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Oscillation Analysis Algorithm for Nonlinear Second-Order Neutral Differential Equations

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Abstract: Differential equations are useful mathematical tools for solving complex problems. Differential equations include ordinary and partial differential equations. Nonlinear equations can express the nonlinear relationship between dependent and independent variables. The nonlinear second-order neutral differential equations studied in this paper are a class of quadratic differentiable equations that include delay terms. According to the t-value interval in the differential equation function, a basis is needed for selecting the initial values of the differential equations. The initial value of the differential equation is calculated with the initial value calculation formula, and the existence of the solution of the nonlinear second-order neutral differential equation is determined using the condensation mapping fixed-point theorem. Thus, the oscillation analysis of nonlinear differential equations is realized. The experimental results indicate that the nonlinear neutral differential equation can analyze the oscillation behavior of the circuit in the Colpitts oscillator by constructing a solution equation for the oscillation frequency and optimizing the circuit design. It provides a more accurate control for practical applications.

Keywords: nonlinear; second order; neutral differential equation; vibration

MSC: 62FXX

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Citation: Song, L.; Chen, S.; Wang, G. Oscillation Analysis Algorithm for Nonlinear Second-Order Neutral Differential Equations. *Mathematics* **2023**, *11*, 3478. https://doi.org/10.3390/math11163478

Academic Editor: Livija Cveticanin

Received: 16 June 2023
Revised: 25 July 2023
Accepted: 1 August 2023
Published: 11 August 2023

1. Introduction

Mathematics was used not only as a tool but also in other sciences, and mathematics led the development of other disciplines. The development of differential equations is one of the important factors. Differential equations are developed on the basis of calculus theory. Differential equations include ordinary differential equations and partial differential equations. Due to its solid background in practice, profound theory, and close relationship with other mathematics, it has developed into a strong branch of mathematics and has been widely used. In history, ordinary differential equations appeared even earlier than the invention of calculus. Some issues required the establishment and solution of differential equations. In fact, however, there are few differential equations that can be expressed by integrating an elementary function, and a large number of differential equations cannot be solved with the elementary integration method. The existence and uniqueness theorem of solutions lays the foundation for the conclusion of calculus. Sturm's work proposed the initial idea of qualitative research on solutions. The past and present of differential equations have provided favorable tools for mechanics, astronomy, physics, chemistry, biology, various technical sciences (such as nuclear energy, rockets, artificial satellites, automatic control, radio electronics, etc.), and several social sciences (such as population issues, economic forecasting, commercial sales issues, transportation scheduling issues, etc.).Second-order differential equations are widely used in fields such as object mechanics, electronics, population ecology, economics, and modern control theory. Nonlinear second-order neutral differential equations include first-order derivative equations and second-order differential equations without derivative terms. Some of these differential equations have delay characteristics, making them suitable for studying dynamic systems with certain delay effects. In recent years, scholars in this field have analyzed the oscillation characteristics of quadratic nonlinear neutral differential equations, and they have obtained a lot of good results [1]. Especially in the study of second-order nonlinear perturbations, damping [2], functionals, and time delays, the research direction of differential equations has been expanded. Experts have found that calculus theory can be used to solve many practical problems, laying the foundation for the development of modern science. For example, calculus is used in astronomy to calculate the orbits of stars, in physics to study the motion and forces of objects, and in economics to study marginal quantity and extreme value problems. These achievements have made calculus an indispensable tool in modern science and technology. Initially, mathematicians focused their main energy on finding general solutions to equations. Later, it was explained that this method was not universal and even impossible to solve a definite solution represented by an elementary solution (in integral form) that met certain conditions. Moreover, most ordinary differential equations cannot find very accurate solutions, only approximate solutions. So, people shifted their attention to qualitative analysis research, and a theory of the existence and uniqueness of equation solutions emerged. In recent years, the oscillation of the solution of ordinary differential equations has also made great progress, greatly enriching the theory of ordinary differential equations. The boundary value problem of differential equations is an important part of differential equations. The research in the seventeenth century laid the physical foundation for dealing with vibration problems. By the eighteenth century, vibration mechanics had become independent of physics. With the joint efforts of scholars in different periods, this discipline has become the most important theoretical achievement of the present. The essence of vibration analysis for nonlinear second-order neutral differential equations lies in studying the oscillation properties and dynamic behavior of the equation solution, as well as the relationship between these behaviors and various parameters and the initial value conditions of the differential equation. Vibration analysis is an important research direction in the field of nonlinear dynamics, which involves many complex mathematical theories and tools, such as Lyapunov stability theory, periodic orbit theory, central manifold theory, etc. For the vibration analysis of nonlinear second-order neutral differential equations, it is necessary to study from different perspectives and adopt different methods and techniques.

In reference [3], the text proposed an oscillation analysis method for third-order neutral distributed delay differential equations, studying the oscillation of solutions for a class of third-order neutral distributed delay differential equations in two different cases. The problem was studied using the generalized Riccati transformation technique and Yang inequality method, and sufficient conditions were established for the oscillation or convergence of each solution of the equation to 0. Based on the obtained results, some famous oscillation criteria in the existing literature were generalized and improved, and examples were given to illustrate the applicability of the obtained results. In reference [4], a new vibration analysis method was proposed. Based on the vibration theory, this method transforms the vibration of a nonlinear delay differential equation into the vibration of a linear differential equation with delay, using linear θ to obtain the corresponding numerical solution. Therefore, the vibration analysis of the numerical solution of a nonlinear differential equation with time delay is carried out. In reference [5], a new method for vibration analysis of even number neutral differential equations was presented. By using Riemann Liouville calculus, Riccati transformation, and inequality methods, the vibration analysis of even number neutral differential equations was achieved. Reference [6] proposed an oscillation analysis method based on nonlinear fractional order differential equations with damping terms. This algorithm utilizes Riemann Liouville calculus, Riccati transformation, and inequality methods to obtain sufficient conditions for the oscillation of nonlinear fractional differential equations with damping terms. It extends existing methods for analyzing oscillations of fractional order differential equations. Thus, the oscillation analysis of solutions to nonlinear fractional order differential equations can be achieved. In reference [7], a vibration analysis method for nonlinear delay differential equations based on a class of independent variables with piecewise continuous arguments was proposed. This method mainly considered the oscillation of numerical solutions for a nonlinear differential equation with piecewise continuous arguments. Through its linearization theory, the vibration of a nonlinear equation was transformed into a method of vibration for a linear equation. The vibration conditions of the nonlinear delay differential equation were obtained, and then the linear θ method was obtained to maintain the vibration of the equation. The oscillation analysis of nonlinear delay differential equations was realized. A vibration analysis method for nonlinear differential equations based on the Burg spectrum estimation of vibration signals was proposed in reference [8]. The method used the Burg spectrum estimation to analyze the vibration signals of differential equations. Through its analysis results, the oscillation analysis of nonlinear differential equations was realized. In reference [9], it mainly studied the oscillation of third-order neutral nonlinear differential equations, and the equations considered the irregular form. Some new vibration criteria were established, and some examples were given to illustrate the main results. Through the analysis of these examples, the vibration problem of the equation is calculated. Reference [10] proposed a vibration analysis method for nonlinear differential equations based on the differential inequality method. The method is constrained by Robin and Dirichlet boundary conditions and has sufficient conditions for the oscillation of differential equation solutions, which can realize the oscillation analysis of nonlinear differential equation solutions. Reference [11] studied the complex dynamic behavior of a new type of chaotic system and introduced a memristor. Based on eigenvalue theory, the stability of a memristor system is analyzed by selecting a key parameter. When the system crosses the critical value, it will exhibit Neimark–Sacker bifurcation and chaotic behavior. The system based on a memristor is simulated to verify the existence of a chaotic attractor. The analog electronic circuit of the memristor chaotic system is designed to ensure that the results of this paper can be applied to practical problems. Reference [12] introduces a two-dimensional discrete-time predator model that studies single-parameter bifurcation and dual-parameter bifurcation by determining fixed points. After strong resonance bifurcation occurs in the model, the branching scenario is determined based on the key coefficients, and the analysis results are validated using MatContM based on numerical continuation methods. Reference [13] analyzed the stability and local bifurcation of the spatiotemporal SI repetitive model from both analytical and numerical perspectives, and based on this, studied the cross-critical bifurcation and other bifurcations. This study calculates the critical normal coefficient to determine its non-degradation condition and uses the numerical continuation method and MatContM to determine the obtained analysis results. Through numerical simulation, the closed invariant curve at the Neimark–Sacker point was solved and decomposed to generate a chaotic singular attractor. Reference [14] utilized the Bayesian VAR model, constant model, time-varying model, and Markov model to measure inflation expectations and study the influencing factors of inflation expectations. Reference [15] solved the spatiotemporal variable-order fractional advection-diffusion equation with a nonlinear source term through the neural network method. According to the property of the derivative of the variation, the loss function of the neural network was solved. If the function satisfies the Lipschitz assumption, the reasonable range of learning rate is determined. The loss function is obtained by repeatedly training the neural network to solve the nonlinear variable fractional order.

Based on the above methods, for the oscillation of solutions for various types of nonlinear differential equations, different methods based on different forms are given. Nonlinear second-order neutral differential equations can solve many practical problems. Therefore, studying the oscillation of its solutions and exploring the oscillation characteristics and behavior laws of solutions to nonlinear neutral differential equations is of great help for practical applications. Thus, this paper studies the oscillation analysis algorithm for solutions for nonlinear second-order neutral differential equations. The whole framework of the method is as follows:

The first is to establish nonlinear second-order neutral differential equations;

The second is to calculate the initial value problem through the value interval in nonlinear second-order neutral differential equations;

The third part determines the existence of global solutions for nonlinear second-order neutral differential equations;

Fourthly, according to the fixed-point theorem of contraction mapping, the existence of solutions for nonlinear second-order neutral differential equations is observed, and the oscillation analysis of solutions for nonlinear second-order neutral differential equations is obtained;

The final part summarizes the analysis algorithms for the oscillation of solutions to nonlinear second-order neutral differential equations and proposes prospects for more possible future research.

To study the oscillation of solutions to differential equations, this study innovatively introduced the fixed-point theorem of condensation mapping to prove the existence of solutions. At the same time, it also inferred the oscillation of three solutions for nonlinear second-order neutral differential equations. To explore the practical application of solving oscillation, the paper also analyzes the nonlinear relationship between capacitance and inductance based on the circuit of the Colpitts oscillator.

In this paper, Section 1 explains the background, conducts the existing literature methods related to the oscillation of differential equation solutions, and elaborates on the research framework of this method. Section 2 introduces the existence of solutions for nonlinear second-order neutral differential equations. Section 3 analyzes the oscillation of solutions for nonlinear second-order neutral differential equations in different situations. Section 4 explains the research process, conclusions, and development trends.

2. Existence of Solutions for Nonlinear Second-Order Neutral Differential Equations

To analyze the oscillation of solutions for nonlinear second-order neutral differential equations, a nonlinear second-order neutral differential equation was first established. Then, the initial value of the differential equation was calculated. Finally, the existence of the solution was analyzed using the fixed-point theorem of condensing mapping.

2.1. Solution of Initial Value of Quadratic Neutral Nonlinear Differential Equation

Nonlinear second-order neutral differential equations are a special form of differential equations that contain neutral terms. Nonlinear second-order neutral differential equations are an important branch of differential equation theory and one of the fundamental problems to be solved in various fields. In solving this equation, several factors must be considered, such as the practical application requirements of the equation, the rationality of the equation form, etc. As a consequence, a quadratic neutral nonlinear differential equation is given. This is a piecewise connectable quadratic differential equation that can satisfy the classical solution of a quadratic neutral nonlinear differential equation. That is, the solution of this second-order differential equation is not globally continuous and differentiable, it is about jumps or discontinuity at certain points. Such equations are often called piecewise smoothing equations. The segmentation can be a spatial segmentation. It can also be a time segmentation. For second-order differential equations that are differentiable or satisfy shards, the solution may have multiple segmented intervals, and the solution in each segmented interval may be in different functional forms [16]. Therefore, for the solutions of such differential equations, they must be solved separately in each segmented interval, and it is necessary to ensure the existence of the derivative at the segmentation point, thus satisfying the initial value or boundary conditions of the equation. Otherwise, the solution of the equation may exhibit discontinuities at the segmentation points and thus fail to satisfy the initial requirements. Therefore, it is possible to reasonably define nonlinear second-order neutral differential equations in the sense of classical solutions. The starting value of the differential equation is selected according to the value interval of ϖ in the function, and the starting value of the differential equation is calculated using the formula of initial value.

The quadratic neutral nonlinear differential equation is:

$$\begin{cases} \left(f(\varpi, x(\varpi-\tau)) + [r(\varpi)x^2(\varpi)]'\right)' = 0, \varpi \geq \varpi_0, \varpi \neq \varpi_k, \varpi_k+\tau, k \in N, \\ x^{(i)}(\varpi_k^+) = g_k^{[i]}(x^{(i)}(\varpi_k)), \varpi_k+\tau \in C(x^{(i)}), i = 0,1,2, k \in N, \\ x(\varpi) = \varphi(\varpi), \varpi \in [\varpi_0-\tau, \varpi_0], \varphi \in C^2([\varpi_0-\tau, \varpi_0]). \end{cases} \quad (1)$$

$C(x^{(i)})$ is the set of all continuous points in the domain of function $x^{(i)}(\varpi)$. N is the set of positive integers. i denotes the order of derivative. R is all real numbers. $k \in N$ is the definite number. τ is a positive number. $r(\varpi)$ is continuous on $[\varpi_0, +\infty)$, and $r(\varpi) > 0$. $f(\varpi, x)$ is continuous on $[\varpi_0, +\infty) \times (-\infty, +\infty)$, and $xf(\varpi, x) > 0 (x \neq 0)$. $\varpi_0 < \varpi_1 < \cdots < \varpi_k < \cdots, \varpi_{k+1} - \varpi_k > \tau, k = 0,1,2,\cdots, \lim\limits_{k\to\infty} \varpi_k = +\infty$. $x^{(0)}(\varpi) = x(\varpi)$, $x^{(i)}(\varpi_k^-)$, $x^{(i)}(\varpi_k^+)$, and $x^{(i)}(\varpi_k)$ are defined as $x^{(i)}(\varpi_k^-) = \lim\limits_{\varpi\to\varpi_k^-} x^{(i)}(\varpi)$, $x^{(i)}(\varpi_k^+) = \lim\limits_{\varpi\to\varpi_k^+} x^{(i)}(\varpi)$, and $x^{(i)}(\varpi_k) = x^{(i)}(\varpi_k^-)$, respectively. $g_k^{[i]}$ is the set of all continuous points in the definition field of function.

The calculation formula for the initial value problem of second-order nonlinear neutral differential equations was derived based on the special properties of this type of differential equation, it will not change due to changes in the type of differential equation. However, the calculation of the initial value problem is affected by the following three aspects:

1. The degree of non-linearity of the equation: The higher the degree of nonlinearity of the equation, the more difficult it is to obtain the analytical formula of its solution and the more difficult the solution process will be. In terms of calculation formulas, it may be necessary to use more advanced numerical calculation methods, such as the iterative method, Runge-Kutta method, etc.
2. Differences in initial conditions: Different initial conditions may affect the selection of calculation formulas, such as the different initial values at different times, the specific values of the initial values, and so on. Different initial conditions may lead to differences in the convergence of the solution and the difficulty of the calculation method, therefore, it is necessary to reconstruct the calculation formula for different initial value conditions;
3. The form of the solution function: The solution function of second-order neutral differential equations can have three forms—one is a scheme that describes the state and the derivative of the previous moment, the second is a scheme that describes only the derivative of the previous moment, and the third is a solution that stores only the state of the previous moment. The calculation formulas for these three forms of solution functions can vary accordingly.

Lemma 1. *Supposing* $x(\varpi) : [\varpi_0 - \tau, +\infty) \to R$, $\varpi_0 \geq 0$ *is called the global solution of the current equation, if the current equation satisfies the condition:*

(1) $x(\varpi) = \varphi(\varpi), \varpi \in [\varpi_0 - \tau, \varpi_0]$.
(2) $\varpi \in [\varpi_0, +\infty), \varpi \neq \varpi_k, \varpi_k + \tau (k \in N), 2x(\varpi)$ *meets:* $[r(\varpi)x^2(\varpi)]' + f(\varpi, x(\varpi - \tau)) = 0$.
(3) $x^{(i)}(\varpi)$ *and* $r(\varpi)x^2(\varpi)$ *are continuous in* $[\varpi_0, +\infty) \backslash \{\varpi_k\}$ *and* $\varpi = \varpi_k$. $x^{(i)}(\varpi_k)$ *satisfies* $x^{(i)}(\varpi_k^+) = g_k^{[i]}(x^{(i)}(\varpi_k)), k \in N$.

By observation, it can be found that the above method avoids the absolute continuous concept of other methods. The range of the value of the whole solution is determined with the ϖ [17].

Proof of Lemma 1. In summary, the calculation formula for the initial value problem of second-order nonlinear neutral differential equations may require different calculation formulas for the degree of nonlinearity of the equation, different initial value conditions, and different forms of solution functions in practical calculations. From this, the initial value calculation method for the second-order nonlinear neutral differential equation given in Formula (1) was derived. □

Assuming that $r(\varpi)$ is continuous on $[\varpi_0, +\infty)$ and $xf(\varpi, x) > 0(x \neq 0)$, the initial value of $x(\varpi)$ is calculated as follows:

$$\begin{cases} f(\varpi, x(\varpi - \tau)) + r(\varpi)x^2(\varpi) = 0, \varpi \geq a, \\ x(\varpi) = \psi(\varpi), \varpi \in [a - \tau, a], \psi \in C^2([a - \tau, a], R), \end{cases} \tag{2}$$

If there is at least one solution to the initial value of any $a \geq \varpi_0$, the initial value problem of $\psi \in C^2([a - \tau, a], R)$ has at least one solution at $[a - \tau, +\infty)$. When $\varphi \in C^2([\varpi_0 - \tau, \varpi_0], R)$ is the initial condition, Formula (1) will have at least one solution of $x(\varpi)$ in $[\varpi_0 - \tau, +\infty)$.

When $x_0(\varpi)$ is the initial value, the formula for calculating its initial value will change [18]. The formula for calculating the initial value after the change is as follows:

$$\begin{cases} f(\varpi, x_0(\varpi - \tau)) + r(\varpi)x_0^2(\varpi) = 0, \varpi \geq \varpi_0, \\ x_0(\varpi) = \varphi(\varpi), \varpi \in [\varpi_0 - \tau, \varpi_0], \end{cases} \tag{3}$$

When a solution on $[\varpi_0 - \tau, +\infty)$ satisfies the condition $\varphi^{(i)}(\varpi_0) = x_0^i$, $x_1(\varpi)$ is the initial value problem. The initial value calculation is as follows:

$$\begin{cases} (x_0^2(\varpi) - \frac{1}{r(\varpi)} r(\varpi_1)x_0^2(\varpi_1) + \frac{1}{r(\varpi)} r(\varpi_1)g_1^{[i]}(x_0^2(\varpi_1)), \\ x_1^i(\varpi_1^+) = g_1^{[i]}(x_0^{(i)}(\varpi_1), \end{cases} \tag{4}$$

The result of Formula (4) is a solution in $(\varpi_1, \varpi_1 + \tau]$. Furthermore, $y_1(\varpi)$ is the initial value problem, and the initial values are calculated as follows:

$$\begin{cases} f(\varpi, y_1(\varpi - \tau)) + r(\varpi)y_1^{(i)}(\varpi) = 0, \varpi \geq \varpi_1 + \tau, \\ y_1(\varpi) = x_1(\varpi), \varpi \in (\varpi_1, \varpi_1 + \tau], y_1(\varpi_1) = x_1(\varpi_1^+), \end{cases} \tag{5}$$

The result of formula (5) is a solution on $(\varpi_1, +\infty]$. To define a function, the function is as follows:

$$z_1(\varpi) = \begin{cases} x_0(\varpi), \varpi \in [\varpi_0 - \tau, \varpi_0], \\ x_1(\varpi), \varpi \in (\varpi_1, \varpi_1 + \tau], \\ y_1(\varpi), \varpi \in (\varpi_1 + \tau, \varpi_2], \end{cases} \tag{6}$$

In the formula, $z_1(\varpi)$ satisfies Formula (1) on the interval $[\varpi_0 - \tau, \varpi_2]$. If $\varpi \in [\varpi_0 - \tau, \varpi_1]$, $z_1(\varpi) = x_0(\varpi)$. According to Formula (3), $z_1(\varpi)$ can also satisfy Formula (1) on interval $[\varpi_0 - \tau, \varpi_1]$. If $\varpi \in (\varpi_1, \varpi_1 + \tau]$, $z_1(\varpi) = x_1(\varpi)$. $z_1^{(i)}(\varpi_1^+) = g_1^{[i]}(z_1^{(i)}(\varpi_1))$ can be derived from the Formulas (3) and (4). When $\varpi \in (\varpi_1, \varpi_1 + \tau]$, the formula is as follows:

$$\begin{aligned} & r(\varpi)z_1^{(i)}(\varpi) - r(\varpi)z_1^{(i)}(\varpi_1^+) \\ & = r(\varpi)x_1^i(\varpi) - r(\varpi_1)g_1^{[i]}(x_0^i + (\varpi_1)) \\ & = r(\varpi)x_0^i(\varpi) - r(\varpi_1)x_0^i + (\varpi_1^+) \\ & = -\int_{\varpi_1}^1 f(s, x_0(s - \tau))ds \\ & = -\int_{\varpi_1}^1 f(s, z_1(s - \tau))ds. \end{aligned} \tag{7}$$

The result of the Formula (7) can be obtained:

$$(r(\varpi)z_1^i(\varpi)) = -f(\varpi, z_1(\varpi - \tau)), \varpi \in (\varpi_1, \varpi_1 + \tau). \tag{8}$$

If $\varpi \in (\varpi_1 + \tau, \varpi_2]$, and when $\varpi \in (\varpi_1 + \tau, \varpi_2]$, the calculation formula is:

$$\begin{aligned} & r(\varpi)z_1^{(i)}(\varpi) - r(\varpi)z_1^{(i)}(\varpi_1 + \tau) \\ & = r(\varpi)y_1^i(\varpi) - r(\varpi_1)y_1^i(\varpi_1 + \tau) \\ & = -\int_{\varpi_1+\tau}^{1} f(s, y_1(s - \tau))ds \\ & = -\int_{\varpi_1+\tau}^{1} f(s, z_1(s - \tau))ds. \end{aligned} \quad (9)$$

Because $z_1(\varpi)$ is continuous on $\varpi_1 + \tau$. According to Formulas (7) and (9), $z_1^i(\varpi)$ is also continuous on $\varpi_1 + \tau$. By using the Formulas (1)–(10), $z_1(\varpi)$ satisfies Formula (1) on $[\varpi_0 - \tau, \varpi_2]$. The calculation of Formula (9) can be obtained when $\varpi \in (\varpi_1 + \tau, \varpi_2)$, and $(r(\varpi)z_1^i(\varpi))$ is:

$$(r(\varpi)z_1^i(\varpi)) = -f(\varpi, z_1(\varpi - \tau)), \varpi \in (\varpi_1 + \tau, \varpi_2). \quad (10)$$

Therefore, based on the value range passed by Formula (10), the value range of the solution was determined.

2.2. Solution of Nonlinear Second Order Neutral Differential Equations

The fixed-point theorem of condensing mapping is a theorem often used in topology, and it refers to the assumption that a compact, convex, and non-empty metric space on which a condensing map is defined has at least one fixed point in the metric space. Among them, a condensing map refers to a mapping that satisfies the following two conditions:

1. Maintains the inclusion relationship between point sets;
2. Maps each connected point set to another connected point set.

The meaning of this theorem is that any mapping that satisfies the above conditions must have at least one fixed point, and this mapping maps a point to itself. In addition, the application of this theorem is very extensive, and it has been applied in many fields of mathematics, physics, and economics. For example, in the field of mathematics, this theorem is often used to prove the judgment of initial value problems. Therefore, for the solution of nonlinear second-order neutral differential equations, using the fixed-point theorem of condensing mapping, the results are used to verify the existence of the initial value problem for nonlinear second-order neutral differential equations. According to its decision results, the existence of the global solution of the equation was judged. The oscillation principle of differential equations was applied to solve the initial value of a quadratic neutral nonlinear differential equation, and the results of the calculation were observed. The oscillation analysis of the solution of the differential equation was realized.

The existence of solutions for neutral differential equations is:

$$\begin{cases} u''(\varpi') = f'(\varpi', u(\varpi'), u''(\varpi'), (Tu)(\varpi')), \varpi' \in J, \\ u(0) = x'_0, u'(0) = x'_1, \end{cases} \quad (11)$$

J is a unit interval, and it is mainly solved with Darbor's fixed-point theorem and Stefan Banach in the early 20th century. This theorem is about real valued functions, describing a continuous function that must have a fixed point under certain conditions.

Lemma 2. *The Darbor fixed-point theorem indicates that any continuous real function has at least one fixed point under certain conditions. $H = [A, B]$ is a closed interval on the real number axis. $F(K)$ is a continuous real function on H, and $F(A) < t < F[B]$. Then, there must be a point $C \in [A, B]$, such that $F(C) = t$. In other words, regardless of the situation, as long as the value interval of the original function is within the defined domain, there must be at least one pointMake, $F(K)$ obtain this value at this point. The results were used to solve the existence of the initial problem* [19–21].

Proof of Lemma 2. $\overline{u}(\varpi')$, $\overline{V}(\varpi')$, $F_0(\varpi', \overline{V}(\varpi'))$, and $\overline{U}_0$ are defined, respectively.

$$\overline{u}(\varpi') = u'(\varpi'), \tag{12}$$

$$\overline{U}(\varpi') = \begin{bmatrix} u(\varpi') \\ \overline{u}(\varpi') \end{bmatrix}, \tag{13}$$

$$F_0(\varpi', \overline{U}(\varpi')) = \begin{bmatrix} \overline{u}(\varpi') \\ f(\varpi', u(\varpi'), \overline{u}(\varpi'), T(\overline{u})(\varpi')) \end{bmatrix}, \tag{14}$$

$$\overline{U}_0 = \begin{bmatrix} x'_0 \\ x'_1 \end{bmatrix}, \tag{15}$$

Since the theory of first-order differential equations is relatively mature [22,23] and the calculation method is simpler, it can more conveniently solve the initial value problem of nonlinear second-order neutral differential equations. In this way, the second order neutral differential equation can be transformed into the first order differential equation.

$$\begin{cases} \overline{U'}(\varpi') = F_0(\varpi', \overline{U}(\varpi')), \\ \overline{U}(0) = \overline{U_0} \end{cases} \tag{16}$$

$$I = J, \\ \begin{cases} u'(\varpi') = f'(\varpi', u, Tu), \varpi' \in I, \\ u(0) = x'_0 \end{cases} \tag{17}$$

In the Formula (17), I is a unit interval.

Therefore, when $I = J$, there exists a solution to a nonlinear second-order neutral differential equation. □

A Banach space is a completely normed vector space, i.e., it is a linear space, and there is a norm that can measure the length between its vectors, and at the same time, it is complete under that norm. A space is complete, meaning that all its Cauchy sequences have a limit (i.e., converge to some vector within the space). Banach space is an important concept in mathematics, it contains a lot of function spaces, such as continuous function space, Hilbert space, etc., and the most famous and important of them is the Hilbert space, which is an inner product space that satisfies the Pacival inequality and a completeness condition. The definition of Banach space is for infinite dimensional spaces, because in a finite dimensional space every norm is equivalent, all linear spaces are complete. On the infinite dimensional space, there are points where Cauchy sequences converge to non-spatial points, which leads us to define the notion of completeness. Banach space is an important branch of modern mathematics, and it is used in functional analysis, partial differential equations, mathematical physics, and other fields. E is a Banach space. Its generalization is $\|\cdot\|$, and the positive element cone is P. $C'(I, E)$, $C''(I, E)$, and $C'''(I, E)$ are defined as the continuous function spaces in I valued at E, respectively. I is the range of J, and $C'(I, E)$ is used to form the Banach space according to generalization number $\|u\|c = \max\|u(\varpi')\|$, where $f'(\varpi', x') : I \times E \times E \to E$, $x'_0 \in E$. T is the linear operator of following definition:

$$(Tu)(\varpi') = \int_0^{\varpi'} k'(\varpi', s')u(s')ds', \tag{18}$$

s' is the bounded set of E, $k'(\varpi', s') \in C'[D, R^+]$, $R^+ = [0, +\infty)$, $D = \{(\varpi', s') \in [J \times J], 0 \le s' \le \varpi' \le a'\}$, $k_0 = \max\{k'(\varpi', s'); \varpi', s' \in D\}$. If $B \subseteq E$ is bounded set, then a bounded set $B_0 \subset B$ that exists in B makes $\alpha(B) \le 2\alpha(B_0)$. If $B \subset C'[I, E]$ is bounded and equicontinuous, $B(\varpi') = \{u(\varpi') | u \in B\}$ is recorded, then

$\alpha(B)(\varpi')$ is continuous on I and $\alpha(B) = \max\limits_{\varpi' \in I} \alpha(B(\varpi'))$; If $B = \{u_n\} \subset C'[I, E]$ is bounded set, then $\alpha(B(\varpi'))$*Lebesgue* is integrable and $\alpha(\left\{\int_0^{\varpi'} u_n(\varpi')d\varpi'\right\}) \leq 2\int_0^{\varpi'} \alpha(B(\varpi'))d\varpi'$.

For the convenience of subsequent research, the following assumptions are established:

(1) D_1: $f : J \times E \times E \to E$ continues;
(2) D_2: there exists a non-negative real number bounded function $L_{i'}(\varpi') \geq 0$, i = 1, 2, which satisfies $4h_0(L_1(\varpi') + k_0L_2(\varpi')) < 1$, so that there is $\alpha(f'(\varpi', B_1, B_2)) \leq L_1(\varpi')\alpha(B_1) + L_2(\varpi')\alpha(B_2)$ for any bounded set $B_{i'} \subseteq E$ and $\varpi' \in J$;
(3) D_3: there is a non-negative real number bounded function $L_{i'}(\varpi') \geq 0, i = 1, 2$. For any bounded set of $B_{i'} \subseteq E$ and $\varpi' \in J$, there are $\alpha(f'(\varpi', B_1, B_2)) \leq L_1(\varpi')\alpha(B_1) + L_2(\varpi')\alpha(B_2)$.

If the nonlinear second-order neutral differential equation satisfies the Conditions (1)–(3), there is a solution to the initial problem [24–27]. From Condition (1), it can be found that $\exists\varepsilon' > 0$ makes f bounded on $[0, \varepsilon'] \times \overline{B}(x'_0, \varepsilon) \times \overline{B}(x'_0, \varepsilon)$, so there is a constant M_0, which makes:

$$\|f'(\varpi', u(\varpi'), (Tu)(\varpi'))\| \leq M_0, \varpi' \in [0, \varepsilon], \tag{19}$$

Take $h_0 = \min\{\varepsilon, \frac{\varepsilon}{M}\}$ and $I = [0, h_0]$. A Ω_0 expression is defined. Through $\Omega_0 = \{u \in C'[I, E] | \|u(\varpi') - x'_0\| \leq \varepsilon\}$, it can be found that Ω_0 defines an operator.

$$(Au)(\varpi') = x'_0 + \int_0^{\varpi'} f(s', u(s'), (Tu)(s'))ds'. \tag{20}$$

The solution of Formula (17) is equivalent to that of Formula (20) [28–30], and there are fixed points. Because of $\Omega_0 = \{u \in C'[I, E] | \|u(\varpi') - x'_0\| \leq \varepsilon_0\}, \varpi' \in [0, h_0]$, the operators defined are as follows:

$$\begin{aligned} &(Au)(\varpi') - x'_0 = x'_0 + \int_0^{\varpi'} f'(s', u(s'), (Tu)(s'))ds' - x'_0 \\ &= \int_0^{\varpi'} f'(s', u(s'), (Tu)(s'))ds'. \end{aligned} \tag{21}$$

Formula (21) can be obtained from the general number.

$$\begin{aligned} &\|(Au)(\varpi') - x'_0\| = \|\int_0^{\varpi'} f'(s', u(s'), (Tu)(s'))ds'\| \\ &\leq \int_0^{\varpi'} \|f'(s', u(s'), (Tu)(s'))\|ds'. \\ &\leq M_0\varpi' \leq M_0h_0 \leq \varepsilon. \end{aligned} \tag{22}$$

According to the definition of Ω_0, $(Au)(\varpi') \in \Omega_0$, or A is an operator of $\Omega_0 \to \Omega_0$. Let the convergent subcolumn n of $u_n \to u$ and $\exists\{\varpi'_{k'}\}$ is a constant, and $n = [0, 2]$. When $\varpi'_k \to \varpi'_1 \in [0, h_0]$, there is $u(\varpi'_k) \to u(\varpi'_1)$, assuming $f'(\varpi', u_{n_{k'}}) \neq\to f'(\varpi', u_1)$, then $\exists\varepsilon_0 > 0$, $\{\varpi'_{k'}\} \in [0, h_0]$. When $n_{k'} \to \infty$, there is:

$$\|f'(\varpi'_{k'}, u_{n_{k'}}(\varpi'_{k'}), (Tu_{n_{k'}})(\varpi'_{k'}) - f'(\varpi'_{k'}, u(\varpi'_{k'}), (Tu)(\varpi'_{k'}))\| \geq \varepsilon_0, \tag{23}$$

From the continuity of f', assuming that f' is continuous at point $(t'_1, u(\varpi'_1))$, $k' \to \infty$, $\overline{\varepsilon}$ is small.

$$\|f'(\varpi'_{k'}, u_{n_{k'}}(\varpi'_{k'}), (Tu_{n_{k'}})(\varpi'_{k'}) - f'(\varpi'_1, u(\varpi'_1), (Tu)(\varpi'_1))\| \leq \overline{\varepsilon}, \tag{24}$$

Because $k' \to \infty$, there are

$$\begin{aligned} &\|f'(\varpi'_{k'}, u_{n_{k'}}(\varpi'_{k'}) - f'(\varpi'_1, u(\varpi'_1)\| \\ &= \|f'(\varpi'_{k'}, u_{n_{k'}}(\varpi'_{k'}) - f'(\varpi'_{k'}, u(\varpi'_{k'}) + f'(\varpi'_{k'}, u(\varpi'_{k'}) - f'(\varpi'_1, u(\varpi'_1))\| \\ &\geq \|f'(\varpi'_{k'}, u_{n_{k'}}(\varpi'_{k'}) - f'(\varpi'_{k'}, u(\varpi'_{k'})\| \\ &= \|f'(\varpi'_{k'}, u(\varpi'_{k'}) - f'(\varpi'_1, u(\varpi'_1)\|. \end{aligned} \tag{25}$$

At this point, Formulas (24) and (25) are contradictory, so the assumption is not true, and the original proposition is established. If $f'(\varpi', u_n(\varpi') \to f'(\varpi'), u(\varpi'))$, $A:\Omega_0 \to \Omega_0$ is a continuous operator. On the other hand, when $\forall u \in \Omega_0$, $\varpi'_1, \varpi'_2 \in [0, h_0]$, $0 \le \varpi'_1 \le \varpi' \le h_0$:

$$\begin{aligned}
&\|(Au)(\varpi'_2) - Au)(\varpi'_1)\| \\
&= \|x'_0 + \textstyle\int_0^{\varpi'_2} f'(s', u(s'), (Tu)(s'))ds'\| \\
&\le M(\varpi_2 - \varpi_1).
\end{aligned} \tag{26}$$

It is known from Formula (26), $A(\Omega_0)$ is constant. Because $\|Au(\varpi')\| \le \|x'_0\| + M\varpi' \le \|x'_0\| + Mh_0$, $A(\Omega_0)$ is bounded, so that $A(\Omega_0)$ is bounded and equicontinuous on $[0, h_0]$. Take $B \subset \Omega_0$, $\exists B_1 = \{u'_n\} \subset B$ and $\alpha(B) \le 2\alpha(B_0)$ can be available:

$$\alpha(A(B)) \le 2\alpha(A(B_1)). \tag{27}$$

Because of $B_1 \subset B \subset \Omega_0$, $A(B_1)$ is a bounded and equicontinuous [31–33]. From $\alpha(B) = \max\limits_{\varpi' \in I} \alpha(B(\varpi'))$, we can see that $\alpha(A(B_1)) = \max\limits_{\varpi' \in I} \alpha(A(B_1)(\varpi'))$, according to the definitions of A and $\alpha(\left\{\int_0^{\varpi'} u_n(\varpi')d\varpi'\right\}) \le 2\int_0^{\varpi'} \alpha(B(\varpi'))d\varpi'$, it is available:

$$\begin{aligned}
&\alpha(A(B_1)(\varpi)) = \alpha(\{x'_0 + \textstyle\int_0^{\varpi'} f'(s', u_n(s'), (Tu_n)(s'))ds' | n \in N\}) \\
&\le (2h_0 L_1(\varpi') + 2h_0^2 k'_0 L_2(\varpi'))\alpha(B_1) \\
&\le 4h_0(L_1(\varpi') + k'_0 h_0 L_2(\varpi'))\alpha(B_1) \\
&< 1.
\end{aligned} \tag{28}$$

As the result of Formula (28), < 1, $A:\Omega_0 \to \Omega_0$ is a condensed mapping. By using the fixed-point theorem, it is proved that A is a fixed point on $[0, h_0]$. Therefore, for a given nonlinear second-order neutral differential equation, if it satisfies Conditions (1) to (3) and there is a condensing mapping $A:\Omega_0 \to \Omega_0$, then the differential equation has a solution, and at least one fixed point exists on $[0, h_0]$.

3. Oscillation Analysis of Solutions for Nonlinear Second-Order Neutral Differential Equations in Different Situations

3.1. Oscillation Inference of Solutions for Nonlinear Second-Order Neutral Differential Equations

There are three possibilities for solving the equation:

Inference 1. *Supposing there is a differentiable function, differentiable function refers to a class of smooth functions in an interval, which can be derived everywhere in the interval* [34–36]. *It makes* $P(\varpi') \ge 0$, $\varpi' \ge \varpi'_0$, $f'(x')/\psi(x') \ge \alpha > 0$, $\psi(x') \ge c > 0$, $x \ne 0$.

$$\begin{aligned}
&\lim_{\varpi' \to \infty} \int_0^{\varpi'} p'(s')[q(s') - \tfrac{a'(s')}{4\partial}(\tfrac{p''(s')}{p'(s')} + \tfrac{p'(s')}{ca'(s')})^2]ds' = +\infty, \\
&\int_0^c \tfrac{\psi(u)}{f'(u)}du < +\infty, \int_{-c}^0 \tfrac{\psi(u)}{f'(u)}du < +\infty,
\end{aligned} \tag{29}$$

$$\limsup_{t' \to \infty} \int_T^{\tau'} \frac{1}{a'(s')p'(s')} \int_T^{\tau'} [p'(\tau')q(\tau') - \frac{a'(\tau')}{4\alpha'}(\frac{p''(\tau')}{p'(\tau')} + \frac{p'(\tau')}{ca'(\tau')})^2]d\tau' ds' = +\infty. \tag{30}$$

If the conditions of Formulas (29) and (30) are established, the solution of the quadratic neutral nonlinear differential equation of this function is given.

Inference 2. *Supposing there is a differentiable function, so* $x'(\varpi') \in [\varpi'_0, +\infty)$. *When* $\varpi' \ge \varpi'_1 \ge \varpi'_0$, $x'(t') > 0, x''(\varpi') > 0, x'''(\varpi') > 0$, *for each constant* $r' \in (0,1)$, *there is* $\varpi'_{r'} \ge \varpi'_1$, $\varpi' \ge \varpi'_{r'}$ *and makes:*

$$x'(\varpi') > r'\varpi' x''(\varpi'), x'(\sigma(\varpi')) \ge r' \frac{\sigma(\varpi')}{\varpi'} x'(\varpi'). \tag{31}$$

$$k'(\varpi', x', y') \leq |y'|^{\beta}, x' > -\infty, y' < +\infty, \beta \geq 0. \tag{32}$$

1. *When* $\beta > 0$:

$$(1 + \int_0^{\varpi'} p'(s')ds')^{-\frac{1}{\beta}} \in L(\varpi_0', +\infty). \tag{33}$$

2. *When* $\beta = 0$:

$$\int_{\varpi_0'}^{+\infty} \exp(-\int_{\varpi_0'}^{s'} p'(\tau')d\tau')ds' = +\infty. \tag{34}$$

When 1 and 2 are established and $f'(x'y') \geq k'f'(x')f'(y')$, *where* $x', y' > 0$ *and* k'*is the positive constant, if* $f'g$ *is strongly sublinear then*

$$\begin{array}{l} \int_{+0} \frac{du'}{f'(u')g(u')} < +\infty, \int_{-0} \frac{du'}{f'(u')g(u')} < +\infty. \\ \int^{+\infty} q(s')f'(\sigma(s'))ds' = +\infty. \end{array} \tag{35}$$

The solution of the second-order neutral nonlinear differential equation is given.

Inference 3. *Supposing there is a differentiable function,* $\psi(x')f'(x') \geq k' > 0$. *If the function satisfies the following conditions, the above three cases are the existence of solutions for nonlinear second-order neutral differential equations.*

$$\int_{\varpi_0}^{+\infty} [q(s') - \frac{P^2(s')}{4ka(s')}]ds' = +\infty. \tag{36}$$

If Formula (36) is satisfied, then the current nonlinear second-order neutral differential equation solution is oscillatory.

On this basis, the second-order neutral nonlinear differential equations are analyzed using steps (1)–(3).

3.2. Oscillation Analysis and Examples of Solutions for Nonlinear Second-Order Neutral Differential Equations

For a given nonlinear second-order neutral differential equation, if it satisfies one of the three possibilities in Section 3.1, it indicates that the equation has an oscillatory solution, thus achieving oscillatory analysis of the solution for the nonlinear second-order neutral differential equation. In the practical application of Sections 2.1, 2.2, and 3.1, this method can be used to analyze the oscillation characteristics of second-order neutral nonlinear differential equations.

Oscillation analysis of solutions to nonlinear second-order neutral differential equations can solve many problems, such as the Colpitts oscillator in circuits that can be described using this differential equation, where the nonlinearity arises from the interaction between capacitors and inductors. The expression is given with Formula (37):

$$L_1 \frac{d^2 x}{dt^2} + \frac{1}{C_1}\left(V(\delta) - \frac{Q}{C_2}\right) + \frac{1}{C_3} \int_{-\infty}^{t} V(\delta)d\delta = 0. \tag{37}$$

In the formula, L_1, C_1, C_2, and C_3 are circuit component parameters. Q is the number of charges. $V(\delta)$ is a voltage function. Set $C_1 = C_2 = C$, find the solution of the equation as shown in Formula (38):

$$\delta = \frac{1}{2Q\sqrt{L_1 C}}. \tag{38}$$

The obtained solution is the oscillation frequency. By analyzing the oscillation behavior of the solution of the nonlinear second-order neutral differential equation in Section 3.1

in three different cases, the oscillation behavior of the circuit can be obtained, thereby optimizing the circuit design.

4. Conclusions

Differential equation is a very important subject. It has a certain practical value. In its application field, nonlinear differential equations are often encountered, which places higher demands on the study of differential equations. Nonlinear differential equation is a difficult and hot point in differential equation theory. As it is a fundamental problem in many fields, it is also very difficult to solve nonlinear differential equations. To solve this problem, the basic theory of ordinary and partial differential equations needs to be strengthened and improved. At the same time, by distinguishing different types of nonlinear differential equations and studying their characteristics and laws, further study of differential equations can be conducted in a wider range of fields. This study focused on the oscillation of differential equations and obtained the following conclusions:

(1) In this paper, a quadratic nonlinear differential equation and its initial value solution were obtained;
(2) The paper innovatively used the fixed-point theorem to provide solutions for a class of differential equations. Using the vibration principle of differential equations, the solution equations of three different nonlinear equations were discussed and derived, and vibration analysis was conducted;
(3) The paper analyzed the circuit problems in the Colpitts oscillator based on the obtained three oscillatory solution equations and obtained the corresponding solution equations to express the nonlinear relationship between capacitance and inductance.

To better solve the problems caused by nonlinear differential equations, there are other combinations of numerical calculation and computer methods to develop corresponding algorithms and software. This can enable the calculation and solution of differential equations in a wider range of fields, promoting the continuous development of differential equation theory and practice. Second-order neutral nonlinear differential equations are widely used in many fields. These problems can be used to solve ordinary differential equations, as well as quadratic neutral nonlinear differential equations. The existing theory cannot meet the needs. In response to this issue, this article provides a better analysis method, which has certain practical significance, especially for exploring wider applications and solving more challenging problems. To this end, research has combined mathematical theory and computational methods to seek more effective analytical and numerical techniques to more accurately solve nonlinear second-order neutral differential equations. Meanwhile, further research is needed on the intrinsic properties, iterative properties, and stability properties of its solution. These further studies will help solve practical problems in a wider range of fields such as physics, chemistry, biology, and engineering and promote the development and application of the discipline.

Author Contributions: Methodology, L.S.; Investigation, S.C.; Resources, G.W.; Writing—original draft, L.S.; Writing—review & editing, L.S. All authors have read and agreed to the published version of the manuscript.

Funding: This paper was funded by the Finite Time Stability and Filtering of Pulse Systems (grant number: 19 A110028) and the Stability and Controller Design of Switching Positive Systems (grant number: 162300410076).

Data Availability Statement: All data generated or analysed during this study are included in this published article.

Conflicts of Interest: The authors declare no conflict of interest.

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