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Aspiration-Based Learning in k-Hop Best-Shot Binary Networked Public Goods Games

Ziyi Chen [1], Kaiyan Dai [1], Xing Jin [1,2], Liqin Hu [1,*] and Yongheng Wang [3,*]

[1] School of Cyberspace, Hangzhou Dianzi University, Hangzhou 310018, China; dky@hdu.edu.cn (K.D.)
[2] Experimental Center of Data Science and Intelligent Decision-Making, Hangzhou Dianzi University, Hangzhou 310018, China
[3] Research Center for Big Data Intelligence, Zhejiang Lab, Hangzhou 310000, China
* Correspondence: huliqin@hdu.edu.cn (L.H.); wangyh@zhejianglab.com (Y.W.)

Abstract: In public goods games, it is common for agents to learn strategies from those who possess the highest utility. However, in reality, because of the lack of information, strategies and utilities from others cannot be obtained or predicted during learning and updating. To address this issue, we introduce a learning update mechanism based on aspirations. To make this model more universal, we study goods that can be shared with k-hop neighbors. Additionally, when a free rider accesses an investor, it is required to pay an access cost to him. We investigate the influence of aspiration, shared scope k, and access cost on the social invest level and utility. It is shown that large shared scope k, moderate aspiration, and moderate access cost are conducive to the maximum utilization of social benefits. However, with low aspiration, the utilities of investors are very close and limited, while both the high aspiration and high access cost could disrupt the social stability.

Keywords: public goods games; best-shot; aspiration; k-hop; access cost

MSC: 91A22

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1. Introduction

With the rapid development of the economy, the demand of people for an improved material standard of living is gradually increasing. However, this demand constantly conflicts with limited social resources, such as urban traffic congestion [1] and the reduction of public space [2]. In response, both corporations and governments have implemented investment measures, such as constructing stadiums [3], where individuals who have not made investments can directly benefit by making reservations. Consequently, self-interested participants are motivated to benefit from the investments made by others in order to fulfill their personal fitness needs. These conflicts of interest between individuals and social investment can be modeled as the Public Goods Games (PGG) [4,5]. Field studies as well as experiments attest to the fact that sustainable development and intact social stability in this circumstance is a direction that deserves further exploration and to be discussed [6].

In traditional Public Goods Games (PGG), agents make investment decisions for a public good and distribute the resulting value of the collective efforts equally. As the research in this field continues to expand, and as the number of players in the games increases and the relationship among them becomes complex, conventional PGG cannot meet the simulation requirement, such as the complex relationships in reality. To meet this requirement, multitude of variants and expansions of PGG have been proposed, including Spatial Public Goods Games (SPGG) [7], Network Public Goods Games (NPGG) [8], and Binary Networked Public Goods Games (BNPGG) [9]. Next, to further study the socially efficient amount of these games, Hirshleifer [10] proposed that social composition functions observed in practice may well involve standards of all three rules, i.e., Summation, Weakest-Link, and Best-shot. Compared to other games and rules, Best-shot BNPGG has the advantage of high utility potential and encouraging participants to compete. Additionally, Best-shot BNPGG presents a more challenging cooperation scenario. This is because in this game, if just one person invests in the entire group, others can take free rides and reap the same benefits. Consequently, it is difficult to achieve a high social payoff that encompasses the aggregate payoffs of all agents. Thus, the exploration of methods to enhance the social payoff in Best-shot PGG has attracted considerable interest among researchers.

In many years of research on Best-shot BNPGG, numerous researchers have focused on Nash equilibrium and dynamic evolution. One of the surprising discoveries was the diversity of Nash equilibrium solutions in Best-shot BNPGG. Chowdhury et al. [11,12] identified multiple equilibria and asymmetric equilibria in a group contest. Boncinelli et al. [13] then expanded multiple equilibria by incorporating stochastic stability into it. Among these diverse solutions, some scholars sought the pure strategy equilibrium (PNE) case, which yields the highest return. Komarovsky et al. [14] utilized the potential function to demonstrate that the PNE of the Best-shot NPGG model is Pareto efficient. They proposed side payment as a means to obtain the optimal result with the greatest social benefits. Levit et al. [15] conducted an initial analysis of the balance between effective PNE and stable PNE in homogeneous scenarios, which was subsequently extended to heterogeneous scenarios by Yu et al. [16]. In the aforementioned studies, Nash equilibrium was considered a consequence of static stability requiring complete information, while achieving dynamic stability during dynamic evolution represented another intriguing research area. Roth et al. [17] observed that both prediction behavior and observation behavior rapidly approach perfect equilibrium in Best-shot models by using simple dynamic models. Building upon this, Duffy et al. [18] incorporated the influence of information on learning balancing strategies. In addition to simple dynamic models, evolutionary game models are also utilized to dynamically update strategies in networks. Moreover, Wang et al. [19] explored the dynamic changes of social income and average social investment under various network topologies using evolutionary game theory. Liu et al. [20] demonstrated in evolutionary dynamics that a pure exclusion strategy can induce cooperation among the three rules, Summation, Weakest-Link, and Best-shot. Besides, Cressman et al. [21] utilized evolutionary dynamics to predict the multiple possibilities of rational behavior under specific incentive schemes. Although dynamic gaming is extensively employed in NPGG research, it is rarely applied in the context of best-shot BNPGG.

In the aforementioned studies on the previous Best-shot BNPGG, both Nash equilibrium analysis and dynamic evolution require obtaining the information for strategy updates. In the case of Nash equilibria, each decision maker most likely needs to have access to global information [12–15], as each decision maker's strategy is interdependent. In the case of dynamic evolution, knowing neighbor information is necessary for policy updates. An agent may randomly select a neighbor to gather the information that could potentially aid their learning process, or he may deliberately choose neighbors who have achieved the highest utilities [19,22,23]. However, it is not easy to get information in reality. What is more, individuals make subjective decisions regarding whether they want to disclose personal information for themselves. They may choose to withhold information deliberately for competitive purposes [24] or to safeguard their privacy [25]. Unobtainable information in Best-shot BNPGG remains a necessity to be studied. As a result, in this paper, we introduce an aspiration-based learning mechanism in Best-shot BNPGG when information is unobtainable. In our mechanism, whether agents update their strategies depends on the difference between their own utilities and aspirations. When the utility cannot meet their aspiration, they choose to learn and update towards strategies opposite their current ones with high possibilities. To make our model more universal, we consider goods that can be shared with k-hop neighbors. Additionally, free riders are required to pay access costs to the corresponding investors. Thus, the three key variables in our experiment are aspiration, k-hop, and access costs.

The rest of this article is structured as follows: Section 2 outlines our proposed model according to an aspiration-based learning mechanism. In Section 3, we present the results of our experiment of aspiration, k-hop, and access costs. In Section 4, we provide analyses respectively of the findings. Finally, Section 5 offers a summary of the conclusions drawn from our study, as well as a discussion of the content of future research.

2. Model

This paper investigates the Best-shot BNPGG model with access costs within the shared scope. The incorporation of access costs in our research stems from its alignment with real-life scenarios, where sharing items often entails a fee [26]. A notable example is the rental fee required for accessing shared bicycles. Furthermore, the shared scope, denoted as k [27], defines the range of agents within k hops from an owner. The value of k is directly linked to the network structure under analysis. When seeking to borrow an item, the search for potential lenders extends beyond the immediate neighbors, encompassing individuals who are not in close proximity [28]. For instance, a college may borrow advanced equipment from a distant school when its neighboring institution lacks such resources.

This section provides detailed explanations of key concepts, such as strategy, utility, cluster, updating mechanism, and simulation process.

2.1. Strategy

The game involves n agents playing on an undirected graph $G = (V, E)$, where $V = \{1, 2, \ldots, n\}$ represents the agents set. Each agent i in the set possesses a strategy defined by (x_i, h_i). Here, x_i represents the agent's strategy: $x_i = 1$ indicates investment, and $x_i = 0$ indicates free riding. Additionally, h_i represents the agent chosen by agent i for access, where $h_i = i$ when $x_i = 1$, and $h_i = j$ when $x_i = 0$ ($i \neq j$). Agent j is supposed to be within the shared scope k of agent i.

2.2. Utility

An agent's utility is impacted mainly by its own strategy and its access agent. The selection of different strategies for an agent implies varying utilities. Accordingly, their utilities are discussed in the following three situations:

1. If agent i chooses to invest ($x_i = 1$), it incurs an investment cost of c, but also receives benefits b from public goods and access costs r from agents who choose to access agent i within shared scope k. Generally, the investment cost c should be lower than benefits b [27,29,30]. Additionally, the investment cost c is supposed to be higher than access cost r [31], that is $r \in (0, c)$. Thus, the utility of agent i is:
$$u(x_i) = b + m_i \times r - c \tag{1}$$
where m_i is the number of agents that choose to access i.
2. If agent i decides to take a free ride($x_i = 0$) and accesses another agent h_i who chooses to invest ($x_{h_i} = 1$), then agent i must pay an access cost r to agent h_i. Then he can enjoy the benefits b of the public goods. The utility of agent i is given by the following expression:
$$u(x_i) = b - r \tag{2}$$
3. If agent i decides to take a free ride ($x_i = 0$) and accesses another agent h_i who is also a free rider ($x_{h_i} = 0$), then agent i gains no utility from the public goods. The utility of agent i is given by the following expression:
$$u(x_i) = 0 \tag{3}$$

Therefore, the utility of i can be concluded as:

$$u(x_i, h_i) = \begin{cases} b + m_i \times r - c & , if\ x_i = 1\ and\ h_i = i \\ b - r & , if\ x_i = 0\ and\ x_{h_i} = 1 \\ 0 & , if\ x_i = 0\ and\ x_{h_i} = 0 \end{cases} \tag{4}$$

2.3. Cluster

In our paper, we refer to the investors who are accessed by free riders as central investors. We take this central investor as the center of a cluster. One cluster includes one central investor and other agents successfully access the same central investor (as illustrated in Figure 1). The utility of agents in the cluster can be concluded as follows:

$$\begin{cases} u(x_{I_i}, h_{I_i}) = b - c + (|A_{I_i}| - 1) \times r & , if\ x_{I_i} = 1\ and\ h_{I_i} = I_i \\ u(x_{A_{I_i} - I_i}, h_{A_{I_i} - I_i}) = b - r & , if\ x_{A_{I_i} - I_i} = 0\ and\ h_{A_{I_i} - I_i} = I_i \end{cases} \tag{5}$$

where I_i represents all central investors in the network. A_{I_i} represents each accessed investment cluster, with cluster investment center I_i and other successful free riders. $|A_{I_i}|$ is the number of agents accessing the cluster (including the central investor).

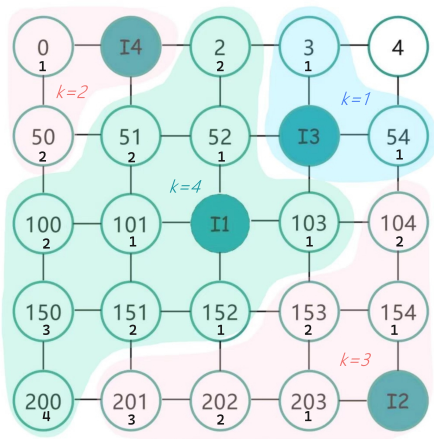


Figure 1. The figure illustrates four green agents, denoted as I_1, I_2, I_3, and I_4, which correspond to central investors. Each agent I_i is the center of a cluster with the same background color, denoted as A_{I_i}. The four clusters have different k-hop scopes. The distance between each agent within a cluster and its respective central investor is indicated below the agent number.

2.4. Updating Mechanism

We use the method of evolutionary games based on aspirations to update the agents' strategies. Because players may engage in hidden or concealed actions, deliberately choosing not to disclose their strategies or behavioral intentions to gain individual advantages, individual agents are unaware of the current strategies of others [24,25]. During the learning update process, each agent compares its own utility with its aspiration and calculates the probability P [32–34] using Equation (6).

$$P(x_i(t) \leftarrow x_{i'}(t)) = \frac{1}{1 + e^{-K(a - u_i)}} \tag{6}$$

where the aspiration level a provides an overall benchmark to evaluate how 'greedy' an agent can be; $x_i(t)$ is the current strategy of agent i; $x_{i'}(t)$ represents the strategy that is the opposite of the current strategy; u_i is the current utility of agent i; K $(K > 0)$ denotes selection intensity in the strategy imitation. In our research, we set K to 1 [19,35,36], which implies that individuals with higher utilities are more inclined to be imitated, while still providing the opportunity for learning from individuals with lower utilities. In particular, a higher value of a indicates that individuals aspire to a higher utility.

Subsequently, a random number p between 0 and 1 is generated. If the calculated probability P exceeds the generated random number p, the agent adjusts its current strategy. For instance, if the agent is previously categorized as a free rider, his strategy shifts to investment. Conversely, if he is categorized as an investment agent before, his strategy turns to free riding, accessing any agent randomly within the k scope range.

2.5. Simulation Process

In this paper, we use the Monte Carlo (MC) method to simulate the process. The pseudo-code is shown in Algorithm 1.

Algorithm 1: Algorithm of model

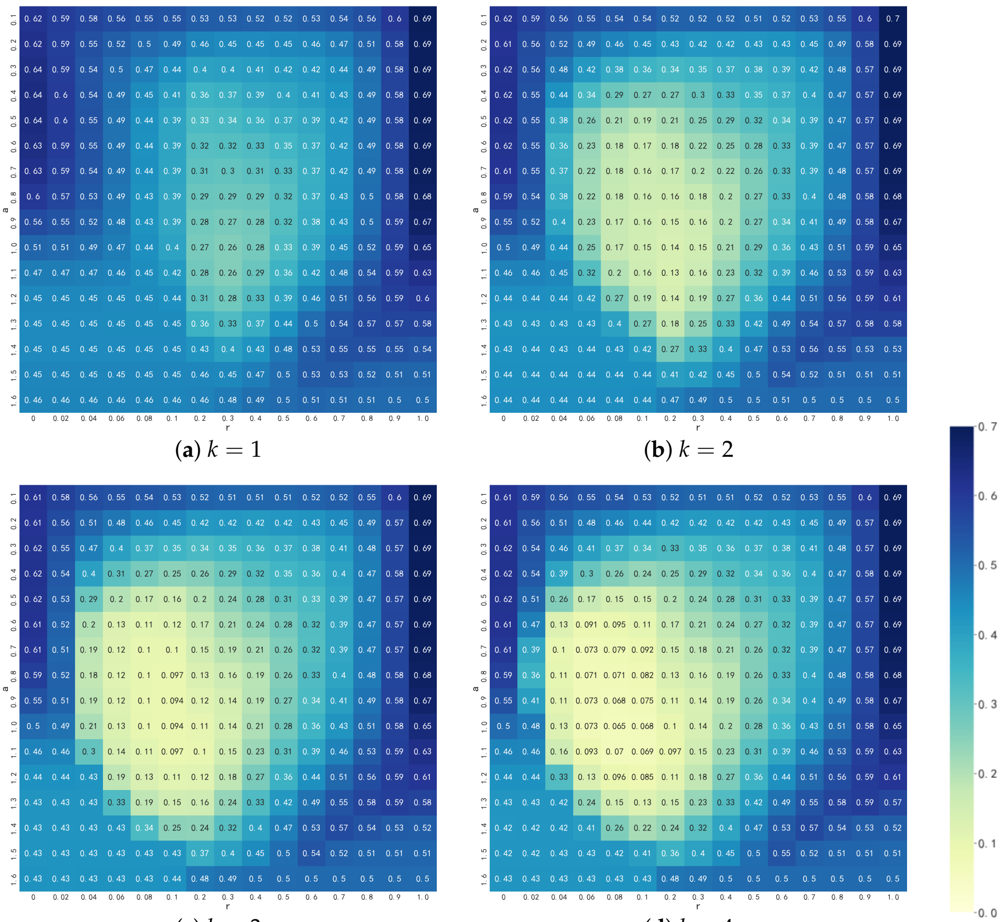
```
Input: undirected graph G = (V, E); Monte Carlo time steps T
Output: network after evolution, utilities of all agents
1: for all agents i ∈ V do
2:   Random initialization strategy x_i ∈ {0, 1}
3:   if x_i = 0 then
4:     Access to another agent j within shared scope k randomly;
5:   end if
6: end for
7: for all agents i ∈ V do
8:   Calculate utility according to Equation (4);
9: end for
10: for t ∈ (0, T) do
11:   for n ∈ [0, |V|) do
12:     Select agent i randomly, select a number p greater than 0 but less than 1 randomly;
13:     Calculate the possibility P that agent i changes strategy according to Equation (6);
14:     if p < P then
15:       if the strategy of agent i is investment then
16:         agent i changes to take a free ride;
17:         agent i access agent j randomly within k scope;
18:       else if the strategy of agent i is non-investment then
19:         agent i changes to invest;
20:         agent i access himself;
21:       end if
22:     end if
23:   end for
24:   for all agents i ∈ V do
25:     Calculate utility according to Equation (4);
26:   end for
27: end for
28: return Outputs
```

At initialization (line 1 to line 9), each agent is given a random investment strategy (investment, non-investment), and an investor is randomly selected for the non-investment agent. The utilities of all agents are updated after their strategy is settled down. The MC iteration then starts (line 10 to line 27). In each MC time step, all agents are selected once on average, namely $|V|$ times in total. One agent i is randomly selected each time, and agent j is randomly selected from the shared scope of agent i. During the learning update process, each agent compares its own utility with its aspiration and calculates the probability P using Equation (6). If the agent i is categorized as an investment agent before, his strategy turns to free riding, accessing any agent j within k scope range. Conversely, if he is previously categorized as a free rider, his strategy shifts to investment. Additionally, he accesses himself. After making $|V|$ selections, their benefits are updated according to the strategies of all agents.

3. Experiments And Results

This paper presents results from 10,000 Monte Carlo simulations conducted in a lattice network with a finite boundary of $L = 50$. The benefits of public goods b are initially set to 2, and the investment cost c is set to 1. The degree k, as shown in Equation (5), plays a crucial role in the agents' utilities, particularly for the central investors. Therefore, we perform experiments on a lattice network of different degrees ranging from 1 to 4. Additionally, according to Equation (5), the access cost r significantly affects the utilities of all players. Hence, we set r from 0 to c. Moreover, the aspiration level, denoted by a, affects the possibility of an agent switching its strategy in Equation (5). When a exceeds $b - c$ (the lowest possible utility of an investment agent), it drives the whole network toward high utility. Therefore, we set the aspiration $a > b - c$—but not too high. In the end, we use social invest level $l_{si} = \sum_{i \in V} \frac{x_i}{|V|}$ as the evaluating indicator to represent the outcome of our model, where x_i and $|V|$ have been defined in the previous section.

To study the impact of k, a, and r on the social invest level, we present Figure 2 and observe the three following main phenomena:



(**a**) $k = 1$ (**b**) $k = 2$ (**c**) $k = 3$ (**d**) $k = 4$

Figure 2. Each figure shows the social invest level of different k-hop with (**a**–**d**) corresponding to $k = 1$, $k = 2$, $k = 3$, and $k = 4$, respectively. a is the aspiration and r means the access cost. The darker blue color represents a higher social invest level while the lighter yellow color represents a lower social invest level. We take the average social invest rate from 9000 to 10,000 MC time steps as the final social invest level.

- ***Phenomenon I:*** As k increases, the color in each subgraph (Figure 2a–d) is lighter (the yellow and white areas in the center grow larger). It indicates that with fixed a and r, as k increases, the social invest level decreases.
- ***Phenomenon II:*** In each subgraph, with fixed r, as a ascends, the color also becomes lighter and then darker. It indicates that with fixed r and k, as a increases, the social invest level goes down and then up.
- ***Phenomenon III:*** In each subgraph, with fixed a, as r ascends, the color becomes lighter and then darker. It indicates that with fixed a and k, as r increases, the social invest level goes down and then up.

Next, we will further analyze the causes of the above three phenomena.

4. Analysis

4.1. Analysis of Phenomenon I

In this section, we examine how the degree k impacts the level of social investment, as well as the relationship between k and a process that we define as Association.

4.1.1. The Impact of K on Social Invest Level

To investigate the effect of k on social resource utilization efficiency precisely, we plot the variations in the social investment level over MC time steps ($t \in (0, 1000)$) for different k in Figure 3. It is discernible from the graph that when t increases, the social investment rate decreases at first and then stabilizes at a relatively low level for all k values. Most notably, as k rises, the reduction of the social invest level becomes steeper. As the utility of agents has an impact on the change in the invest level, we aim to investigate the utility of agents in the network across various k values.

A heat map displaying the utilities of all agents does not clearly reveal the strategic relationships between them. Thus, we focus on clusters as the unit of analysis, with the utilities of the central investors serving as a proxy for the cluster's utility. This allows us to compare yields across clusters and observe strategy relationships within each cluster. Based on that, we create lattice network snapshots (Figure 4) for various k values at time step $(t = 1000)$ when the investment rate is numerically stable. It is observed that a decrease in k leads to a lower utility for central investors and a smaller cluster size with scattered distribution, particularly in subgraph Figure 4a. Conversely, an increase in k leads to a higher utility for central investors and a larger cluster size with blocky distribution, particularly in subgraph Figure 4b–d, which illustrate the process of morphological change from scattered distribution to blocky distribution. It is evident that Figure 4b has assumed a cloud-like shape in contrast to Figure 4a. Within each cluster, the utility of a free rider depends entirely on the central investor. It is worth considering how the surrounding free rider agents will adjust their strategies if the central investors alter their strategies.

To investigate the impact of changes in the central investor's strategy on the surrounding free riders, we study a comparative example. In Figure 5a, we select a specific moment randomly as the initial set of all agents' strategies. In Figure 5b, we keep the investors' strategies from Figure 5a unchanged and perform strategy learning updates in the next time step. At this point, most of the surrounding free riders' strategies do not change. On the contrary, in Figure 5c, we change all the investors' strategies of Figure 5a to ride along and perform strategy update learning in the next time step for all agents. We find that most of the surrounding free riders change their original strategies. Thus, it demonstrated that changes in the central investors do influence the strategies of the surrounding free riders, a phenomenon we refer to as **Association**. It is hypothesized that the varying degrees of **Association** for different k values result in differences in investment rates. We will examine this hypothesis in the next subsection.

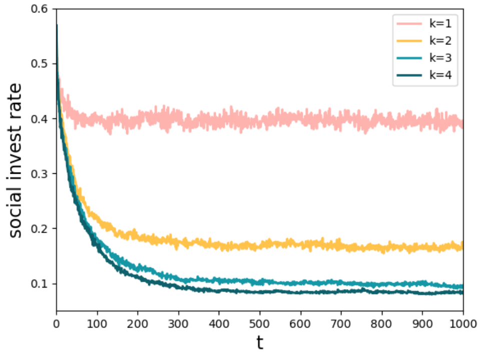
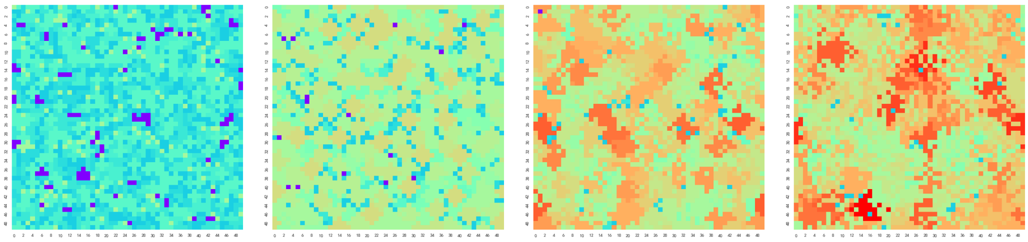
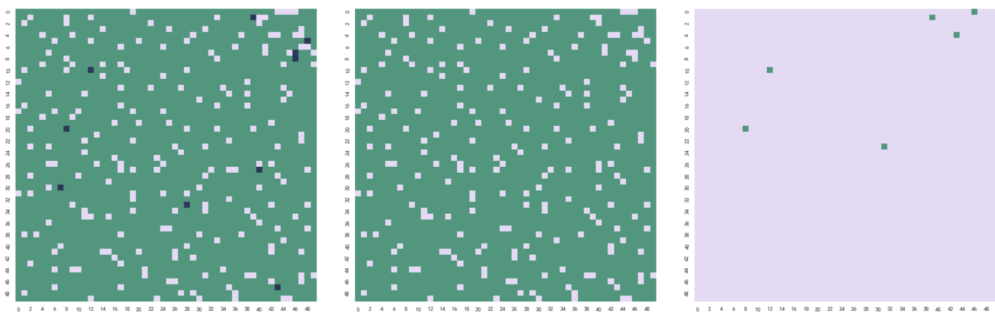


Figure 3. The effect of different levels k on the investment rate. Fixed parameters: $a = 1.0$, $r = 0.1$.



(**a**) $k = 1$ (**b**) $k = 2$ (**c**) $k = 3$ (**d**) $k = 4$

Figure 4. The figures are identified by (**a**–**d**), corresponding respectively to $k = 1$, $k = 2$, $k = 3$, and $k = 4$. Agents who access the same center are set in the same color scheme, with warm colors representing high utility of central investors and cool colors representing low utility of central investors. Fixed parameters: $r = 0.1$, $a = 1.0$.



(**a**) initial (**b**) strategy maintained (**c**) strategy changed

Figure 5. Subgraph (**a**) depicts the network agents' strategies at time step ($t = 1000$). (**b**) illustrates the strategies of the agents in the network at the subsequent moment after the central investment agent's strategy in (**a**) changes. (**c**) demonstrates the changes in network agent strategy after modifying the strategy of all central investors in (**a**). The central investors is represented by purple, successful free riders by green, and failed free riders by dark blue. Fixed parameters: $r = 0.1$, $a = 1.0$.

4.1.2. The Relation between K and Association

Firstly, we provide a specific example to illustrate the microscopic process of **Association** formation. Figure 6 displays the association's emerging procedures, where the central agent is 102 with neighboring agents 52, 101, 103, and 152. In subgraph (a), agents 101, 103, and 152 access central agent 102, leading to successful investment, while agent 52 chooses to invest. Subsequently, in subgraph (b), central agent 102 modifies its strategy by accessing agent 52 with low probability. Ironically, agent 52 also changes strategies and attempts to access agent 2, as the remaining agents maintain their prior strategy.

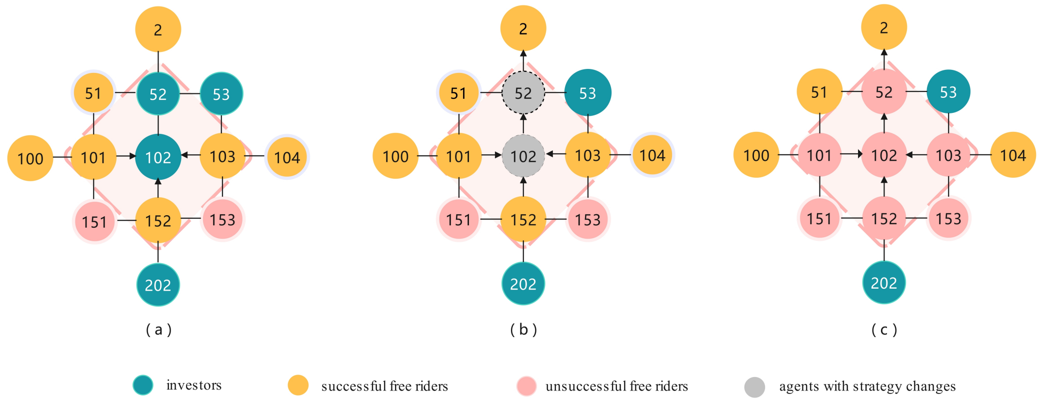


Figure 6. In the network, (**a**,**c**) act as partial agents with their respective strategies at t_0 and t_0 + 1. At t_0 + 1, (**b**) reflects the altered strategies of (**a**,**c**) from t_0. Green nodes represent the investing agents, yellow nodes indicate successful free riders, red nodes depict unsuccessful free riders, and gray nodes signify strategy changers. The arrows show the sequence of free riding, beginning with free riders and concluding with investors. Fixed parameters: $k = 1$, $r = 0.1$, $a = 1.0$.

Consequently, it elicits the association formation in subgraph (c). Agent 52 cannot take a free ride to agent 2, resulting in the failure of agent 102's free ride to agent 52. Due to agent 102's inability to take a free ride to agent 52, agents 101, 103, and 152 cannot gain free rides to agent 102 either. At the next simulation, these unsuccessful free riders will invest and contribute to improving social investment levels.

To mark the degree of **Association**, we use χ in Equation (7) to find the statistics of the number of strategy-changing agents in 1000 time steps.

$$\chi = \sum_{t=1}^{1000} \sum_{i=1}^{|I|} (|A_{I_i}| - 1) \tag{7}$$

$$\begin{cases} x_{I_i(t)} = 1, h_{I_i(t)} = I_i \\ x_{I_i(t+1)} = 0 \\ x_{A_{I_i}-I_i}(t+1) = 0, h_{A_{I_i}-I_i}(t+1) = I_i \\ x_{A_{I_i}-I_i}(t+2) = 1 \end{cases} \tag{8}$$

where I_i represents a central investor in the network. These agents are investment strategy at time t, but change strategy at $t+1$ as free riders. A_{I_i} represents each accessed investment cluster, with the cluster investment center I_i and the other successful free rider. $|A_{I_i}|$ is the number of agents accessing the cluster (including the central investors).

The degree of association in different values of k can be reflected by χ. Figure 7 illustrates the variation of the number of associated agents in different k over the simulation. The number of associated agents decreases more rapidly as k increases, with the number sinking from 150. At $k = 1$, there are still approximately 100 associated agents, but at $k = 4$ they almost vanish. In conclusion, a smaller k results in a greater number of agents being associated in the network, and it is necessary to further investigate the reason for this result.

Then, we aim to explain the relationship between k and **Association**. Figure 8a depicts the scenario where A is the central investor. Given $k = 1$, only agents 52, 101, 103, and 152 can connect to A. Therefore, A's maximum utility is $u_A = 1 + 0.1 \times 4 = 1.4$. In Figure 8b, B is the central investor, and given that $k = 2$, there are at most 12 agents surrounding B. Hence, B's maximal utility is $u_B = 1 + 0.1 \times 12 = 2.2$. It is evident that $u_A < a$ (a = 1.5), while $u_B > a$. Accordingly, A is more likely to alter its strategy and associate with all adjacent agents, while B has a higher tendency to preserve its current investment plan. Therefore, an increase in k enhances the stability of the central investor, thereby reducing the likelihood of **Association** and promoting the social investment level, which is the general conclusion. Equation (5) illustrates the general case, where the value of k determines the magnitude of $|A_{I_i}|$. Hence, a larger k leads to a bigger $|A_{I_i}|$, thereby promoting $u(x_{I_i}, h_{I_i})$. Therefore, central investors with aspiration a are highly probable to retain their investment strategies.

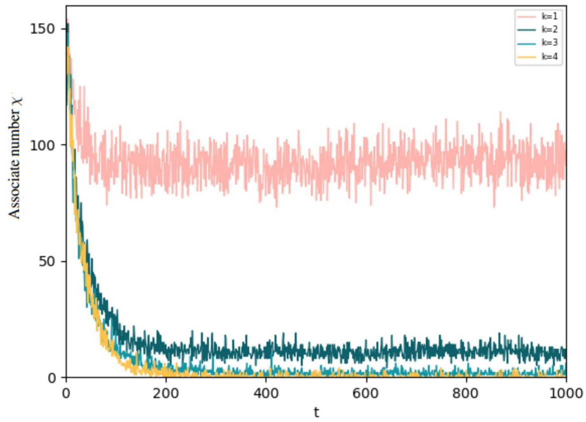


Figure 7. Associate number χ with different k over 1000 MC time step. Fixed parameters: $a = 1.0$, $r = 0.1$.

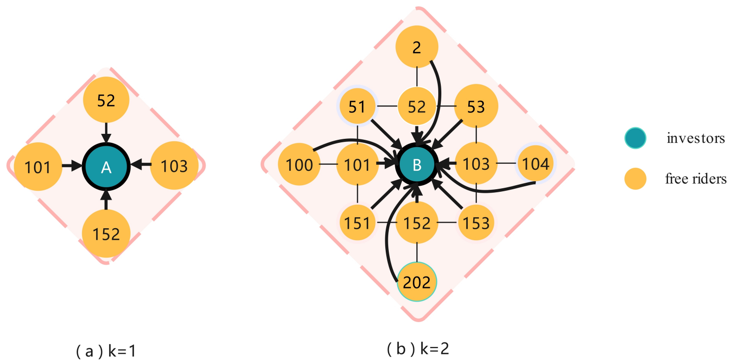


Figure 8. Each figure shows an access cluster with (**a**,**b**) corresponding to $k = 1$ and $k = 2$. Nodes in green represent central investors while yellow nodes represent free riders. Fixed parameters: $a = 1.5$, $r = 0.1$.

So far, our analysis of χ at varying k values has demonstrated that as k increases, central investors tend to adhere more strictly to their respective strategy due to weak **Associations**.

4.2. Analysis of Phenomenon II

Because of the independence of the individual, the expectant utility of an agent will have an impact on the change of its strategy and ultimately affect the overall social utility. In this section, we discuss how these three level aspirations a impact the social invest level.

4.2.1. Aspiration Impact on Social Invest Level

In Phenomenon II, we discover that when r and k are fixed, the social investment level demonstrates a U-shape pattern as a increases. In Figure 2, we note that for $r = 0.1$, the social investment level fluctuates more significantly compared to other values. To investigate the level of social investment when $r = 0.1$, we illustrate Figure 9 for different values of a, $r = 0.1$, and $k = 1, 2, 3, 4$. It indicates that, as a increases, the social investment level initially decreases and then rises. Remarkably, when a surpasses 1.5, the social investment level stabilizes around 0.5. As the utilities of agents have an impact on the change in invest level, similarly, we aim to investigate the utility of agents in the network. Moreover, because the investment level in $k = 4$ experiences the most significant changes, we capture the social network's stable utility by taking snapshots of 12 different a at $t = 1000$ in Figure 10.

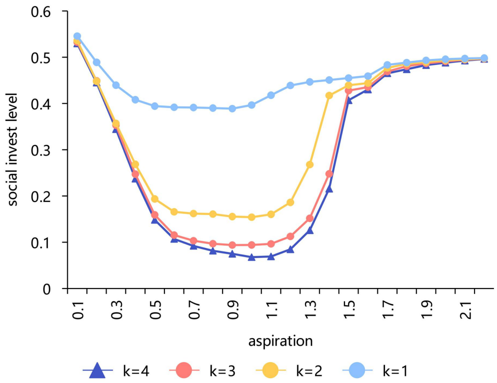


Figure 9. The effect of different levels aspirations on the investment rate. Fixed parameter: $r = 0.1$.

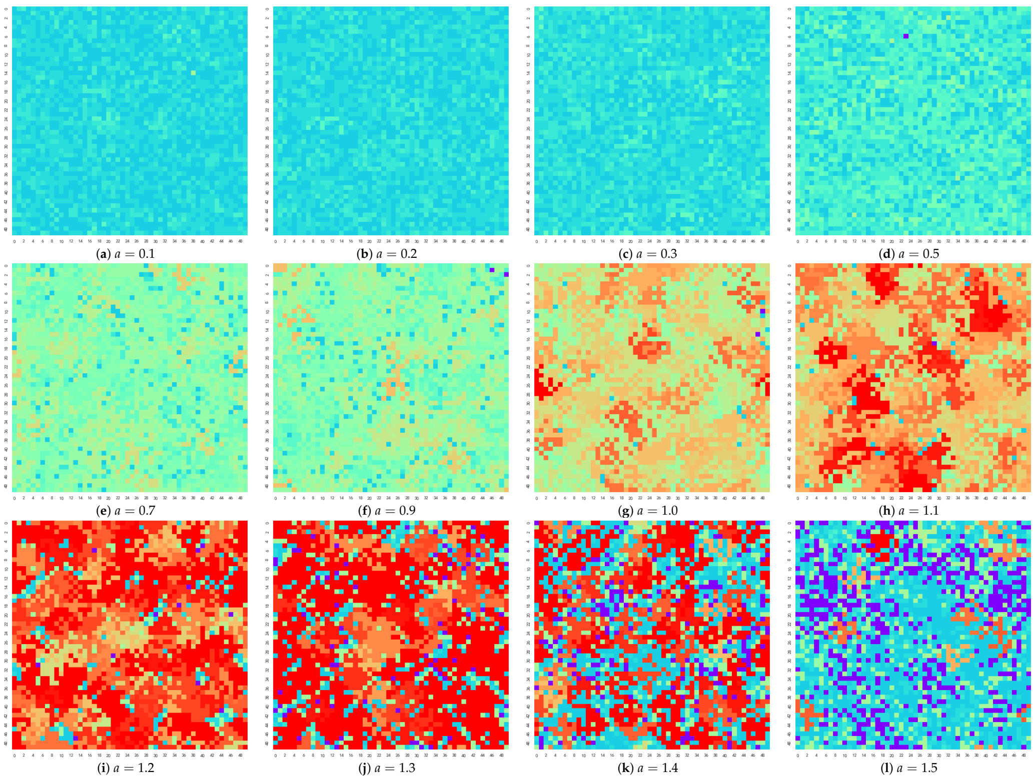


Figure 10. Each figure indicates the access clusters in the network for different values of aspirations. Agents that access the same central investors are depicted in the same color. Nodes represented by warm colors indicate access to high-utility investors, while nodes represented by cool colors indicate access to low-utility investors. The values of r and k are held constant at 0.1 and 4, respectively.

In Figure 10a–f, the subgraphs display nearly uniform colors when a takes a small value, such as $a < 1.0$. Conversely, when a increases, such as $a > 1.0$, the subgraphs Figure 10h–k exhibit a polarization phenomenon, characterized by the concurrent existence of dark red and dark blue colors within the same subgraphs. Finally, as a continues to increase, dark red clusters nearly disappear in Figure 10i. The phenomena above can be observed to occur within three distinct intervals of aspirations a. When a is within a small range, central investors possess almost the same level of utility. Nonetheless, as a continues to increase to a middle level, the network gets populated with highly- and lowly-utility agents concurrently. Finally, when a increases to a larger range, highly-utility agents decline, while the lowly-utility agents dominate the network.

Figure 10 also illustrates the pattern of social investment change depicted in Figure 9. As shown in Figure 10, by examining the color distribution of the clusters, we observe that as the aspiration a increases, the network transitions from a scattered pattern to a blocked one, and then returns to a scattered pattern. This suggests that the count of central investors initially decreases and subsequently increases, implying the U-shape pattern in social investment (Figure 9).

In the next section, we will analyze the reasons behind the observed phenomena corresponding to these three aspiration intervals.

4.2.2. The Impact of Varied Levels of Aspiration on Social Development

We proceed to analyze the theoretical basis for the existence of the three aspiration intervals depicted in Figure 10 and describe the reasons behind the occurrence of the corresponding phenomena in the network.

As shown in Table 1, we examine the significance of three distinct levels of aspirations concerning the strategy of investment, successful free riding, and failed free riding in terms of their conformity to aspirations. Firstly, within the **low** aspiration range, both the investors and the successful free riders are likely to be content with the current strategy, while only the failed free riders would alter its strategy. That is when $a \leq b - c$. Secondly, during the **middle** range, as the aspirations already surpass the minimum utility on investment ($b - c$), whether the investors meet their own aspirations depends on the access number of surrounding successful free riders. If the utility on the successful free riders, denoted as $b - r$, exceeds a, then these successful free riders are satisfied with their respective utility. In this case, central investors with high levels of visits by successful free riders are likely to maintain their strategies to a large extent, while those with low levels of visits are more prone to changing strategies. That is when $b - c < a \leq b - r$. Thirdly, during the **high** range, if $b - r$ falls below a, there exists instability in the strategy of the successful free riders. With low visits by successful free riders, central investors intend to fail to reach their aspirations, leading the instability in the current strategy. In this case, irrespective of successful or failed free riding, or the investors, the strategies prove to be unsatisfactory. That is when $a > b - r$.

Table 1. Three aspiration levels .

Low	$a \leq b - c$
Middle	$b - c < a \leq b - r$
High	$b - r < a$

After analyzing the theoretical basis for the existence of the three aspirations intervals, we then describe the reasons behind the occurrence of the corresponding phenomena in Figures 2 and 10.

Low Aspiration. Here we explain why in low aspirations, the utilities of all central investors are very close and the social invest level is declining.

Firstly, we interpret why the utilities of all central investors are relatively close. According to Equation (5), the utility of a central investor is $b - c + (|A_{I_i}| - 1) \times r$ and that of free riders around the central investor would be $b - r$, $r \in (0, c)$. When $a < b - c$, now the utility of free riders around central investor $u(x_{A_{I_i}}, h_{A_{I_i}}) = b - r > b - c > a$, which meets the aspirations. So for them, based on Equation (6), it is difficult to alter strategies. At the same time, the central investors' utility is $u(x_{I_i}, h_{I_i}) = b - c + (|A_{I_i}| - 1) \times r > b - c > a$. As a consequence, even when $|A_{I_i}| - 1 = 0$, central investors would be satisfied with the current utility. Similarly, for these central investors, it is impossible to alter strategies. In this scenario, both investors and successful free riders achieve their aspirations, while free riders are not likely to access other investors during the updates. As a result, as shown in the subgraphs in Figure 10a–f, the utility values of central investors are similar.

Figure 10a–f depicts the phenomenon of the average utility of central investors for $k = 4$ when a takes a small value. We hypothesize that similar situations would occur when $k = 1, 2$, and 3. As the utility of central investors surpasses a, it leads to the insignificance of k regarding the impact on the access number $(|A_{I_i}| - 1)$. To support this claim, we present the average access number of investors by successful free riders at different values of k, while holding $r = 0.1$, $a = 0.5$, and $t = 1000$, as illustrated in Figure 11a. Direct evidence, provided by Figure 11a and supporting our assertion, is provided by the maintenance of the access number of the investors between 1 and 2, irrespective of the value of k.

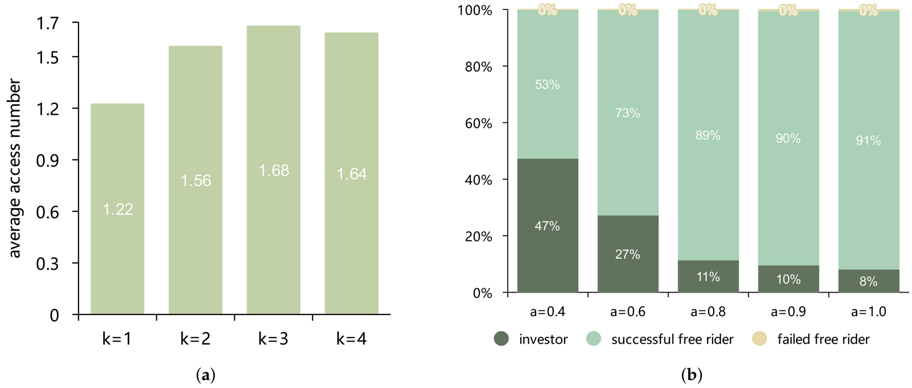


Figure 11. Subgraph (**a**) shows the average access number of an investor in different k. Fixed parameter: $r = 0.1$, $a = 0.5$, $t = 1000$. Subgraph (**b**) shows the proportion of three types of agents in the network. Fixed parameter: $r = 0.1$, $k = 4$, $t = 100$.

Secondly, as for why the social invest level continues to decrease, when $0 < a < b - c$, central investors with low $|A_{I_i}|$ are far less profitable than successful free riders, i.e., $b - c + (|A_{I_i}| - 1) \times r < b - r$. As a rises, $(b - c + (|A_{I_i}| - 1) \times r - a)$ becomes smaller, which increases the probability that the central investors will turn to free riders. Only when $|A_{I_i}|$ is large enough can central investors keep their investment strategy unchanged. So when a increases to $b - c$, the growth of $|A_{I_i}|$ can stabilize the network. In Table 2, when $a = 0.5$, the corresponding $|A_{I_i}|$ is only 2.64; when a increases to 1.0, $|A_{I_i}|$ rises to 11.4. This suggests that as the aspirations continue to increase, only those investors with sufficient access by free riders can survive in the network. Moreover, when a reaches $b - c$, central investors in the network are in the minority, while successful free riders are in the majority. As a result, the number of investors in this process decreases, while the number of free riders around the central investors increases, as shown in Figure 11b.

Overall, we prove that the utilities of all central investors are very close through Figure 11a and explain why the social invest level is declining through Figure 11b as well as Table 2.

Table 2. Change of $|A_{I_i}|$ with different a. Fixed parameter: $r = 0.1$, $k = 4$, $t = 1000$.

a	0.5	0.6	0.7	0.8	0.9	1.0
$\lvert A_{I_i} \rvert$	2.64	3.55	5.76	8.7	11.1	11.4

Middle Aspiration. Here we explain the reason for the polarization phenomenon in Figure 10h,k and why the social invest level starts to rise at this time.

Firstly, we interpret the reason for the central investors' utility polarization. At this time, free riders can realize their aspirations. They will therefore stick to their strategies with high probabilities. However, the comparison between central investors' utility $b - c + (|A_{I_i}| - 1) \times r$ and aspiration a mainly depends on $|A_{I_i}|$. Central investors who have a vast number of visitors, denoted as α, can achieve a high level of performance $(b - c +$

$(|A_{I_\alpha}| - 1) \times r)$ that exceeds a. In these cases, their clusters are generally stable, with a high probability rate, as shown by the dark red blocks in Figure 10. Conversely, central investors who have fewer visitors, denoted as β, are likely to experience a utility level $(b - c + (|A_{I_\beta}| - 1) \times r)$ that is lower than a. This type of occurrence is represented by the dark blue blocks in Figure 10, and the coexistence of agents with high and low utility is the primary cause of utility polarization.

Secondly, the social invest level starts to rise at this time because of the emergence of β, $b - c + (|A_{I_\beta}| - 1) \times r$, leading to the instability of the central investors' strategies and the occurrence of **Association**. The degree of association in different values of k can be reflected by χ. Table 3 indicates that as a increases, so does the number of associated agents. This results in an increase in the number of strategies shifting from non-investing to investing. Consequently, the level of social investment increases.

Table 3. The degree of association with different a. Fixed parameter: $r = 0.1,\ k = 4$.

a	1.1	1.2	1.3	1.4	1.5
$\chi(\times 10^4)$	1.5	4.3	15.9	48.2	82.9

High Aspiration. Here we explain the reasons behind the donations made by agents with low utility in Figure 10i.

Firstly, we explain the reason for the donation of low-utility agents in the network. At this time, the utility of the surrounding free riders is $b - r < a$, which cannot meet their aspirations. So they intend to change to invest. It affects the utility of central investors $b - c + (|A_{I_i}| - 1) \times r$ because the access number $(|A_{I_i}| - 1)$ is quite small. As a result, $b - c + (|A_{I_i}| - 1) \times r$ is about equal to $b - c$. With $b - c < a$, the investors are unable to achieve the expected utility, leading them to be inclined to change their strategies to free riding. In this case, the central investors' utility can be low.

Accordingly, a network dilemma occurs: neither investors nor free riders are pleased with their present utility, leading to an unceasing transition of a strategic shift. This phenomenon is called **Fluctuation**.

Furthermore, we will show more details about **Fluctuation**. Microscopically, **Fluctuation** represents the strategy changes of agents under each time step in the network, as shown in Figure 12a. It shows that the amplitude of the yellow line always lags behind the green line by a time step, meaning that, in the network, if there are n free riders turned to invest at time t_0, then at time $t_0 + 1$, there would be n investors turned to free riders. No matter what strategy the agent makes, it would be changed in the next time step. It is therefore equiprobable for both changers, i.e., investors changing to free riders and free riders changing to investors. Thus, the average social invest level could always remain at 0.5, which is shown in Figure 9. To be more specific about social invest level, we explore it within 1000 simulations in Figure 12b. It is shown that the social invest level curve of odd time steps and even time steps are approximately symmetrical about 0.5. This explains why the social investment level is around 50% when the access costs are high, as depicted in Figure 9.

However, it is important to note that, despite the stable display of an average social investment rate around 50% in Figure 9, this does not imply stability in the overall societal strategy changes. Figure 12a shows thousands of agents changing the current state between adjacent time intervals, indicating extremely poor social stability at that time.

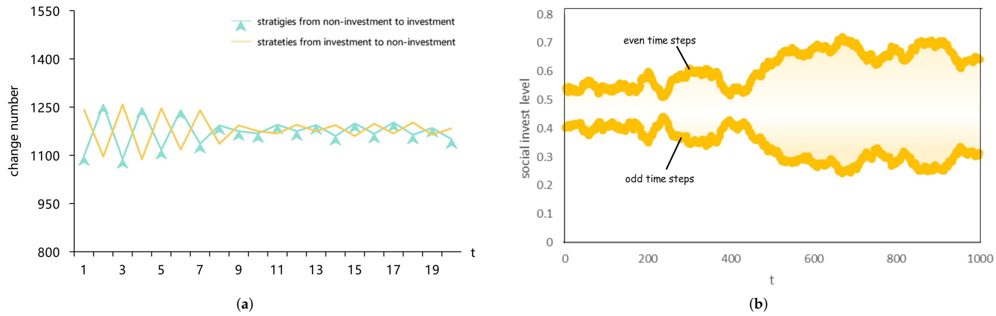


Figure 12. Subgraph (**a**) shows the number of strategy-changing agents in the first 20 simulations, where the green line represents those changing to invest, and the yellow line represents those changing to non-invest. Fixed parameters: $a = 2.0$, $r = 0.1$, $k = 4$. Subgraph (**b**) shows the average social invest level of 1000 simulations. The line above represents the social invest level corresponding to an even number of time steps. The line below represents the social invest level corresponding to an odd number of time steps. Fixed parameter: $a = 2.0$, $r = 0.1$, $k = 4$.

4.3. Analysis of Phenomenon III

The access cost significantly impacts the utilities of both investors and free riders. This influences their decision to uphold their initial strategy. In this section, we examine how the access cost for two levels of entry, denoted as r, affects the social investment level.

4.3.1. Access Cost Impact on Social Invest Level

In Phenomenon III, we discuss that when a and k are fixed, the social investment level demonstrates a U-shape pattern as r increases. Additionally, in Section 4.2.2, we discussed three levels of aspirations. Here we select the aspiration threshold, which is $a = 1.3$, to avoid upcoming fluctuations as a fixed parameter in this section. As shown in Figure 13, with fixed a and k, as r increases, the social invest level goes down and then up. Because the trend of the social invest level for $k = 2$ in Figure 13 is relatively obvious, we take snapshots of the network for eight different r in Figure 14 to observe the utilities of central invest agents. It shows in Figure 14 that when $r = 0.2$, the central investors profit the most, with the highest social utility level. Based on this, we divide the access costs into two intervals for investigation. Firstly, when the access costs are low, the returns of the central investors are relatively small. Secondly, when the access costs are high, the returns of the central investors are also relatively low.

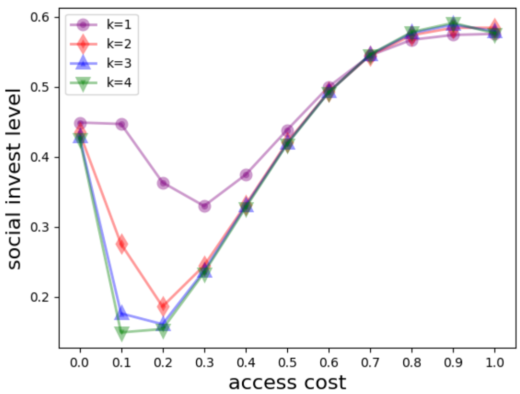


Figure 13. The variation of social invest level over access cost r for $k = 1, 2, 3, 4$. It shows that as r increases, the social invest level goes down and then up. Fixed parameter: $a = 1.3$.

4.3.2. The Impact of Varied Levels of Access Cost on Social

In Section 4.2, we mentioned that low aspirations could lead to the equalization of central investors' utilities while exorbitant aspirations most likely will cause network fluctuation. Thus, we choose a more moderate expectation period $b - c < a < b$ to explain phenomena with different access costs r. Assume that there exists $\theta > 1$, when $r = \frac{c}{\theta}$, the equation $b - r = a$ can be satisfied. Particularly, in Figure 14, the lowest social invest level can be obtained when θ is set to 5 ($r = 0.2$). In fact, the optimal r corresponding to θ is related to a, k and the probability parameter K in Equation (6).

As shown in Table 4, we examine the significance of two distinct access cost levels concerning the strategy of investment, successful free riding, and failed free riding in terms of their conformity to aspirations. Firstly, in the case of **cheap** access costs, free riders tend to satisfy their current utility and are therefore highly likely to maintain their strategy. Investors, due to their large visits by free riders, tend to satisfy their current utility as well. Secondly, in the case of **expensive** access costs, free riders are likely to not satisfy their current utility, resulting in a substantial change in their strategy. Investors, due to their lower visits by free riders, do not satisfy their current utility.

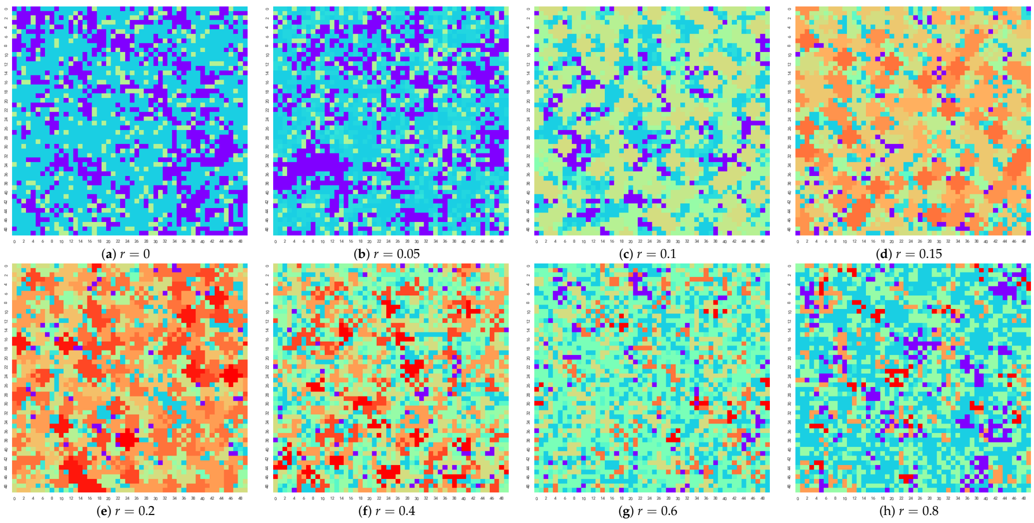


Figure 14. Each figure indicates the access clusters in the network for different values of access costs. Agents that access the same central investors are depicted in the same color. Nodes represented by warm colors indicate access to high-utility investors, while nodes represented by cool colors indicate access to low-utility utility investors. When r increases from 0 to 0.2, the color gradually changes from dark blue to dark red. When r reaches 0.2, the most obvious blocky dark red color appears among the snapshots. Next, r consistently grows to 1, and dark red clusters gradually disappear and then turn to dark blue scatters. Fixed parameters: $k = 2,\ t = 1000$.

Table 4. Two access cost levels.

Cheap	$0 < r < \frac{c}{\theta}$
Expensive	$\frac{c}{\theta} < r < c$

Cheap Access Cost. When r approaches zero, the utility of the surrounding free riders $b - r = b > a$, and they will then not change the original strategy with high possibility. Meanwhile the utility of the central investors is $b - c + (|A_{I_i}| - 1) \times r = b - c < a$. They can then easily change strategy. Therefore, the change of the cluster's strategy depends on the change of the central investors' strategy, and the probability of the occurrence of **Association** is very high.

When $0 < r < \frac{c}{\theta}$, the utility of the surrounding free riders $b - r \geq a$, so they keep their original strategy with high possibility. Meanwhile, due to the increase of r, the difference between the utility of the central investors and a gets smaller, which firms the investors' strategy. The degree of **Association** in different values of k can be reflected by χ. Table 5 shows that as the variable r increases to 0.2, the **Association** number χ diminishes in this situation. This indicates that the strategy of the central investors becomes more and more stable during this process.

Table 5. The number of associate agents in different access costs.

r	0	0.05	0.1	0.15	0.2
$\chi(\times 10^4)$	87.1	85.2	38.4	11.5	5.8

Expensive Access Cost. When $\frac{c}{\theta} < r < c$, the utility of the surrounding free riders $b - r < a$, and they will then choose to change their strategy to invest with possibility, which leads to a decrease in $|A_{I_i}|$ (Table 6). However, at the same time, r consistently climbs. Thus, we need to analyze two situations: (1) $b - c + (|A_{I_i}| - 1) \times r > a$, (2) $b - c + (|A_{I_i}| - 1) \times r < a$.

Table 6. The average free riders for an investor. Fix parameters: $a = 1.3$, $k = 4$, $t = 1000$.

r	0.2	0.3	0.4	0.5	0.6	0.7
$\lvert A_{I_i}\rvert$	4.545	3.185	2.434	1.947	1.701	1.477

1. If r can still meet the condition that $b - c + (|A_{I_i}| - 1) \times r > a$, investors keep their strategy with high possibility. So at this time, it is the strategy change by the free riders that contributes to the increase of social invest level. In Figure 15a, the number of successful free riders decreases with the increase of r, while the number of investors increases. This indicates that the surrounding free riders are gradually separating from the central investor due to the increasing access costs, and their strategy change to invest.
2. As r increases, the number of free riders $(|A_{I_i}| - 1)$ tends towards zero because of the high access cost. Then the utility of the central investors would be $b - c + (|A_{I_i}| - 1) \times r = b - c < a$. So the central investors want to change their strategy to take a free ride. In addition, at this time, the surrounding free riders will most likely change their strategy, separating them from central investors. Therefore, **Fluctuations** occur, as depicted in Figure 15b.

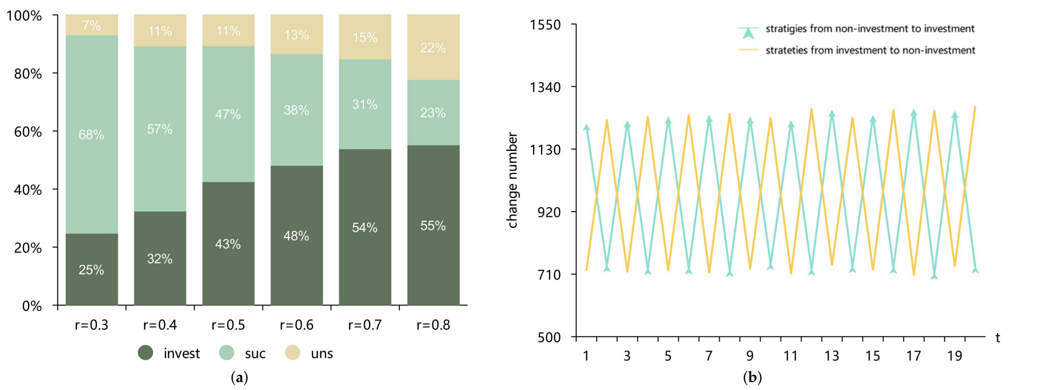


Figure 15. Subgraph (**a**) shows the proportion of three types of agents in the network. Fixed parameter: a = 1.3, k = 4, t = 1000. Subgraph (**b**) shows the number of strategy-changing agents in the first 20 simulations, where the green and yellow lines represent the same as Figure 12a. Fix parameters: $a = 1.3$, $r = 1.0$, $k = 4$.

5. Conclusions and Future Work

In this paper, we introduce an aspiration-based learning mechanism in Best-shot BNPGG when information is unobtainable. In our mechanism, whether agents update their strategies depends on the difference between their own utilities and aspirations. We identify and provide reasonable explanations for three phenomena. Firstly, increasing the access scope k benefits the stability of clusters, leading to a lower investment rate. Secondly, excessively high or low access costs both lead to higher investment rates, which are detrimental to societal development. Excessive access costs can cause significant oscillations in individual strategies, posing a serious threat to social stability. Therefore, moderate access cost is suggested to consolidate healthy and stable social development. Thirdly, and most importantly, aspiration plays an indispensable role in driving social development. When aspirations are too low, the overall social utility diminishes as in a downturn. In such cases, goals are easily achieved, and individuals lack the motivation to pursue more challenging tasks or realize their ambitions, resulting in stagnant development. Or, expanding the access scope does not alleviate such a diminishment. Conversely, when aspirations are excessively high, it leads to polarization between the wealthy and the impoverished and even oscillations of individual strategies. When high aspirations fall within an acceptable degree, capable clusters can achieve challenging goals, while incompetent clusters face challenges in meeting the targets. However, once aspirations exceed an acceptable degree, the entire network experiences strategy oscillations, resulting in societal instability. Hence, moderate overall societal expectations are preferred to mitigate the issues of downturn, polarization, and oscillations in the network. We hope that our findings are useful for corporations and organizations to develop strategies. In the future, except for lattice networks, other types of networks can be introduced in dynamic evolutionary, including ErdOs-Renyi networks, and scale-free networks.

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