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Editorial

Preface to the Special Issue on "Hypergroup Theory and Algebrization of Incidence Structures"

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This work contains the accepted papers of a Special Issue of the MDPI journal *Mathematics* entitled "Hypergroup Theory and Algebrization of Incidence Structure". As Guest Editors of this Special Issue, we have invited significant and original contributions dealing with algebraic hyperstructures in a broad sense, of which hypergroups and adjacency structures are two prominent representatives.

Algebraic hyperstructures are natural generalizations of ordinary algebraic structures when the composition operator is multivalued. For this reason, the study of algebraic hyperstructures is also called hypercompositional algebra. The first results of this theory appeared in the 1940s. Since then, it has undergone a lively development, beginning in the 1970s with the work of various research groups in France, Greece, and Italy on the theory of hypergroups, hyperrings, and hyperfields. At present, hypercompositional algebra is being studied by many researchers on almost all continents, and is characterized by a great diversity of style and subject matter, as evidenced by the contributions collected in this work.

Since its origin, hypercompositional algebra has had a close relationship with classical algebra, borrowing concepts and methods from it. For example, an important class of hypergroups is constructed from the quotient structure of a classical group with respect to a non-normal subgroup. As studies progressed, the theory found new and profound relationships with Galois theory, geometry, and, in particular, incidence structures. Today, hypercompositional algebra has a fruitful variety of connections with other areas of mathematics, such as fuzzy set theory, combinatorics, and probability, with applications in various other sciences such as computer science, artificial intelligence and physics, as well as other natural and social sciences.

The Special Issue received 24 distinct submissions of which 10 (42%) were published after peer review; 13 contributions (54%) were rejected by the MDPI Editorial Board and only one (4%) after peer review. The published papers were written by 24 different authors from 10 different countries, with an average number of 2.9 authors per paper, and address theoretical aspects, applications, and related computational issues of algebraic hyperstructures. The names of the authors in alphabetical order are: Jan Chvalina, Irina Cristea, Bijan Davvaz, Henri De Boutray, Mario De Salvo, Dario Fasino, Domenico Freni, Alain Giorgetti, Frédéric Holweck, Šárka Hošková-Mayerová, Stefano Innamorati, Milica Kankaraš, Osman Kazancı, Sorasak Leeratanavalee, Giovanni Lo Faro, Christos Massouros, Gerasimos Massouros, Salvatore Milici, Anak Nongmanee, Metod Saniga, Bedrich Smetana, Antoinette Tripodi, Jana Vyroubalová, and Fulvio Zuanni. We provide hereafter a brief overview of their contributions. The reader will find here a plethora of different problems, focuses and methods that provide insight into the variegated features of hypercompositional algebra.

In [1], the authors define new classes of hyperfields and hyperrings. They classify finite hyperfields as quotient hyperfields or non-quotient hyperfields, and analyze structures resulting from the subtraction of a multiplicative subgroup from a field. This paper includes an extensive bibliography on the subject, which not only provides the interested reader with a detailed roadmap of hyperfield theory but also opens up further investigation into the boundary between classical and hypercompositional algebra.

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The authors of [2] analyze the relations between multi-fuzzy soft sets and polygroups. Moreover, they extend some algebraic properties of fuzzy soft polygroups and soft polygroups to multi-fuzzy soft polygroups. Finally, they define new operations on a multi-fuzzy soft set and present some of their algebraic properties.

The main result in [3] is the establishment of a novel combinatorial characterization of $H(4, q^2)$, a Hermitian variety embedded in the projective space $PG(4, q^2)$. This characterization makes it possible to remove an unnecessary hypothesis that is present in a previously known analogous characterization, except for a few small cases.

The authors of [4] consider a class of linear differential operators with time-dependent coefficients inspired by artificial neurons, called differential neurons. With these objects, they first define an infinite cyclic group isomorphic to $(\mathbb{Z}, +)$, extending the monoid of differential neurons with their negative powers. This construction is then extended by successive steps until a non-commutative join space is defined.

The paper [5] introduces a family of hypergroups, here called weakly complete, that generalizes the construction of complete hypergroups. Furthermore, the authors define the degree of completeness of finite hypergroups which, in some sense, quantifies the extent to which the hypergroup is close to being complete. For weakly complete hypergroups, this degree can be computed by explicit formulas.

In the context of graph factorizations, the paper [6] provides a complete solution to an unsolved existence problem for uniform factorizations of complete simple graphs in terms of cycles and paths with certain specified sizes. The solution crucially relies on two constructions that allow to derive factorizations of larger graphs from the knowledge of simpler cases.

The objective of [7] is to take the first step in the classification of G-hypergroups, that is, hypergroups whose heart is a non-trivial group. This research has an emphasis on G-hypergroups whose heart is a torsion group. In particular, the authors characterize G-hypergroups which are of type U on the right or right cogroups. The paper also includes the hyperproduct tables of all G-hypergroups with sizes up to 5, up to isomorphisms.

In [8], the authors introduce the notion of v-regular ternary Menger algebras, which generalizes the notion of regular ternary semigroups. Furthermore, they consider a special class of n-ary functions, the so-called left translations, and prove that the set of left translations can be endowed with the structure of a ternary Menger algebra.

The authors of [9] consider certain physically relevant finite geometries of binary symplectic polar spaces of small rank, when the points of these spaces canonically encode multi-qubit observables. In particular, they present a complete taxonomy of polar subspaces of $W(2N-1, 2)$ for $2 \leq N \leq 4$, whose rank is $N-1$. The results required extensive computer-aided proofs.

The aim of [10] is to extend the concept of reducibility to hyperrings, which is a well-known concept in hypergroup theory. The authors define this extension in general hyperrings, where addition and multiplication are both multivalued operations, and then apply this novel definition to particular classes of hyperrings, e.g., hyperrings of formal series, hyperrings with P-hyperoperations, and complete hyperrings. The main results provide conditions under which these hyperrings are or are not reduced.

We are grateful to all authors who contributed their manuscripts. We would also like to thank all the reviewers for their valuable comments, which improved the quality of the submissions. Finally, we express our warmest gratitude to the MDPI Editor, Dr Syna Mu, who assisted us at every stage of the editorial process with excellent timeliness and professionalism.

The aim of this Special Issue was to attract high-quality, novel papers in the field of hypercompositional algebra. The response from the international scientific community and the number of manuscripts submitted for consideration exceeded our expectations. We would like to mention that the MDPI publishing house has already issued two books from Special Issues that have dealt explicitly with algebraic hyperstructures, namely that edited by C. Massouros in 2021, *Hypercompositional Algebra and Applications*, and *Symmetry* *in Classical and Fuzzy Algebraic Hypercompositional Structures* edited by I. Cristea in 2020. We hope that the papers included in all these collections will be influential for the scientific community and will motivate further research in this exciting, active, and engaging research area.

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