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Quality Analysis of Natural Gas Using the Structural Reliability of an Analytical Information System

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Abstract: In this study, the authors first attempted to evaluate the efficiency of available systems for natural gas quality analysis using various examples. For this purpose, a model for such gas analysis systems was designed and the structural reliability of these systems were calculated. In the following, the main shortcomings of the existing methods for evaluating the reliability of gas analysis systems were discussed. Finally, a new probabilistic approach for the reliability assessment of such systems was proposed. This approach included a subsystem of measuring instruments that depended on the number of measured parameters. Specifically, it was suitable for measuring a single parameter of a gas mixture, but in order to check its effectiveness, a number of criteria were considered to identify and record system failures. For each criterion, various mathematical equations were constructed for reliability indices, including an operating time distribution function, reliability function, and average time to failure function. Finally, the obtained values and the reliability evaluation of gas analysis systems were discussed. Additionally, the main advantages of using the new method compared to the existing methods were enumerated. Furthermore, instead of assessing the standard structural reliability, a probabilistic assessment of reliability based on the accuracy of measurements was proposed.

Keywords: quality analysis of natural gas; reliability assessment; gas analysis systems; probabilistic approach; failure detection

MSC: 68M15

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1. Introduction

Natural gas is one of the most essential resources in human daily life and even some large industries and factories. Therefore, the study of issues related to natural gas, such as its extraction and transmission, has been the focus of many researchers. Also, the design, construction, and optimization of its storage tanks for different purposes have been considered, because these tanks are at risk of bursting and exploding [1–3]. In addition to the above-mentioned cases, the quality of natural gas is also important and can significantly affect the efficiency of the system and the quality of the final product. Therefore, gas quality analysis is also very important. In this regard, various information processing methods have been presented for gas quality analysis that can be used in the design of an information computing system. Choosing the most effective method to build such a system based on theoretical considerations reduces the time and cost of designing an automatic information system. Next, it is necessary to check the reliability of the selected system and study its structure, including the software and hardware tools used to develop the system requirements. In the following, the implementation of such an automatic information computing system model is discussed. In our previous studies, the present model was used, and very good efficiency results were reported [4,5]. In fact, this system was based on an analytical method of analysis to determine gas quality. It is true that many similar systems are offered in different industries. However, a problem that exists in full-scale and semi-natural tests, in addition to understanding how these types of system work, is to evaluate the reliability of a system in order to make managerial decisions as to whether it is sufficient, needs to be developed, or should be completely abandoned and replaced by more advanced systems. Also, the service life of such systems is usually around 10 years, leading to relatively high costs in terms of equipment. However, existing methods are generally not suitable for such evaluations. To address this issue, in this study, a suitable new model of system reliability was created, and formulas for reliability indicators were presented herein. These reliability metrics are used in a wide range of scientific research, and we used them to monitor a gas analysis system. For this purpose, the probabilistic method for assessing the reliability of gas analysis systems was modified.

2. Materials and Methods

2.1. General Structure of the Proposed System

The method developed in this article enabled the design of specialized structural diagrams of gas analysis systems. Furthermore, by studying such block diagrams, it was possible to evaluate the reliability characteristics of the target systems by probabilistic methods. Additionally, problematic regions of gas evaluation systems could be recognized in terms of reliability and protection characteristics. Finally, the gas analysis system developed in this research was evaluated from the reliability viewpoint. To achieve this, it was most important to determine the fundamental terms and reliability signs for deciding the degree of reliability of such systems.

It can be said that reliability is the property of an object that allows it to maintain (inside an established range) the values of all parameters that are vital to carrying out the specified tasks in the situations of use, upkeep, storage, and transportation [6]. The main conditions and indicators of system reliability are related evaluating the performance of the system. To this end, it is essential to distinguish the type of failures that occur in the system. Failure refers to a crash in the system, i.e., it stops operating. In the system proposed in this article, failures were considered as parameters that had a working range, and when their value exceeded the permissible limit, this was recorded as a failure event. Furthermore, the main reliability indicators considered in the current system are the probability of system failure (i.e., the probability of failure occurring in the operational period), the average system operating time, the failure rate, and the distribution density of time to failure. In addition, the occurrence of errors and the number and intensity of failures are also considered. Finally, methods are used to correct the errors found during the development and operation of the system [7].

Many techniques for assessing the reliability of technical systems are related to the probabilistic approach, for example, Markov schemes. These methods provide the possibility of evaluating the reliability of machine, instrument, and device functioning in various fields. Analytical and simulation strategies are also utilized to obtain reliability indicators for technical systems [8]. These include the techniques of random process theory; expert assessment (exploratory prediction); decomposition (equivalence); and logical-probabilistic, asymptotic, analytical, and statistical strategies. Meanwhile, random process theory strategies are employed to determine the reliability of various systems with continuous technological processes by industrial companies, like gas companies. Generally, in the design phase of a distributed system, the reliability is calculated based on the data of the failure rate of the elements constituting the system, and the data are determined empirically. However, to draw conclusions about the reliability of the system as a whole, one must consider the structure of the system and improve the system if possible. The methods of calculating structural reliability based on data related to the failure rate are widely known [9]. Examples of the development and optimization of such models for distributed computing and network control systems can be found in [10–12].

However, for gas quality analysis, these methods have some shortcomings. The concept of structural reliability as "the ability of a system to perform specified functions within a specified period of time" is very popular in the gas analysis field. Therefore, it was necessary to determine which functions are important in the considered case. The primary task of this work was natural gas quality analysis. If the goal was to describe the way in which such systems work using the discrete finite space of the system states, it would not be possible to build a model of the systems that could be practically applied, because the accuracy of the measurements would change continuously and could vary depending on the task. Therefore, in this section, the authors present a novel methodology based on the precise determination of gas parameters. To calculate the structural reliability of the proposed system, the distributed system was divided into elements. In addition, to obtain reliability indicators, structural-logical diagrams of reliability were employed. These diagrams assisted us in graphically displaying the relationships of the elements in the system and specifying how the elements effected the overall performance of the system. A structural-logical diagram is a set of elements connected to each other in series and/or in parallel. Methods for calculating structural reliability are widely recognized, and this method worked well under normal situations [13,14]. The block diagram of a distributed data collection system is illustrated in Figure 1. This system consisted of four main subsystems: the measurement instrument subsystem comprising *N* measurement instruments (MI), the data collection subsystem, the data processing subsystem, and the data analysis subsystem.

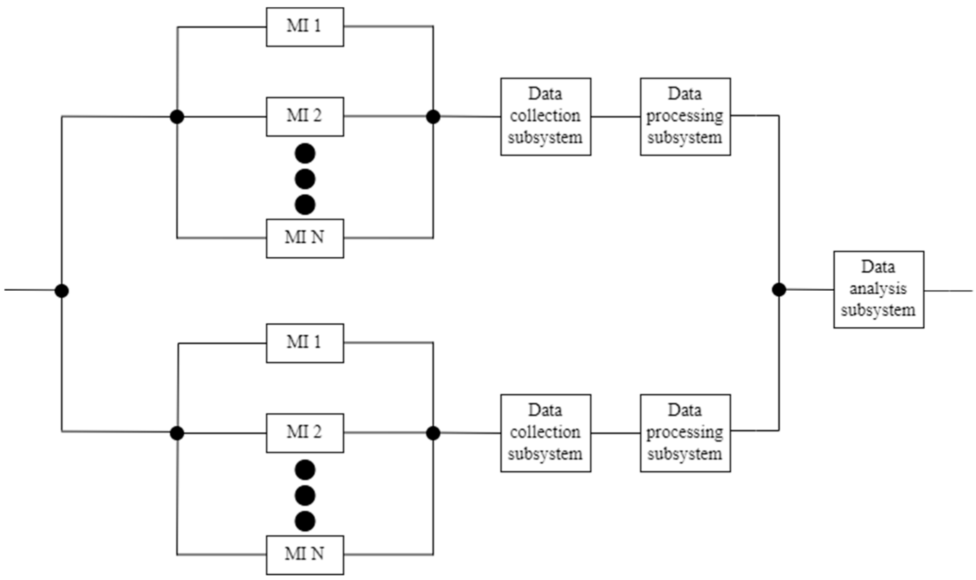

Figure 1. A structural model of the gas quality analysis system proposed in the present research.

The measurement information subsystem consisted of measuring instruments and equipment that could be used in a standard task to obtain the required physical parameters and reference data to evaluate the system performance. It should be noted that this subsystem was tested on a simulation model that had all the features of the proposed system, especially the possibility of achieving high accuracy in analysis by preparing mixtures using mass-flow controllers, producing measurements with commercial applicability. One of the advantages of the above-mentioned measurement instruments is their availability and relatively cheap price. Moreover, the studied measurement information subsystem included the visualization of the main measured parameters, especially the speed of sound, thermal conductivity, and carbon dioxide concentration, for the visual representation of the measurement process.

Here, the information entered to the information analysis system through two independent measurement channels. The circuit could be enlarged by adding further measurement channels to increase both the reliability characteristics and the accuracy of the distributed system. As seen in the circuit presented in Figure 1, the measuring instruments were connected in parallel, because it was assumed that if one of the measuring instruments failed, the system would continue to work but with lower accuracy. To calculate the structural reliability, a state graph was constructed according to the structural scheme. To achieve this, we considered that all the elements had different reliability indicators. After that, the nature of the failure flow, the severity of the failure, and the restoration intensity were determined separately for the system elements. Accordingly, the probability of finding each element in a working or inoperable state at a specified time t was determined. Then, a system of differential equations was formulated. In addition to solving this system of equations, it was possible to obtain the required reliability indicators, e.g., the availability coefficient of the stationary system. The reliability of a redundant system $p_r(t)$ (the probability that a redundant system will operate regularly during time t), where the system consists of elements operating independently in the load mode, could be evaluated by the following equation:

$$p_r(t) = 1 - \prod_{k=1}^{n} \left(\frac{\lambda_k}{\lambda_k + \mu_k} - \frac{\lambda_k}{\lambda_k + \mu_k} e^{-(\lambda_k + \mu_k)t}\right) \tag{1}$$

in which λ_k and μ_k represent the failure and recovery rates of the k-th element, respectively. Therefore, the reliability (probability of operating regularly) $p_{rm}(t)$ of a system of m consecutive blocks with $(n - 1)$-fold redundancy in a general case could be calculated using Equation (2) [15]:

$$p_{rm}(t) = \prod_{i=1}^{m} \left(1 - \prod_{j=1}^{n} \left(\frac{\lambda_{ij}}{\lambda_{ij} + \mu_{ij}} - \frac{\lambda_{ij}}{\lambda_{ij} + \mu_{ij}} e^{-(\lambda_{ij} + \mu_{ij})t}\right)\right) \tag{2}$$

2.2. Numerical Results for Structural Reliability

For the block diagram of the gas analysis system shown in the Figure 1, the reliability index (the stationary availability factor) was calculated. Since gas analysis systems operate for a long time (more than 5 years), the second term in the above equations was omitted, and the index was calculated utilizing simplified formulas. In this regard, the values of the parameters for reliability evaluation are given in Table 1.

Table 1. Values of parameters for reliability assessment.

Parameter	Variable	Value (1/h)
Failure rate of temperature sensor	λ_T	5×10^{-5}
Failure rate of pressure sensor	λ_P	5×10^{-5}
Failure rate of sound speed sensors	λ_{VOS}	6.7×10^{-5}
Failure rate of thermal conductivity sensors	λ_{TCD}	6.7×10^{-5}
Failure rate of carbon dioxide concentration sensors	λ_{CO2}	6.7×10^{-5}
Recovery rate for sensors	μ_{sensor}	0.1
Recovery intensity for systems to collect, process, and analyze information	μ_{comp}	0.05
Computer failure rate	λ_{comp}	3.9×10^{-5}

Next, to simplify the implementation of the proposed method and to calculate the reliability, the circuit shown in Figure 1 was divided into several simpler circuits (see Figure 2).

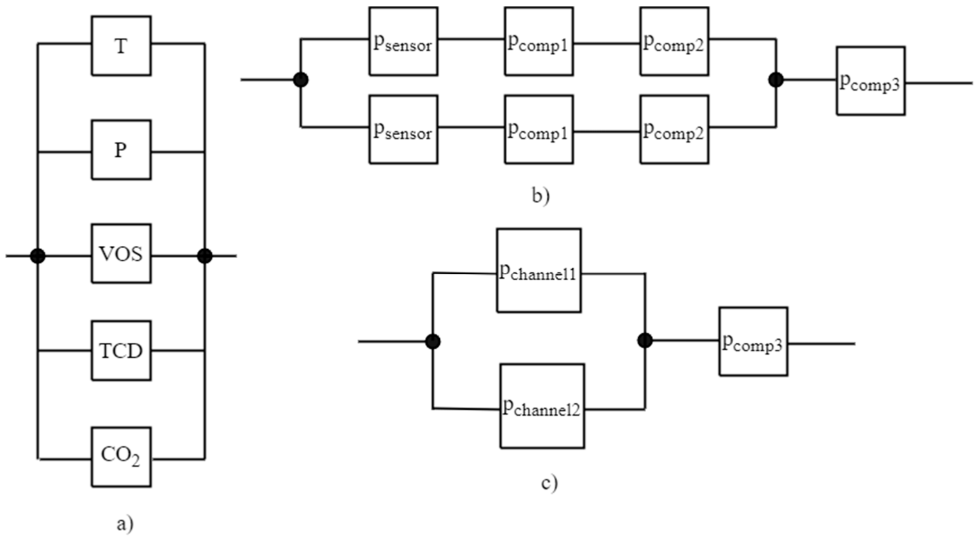

Figure 2. Additional instruments to compute the reliability, including breaking the circuit into simpler circuits.

The probability of the regular operation of parallel-connected sensors p_{sensor} (Figure 2a) was obtained as follows:

$$p_{sensor} = 1 - \left(\frac{\lambda_T}{\lambda_T + \mu_T} \frac{\lambda_P}{\lambda_P + \mu_P} \frac{\lambda_{VOS}}{\lambda_{VOS} + \mu_{VOS}} \frac{\lambda_{TCD}}{\lambda_{TCD} + \mu_{TCD}} \frac{\lambda_{CO_2}}{\lambda_{CO_2} + \mu_{CO_2}} \right) \quad (3)$$

Moreover, the probability of the regular operation of the system that collected, processed, and analyzed information (CPAS), p_{comp} (Figure 2b), was obtained as follows:

$$p_{comp} = 1 - \frac{\lambda_{comp}}{\lambda_{comp} + \mu_{comp}} \quad (4)$$

It should be noted that for the CPAS system, the probability of regular operation was equal to $p_{comp1} = p_{comp2} = p_{comp3}$. After simplifying the formula and converting the circuit to that shown in Figure 2b, the probability of regular operation $p_{channel1}$ for one measurement channel was calculated by the following formula:

$$p_{channel_1} = p_{sensor} \times p_{comp_1} \times p_{comp_2} \quad (5)$$

Also, the probability of the regular operation of two measurement channels $p_{channel}$ was calculated as follows:

$$p_{channel} = 1 - (1 - p_{channel_1})^2 \quad (6)$$

The final probability of the regular operation of the system p_{final} was calculated by employing Equation (7).

$$p_{final} = p_{channel} \times p_{comp_3} \quad (7)$$

According to the stated conditions, the probability of free-failure operation was equal to 0.9992. This value shows that the presented model was very reliable for gas analysis systems. It is true that this scheme is applicable for general processes; however, it did not allow us to make accurate measurements of certain gas parameters.

The instrumentation subsystem played a key role here. Therefore, it was necessary to introduce new definitions and then create a new model with a different approach to reliability for a measuring instrument system. The standard scheme for calculating reliability in the scenario explored in this research had drawbacks that made it unsuitable for the current work.

The main disadvantage of the standard approach and Equations (1)–(7) is that it is impossible to clearly identify if some elements have failed. If the system fails to ensure the specified measurement accuracy, this is essentially a hidden failure. On the other hand, if an external measuring device sends incorrect data, this is still considered a failure from the viewpoint of how the system works. The consequences of such a failure are very critical, because the main function of the system, i.e., the determination of gas energy characteristics, is not performed. If it is not possible to explicitly detect how the system would fail to collect, process, and analyze information (computer failure), then this cannot be immediately detected by measuring instruments. Thus, if the measuring instrument fails, one may not receive obvious external signs of failure. Thus, this phenomenon is not observable, and it can only be judged by the output data. This is why all measuring instruments are duplicated. If the data sent from the measuring devices are not correct, it is assumed that a failure has occurred. This, in turn, is identified by the difference between the readings of the main and backup measuring devices (that is, if the readings between the first and second measuring devices are very different and exceed a threshold). This requires the development of a new approach to evaluate the reliability of the measurement instrument subsystem (exactly what is presented in this paper).

3. Results

3.1. Preliminary Definitions

In this research, it was assumed that for the analysis of each physical parameter of the gas mixture, two measuring instruments (MIs) were needed, namely the main and the backup instruments. It should be noted that the instruments operated independently of each other. In this way, the measuring instruments independently transmitted information through two different channels (i.e., the main and backup channels) to the information-gathering system and, next, to the information-processing system—these data together constituted a measurement channel (MC). The main MIs transmitted information to the main MC. The redundant MIs transmitted information to the redundant MC that operated in a hot mode. Thus, the distributed system gathered data from 2N measuring gadgets (N main and N backup devices).

Due to the fact that the primary duty of the system was to determine energy characteristics, we considered the system reliable if it provided a high level of accuracy in distinguishing the required gas characteristics. Consequently, we determined the reliability of the measuring channel–computer subsystem using the scheme described, for instance, in [12,14]. Due to the fact that all subsystems operated without dependency, their reliability could be assessed independently.

At this time, it was assumed that the subsystem for measuring gas mixture parameters was working regularly if both measuring instruments, i.e., the main and backup devices, had the same readings for an instant of time at the same measuring point. Now, if the readings from two devices varied insignificantly, the measurement was considered reliable. On the other hand, if the output differed significantly, i.e., by more than 10%, from the output of another MI, a failure was assumed to have occurred. In other words, an MI was out of order. In the following, a detailed description of the model proposed in this paper is presented. First of all, it is necessary to introduce the following definitions. Also, the reliability of the measurement system was defined as the ability of the system to carry out defined functions for a certain period of time. Hereon, the "given performance of the system" refers to its capacity to measure gas parameters with high accuracy.

Definition 1. *If the values obtained from the main and backup measuring device differ (in absolute value) by more than a certain value of* ε_i *(i.e.,* $\varepsilon_i > 0$*), we consider the measurement for the* i-th *gas parameter to be accurate. In cases where it is important to define* N *parameters, we specify a vector of positive values,* $\varepsilon = (\varepsilon_1, \ldots, \varepsilon_N)$, $\varepsilon_i > 0\ \forall i = 1, \ldots, N$, *that is, for each measuring device, a certain range of admissible values* $(-\varepsilon_i, \varepsilon_i)$ *is specified.*

Definition 2. *If the accuracy indicator exceeds the allowable limits, the system will fail.*

Furthermore, in practice, every time a specific problem was solved, the concept of failure had to be clarified. For example, for an N-component mixture, the system would fail if k MI subsystems out of N exceeded the limit, where k is in the range of 1 to N (depending on the operating conditions and other factors, e.g., financial factors and customer necessities). If k = 1, the maximum stringent necessities were imposed on the MI subsystem, i.e., the system failed if the accurate measurement of at least one gas parameter was impossible. For k = N, for the MI subsystem to operate regularly, it was sufficient that a minimum of one gas parameter could be measured. The nature of the proposed reliability evaluation technique, which determined the extent to which the readings of the main and backup MIs deviated from each other, is shown in Figure 3 and Equation (8).

$$\begin{gathered} \left|Y_1^{(1)} - Y_2^{(2)}\right| = \xi_1 \\ \bullet\bullet\bullet \\ \left|Y_1^{(N)} - Y_2^{(N)}\right| = \xi_N \end{gathered} \tag{8}$$

where $Y^1{}_i$ is the i-th measurement of the main MI, $Y^2{}_i$ is the i-th measurement of the backup MI, and ξ_i is the deviation value for the i-th measurement.

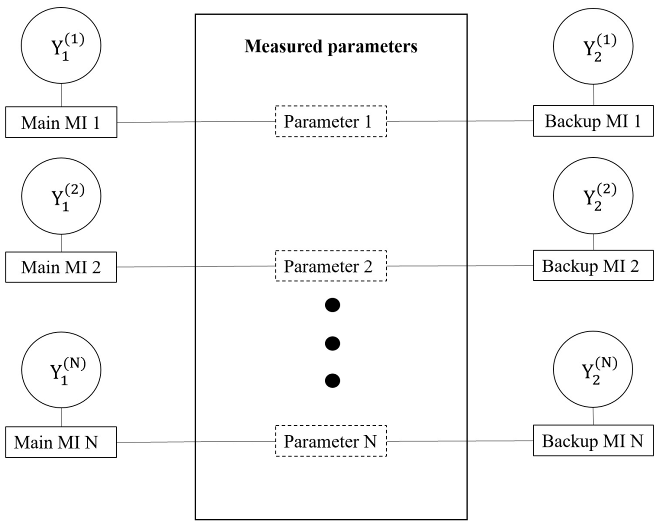

Figure 3. The proposed method for evaluating reliability through the difference between the measured values of the main and backup measuring instruments.

3.2. The Proposed Reliability Model for the Measurement of a Single Parameter of Gas Mixture

If the assumption was that only one parameter of the gas mixture would be measured, the readings would be received at discrete times from two measuring instruments (the main and backup). In this case, if the measuring instruments operated in a regular mode, the readings of the instruments would be within a pre-selected range. Therefore, we would have a discrete sequence of random variables: $\{\xi_i\}$, i = 0, 1, 2... These variables were interpreted as the differences between the readings of the measuring instruments.

We anticipated that the values of this series would fluctuate around zero if the readings of the measuring instruments were slightly different. At the same time, if absolute measurements were taken, the observations could be dependent. Due to this fact, we considered the special modulus and the perspective of fluctuations within the actual composition of the gas. This allowed us to confirm that $\{\xi_i\}$, = 0, 1, 2... were unbiased and identically allotted random variables. Also, we considered ξ_i to represent a few dimension errors. Hence, we anticipated that the elements of the sequence were normal random variables: $\xi_i \sim N(0, \sigma^2)$, i = 0, 1, 2... without a lack of generality, we anticipated the variance of the random variable $\sigma^2 = 1$, i.e., the elements of the sequence were standard normal random variables.

Let us present the following hypothesis about the proposed measuring instruments: as the number of measurements increases (the operating time increases), the measuring instruments will wear out more, and the error will mean that the distinction between the readings of the MIs grows; the error will increase at a non-steady rate in the general case, and the rate will rely upon the time and the state of the system $m(i, X_i)$. Eventually, we modeled the behavior of the system as a discrete-time stochastic process, which summarized the accumulated errors:

$$X_i = m(i, X_i)i + \sum_{k=0}^{i} \xi_k \quad (9)$$

In the above equation, i = 0, 1, 2..., $m(i, X_i)$ may be interpreted as the rate of error accumulation with the wear of the measuring device, and $\xi_i \sim N(0, 1)$. Without a lack of generality, it could be assumed that the measurements were acquired continuously, accordingly deriving from (9) a non-stop version of the process. In this case, the Herbal model was based on Brownian motion [16]. Any such model may be checked using the well-known results from the theory of random processes [17].

3.3. *Preliminary Study: The Reliability of a Measurement Subsystem When* n = 1

We considered a random process in non-stop time X(t) that could be defined via an easy differential equation [17]:

$$dX(t) = \mu[t, X(t)]dt + \sigma[t, X(t)]dW(t) \quad (10)$$

This method could also be interpreted as follows: the error (difference) could be modeled via 1D Brownian movement X(t) = W(t), or, more formally, dX(t) = dW(t). After generalizing, we obtained $dX(t) = \sigma(t, X(t))dW(t)$, wherein the coefficient σ is regular in the general case; however, it may also depend upon the state of the system. Therefore, our measurements were assumed to be somewhat random; that is, either the main measuring device would produce a value greater than that of the backup device, or the backup device would produce a value greater than that of the main device, so the error would either increase or decrease. It was also assumed that the metrological characteristics and the reliability parameters of the measuring devices would decrease with time, so the error would grow on average at a certain rate μ. At an intuitive level, X (the error change value) was modeled by a random walk; as the time step tends to zero, X is described by Equation (10). However, we further considered $\sigma > 0$ and $\mu \geq 0$ as constants:

$$dX(t) = \mu dt + \sigma dW(t) \quad (11)$$

In this step, we assumed that each measuring instrument had a certain range of readings $(-A, A)$, $A > 0$; this interval determined the regular operation of the system [13]. If the system moved outside the interval, a failure state was recorded.

Definition 3. *The system operates regularly if* $X(t) \in (-A, A)$.

Definition 4. *The system fails if* $X(t) \notin (-A, A)$: $X(t) \leq -A$ *or* $X(t) \geq A$.

Definition 5. *The MTBF is the first point at which* X *reaches* A: $\tau_A = \inf\{t \geq 0\colon |X(t)| = A\}$, *(for a strictly positive* μ, *the probability* $P\{X(t) = A\} \to 0$).

Definition 6. *The system reliability function (generally) is expressed as* $R(t) = P(\tau > t)$.

Definition 7. *The distribution function of the system operation time (generally) is obtained as* $F(t) = 1 - R(t) = P(\tau \leq t)$.

Problem 1. *For a system described (in the general case) by Equation (11), we estimate the mean time between failures (MTTF) as* E_τ *(in the simplest case, we find the time explicitly by analysis).*

To pick the threshold values of Ai for each of the measured parameters, it was most important to set how the physical parameters of the gas should be determined. Most standards [18] specify a low error level when determining the required parameters. Thus, we had to bring the values of the maximum absolute error of the MI to the threshold value at which the differences in the readings of the measuring instruments would not exceed the permissible values. For these values, it was proposed to use the reproducibility value of the MI measurements employed for the experimental measurements. In this regard, Table 2 presents the threshold values for all input and output parameters.

Table 2. Threshold values for gas parameters.

Physical Gas Parameter	Unit	Threshold Value of A_i
Methane concentration (X_{CH4})	Mole fraction (%)	0.1
Propane concentration (X_{C3H8})	Mole fraction (%)	0.1
Nitrogen concentration (X_{N2})	Mole fraction (%)	0.1
Carbon dioxide concentration (X_{CO2})	Mole fraction (%)	0.1
Speed of sound (c)	m/s	0.2
Thermal conductivity (χ)	$W/m \times K$	0.009

4. Discussion

One of the features of the model proposed in the current paper was that if a measuring device failed, the system still worked, but with a lower accuracy. To achieve this goal, the measuring devices were connected in parallel. Calculating the reliability as the structural reliability of the parallel subsystem would be incorrect, because if the $N - 1$ subsystem failed in this case, the system would continue to operate. However, only one gas parameter could be determined. Therefore, for determining how the failure of individual measuring instruments would affect the accuracy of the calculations, the reliability model "k out of N" ($k \leq N$) was correct. In addition, N = 5 was assumed everywhere, and the following criteria for how the system operated were considered:

Criterion 1. *If the accuracy indicators decrease to a certain threshold for all parameters, the system will fail.*

Criterion 2. *If the accuracy of at least one parameter is reduced, the system will fail.*

Criterion 3. *If the accuracy indicators decrease for* k ($k < 5$) *parameters, the system will fail.*

At this stage, the first criterion for system operation was considered. The distribution function of the system operation time for the first criterion is denoted as $F_1(t)$.

Theorem 1. *The distribution function of the subsystem operation time* $F_1(t)$ *for Criterion 1 is calculated by the following equation:*

$$F_1(t) = \prod_{i=1}^{5}(1 - \Phi(\frac{A_i - \mu_i t}{\sigma\sqrt{t}}) + e^{\frac{-2\mu_i A_i}{\sigma^2}}\Phi(\frac{-A_i - \mu_i t}{\sigma\sqrt{t}})) \quad (12)$$

Proof. It was assumed that for each parameter (i = 1, 2, 3, 4, 5) in the gas compositions, the error variation model was valid and the parameters A_i, μ_i, σ were given.

Let us use the following well-known formula for the intersection probability, where a Wiener process W(t) crosses a linear boundary a for the above-mentioned μ and σ [19]:

$$P\{\sup_{0\leq s\leq t} W(t) \geq A\} \approx 1 - \Phi(\frac{a-\mu t}{\sigma\sqrt{t}}) + e^{\frac{-2\mu a}{\sigma^2}}\Phi(\frac{-a-\mu t}{\sigma\sqrt{t}}) \quad (13)$$

where Φ is the distribution function for the standard normal law.

Using Equation (13), Equation (12) could be rewritten as follows:

$$F_1(t) = \prod_{i=1}^{5}(1 - \Phi(\frac{A_i-\mu_i t}{\sigma\sqrt{t}}) + e^{\frac{-2\mu_i A_i}{\sigma^2}}\Phi(\frac{-A_i-\mu_i t}{\sigma\sqrt{t}})) \quad (14)$$

□

Corollary 1. *As a consequence of Theorem 1, the reliability function and mean time to failure related to Criterion 1 are as follows:*

$$R_1(t) = 1 - F_1(t) \quad (15)$$

$$MTTF_1 = \int_0^\infty R_1(t)dt \quad (16)$$

Corollary 2. *The distribution function of the subsystem operation time* $F_2(t)$*, the reliability function, and the mean time to failure related to Criterion 2 are*

$$F_2(t) = \prod_{i=1}^{n}(1 - \Phi(\frac{A_i-\mu_i t}{\sigma\sqrt{t}}) + e^{\frac{-2\mu_i A_i}{\sigma^2}}\Phi(\frac{-A_i-\mu_i t}{\sigma\sqrt{t}})) \quad (17)$$

$$R_2(t) = 1 - F_2(t) \quad (18)$$

$$MTTF_2 = \int_0^\infty R_2(t)dt \quad (19)$$

Theorem 2. *The distribution function of the subsystem operation time* $F_3(t)$ *considering Criterion 3 is calculated as follows:*

$$F_3(t) = \prod_{i=1}^{5}(1 - \Phi(\frac{A_i-\mu_i t}{\sigma\sqrt{t}}) + e^{\frac{-2\mu_i A_i}{\sigma^2}}\Phi(\frac{-A_i-\mu_i t}{\sigma\sqrt{t}})) \quad (20)$$

As deduced from these formulas, the key to ensuring high reliability was to increase the uptime of the measuring instrument subsystem. Therefore, the use of primary converters with highly reliable indicators was considered appropriate. The converters provided the required measurement information. This information was used to accurately evaluate the required parameters. In the case of a failure to provide the required indicators of reliability and accuracy, it was proposed to use more expensive converters with more reliable and accurate indicators.

Figure 4 compares the measurement results of the main and backup instruments for the sound speed parameter. Based on the study, it could be concluded that the measurement data were reliable. Thus, we used this fact to experimentally test the automated system.

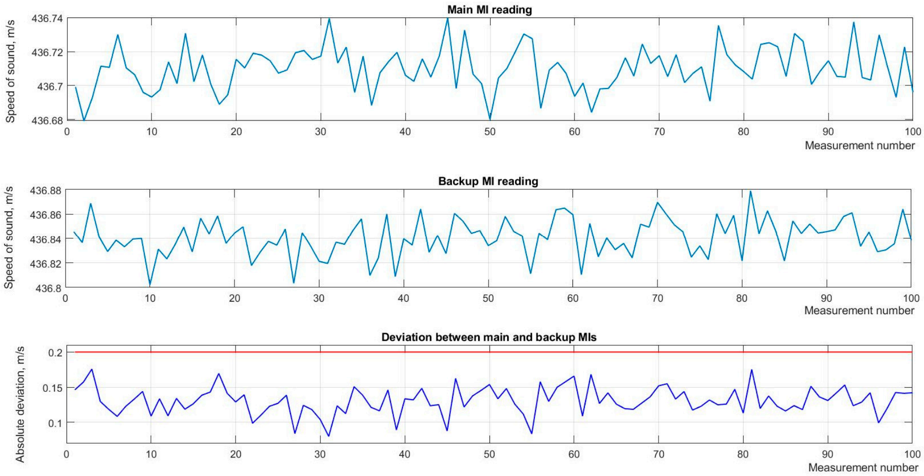

Figure 4. Comparison of measurements of the main and backup measuring instruments (MIs).

To calculate the numerical values of the indicators and to plot the reliability functions, the parameters of the real gas mixture were measured. To this end, the operating time distribution function of the measuring instrument subsystem was calculated using Equation (14). Also, the threshold values for the measuring instruments were taken from Table 1. The parameters μ and σ were determined as the difference between the measurements of the main and backup instruments exceeding the threshold (see Figures 5 and 6).

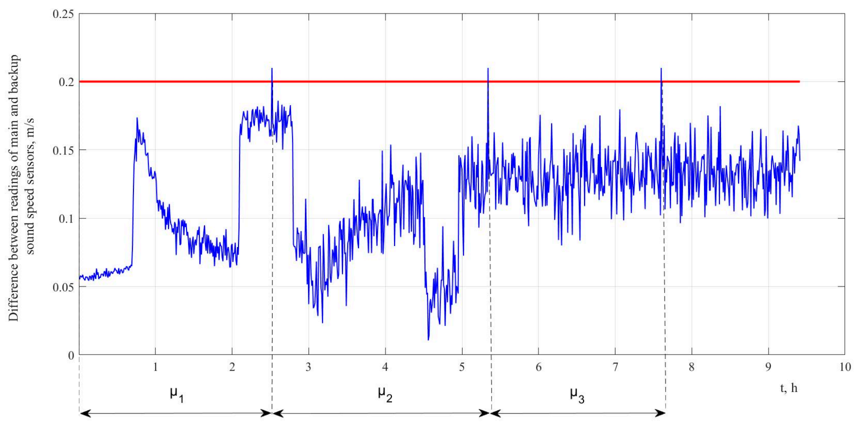

Figure 5. Comparison of sound speed measurements of the main and backup measuring instruments.

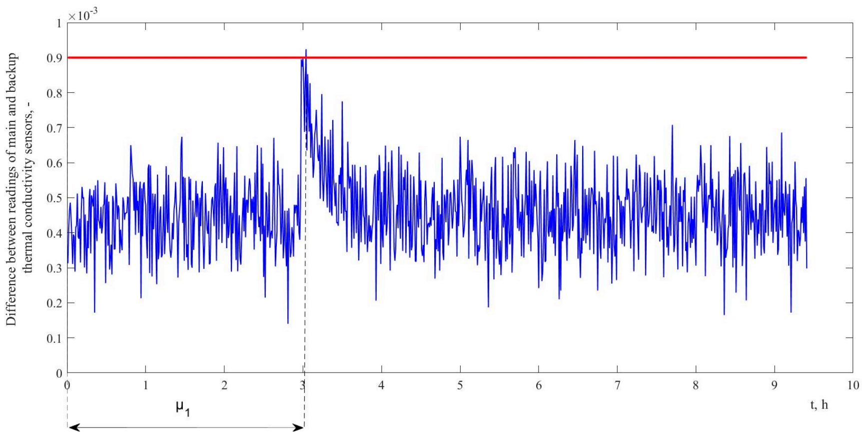

Figure 6. Comparison of thermal conductivity measurements of the main and backup measuring instruments.

The red line in Figures 4–6 indicates the threshold values for measuring the corresponding parameters. Moreover, the blue line displays the measurements of the corresponding parameters (i.e., sound speed and thermal conductivity). For the entire set of experimental data, the values of μ and σ were considered to be the same. Furthermore, the distribution function of the MI subsystem's operating time F(t) (Figure 7), the reliability function R(t) (Figure 8), and the mean time to failure were determined based on Equations (14)–(16). Also, the probability of the free-failure operation of the system was considered for different cases. Finally, Table 3 compares the developed model of an automated information system with the average characteristics of the existing systems. From the data reported in this table, it is obvious that using the new method instead of the existing methods caused the duration of analysis to decrease. Moreover, it caused the mean time to failure to increase. Ultimately, both these results led to a reduction in cost.

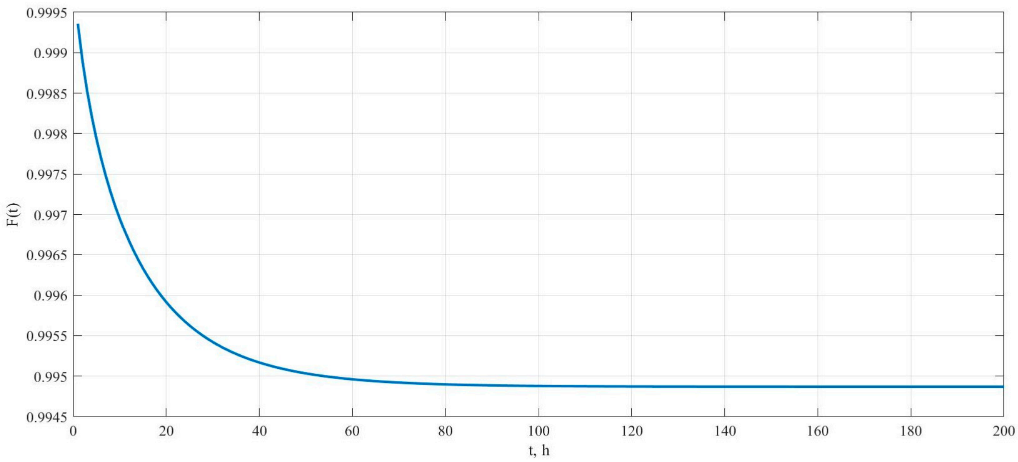

Figure 7. The MI subsystem operating time distribution function vs. time based on the proposed method.

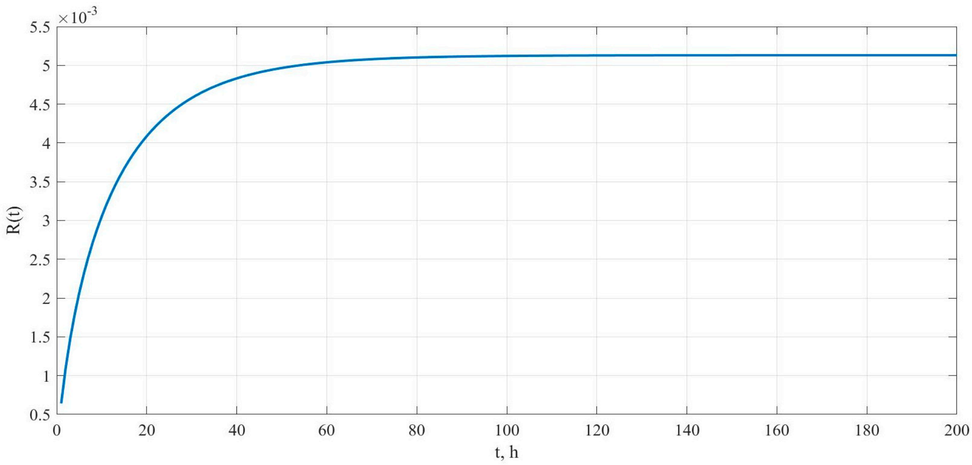

Figure 8. The MI subsystem reliability function vs. time based on the proposed method.

Table 3. Comparison of automated information systems.

Criterion	Developed System	Existing Systems
Order of analysis time (s)	10^0	10^3
Accuracy (MJ/m^3)	$\approx\pm0.5$	$\pm(0.1–0.2)$
Order of cost (thousand RUB)	10^2	10^3
Mean time to failure of technical components (h)	$\approx 2.6 \times 10^4$	$\approx 10^4$

5. Conclusions

In the present research, the authors investigated a method to evaluate the reliability of an information computing system. In this regard, the probabilistic method was used to set the distribution pattern of the operating time, the reliability function, and the mean time to failure. For the constructed block diagram of the proposed system, basic formulas were derived to distinguish the required functions and reliability indicators, in particular, the distribution pattern of the operating time, the reliability function, and the mean time to failure. An evaluation was carried out in terms of different system failure criteria depending on the number of parameters, and the accuracy of the indicators was seen to decrease. For each criterion, formulas for the required reliability functions were derived. It was also shown that the proposed method differed from existing methods in that it used several independent measurement channels (main and backup), which made the received measurement data more reliable compared to the measurement results of the main and backup measuring instruments. Further research will be dedicated to the investigation of approaches to reliability assessment for modern systems from the perspective of different fields of knowledge [20–23]. The existing methods were used to solve the discussed problems. The described method had several drawbacks that should be noted. This method was not tested on a large amount of experimental data. The best way to improve the method would be to increase the amount of experimental data. This class of method is not widely used in industry, and it would be difficult to implement it in a real-world application. Further research will include the development of a prototype of an automated information system, its validation to determine the quality indicators of natural gas, and the study of its reliability using the proposed method.

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