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Building Up of Fuzzy Evaluation Model of Life Performance Based on Type-II Censored Data

Kuo-Ching Chiou

Department of Finance, Chaoyang University of Technology, Taichung 413310, Taiwan; kcchiou@cyut.edu.tw

Abstract: The semiconductor industry is a rapidly growing sector. As collection technologies for production data continue to improve and the Internet of Things matures, production data analysis improves, thus accelerating progress towards smart manufacturing. This not only enhances the process quality, but also increases product lifetime and reliability. Under the assumption of exponential distribution, the ratio of lifetime and warranty has been proposed as a lifetime performance index for electronic products. As unknown parameters of the index, to use point estimates to assess lifetime performance may cause misjudgment due to sampling errors. In addition, cost and time limitations often lead to small sample sizes that can affect the results of the analysis. Type-II censored data are widely applied in production and manufacturing engineering. Thus, this paper proposes an unbiased and consistent estimator of lifetime performance based on type-II censored data. The $100(1-\alpha)\%$ confidence interval of the proposed index is derived based on its probability density function. Overly small sample sizes not only make the length estimates of lifetime performance index intervals for electronic products too long, but they also increase sampling errors, which distort the estimation and test results. We therefore used the aforementioned interval to construct a fuzzy test model for the assessment of product lifetime and further help manufacturers to be more prudent and precise to evaluate the performance of product life cycles. A numerical example illustrates the applicability of the proposed model.

Keywords: relative lifetime performance index; type II censoring data; unbiased estimator; consistent estimator; confidence-interval-based fuzzy testing method

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1. Introduction

The semiconductor industry is involved in the wafer manufacturing, integrated circuit (IC) design, packaging, and peripheral components necessary for end products such as smartphones, tablet computers, and smart internet end devices [1,2]. Industry clusters in Taiwan represent a crucial industry chain for consumer electronics worldwide [3–6]. Offering good product quality not only enhances its product lifespan and reliability, but also bolsters user satisfaction and willingness to use it [2,7]. As the collection technologies for production data continue to improve and the Internet of Things matures, production data analysis improves, thus accelerating progress toward smart manufacturing. This not only enhances the process quality, but also increases the product lifetime and reliability [8]. Furthermore, owing to the limitation of the cost and time, the estimation accuracy of the samples in the study leads to not being significant. Thus, in order to increase its estimation accuracy and eliminate the uncertain measurement data, confidence interval-based fuzzy evaluation models were built up via the confidence interval of indices in the study [9,10]. In order to prevent the risk of misjudgment caused not only by sampling errors but also by factoring in expert experiences and past data into consideration, it becomes necessary to increase the accuracy of each case with smaller sample sizes and analyze data with confidence intervals. Constructing a fuzzy test model to evaluate the product lifetime is also a way to compensate for sampling errors in small sample sizes [7,9].

As for product marketing, the term of warranty is shown to be a crucial index. Chen and Yu [11] indicated that whether customers feel satisfied with the products and be willing to use them lies in the good quality of the product with longer product lifetime and its reliability. Many researchers have confirmed the convenience and efficacy of process capability indices (PCIs) for the assessment of process quality in practice [12]. PCIs have also been applied to the lifetime and reliability of electronic products [13]. On the basis of some studies shown, it has been proved that ameliorating the process of quality check is able to shun off some unnecessary cost caused by the rework and defective products. Furthermore, it is also able to decrease energy consumption and carbon emissions [14,15]. It is of importance to manufacture all parts of the product with high quality. In order to make all the final products meet the quality standard, forming stringent requirements becomes necessary [16].

Additionally, in the industrial field, on account of the limitation of cost and time, noticing small-size samples implemented in the survey is not uncommon [17]. According to some previous studies conducted by the experts, it has been argued that utilizing the analyzing tool, fuzzy evaluation model, to analyze the sample with small data is able to make the result of the survey reach its reliability and validity [9,10]. Additionally, in order to lower the risk of misjudgment caused by sampling errors, putting interval estimates into practice has been proved to be much more accurate compared to the point estimates [7].

Product lifetime is exponentially distributed with mean λ. Tong et al. [18] proposed the following lifetime performance index C_L:

$$C_L = \frac{\mu_T - L}{\mu_T} = 1 - \frac{L}{\lambda} \tag{1}$$

where L denotes the minimum number of time units that the lifetime of each electronic component is required to reach, and parameter λ is the expected value μ_T of the electronic component lifetime. We assume that the lifetime of the electronic component (T) follows an exponential distribution with the mean λ; thus, the probability density function of T is as follows:

$$f_T(t) = \frac{1}{\lambda} e^{-\frac{t}{\lambda}},\ t\ >\ 0 \tag{2}$$

As noted by Chen and Yu [19], when the mean lifetime of the electronic component $\lambda \geq L$, then the lifetime performance index $C_L \geq 0$. Clearly, the greater the lifetime performance index C_L is, the better its lifetime performance is. However, the warranty period of a product is generally only three years (L = 3), yet only when the mean lifetime λ approaches infinity does the lifetime performance index C_L of the electronic component approach 1. This does not fit the conventions of the industry. Chen et al. [20] therefore proposed a relative lifetime performance index. This index is defined as follows:

$$\beta_L = \frac{\mu_T}{L} = \frac{\lambda}{L} \tag{3}$$

As noted by Chen and Yu [19], the lifetime performance index is the ratio of λ and L. The one-to-one relationship between both index β_L and C_L is $\beta_L = (1 - C_L)^{-1}$. If the relative lifetime is $X = T/L$, then (1) when random variable $X < 1$, the lifetime of electronic component is denoted as equal to the warranty ($T < L$), (2) when random variable $X = 1$, the lifetime of the electronic component is denoted as equal to the warranty ($T = L$), (3) when random variable $X > 1$, the lifetime of the electronic component is denoted as longer than the warranty ($T > L$). Thus, X is the only value required for managers to assess if product lifetime is sufficient.

The probability density function of relative lifetime X is as follows:

$$f_X(x) = \frac{1}{\beta_L} exp\left(-\frac{x}{\beta_L}\right),\ x\ >\ 0 \tag{4}$$

Relative lifetime X is an exponential distribution with mean β_L. Therefore, the failure rate is $r_X(x) = \beta_L^{-1}$ and product reliability $p_r = S_X(1) = \exp\left(\beta_L^{-1}\right)$ where $S_X(x)$ is the survival function of relative lifetime X as follows:

$$S_X(x) = p(X > x) = exp\left(-\frac{x}{\beta_L}\right),\ x\ >\ 0 \tag{5}$$

As pointed out by Chen et al. [21], the unknown parameters in the index decrease its accuracy if the point estimates are simply utilized to evaluate the data with small-size samples [7,19,21–23]. As the results of statistical tests tend to vary with sample size, censoring can be applied to achieve consistent results in a short time [22–26]. Type-II censoring is widely applied in production and manufacturing data. Thus, this paper proposes an unbiased and consistent estimator for the lifetime performance index β_L based on type-II censored data. The $100(1-\alpha)\%$ confidence interval of the index β_L is derived based on its probability density function. Using this interval and the method proposed by Chen and Yu [19], a fuzzy test model is constructed to assess whether product lifetime performance reaches the required level. The application of the model proposed in the study is demonstrated through a numerical example. The final section presents our conclusions.

The rest of the present paper would be arranged as follows. In Section 2, we derive the estimator and find the confidence interval of the lifetime performance index. Section 3 presents the fuzzy test method for lifetime performance index. We employ an application to demonstrate the efficacy of the proposed approach in Section 4. Conclusions are given in Section 5.

2. Estimation of Ratio for Lifetime Performance Index

Incomplete data collection due to external or human factors during product development can reduce the reliability of analysis results. Censoring type is a form of data collection that is accurate as well as cost-effective and quick [14]. Censoring type can be divided into three types: type-I censoring, type-II censoring, and random censoring [27]. Type-II censoring is the most widely applied in production and manufacturing engineering [14,27]. Furthermore, type-II progressive censoring has become a common approach to the analysis of lifetime data for highly reliable products [14,28–31].

The proposed index must be estimated based on sample data. The lifetime T follows an exponential distribution with mean λ, denoted as $T\ \sim exp(\lambda)$. The relative lifetime $X = T/L$ is an exponential distribution with mean β_L, denoted as $X\ \sim exp(\beta_L)$. $(T_1, T_2, \ldots, T_n)$ and $(X_1, X_2, \ldots, X_n)$ are random samples of T and X, respectively. $(Y_1, Y_2, \ldots, Y_n)$ is a sample set of the type-II censored data, $Y_j = min\left(X_j,\ X_{(r)}\right) = \ min\ \left(T_j/L,\ T_{(r)}/L\right), j = 1, 2, \ldots, n$, where the number of uncensored data is denoted by r and the order statistics are denoted by $X_{(r)}$ and $T_{(r)}$. The estimator $\hat{\beta}_L$ of β_L is as follows:

$$\hat{\beta}_L = \frac{\hat{\lambda}}{L} = \frac{1}{r}\sum_{i=1}^{n} Y_i \tag{6}$$

where

$$\hat{\lambda}\ =\ \frac{L}{r}\sum_{i=1}^{n} Y_i \tag{7}$$

If random variable $W = 2r\hat{\beta}_L/\beta_L$, according to Chiou and Chen [14], W follows a chi-square distribution with $2r$ degrees of freedom, denoted by $W \sim \chi^2_{(2r)}$. Therefore, the expected value of the estimator $\hat{\beta}_L$ is as follows:

$$E\left[\hat{\beta}_L\right] = E[W] \times \left(\frac{\beta_L}{2r}\right) = (2r) \times \left(\frac{\beta_L}{2r}\right) = \beta_L \tag{8}$$

$\hat{\beta}_L$ is an unbiased estimator of the lifetime performance index β_L. Its variance is calculated as follows:

$$Var\left[\hat{\beta}_L\right] = Var[W] \times \left(\frac{\beta_L}{2r}\right)^2 = (4r) \times \left(\frac{\beta_L^2}{4r^2}\right) = \frac{\beta_L^2}{r} \tag{9}$$

For large samples,

$$\lim_{n\to\infty} E\left(\hat{\beta}_L - \beta_L\right)^2 = \lim_{n\to\infty} Var\left(\hat{\beta}_L\right) = \lim_{r\to\infty} \frac{\beta_L^2}{r} = 0 \tag{10}$$

Based on Equations (8) and (10), $\hat{\beta}_L$ is an unbiased and consistent estimator of the lifetime performance index β_L. The $100(1-\alpha)\%$ confidence interval of the lifetime performance index β_L is derived as follows:

$$\begin{aligned} 1-\alpha &= p\left\{\chi^2_{(2r),\,\alpha/2} \le W \le \chi^2_{(2r),\,1-\alpha/2}\right\} = p\left\{\chi^2_{(2r),\,\alpha/2} \le \frac{2r\hat{\beta}_L}{\beta_L} \le \chi^2_{(2r),\,1-\alpha/2}\right\} \\ &= p\left\{\left(\frac{2r}{\chi^2_{(2r),\,1-\alpha/2}}\right)\hat{\beta}_L \le \beta_L \le \left(\frac{2r}{\chi^2_{(2r),\,\alpha/2}}\right)\hat{\beta}_L\right\} \end{aligned} \tag{11}$$

where $\chi^2_{(2r),\alpha/2}$ is the lower $\alpha/2$ quantiles of $\chi^2_{(2r)}$ and $\chi^2_{(2r),1-\alpha/2}$ is the lower $1-\alpha/2$ quantiles of $\chi^2_{(2r)}$. Therefore, the lower confidence of the lifetime performance index β_L is

$$L\beta_L = \left(\frac{2r}{\chi^2_{(2r),\,1-\alpha/2}}\right)\hat{\beta}_L \tag{12}$$

Similarly, the upper confidence of the lifetime performance index β_L is

$$U\beta_L = \left(\frac{2r}{\chi^2_{(2r),\,\alpha/2}}\right)\hat{\beta}_L \tag{13}$$

The length of the $100(1-\alpha)\%$ confidence interval of the lifetime performance index β_L is

$$l\beta_L = U\beta_L - L\beta_L = \left(\frac{2r}{\chi^2_{(2r),\,\alpha/2}} - \frac{2r}{\chi^2_{(2r),\,1-\alpha/2}}\right)\hat{\beta}_L \tag{14}$$

Since $\hat{\beta}_L$ is an unbiased estimator of the lifetime performance index β_L, the following defines the expected length of the $100(1-\alpha)\%$ confidence interval $l\beta_L$:

$$E(l\beta_L) = \left(\frac{2r}{\chi^2_{(2r),\,\alpha/2}} - \frac{2r}{\chi^2_{(2r),\,1-\alpha/2}}\right)\beta_L \tag{15}$$

For fixed $(1-\alpha) \times 100\% = 95\%$, sample $n = 100$, $r = 10\ (10)\ 100$, and $\beta_L = 1\ (1)\ 5$, the expected value $E(l\beta_L)$ is shown in Figure 1, where $r = 10\ (10)\ 100$ indicates that the value of r begins at 10 and increases by 10 each time until its value equals 100. Similarly, index $\beta_L = 1\ (1)\ 5$ means that the value of the index β_L begins at 1 and increases by 1 each time until its value equals 5.

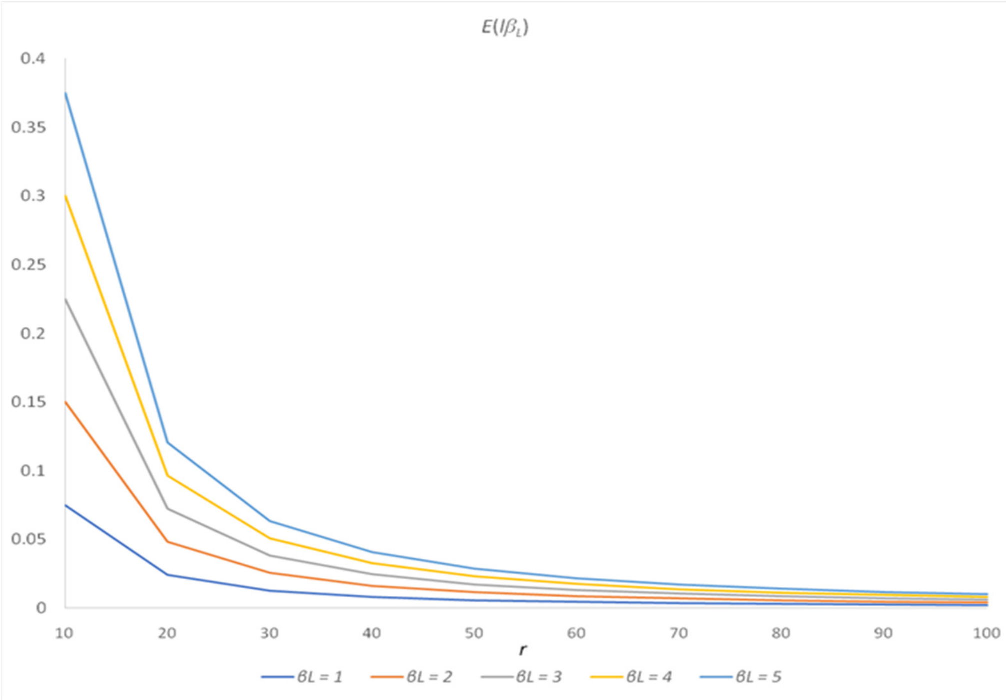

Figure 1. $E(l\beta_L)$ curves for β_L = 1 (1) 5, r = 10 (10) 100, and α = 0.05.

Given confidence level ($(1-\alpha) \times 100\%$) and sample size n, the smaller the mean length of confidence interval $E(l\beta_L)$ is, the better estimation of the index β_L under different numbers of uncensored data r is. As noted in Figure 1, when index β_L is fixed, the mean length of the confidence interval $E(l\beta_L)$ is inversely proportional to the number of uncensored data r. This means that the better the estimate of the index β_L is, the more uncensored data have been collected.

3. Fuzzy Test Method for Lifetime Performance Index

In this section, for the purpose of determining whether lifetime performance meets its requirement, a fuzzy test method is utilized. The hypothesis is H_0:$\beta_L \geq c$ vs. H_1:$\beta_L < c$ [19], where c is the minimal value of relative lifetime performance index β_L required by customers. The following statistical testing rules are taken into consideration:

(1) If $\hat{\beta}_L < C_R$, then $\beta_L < c$ (i.e., the null hypothesis is rejected).
(2) If $\hat{\beta}_L \geq C_R$, then $\beta_L \geq c$ (i.e., the null hypothesis is not rejected).

C_R is the critical value determined by

$$p\{\hat{\beta}_L < C_R \,|\beta_L = \ c \in \ H_0\} = p\left\{W < \frac{2r\,C_R}{c}\right\} = \alpha \tag{16}$$

Hence, C_R can be calculated as follows:

$$C_R \ = \frac{c\,\chi^2_{(2r),\,\alpha}}{2r} \tag{17}$$

If we let $y_1, y_2, \ldots, y_n$ be the observed value of $Y_1, Y_2, \ldots, Y_n$, then the observed value of the estimator is

$$\hat{\beta}_{L0} = \frac{\hat{\lambda}_0}{L} = \ \frac{1}{r}\sum_{i=1}^{n} y_i \tag{18}$$

where

$$\hat{\lambda}_0 = \frac{L}{r}\sum_{i=1}^{n} y_i \tag{19}$$

As noted by Buckley [32], the α-cuts of triangular fuzzy numbers $\widetilde{\beta}_{L0}$ are [19,22]

$$\widetilde{\beta}_{L0}[\alpha] = \begin{cases} \left[\hat{\beta}_{L01}(\alpha), \hat{\beta}_{L02}(\alpha)\right], \text{ for } 0.01 \le \alpha \le 1 \\ \left[\hat{\beta}_{L01}(0.01), \hat{\beta}_{L02}(0.01)\right], \text{ for } 0 \le \alpha \le 0.01 \end{cases} \tag{20}$$

where

$$\hat{\beta}_{L01}(\alpha) = \frac{2r}{\chi^2_{(2r),\,1-\alpha/2}}\hat{\beta}_{L0} \tag{21}$$

and

$$\hat{\beta}_{L02}(\alpha) = \frac{2r}{\chi^2_{(2r),\,\alpha/2}}\hat{\beta}_{L0} \tag{22}$$

Obviously, the value of $\hat{\beta}_{L01}(\alpha)$ is not equal to the value of $\hat{\beta}_{L02}(\alpha)$ with $\alpha < 1$. As $\alpha = 1$, $\hat{\beta}_{L01}(1) = \hat{\beta}_{L02}(1) = \left(\frac{2r}{\chi^2_{(2r),\,0.5}}\right)\hat{\beta}_{L0} \neq \hat{\beta}_{L0}$.

Therefore, this paper let

$$\beta^*_{L0} = \frac{\chi^2_{(2r),\,0.5}}{2r}\hat{\beta}_{L0} \tag{23}$$

Then, the α-cuts of new triangular fuzzy numbers $\widetilde{\beta}^*_{L0}$ are

$$\widetilde{\beta}^*_{L0}[\alpha] = \begin{cases} \left[\beta^*_{L01}(\alpha), \beta^*_{L02}(\alpha)\right], \text{ for } 0.01 \le \alpha \le 1 \\ \left[\beta^*_{L01}(0.01), \beta^*_{L02}(0.01)\right], \text{ for } 0 \le \alpha \le 0.01 \end{cases} \tag{24}$$

where

$$\beta^*_{L01}(\alpha) = \frac{\chi^2_{(2r),0.5}}{\chi^2_{(2r),\,1-\alpha/2}}\hat{\beta}_{L0} \tag{25}$$

and

$$\beta^*_{L02}(\alpha) = \frac{\chi^2_{(2r),0.5}}{\chi^2_{(2r),\,\alpha/2}}\hat{\beta}_{L0} \tag{26}$$

Obviously, the value of $\beta^*_{L01}(\alpha)$ is equal to the value of $\beta^*_{L02}(\alpha)$ with $\alpha = 1$ ($\beta^*_{L01}(1) = \beta^*_{L02}(1) = \hat{\beta}_{L0}$) and there is a new triangular fuzzy number, denoted as $\widetilde{\beta}^{**}_{L0} = \Delta(\beta_{L0}, \beta_{M0}, \beta_{R0})$, where $\beta_{M0} = \hat{\beta}_{L0}$,

$$\beta_{L0} = \frac{\chi^2_{(2r),0.5}}{\chi^2_{(2r),0.995}}\hat{\beta}_{L0} \tag{27}$$

and

$$\beta_{R0} = \frac{\chi^2_{(2r),0.5}}{\chi^2_{(2r),0.005}}\hat{\beta}_{L0} \tag{28}$$

The following defines the membership function of fuzzy number $\widetilde{\beta}^{**}_{L0}$:

$$h(x) = \begin{cases} 0 & , if\ x < \beta_{L0} \\ 2\left(1 - F_W\left(\frac{\hat{\beta}_{L0}}{x}\chi^2_{(2r),0.5}\right)\right) & , if\ \beta_{L0} \le x < \hat{\beta}_{L0} \\ 1 & , if\ x = \hat{\beta}_{L0} \\ 2F_W\left(\frac{\hat{\beta}_{L0}}{x}\chi^2_{(2r),0.5}\right) & , if\ \hat{\beta}_{L0} < x \le \beta_{R0} \\ 0 & , if\ \theta_{R0} < x \end{cases} \tag{29}$$

where the cumulative distribution function of random variable W is denoted by F_W. Similarly to fuzzy numbers $\widetilde{\beta}^*_{L0}$, the α-cuts of triangular fuzzy critical values $\widetilde{C}_R$ are

$$\widetilde{C}_R[\alpha] = \begin{cases} [C_{R1}(\alpha), C_{R2}(\alpha)], \text{ for } 0.01 \leq \alpha \leq 1 \\ [C_{R1}(0.01), C_{R2}(0.01)], \text{ for } 0 \leq \alpha \leq 0.01 \end{cases} \tag{30}$$

where

$$C_{R1}(\alpha) = \frac{\chi^2_{(2r),0.5}}{\chi^2_{(2r),1-\alpha/2}} C_R \tag{31}$$

and

$$C_{R2}(\alpha) = \frac{\chi^2_{(2r),0.5}}{\chi^2_{(2r),\ \alpha/2}} C_R \tag{32}$$

Obviously, the value of $C_{R1}(\alpha)$ is equal to the value of $C_{R2}(\alpha)$ with $\alpha = 1$ ($C_{R1}(1) = C_{R2}(1) = C_R$) and the new triangular fuzzy number is $\widetilde{C}_0 = \Delta(C_{LR}, C_{MR}, C_{RR})$, where $C_{MR} = C_R$,

$$C_{LR} = \frac{\chi^2_{(2r),0.5}}{\chi^2_{(2r),0.995}} C_R \tag{33}$$

and

$$C_{RR} = \frac{\chi^2_{(2r),0.5}}{\chi^2_{(2r),0.005}} C_R \tag{34}$$

The following defines the membership function of fuzzy $\widetilde{C}_R$:

$$g(x) = \begin{cases} 0 & , if\ x < C_{LR} \\ 2\left(1 - F_W\left(\frac{C_R}{x}\chi^2_{(2r),0.5}\right)\right) & , if\ C_{LR} \leq x < C_R \\ 1 & , if\ x = C_R \\ 2F_W\left(\frac{C_R}{x}\chi^2_{(2r),0.5}\right) & , if\ C_R < x \leq C_{RR} \\ 0 & , if\ C_{RR} < x \end{cases} \tag{35}$$

As noted, the cumulative distribution function of random variable W is denoted by F_W. Membership functions $h(x)$ and $g(x)$ are presented in Figure 2:

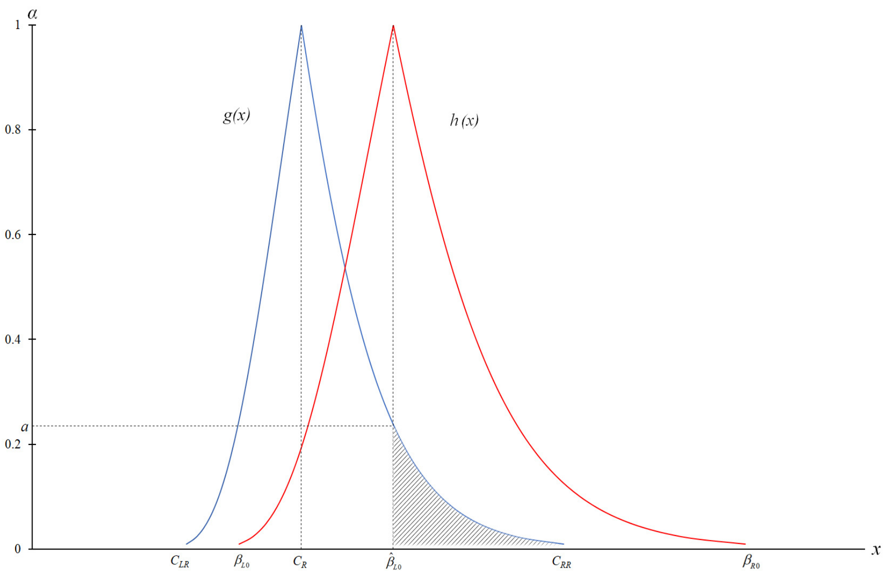

Figure 2. Membership functions $h(x)$ and $g(x)$.

Based on Chen and Yu [19], this paper let set B_T be the area under the graph of $g(x)$. That is,

$$B_T = \{(x,\alpha)|C_{R1}(\alpha) \le x \le C_{R2}(\alpha), 0 \le \alpha \le 1\} \tag{36}$$

As noted by Chen and Chang [13] and Chen and Yu [19], it is difficult to use integration to calculate the area of a set B_T. The approach, trapezoidal rule, is implemented in the study in order to build up the area of the block B_T. The procedures are following: (1) we classify the block B_T, n = 100, into several equal horizontal blocks. (2) Each section of the blocks would be calculated through the approximate trapezoid area. Then, (3) the sum of the areas for these 100 horizontal blocks is calculated. For this reason, $i = [100 \times \alpha]$ is considered. Then, $i = 0, 1, 2, \ldots, 100$ for $0 \le \alpha \le 1$, where $[100 \times \alpha]$ represents the largest integer less than or equal to $100 \times \alpha$. Similarly, $\alpha = i \times 0.01$, $i = 0, 1, 2, \ldots, 100$. These 101 horizontal lines are cut B_T into 100 trapezoidal blocks. Then, the following denotes the ith block:

$$B_{Ti} = \{(x,\alpha)|C_{R1}(0.01 \times i) \le x \le C_{R2}(0.01 \times i), 0.01 \times (i-1) \le \alpha \le 0.01 \times i\}, i = 1, \ldots, 100 \tag{37}$$

The following definition for the length of ith horizontal line d_i as follows:

$$d_i = \left(\frac{\chi^2_{(2r),0.5}}{\chi^2_{(2r),\,0.005\times i}} - \frac{\chi^2_{(2r),0.5}}{\chi^2_{(2r),\,1-0.005\times i}}\right)C_R = 1, 2, \ldots, 100 \tag{38}$$

Obviously, $d_0 = d_1$ and d_{100} = 0, so the area B_T is

$$B_T = \sum_{i=1}^{100}(0.01) \times \left(\frac{d_{i-1}+d_i}{2}\right) = 0.01\left(\frac{d_1}{2} + \sum_{i=1}^{99} d_i\right) \tag{39}$$

If B_R denotes the area under graph $g(x)$ to the right of $x = \hat{\beta}_{L0}$, then

$$B_R = \{(x,\alpha)|\hat{\beta}_{L0} \le x \le C_{R2}(\alpha), 0 \le \alpha \le a\} \tag{40}$$

where $\alpha = a$ such that $C_{R2}(a) = \hat{\beta}_{L0}$. Similarly B_T, $k = [100 \times a]$. Then, for $0 \le \alpha \le a$, where $[100 \times a]$ represents the largest integer less than or equal to $100 \times a$. Obviously, $a = 0.01 \times k$ and $\alpha = i \times 0.01$, $(i = 0, 1, 2, \ldots, k)$ horizontal lines cut B_R into k trapezoidal blocks. Then, the ith block can be expressed as follows:

$$B_{Ri} = \{(x,\alpha)|\hat{\beta}_{L0} \le x \le C_{R2}(0.01 \times i), 0.01 \times (i-1) \le \alpha \le 0.01 \times i\} = 1, 2, \ldots, k \tag{41}$$

The following defines the length of ith horizontal line r_i:

$$r_i = \frac{\chi^2_{(2r),0.5}}{\chi^2_{(2r),\,0.005\times i}} C_R - \hat{\beta}_{L0} = 1, 2, \ldots, k \tag{42}$$

This indicates that $r_0 = r_1$ and $r_k = 0$, so the area of B_R is

$$B_R = \sum_{i=1}^{k}(0.01) \times \left(\frac{r_{i-1}+r_i}{2}\right) = 0.01\left(\frac{r_1}{2} + \sum_{i=1}^{k-1} r_i\right) \tag{43}$$

The ratio of B_R/B_T can be usefully applied to fuzzy decision-making:

$$B_R/B_T = \frac{0.01\left(\frac{r_1}{2} + \sum_{i=1}^{k-1} r_i\right)}{0.01\left(\frac{d_1}{2} + \sum_{i=1}^{99} d_i\right)} \tag{44}$$

However, calculation of B_R/B_T is complicated.

According to Equations (39) and (43), these have calculated that, respectively, obtaining the block areas of B_T and B_R is extremely complicated. Therefore, for the purpose of simplifying the complicated calculating process of ratio B_R/B_T, the technique, membership functions $g(x)$ and $h(x)$ with asymmetry (in Figure 2), proposed by Chen and Chang [13] is utilized in the present study. The method suggested by Chen and Chang [13], to replace d_R (the length of the base of the set B_R) with the area of B_R, facilitates industrial applications. Similarly, d_T (the length of the base of the set B_T) is replaced with the area of B_T. As the membership functions are asymmetric, $d_T = 2(C_{RR} - C_R)$ on the basis of Chen and Chang [13] and Chen et al. [20]. In Figure 2, by using the principle of similar shapes, the square of the side length ratio is equal to the area ratio. Next, B_R/B_T was replaced with d_R/d_T as the fuzzy evaluation tool, where d_R and d_T are calculated as follows [13,20,33]:

$$d_R = C_{RR} - \hat{\beta}_{L0} = \frac{\chi^2_{(2r),0.5}}{\chi^2_{(2r),\,0.995}} C_R - \hat{\beta}_{L0} \tag{45}$$

and

$$d_T = \ 2(C_{RR} - C_R = 2\left(\frac{\chi^2_{(2r),0.5}}{\chi^2_{(2r),0.005}} C_R - C_R\right) \tag{46}$$

Based on their past experiences originating from other experts and the past data over the certain products [34], manufacturing engineers are allowed to define the values of δ_1 and δ_2. The following two numbers $0 < \delta_1 < \delta_2 < 0.5$ and $\delta = d_R/d_T$, the fuzzy test rules are as follows [13,20,35]:

(1) If $\delta < \delta_1$, then conclude that $\beta_L \geq c$ (i.e., do not reject H_0).
(2) If $\delta_1 \leq \delta \leq \delta_2$, then make no decision; more information is needed.
(3) If $\delta_2 < \delta < 0.5$, then conclude that $\beta_L < c$ (i.e., reject H_0).

4. Practical Example

This section presents a numerical example to demonstrate the proposed fuzzy test method. The required value of the lifetime performance index is at least 3; thus, the null hypothesis is H_0:$\beta_L \geq 3$ vs. the alternative hypothesis H_1:$\beta_L < 3$ [19]. If $y_1, y_2, \ldots, y_{30}$ is the observed value of $Y_1, Y_2, \ldots, Y_{30}$ with number of the uncensored data $r = 18$ ($r/n = 60\%$), then the observed value of the estimator is

$$\hat{\beta}_{L0} = \frac{1}{r}\sum_{i=1}^{n} y_i = \frac{41.6894}{18} = 2.316 \tag{47}$$

The values of β_{L0} and β_{R0} are then calculated as follows:

$$\beta_{L0} = \frac{\chi^2_{(18),0.5}}{\chi^2_{(18),0.995}} \times 2.316 \ = \ 1.329 \tag{48}$$

and

$$\beta_{R0} = \frac{\chi^2_{(18),0.5}}{\chi^2_{(18),0.005}} \times 2.316 \ = \ 4.576 \tag{49}$$

Furthermore, the membership function of fuzzy numbers $\widetilde{\beta}^{**}_{L0}$ is

$$h(x) = \begin{cases} 0 & , if \ x < 1.329 \\ 2\left(1 - F_W\left(\frac{1.3290}{x} \times \chi^2_{(36),0.5}\right)\right) & , if \ 1.329 \leq x < \ 2.316 \\ 1 & , if \ x = 2.316 \\ 2F_W\left(\frac{1.3290}{x} \times \chi^2_{(36),0.5}\right) & , if \ 2.316 < x \leq 4.576 \\ 0 & , if \ 4.576 < x \end{cases} \tag{50}$$

where $\chi^2_{(36),0.005}$ = 17.887, $\chi^2_{(36),0.995}$ = 61.581, and $\chi^2_{(36),0.5}$ = 35.336. As the significance level is $\alpha = 0.05$, then

$$C_R = \frac{c\chi^2_{(2r),\,\alpha}}{2r} = \frac{3\chi^2_{(36),0.05}}{36} = 1.939 \tag{51}$$

The values of C_{LR} and C_{RR} are calculated as follows:

$$C_{LR} = \frac{\chi^2_{(36),0.5}}{\chi^2_{(36),0.995}} \times 1.9391 = 1.113 \tag{52}$$

and

$$C_{RR} = \frac{\chi^2_{(36),0.5}}{\chi^2_{(36),0.005}} \times 1.9391 = 3.831 \tag{53}$$

Furthermore, the membership function of fuzzy number $\widetilde{C}_R$ is

$$g(x) = \begin{cases} 0 & , if\ x \le 1.113 \\ 2\left(1 - F_W\left(\frac{1.9391}{x}\chi^2_{(36),0.5}\right)\right) & , if\ 1.113 < x < 1.939 \\ 1 & , if\ x = 1.939 \\ 2F_W\left(\frac{1.9391}{x}\chi^2_{(36),0.5}\right) & , if\ 1.939 < x \le 3.831 \\ 0 & , if\ 3.831 < x \end{cases} \tag{54}$$

By Equations (50) and (54), we have the graphs of $h(x)$ and $g(x)$ in Figure 3. From Equation (54), we obtain α = $g(x)$. When x = $\hat{\beta}_{L0}$ = 2.316, $\alpha \in$ (0.46, 0.47), a = 0.468 could be obtained by interpolation method.

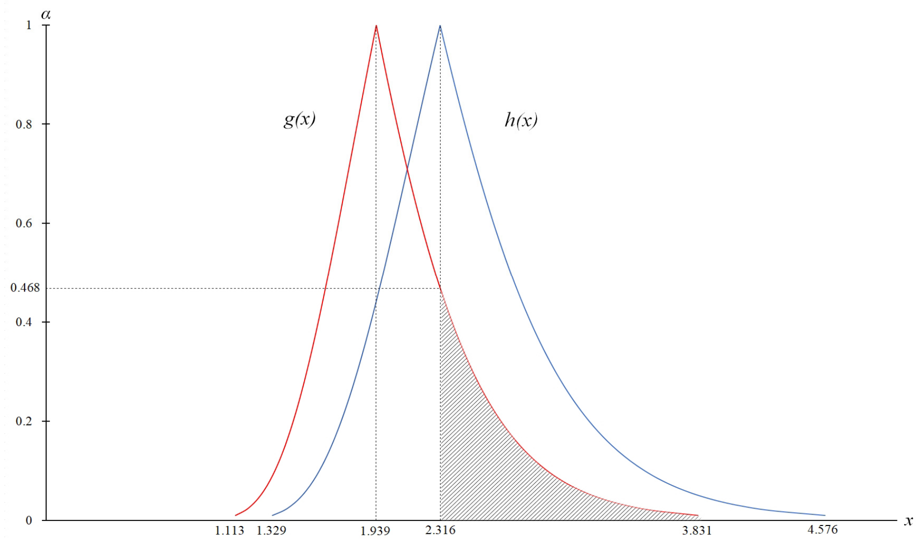

Figure 3. Membership functions $g(x)$ and $h(x)$ for numerical example.

The values of d_R and d_T are calculated as follows [13,20,33]:

$$d_R = C_{RR} - \hat{\beta}_{L0} = 3.831 - 2.316 = 1.515 \tag{55}$$

and

$$d_T = 2(C_{RR} - C_R) = 2(3.831 - 1.939) = 3.784 \tag{56}$$

Therefore,

$$\delta = d_R/d_T = 1.515/3.784 = 0.4004 \tag{57}$$

This leads to the conclusion that for $\hat{\beta}_{L0} = 2.316 > C_R = 1.939$, $\beta_L \geq 3$ (i.e., the null hypothesis should not be rejected). However, $\hat{\beta}_{L0} = 2.316$ is far less than $\beta_L = 3$. Thus, for $\delta_1 = 0.2$ and $\delta_2 = 0.4$ [13], $\beta_L < 3$ (i.e., the null hypothesis should be rejected). This is the risk of misjudgment caused by sampling errors in small sample sizes [7,9]. The proposed fuzzy method therefore provides a more reasonable conclusion.

5. Conclusions

This paper proposes an evaluation approach for product lifetime performance under type-II censoring. This evaluation enables the improvement of lifetime performance, which enhances the value of products as well as attains green goals such as energy efficiency and waste reduction. The proposed index is easy to use as its value increases with performance. Examination of the probability density function, cumulative distribution function, and reliability function of relative lifetime X indicated that reliability increased with the value of the index, as did the probability of the product lifetime surpassing the minimum with value L. An unbiased consistent estimator of the proposed index is also presented alongside a fuzzy test model based on the derived confidence interval. This model reduces the probability of misjudgment caused by sampling errors [7,9]. Additionally, many benefits will be gained by seizing the chance to improve, such as decreasing the testing cost and meeting the certain requirements in a short time. Furthermore, doing so is said to expand the possibility of using less paper, saving social resources, decreasing the carbon footprint and so forth [36]. In the electronics industry, passive components have long been indispensable parts that stimulate peripheral equipment industries. The proposed model thus focuses on passive components, with applicability demonstrated through a numerical example.

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