularly in the context of power systems.

In future work, we plan to extend these findings by generalizing our results to include tempered fractional-order systems with time delays.

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