

Article

# Optimal Multi-Attribute Auctions Based on Multi-Scale Loss Network

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**Abstract:** There is a strong demand for multi-attribute auctions in real-world scenarios for non-price attributes that allow participants to express their preferences and the item's value. However, this also makes it difficult to perform calculations with incomplete information, as a single attribute—price—no longer determines the revenue. At the same time, the mechanism must satisfy individual rationality (IR) and incentive compatibility (IC). This paper proposes an innovative dual network to solve these problems. A shared MLP module is constructed to extract bidder features, and multiple-scale loss is used to determine network status and update. The method was tested on real and extended cases, showing that the approach effectively improves the auctioneer's revenue without compromising the bidder.

**Keywords:** optimal mechanism; multi-attribute auction; multi-scale loss

**MSC:** 91-10; 68T07



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## 1. Introduction

The multi-attribute auction is a practical tool widely used on various occasions, such as government auctions of rare resources such as minerals, land, and spectra; online advertising auctions [1]; and supply chain management [2]. In a single-attribute auction, bidders only need to consider one bidding factor, such as price, to determine their bidding strategy. Such auctions lack universality. Multi-attribute auctions provide participants with more options. Bidders can consider multiple factors, such as, in the auction of transportation services, to ensure that the transported items arrive at the designated location more safely. Participants have special needs in terms of price, service quality, delivery time, etc.; this makes the formulation of bidding strategies more complex.

Myerson [3] designed a unique mechanism: the single-item optimal auction mechanism. This is in line with the pursuit of maximizing the interests of one party in the auction. For example, in the aforementioned public resource auction scenario, the optimal auction can pursue the maximization of public welfare. But the optimal mechanism design is complex. In terms of item quantity, the optimal mechanism design for a single item is easy, but it is difficult for multiple items. Dütting [4] solved the 40-year stagnation problem of multiple items using the deep learning method and subsequently derived more complex single-attribute optimal auction mechanisms.

However, the problem of multiple attributes has yet to be solved. Previous papers have proven that the multi-attribute optimal mechanism for single bidders is highly complex [5]. Furthermore, attributes contain private information belonging to participants and cannot be directly converted to one attribute. This paper proposes a new network model and a shared module in Section 4.2 to address this issue.

We noticed that maximizing expected utility implies no labels for network training. Additionally, multi-attribute optimal auctions must satisfy individual rationality (IR) and incentive compatibility (IC) [6] constraints, where IR means that individuals make decisions

that they believe will lead to optimal rewards and IC means that each participant can achieve their best outcome by acting according to their true preferences. These require the network to be updated within a certain range. Therefore, a multi-scale loss network optimization method (MLN) was designed.

Then, the MLN method was tested on real reverse-auction cases. The results indicate that this method could effectively reduce auctioneers' expenses while not causing harm to bidders, ensuring the sustainability of the auction. Moreover, the method was tested with extended experiments, demonstrating its generalization performance and robustness.

## 2. Contributions

1. This paper proposes a dual network structure that includes a shared module. This module extracts multiple non-price attributes from multi-attribute optimal auctions as standard features, which can handle bidding with different preferences and settings.

2. A multi-scale loss method is proposed to optimize the networks. IR, IC, and additional constraints in special auction scenarios are mapped to multi-scale loss functions, ensuring that the auction rule satisfies all parties' interests.

## 3. Related Work

The optimal auction is a special auction mechanism and concept, with the core of maximizing revenue for one party. Myerson solved the problem of maximizing seller returns in a single-indivisible-item, multi-bidder auction while satisfying the incentive compatibility mechanism for bidders to submit true valuations, which is a great innovation. Although Myerson's method cannot achieve the mechanism design of a multi-item, multi-bidder auction, it has indeed been proven to be difficult to calculate [7,8]. With the increase in the number of bidders and items and the complexity of auction forms, especially when bidders' submissions are continuous, the design and verification of mechanisms become extremely difficult [9,10].

Machine learning methods have brought about a turnaround in this matter. For example, Duting [11] designed a simple three-layer MLP network (RochetNet) based on Rochet's idea [12], successfully solving the single-attribute optimal auction problem for multiple items and single bidders. Subsequently, the author proposed a new network structure (RegretNet) to solve the optimal auction problem of multiple items and bidders. The author designed two networks for allocation and payment, where the networks' input is the bidding of multiple bidders for multiple lots, and the output is the probability of each bidder obtaining each item and the price that should be paid. With the emergence of RegretNet, many mechanism design methods for dealing with more complex scenarios have been derived, such as considering the budget of bidders [13], coding, and classifying participants' preferences [14,15]. Meanwhile, machine learning methods have proven effective in practice. Zhe [16] applied the design of an optimal auction mechanism based on neural networks to allocate vehicular edge computing resources. Liu [1] used an optimal auction mechanism based on neural networks in advertising bidding in e-commerce.

The earliest multi-attribute mechanism, consisting of two attributes, i.e., the cost and time of the bidder, was proposed by Ellis [17] and was applied to the auction of highway contracting. Although the auction content is relatively simple and the time factors can be converted to calculate profits, it initiated formal research on multi-attribute auctions. Compared with the single-attribute mechanism, multiple attributes can better take care of the needs of participants [18,19]. Therefore, this mechanism is widely used in real life. But multiple attributes also bring more uncertainty. In 1991, Staschus et al. [20] proposed a multi-attribute auction framework, which was not verified with experiments, that converts all bid attributes of bidders into a single price attribute of the auctioneer. But this method is difficult to calculate in complex scenarios.

On the other hand, attributes and utility functions represent participants' private information. On this topic, Chen Ritzo et al. [21] proposed a multi-attribute reverse-auction mechanism with limited information feedback. Gupta et al. [22] analyzed the information

disclosure mechanism in multi-attribute auctions and designed a multi-attribute auction mechanism with changeable feedback information for experiments. However, the problem of multi-attribute optimal mechanism design has not been solved. Existing research has proved that it is difficult to calculate the multi-dimension of a single bidder [5,23], let alone multiple bidders.

#### 4. Methodology

##### 4.1. Optimal Multi-Attribute Auction

Let us suppose a multi-attribute, multi-item, multi-bidder auction scenario: There are a set of  $N$  bidders  $\{1, 2, \dots, n\}$  with additive valuations and individual rationality, and  $G$  items  $\{1, 2, \dots, g\}$ . Each bidder  $i$  has  $t$  non-price attributes requirements for each item  $\{p_{ij}, q_{ij1}, \dots, q_{ijt}\}$ , where  $p_{ij}$  is the price of the item and  $q_{ijt}$  are non-price attributes. The bid submitted by bidder  $i$  in the auction is  $b_i = \{b_{i1}, b_{i2}, \dots, b_{ig}\}$ , where  $b_{ij} = \{p_{ij}, q_{ij1}, \dots, q_{ijt}\}$  and  $b_i : 2^G \rightarrow \mathbb{R} \geq 0$ .

After receiving bids from all bidders, the auctioneer decides the probability of each person winning each item and the fee. Then, the bidder's expected utility ( $u_i$ ) can be expressed as

$$u_i = \sum_j Pr_{ij}(b_{ij})p_{ij} - Pay_{ij} \tag{1}$$

where the formula indicates that the expected return of the bidder is calculated by subtracting the actual expenditure from the expected expenditure.

In this simple scenario, let us assume that the auctioneer has  $t$  reserved attributes  $R = (r_{j1}, \dots, r_{jt})$  for item  $j$ . If the submitted attribute  $q_i$  exceeds the reserved attribute, it harms the auctioneers' revenue, and the weight is  $W_{jk}$ . Then, the auctioneer's expected utility ( $u_0$ ) can be expressed as follows:

$$u_0 = \sum_i \sum_j Pr_{ij}(b_j)Pay_{ij} - \sum_k^t W_{ijk}(r_{jk} - q_{ijk}) \tag{2}$$

where the formula states that the auctioneer's expected utility is calculated by subtracting the expected revenue from the loss due to the non-price attributes being lower than the reserved attributes.

Due to IR, which also conforms to the characteristics of the economic behavior of the auction, the bidders' purpose must be to maximize profit, at least not to cause losses to themselves. It is foreseeable that if there are no conditional restrictions, the bidder obtains extra income  $e_i$  by submitting an untrue bid  $b'_i$ .

$$e_i = \sum_j \sum_k \varphi_{ijk}(q'_{ijk} - q_{ijk}) \tag{3}$$

where  $\varphi$  represents the weight of lying and  $q$  are untrue non-price attributes. Then, the bidder's expected utility ( $u_i$ ) is

$$u_i = \sum_j Pr_{ij}(b_{ij})p_{ij} + e_{ij} - Pay_{ij} \tag{4}$$

The purpose of the optimal mechanism is to maximize the auctioneer's expected utility; hence, the most direct way to satisfy this purpose is to let bidders submit their actual values. In order to satisfy this condition and make the mechanism sustainable, IC must be satisfied, whereby bidder  $i$ 's income from submitting a true bid must not be lower than that from submitting an untrue one.

$$u_i(b_i, b_{-i}) \geq u'_i(b'_i, b_{-i}) \tag{5}$$

At the same time, the mechanism needs to satisfy IR, whereby it cannot damage the participants' benefits.

$$u_i(b_i, b_{-i}) \geq 0 \tag{6}$$

#### 4.2. Network Design

Since the design of the multi-attribute optimal mechanism mainly consists of payment rules and allocation rules, this paper designed a dual network with reference to Dütting [4]. The Allocation Network and Payment Network were constructed to determine the probability of bidders obtaining items and the proportion of their payments, respectively (Figure 1). The Allocation Network is denoted by  $A^\gamma$ , and the Payment Network is denoted by  $P^\delta$ . Among them,  $\gamma$  and  $\delta$  represent the parameters of the network. These two networks together constitute our optimal mechanism or rules  $(A^\gamma, P^\delta)$ . The input multi-attribute bidding data are extracted into one feature using shared modules. The extracted features are then processed by the Allocation Network and the Payment Network to obtain allocation and payment results, respectively. The two results are used for the computerized status and for updating the networks.

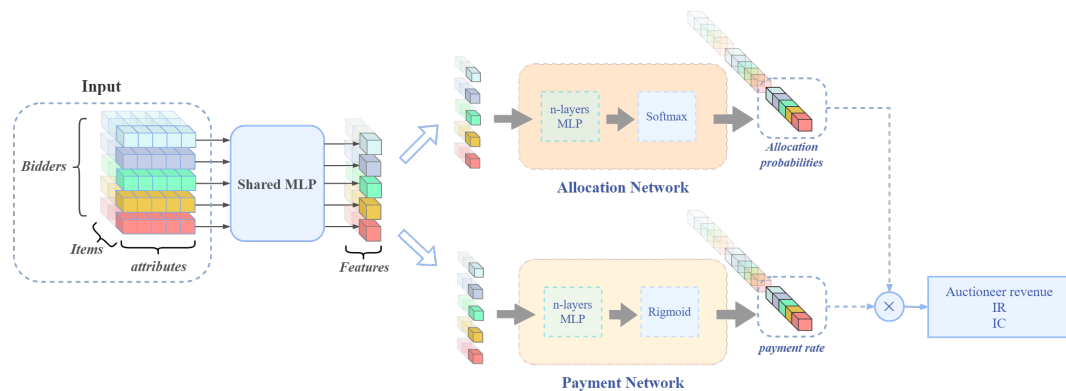


Figure 1. Figure of network structure.

The purpose of the optimal auction is to maximize the auctioneer's expected utility ( $u_0$ ) (Formula (7)) while satisfying the IR and IC conditions. Usually, the artificially designed mechanism considers the participation enthusiasm of bidders and the sustainability of the auction by restricting rules based on IC, such as adjusting payment prices based on ranking or bidding content to satisfy IC constraints.

$$\max u_0 = A^\gamma(b) \times P^\delta(b) \times (\text{price} + W(q - r)) \tag{7}$$

Without constraints, the payment rules would cause significant losses to bidders for deep learning networks. If there were only the IC constraint, it would only make misreporting lose meaning for bidders, as the auctioneer could infinitely increase the payment ratio without considering true and untrue bids. Therefore, Formulas (8) and (9) are used to measure whether the degrees of incentive compatibility and individual rationality are satisfied, respectively. Then, these two parameters are used to assist in network optimization.

$$IC = A^\gamma(b'_i, b_i) \times P^\delta(b'_i, b_i) \times (\text{price}'_i + \varphi_i(q'_i - q_i)) - A^\gamma(b_i) \times P^\delta(b_i) \times \text{price}_i \tag{8}$$

$$IR = A^\gamma(b_i, b_{-i}) \times P^\delta(b_i, b_{-i}) \times \text{price}_i - A^\gamma(b_i) \times \text{price}_i \tag{9}$$

$$\begin{aligned} \max \quad & A^\gamma(b) \times P^\delta(b) \times (\text{price} + W(q - r)) \\ \text{s.t.} \quad & IR \geq 0 \\ & IC = 0 \end{aligned} \tag{10}$$

In previous mechanism design research based on deep learning, researchers commonly used a sigmoid function as the activation function of the Payment Network to convert the specific payment price into a payment proportion between 0 and 1, which increases the generalization performance and robustness of the model. However, due to the limitations of sigmoid functions, the network does not generate a penalty or incentive payment ratio exceeding 100%.

To solve this problem, this paper introduces the Rigmoid function.

$$Rigmoid = \frac{1}{0.5 + e^{-x}} \tag{11}$$

Although this expands the payment ratio to between 0 and 2, the payment ratio does not reach an astonishing 200% due to the limitation of IR. It is even possible to create a “win-win space” without infringing the interests of both parties when the utility functions of bidders and auctioneers are significantly different.

As for the output of the Allocation Network, since the allocated probability of an item is at most 1 and there is no case where all bidders are unqualified, we use a simple SoftMax function as the activation function of the output layer of the Allocation Network.

In addition, compared with single-attribute auction research, this paper faces the problem of mapping multiple attributes. As noted above, the auctioneer does not know the bidders’ true bids or utility function. It would violate the rule to suppose that multiple attributes are mapped as a single attribute by directly using the utility function of the bidder in the data pre-processing stage. If all the attributes submitted by all bidders were added in the hidden layer without processing, in that case, it would result in (1) slow training due to the increase in network parameters and (2) possible over-fitting.

In order to solve the above two foreseeable problems, this paper created a shared encoder to extract the characteristics of bidders’ bids and then output the allocation and payment results that satisfy each bidder of Formulas (5) and (6) with the Allocation Network and Payment Network. Finally, the network is used for optimization according to the feedback of Formulas (5) and (6), and the auctioneer’s income. In this paper, the multi-attribute bidding of each bidder has the same nature, so it can be processed by sharing weights. A four-layer MLP (Multi-Layer Perceptron) module was built. This module was placed before the Allocation and Payment Networks to extract the original  $\{G \text{ items}, N \text{ bids}, t + 1 \text{ attributes}\}$  data into the features of  $\{G \text{ items}, N \text{ bidders}\}$ .

#### 4.3. Model Adjustment

Multi-attribute auctions are complex. To demonstrate the effectiveness of the MLN method, we chose the “Yili” case with rich parameters for the experiments [24]. It is an auction of about 100 units of dairy transportation rights with detailed information on the auctioneer (shipper) and bidders (carriers) (carrier’s shipping cost ( $cost_i$ ), shipping time ( $time_i$ ), damage rate during shipping ( $det_i$ ), and carrier’s shipping capacity ( $cap_i$ )) (Table 1); three preferences for bidding attributes (cases 1, 2, 3); and two preferences for time requirements (cases A, B) (Table 2). The shipper expects that shipping time  $t_a$  is completed within 5 days, and the deterioration rate ( $m_a$ ) of shipping dairy products is less than 5%. The bidders’ delivery performance impacts the auctioneer’s costs, and the revised cost is

$$S = \sum_i cost_i + \Delta t \times t_i + \Delta m \times m_i \tag{12}$$

where  $\Delta m_i = m_i - m_a$ . The revised costs are subsequently deemed to be the auctioneer’s costs.

**Table 1.** Bidders’ information.

Attribute	a1	a2	a3	a4	a5	a6	a7	a8	a9
Cost (USD 100)	9.5	9	10	10	7	8	9.5	9	8.5
Transportation time (day)	5	4	2	3	6	8	6	4	7
Deteriorate rate (percent)	5	6	4	2	7	6	3	1	7
Capacity (truckload)	30	25	20	30	35	20	25	30	30

**Table 2.** Auction settings based on auctioneer’s preference.

Case A	$\Delta ti = 2 \times (\max\{0, (ti - ta)\}) - 2 \times (\max\{0, (ta - ti)\})$ (focus on speed)
Case B	$\Delta ti =  ti - ta ^{0.5}$ (focus on punctuality)
Case 1	$W_{\text{time}} = 0.1, W_{\text{Deteriorate rate}} = 0.1$ (focus on cost)
Case 2	$W_{\text{time}} = 2, W_{\text{Deteriorate rate}} = 0.1$ (focus on time)
Case 3	$W_{\text{time}} = 0.1, W_{\text{Deteriorate rate}} = 2$ (focus on service quality)

As mentioned above, the model’s loss function should be composed of three constraints, auctioneer expenditure, IC, and IR, which is different from the conventional model training process. In addition, in the “YILI” case, each bidder has transportation capacity limitations. Using the SoftMax function in the output layer of the Allocation Network is likely to output results that exceed the capabilities of bidders.

Constructing several loss functions can solve these problems. In this case, the higher the payment ratio of the bidder, the higher the auctioneer’s expenses. Therefore, maximizing the expected revenue is changed to minimizing the expected fees ( $p_0$ ) (Formula (13)).

$$\min p_0 = A^\gamma(b) \times P^\delta(b) \times S_i \tag{13}$$

$ext_i$  stands for the extra benefits that bidders obtain by misreporting (Formula (14)).  $ext_i$  is used to limit the motivation of bidders to “lie” and ensure that bidders do not lose money in the auction as much as possible.

$$ext_i = \text{ReLU}\left(A^\gamma(b'_i, b_i) \times P^\delta(b'_i, b_i) \times \text{price}'_i - A^\gamma(b_i) \times P^\delta(b_i) \times \text{price}_i\right) \tag{14}$$

$def_i$  is a new IR constraint, which means that the deficit calculation function of the profit part is weakened (Formula (15)).

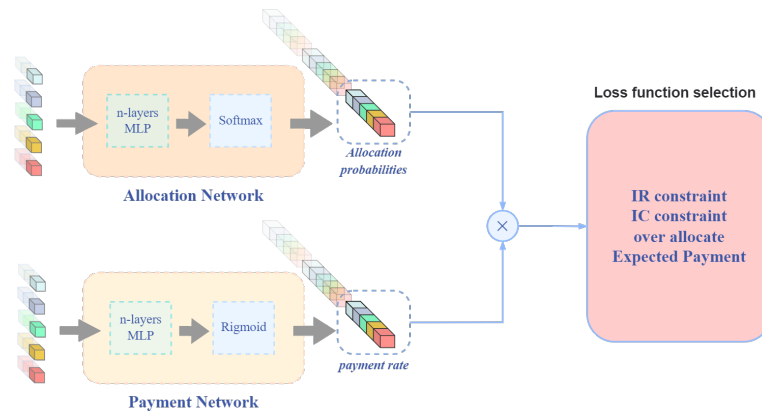
$$def_i = \text{LeakyReLU}\left(A^\gamma(b'_i) \times \text{price}'_i - A^\gamma(b'_i) \times P^\delta(b'_i) \times \text{price}'_i\right) \tag{15}$$

$rev_i$  measures whether the allocation result is over-allocate (OA). Then, it is necessary to determine which loss function to use to optimize the model based on the priority of OA = IR = IC > goal (Figure 2).

$$rev_i = \text{ReLU}(A^\gamma(b_i) \times 100 - \text{Capacity}) \tag{16}$$

Bidders explore how to adjust misreport  $b'_i$  under the rules during the auction process to pursue higher profits. The problem is solved by calculating the gradient of the bidding content based on the earnings from misreports.

$$\nabla_{b'_i} A^\gamma(b'_i, b_i) \times P^\delta(b'_i, b_i) \times \text{price}_i \tag{17}$$



**Figure 2.** The network state is judged according to the loss function; then, the corresponding loss function is selected to optimize the network.

The training process of the MLN model is described below.

Training Process
<b>hyper-parameters:</b> learning rate of the Network $\eta = 0.0001$ , learning rate of updating misreport $\xi = 0.01$ , batch size = 20, Penalty weight $\rho > 0$
<b>Initialize:</b> Lagrange multipliers $\lambda, \mu, \nu \in \{R\}$ , network parameters $\gamma, \delta \in \{R\}$
For 0 to data size/batch size:
Batch $B = \{b^1, b^2, \dots, b^{20}\}$
For 0 to B
Input true bid $b_i$ into Allocation and Payment Network $\rightarrow A^\gamma(b), P$
Update misreport $b'_i$ by calculating $b_i$ gradient:
$b'_i = b_i - \xi \nabla_{b'_i} A^\gamma(b'_i, b_i) \times P^\delta(b'_i, b_i) \times \text{price}_i$
Input misreport $b'_i$ into Allocation and Payment Network $\rightarrow A^\gamma(b'_i), P^\delta(b'_i)$
Calculate revised cost: $S = \sum_i \text{cost}_i + \Delta t \times t_i + \Delta m \times m_i$
Calculate the bidder's deficit $def$ (15).
Calculate the additional benefit bidders gain by misrepresenting $ext$ (14).
Calculate the degree to which the allocation results exceed the bidder's transportation capacity $rev$ (16).
loss function judgment:
if $def > 0$ : $loss = def$
else if $ext > 0$ : $loss = ext$
else if $rev > 0$ : $loss = rev$
else: $loss = p_0$ (13)
loss.backward()
Updating model

### 5. Experiments

The range of data attributes was determined with reference to Table 1:

$$\text{cost}_i \sim \frac{1}{2} \times U_{\text{cost}}[7, 10] \in \mathbb{Z} \tag{18}$$

$$\text{time}_i \sim U_{\text{time}}[2, 8] \in \mathbb{Z} \tag{19}$$

$$\det_i \sim U_{\det}[1,7] \in \mathbb{Z} \tag{20}$$

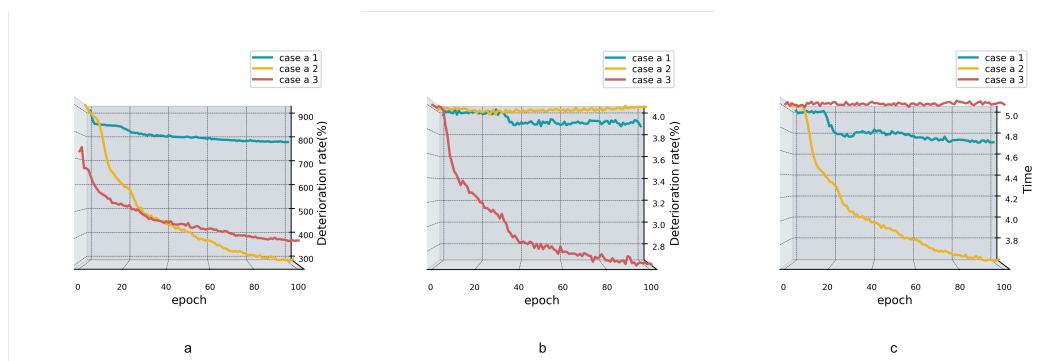
$$\text{cap}_i \sim 4 \times U_{\text{cap}}[4,8] \in \mathbb{Z} \tag{21}$$

Then, auction datasets (1), (2), and (3) were generated with uniform distribution, and three experiments were conducted on each dataset corresponding to the same cases (A1, A2, A3) of auction preferences as the original.

(1) One item, nine bidders.

Randomly generated bidder data were used for model training. Then, we used the same information as Zhang as the input data for the model during validation and compared the results of testing with other methods.

Figure 3 and Table 3 show that the performance of the rules generated with the MLN method in terms of time and deterioration rate was similar to that of the method by Zhang and was subject to IC constraints. In case A1, the auctioneer had a low weight of 0.1 for both time and deterioration rate, which had little impact on the correction cost function. MLN maintained a good level of time and deterioration rate and reduced expenses for auctioneers by approximately 55 (USD 100) without causing deficits to bidders.



**Figure 3.** Experimental results. (a) Auctioneer’s expenditure results caused by the model’s output rules during training under three case settings. (b) Time results. (c) Deterioration rate results.

**Table 3.** Results of experiment (1).

	A1			A2			A3		
	Ours	Zhang	P-VCG [25]	Ours	Zhang	P-VCG	Ours	Zhang	P-VCG
Time (day)	5.64	5.5	6.25	3.78	3.25	6.25	4.24	3.75	6.25
Deterioration (%)	5.6	5	5.25	3.6	3.25	5.25	2.7	2.5	5.25
Cost	848.50	904	937	441.86	884	1469	483.72	918.5	1108
IC	0	0	None	0	0	None	0	0	None

In case A2, the time and deterioration rate weights were 2 and 0.1, respectively, due to the auctioneer’s preference for time being “faster is better” and four out of nine bidders having transportation times shorter than  $ta = 5$  days, which could create more revenue for the auctioneer. This gave the MLN method a better chance at improving performance, ultimately resulting in a reduction of more than half in the revised cost of the auctioneer. In case A3, the time and deterioration rate weights were 0.1 and 2, respectively. The revised cost was higher than that in case A2, for the magnification of  $\Delta mi$  was smaller than  $\Delta ti$ , but our method still made significant progress.

The MLN method could achieve good time and deterioration rates and reduce expenses for auctioneers under all three preferences. The reason is that it did not transfer the profits obtained thanks to the bidder’s good performance to the bidder, and it can be seen

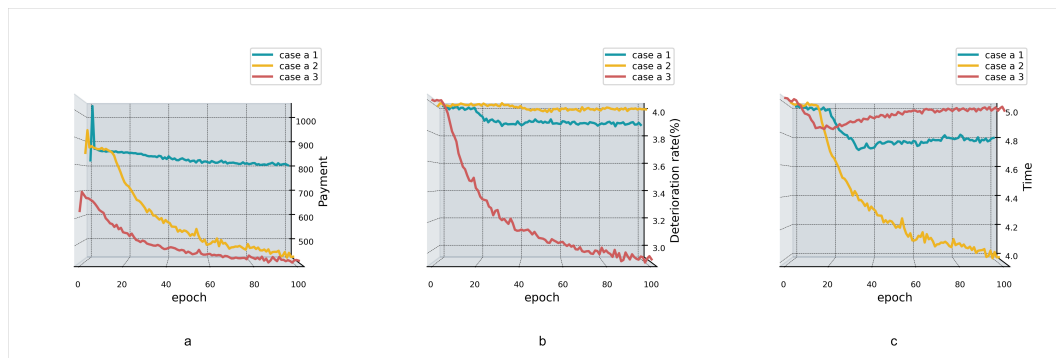


that the bidder had no deficit in the auction and received an higher average price than the bid. Overall, the auction rules generated by the model are sustainable and can significantly reduce auctioneers' expenses.

The mechanism design of multi-attribute, multi-item auctions is complex. To further expand the experiment, the original numbers of items and participants were modified to demonstrate the robustness of the MLN model and its contribution to multi-attribute, multi-item auctions.

(2) Three items, nine bidders.

The number of items was increased to three. The experiment used randomly generated data that followed the data distribution for model training (Figure 4) and testing and presented the last epoch's results (Table 4).



**Figure 4.** Experimental results. (a) Auctioneer's expenditure results caused by the model's output rules during training under three case settings. (b) Time results. (c) Deterioration rate results.

**Table 4.** Results of experiment (2).

	A1	A2	A3
Time	4.79	3.89	4.92
Deterioration rate (%)	3.90	3.96	2.96
Payment (USD 100)	804.86	385.15	450.85
IR	−13.2	−15.3	−37.3
IC	0	0	0

Similar to experiment (1), the average expenditure of auctioneers under the three cases was the same as the trend in (1) due to the weights of time and damage rate preferences. After increasing the number of items, MLN also achieved good results. It was found that the average cost of testing results was relatively lower than that obtained using nine-bidder original data because there was a strong correlation between the transportation time and damage rate of bidders in the original dataset, unlike in random bidding generated based on the distribution.

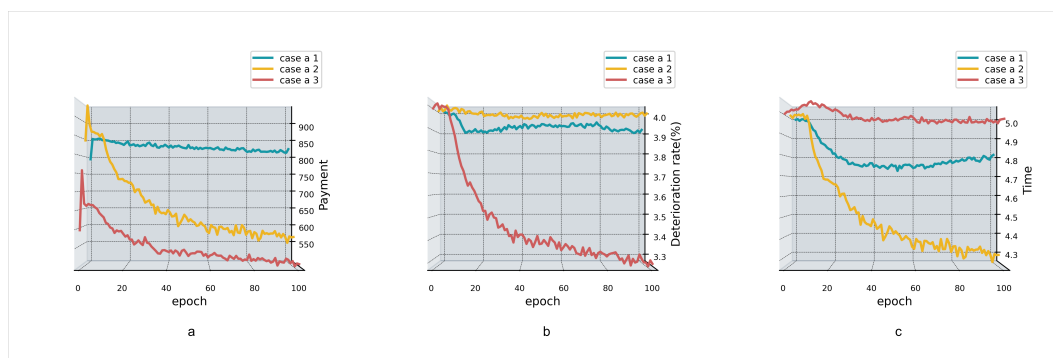
(3) Four items, seven bidders.

To further test the generalization performance of MLN, this paper conducted experiments on four items and seven bidders. The bidder data were still random, followed the same distribution, and were used during testing.

It was noticed that compared with the setting of nine bidders, the experimental performance of seven bidders was slightly inferior (Table 5), especially in cases A2 and A3, where the average payment of the auctioneer in experiment (3) increased compared with experiment (2) (Figure 5). The reason is that as the number of bidders increases, the bidding base conducive to auctioneers also increases relatively.

**Table 5.** Results of experiment (3).

	A1	A2	A3
Time	4.81	4.36	4.93
Deterioration rate (%)	3.91	3.99	3.32
Payment (USD 100)	817.76	595.60	508.24
IR	−19.26	−24.04	−6.22
IC	0	0	0

**Figure 5.** Experimental results. (a) Auctioneer's expenditure results caused by the model's output rules during training under three case settings. (b) Time results. (c) Deterioration rate results.

## 6. Conclusions

This paper proposes a dual network based on multi-scale loss and shared modules that encodes the inputs of multiple attributes of bidders into a single feature, solving the problem of incomplete information and computation in designing multi-attribute mechanisms. The scene settings for multi-attribute auctions are complex and diverse. In the experimental phase, the post-paid "YILI" case was used. The loss function of the network was adjusted according to the limit of the number of bidders allocated, and the model significantly reduced the expenditure of the auctioneer. Subsequently, in the expansion experiment, the model also performed well when dealing with different combinations of bidders, and numbers of bidders and items, demonstrating its generalization performance and robustness.

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**Conflicts of Interest:** We declare that we have no financial and personal relationships with other people or organizations that can inappropriately influence our work, and there are no professional or other personal interests of any nature or kind in any product, service, and/or company.

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