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Joint Optimal Production Planning in Supply Chains under Diverse Low-Carbon Policies

Chenglin Hu [1], Junsong Bian [2,*] and Kin-Keung Lai [3,*]

[1] College of Intelligent Manufacturing, Tianjin Sino-German University of Applied Sciences, Tianjin 300350, China; hclin112@126.com
[2] Department of Supply Chain Management and Information Systems, Rennes School of Business, 2 Rue Robert d'Arbrissel, 35065 Rennes, France
[3] Industrial and Manufacturing Systems Engineering Department, Hong Kong University, Hong Kong
* Correspondence: jason.bian@rennes-sb.com (J.B.); mskklai@outlook.com (K.-K.L.)

Abstract: This paper studies the impact of diverse carbon emission regulations on optimal operation decisions by exploiting the economic production lot model of a multi-manufacturer system in a low-carbon environment. The optimal production planning for multiple enterprises in terms of cost optimization and carbon emission optimization are presented in various scenarios of the fully decentralized system under carbon taxation, cap and trade, and mandatory carbon cap. We prove a unified framework for modeling the scenarios enabled by carbon tax and cap-and-trade policies. Furthermore, the decision vector of the optimal production lot is obtained by analyzing the joint decision scenario based on shared carbon emission caps. We give a comparative analysis of two settings of fully decentralized and carbon quota sharing. The results show that the decision of joint production and stock preparation can reduce the total operational costs of the multi-enterprise system, but the realized carbon emissions may not change or only increase slightly. Our study provides a useful reference for government carbon emission regulation and enterprise operation decision optimization under carbon neutrality and a carbon peak environment.

Keywords: low-carbon supply chain; carbon emission regulations; decentralized decision-making; carbon quota sharing; joint production planning

MSC: 90-11

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1. Introduction

In a low-carbon economy, firms can reduce carbon emissions by either adopting direct emission reduction technology in response to the pressure of carbon emissions or adjusting their operations strategies to meet various low-carbon constraints. In this paper, the typical economic production quantity (EPQ) model is used to examine production tactics under the constraint of diverse low-carbon policies, such as mandatory carbon cap, cap and trade, and carbon taxation. Although a couple of basic carbon emission regulations have been explored in the literature, these policies, including mandatory carbon emission policy, are among the most common and easy to operate. This study shows the feasibility of adjusting operation strategy to realize carbon emission control by taking the cost optimization problem under low-carbon regulations.

Supply chain multi-links like procurement, production, and inventory usually generate carbon emissions in different forms, and the trade-off among the links also has an important impact on carbon emissions control, so the joint decision-making among these functions has attracted much attention. For example, production quantity, equipment installation frequency, and inventory level are interrelated, and affect the cost and carbon emission level simultaneously. To solve these problems, the economic order quantity model under carbon emission constraints is used to discuss the optimal order or production quantity and emission control, which is also extended to the newsvendor environment. Furthermore, there is research on joint decision-making of different supply chain functions based on cost and emission reduction considerations. Similarly, research concerning the optimal production-inventory decision of closed-loop supply chains under carbon tax policy or considering the comprehensive decision of inventory path and operation adjustment emerges. Hence, researchers have refined from general goods to special goods inventory and extended from a simple secondary to a closed-loop supply chain. However, the issues related to carbon emission quota sharing under the mandatory carbon cap policy that the present study pays attention to have not been reflected yet in the existing literature.

Driven by practice and theory, researchers have realized the importance of integrating carbon emissions and sustainability into the supply chain from the relationship between supply chain management and environmental protection. From the angle of measuring the carbon footprint of the supply chain, carbon emission reduction investment often brings great cost pressure to enterprises and affects supply chain efficiency and sustainable development. Hence, there are studies on carbon emission reduction technology investment and carbon emission reduction initiative choice at the supply chain level, such as discussing the joint investment behavior of supply chain members in emission reduction technology. Although these researches pay attention to the interaction of carbon emission reduction strategies at the supply chain level, which provides new insights for inter-enterprise emission reduction, it seldom involves the influence of carbon emission quota sharing on inter-enterprise decision-making interaction.

Low carbon policy can also influence strategic supply chain decisions like supply chain network design. The existing studies usually relate carbon emissions to transportation links for investigating how to choose and optimize transportation modes to reduce carbon emissions as much as possible. Furthermore, research exists in response to the tactical level through operations integration, such as order quantity and inventory when considering carbon emission regulation and carbon trading, which points out the necessity and importance of considering carbon emissions in supply chain operations. Along this vein, this paper integrates carbon emissions into the traditional EPQ model to explore optimal production decisions under diverse scenarios.

The distinct effects of carbon policy heterogeneity on supply chain operations are also stressed in the literature. In view of the effect of emission reduction achieved in practice, examining the influence of different carbon regulations on a firm's decision-making and social welfare not only helps enterprises to cope with various policy constraints flexibly but also inspires the government to formulate optimal policies. In this paper, we use nonlinear optimization theory to build a mathematical programming model to study the impact of various low-carbon policies on multi-enterprise operations. In this sense, our models show the power of simple models to describe low-carbon operation problems and the significance of describing low-carbon joint decision-making problems among enterprises in some typical scenarios. Particularly, the influence of different strategies of using carbon caps on the operations in this paper has not been covered previously.

The remainder of the paper is organized as follows. A literature review is given in Section 2. In Section 3 presents the multi-firm decentralized system under various regulative low-carbon policies including carbon tax, cap and trade and mandatory cap policies. Section 4 proceeds to discuss the joint economic production decisions under caps sharing. A complete comparison and numerical analysis of these two scenarios come in Section 5. Concluding remarks are in Section 6.

2. Literature Review

This study is related to several streams of literature on low-carbon environment, which will be reviewed as follows.

First, our work is relevant to the literature on considering factors of carbon emissions and sustainability in EOQ/inventory models. Ping He et al. found the optimal production and emissions of a firm under the regulations of total control and trading and carbon tax [1]. Vincent Hovelaque et al. proposed an EOQ model with carbon constraints. The model considered the link between inventory policy, carbon emissions, price, and environment-related demand to maximize the firm's profits while reducing emissions [2]. Ali Bozorgi et al. proposed an inventory model applicable to items that must be stored at a specific temperature [3]. Taleizadeh et al. constructed four new sustainable EPQ models to address different inventory shortage situations in the production system [4]. Liao et al. incorporated demand uncertainty into the EOQ model and solved the multi-objective optimization problem by deducing the optimal acquisition strategies for remanufactured products under a cap-and-trade policy [5]. S. Wang et al. compared the EOQ and JIT inventory models considering carbon emissions generated during inventory and transportation processes and obtained the cost difference point [6]. Rabta constructed an EOQ model under the circular economy and represented the circularity level of product production in exponential form [7]. Gharaei et al. developed a sustainable EOQ model for growing products under environmental costs and constraints. They found that planning projects considering sustainable development factors not only increased order quantity but also significantly shortened the growth cycle [8].

Our study is related to the research comparing inventory management under different carbon policies. Gökce Palak et al. analyzed the impact of different carbon regulation mechanisms on the costs and emissions of replenishment programs and supply chains in the context of biofuels [9]. Chung-Yuan Dye and Chih-Te Yang investigated the effects of trade credits and different carbon policies on inventory management models [10]. A bi-objective continuous review inventory model considering cost and carbon emission was developed by Konur et al. [11]. Tang et al. studied the (R, Q) inventory control models under three carbon policies: restricted percentage reduction target, carbon tax, and carbon offset [12]. Yu et al. innovatively considered supply strategy and investment in preservation technology to reduce the deterioration rate [13]. Mishra et al. constructed three inventory models under different shortage situations in the context of manufacturers investing in green technology and found that the sustainable EPQ carbon tax and cap partial backlogging model has higher profit and lower green investment cost [14]. Ruidas et al. established incomplete production inventory models under four popular carbon policies. They compensated for the shortcomings of previous studies by setting carbon emission parameters as interval numbers rather than fixed values. In contrast to the studies considering multiple carbon policies [15], As'ad et al. investigated the impact of different implementation methods of carbon quota policy (i.e., implementing in the entire planning scope or each period) on the sustainable, dynamic batch model for frozen products [16]. Turken et al. constructed a single-vendor-multi-buyer-multi-product model in which the buyer and the vendor are subject to various environmental regulations [17].

Our study is also relevant to the literature on the investment behavior of companies in emissions reduction and inventory management issues. Ayşegül Toptal et al. analyzed the inventory replenishment and carbon reduction investment decisions of retailers under three different carbon regulatory policies [18]. Li et al. developed a single-warehouse multi-retailer inventory management model and found that considering carbon emission costs increases the reorder interval for both the warehouse and retailers [19]. Lu et al. investigated the joint investment behavior of supply chain companies in reducing emissions under cap-and-trade and carbon offset policies [20]. J.-Y. Lee studied the optimal order quantity and carbon reduction investment in the EOQ model under the cap-and-price policy and found that investing in carbon reduction reduces carbon emissions per replenishment and per production unit [21]. Mishra et al. studied the replenishment problem of non-instantaneous deteriorating seasonal products based on joint pricing [22]. Mashud et al. investigated sustainable inventory issues for retailers selling non-instantaneous deteriorating products, considering green technology investment in both owned and leased warehouses, and found that investing in green technology can reduce emission costs and increase total profits [23]. Qi et al. developed a joint decision-making model for emission reduction investment and ordering under the cap-and-trade policy and investigated the impact of risk aversion behavior and investment coefficients on the optimal decision for risk-averse companies [24]. Halat et al. studied the inventory cooperation game problem of multiple supply chains under carbon tax policy, considering four structures: decentralized, downward vertical, upward vertical, and horizontal cooperation [25]. Fu et al. innovatively examined the impact of emission asymmetry and carbon tax on the production and green technology investment of competing companies [26].

The literature on production strategies and pricing issues in the context of carbon emissions is also close to our work. Du et al. considered a low carbon premium for consumers in an aggregate control and trading mechanism to promote manufacturers' participation in low-carbon production [27]. Arda Yenipazarli used the Stackelberg game model to investigate the effect of an emissions tax on manufacturers' optimal production and pricing strategies [28]. Xu et al. investigated the production and pricing problems of a supply chain comprising manufacturers and retailers of two made-to-order (MTO) products [29]. Liu et al. proposed three optimization models to analyze the impact of three regulatory models, mandatory carbon capacity, carbon tax, and cap-and-trade, on remanufacturers' production quantity decisions [30]. Wang et al. investigated the manufacturing and remanufacturing problems of manufacturers considering capital and carbon emission constraints [31].

Other literature close to our study is about integrated inventory control and transportation planning issues in the context of carbon emissions. Konur and Schaefer obtained optimal order quantities for retailers under different carrier regulations and illustrated the impact of regulatory parameters on carrier preferences [32]. Hariga et al. developed three models to evaluate the impact of carbon emissions in the transportation and storage of cold chain products in a multi-stage supply chain [33]. Rout et al. constructed a sustainable supply chain inventory management model for a single supplier and multiple buyers, considering product deterioration and imperfect production under carbon emission constraints [34]. Konur et al. constructed a mixed-integer nonlinear programming model considering zoning to study integrated regional partitioning and fleet composition [35].

Several other relevant studies have explored existing emissions reduction methods or policies. Park et al. discussed the impact of levying carbon costs on supply chain structure and social welfare from the perspective of central policymakers in three different monopolistic competitive environments [36]. Zhao et al. studied the long-term joint emission reduction and low carbon propaganda in the supply chain by constructing three differential games [37]. Zhou et al. developed a three-stage game model involving consumers, firms, and the government [38]. Xu et al. considered a two-tier supply chain under aggregate control and transaction regulation and compared the profits of decentralized and centralized systems [39]. Chen et al. proposed that punishing the carbon emissions of individual companies may increase the carbon emissions of the entire supply chain due to negative externalities [40]. Lee et al. suggested that one company in the supply chain supporting another company's carbon emissions reduction efforts benefits all participants [41]. Gao et al. compared two methods for companies to reduce emissions, namely direct investment in green technology and indirect carbon offsetting [42]. Guo et al. constructed a dynamic game theory model and compared the business models of credit wholesalers and voluntary brokers for public emissions reduction projects [43]. Kaur et al. proposed a sustainable disaster-resistant supply chain management model for procurement and logistics under a cap-and-trade policy, establishing an effective and optimal balance between a firm's economic benefits and environmental responsibility [44]. Yenipazarli built a two-stage duopoly model to investigate the impact of consumer demand with brand preferences, competitor actions, and carbon tax policy on firms' environmental R&D under uncertainty [45]. Y. Wang et al. studied a low-carbon supply chain consisting of a small and medium-sized manufacturer and a leading retailer. They found that altruism can improve the efficiency of the supply chain [46]. Feng et al. investigated the profit distribution rules for joint replenishment of retailers under cap-and-trade and divided retailers into two categories, efficient with altruistic behavior and inefficient. They found that the altruism parameter plays a critical role in carbon quota transfers [47]. More research articles related to this field and green emissions reduction are found in the literature review papers [48–51].

Though closely related to our study is the Chen et al. (2019) [52], their work only considers single firm EOQ production problem. Moreover, they never consider the joint production optimization and carbon caps sharing situation, which is exactly what we examine in the EPL setting in this paper. Other close work in some studies discusses integrating carbon emissions and sustainability into the supply chain from the relationship between supply chain management and environmental protection. For example, Carbon-Trust (2006) focused on developing carbon footprint measurement tools at the supply chain level [53]. After obtaining the energy consumption data of each link in the supply chain of different products, the carbon footprint of each product can be obtained by this tool. Other researchers focus on and measure the carbon footprint of the supply chain, such as Cholette and Venkat (2009) [54], Sundarakani et al. (2010) [55], and Chaabane et al. (2012) [56]. Furthermore, considering that carbon emission reduction investment often brings great cost pressure to enterprises and affects supply chain efficiency and sustainable development, there are studies on carbon emission reduction technology investment and carbon emission reduction initiative choice at the supply chain level. Chen et al. (2019) studied the EOQ problem of a single supplier and a single buyer. They found that punishing each enterprise for emissions alone might lead to higher overall emissions of the supply chain [40].

3. Decentralized System under Regulative Low-Carbon Policies

This study starts from the classic economic production quantity (EPQ) model to focus on multiple firms constrained by regulative low-carbon policies. Consider N manufacturers make production decisions independently in a low-carbon environment. As stated in the EPQ model, manufacturer i faces his own deterministic market demand d_i. And his production rate is a constant p_i, which can be regarded as production capacity. To let the following formulation make sense, we need condition $d_i/p_i < 1$. Manufacturers organize production and seek optimal output or supply levels by considering demand characteristics and production capacity comprehensively. In the framework of the joint inventory optimization model based on the extended model of economic production quantity (EPQ) model, we consider that each manufacturer produces one kind of product with the demand $d_i > 0$ and variable production cost c_i. Similarly, we assume the supply rate a constant $p_i \geq 0$. The ordering cost and the storage cost rates are $a_i \geq 0$ and $h_i \geq 0$, respectively. We eliminate the impact of environmental change on inventory costs as well as the limit of order quantity and storage capacity.

In the low-carbon era, the regulative carbon policies will substantially affect the business and economic activities relative to the situation without carbon constraints. We, therefore, consider three common policies, i.e., mandatory cap, cap and trade, and carbon taxation, and examine their impacts on supply chain operations. To reflect the acting effect of carbon policies, we assume that all ordering, inventory, and production will generate carbon emissions, the corresponding rate coefficient of which is shown in Table 1. Furthermore, assume that these coefficients do not change over time.

Note that the time dimension of the above notations is measured in years for all $i \in N$. Throughout the paper, we attach an embellishment hat "$\wedge$" to indicate their relationship to carbon emission.

In this part, we first examine the scenario of completely decentralized decision-making for a dyadic supply chain and then consider the situation with a single supplier and multiple manufacturers. As mentioned above, the order quantity Q_i implies that the order period and frequency satisfy $L_i = Q_i/d_i$ and $m_i = d_i/Q_i$, respectively. The retailer's costs are made up of the annual ordering cost $a_i d_i/Q_i$, the annual inventory cost $h_i Q_i(p_i - d_i)/(2p_i)$, and the annual production cost $c_i d_i$, where $Q_i(p_i - d_i)/(2p_i)$ represents the annual average inventory level. The carbon emission associated with the supply chain operations includes ordering-incurred emission $\hat{a}_i d_i/Q_i$, inventory-incurred emission $\hat{h}_i Q_i(p_i - d_i)/(2p_i)$, and production-incurred emission $\hat{c}_i d_i$.

Table 1. Classification of relevant literature.

Problem Presented in the Literature	Sources
Optimization of EOQ/inventory model considering carbon emissions	[1–8,52]
Comparison of Inventory Management under different carbon policies	[9,17]
The investment behavior of companies in emissions reduction and inventory management issues	[18,26]
Production strategies and pricing issues in the context of carbon emissions	[27–31]
Integrated inventory control and transportation planning issues in the context of carbon emissions	[32–35]
Research exploring existing emission reduction methods or policies	[36–43]
Sustainable disaster resilient supply chain management under carbon policy	[44]
Impact of carbon tax policy and other policies on corporate environmental R&D	[45]
The role of altruism in low carbon supply chains	[46,47]
Review of literature in the field of carbon constraint	[48–51]

3.1. Carbon Tax or Cap-and-Tarde Policy-Based Optimal Production Quantity

In a low-carbon economy, it is natural to think about the impact of low-carbon policies on economic activities and operations management. Therefore, we consider the impact of two typical decarbonization schemes, i.e., carbon taxation and subsidy, on supply chain operations. We refer to carbon taxation and cap-and-trade policies as carbon pricing policies since both charge costs according to the units of emission permits. The pricing of carbon taxation reflects the emission penalty, while the pricing in cap and trade regulation represents the opportunity cost of consuming carbon rights. Unlike prior literature, we mathematically formulate these two schemes in a unified framework as follows, except that the term C_{GHG_i} vanishes under the cap taxation policy. We give the n-firm system total cost $AC(Q_i, \cdots, Q_n)$ as follows:

$$\min_{Q_i \geq 0} AC(Q_i, \cdots, Q_n) = \sum_{i=1}^{N} \left[a_i \frac{d_i}{Q_i} + h_i \frac{p_i - d_i}{2p_i} Q_i + c_i d_i + \tau \left(\hat{a}_i \frac{d_i}{Q_i} + \hat{h}_i \frac{p_i - d_i}{2p_i} Q_i + \hat{c}_i d_i - C_{GHG_i} \right) \right] \tag{1}$$

where τ ($\tau > 0$) represents the tax rate under carbon tax policy or emission permit price under cap and trade policy, which can be regarded as the governmental penalty rate on carbon emission. Here we denote the term $\hat{E}_i = \hat{a}_i \frac{d_i}{Q_i} + \hat{h}_i \frac{p_i - d_i}{2p_i} Q_i + \hat{c}_i d_i - C_{GHG_i}$ as the net carbon emission generated under each policy, and C_{GHG_i} disappears as cap tax regulation applies. Under the cap-and-trade policy, the positive (negative) $\hat{E}_i$ means permits purchasing cost (sales revenue). Analyzing the general total cost incorporating net emission under taxation or cap and trade gives the proposition as follows:

Corollary 1. *The optimal solutions of the economic production lot model for a single firm under carbon tax policy or cap and trade policy gives optimal production quantity* $Q_{i,\tau} = \sqrt{\frac{a_i + \tau\hat{a}_i}{h_i + \tau\hat{h}_i} \frac{2p_i d_i}{p_i - d_i}}$, *frequency* $m_{i,\tau} = \sqrt{\frac{h_i + \tau\hat{h}_i}{a_i + \tau\hat{a}_i}} \sqrt{\frac{d_i(p_i - d_i)}{2p_i}}$, *emission* $\hat{E}_{i,\tau} = \left(\hat{a}_i \sqrt{\frac{h_i + \tau\hat{h}_i}{a_i + \tau\hat{a}_i}} + \hat{h}_i \sqrt{\frac{a_i + \tau\hat{a}_i}{h_i + \tau\hat{h}_i}} \right) \sqrt{\frac{d_i(p_i - d_i)}{2p_i}} + \hat{c}_i d_i$ *and total cost* $AC = 2\sqrt{(a_i + \tau\hat{a}_i)(h_i + \tau\hat{h}_i)} \sqrt{\frac{d_i(p_i - d_i)}{2p_i}} + c_i d_i + \tau(\hat{c}_i d_i - C_{GHG_i})$ *with the following features: (1)* $\frac{\partial AC}{\partial \tau} \geq 0$ *holds if* $C_{GHG_i} \leq \sqrt{\frac{2\hat{a}_i \hat{h}_i d_i (p_i - d_i)}{p_i}} + \hat{c}_i d_i$; *(2)* $\frac{\partial AC_\tau}{\partial \tau} < 0$ *if* $C_{GHG_i} > \left(\hat{h}_i \sqrt{\frac{a_i}{h_i}} + \hat{a}_i \sqrt{\frac{h_i}{a_i}} \right) \sqrt{\frac{d_i(p_i - d_i)}{2p_i}} + \hat{c}_i d_i$; *(3)* $\frac{\partial AC_\tau}{\partial \tau} > 0$ *for* $0 < \tau < \tau_0$ *and* $\frac{\partial AC_\tau}{\partial \tau} \geq 0$ *for* $\tau \geq \tau_0$ *if* $\sqrt{\frac{2\hat{a}_i \hat{h}_i d_i (p_i - d_i)}{p_i}} + \hat{c}_i d_i + \hat{c}_i d_i < C_{GHG_i} < \left(\hat{h}_i \sqrt{\frac{a_i}{h_i}} + \hat{a}_i \sqrt{\frac{h_i}{a_i}} \right) \sqrt{\frac{d_i(p_i - d_i)}{2p_i}} + \hat{c}_i d_i$; *(4) Systemwide emission* $\hat{E}_\tau$ *always decreases in emission penalty rate* τ, *where* τ_0 *is the unique solution of equation* $\left(\hat{h}_i \sqrt{\frac{a_i + \tau\hat{a}_i}{h_i + \tau\hat{h}_i}} + \hat{a}_i \sqrt{\frac{h_i + \tau\hat{h}_i}{a_i + \tau\hat{a}_i}} \right) \sqrt{\frac{d_i(p_i - d_i)}{2p_i}} = C_{GHG_i} - \hat{c}_i d_i$ *if the following condition* $\sqrt{\frac{2\hat{a}_i \hat{h}_i d_i (p_i - d_i)}{p_i}} < C_{GHG_i} - \hat{c}_i d_i < \left(\hat{h}_i \sqrt{\frac{a_i}{h_i}} + \hat{a}_i \sqrt{\frac{h_i}{a_i}} \right) \sqrt{\frac{d_i(p_i - d_i)}{2p_i}}$ *holds.*

Proposition 1. *The optimal solutions of economic production lot model for multiple firms under carbon tax policy or cap and trade policy gives optimal production quantity* $Q_{i,\tau} = \sqrt{\frac{a_i+\tau\hat{a}_i}{h_i+\tau\hat{h}_i}\frac{2p_id_i}{p_i-d_i}}$*, total emission* $\hat{E}_\tau = \sum_{i\in N}\left[\left(\hat{a}_i\sqrt{\frac{h_i+\tau\hat{h}_i}{a_i+\tau\hat{a}_i}} + \hat{h}_i\sqrt{\frac{a_i+\tau\hat{a}_i}{h_i+\tau\hat{h}_i}}\right)\sqrt{\frac{d_i(p_i-d_i)}{2p_i}} + \hat{c}_id_i\right] = \widetilde{E}_\tau(\tau) + \sum_{i\in N}\hat{c}_id_i$ *and systemwide cost* $AC = \sum_{i=1}^n\left[2\sqrt{(a_i+\tau\hat{a}_i)(h_i+\tau\hat{h}_i)}\sqrt{\frac{d_i(p_i-d_i)}{2p_i}} + c_id_i + \tau(\hat{c}_id_i - C_{GHG_i})\right]$ *with following features: (1)* $\frac{\partial AC}{\partial\tau} \geq 0$ *holds if* $\sum_{i\in N}C_{GHG_i} \leq \inf\left\{\widetilde{E}_\tau\right\} + \sum_{i\in N}\hat{c}_id_i$*; (2)* $\frac{\partial AC}{\partial\tau} < 0$ *if* $\sum_{i\in N}C_{GHG_i} > \sup\left\{\widetilde{E}_\tau\right\} + \sum_{i\in N}\hat{c}_id_i$*; (3)* $\frac{\partial AC_\tau}{\partial\tau} > 0$ *for* $0 < \tau < \tau_0$ *and* $\frac{\partial AC_\tau}{\partial\tau} \leq 0$ *for* $\tau \geq \tau_0$ *if* $\inf\left\{\widetilde{E}_\tau\right\} + \sum_{i\in N}\hat{c}_id_i < \sum_{i\in N}C_{GHG_i} < \sup\left\{\widetilde{E}_\tau\right\} + \sum_{i\in N}\hat{c}_id_i$*; (4) Systemwide emission* $\hat{E}_\tau$ *always decreases in emission penalty rate* τ*, where* τ_0 *is the unique solution of equation* $\sum_{i\in N}\left[\left(\hat{h}_i\sqrt{\frac{a_i+\tau\hat{a}_i}{h_i+\tau\hat{h}_i}} + \hat{a}_i\sqrt{\frac{h_i+\tau\hat{h}_i}{a_i+\tau\hat{a}_i}}\right)\sqrt{\frac{d_i(p_i-d_i)}{2p_i}}\right] = \sum_{i\in N}(C_{GHG_i} - \hat{c}_id_i)$ *if the following condition* $\inf\left\{\widetilde{E}_\tau\right\} + \sum_{i\in N}\hat{c}_id_i < \sum_{i\in N}C_{GHG_i} < \sup\left\{\widetilde{E}_\tau\right\} + \sum_{i\in N}\hat{c}_id_i$ *holds, where we denote* $\inf\{\widetilde{E}_\tau\} = \sum_{i=1}^N\left[\sqrt{\frac{2\hat{a}_i\hat{h}_id_i(p_i-d_i)}{p_i}}\right]$ *and* $\sup\{\widetilde{E}_\tau\} = \sum_{i=1}^N\left[\left(\hat{h}_i\sqrt{\frac{a_i}{h_i}} + \hat{a}_i\sqrt{\frac{h_i}{a_i}}\right)\sqrt{\frac{d_i(p_i-d_i)}{2p_i}}\right]$.

The proofs of Corollary 1 and Proposition 1 are shown in Appendix A. The above summary shows that the joint production decision among multiple enterprises can overcome the problem caused by the insufficient carbon emission of a single enterprise. Under this situation, enterprises have a broader decision-making space. How the carbon penalty rate affects the total cost depends on the comparison between the total emission quota and several thresholds. Specifically, increasing the penalty rate will reduce (resp. increase) the total cost when the total carbon quota is greater (resp. less) than a certain threshold, while increasing the rate will make the total cost increase and decrease when the total carbon quota is between certain thresholds. However, the associated optimal emission is always decreasing with respect to the carbon price. This is intuitive since the higher carbon price can better curb the total emission. Nevertheless, it does not always benefit the control of overall cost since a balance exists between the operational and emission costs.

Proposition 2. *The impact of the emission penalty rate* τ *on the optimal production for multiple firms exhibits the following features:* $\frac{\partial Q_{i,\tau}}{\partial\tau} \geq 0$ *if* $\frac{a_i}{h_i} \leq \frac{\hat{a}_i}{\hat{h}_i}$*, and* $\frac{\partial Q_{i,\tau}}{\partial\tau} < 0$ *otherwise.*

For manufacturer i, the problem of solely optimizing operational cost is equivalent to the situation without carbon emission constraints. Therefore, from Equation (1) can be directly derived the associated order quantity $Q_i^* = \lim_{\tau\to 0}Q_{i,\tau} = \sqrt{\frac{2a_id_ip_i}{h_i(p_i-d_i)}}$, optimal ordering frequency $m_i^* = \frac{d_i}{Q_i^*} = \sqrt{\frac{h_i(p_i-d_i)d_i}{2a_ip_i}}$ and minimum average cost $AC(Q_i^*) = 2a_im_i^* + c_id_i$. Further analysis shows the equal optimal ordering cost and inventory cost $a_im_i^*$. The carbon emission of solely operational cost optimization can be expressed as follows:

$$EM(Q_i^*) = \lim_{\tau\to 0}\hat{E}_{i,\tau} = (\hat{a}_i\sqrt{\frac{h_i}{a_i}} + \hat{h}_i\sqrt{\frac{a_i}{h_i}})\sqrt{\frac{d_i(p_i-d_i)}{2p_i}} \tag{2}$$

Accordingly, we can get the optimal inventory cost $AC_o(Q_i^*)$ and optimal ordering cost $AC_h(Q_i^*)$ as shown in Table 2.

Optimizing the emission cost solely is the other angle to observe the objective function of total cost in Formula (1). Taking the same manner as in the scenario of solely optimizing operational cost to examine the obtained $Q_{i,\tau}$, $\hat{E}_{i,\tau}$ and $AC_{i,\tau}$ yields the emission cost solely optimization-based optimal values, such as production quantity $\hat{Q}_{\min,i} = \lim_{a_i,h_i\to 0}Q_{i,\tau} = \sqrt{\frac{2\hat{a}_ip_id_i}{\hat{h}_i(p_i-d_i)}}$, ordering frequency $\hat{m}_{\min,i} = \sqrt{\frac{\hat{h}_id_i(p_i-d_i)}{2\hat{a}_ip_i}}$, average cost $AC(\hat{Q}_{\min,i}) = \lim_{a_i,h_i \to 0} AC_{i,\tau} = \tau[\sqrt{2\hat{a}_i\hat{h}_i d_i(p_i - d_i)/p_i} + (\hat{c}_i d_i - C_{GHG_i})]$, and carbon emission $EM(\hat{Q}_{\min,i}) = \lim_{a_i,h_i \to 0} \hat{E}_{i,\tau} = \sqrt{2\hat{a}_i\hat{h}_i d_i(1 - d_i/p_i)} + \hat{c}_i d_i$. $EM(\hat{Q}_{\min,i})$ is increasing in production rate p_i, which is consistent with intuition. Furthermore, $AC(\hat{Q}_{\min,i})$ and $EM(\hat{Q}_{\min,i})$ degenerate into economic order quantities when $d_i/p_i \to 0$, while the cost and carbon emission tend to be infinite when $d_i/p_i \to 1$. Therefore, our model under EPL situation is substantially general to incorporate the EOQ-based settings, which can be considered a special case of ours. The pairs of system decision variables from the perspective of cost optimization and carbon emission are shown in Table 3.

Table 2. Summary of notation.

Notation	Explanation
p_i	Annual supply (production) rate of product *i* per year
d_i	Annual demand rate of product *i* per year
a_i	Single order cost per batch
h_i	Annual inventory holding cost per unit product
c_i	Variable production cost per unit product
$\hat{a}_i$	Carbon emissions from a single batch order
$\hat{h}_i$	Carbon emission caused by inventory holding product per unit
$\hat{c}_i$	Carbon emissions generated by production of unit product
C_{GHG_i}	Firm *i*'s emission quota/cap
Q_i	Economic production quantity, decision variable
Q_i^*	Optimal economic production quantity without carbon emission reduction constraints
$Q_{\min}^*$	Optimal economic production quantity size considering only the minimization of carbon emissions
$\hat{Q}_i^*$	Optimal economic production quantity with consideration of carbon emission reduction constraints and cost minimization
$\widetilde{Q}_i^*$	Optimal economic production quantity size in joint operation decision

Table 3. Comparison of system decision variables from two perspectives.

Item	Optimizing Operational Cost Solely	Optimizing Carbon Cost Solely
Optimal order quantity Q_i^*, $\hat{Q}_{i,\min}$	$\sqrt{\frac{2a_i d_i p_i}{h_i(p_i - d_i)}}$	$\sqrt{\frac{2\hat{a}_i d_i p_i}{\hat{h}_i(p_i - d_i)}}$
Optimal ordering frequency m_i^*, $\hat{m}_{i,\min}$	$\sqrt{\frac{h_i d_i(p_i - d_i)}{2a_i p_i}}$	$\sqrt{\frac{\hat{h}_i d_i(p_i - d_i)}{2\hat{a}_i p_i}}$
Optimal total cost AC	$\sqrt{\frac{2a_i h_i d_i(p_i - d_i)}{p_i}} + c_i d_i$	$\tau[\sqrt{\frac{2\hat{a}_i\hat{h}_i d_i(p_i - d_i)}{p_i}} + (\hat{c}_i d_i - C_{GHG_i})]$
Optimal ordering cost AC_o	$\sqrt{\frac{a_i h_i(p_i - d_i)d_i}{2p_i}}$	$a_i\sqrt{\frac{\hat{h}_i d_i(p_i - d_i)}{2\hat{a}_i p_i}}$
Optimal inventory cost AC_h	$\sqrt{\frac{a_i h_i(p_i - d_i)d_i}{2p_i}}$	$h_i\sqrt{\frac{\hat{a}_i d_i(p_i - d_i)}{2\hat{h}_i p_i}}$
Carbon emission EM_i	$(\hat{a}_i\sqrt{\frac{h_i}{a_i}} + \hat{h}_i\sqrt{\frac{a_i}{h_i}})\sqrt{\frac{d_i(p_i - d_i)}{2p_i}} + \hat{c}_i d_i$	$\sqrt{2\hat{a}_i\hat{h}_i d_i(1 - \frac{d_i}{p_i})} + \hat{c}_i d_i$

3.2. Mandatory Cap Policy-Based Optimal Production Quantity

In this section, we check the same system under the situation with another regulative policy, the mandatory cap scheme, which differs from the previous carbon pricing policy. Actually, the mandatory cap means a very strict constraint on the total emission, which has a distinct acting principle and mechanism compared with the carbon pricing regime. Therefore, the economic production quantity optimization problem constrained by the mandatory carbon emission cap can be expressed as the following nonlinear optimization model:

$$\begin{aligned}&\min_{Q_i \geq 0} AC(Q_i) = a_i \frac{d_i}{Q_i} + h_i \frac{p_i - d_i}{2p_i} Q_i + c_i d_i \\ &s.t.\ EM(Q_i) = \hat{a}_i \frac{d_i}{Q_i} + \hat{h}_i \frac{p_i - d_i}{2p_i} Q_i + \hat{c}_i d_i \leq C_{GHG_i} \text{ for } \forall i \in N\end{aligned} \tag{3}$$

Note that $AC(Q_i)$ is the annual total operating cost when the production quantity is Q_i, and $EM(Q_i)$ represents the corresponding annual total carbon emissions. The constraint in the above model means that the annual total carbon emissions cannot exceed the total constraint C_{GHG_i}. The left-hand side expression of constraint contains emissions incurred by delivering and storing goods, respectively.

From the above description, it can be seen that to solve the inequality, the limit of mandatory carbon emission constraint should satisfy the inequality below:

$$C_{GHG_i} \geq EM(Q_{\min,i}) = \sqrt{2\hat{a}_i \hat{h}_i d_i (p_i - d_i)/p_i} + \hat{c}_i d_i \tag{4}$$

Otherwise, enterprises will be unable to meet the carbon emission constraint no matter how they organize production and order, and this situation will also make the mandatory carbon emission reduction regulation meaningless. Moreover, the mandatory carbon cap requires the actual production Q to meet the condition $EM(Q_i) \leq C_{GHG_i}$.

Therefore, solving the above model yields the optimal economic production quantity Q_i^* for minimizing annual operational costs under the constraint of a mandatory cap. A simple analysis of the constrained nonlinear optimization model shows that the goal of enterprise operation is to minimize the operational cost and meet the carbon emission constraints. To better reflect these two goals, we need to analyze them separately.

Based on the above perspectives, we investigate the nonlinear optimization problem considering both carbon emission and cost optimization constraints. First, observing the constraint, we can rewrite the inequality into the following expression:

$$\hat{h}_i \frac{p_i - d_i}{2p_i} Q_i^2 - (C_{GHG_i} - \hat{c}_i d_i) Q_i + \hat{a}_i d_i \leq 0 \tag{5}$$

Two roots of the equation are $Q_{i,1} = \frac{p_i[\hat{C}_{GHG_i} - \sqrt{\hat{C}^2{}_{GHG_i} - 2\hat{a}_i \hat{h}_i (p_i - d_i) d_i / p_i}]}{\hat{h}_i (p_i - d_i)}$ and $Q_{i,2} = \frac{p_i[\hat{C}_{GHG_i} + \sqrt{\hat{C}^2{}_{GHG_i} - 2\hat{a}_i \hat{h}_i (p_i - d_i) d_i / p_i}]}{\hat{h}_i (p_i - d_i)}$, where we denote $\hat{C}_{GHG_i} = C_{GHG_i} - \hat{c}_i d_i$. The solution of equation (5) shows that the value range of the production quantity that meets the constraint is:

$$Q_{i,1} \leq Q_i \leq Q_{i,2} \tag{6}$$

In this way, the requirements for the value of economic production quantity are very clear.

Considering the objective function $AC(Q_i)$, a convex function of production quantity Q_i, and the evaluation of decision variables from the perspective of solely cost optimization in Table 2, the optimal solution of above constrained nonlinear programming can be obtained as follows:

$$\hat{Q}_i^* = \begin{cases} Q_i^*, \text{ if } Q_{i,1} \leq Q_i^* \leq Q_{i,2}; \\ Q_{i,1}, \text{ if } Q_i^* \leq Q_{i,1}; \\ Q_{i,2}, \text{ if } Q_i^* \geq Q_{i,2}; \end{cases} \quad \text{for } \forall i \in \{1, 2, \cdots, n\} \tag{7}$$

Equation (7) shows that the solely cost optimization-based production quantity and the production boundary determined by pure carbon emission constraints together determine the optimal economic production quantity in the supply chain subject to mandatory cap policy. The above formula shows that the optimal economic production quantity problem in a low-carbon environment can be equivalent to the problem without carbon constraint only when the feasible range of emission constraint meets certain conditions, that is, the emission constraint fails to act in affecting the production quantity from the perspective of solely cost optimization. From the opposite point of view, the governmental mandatory carbon emission restriction policy can play its due role only if it is set at a reasonable level. The inequalities $AC(Q_i^*) \leq AC(\hat{Q}_{i,\min})$ and $EM_i(Q_i^*) \geq EM_i(\hat{Q}_{i,\min})$ show a contradictory relationship between the optimal operational cost of solely cost optimization and optimal carbon emission of solely emission optimization.

Considering system decision from the perspective of utility, we can incorporate the emission constraint as a penalty term in the objective function as follows, which has a similar format to Formula (1).

$$\begin{aligned}\min_{Q_i \geq 0} AC_s(Q_i) &= a_i \frac{d_i}{Q_i} + h_i \frac{p_i - d_i}{2p_i} Q_i + c_i d_i + s_i[EM(Q_i) - C_{GHG_i}] \\ &= (a_i + s_i \hat{a}_i) \frac{d_i}{Q_i} + (h_i + s_i \hat{h}_i) \frac{p_i - d_i}{2p_i} Q_i + c_i d_i + s_i(\hat{c}_i d_i - C_{GHG_i})\end{aligned} \tag{8}$$

In the above formula, s_i can be regarded as the unit penalty rate of carbon emission rights exceeding the quota or the equivalence benefit of quota surplus. When there are no other constraints in Equation (8), the optimal value of the corresponding decision variable is $Q_i^{**} = \sqrt{\frac{2p_i d_i}{p_i - d_i} \frac{a_i + s_i \hat{a}_i}{h_i + s_i \hat{h}_i}}$, having a similar mathematical structure with the constraint-free optimal decision $Q_i^* = \sqrt{\frac{2p_i d_i}{p_i - d_i} \frac{a_i}{h_i}}$ and the solely emission optimization-based decision $Q_{EM_i}^* = \sqrt{\frac{2p_i d_i}{p_i - d_i} \frac{\hat{a}_i}{\hat{h}_i}}$. Comparing these optimal decisions gives the following Proposition 3 from a utility perspective.

Proposition 3. *Under mandatory carbon cap, investigating the relationship between the ratio of ordering and inventory holding associated carbon emission rates and the ratio of cost rates of the same kind yields: (1) If* $\frac{\hat{a}_i}{\hat{h}_i} \geq \frac{a_i}{h_i}$*, according to the above analysis,* C_{GHG_i} *can enable* $Q_{EM_i}^*$ *available, and accordingly* $\hat{Q}_i^* = Q_i^{**}$ *will be the systemwide optimal value; (2) If* $\frac{\hat{a}_i}{\hat{h}_i} < \frac{a_i}{h_i}$*,the systemwide optimal solution in this situation is:*

$$\hat{Q}_i^* = \begin{cases} Q_i^{**}, \text{ if } Q_{i,1} \leq Q_i^{**} \leq Q_{i,2}; \\ Q_{i,1}, \text{ if } Q_i^{**} \leq Q_{i,1}; \\ Q_{i,2}, \text{ if } Q_i^{**} \geq Q_{i,2}; \end{cases} \tag{9}$$

Proof. According to $\frac{a_i + s_i \hat{a}_i}{h_i + s_i \hat{h}_i} \geq \frac{a_i}{h_i} \Leftrightarrow \frac{\hat{a}_i}{\hat{h}_i} \geq \frac{a_i}{h_i}$ and $\frac{a_i + s_i \hat{a}_i}{h_i + s_i \hat{h}_i} \geq \frac{\hat{a}_i}{\hat{h}_i} \Leftrightarrow \frac{a_i}{h_i} \geq \frac{\hat{a}_i}{\hat{h}_i}$, the relationship between the optimal order decision in the three scenarios is $Q_{EM_i}^* \geq Q_i{}^{**} \geq Q_i{}^*$ if $\frac{\hat{a}_i}{\hat{h}_i} \geq \frac{a_i}{h_i}$ and $Q_i{}^* \geq Q_i{}^{**} \geq Q_{EM_i}^*$ if $\frac{a_i}{h_i} \geq \frac{\hat{a}_i}{\hat{h}_i}$. Therefore, the above proposition can be obtained. □

4. Joint Production Stocking Decision under Carbon Caps Sharing

This Section still considers the same supply chain structure as Section 2 but investigates different scenarios. Because retailers have certain differences in various operating parameters and carbon emission constraint parameters, they greatly differ in considering carbon emission constraints in simultaneously optimizing operations. This difference allows enterprises to take advantage of the differences in emission reduction endowments to better organize and make operational decisions. Therefore, this section examines the situation when retailers make completely independent decentralized decisions among operators but share carbon emissions. This situation enables enterprises to maintain good independence in operation but form an alliance to deal with carbon emissions and jointly face carbon emission regulations.

Based on the results in the above section, we can obtain the optimal decentralized operational decision problem for joint emission reduction under the scenario of decentralized operational decision-making but shared carbon emission allowances, as shown in the constrained nonlinear programming below:

$$\min AC(Q_1, Q_2, \cdots, Q_n) = \sum_{i\in N} [a_i \frac{d_i}{Q_i} + h_i \frac{p_i - d_i}{2p_i} Q_i + c_i d_i] \tag{10}$$

$$s.t.\ \ EM(Q_1, Q_2, \cdots, Q_n) = \sum_{i\in N} [\hat{a}_i \frac{d_i}{Q_i} + \hat{h}_i \frac{p_i - d_i}{2p_i} Q_i + \hat{c}_i d_i] \le \widetilde{C}_{GHG} \tag{11}$$

Including $\widetilde{C}_{GHG} = \sum_{i\in N} C_{GHG_i}$, $\sum_{i\in N} [\sqrt{2\hat{a}_i \hat{h}_i d_i (p_i - d_i)/p_i} + \hat{c}_i d_i] \le \widetilde{C}_{GHG}$ is satisfied.

The Hessian matrix of the above formula is:

$$\begin{bmatrix} \partial^2 AC/\partial {Q_1}^2,\ \partial^2 AC/\partial Q_1 \partial Q_2,\ \cdots, \partial^2 AC/\partial Q_1 \partial Q_n \\ \partial^2 AC/\partial Q_1 \partial Q_2,\ \partial^2 AC/\partial {Q_2}^2,\ \cdots, \partial^2 AC/\partial Q_2 \partial Q_n \\ \vdots \qquad\qquad \ddots \qquad\qquad \vdots \\ \partial^2 AC/\partial Q_n \partial Q_1,\ \partial^2 AC/\partial Q_n \partial Q_2,\ \cdots, \partial^2 AC/\partial {Q_n}^2 \end{bmatrix} = \begin{bmatrix} 2a_1 d_1 Q_1^{-3},\ 0,\ \cdots, 0 \\ 0,\ 2a_2 d_2 {Q_2}^{-3}, \cdots, 0 \\ \vdots \qquad \ddots \qquad \vdots \\ 0,\ 0, \cdots, 2a_n d_n Q_n^{-3} \end{bmatrix}$$

Therefore, the function $AC(Q_1, Q_2, \cdots, Q_n)$ is convex since the Hessian matrix H_{AC} is positively definite, and there is a global minimum. Similarly, the Hessian matrix of the function $EM(Q_1, Q_2, \cdots, Q_n)$ is also positively definite with a global minimum.

The solving the nonlinear programming problem under this situation yields Propositions 4–6, and their related proofs.

Proposition 4. *The solution of Equations (10) and (11) is:*

$$\widetilde{Q}_i^* = \begin{cases} \sqrt{\frac{2a_i p_i d_i}{h_i(p_i - d_i)}} = {Q_i}^*, \text{ when } Q^* \in \Gamma \\ \sqrt{\frac{a_i + \gamma \hat{a}_i}{h_i + \gamma \hat{h}_i} \frac{2p_i d_i}{p_i - d_i}}, \text{ when } Q^* \in \partial\Gamma \cup (\Re_+^n \backslash \Gamma) \end{cases} \tag{12}$$

where $\Gamma = \left\{(Q_1, Q_2, \cdots, Q_n) | \sum_{i\in N} [\hat{a}_i \frac{d_i}{Q_i} + \hat{h}_i \frac{p_i - d_i}{2p_i} Q_i + \hat{c}_i d_i] \le \widetilde{C}_{GHG}\right\}$, $Q^* = (Q_1^*, Q_2^*, \cdots, Q_n^*)$, $\partial\Gamma$ *is the boundary of* Γ *and* γ *the Lagrange multiplier determined by Equation (15).*

Proof. The Lagrange function is $L_{AC}(Q_1, Q_2, \cdots, Q_n) = \sum_{i\in N} [a_i \frac{d_i}{Q_i} + h_i \frac{p_i - d_i}{2p_i} Q_i + c_i d_i] - \gamma\left\{\widetilde{C}_{GHG} - \sum_{i\in N} [\hat{a}_i \frac{d_i}{Q_i} + \hat{h}_i \frac{p_i - d_i}{2p_i} Q_i + \hat{c}_i d_i]\right\}$, the corresponding K-T condition includes $\gamma\left\{\widetilde{C}_{GHG} - \sum_{i\in N} [\hat{a}_i \frac{d_i}{Q_i} + \hat{h}_i \frac{p_i - d_i}{2p_i} Q_i + \hat{c}_i d_i]\right\} = 0$, $\nabla L_{AC} = \nabla AC - \gamma \nabla EM = 0$ and $\gamma \ge 0$. Considering $\nabla L_{AC} = [\cdots, -(a_i + \gamma \hat{a}_i) \frac{d_i}{Q_i^2} + (h_i + \gamma \hat{h}_i) \frac{p_i - d_i}{2p_i}, \cdots]$, that is $\widetilde{Q}_i^* = \sqrt{\frac{a_i + \gamma \hat{a}_i}{h_i + \gamma \hat{h}_i} \frac{2p_i d_i}{p_i - d_i}}$, which is derived from the function $-(a_i + \gamma \hat{a}_i) \frac{d_i}{Q_i^2} + (h_i + \gamma \hat{h}_i) \frac{p_i - d_i}{2p_i} = 0$.

If $\gamma = 0$, $\widetilde{Q}_i^* = \sqrt{\frac{2a_i p_i d_i}{h_i(p_i - d_i)}} = {Q_i}^*$, then Equation (13) is established.

$$({Q_1}^*, {Q_2}^*, \cdots, {Q_n}^*) \in \Gamma = \left\{(Q_1, Q_2, \cdots, Q_n) | \sum_{i\in N} [\hat{a}_i \frac{d_i}{Q_i} + \hat{h}_i \frac{p_i - d_i}{2p_i} Q_i + \hat{c}_i d_i] \le \widetilde{C}_{GHG}\right\} \tag{13}$$

and there is $AC(Q_1, Q_2, \cdots, Q_n) = \sum_{i\in N} AC_i(Q_i^*) = \sum_{i\in N} [\sqrt{2a_i h_i d_i (p_i - d_i)/p_i} + c_i d_i]$.

$$EM(Q_1, Q_2, \cdots, Q_n) = \sum_{i\in N} EM_i(Q_i^*) = \sum_{i\in N} [\hat{a}_i \sqrt{\frac{h_i d_i (p_i - d_i)}{2a_i p_i}} + \hat{h}_i \sqrt{\frac{a_i d_i (p_i - d_i)}{2h_i p_i}} + \hat{c}_i d_i]$$

If $\gamma > 0$, then

$$\widetilde{Q}_i^* = Q_{i,\gamma} = \sqrt{\frac{a_i + \gamma \hat{a}_i}{h_i + \gamma \hat{h}_i} \frac{2p_i d_i}{p_i - d_i}} \tag{14}$$

where $(\widetilde{Q}_1{}^*, \widetilde{Q}_2{}^*, \cdots, \widetilde{Q}_n{}^*) \in \left\{(\widetilde{Q}_1, \widetilde{Q}_2, \cdots, \widetilde{Q}_n) | \widetilde{C}_{GHG} = \sum_{i\in N} \left[\hat{a}_i \frac{d_i}{\widetilde{Q}_i{}^*} + \hat{h}_i \frac{p_i - d_i}{2p_i} \widetilde{Q}_i{}^* + \hat{c}_i d_i\right]\right\}$, that is

$$\widetilde{C}_{GHG} = \sum_{i\in N} \left[\left(\hat{a}_i \sqrt{\frac{h_i + \gamma \hat{h}_i}{a_i + \gamma \hat{a}_i}} + \hat{h}_i \sqrt{\frac{a_i + \gamma \hat{a}_i}{h_i + \gamma \hat{h}_i}}\right) \sqrt{\frac{(p_i - d_i) d_i}{2p_i}} + \hat{c}_i d_i\right] \tag{15}$$

The optimal yield and Lagrange multiplier values for decentralized decision-making but shared carbon emission constraints are determined by Equations (14) and (15) simultaneously, that is, the optimal yield can be obtained by substituting γ into (14). □

Proposition 5. *The prerequisite for Equation (15) to have a positive real solution is:*

$$\sum_{i\in N} \left(\sqrt{2\hat{a}_i \hat{h}_i (p_i - d_i) d_i / p_i} + \hat{c}_i d_i\right) \le \widetilde{C}_{GHG} \le \sum_{i\in N} \left[\left(\hat{a}_i \sqrt{\frac{h_i}{a_i}} + \hat{h}_i \sqrt{\frac{a_i}{h_i}}\right) \sqrt{\frac{(p_i - d_i) d_i}{2p_i}} + \hat{c}_i d_i\right] \tag{16}$$

Proof. Let the right side of the Equation (15) be expressed as $f(\gamma) = \sum_{i\in N} \left[\left(\hat{a}_i \sqrt{\frac{h_i + \gamma \hat{h}_i}{a_i + \gamma \hat{a}_i}} + \hat{h}_i \sqrt{\frac{a_i + \gamma \hat{a}_i}{h_i + \gamma \hat{h}_i}}\right) \sqrt{\frac{(p_i - d_i) d_i}{2p_i}} + \hat{c}_i d_i\right]$, then get

$$f'(\gamma) = -\sum_{i\in N} \left[\frac{\left(\hat{h}_i a_i - h_i \hat{a}_i\right)^2}{2(a_i + \gamma \hat{a}_i)^{3/2} (h_i + \gamma \hat{h}_i)^{3/2}} \sqrt{\frac{(p_i - d_i) d_i}{2p_i}}\right] \le 0 \tag{17}$$

$$f''(\gamma) = \sum_{i\in N} \left[\left(\frac{\hat{a}_i}{a_i + \gamma \hat{a}_i} + \frac{\hat{h}_i}{h_i + \gamma \hat{h}_i}\right) \frac{3\left(\hat{h}_i a_i - h_i \hat{a}_i\right)^2}{4(a_i + \gamma \hat{a}_i)^{3/2} \left(h_i + \gamma \hat{h}_i\right)^{3/2}} \sqrt{\frac{(p_i - d_i) d_i}{2p_i}}\right] \ge 0 \tag{18}$$

From Equations (17) and (18), the function $f(\gamma)$ decreases monotonically. There is a unique real root γ_0 for Equation (15) if it exists.

$$\lim_{\gamma\to\infty} f(\gamma) \le f(\gamma) \le f(0) = \sum_{i\in N} \left[\left(\hat{a}_i \sqrt{\frac{h_i}{a_i}} + \hat{h}_i \sqrt{\frac{a_i}{h_i}}\right) \sqrt{\frac{(p_i - d_i) d_i}{2p_i}} + \hat{c}_i d_i\right]$$

where $\lim_{\gamma\to\infty} f(\gamma) = \sum_{i\in N} \left(\sqrt{2\hat{a}_i \hat{h}_i (p_i - d_i) d_i / p_i} + \hat{c}_i d_i\right)$, therefore, Formula (16) is established. □

To make a more detailed explanation of the above propositions, without losing generality, we give the following proposition reflecting the optimal decision of homogeneous enterprises.

Proposition 6. *For N homogeneous enterprises in the situation of joint carbon emission reduction, the following expression can be established:*

$$\sqrt{\frac{a_i + \gamma \hat{a}_i}{h_i + \gamma \hat{h}_i}} = \begin{cases} \frac{1}{2\hat{h}_i} \left(\frac{1}{n} \widetilde{C}_{GHG} - \hat{c}_i d_i\right) \sqrt{\frac{2p_i}{(p_i - d_i) d_i}} - \sqrt{\frac{2p_i}{4\hat{h}_i{}^2 (p_i - d_i) d_i} \left(\frac{1}{n} \widetilde{C}_{GHG} - \hat{c}_i d_i\right)^2 - \frac{\hat{a}_i}{\hat{h}_i}}, \text{ if } \frac{\hat{a}}{\hat{h}} \ge \frac{a}{h} \\ \frac{1}{2\hat{h}_i} \left(\frac{1}{n} \widetilde{C}_{GHG} - \hat{c}_i d_i\right) \sqrt{\frac{2p_i}{(p_i - d_i) d_i}} + \sqrt{\frac{2p_i}{4\hat{h}_i{}^2 (p_i - d_i) d_i} \left(\frac{1}{n} \widetilde{C}_{GHG} - \hat{c}_i d_i\right)^2 - \frac{\hat{a}_i}{\hat{h}_i}}, \text{ if } \frac{a}{h} \ge \frac{\hat{a}}{\hat{h}} \end{cases} \tag{19}$$

$$\gamma = \begin{cases} \dfrac{h_i\left[\sqrt{\frac{2p_i}{4\hat{h}_i^2(p_i-d_i)d_i}\left(\frac{1}{n}\widetilde{C}_{GHG}-\hat{c}_id_i\right)^2}-\sqrt{\frac{2p_i}{4\hat{h}_i^2(p_i-d_i)d_i}\left(\frac{1}{n}\widetilde{C}_{GHG}-\hat{c}_id_i\right)^2-\frac{\hat{a}_i}{\hat{h}_i}}\right]^2-a_i}{\hat{a}_i-\hat{h}_i\left[\sqrt{\frac{2p_i}{4\hat{h}_i^2(p_i-d_i)d_i}\left(\frac{1}{n}\widetilde{C}_{GHG}-\hat{c}_id_i\right)^2}-\sqrt{\frac{2p_i}{4\hat{h}_i^2(p_i-d_i)d_i}\left(\frac{1}{n}\widetilde{C}_{GHG}-\hat{c}_id_i\right)^2-\frac{\hat{a}_i}{\hat{h}_i}}\right]^2}, \text{if } \frac{\hat{a}}{\hat{h}} \geq \frac{a}{h} \\ \dfrac{h_i\left[\sqrt{\frac{2p_i}{4\hat{h}_i^2(p_i-d_i)d_i}\left(\frac{1}{n}\widetilde{C}_{GHG}-\hat{c}_id_i\right)^2}+\sqrt{\frac{2p_i}{4\hat{h}_i^2(p_i-d_i)d_i}\left(\frac{1}{n}\widetilde{C}_{GHG}-\hat{c}_id_i\right)^2-\frac{\hat{a}_i}{\hat{h}_i}}\right]^2-a_i}{\hat{a}_i-\hat{h}_i\left[\sqrt{\frac{2p_i}{4\hat{h}_i^2(p_i-d_i)d_i}\left(\frac{1}{n}\widetilde{C}_{GHG}-\hat{c}_id_i\right)^2}+\sqrt{\frac{2p_i}{4\hat{h}_i^2(p_i-d_i)d_i}\left(\frac{1}{n}\widetilde{C}_{GHG}-\hat{c}_id_i\right)^2-\frac{\hat{a}_i}{\hat{h}_i}}\right]^2}, \text{if } \frac{a}{h} \geq \frac{\hat{a}}{\hat{h}} \end{cases} \tag{20}$$

Proof. Make $x = \sqrt{(a_i + \gamma\hat{a}_i)/(h_i + \gamma\hat{h}_i)}$, the roots of Equation (15) are x_1 and x_2, where we have $x_1 = \frac{1}{2\hat{h}_i}\left(\frac{1}{n}\widetilde{C}_{GHG} - \hat{c}_id_i\right)\sqrt{\frac{2p_i}{(p_i-d_i)d_i}} - \sqrt{\frac{2p_i}{4\hat{h}_i^2(p_i-d_i)d_i}\left(\frac{1}{n}\widetilde{C}_{GHG} - \hat{c}_id_i\right)^2 - \frac{\hat{a}_i}{\hat{h}_i}}$ and $x_2 = \frac{1}{2\hat{h}_i}\left(\frac{1}{n}\widetilde{C}_{GHG} - \hat{c}_id_i\right)\sqrt{\frac{2p_i}{(p_i-d_i)d_i}} + \sqrt{\frac{2p_i}{4\hat{h}_i^2(p_i-d_i)d_i}\left(\frac{1}{n}\widetilde{C}_{GHG} - \hat{c}_id_i\right)^2 - \frac{\hat{a}_i}{\hat{h}_i}}$.

As mentioned previously, $\frac{a}{h} \leq \frac{a+s\hat{a}}{h+s\hat{h}} \leq \frac{\hat{a}}{\hat{h}}$ if $\frac{\hat{a}}{\hat{h}} \geq \frac{a}{h}$ and $\frac{\hat{a}}{\hat{h}} \leq \frac{a+s\hat{a}}{h+s\hat{h}} \leq \frac{a}{h}$ if $\frac{a}{h} \geq \frac{\hat{a}}{\hat{h}}$ were established. The inequality $-\sqrt{\frac{2p_i}{4\hat{h}_i^2(p_i-d_i)d_i}\left(\frac{1}{n}\widetilde{C}_{GHG} - \hat{c}_id_i\right)^2 - \frac{\hat{a}_i}{\hat{h}_i}} < \sqrt{\frac{2p_i}{4\hat{h}_i^2(p_i-d_i)d_i}\left(\frac{1}{n}\widetilde{C}_{GHG} - \hat{c}_id_i\right)^2} - \sqrt{\frac{\hat{a}_i}{\hat{h}_i}} \leq \sqrt{\frac{2p_i}{4\hat{h}_i^2(p_i-d_i)d_i}\left(\frac{1}{n}\widetilde{C}_{GHG} - \hat{c}_id_i\right)^2 - \frac{\hat{a}_i}{\hat{h}_i}}$ and $\sqrt{\frac{\hat{a}_i}{\hat{h}_i}} \leq \sqrt{\frac{2p_i}{4\hat{h}_i^2(p_i-d_i)d_i}\left(\frac{1}{n}\widetilde{C}_{GHG} - \hat{c}_id_i\right)^2}$ also holds. There are $x = x_1$ if $\frac{\hat{a}}{\hat{h}} \geq \frac{a}{h}$ and $x = x_2$ if $\frac{a}{h} \geq \frac{\hat{a}}{\hat{h}}$. Then solving equation $x_1 = \sqrt{\frac{a_i+\gamma\hat{a}_i}{h_i+\gamma\hat{h}_i}}$ and $x_2 = \sqrt{\frac{a_i+\gamma\hat{a}_i}{h_i+\gamma\hat{h}_i}}$ gives the Equation (21). Therefore, the above proposition can be proved.

Substituting $\widetilde{Q}_i{}^*$ into constraint (11) yields inequality $EM\left(\widetilde{Q}_1{}^*, \widetilde{Q}_2{}^*, \cdots, \widetilde{Q}_n{}^*\right) \geq \sum_{i\in N} EM_i(\hat{Q}_{i,\min})$. Thus, we have $AC(Q_1, Q_2, \cdots, Q_n) = \sum_{i\in N}\left[\left(a_i\sqrt{\frac{h_i+\gamma\hat{h}_i}{a_i+\gamma\hat{a}_i}} + h_i\sqrt{\frac{a_i+\gamma\hat{a}_i}{h_i+\gamma\hat{h}_i}}\right)\sqrt{\frac{(p_i-d_i)d_i}{2p_i}} + c_id_i\right]$ and total emissions $\widetilde{C}_{GHG}$. Retailer i's emissions is $EM_i(\widetilde{Q}_i^*) = \left(\hat{a}_i\sqrt{\frac{h_i+\gamma\hat{h}_i}{a_i+\gamma\hat{a}_i}} + \hat{h}_i\sqrt{\frac{a_i+\gamma\hat{a}_i}{h_i+\gamma\hat{h}_i}}\right)\sqrt{\frac{(p_i-d_i)d_i}{2p_i}} + \hat{c}_id_i \geq EM_i(\hat{Q}_{i,\min})$. □

The comparison of centralized and decentralized situations is shown in Table 4.

Observation 1. *From previous Propositions, we can have* $\widetilde{C}_{GHG} \geq \sum_{i\in N}\left[\left(\hat{a}_i\sqrt{\frac{h_i}{a_i}} + \hat{h}_i\sqrt{\frac{a_i}{h_i}}\right)\sqrt{\frac{(p_i-d_i)d_i}{2p_i}} + \hat{c}_id_i\right]$. *The optimal solution satisfying (10) and (11) is* $\widetilde{Q}^* = (Q_1^*, Q_2^*, \cdots, Q_n^*)$,*where the constraint (11) is inactive and retailers decide according to the state without carbon emission constraint. Hence, to reduce the carbon emission of the whole supply chain system, the total carbon emission limit of the system should meet the condition Formula (16). Particularly, Formula (16) does not imply that the constraint in Formula (3) will be satisfied for every retailer (e.g., i). On the contrary, it is possible that the retailer does not satisfy Formula (3) but satisfies the overall carbon emission constraint (16) when considering systemwide optimization. That is,* $\hat{Q}_i^* = Q_i^*$ *while* $\widetilde{Q}_i^* = Q_{i,\gamma}$ *or* $\hat{Q}_i^* = Q_{i,1}(\text{or } Q_{i,2})$ *while* $\widetilde{Q}_i^* = Q_{i,\gamma}$ *for retailer i.*

The following discussion mainly focuses on the situations with condition (16) established.

Proposition 7. *In the situation of carbon quota sharing, as follows is the relationship between carbon emissions-oriented optimal decision* $\widetilde{Q}_i{}^*$*, total cost-oriented optimal decision* $Q_i{}^*$*, and optimal systemwide decision* $\hat{Q}_i^*$ *under different parameters configurations:*

(1) If $\frac{\hat{a}}{\hat{h}} \geq \frac{a}{h}$ *for retailer i, then* $Q_i{}^* < \widetilde{Q}_i{}^* = \sqrt{\frac{a_i+\gamma\hat{a}_i}{h_i+\gamma\hat{h}_i}\frac{2p_id_i}{p_i-d_i}} = \sqrt{\frac{a_i(1+\gamma\hat{a}_i/a_i)}{h_i(1+\gamma\hat{h}_i/h_i)}\frac{2p_id_i}{p_i-d_i}} < \hat{Q}_{i,\min}$.

(2) If $\frac{\hat{a}}{\hat{h}} < \frac{a}{h}$ *for retailer i, then* $\hat{Q}_{i,\min} < \widetilde{Q}_i{}^* = \sqrt{\frac{a_i+\gamma\hat{a}_i}{h_i+\gamma\hat{h}_i}\frac{2p_id_i}{p_i-d_i}} = \sqrt{\frac{a_i(1+\gamma\hat{a}_i/a_i)}{h_i(1+\gamma\hat{h}_i/h_i)}\frac{2p_id_i}{p_i-d_i}} < Q_i{}^*$.

(3) *If* $\frac{\hat{a}}{\hat{h}} = \frac{a}{h}$, *then* $\widetilde{Q}_i{}^* = Q_i{}^* = \hat{Q}_{i,\min}$.

Table 4. Comparison of decisions under completely decentralized and quota-sharing strategies.

	Completely Decentralized Situation	Decentralized But Shared Carbon Emission Allowance
Optimal order quantity	$\hat{Q}_i^* = \begin{cases} Q_i^*, \text{ if } Q_{i,1} \le Q_i^* \le Q_{i,2}; \\ Q_{i,1}, \text{ if } Q_i^* \le Q_{i,1}; \\ Q_{i,2}, \text{ if } Q_i^* \ge Q_{i,2}; \end{cases}$, $i = 1, 2, \cdots, n$	$\widetilde{Q}_i{}^* = \begin{cases} \sqrt{\frac{2a_i p_i d_i}{h_i(p_i - d_i)}} = Q_i{}^*, \text{ if } Q \in \Gamma \\ \sqrt{\frac{a_i + \gamma\hat{a}_i}{h_i + \gamma\hat{h}_i} \frac{2p_i d_i}{p_i - d_i}} = Q_{i,\gamma}, \text{ if } Q \in \partial\Gamma \cup (\Re_+^n \backslash \Gamma) \end{cases}$
Optimal order frequency	$\hat{m}_i^* = \frac{d_i}{\hat{Q}_i^*}$	$\widetilde{m}_i^* = \frac{d_i}{\widetilde{Q}_i{}^*}$
Optimal total cost	$AC_i(Q_i^*) = \begin{cases} \sqrt{\frac{2a_i h_i d_i (p_i - d_i)}{p_i}} + c_i d_i, \\ \text{if } Q_{i,1} \le Q_i^* \le Q_{i,2}; \\ AC_i(Q_{i,1}^*), \quad \text{if } Q_i^* \le Q_{i,1}; \\ AC_i(Q_{i,2}^*), \quad \text{if } Q_i^* \ge Q_{i,2}; \end{cases}$	$AC_i(\widetilde{Q}_i{}^*) = \begin{cases} \sqrt{\frac{2a_i h_i d_i (p_i - d_i)}{p_i}} + c_i d_i, \text{ if } Q \in \Gamma; \\ \left(a_i \sqrt{\frac{h_i + \gamma\hat{h}_i}{a_i + \gamma\hat{a}_i}} + h_i \sqrt{\frac{a_i + \gamma\hat{a}_i}{h_i + \gamma\hat{h}_i}} \right) \sqrt{\frac{(p_i - d_i) d_i}{2p_i}} + c_i d_i, \text{ if } Q \in \partial\Gamma \cup (\Re_+^n \backslash \Gamma). \end{cases}$
Optimal order cost	$AC_o(Q_i^*) = a_i m_i^* = \sqrt{\frac{a_i h_i (p_i - d_i) d_i}{2p_i}}$	$AC_o(\widetilde{Q}_i^*) = \begin{cases} \sqrt{\frac{2a_i h_i d_i (p_i - d_i)}{p_i}} + c_i d_i, \text{ if } Q \in \Gamma; \\ a_i \sqrt{\frac{h_i + \gamma\hat{h}_i}{a_i + \gamma\hat{a}_i}} \sqrt{\frac{(p_i - d_i) d_i}{2p_i}}, \text{ if } Q \in \partial\Gamma \cup (\Re_+^n \backslash \Gamma). \end{cases}$
Optimal inventory cost	$AC_h(Q_i^*) = a_i m_i^* = \sqrt{\frac{a_i h_i (p_i - d_i) d_i}{2p_i}}$	$AC_h(\widetilde{Q}_i^*) = \begin{cases} \sqrt{\frac{a_i h_i d_i (p_i - d_i)}{p_i}} + c_i d_i, \text{ if } Q \in \Gamma; \\ h_i \sqrt{\frac{a_i + \gamma\hat{a}_i}{h_i + \gamma\hat{h}_i}} \sqrt{\frac{(p_i - d_i) d_i}{2p_i}}, \text{ if } Q \in \partial\Gamma \cup (\Re_+^n \backslash \Gamma). \end{cases}$
Total carbon emission	$EM_i(Q_i^*) = \hat{a}_i \sqrt{\frac{h_i d_i (p_i - d_i)}{2a_i p_i}} + \hat{h}_i \sqrt{\frac{a_i d_i (p_i - d_i)}{2h_i p_i}} + \hat{c}_i d_i$	$EM_i(\widetilde{Q}_i{}^*) = \begin{cases} \sqrt{\frac{2\hat{a}_i \hat{h}_i d_i (p_i - d_i)}{p_i}} + \hat{c}_i d_i \\ \left(\hat{a}_i \sqrt{\frac{h_i + \gamma\hat{h}_i}{a_i + \gamma\hat{a}_i}} + \hat{h}_i \sqrt{\frac{a_i + \gamma\hat{a}_i}{h_i + \gamma\hat{h}_i}} \right) \sqrt{\frac{(p_i - d_i) d_i}{2p_i}} + \hat{c}_i d_i \end{cases}$

The above Proposition 7 shows the influence of different parameter Settings on the optimal decisions of three scenarios; that is, comparing the ratio between carbon emission rates caused by production lot and inventory holding and the ratio between cost rates caused by them will determine the size relationship between optimal decisions.

The above formulation mainly discusses and compares the optimal production plan and corresponding cost under the two situations of individual decision-making and joint decision-making. Joint decision-making means that all enterprises implement centralized decision-making based on sharing carbon emission quota, that is, to make joint optimal production arrangements from the perspective of maximizing system benefits. The purpose of exploring the significance of centralized decision-making is to demonstrate that the joint optimal decision-making in this situation can achieve better operational performance than the traditional decentralized decision-making situation to demonstrate that the government's relaxation of the carbon quota use standard has important social and economic benefits. This analysis lays a good foundation for designing a reasonable and feasible guarantee mechanism for implementing carbon quota sharing and joint optimal production decision-making among enterprises in the case of low-carbon policy improvement.

5. The Comparative Analysis of Two Scenarios

5.1. Comparison of Two Decision-Making Situations under Carbon Emission Constraints

In this part, we mainly analyze the optimal operational cost and carbon emission under two situations, i.e., completely decentralized decision and joint emission reduction.

(1) If $\hat{Q}_i^* = Q_i^*$ for $\forall i \in N$, that is, constraint (16) is established as constraint (2) is satisfied, thus $\widetilde{Q}_i^* = Q_{i,\gamma}$.

(2) $\hat{Q}_i^* = Q_{i,1} \text{ or } Q_{i,2}$ holds for $i \in W$, where $W = \left\{ i \middle| \hat{a}_i \sqrt{\frac{h_i d_i (p_i - d_i)}{2a_i p_i}} + \hat{h}_i \sqrt{\frac{a_i d_i (p_i - d_i)}{2h_i p_i}} + \hat{c}_i d_i \ge C_{GHG_i} \right\}$, and $\hat{Q}_j^* = Q_j^*$ for $j \in N \backslash W$,

then. In the case of joint production decisions with carbon emission caps sharing for retailers $i \in N$, there are two situations concerning whether Formula (16) is established or not:

(2-i) If the following inequality holds $\sum_{i \in N}\left(\sqrt{2\hat{a}_i\hat{h}_i(p_i - d_i)d_i/p_i} + \hat{c}_i d_i\right) \le \widetilde{C}_{GHG} \le \sum_{i \in N}\left[\left(\hat{a}_i\sqrt{\frac{h_i}{a_i}} + \hat{h}_i\sqrt{\frac{a_i}{h_i}}\right)\sqrt{\frac{(p_i-d_i)d_i}{2p_i}} + \hat{c}_i d_i\right]$, i.e., Formula (16) still established. And then the optimal production economic batch of retailers is: $\widetilde{Q}_i^* = Q_{i,\gamma}$.

(2-ii) If $\widetilde{C}_{GHG} > \sum_{i \in N}\left[\left(\hat{a}_i\sqrt{\frac{h_i}{a_i}} + \hat{h}_i\sqrt{\frac{a_i}{h_i}}\right)\sqrt{\frac{(p_i-d_i)d_i}{2p_i}} + \hat{c}_i d_i\right]$, then Formula (16) is not established. At this point, in Formula (15) have no positive real solution according to prior propositions. In this situation there is $\widetilde{Q}_i{}^* = \sqrt{\frac{2a_i p_i d_i}{h_i(p_i-d_i)}} = Q_i{}^*$, that is, all retailers can make decisions based solely on cost optimization.

In situation (2-ii), the carbon emission constraint does not affect the cost-optimal decision, and there is no difference between scenarios of solely cost-optimizing and solely emission decision. When some enterprises belong to set W and others belong to set $N \backslash W$, sharing carbon emissions will realize the reallocation of carbon emissions in the retailer alliance, regardless of the case (2-i) or (2-ii).

Proposition 8. *Let the left-hand side of inequality (11) be function* $EM\left(\overrightarrow{Q}\right) = \sum_{i \in N} EM_i(Q_i)$, *including vector* $\overrightarrow{Q} = [Q_1^*, Q_2^*, \cdots, Q_n^*]$, $EM_i(Q_i)$ *is expressed as in (2). The lower bound of total system cost under completely decentralized and joint emission reduction decision makings is related as follows:*

$$\inf\left\{AC\left(\overrightarrow{Q}\right) : EM\left(\overrightarrow{Q}\right) \le \widetilde{C}_{GHG}\right\} \le \inf\left\{\sum_{i \in N} AC_i(Q_i) : EM_i(Q_i) \le C_{GHG_i}\right\} \tag{21}$$

Proof. First, we examine the domain of the objective function defined by the constraints of the two situations. Obviously, there is

$$\left\{(Q_1^*, Q_2^*, \cdots, Q_n^*)|\hat{a}\frac{d}{Q_i} + \hat{h}\frac{p-d}{2p}Q_i + \hat{c}d \le C_{GHG_i}, \text{ for all } i \in N\right\}$$
$$\subseteq \left\{(Q_1^*, Q_2^*, \cdots, Q_n^*)|\sum_{i \in N}\left[\hat{a}_i\frac{d_i}{Q_i} + \hat{h}_i\frac{p_i-d_i}{2p_i}Q_i + \hat{c}_i d_i\right] \le \sum_{i \in N} C_{GHG_i}\right\}$$

That is, $\{(Q_1^*, Q_2^*, \cdots, Q_n^*)|EM_i(Q_i) \le C_{GHG_i} \text{ for all } i \in N\} \subseteq \left\{(Q_1^*, Q_2^*, \cdots, Q_n^*)|EM\left(\overrightarrow{Q}\right) \le \widetilde{C}_{GHG}\right\}$ holds.

Although $AC\left(\overrightarrow{Q}\right)$ and $\sum_{i \in N} AC_i(Q_i)$ are equal in value, using $AC\left(\overrightarrow{Q}\right)$ instead of the latter expression aims to distinguish two decision scenarios. Therefore, when the function expressions are equal, the lower bound of the function with a large value space of independent variables must not be greater than the lower bound of the function with a small value space, so Equation (21) is established and Proposition 8 is proved. □

The discussion above and Proposition 8 show that enterprises strictly constrained by carbon emission allowances in the case of independent decision-making can relax the carbon emission constraints by sharing emission allowances so that all retailers in their system make decisions according to solely cost optimization. Essentially, the enterprises with more carbon emission limits (in the relaxed state of constraints) will transfer part of their allowances to the enterprises with tight emission limits, making the cost of the whole system lower. In essence, firms with relatively more carbon allowances (the constraints are relaxed) will transfer some of their allowances to firms with tight allowances, making the systemwide cost less.

$$AC\left(\widetilde{Q}_1{}^*, \widetilde{Q}_2{}^*, \cdots, \widetilde{Q}_n{}^*\right) = \inf\left\{AC\left(\overrightarrow{Q}\right) : EM\left(\overrightarrow{Q}\right) \le \widetilde{C}_{GHG}\right\} \le \\ \inf\left\{\sum_{i\in N} AC_i(Q_i) : EM_i(Q_i) \le C_{GHG_i}\right\} \le \sum_{i\in N} AC_i\left(\hat{Q}_i{}^*\right)$$

Therefore, the difference between optimal total operational costs under two situations is:

$$\begin{aligned} \Delta AC &= \sum_{i\in N} AC_i\left(\hat{Q}_i{}^*\right) - AC\left(\widetilde{Q}_1{}^*, \widetilde{Q}_2{}^*, \cdots, \widetilde{Q}_n{}^*\right) \\ &= \begin{cases} \sum_{i\in W}\left[a_i \frac{d_i}{Q^*_{i,k}} + h_i \frac{p_i - d_i}{2p_i} Q^*_{i,k} + c_i d_i\right] + \sum_{i\in N\setminus W}\left[\sqrt{2a_i h_i d_i (p_i - d_i)/p_i} + c_i d_i\right] & \\ -\sum_{i\in N}\left[\left(a_i\sqrt{\frac{h_i+\gamma\hat{h}_i}{a_i+\gamma\hat{a}_i}} + h_i\sqrt{\frac{a_i+\gamma\hat{a}_i}{h_i+\gamma\hat{h}_i}}\right)\sqrt{\frac{(p_i-d_i)d_i}{2p_i}} + c_i d_i\right], & \text{if } W \ne \varnothing; \\ 0, & \text{if } W = \varnothing. \end{cases} \end{aligned} \tag{22}$$

where $k \in \{1, 2\}$.

It can be seen that the carbon quotas sharing scenario can meet the overall carbon emission constraint while reducing the systemwide operational cost of the downstream retailer's alliance.

5.2. Numerical Analysis

This subsection takes a one-supplier triple-retailer supply chain as an example for computational study. Assume that only one product is produced and sold and that the market demand and supply characteristics are consistent with the assumptions made at the beginning. The setting of parameters is shown as follows:

Notation	Description
p	[2.5, 5.6, 4.8]
d	[1.2, 4.1, 2.9]
a	[12.3, 13.4, 15.7]
h	[0.61, 0.38, 0.50] Annual inventory carrying cost per unit product
c	[5.8, 4.2, 4.6] Variable cost of production per unit of product
$\hat{a}$	[2.3, 4.7, 3.6] Carbon emissions produced by a single order
$\hat{h}$	[0.017, 0.023, 0.033] Carbon emissions caused by annual inventory holding per unit of product
$\hat{c}$	[0.25, 0.18, 0.22] Carbon emissions per unit product
C_{GHG}	[4.7, 6.3, 5.8] Annual total inventory emissions
Q	Economic production batch, decision variables
Q^*	Optimal economic production batch without carbon emission constraint
$Q_{\min}$	Optimal economic production batch with carbon emission minimized
$\hat{Q}^*$	The optimal economic production batch with carbon emission constraint and cost minimization is considered simultaneously
$\widetilde{Q}^*$	Optimal economic production batch for joint operation decisions

The numerical results in Table 5 verify the propositions and related conclusions in the preceding section. The results of completely decentralized decision-making and carbon quota-sharing decision-making show that sharing carbon emission allowances can reduce the total operating cost of the overall system of the alliance but, at the same time, meet the total carbon emission quota constraints of the system. At the same time, the actual carbon emission may be increased, but the total social utility may still be improved.

Table 5. Numerical analysis of cost and carbon emission in settings of decentralized decision making and joint emission reduction.

Cap vector	C_{GHG}	[2.2, 3.0, 4.5]	[1.5, 1.8, 2.0]	[1.3, 1.5, 1.6]	[0.83, 1.27, 1.17]	[0.77, 1.25, 1.20]	[0.74, 1.24, 1.18]	[0.72, 1.25, 1.17]
Critical decentralized production	$Q_{\cdot,1} = [Q_{1,1}\ Q_{2,1}\ Q_{3,1}]$	[1.46, 8.62, 2.72]	[2.32, 19.22, 7.97]	[2.79, 28.59, 11.80]	[5.46, 51.70, 32.97]	[6.24, 57.59, 27.13]	[6.73, 61.89, 30.39]	[7.10, 57.59, 32.97]
	$Q_{\cdot,2} = [Q_{1,2}\ Q_{2,2}\ Q_{3,2}]$	[428.4, 725.7, 588.6]	[269.2, 325.55, 200.6]	[223.5, 218.8, 135.5]	[114.45, 121, 48.49]	[100.1, 108.62, 58.92]	[92.82, 101.08, 52.59]	[87.92, 108.62, 48.49]
Production for minimum emission	$Q_{\min} = [Q_{\min,1} Q_{\min,2} Q_{\min,3}]$	[24.99, 79.09, 39.98]	[24.99, 79.09, 39.98]	[24.99, 79.09, 39.98]	[24.99, 79.09, 39.98]	[24.99, 79.09, 39.98]	[24.99, 79.09, 39.98]	[24.99, 79.09, 39.98]
Production for minimum total cost	$Q^* = [Q_1^*\ Q_2^*\ Q_3^*]$	[9.65, 32.86, 21.45]	[9.65, 32.86, 21.45]	[9.65, 32.86, 21.45]	[9.65, 32.86, 21.45]	[9.65, 32.86, 21.45]	[9.65, 32.86, 21.45]	[9.65, 32.86, 21.45]
Optimal decentalized production	$\hat{Q}^* = [\hat{Q}_1^*\ \hat{Q}_2^*\ \hat{Q}_3^*]$	[9.65, 32.86, 21.45]	[9.65, 32.86, 21.45]	[9.65, 32.86, 21.45]	[9.65, 51.70, 32.97]	[9.65, 57.59, 27.13]	[9.65, 61.89, 30.39]	[9.65, 57.59, 32.97]
Optimal production with caps sharing	$\widetilde{Q}^* = [\widetilde{Q}_1^*\ \widetilde{Q}_2^*\ \widetilde{Q}_3^*]$	[9.65, 32.86, 21.45]	[9.65, 32.86, 21.45]	[9.65, 32.86, 21.45]	[10.00, 34.96, 22.23]	[10.42, 37.37, 23.14]	[11.05, 40.72, 24.44]	[11.30, 41.98, 24.94]
Total cost with minimum emission	$AC(Q_{\min})$	[11.51, 21.94, 18.44]	[11.51, 21.94, 18.44]	[11.51, 21.94, 18.44]	[11.51, 21.94, 18.44]	[11.51, 21.94, 18.44]	[11.51, 21.94, 18.44]	[11.51, 21.94, 18.44]
Total cost with minimum cost	$AC(Q^*)$	[10.02, 20.56, 17.59]	[10.02, 20.56, 17.59]	[10.02, 20.56, 17.59]	[10.02, 20.56, 17.59]	[10.02, 20.56, 17.59]	[10.02, 20.56, 17.59]	[10.02, 20.56, 17.59]
Optimal decentralized production cost	$AC(\hat{Q}^*)$	[10.02, 20.56, 17.59]	[10.02, 20.56, 17.59]	[10.02, 20.56, 17.59]	[10.02, 20.91, 17.98]	[10.02, 21.10, 17.70]	[10.02, 21.26, 17.85]	[10.02, 21.10, 17.98]
Optimal caps-sharing production cost	$AC(\widetilde{Q}^*)$	[10.02, 20.56, 17.59]	[10.02, 20.56, 17.59]	[10.02, 20.56, 17.59]	[10.02, 20.57, 17.59]	[10.03, 20.59, 17.60]	[10.05, 20.64, 17.62]	[10.06, 20.67, 17.63]
Optimal costs comparison analysis	$\sum_{i\in N} [AC_i(\hat{Q}^*) - AC_i(\widetilde{Q}^*)]$	0 (→)	0 (→)	0 (→)	0.73 (↓)	0.60 (↓)	0.82 (↓)	0.74 (↓)
Targeted minimum emission	$EM(Q_{\min})$	[0.52, 1.23, 1.16]	[0.52, 1.23, 1.16]	[0.52, 1.23, 1.16]	[0.52, 1.23, 1.16]	[0.52, 1.23, 1.16]	[0.52, 1.23, 1.16]	[0.52, 1.23, 1.16]
Emission with minimum total cost	$EM(Q^*)$	[0.63, 1.43, 1.26]	[0.63, 1.43, 1.26]	[0.63, 1.43, 1.26]	[0.63, 1.43, 1.26]	[0.63, 1.43, 1.26]	[0.63, 1.43, 1.26]	[0.63, 1.43, 1.26]
Emission of decentralized production	$EM(\hat{Q}^*)$	[0.63, 1.43, 1.26]	[0.63, 1.43, 1.26]	[0.63, 1.43, 1.26]	[0.63, 1.27, 1.17]	[0.63, 1.25, 1.20]	[0.63, 1.24, 1.18]	[0.63, 1.25, 1.17]
Emission of caps-sharing production	$EM(\widetilde{Q}^*)$	[0.63, 1.43, 1.26]	[0.63, 1.43, 1.26]	[0.63, 1.43, 1.26]	[0.62, 1.40, 1.25]	[0.61, 1.37, 1.24]	[0.60, 1.34, 1.22]	[0.59, 1.33, 1.22]
Comparison analysis of two- scenario emissions	$\sum_{i\in N} [EM_i(\hat{Q}^*) - EM_i(\widetilde{Q}^*)]$	0 (→)	0 (→)	0 (→)	−0.20 (↑)	−0.14 (↑)	−0.11 (↑)	−0.09 (↑)
Feasibility test of caps sharing production	$C_{GHG_i} - EM_i(Q_{\min})$	[1.68, 1.77, 3.34]	[0.98, 0.57, 0.84]	[0.78, 0.27, 0.44]	[0.31, 0.04, 0.01]	[0.25, 0.02, 0.04]	[0.22, 0.01, 0.02]	[0.20, 0.02, 0.01]
	$\sum_{i\in N} C_{GHG_i} - \sum_{i\in N} EM_i(Q_{\min})$	6.79	2.39	1.49	0.36	0.31	0.25	0.23
	$\sum_{i\in N} C_{GHG_i} - \sum_{i\in N} EM_i(Q_i^*)$	6.38	1.98	1.08	−0.05	−0.10	−0.16	−0.18
Parameters for KKT condition	γ	NaN	NaN	NaN	0.47	1.08	2.08	2.51

6. Concluding Remarks

Inspired by emission reduction practices and governmental carbon regulations, we study in this paper a two-layer supply chain consisting of a single supplier and multi-retailer under carbon emission constraints. The optimal output under completely decentralized decision and joint emission reduction decision are given, respectively. At the same time, the carbon emission reduction effects under the two decision-making situations are compared and analyzed. The results show that sharing a carbon emission quota can effectively make full use of carbon emission resources, minimize the overall cost of the supply chain, and satisfy carbon emission constraints. This paper also refers to the condition of joint decision, that is, the effective range of the carbon emission constraint coefficient. The research of this paper has certain enlightening significance for enterprise operation decisions in a low-carbon environment. Enterprises can realize effective utilization of carbon emission resources using horizontal alliance. One limitation is that we formulate the focused problem in a deterministic setting. Therefore, studying joint production planning with caps sharing facing stochastic demand can be a future research direction.

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Appendix A

Proof of Proposition 1.

Consider

$$\min_{Q_i \geq 0} AC(Q_i, \cdots, Q_n) \; = \; \sum_{i=1}^{N} \left[a_i \frac{d_i}{Q_i} \; + \; h_i \frac{p_i - d_i}{2p_i} Q_i \; + \; c_i d_i \; + \; \tau \left(\hat{a}_i \frac{d_i}{Q_i} + \hat{h}_i \frac{p_i - d_i}{2p_i} Q_i + \hat{c}_i d_i - C_{GHG_i} \right) \right].$$

Its Hessian matrix can be shown as follows:

$$\begin{aligned} AC &= \textstyle\sum_{i=1}^{N} \left[a_i \frac{d_i}{Q_i} + h_i \frac{p_i - d_i}{2p_i} Q_i + c_i d_i + \tau \left(\hat{a}_i \frac{d_i}{Q_i} + \hat{h}_i \frac{p_i - d_i}{2p_i} Q_i + \hat{c}_i d_i - C_{GHG_i} \right) \right] \\ &= \textstyle\sum_{i=1}^{N} \left[(a_i + \tau \hat{a}_i) \frac{d_i}{Q_i} + (h_i + \tau \hat{h}_i) \frac{p_i - d_i}{2p_i} Q_i + c_i d_i + \tau \left(\hat{c}_i d_i - C_{GHG_i} \right) \right] \end{aligned}$$

$$H_{AC} = \begin{bmatrix} \partial^2 AC/\partial {Q_1}^2, \, \partial^2 AC/\partial Q_1 \partial Q_2, \cdots, \partial^2 AC/\partial Q_1 \partial Q_n \\ \partial^2 AC/\partial Q_1 \partial Q_2, \, \partial^2 AC/\partial {Q_2}^2, \cdots, \partial^2 AC/\partial Q_2 \partial Q_n \\ \vdots \qquad \ddots \qquad \vdots \\ \partial^2 AC/\partial Q_n \partial Q_1, \, \partial^2 AC/\partial Q_n \partial Q_2, \cdots, \partial^2 AC/\partial {Q_n}^2 \end{bmatrix} = \begin{bmatrix} 2(a_1 + \tau \hat{a}_1) d_1 Q_1^{-3}, \; 0, \; \cdots, 0 \\ 0, \; 2(a_2 + \tau \hat{a}_2) d_2 Q_2^{-3}, \cdots, 0 \\ \vdots \qquad \ddots \qquad \vdots \\ 0, \; 0, \cdots, 2(a_n + \tau \hat{a}_n) d_n Q_n^{-3} \end{bmatrix}$$

which implies the existence of a global optimizer. Hence, we can obtain the optimal solution as below from the first order condition of the objective function.

$$\frac{\partial AC}{\partial Q_i} = -(a_i + \tau \hat{a}_i) \frac{d_i}{{Q_i}^2} + (h_i + \tau \hat{h}_i) \frac{p_i - d_i}{2p_i} \Rightarrow {Q_i}^* = \sqrt{\frac{a_i + \tau \hat{a}_i}{h_i + \tau \hat{h}_i} \frac{2p_i d_i}{p_i - d_i}}$$

□

Proof of Corollary 1.

$$AC_{i,\tau} = 2(a_i + \tau\hat{a}_i)m_{i,\tau} = 2\sqrt{(a_i + \tau\hat{a}_i)(h_i + \tau\hat{h}_i)}\sqrt{\tfrac{d_i(p_i-d_i)}{2p_i}} + c_i d_i + \tau(\hat{c}_i d_i - C_{GHG_i}),$$
$$\tfrac{\partial AC_{i,\tau}}{\partial\tau} = \left(\hat{h}_i\sqrt{\tfrac{a_i+\tau\hat{a}_i}{h_i+\tau\hat{h}_i}} + \hat{a}_i\sqrt{\tfrac{h_i+\tau\hat{h}_i}{a_i+\tau\hat{a}_i}}\right)\sqrt{\tfrac{d_i(p_i-d_i)}{2p_i}} + (\hat{c}_i d_i - C_{GHG_i}) \Leftrightarrow$$
$$\begin{cases} \tfrac{\partial AC_{i,\tau}}{\partial\tau} > 0, \text{ if } \hat{c}_i d_i > C_{GHG_i}; \\ \tfrac{\partial AC_{i,\tau}}{\partial\tau} > 0, \text{ if } \hat{c}_i d_i < C_{GHG_i} \text{ and } [\hat{a}_i h_i + a_i\hat{h}_i + \tau(\hat{a}_i\hat{h}_i + \hat{a}_i\hat{h}_i)]^2\tfrac{d_i(p_i-d_i)}{2p_i} > (C_{GHG_i} - \hat{c}_i d_i)^2(a_i+\tau\hat{a}_i)(h_i+\tau\hat{h}_i) \\ \tfrac{\partial AC_{i,\tau}}{\partial\tau} \le 0, \text{ if } \hat{c}_i d_i < C_{GHG_i} \text{ and } [\hat{a}_i h_i + a_i\hat{h}_i + \tau(\hat{a}_i\hat{h}_i + \hat{a}_i\hat{h}_i)]^2\tfrac{d_i(p_i-d_i)}{2p_i} < (C_{GHG_i} - \hat{c}_i d_i)^2(a_i+\tau\hat{a}_i)(h_i+\tau\hat{h}_i) \end{cases}$$
$$\because [\hat{a}_i h_i + a_i\hat{h}_i + \tau(\hat{a}_i\hat{h}_i + \hat{a}_i\hat{h}_i)]^2\tfrac{d_i(p_i-d_i)}{2p_i} > (C_{GHG_i} - \hat{c}_i d_i)^2(a_i+\tau\hat{a}_i)(h_i+\tau\hat{h}_i)$$
$$\Leftrightarrow [\hat{h}_i\sqrt{\tfrac{a_i+\tau\hat{a}_i}{h_i+\tau\hat{h}_i}} + \hat{a}_i\sqrt{\tfrac{h_i+\tau\hat{h}_i}{a_i+\tau\hat{a}_i}}] > (C_{GHG_i} - \hat{c}_i d_i)\sqrt{\tfrac{2p_i}{d_i(p_i-d_i)}}$$

Thus, the above inequality holds always, if

$$\hat{h}_i\sqrt{\frac{a_i+\tau\hat{a}_i}{h_i+\tau\hat{h}_i}} + \hat{a}_i\sqrt{\frac{h_i+\tau\hat{h}_i}{a_i+\tau\hat{a}_i}} \ge 2\sqrt{\hat{a}_i\hat{h}_i} \ge (C_{GHG_i} - \hat{c}_i d_i)\sqrt{\frac{2p_i}{d_i(p_i-d_i)}}$$

and there exists two solutions if

$$\hat{h}_i\sqrt{\frac{a_i+\tau\hat{a}_i}{h_i+\tau\hat{h}_i}} + \hat{a}_i\sqrt{\frac{h_i+\tau\hat{h}_i}{a_i+\tau\hat{a}_i}} \ge (C_{GHG_i} - \hat{c}_i d_i)\sqrt{\frac{2p_i}{d_i(p_i-d_i)}} \ge 2\sqrt{\hat{a}_i\hat{h}_i}$$

i.e.,

$$\frac{a_i+\tau\hat{a}_i}{h_i+\tau\hat{h}_i} \ge \left[\sqrt{\frac{p_i(C_{GHG_i} - \hat{c}_i d_i)^2}{2\hat{h}_i{}^2 d_i(p_i-d_i)}} + \sqrt{\frac{p_i(C_{GHG_i} - \hat{c}_i d_i)^2}{2\hat{h}_i{}^2 d_i(p_i-d_i)} - \frac{\hat{a}_i}{\hat{h}_i}}\right]^2$$

and

$$0 < \frac{a_i+\tau\hat{a}_i}{h_i+\tau\hat{h}_i} \le \left[\sqrt{\frac{p_i(C_{GHG_i} - \hat{c}_i d_i)^2}{2\hat{h}_i{}^2 d_i(p_i-d_i)}} - \sqrt{\frac{p_i(C_{GHG_i} - \hat{c}_i d_i)^2}{2\hat{h}_i{}^2 d_i(p_i-d_i)} - \frac{\hat{a}_i}{\hat{h}_i}}\right]^2$$

Therefore, we have

$$0 \le \tau \le \frac{a_i - h_i\left[\sqrt{\frac{p_i(C_{GHG_i} - \hat{c}_i d_i)^2}{2\hat{h}_i{}^2 d_i(p_i-d_i)}} + \sqrt{\frac{p_i(C_{GHG_i} - \hat{c}_i d_i)^2}{2\hat{h}_i{}^2 d_i(p_i-d_i)} - \frac{\hat{a}_i}{\hat{h}_i}}\right]^2}{\hat{h}_i\left[\sqrt{\frac{p_i(C_{GHG_i} - \hat{c}_i d_i)^2}{2\hat{h}_i{}^2 d_i(p_i-d_i)}} + \sqrt{\frac{p_i(C_{GHG_i} - \hat{c}_i d_i)^2}{2\hat{h}_i{}^2 d_i(p_i-d_i)} - \frac{\hat{a}_i}{\hat{h}_i}}\right]^2 - \hat{a}_i}$$

and

$$\tau \ge \frac{a_i - h_i\left[\sqrt{\frac{p_i(C_{GHG_i} - \hat{c}_i d_i)^2}{2\hat{h}_i{}^2 d_i(p_i-d_i)}} - \sqrt{\frac{p_i(C_{GHG_i} - \hat{c}_i d_i)^2}{2\hat{h}_i{}^2 d_i(p_i-d_i)} - \frac{\hat{a}_i}{\hat{h}_i}}\right]^2}{\hat{h}_i\left[\sqrt{\frac{p_i(C_{GHG_i} - \hat{c}_i d_i)^2}{2\hat{h}_i{}^2 d_i(p_i-d_i)}} - \sqrt{\frac{p_i(C_{GHG_i} - \hat{c}_i d_i)^2}{2\hat{h}_i{}^2 d_i(p_i-d_i)} - \frac{\hat{a}_i}{\hat{h}_i}}\right]^2 - \hat{a}_i}.$$

□

Proof of Proposition 1.

$$\because AC_{i,\tau} = 2(a_i + \tau\hat{a}_i)m_{i,\tau} = 2\sqrt{(a_i + \tau\hat{a}_i)(h_i + \tau\hat{h}_i)}\sqrt{\frac{d_i(p_i - d_i)}{2p_i}} + c_i d_i + \tau(\hat{c}_i d_i - C_{GHG_i}),$$
$$\frac{\partial AC}{\partial \tau} = \sum_{i\in N}\left[\left(\hat{h}_i\sqrt{\frac{a_i+\tau\hat{a}_i}{h_i+\tau\hat{h}_i}} + \hat{a}_i\sqrt{\frac{h_i+\tau\hat{h}_i}{a_i+\tau\hat{a}_i}}\right)\sqrt{\frac{d_i(p_i-d_i)}{2p_i}}\right] - \sum_{i\in N}(C_{GHG_i} - \hat{c}_i d_i)$$
$$= \widetilde{E}(\tau) + \sum_{i\in N}(\hat{c}_i d_i - C_{GHG_i}) \Leftrightarrow$$
$$\begin{cases} \frac{\partial AC}{\partial \tau} > 0, \text{ if } \sum_{i\in N}\hat{c}_i d_i > \sum_{i\in N} C_{GHG_i}; \\ \frac{\partial AC}{\partial \tau} > 0, \text{ if} \sum_{i\in N}\hat{c}_i d_i < \sum_{i\in N} C_{GHG_i} \text{ and } \sum_{i\in N}\widetilde{E}(\tau) > \sum_{i\in N} C_{GHG_i} - \sum_{i\in N}\hat{c}_i d_i; \\ \frac{\partial AC}{\partial \tau} \le 0, \text{ if } \sum_{i\in N}\hat{c}_i d_i < \sum_{i\in N} C_{GHG_i} \text{ and } \sum_{i\in N}\widetilde{E}(\tau) < \sum_{i\in N} C_{GHG_i} - \sum_{i\in N}\hat{c}_i d_i. \end{cases}$$
$$\frac{\partial^2 AC}{\partial \tau^2} = \frac{\partial \widetilde{E}(\tau)}{\partial \tau} < 0$$
$$\because \sum_{i\in N}\widetilde{E}(\tau) > \sum_{i\in N} C_{GHG_i} - \sum_{i\in N}\hat{c}_i d_i$$

Therefore, we have: (1) $\frac{\partial AC}{\partial \tau} \ge 0$ holds if $\sum_{i\in N} C_{GHG_i} \le \sum_{i\in N}\left[\sqrt{\frac{2\hat{a}_i\hat{h}_i d_i(p_i-d_i)}{p_i}}\right] + \sum_{i\in N}\hat{c}_i d_i$; (2) $\frac{\partial AC_\tau}{\partial \tau} < 0$ if $\sum_{i\in N} C_{GHG_i} > \sum_{i\in N}\left[\left(\hat{h}_i\sqrt{\frac{a_i}{h_i}} + \hat{a}_i\sqrt{\frac{h_i}{a_i}}\right)\sqrt{\frac{d_i(p_i-d_i)}{2p_i}}\right] + \sum_{i\in N}\hat{c}_i d_i$; (3) $\frac{\partial AC_\tau}{\partial \tau} > 0$ for $0 < \tau < \tau_0$ and $\frac{\partial AC_\tau}{\partial \tau} \ge 0$ for $\tau \ge \tau_0$ if $\inf\{\widetilde{E}_\tau\} + \sum_{i\in N}\hat{c}_i d_i < \sum_{i\in N} C_{GHG_i} < \sup\{\widetilde{E}_\tau\} + \sum_{i\in N}\hat{c}_i d_i$; (4) Systemwide emission $\hat{E}_\tau$ always decreases in emission penalty rate τ. For emission

$$\sum_{i\in N}\hat{E}_{i,\tau} = \sum_{i\in N}\left[\left(\hat{a}_i\sqrt{\frac{h_i+\tau\hat{h}_i}{a_i+\tau\hat{a}_i}} + \hat{h}_i\sqrt{\frac{a_i+\tau\hat{a}_i}{h_i+\tau\hat{h}_i}}\right)\sqrt{\frac{d_i(p_i-d_i)}{2p_i}} + \hat{c}_i d_i\right] = \widetilde{E}_\tau + \sum_{i\in N}\hat{c}_i d_i$$
$$\hat{a}_i\sqrt{\frac{h_i+\tau\hat{h}_i}{a_i+\tau\hat{a}_i}} \ge \hat{h}_i\sqrt{\frac{a_i+\tau\hat{a}_i}{h_i+\tau\hat{h}_i}} \Leftrightarrow \frac{\hat{a}_i}{\hat{h}_i}(h_i + \tau\hat{h}_i) \ge a_i + \tau\hat{a}_i \Leftrightarrow \frac{\hat{a}_i}{\hat{h}_i} \ge \frac{a_i}{h_i}$$
$$\frac{\partial \hat{E}_{i,\tau}}{\partial \tau} = \frac{(a_i\hat{h}_i - \hat{a}_i h_i)}{2}\left[\sqrt{\frac{a_i+\tau\hat{a}_i}{h_i+\tau\hat{h}_i}}\frac{\hat{a}_i}{(a_i+\tau\hat{a}_i)^2} - \sqrt{\frac{h_i+\tau\hat{h}_i}{a_i+\tau\hat{a}_i}}\frac{\hat{h}_i}{(h_i+\tau\hat{h}_i)^2}\right]$$
$$\because \sqrt{\frac{a_i+\tau\hat{a}_i}{h_i+\tau\hat{h}_i}}\frac{\hat{a}_i}{(a_i+\tau\hat{a}_i)^2} > \sqrt{\frac{h_i+\tau\hat{h}_i}{a_i+\tau\hat{a}_i}}\frac{\hat{h}_i}{(h_i+\tau\hat{h}_i)^2} \Leftrightarrow \frac{\hat{a}_i}{(a_i+\tau\hat{a}_i)} > \frac{\hat{h}_i}{(h_i+\tau\hat{h}_i)} \Leftrightarrow \hat{a}_i h_i > a_i\hat{h}_i$$

$\therefore \frac{\partial \hat{E}_{i,\tau}}{\partial \tau} < 0$ always holds. □

Proof of Proposition 2.

$$\min_{Q_i\ge 0} AC(Q_i) = a_i\frac{d_i}{Q_i} + h_i\frac{p_i-d_i}{2p_i}Q_i + c_i d_i + \tau\left(\hat{a}_i\frac{d_i}{Q_i} + \hat{h}_i\frac{p_i-d_i}{2p_i}Q_i + \hat{c}_i d_i - C_{GHG_i}\right)$$
$$= (a_i + \tau\hat{a}_i)\frac{d_i}{Q_i} + (h_i + \tau\hat{h}_i)\frac{p_i-d_i}{2p_i}Q_i + (c_i + \tau\hat{c}_i)d_i - \tau C_{GHG_i}$$
$$\Rightarrow Q_{i,\tau} = \sqrt{\frac{a_i+\tau\hat{a}_i}{h_i+\tau\hat{h}_i}\frac{2p_i d_i}{p_i-d_i}} = \sqrt{\frac{\hat{a}_i}{\hat{h}_i}\frac{\tau+\frac{a_i}{\hat{a}_i}}{\tau+\frac{h_i}{\hat{h}_i}}\frac{2p_i d_i}{p_i-d_i}}$$
$$= \sqrt{\frac{\hat{a}_i}{\hat{h}_i}\left[1 + \frac{a_i/\hat{a}_i - h_i/\hat{h}_i}{\tau + h_i/\hat{h}_i}\right]\frac{2p_i d_i}{p_i-d_i}}$$
$$\frac{a_i}{h_i} < \frac{\hat{a}_i}{\hat{h}_i} \Leftrightarrow \frac{a_i}{\hat{a}_i} < \frac{h_i}{\hat{h}_i} \Rightarrow \frac{\partial Q_{i,\tau}}{\partial \tau} > 0$$
$$\frac{a_i}{h_i} > \frac{\hat{a}_i}{\hat{h}_i} \Leftrightarrow \frac{a_i}{\hat{a}_i} > \frac{h_i}{\hat{h}_i} \Rightarrow \frac{\partial Q_{i,\tau}}{\partial \tau} < 0$$

□

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