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On the Relationship between the Fractal Dimension of Geomagnetic Variations at Altay and the Space Weather Characteristics

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Abstract: The fractal dimension of geomagnetic field component variations (horizontal—H, vertical—Z and magnetic declination—D) at the Baigazan magnetic station at Russian Altay, for the period 2011–2013, were calculated using the Higuchi method. The daily variation of Higuchi Fractal Dimension (HFD) for the D, H, Z components of the geomagnetic field were investigated, and its contribution to the variability of HFD was found to be from 30 to 40 percent of the total variance. A correlation analysis of the fractal dimension of the variations of the D, H, Z components with the Auroral Electrojet (AE) index and solar wind characteristics was carried out. Negative correlations with logarithms of the AE-index, interplanetary magnetic field (IMF) strength and solar wind velocity were found. About 25 percent of the HFD variance is controlled by the variability of these characteristics. Pair and partial correlation coefficients for these parameters were calculated for every month of 2011–2013.

Keywords: Higuchi fractal dimension; geomagnetical variation; AE-index; solar wind; Altay

MSC: 28A80; 42A38; 86A25; 62M10

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1. Introduction

Many natural systems, and the signals that determine them, have complex nonlinear characteristics and cannot be fully studied using only the theory of linear systems. Therefore, it is necessary to use the methods of nonlinear dynamics.

As a nonlinear measure for signal analysis, one can use the method of calculating the Higuchi fractal dimension (HFD) [1]. The FRF method estimates the fractal dimension (or Hausdorff space) D of the time series, wherein this dimension measures the degree of "roughness" of the fractal shape and varies in a range of values from 1 to 2.

In the work [2], it is shown that the HFD method has advantages in comparison with methods based on the construction of the signal power spectrum. In [3], the HFD method gave the same results compared to the box counting method. However, faster data preprocessing, while using fewer computing resources, makes it possible to make a choice in favor of the HFD method.

The relationship between the fractal dimension of geomagnetic variations and magnetic storms has been studied by many researchers [4–10]. Moreover, the fractal dimension of fluctuations in geomagnetic indices, such as AE [11], Sym-H [12],and Dst [13], which are the result of the processing of measurements at several magnetic observatories, were studied. The majority of studies noted the effect of reducing the fractal dimension of geomagnetic variations or fluctuations in geomagnetic indices during geomagnetic disturbances at high [11] and low latitudes [5,7]. Wanliss and co-authors [8] showed that the fractal dimension of geomagnetical variation increased with geomagnetical latitude. It was noted in [9] that the temporal variation of geomagnetic field fractal dimensions at medium and high latitudes is different.

In [10], wavelet analysis was used to study fluctuations of the horizontal component of the Earth's magnetic field (H-component) in order to determine the scaling properties of the behavior of the temporal variability of geomagnetic data during periods of magnetic storms during a 23-year solar cycle. It was shown that, during periods of magnetic storms, there was a rapid and unidirectional change in the spectral scaling index at the time of the onset of the storm.

Usually, in the studies mentioned above, the fractal dimension data were compared under quiet and disturbed geomagnetic conditions. The objective of this paper was to use correlation analysis to study a relationship between the Higuchi fractal dimension (HFD) of geomagnetic variations at Russian Altay (at middle latitude) and the space weather characteristics, in particular, with the auroral electrojet (AE) index and solar wind characteristics.

The structure of the article consists of an introduction, which describes the methods for studying geomagnetic field variations and provides links to articles on the subject of the study. Section 2 describes the materials and methodology of the study. Information is provided on the point of registration of the D,H,Z-component of the geomagnetic field (horizontal—H, vertical—Z and magnetic declination—D), the method for constructing the fractal dimension, according to Higuchi, is described, and the preliminary data processing is addressed. Section 3 presents the results of applying the Higuchi fractal dimension to study variations in the D,H,Z components of the geomagnetic field in 2011 at the Baigazan station in Altay, and an interpretation of the study results is provided.

2. Materials and Methods

2.1. Data Source

Since 2009, geomagnetic field variations have been recorded at the magnetic station of Gorno-Altaisk State University, "Baigazan" [14]. The station is located at the cordon of the same name of the Altaiskiy Reserve on the northern shore of Lake Teletskoye (51°45.596′ N, 87°25.916′ E). The territory of the reserve is protected by the law of the Russian Federation; therefore, there is no magnetic pollution and technogenic noises. Measurements at the station are carried out using a quartz variometer, based on the sensor of CMVS "Quartz-3EM", with the rate equaled to 5 Hz. The accuracy of sec mean values of the D,H,Z components was 10 pT. The station is powered from alternative energy sources, installed 120 m from the station near the inspector's house. The power consumption was 27 W.

There were data losses at the following periods: 28 January 2012–7 March 2012, 27 July 2012–11 October 2012, 24 May 2013–3 July 2013, and 15 August 2013–20 October 2013. The summer losses were caused by lightning activity, which destroyed the power system elements of the station and stopped the measurements. From 11–23 December 2012, as well as 16–19 February 2013, and 9–24 May 2013, the H-component sensor was in the second steady state, which is characterized by an increased noise level, so the calculations gave anomalously high values of the HFD. On 7–9 September 2011 and 20–26 September 2011, a similar problem was observed with the D-sensor. Data for these periods were excluded from the analysis.

2.2. Higuchi Method of Fractal Dimension Calculation

For each 45-min interval of the quartz variometer data for the years 2011–2013, the fractal dimensions of D,H,Z-geomagnetic variations were calculated using the Higuchi method [1] by means of MATLAB R2023a.

The experience of using various methods for estimating the fractal dimension time series has shown that the Higuchi method is one of the most robust methods [9,15]. In this method, the mean distance between the elements of the time series $X(j), j = 1, \dots N$ spaced by k elements, for different values of the number of the initial element m was calculated as follows:

$$L_m(k) = \frac{\sum_{i=1}^{\left[\frac{(N-m)}{k}\right]} |X(m+ik) - X(m+(i-1)k)| \cdot (N-1)}{\left[\frac{(N-m)}{k}\right] k^2}, m = 1, 2, \dots, k \quad (1)$$

where N is the length of a series, and $[z]$ denotes the integer part of z.

This distance is averaged over all numbers of the initial element m

$$L(k) = \frac{1}{k} \sum_{m=1}^{k} L_m(k). \quad (2)$$

The dependence of the average distance between spaced elements on time space k has a power-law form with a coefficient equal to the fractal dimension D_0: $L(k) \sim k^{-D_0}$.

This corresponds to a linear relationship on a log scale

$$\lg(L(k)) = -D_0 \lg(k) + \lg(L(1)). \quad (3)$$

The fractal dimension was calculated as a linear regression coefficient between the logarithms L and $k = 2^l, l = 1, 2, \dots, 10$.

The function to calculate the average distance L in MATLAB is given below

```
function Lk=higuchi(H,a_max)
N=length(H);
for a=1:a_max;
    k=2^a;
  for m=1:k
    for i=1:(N-m)/k
        dH(i)=abs((H(m+i*k)-H(m+(i-1)*k)));
    end
      L(m)=1/k*mean(dH)*(N-1)/k;
      clear dH
  end
    Lk(a)=mean(L);
clear L
end
```

The calculation is not particularly complicated and consists of three nested cycles, the first of which calculates the distance between the elements, the second averages it over all m, and the third changes the space between the elements. The accuracy of the HFD estimation (a median of regression coefficient errors at 0.95 confidence level for year) equals 0.08. The example of the variation, its FFT spectrum and $L(k)$—dependence are shown in Figure 1. As a result of calculations, a series of geomagnetic variations HFD were obtained (32 values for each day).

Note that HFD describes a continual part of the geomagnetic variation spectrum and, therefore, a spectrum slope coefficient can be calculated by a formula: $|\beta_H| = 5 - 2D_0$ [15].

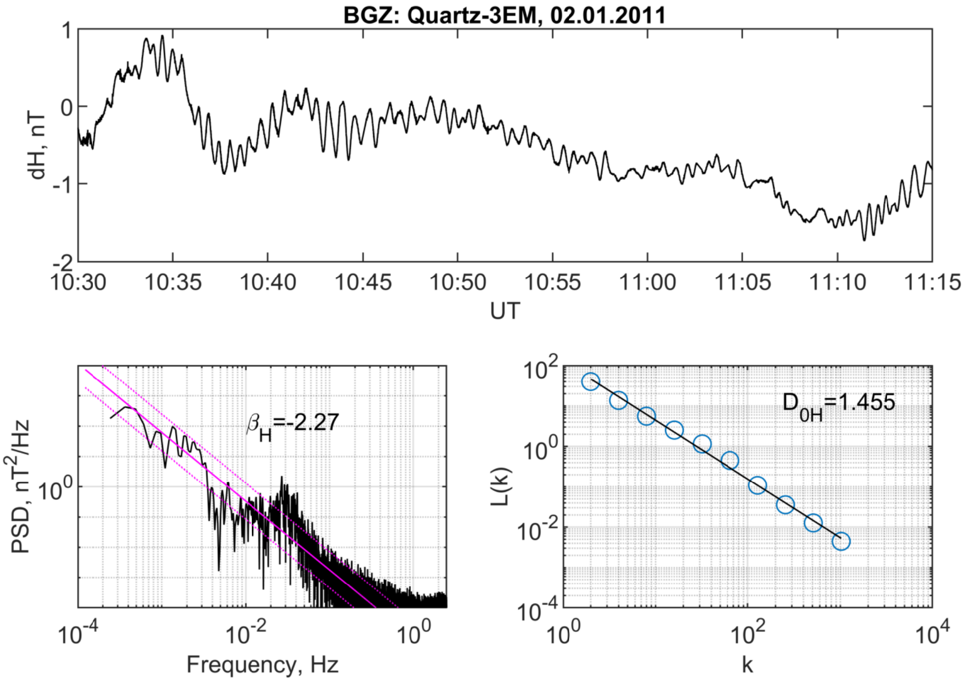

Figure 1. Variations of geomagnetic field horizontal component on 2 January 2011, at Altay (**upper panel**), its FFT spectrum (**low left panel**) and dependence of distance between spaced elements on the time space k (**low right panel**). HFD and β_H—spectrum slope coefficient are shown on the graphs.

2.3. Dependence of HFD Estimation on Maximal Time Space $k_{\max}$

It has been shown that HFD estimation value depends on maximal time space $k_{\max}$ and data type [16].

HFD was calculated at various values of its parameter ($k_{\max} = 2^{l_{\max}}, l_{\max} = 7, \ldots, 12$) for 6 h time intervals to study HFD estimation dependencies on $k_{\max}$ (see Figure 2).

These estimations were compared to the mean value of 8 values of HFD, calculated at $l_{\max} = 10$ for 45 min intervals within this 6 h interval. An example of these calculations is shown in Figure 2.

Similar calculations were carried out for 20 days, both in quiet and disturbed geomagnetic conditions. The analysis showed that the mean value was in the error band of the $D_0(k_{\max})$—curve with the quiet geomagnetic field (the example is shown in Figure 2 for time period 00-06 UT).

However, the difference between the mean value and $D_0(k_{\max})$—curve could reach 0.1 when HFD changed rapidly. The example at time period 06–12 UT is shown in Figure 2, which contains the beginning of the storm, and the HFD changed from 1.3 to 1.1 at this time. Meanwhile, when the storm developed and the fractal dimension was consistently low (12–24 UT in Figure 2), the average HFD value was again within the error interval of the $D_0(k_{\max})$ curve. Thus, under stable geomagnetic conditions, the calculation method did not significantly affect the HFD value.

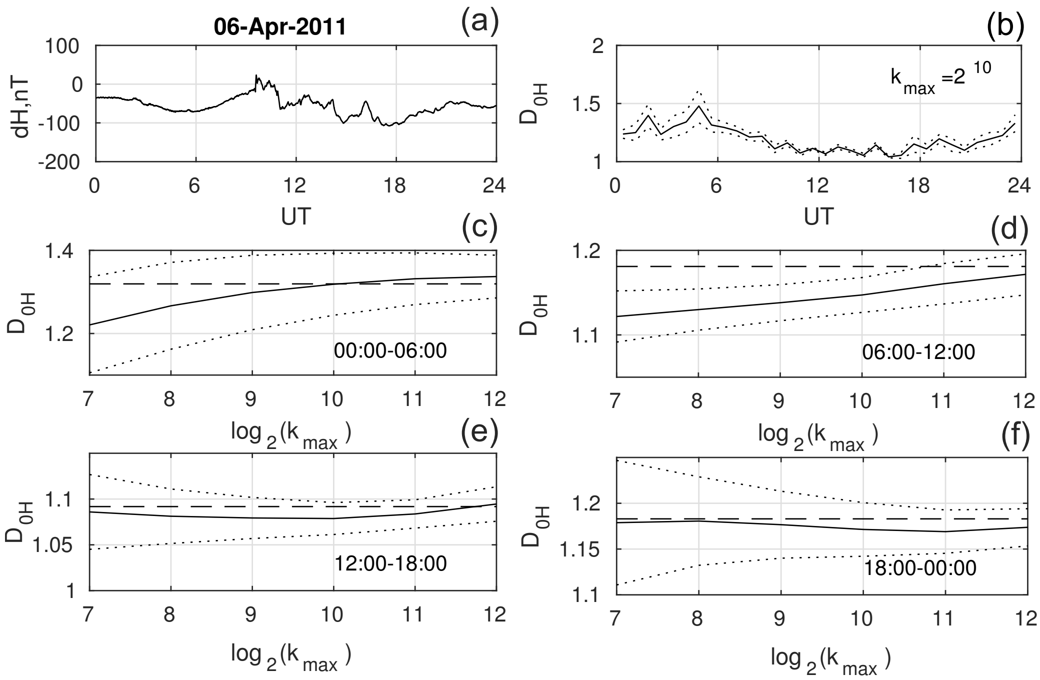

Figure 2. The dependence of HFD estimation on maximal time space k_{max}. H-component variation 6 April 2011, (**a**) and dependence of HFD estimation values on time at $k_{\max} = 2^{10}$ (**b**). Panels (**c**–**f**) show dependencies of HFD estimation on maximal time space $k_{\max}$ at various 6 h time intervals. The horizontal line corresponds to a mean value of 8 estimations at $k_{\max} = 2^{10}$.

2.4. Diurnal Variation of HFD

The analysis showed that the fractal dimension was subject to periodic variations, including daily variations. A spectrum of HFD variations for February 2011, data and diurnal variation of H, D, Z-component fractal dimensions are shown in Figure 3a. Daily variations of HFD for each month were calculated using the following formula [17]:

$$\Delta D_{0Xj}(UT) = \frac{1}{n_j}\sum_{i=1}^{n_j}\left(D_{0Xi}(UT) - \overline{D_{0X}}\right), \quad (4)$$

where $i = 1 \ldots n_j$ is the day number in a month, $j = 1, 2, \ldots, 12$ is the month number, $X = H, D, Z$ are geomagnetic components, $\overline{D_{0X}}$ is the daily average of fractal dimension, UT—Universal Time. A seasonal variation of HFD diurnal variation at Altay is shown in Figure 3b–d. It is shown that the diurnal variations for the fractal dimensions of D- and H-components were roughly opposite. The maximum value of ΔD_{0Hj} (for the H-component) was distinguished near the sunset terminator and was at a minimum—at night time. The maximum ΔD_{0Dj} (for D-components) was observed at 14 LT and minimum—in 6 LT and 11 LT (LT—Local Time). The fractal dimension of the Z-component decreased during the day and increased at night.

The diurnal variation amplitude was close to 0.1, and its variance ratio to the total variance of the fractal dimension for the H, D, Z-components equaled 0.30, 0.31 and 0.41, respectively.

Then, the diurnal variation of fractal dimension was removed from the data (4)

$$D^*_{0Xj}(UT) = D_{0Xj}(UT) - \Delta D_{0Xj}(UT)$$

and the pair and partial correlation coefficients, with mean values of the space weather characteristics over 45 min, were calculated. The data of solar wind parameters was obtained at OMNIWEB, data of AE-index—at the World Data Center for Geomagnetism, Kyoto. This data was averaged for each 45-min interval.

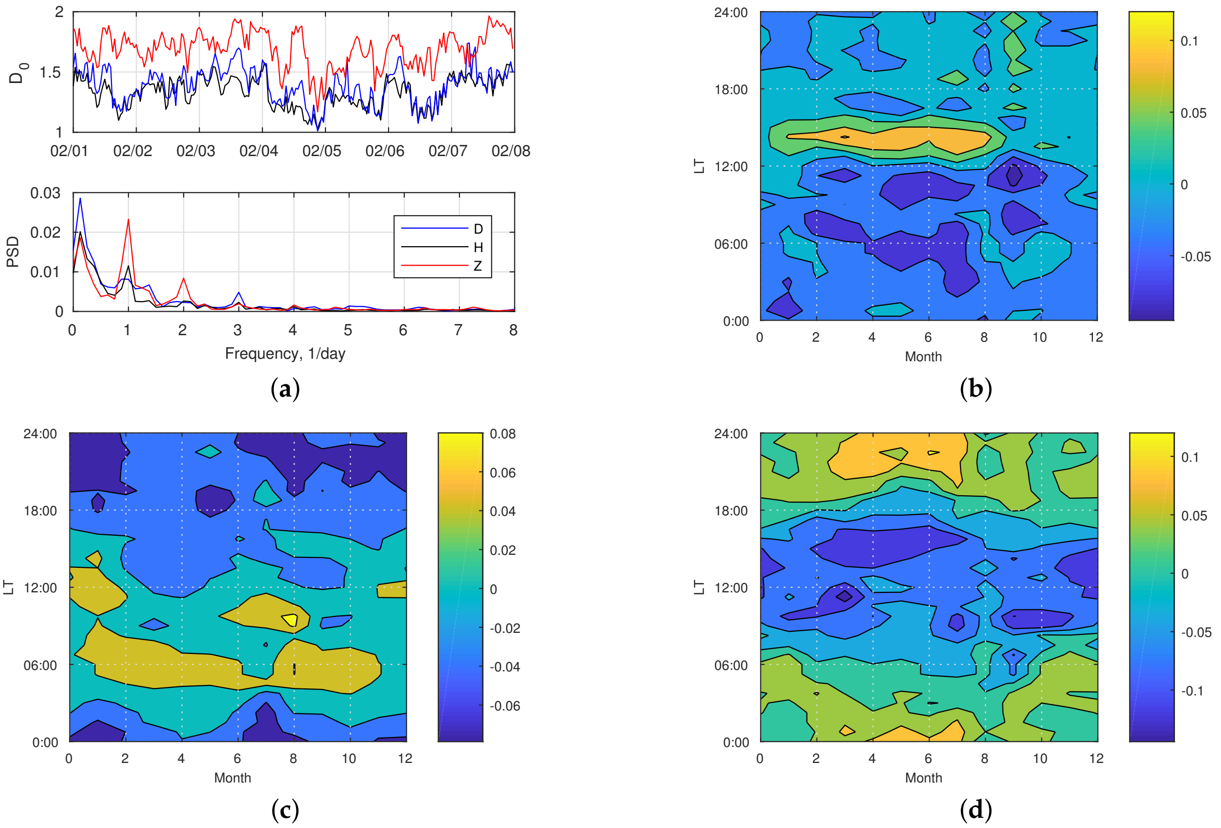

Figure 3. The time series of geomagnetic variations HFD 1–7 February, 2011, and its spectra for February, 2011 (**a**), and daily–seasonal dependence of deviation from the daily average of geomagnetic variation fractal dimension at Altay ΔD_{0Xj}: (**b**)—D-component, (**c**)—H-component, (**d**)—Z-component. $LT = UT + 6$.

3. Results and Discussion

It is known that the HFD of geomagnetical variations can be described as a normally distributed random value [8]. Annual averaged values and standard deviations of HFD for H,D,Z-components in 2011, as parameters of normal distribution, were calculated. These values are presented in Table 1.

Table 1. Annual averaged values and standard deviations of geomagnetic variation HFD in 2011.

Parameters of Normal Distribution	*H*	*D*	*Z*
Mean value	1.365	1.420	1.662
Standard deviation	0.124	0.144	0.142

It can be seen that the HFD of the Z-component was significantly higher than those of the other components. The Z-components can be described as anti-persistent noise. The averaged values for the H,D-components coincided with the results of [8]. The standard deviations were less than in [8], which considered data on 40 magnetic stations at various geomagnetic latitudes for the solar cycle. Apparently, the missing part of the variance controls the latitude dependence of the fractal dimension.

For the fractal dimension with the AE index, the characteristics of the solar wind Pearson pair correlation coefficients were calculated as well. The correlation coefficients for the H, D, Z-components for the data of 2011 are presented in Table 2.

Table 2. Pair correlation coefficients of geomagnetic variation fractal dimension and space weather parameters in 2011.

Parameters	*H*	*D*	*Z*
Logarithm of solar wind speed $lg(v)$	−0.286	−0.360	−0.366
Logarithm of plasma density $lg(n)$	−0.184	−0.091	−0.138
Logarithm of interplanetary magnetic field strength $lg(IMF)$	−0.355	−0.314	−0.314
North component of IMF Bz	0.087	0.143	0.116
Logarithm of AE-index $lg(AE)$	−0.439	−0.541	−0.509

In order to ensure data agreement with a normal distribution, the AE index, plasma density, solar wind velocity and IMF strength were derived logarithmically. Negative correlations of the geomagnetic variation fractal dimension, with an averaged logarithm of solar wind velocity, logarithm IMF and logarithm of AE-index, were detected.

In general, this result coincided with the results of the papers mentioned in the introduction. It should be noted that, in our calculations, a statistically reliable relationship of HFD with the space weather characteristics was revealed for all components of the geomagnetic field (H, D, Z), while in previous studies [6–9], a significant decrease in HFD with geomagnetic disturbances was noted only for the horizontal component.

Since the AE-index, solar wind speed and IMF correlated (except v and IMF), partial correlation coefficients were calculated for them. Partial correlation coefficients for four variables: $(D_{0X}, lg(AE), lg(IMF), lg(v))$—are shown in Table 3.

Table 3. Partial correlation coefficients of geomagnetic variation fractal dimension of geomagnetic variations for H, D, Z-component and logarithms of auroral electrojet index (AE), interplanetary magnetic field (IMF) and solar wind velocity (v) in 2011.

Partial Correlation Coefficients	$X = H$	$X = D$	$X = Z$
$R_{D_{0X},AE\|IMF,v}$	−0.274	−0.389	−0.344
$R_{D_{0X}.IMF\|AE,v}$	−0.242	−0.164	−0.175
$R_{D_{0X}.v\|AE,IMF}$	−0.113	−0.143	−0.166
Determination coefficient R^2	0.252	0.329	0.312

Here, X means one of the components of the geomagnetic field. The partial correlation coefficients were smaller than the pair coefficients for all characteristics. A determination coefficient showed that at least a quarter of the fractal dimension variance was controlled by the variability of the AE index, IMF and v. Thus, the part of variance controlled by these parameters had approximately the same value as the daily variation. Probably the variation of these parameters caused a harmonic with a period approximately equal to a week in Figure 3a.

Partial correlation coefficients of HFD with logarithms of AE-index, IMF and solar wind velocity were calculated for each month of 2011–2013. The example of temporal dependencies of HFD, $lg(AE)$, $lg(IMF)$ and $lg(v)$ for a month, and the scatter plots, are shown in Figure 4.

The figure illustrates that high values of the AE-index and maxima of interplanetary magnetic fields corresponded to low values of HFD in February 2011.

The calculation results of the monthly partial correlation coefficient of HFD with $lg(AE)$, $lg(IMF)$ and $lg(v)$ for the period 2011–2013 are shown in Figure 5.

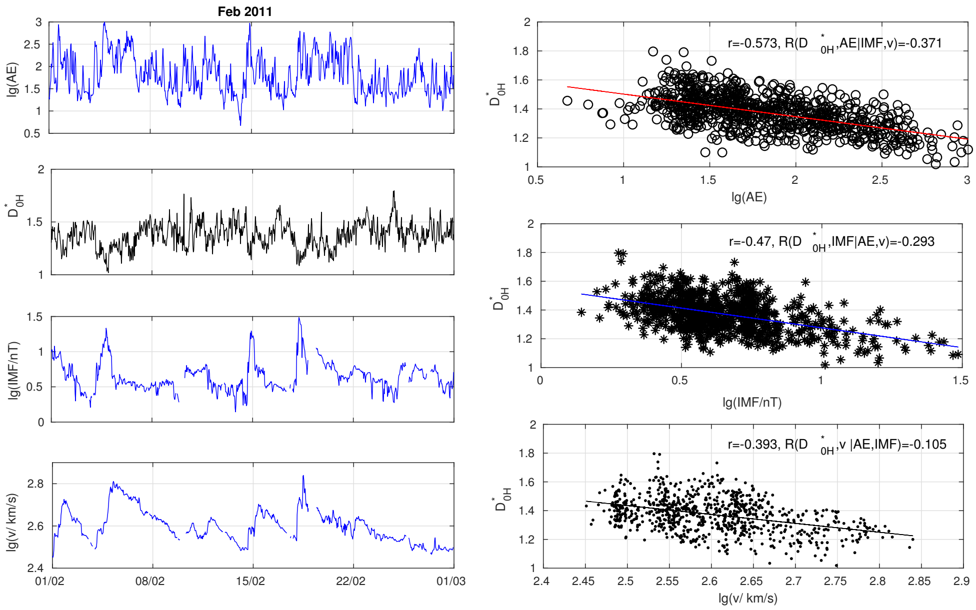

Figure 4. The correlation of geomagnetic variation HFD and space weather characteristics in February, 2011. The following are shown: (**left**)—temporal dependencies of logarithms of AE-index, HFD of H-component geomagnetic variation, logarithms of IMF and solar wind velocity (v), (**right**)—scatter plots for HFD and $lg(AE)$ (**upper**), $lg(IMF)$ (**medium**) and $lg(v)$ (**lower**). Pair and partial correlation coefficients are shown at the plots.

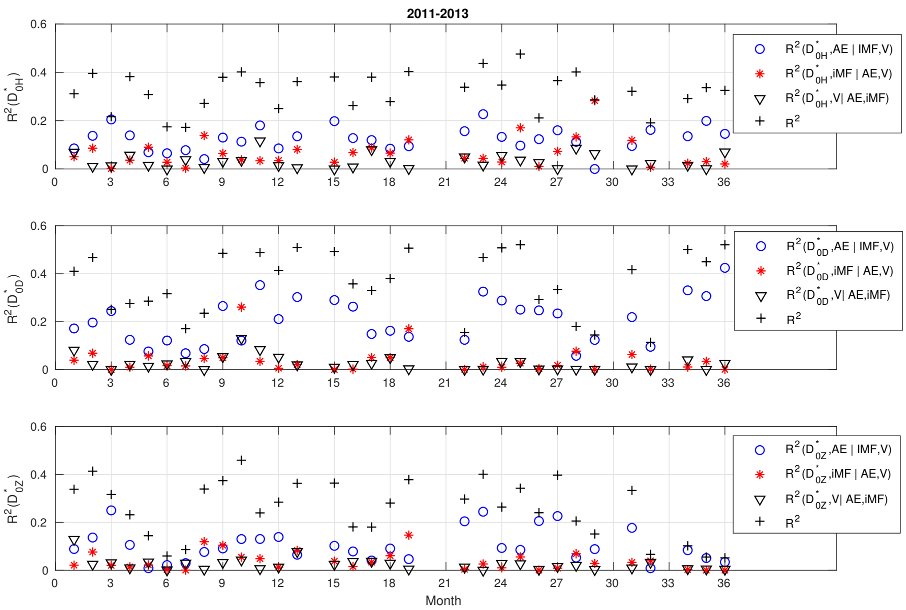

Figure 5. The results of calculation of the square of partial correlation coefficients of geomagnetic variation fractal dimension at Altay with logarithms of AE-index, interplanetary magnetic field strength and solar wind velocity (2011–2013). The crosses show the determination of coefficient values.

It can be seen that partial correlation coefficients for IMF and solar wind velocity were usually smaller than for the AE-index. The crosses show the part of the HFD variance controlled by the variability of these parameters (determination coefficient). It varied widely from 0.06 (Z-component, June 2011) to 0.52 (D-component, January 2013). On average, it was 0.32, 0.38, 0.26 for the H, D, Z-components, respectively. The reasons for these changes and changes in the partial correlation coefficients require additional research.

4. Conclusions

The fractal dimension of geomagnetic variations at the Baigazan magnetic station at Russian Altay for 2011–2013 was calculated using the Higuchi method. The daily variation of HFD for the DHZ-components was investigated. Its contribution to the variability of HFD was from 30 to 40 percent of the total variance. A correlation analysis of the fractal dimension of the variations of the D, H, Z-components with AE-index and solar wind characteristics was carried out. Negative correlations with logarithms of AE-index, IMF, strength and solar wind velocity were found. About one quarter of the HFD variance is controlled by the variability of these space weather characteristics. Pair and partial correlation coefficients for these parameters were calculated for every month of 2011–2013.

Further development of the research may involve the application of the Katz method to calculate the fractal dimension of the variation in the D, H, Z components of the geomagnetic field [18]. It should be noted here that the Hurst exponent method could also be used [19].

Another direction of development for this work is the use of a multi-fractal approach in the study of, by analogy with [20], the dynamics of changes in the H, D, Z-components of the geomagnetic field.

In the future, it is also planned to make calculations for the Kamchatka Territory, where the complex geophysical observatory "Paratunka" is located.

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Data Availability Statement: The data concerning the Gorno-Altaisk State University (GASU) magnetic station «Baygazan» are available at request. The data of solar wind parameters are available at OMNIWEB https://omniweb.gsfc.nasa.gov/form/om_filt_min.html (accessed on 1 May 2023). The data of AE-ndex are available at World Data Center for Geomagnetism, Kyoto, https://wdc.kugi.kyoto-u.ac.jp/aeasy/index (accessed on 1 May 2023).

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Conflicts of Interest: The author declares no conflict of interest.

Abbreviations

The following abbreviations are used in this manuscript:

HFD	Higuchi Fractal Dimension
AE	Auroral Electrojet
IMF	Interplanetary Magnetic Field
FFT	Fast Fourier Transform
UT	Universal Time
LT	Local Time

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