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Article

Faculty Performance Evaluation through Multi-Criteria Decision Analysis Using Interval-Valued Fermatean Neutrosophic Sets

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Abstract: The Neutrosophic Set (N_{set}) represents the uncertainty in data with fuzzy attributes beyond true and false values independently. The problem arises when the summation of true $(\mathcal{T}r)$, false $(\mathcal{F}a)$, and indeterminacy $(\mathcal{I}n)$ values crosses the membership value of one, that is, $\mathcal{T}r + \mathcal{I}n + \mathcal{F}a < 1$. It becomes more crucial during decision-making processes like medical diagnoses or any data sets where $\mathcal{T}r + \mathcal{I}n + \mathcal{F}a < 1$. To achieve this goal, the FN_{set} is recently introduced. This study employs the Interval-Valued Fermatean Neutrosophic Set $(IVFN_{set})$ as its chosen framework to address instances of partial ignorance within the domains of truth, falsehood, or uncertainty. This selection stands out due to its unique approach to managing such complexities within multi-decision processes when compared to alternative methodologies. Furthermore, the proposed method reduces the propensity for information loss often encountered in other techniques. IVFNS excels at preserving intricate relationships between variables even when dealing with incomplete or vague information. In the present work, we introduce the $IVFN_{set}$, which deals with partial ignorance in true, false, or uncertain regions independently for multi-decision processes. The $IVFN_{set}$ contains the interval-valued $\mathcal{T}r_{membership}$ value, $\mathcal{I}n_{membership}$ value, and $\mathcal{F}a_{membership}$ for knowledge representation. The algebraic properties and set theory between the interval-valued FN_{set} have also been presented with an illustrative example.

Keywords: Fermatean neutrosophic sets; interval-valued Fermatean neutrosophic sets; faculty performance evaluation; multicriteria decision analysis

MSC: 03E72; 05C72; 90B50

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1. Introduction

The acronyms given in the following Table 1 are used throughout the entire manuscript. For the computation of linguistic words like tall or young, Zadeh proposed FS in 1965 [1]. F_{set} are used to represent the acceptance and rejection of fuzzy attributes by membership values that lie in [0, 1]. The $\mathrm{N_{set}}$ helps to represent the hesitant part with the independent values of $\mathcal{T}\mathrm{r}_{\mathrm{membership}}$, $\mathcal{I}\mathrm{m}_{\mathrm{membership}}$, and $\mathcal{F}\mathrm{a}_{\mathrm{membership}}$ such that $^{-}0 < \mathcal{T}\mathrm{r}_{\mathrm{membership}} + \mathcal{I}\mathrm{m}_{\mathrm{membership}} + \mathcal{F}\mathrm{a}_{\mathrm{membership}} < 3^{+}$ [2]. Later, interval-valued membership sets were introduced, which dealt with the ignorance of partial data about the membership values [3,4]. Yager [5–7] coined a new kind of F_{set} called the PF_{set} as an extension of the IF_{set}. It has many practical applications in MCDM [8,9]. It is based on the Fermatean fuzzy set [10], which was recently hybridized with the N_{set} [11–13] and Fermatean fuzzy graph [14]. Rani and Mishra [15] studied the $IVFF_{set}$. The $IVFF_{set}$ [16,17] is used in several fields for the DM process because of its extensive properties [18,19].

Table 1. Acronyms.

Abbreviations	Full Phrase
Fuzzy Set	FS
Intuitionistic Fuzzy Set	IFS
Neutrosophic Set	NS
Interval valued Pythagorean neutrosophic sets	IVPNS
Interval-valued Fermatean neutrosophic set	IVFNS
Interval valued neutrosophic sets	IVNS
Pythagorean Fuzzy Set	PFS
Fermatean Fuzzy set	FFS
Interval-valued Fermatean Fuzzy numbers.	IVFFN
Interval-valued Fermatean Fuzzy sets	IVFFS
Hesitant Fermatean fuzzy sets	HFFS
Interval valued hesitant Fermatean fuzzy sets	IVHFFS
Multi-Criteria decision-making	MCDM
Technique for Order of Preference by Similarity to Ideal Solution	TOPSIS
Interval-valued Fermatean fuzzy TOPSIS	IVFFTOPSIS
Single valued neutrosophic set	SVNS
Fermatean neutrosophic graph	FN_{graph}

The N_{set} theory is introduced by Smarandache [2] as an extension of the IF_{set} theory to deal with indeterminacy. Wang [4] defined the $IVSN_{set}$ in 2010 as an extension of interval fuzzy sets [20]. Zhang et al. [21] applied the concept of Interval neutrosophic sets in multicriteria decision-making problems. Wang, T [22] introduced a projection model with unknown weight information within an interval neutrosophic environment and applied it to software quality-in-use evaluation. Another class of the N_{set} is the $IVNP_{set}$ with the dependent interval-valued Pythagorean component, proposed by Stephy and Helen [13]. Clearly, it is a generalization of the $IVPN_{set}$ and can handle more information than the IVN_{set}. Motivated by the FF_{set} Jansi [11] defined the FN_{set} and provided its various properties. Jeevaraj [16] introduced the concept of the $IVFF_{set}$s and derived mathematical operations on the class of the $IVFF_{set}$. Score functions in the $IVFF_{set}$ are introduced and their properties are studied. Recently, PalaniKumar and Iampan [17] proposed the concept of the spherical $IVFF_{soft\ set}$. Liu et al. [18] discussed Fermatean fuzzy linguistic term sets, their basic operational laws, and aggregate functions. Broumi et al. [19] proposed the $IVFN_{graph}$ and presented some basic operational laws. He also [23] introduced the FN_{graph} and RFN_{graph}, SFN_{graph}, and F_{number} product graphs.

For DM problems in the Neutrosophic context, the value of times squared of the sum of the $\mathcal{T}r, \mathcal{I}m,$ and $\mathcal{F}a$ degrees does not exceed two. To deal with this issue, Sweety and Jansi introduced the FN_{set} [11]. Also, the FN_{set} is a generalization of the PN_{set} and it is characterized by the condition that the cubes of their sum of their $\mathcal{T}\mathbf{r}_{\mathbf{membership}}$, $\mathcal{F}\mathbf{a}_{\mathbf{membership}}$, and $\mathcal{I}\mathbf{m}_{\mathbf{membership}}$ degrees do not exceed them twice. Motivated by the above literature, we develop the idea of the $IVFN_{set}$ and its algebraic operations. The major findings of the present article are as follows:

- To establish and study the $IVFN_{set}$ and its algebraic operations.
- To introduce the accuracy and score functions $(AF$ and $SF)$ of the $IVFN_{number}$.
- To illustrate the applications of the $IVFN_{set}$.

Section 1 includes an introductory part; Section 2 deals with the basic algebraic operations related to the $IVFN_{set}$; Section 3 defines the AF and SF of the $IVFN_{set}$; and Section 4 discusses the application of the $IVFN_{set}$ and delivers recommendations for future research.

2. Prerequisites

In this section, we briefly introduce the necessary basic definitions and preliminary results.

A F_{set} [1] A on $\mathfrak{A}$ is of the form: $A_{PFS} = \{\langle \hbar, \mu_A(\hbar)\rangle | \hbar \in \mathfrak{A}\}$ where $\mu_A(\hbar) : \mathfrak{A} \to [0,1]$. A PFS [5–7] A on $\mathfrak{A}$ is of the form: $A_{PFS} = \{\langle \hbar, \boldsymbol{\mathcal{T}r}_A(\hbar), \boldsymbol{\mathcal{F}a}_A(\hbar)\rangle | \hbar \in \mathfrak{A}\}$, where $\boldsymbol{\mathcal{T}r}_A(\hbar) : \mathfrak{A} \to [0,1]$ denotes the membership degree (***md***) and $\boldsymbol{\mathcal{F}a}_A(\hbar) : \mathfrak{A} \to [0,1]$ denotes the $\boldsymbol{non-md}, \forall \hbar \in \mathfrak{A}$ to the set A_{PFS}, respectively, such that $0 \le (\boldsymbol{\mathcal{T}r}_A(\hbar))^2 + (\boldsymbol{\mathcal{F}a}_A(\hbar))^2 \le 1$. Corresponding to its ***mf***, the indeterminacy degree is given by $\phi_A(\hbar) = \sqrt{1 - \boldsymbol{\mathcal{T}r}_A(\hbar)^2 - \boldsymbol{\mathcal{F}a}_A(\hbar)^2}, \forall \hbar \in \mathfrak{A}$. A FF_{set} [11] A on $\mathfrak{A}$ is of the form as $A_{FFS} = \{\langle \hbar, \boldsymbol{\mathcal{T}r}_A(\hbar), \boldsymbol{\mathcal{F}a}_A(\hbar)\rangle | \hbar \in \mathfrak{A}\}$ where $\mathcal{T}r_A(\hbar) : \mathfrak{A} \to [0,1]$ represents the ***md***, and $\boldsymbol{\mathcal{F}a}_A(\hbar) : \mathfrak{A} \to [0,1]$ represents the $\boldsymbol{non-md}, \forall \hbar \in \mathfrak{A}$ to the set A, respectively, such that $0 \le (\boldsymbol{\mathcal{T}r}_A(\hbar))^3 + (\boldsymbol{\mathcal{F}a}_A(\hbar))^3 \le 1$. Corresponding to its ***f***, the indeterminacy degree is given by $\pi_A(\hbar) = \sqrt{1 - \boldsymbol{\mathcal{T}r}_A(\hbar)^3 - \boldsymbol{\mathcal{F}a}_A(\hbar)^3}$, $\forall \hbar \in \mathfrak{A}$.A N_{set} [2] A on $\mathfrak{A}$ is defined by its truth $(\boldsymbol{\mathcal{T}r}_A(\hbar))$, indeterminacy $(\mathcal{I}m_A(\hbar))$ and falsity membership function $(\boldsymbol{\mathcal{F}a}_A(\hbar))$ such that $0^- \le \boldsymbol{\mathcal{T}r}_A(\hbar) + \mathcal{I}m_A(\hbar) + \boldsymbol{\mathcal{F}a}_A(\hbar) \le 3^+$ for all $\hbar \in \mathfrak{A}$, whose all the subset of $[0^-, 1^+]$.

In the following, Figure 1 depicted the graphical visualization between the Intuitionistic, Pythagorean, and Fermatean Fuzzy sets.

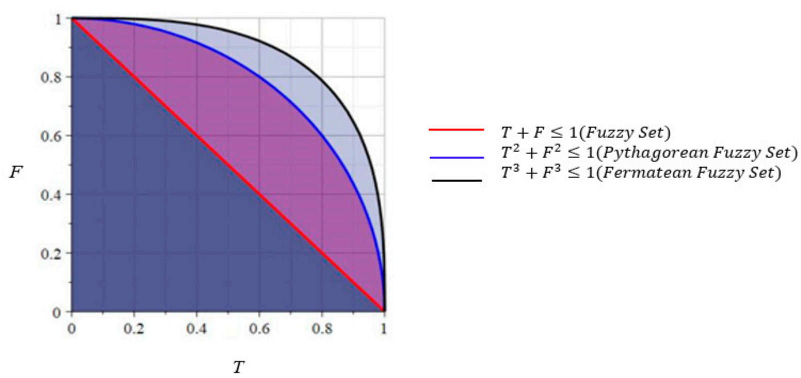


Figure 1. A graphical visualization of the Intuitionistic, Pythagorean, and Fermatean Fuzzy sets.

The SVN_{set} [3] A on $\mathfrak{A}$ is is of the form: $A_{SVNS} = \{\langle \hbar, \boldsymbol{\mathcal{T}\mathbf{r}}_A(\hbar), \boldsymbol{\mathcal{I}\mathbf{m}}_A(\hbar), \boldsymbol{\mathcal{F}a}_A(\hbar)\rangle | \hbar \in \mathfrak{A}\}$, where $\boldsymbol{\mathcal{T}\mathbf{r}}_A(\hbar) : \mathfrak{A} \to [0,1]$ represents the ***md***, $\boldsymbol{\mathcal{I}\mathbf{m}}_A(\hbar) : \mathfrak{A} \to [0,1]$ represents the $\boldsymbol{indeterminacy - md}$, and $\boldsymbol{\mathcal{F}a}_A(\hbar) : \mathfrak{A} \to [0,1]$ represents the $\boldsymbol{non-md}, \forall \hbar \in \mathfrak{A}$ to the setA, respectively, with $0 \le \boldsymbol{\mathcal{T}\mathbf{r}}_A(\hbar) + \boldsymbol{\mathcal{I}\mathbf{m}}_A(\hbar) + \boldsymbol{\mathcal{F}a}_A(\hbar) \le 3$. The PN_{set} [8] is defined as, $0 \le (\boldsymbol{\mathcal{T}r}_A(\hbar))^2 + (\boldsymbol{\mathcal{I}m}_A(\hbar))^2 \le 1$ and $0 \le (\boldsymbol{\mathcal{F}a}_A(\hbar))^2 \le 1$ then $0 \le (\boldsymbol{\mathcal{T}\mathbf{r}}_A(\hbar))^2 + (\boldsymbol{\mathcal{I}\mathbf{m}}_A(\hbar))^2 + (\boldsymbol{\mathcal{F}a}_A(\hbar))^2 \le 2$. Sweety et al. [11] introduced the FN_{set} as: $0 \le (\boldsymbol{\mathcal{T}\mathbf{r}}_A(\hbar))^3 + (\boldsymbol{\mathcal{I}\mathbf{m}}_A(\hbar))^3 \le 1$ and $0 \le (\boldsymbol{\mathcal{F}a}_A(\hbar))^3 \le 1$ then $0 \le \hbar \in \mathfrak{A}$ to the set A, with $(\boldsymbol{\mathcal{T}\mathbf{r}}_A(\hbar))^3 + (\boldsymbol{\mathcal{I}\mathbf{m}}_A(\hbar))^3 + (\boldsymbol{\mathcal{F}a}_A(\hbar))^3 \le 2$ $\forall \hbar \in \mathfrak{A}$. An IVF_{set} [19] set $\widetilde{\mathrm{A}}$ on $\mathfrak{A}$ is a function $\widetilde{\mathrm{A}} : \mathfrak{A} \to Int([0,1])$ and the set of all IVF_{set} on $\mathfrak{A}$ is denoted by $\hbar(\mathfrak{A})$. Suppose that $\widetilde{\mathrm{A}} \in \hbar(\mathfrak{A}), \forall \hbar \in \mathfrak{A}, \mu_{\widetilde{\mathrm{A}}}(\hbar) = \left[\mu^-_{\widetilde{\mathrm{A}}}(\hbar), \mu^+_{\widetilde{\mathrm{A}}}(\hbar)\right]$ is the ***md*** of an element $\hbar$ to $\widetilde{\mathrm{A}}$, $\mu^-_{\widetilde{\mathrm{A}}}(\hbar)$, $\mu^+_{\widetilde{\mathrm{A}}}(\hbar)$ are the least and greatest bounds of ***md***$\hbar$ to $\widetilde{\mathrm{A}}$, where $0 \le \mu^-_{\widetilde{\mathrm{A}}}(\hbar) \le \mu^+_{\widetilde{\mathrm{A}}}(\hbar) \le 1$. The $IVPFS$ [10] A on $\mathfrak{A}$ is of the form as: $A_{IVNP_{set}} = \left\{\left\langle \hbar, \left[\boldsymbol{\mathcal{T}\mathbf{r}}^-_A(\hbar), \boldsymbol{\mathcal{T}\mathbf{m}}^+_A(\hbar)\right], \left[\boldsymbol{\mathcal{F}a}^-_A(\hbar), \boldsymbol{\mathcal{F}}^+_A(\hbar)\right]\right\rangle : \hbar \in \mathfrak{A}\right\}$ where $0 \le \boldsymbol{\mathcal{T}\mathbf{r}}^-_A(\hbar) \le \boldsymbol{\mathcal{T}\mathbf{r}}^+_A(\hbar) \le 1, 0 \le \boldsymbol{\mathcal{F}a}^-_A(\hbar) \le \boldsymbol{\mathcal{F}a}^+_A(\hbar) \le 1$ and $0 \le \left(\boldsymbol{\mathcal{T}\mathbf{r}}^+_A(\hbar)\right)^2 + \left(\boldsymbol{\mathcal{F}a}^+_A(\hbar)\right)^2 \le 1, \forall \hbar \in \mathfrak{A}$. $IVFF_{set}$ [13] A on $\mathfrak{A}$ of the form as: $A_{IVNP_{set}} = \left\{\left\langle \hbar, \left[\boldsymbol{\mathcal{T}\mathbf{r}}^-_A(\hbar), \boldsymbol{\mathcal{T}\mathbf{r}}^+_A(\hbar)\right], \left[\boldsymbol{\mathcal{F}a}^-_A(\hbar), \boldsymbol{\mathcal{F}a}^+_A(\hbar)\right]\right\rangle : \forall \hbar \in \mathfrak{A}\right\}$ where $0 \le \boldsymbol{\mathcal{T}\mathbf{r}}^-_A(\hbar) \le \boldsymbol{\mathcal{T}\mathbf{r}}^+_A(\hbar) \le 1, 0 \le \boldsymbol{\mathcal{F}a}^-_A(\hbar) \le \boldsymbol{\mathcal{F}a}^+_A(\hbar) \le 1$ and $0 \le \left(\boldsymbol{\mathcal{T}\mathbf{r}}^+_A(\hbar)\right)^3 + \left(\boldsymbol{\mathcal{F}a}^+_A(\hbar)\right)^3 \le 1, \forall \hbar \in \mathfrak{A}$. A IVN_{set} [24] ***A*** for every point $x \in \mathfrak{A}, \boldsymbol{\mathcal{T}\mathbf{r}}_A(\hbar), \boldsymbol{\mathcal{I}\mathbf{m}}_A(p), \boldsymbol{\mathcal{F}a}_A(\hbar) \subseteq [0,1]$.

$A_{\mathrm{IVN_{set}}} = \{\langle [\mathcal{T}\mathbf{r}_A^-(\hbar),\ \mathcal{T}\mathbf{r}_A^+(\hbar)], [\mathcal{I}\mathbf{m}_A^-(\hbar), \mathcal{I}\mathbf{m}_A^+(\hbar)], [\mathcal{F}\boldsymbol{a}_A^-(\hbar),\ \mathcal{F}\boldsymbol{a}_A^+(\hbar)]\rangle : \forall \hbar \in \mathfrak{A}\}$ with $0 \leq\ \mathcal{T}\mathbf{r}_A^+(\hbar) + \mathcal{I}\mathbf{r}_A^+(\hbar) + \mathcal{F}\boldsymbol{a}_A^+(\hbar)\ \leq 3$. An $IVNP_{set}$ [25] A on $\mathfrak{A}$ is of the form as $A_{IVNP_{set}} = \{\langle [\mathcal{T}\mathbf{r}_A^-(\hbar),\ \mathcal{T}\mathbf{r}_A^+(\hbar)], [\mathcal{I}\mathbf{m}_A^-(\hbar), \mathcal{I}\mathbf{m}_A^+(\hbar)], [\mathcal{F}\boldsymbol{a}_A^-(\hbar),\ \mathcal{F}\boldsymbol{a}_A^+(\hbar)]\rangle : \forall \hbar \in \mathfrak{A}\}$ where $\mathcal{T}\mathbf{r}_A^-(\hbar),\ \mathcal{T}\mathbf{r}_A^+ : \mathfrak{A} \to [0,\ 1]$ represents the least and greatest bounds of truth ***md***, $\mathcal{I}\mathrm{m}_A^-(\hbar), \mathcal{I}\mathrm{m}_A^+(\hbar) : \mathfrak{A} \to [0,\ 1]$ represents the least and greatest bounds of ***indeterminacy md***, and $\mathcal{F}a_A^-(\hbar),\ \mathcal{F}a_A^+(\hbar) : \mathfrak{A} \to [0,\ 1]$ represents the least and greatest bounds of ***falsity md***, $\forall \hbar \in \mathfrak{A}$ to the set A, with $0 \leq \left[\frac{\mathcal{T}\mathbf{r}_A^-(\hbar) + \mathcal{T}\mathbf{r}_A^+(\hbar)}{2}\right]^2 + \left[\frac{\mathcal{I}\mathbf{m}_A^-(\hbar) + \mathcal{I}\mathbf{m}_A^+(\hbar)}{2}\right]^2 + \left[\frac{\mathcal{F}\boldsymbol{a}_A^-(\hbar) + \mathcal{F}\boldsymbol{a}_A^+(\hbar))}{2}\right]^2 \leq 2$. In Zhang et al. [21], the operators of set-theoretic on the IN_{set} are defined as follows:

The IVN_{set} is contained in another $IVN_{set}\mathrm{B_{IVN_{set}}}$, $\mathrm{A_{IVN_{set}}} \subseteq \mathrm{B_{IVN_{set}}}$, $\Leftrightarrow \mathcal{T}\mathbf{r}_A^-(\hbar) \leq \mathcal{T}\mathbf{r}_B^-(\hbar),\ \mathcal{T}\mathbf{r}_A^+(\hbar) \leq \mathcal{T}\mathbf{r}_B^+(\hbar);\ \mathcal{I}\mathbf{m}_A^-(\hbar) \geq \mathcal{I}\mathbf{m}_B^-(\hbar), \mathcal{I}\mathbf{m}_A^+(\hbar) \geq \mathcal{I}\mathbf{m}_B^+(\hbar); \mathcal{F}\boldsymbol{a}_A^-(\hbar) \geq \mathcal{F}\boldsymbol{a}_B^-(\hbar)$,
$\mathcal{F}\boldsymbol{a}_A^+(\hbar) \geq \mathcal{F}\boldsymbol{a}_{\mathbf{B}}^+(\hbar), \forall \hbar \in \mathfrak{A}$.

Two IVN_{set}, $A_{\mathrm{IVN_{set}}} = B_{\mathrm{IVN_{set}}} \Leftrightarrow A_{\mathrm{IVN_{set}}} \subseteq B_{\mathrm{IVN_{set}}}$ and $B_{\mathrm{IVN_{set}}} \subseteq A_{\mathrm{IVN_{set}}}$,

That is,

$$\begin{gathered}\mathcal{T}\mathbf{r}_A^-(\hbar) = \mathcal{T}_{rB}^-(\hbar),\ \mathcal{T}\mathbf{r}_A^+(\hbar) = \mathcal{T}\mathbf{r}_B^+(\hbar)\ ; \mathcal{F}\boldsymbol{a}_A^-(\hbar) = \mathcal{F}\boldsymbol{a}_B^-(\hbar), \mathcal{F}\boldsymbol{a}_A^+(\hbar) \geq \mathcal{F}\boldsymbol{a}_B^+(\hbar),\\ \mathcal{I}\mathbf{m}_A^-(\hbar) = \mathcal{I}\mathbf{m}_B^-(\hbar), \mathcal{I}\mathbf{m}_A^+(\hbar) = \mathcal{I}\mathbf{m}_B^+(\hbar), \text{for all } \hbar \in \mathfrak{A}.\end{gathered}$$

The IVN_{set} A is empty $\Leftrightarrow \mathcal{T}\mathbf{r}_A^-(\hbar) = \mathcal{T}\mathbf{r}_A^+(\hbar) = 0,\ \mathcal{F}\boldsymbol{a}_A^-(\hbar) = \mathcal{F}\boldsymbol{a}_A^+(\hbar) = 1$ and $\mathcal{I}\mathbf{m}_A^-(\hbar) = \mathcal{I}\mathbf{m}_A^+(\hbar) = 0$, for all $\hbar \in \mathfrak{A}$.

A complement of the INV_{set} is

$$A_{\mathrm{IVN_{set}^C}} = \left\{\begin{matrix} p,\ [\mathcal{T}\mathbf{r}_A^-(\hbar), \mathcal{T}\mathbf{r}_A^+(\hbar)], \\ [1 - \mathcal{I}\mathbf{m}_A^+(\hbar), 1 - \mathcal{I}\mathbf{m}_A^-(\hbar)], \\ [\mathcal{F}\boldsymbol{a}_A^-(\hbar), \mathcal{F}\boldsymbol{a}_A^+(\hbar)] \end{matrix}\right\}, \hbar \in \mathfrak{A}$$

$A_{IVN_{set}} \cap B_{IVN_{set}}$, defined as follows:

$$A_{IVN_{set}} \cap B_{IVN_{set}} = \left\{\left\langle \begin{matrix} p, [\mathcal{T}\mathbf{r}_A^-(\hbar) \wedge \mathcal{T}\mathbf{r}_B^-(\hbar), \mathcal{T}\mathbf{r}_A^+(\hbar) \wedge \mathcal{T}\mathbf{r}_B^+(\hbar)], \\ [\mathcal{I}\mathbf{m}_A^-(\hbar) \vee \mathcal{I}\mathbf{m}_{\mathrm{B}}^-(\hbar), \mathcal{I}\mathbf{m}_A^+(\hbar) \vee \mathcal{I}\mathbf{m}_B^+(\hbar)], \\ [\mathcal{F}\boldsymbol{a}_A^-(\hbar) \vee \mathcal{F}\boldsymbol{a}_{\mathrm{B}}^-(\hbar), \mathcal{F}\boldsymbol{a}_A^+(\hbar) \vee \mathcal{F}\boldsymbol{a}_B^+(\hbar)] \end{matrix} \right\rangle\right\}, \hbar \in \mathfrak{A}$$

$A_{\mathrm{IVN}_{set}} \cup B_{\mathrm{IVN}_{set}}$, defined as follows:

$$A_{IVN_{set}} \cup B_{IVN_{set}} = \left\{\left\langle \begin{matrix} k, [\mathcal{T}\mathbf{r}_A^-(\hbar) \vee \mathcal{T}\mathbf{r}_B^-(\hbar), \mathcal{T}\mathbf{r}_A^+(\hbar) \vee \mathcal{T}\mathbf{r}_B^+(\hbar)], \\ [\mathcal{I}\mathbf{m}_A^-(\hbar) \wedge \mathcal{I}\mathbf{m}_{\mathrm{B}}^-(\hbar), \mathcal{I}\mathbf{m}_A^+(\hbar) \wedge \mathcal{I}\mathbf{m}_B^+(\hbar)] \\ [\mathcal{F}\boldsymbol{a}_A^-(\hbar) \wedge \mathcal{F}\boldsymbol{a}_{\mathrm{B}}^-(\hbar), \mathcal{F}\boldsymbol{a}_A^+(\hbar) \wedge \mathcal{F}\boldsymbol{a}_B^+(\hbar)] \end{matrix} \right\rangle\right\},\ \hbar \in \mathfrak{A}.$$

The difference between two $IVN_{set}A$ and $IVN_{set}\ B$ is the $IVN_{set}A_{\mathrm{IVN_{set}}} \ominus B_{\mathrm{IVN_{set}}}$, defined as A⊖B = < $[\mathcal{T}\mathbf{r}^-_{A_{\mathrm{IVN_{set}}} \ominus B_{\mathrm{IVN_{set}}}}, \mathcal{T}\mathbf{r}^+_{A_{\mathrm{IVN_{set}}} \ominus B_{\mathrm{IVN_{set}}}}]$, $[\mathcal{I}\mathbf{m}^-_{A_{\mathrm{IVN_{set}}} \ominus B_{\mathrm{IVN_{set}}}}, \mathcal{I}\mathbf{m}^+_{A_{\mathrm{IVN_{set}}} \ominus B_{\mathrm{IVN_{set}}}}]$, $[\mathcal{F}\boldsymbol{a}^-_{A_{\mathrm{IVN_{set}}} \ominus B_{\mathrm{IVN_{set}}}}, \mathcal{F}\boldsymbol{a}^+_{A_{\mathrm{IVN_{set}}} \ominus B_{\mathrm{IVN_{set}}}}]$> where

$$\begin{aligned}\mathcal{T}\mathbf{r}^L_{A_{\mathrm{IVN_{set}}} \ominus B_{\mathrm{IVN_{set}}}} &= \max(\mathcal{T}\mathbf{r}_A^-(\hbar), \mathcal{T}\mathbf{r}_B^-(\hbar)),\ \mathcal{T}\mathbf{r}^+_{A_{\mathrm{IVN_{set}}} \ominus B_{\mathrm{IVN_{set}}}} = \max(\mathcal{T}\mathbf{r}_A^+(\hbar), \mathcal{T}\mathbf{r}_B^+(\hbar))\\ \mathcal{I}\mathbf{m}^L_{A_{\mathrm{IVN_{set}}} \ominus_2 B_{\mathrm{IVN_{set}}}} &= \max(\mathcal{I}\mathbf{m}_A^+(\hbar), 1 - \mathcal{I}\mathbf{m}_B^+(\hbar)),\ \mathcal{I}\mathbf{m}^U_{A_{\mathrm{IVN_{set}}} \ominus_2 B_{\mathrm{IVN_{set}}}} = \max(\mathcal{I}\mathbf{m}_A^+(\hbar), 1 - \mathcal{I}\mathbf{m}_{\mathrm{B}}^-(\hbar))\\ \mathcal{F}\boldsymbol{a}^L_{A_{\mathrm{IVN_{set}}} \ominus_2 B_{\mathrm{IVN_{set}}}} &= \max(\mathcal{F}\boldsymbol{a}_A^-(\hbar), \mathcal{T}\mathbf{r}_B^-(\hbar)),\ \mathcal{F}\boldsymbol{a}^U_{A_{\mathrm{IVN_{set}}} \ominus_2 B_{\mathrm{IVN_{set}}}} = \max(\mathcal{F}\boldsymbol{a}_A^+(\hbar), \mathcal{T}\mathbf{r}_B^+(\hbar))\end{aligned}$$

The scalar division of the IVN_{set}A is $A_{\mathrm{IVN_{set}}}/$a, defined as follows:

$$A_{\mathrm{IVN_{set}}}/a == \left\{\left\langle \begin{matrix} \hbar, [\max(\mathcal{T}\mathbf{r}_A^-(\hbar)/a, 1), \max(\mathcal{T}\mathbf{r}_A^+(\hbar)/a, 1)], \\ [\min(\mathcal{I}\mathbf{m}_A^-(\hbar)/a, 1), \min(\mathcal{I}\mathbf{m}_A^+(\hbar)/a, 1)], \\ [\min(\mathcal{F}\boldsymbol{a}_A^-(\hbar)/a, 1), \min(\mathcal{F}\boldsymbol{a}_A^+(\hbar)/a, 1)] \end{matrix} \right\rangle\right\},\ \hbar \in \mathfrak{A},\ a \in R^+$$

$A_{\mathrm{IVN_{set}}}.a$, defined as follows:

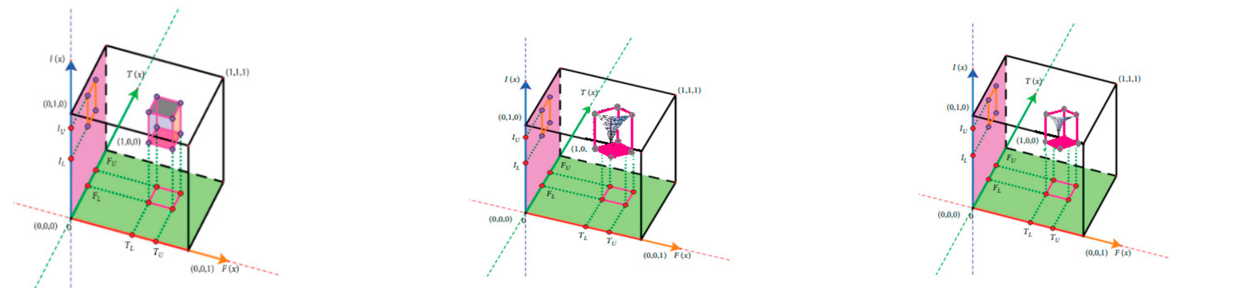
$$A_{\mathrm{IVN_{set}}}.a = \left\{ \left\langle \begin{array}{c} p, \left[\max\left(\mathcal{T}\mathbf{r}_A^-(\hbar).a,1\right), \max\left(\mathcal{T}\mathbf{r}_A^+(\hbar).a,1\right)\right], \\ \left[\min\left(\mathcal{I}\mathbf{m}_A^-(\hbar).a,1\right), \min\left(\mathcal{I}\mathbf{m}(\hbar).a,1\right)\right] \\ \left[\min\left(\mathcal{F}\boldsymbol{a}_A^-(\hbar).a,1\right), \min\left(\mathcal{F}\boldsymbol{a}_A^+(\hbar).a,1\right)\right] \end{array} \right\rangle \right\}, \hbar \in \mathfrak{A},\ \mathrm{a}\ \in R^+.$$

3. Interval-Valued Fermatean Neutrosophic Sets ($IVFN_{set}$)

The concept of the $IVFN_{set}$, $IVFN_{umber}$, and their basic properties are introduced in this section.

Definition 1. *The $IVFN_{set}$ A on $\mathfrak{A}$ is of the form $A = \{\langle[\mathcal{T}\boldsymbol{r}_A^-(\hbar),\ \mathcal{T}\boldsymbol{r}_A^+(\hbar)], [\mathcal{I}\boldsymbol{m}_A^-(\hbar), \mathcal{I}\boldsymbol{m}_A^+(\hbar)],\ [\mathcal{F}\boldsymbol{a}_A^-(\hbar),\ \mathcal{F}\boldsymbol{a}_A^+(\hbar)]\rangle \mid \hbar \in \mathfrak{A}\}$ where $\mathcal{T}\boldsymbol{r}_A(p) = \mathcal{T}\boldsymbol{r}_A^-(\hbar),\ \mathcal{T}\boldsymbol{r}_A^+(\hbar) : \mathfrak{A} \to [0,1]$ represents the least and greatest bounds of **truthmd**, $\mathcal{I}\boldsymbol{m}_A(\hbar) = [\mathcal{I}\boldsymbol{m}_A^-(\hbar), \mathcal{I}\boldsymbol{m}_A^+(\hbar)] : \mathfrak{A} \to [0,1]$ represents the least and greatest bounds of **indeterminacy md**, and $\mathcal{F}\boldsymbol{a}_A(\hbar) = [\mathcal{F}\boldsymbol{a}_A^-(\hbar),\ \mathcal{F}\boldsymbol{a}_A^+(\hbar)] : \mathfrak{A} \to [0,1]$ represents the least and greatest bounds of **falsity md**, $\forall\ \hbar \in \mathfrak{A}$ to the set A, respectively, with $0 \le (\mathcal{T}\boldsymbol{r}_A(\hbar))^3 + (\mathcal{F}\boldsymbol{a}_A(\hbar))^3 \le 1$ and $0 \le (\mathcal{I}\boldsymbol{m}_A(\hbar))^3 \le 1$, $0 \le (\mathcal{T}\boldsymbol{r}_A(\hbar))^3 + (\mathcal{F}\boldsymbol{a}_A(\hbar))^3 + (\mathcal{I}\boldsymbol{m}_A(\hbar))^3 \le 2$ means $0 \le (\mathcal{T}\boldsymbol{r}_A(\hbar))^3 + (\mathcal{F}\boldsymbol{a}_A(\hbar))^3 + (\mathcal{I}\boldsymbol{m}_A(\hbar))^3 \le 2 \forall\ \hbar \in \mathfrak{A}$.*

In the following, Figure 2 depicted the Geometric representation of the IVN_{set}, $IVPN_{set}$, and $IVFN_{set}$.



Geometric representation of the IVN_{set} *Geometric representation of the $IVPN_{set}$* *Geometric representation of the $IVFN_{set}$*

Figure 2. Geometric representation of the IVN_{set}, $IVPN_{set}$, and $IVFN_{set}$.

Definition 2. *For an $IVFN_{number}\alpha = \left(\left[\mathcal{T}\boldsymbol{r}_\alpha^{A-}, \mathcal{T}\boldsymbol{r}_\alpha^{A+}\right], \left[\mathcal{I}\boldsymbol{m}_\alpha^{A-}, \mathcal{I}\boldsymbol{m}_\alpha^{A+}\right], \left[\mathcal{F}\boldsymbol{a}_\alpha^{A-}, \mathcal{F}\boldsymbol{a}_\alpha^{A+}\right]\right)$, which satisfies $\left(\mathcal{T}\boldsymbol{r}_\alpha^{+A}\right)^3 + \left(\mathcal{I}\boldsymbol{m}_\alpha^{+A}\right)^3 + \left(\mathcal{F}\boldsymbol{a}_\alpha^{+A}\right)^3 \le 2$. Consider $\alpha = \left(\left[\mathcal{T}\boldsymbol{r}_\alpha^{A-}, \mathcal{T}\boldsymbol{r}_\alpha^{A+}\right], \left[\mathcal{I}\boldsymbol{m}_\alpha^{A-}, \mathcal{I}\boldsymbol{m}_\alpha^{A+}\right], \left[\mathcal{F}\boldsymbol{a}_\alpha^{A-}, \mathcal{F}\boldsymbol{a}_\alpha^{A+}\right]\right) = ([a,b],[c,d],[e,f])$ is a $IVFN_{number}$.*

Remark 1. *The $IVFN_{set}$ is an extension of the $IVFF_{set}$. The $IVFN_{number}$ occupies more space than the $IVFF_{number}$, $IVIF_{number}$, and $IVPF_{number}$. There is no doubt that the $IVFN_{set}$ is the more appropriate tool for finding the best alternative in complex MCDM uncertainty problems rather than the $IVFF_{set}$, $IVPF_{set}$, and $IVIF_{set}$.*

Definition 3. *Let $\mathfrak{K}$ and $\mathfrak{L}$ be two $IVFN_{set}$ on $\mathfrak{A}$, defined by:*

$$\mathfrak{K} = \{\langle \hbar, \mathcal{T}\mathbf{r}_{\mathfrak{K}}(\hbar), \mathcal{I}\mathbf{m}_{\mathfrak{K}}(\hbar), \mathcal{F}\mathbf{a}_{\mathfrak{K}}(\hbar)\rangle \mid \hbar \in \mathfrak{A}\}$$
$$\mathfrak{L} = \{\langle \hbar, \mathcal{T}\mathbf{r}_{\mathfrak{L}}(\hbar), \mathcal{I}\mathbf{m}_{\mathfrak{L}}(\hbar), \mathcal{F}\mathbf{a}_{\mathfrak{L}}(\hbar)\rangle \mid \hbar \in \mathfrak{A}\}$$

where $\mathcal{T}\mathbf{r}_{\mathfrak{K}}(\hbar) = [\mathcal{T}\mathbf{r}_{\mathfrak{K}}^-(\hbar), \mathcal{T}\mathbf{r}_{\mathfrak{K}}^+(\hbar)]$, $\mathcal{I}\mathbf{m}_{\mathfrak{K}}(\hbar) = [\mathcal{I}\mathbf{m}_{\mathfrak{K}}^-(\hbar),\ \mathcal{I}\mathbf{m}_{\mathfrak{K}}^+(\hbar)]$, $\mathcal{F}\mathbf{a}_{\mathfrak{K}}(\hbar) = \left[\mathcal{F}\mathbf{a}_{\mathfrak{K}}^-(\hbar), \mathcal{F}\mathbf{a}_{\mathfrak{K}}^+(\hbar)\right]$ and $\mathcal{T}\mathbf{r}_{\mathfrak{L}}(\hbar) = [\mathcal{T}\mathbf{r}_{\mathfrak{L}}^-(\hbar), \mathcal{T}\mathbf{r}_{\mathfrak{L}}^+(\hbar)]$, $\mathcal{I}\mathbf{m}_{\mathfrak{L}}(\hbar) = [\mathcal{I}\mathbf{m}_{\mathfrak{L}}^-(\hbar), \mathcal{I}\mathbf{m}_{\mathfrak{L}}^+(\hbar)]$, $\mathcal{F}\mathbf{a}_{\mathfrak{L}}(\hbar) = \left[\mathcal{F}\mathbf{a}_{\mathfrak{L}}^-(\hbar), \mathcal{F}\mathbf{a}_{\mathfrak{L}}^+(\hbar)\right]$.

Then for all $\hbar \in \mathfrak{A}$

i. $\mathfrak{K}$ *is contained in* $\mathfrak{L}$ *if and only if* $\mathcal{T}\mathbf{r}^-_{\mathfrak{K}}(\hbar) \leq \mathcal{T}\mathbf{r}^-_{\mathfrak{L}}(\hbar)$

$$\begin{gathered}\mathcal{T}\mathbf{r}^-_{\mathfrak{K}}(\hbar) \leq \mathcal{T}\mathbf{r}^-_{\mathfrak{L}}(\hbar), \mathcal{T}\mathbf{r}^+_{\mathfrak{K}}(\hbar) \leq \mathcal{T}\mathbf{r}^+_{\mathfrak{L}}(\hbar),\\ \mathcal{I}\mathbf{m}^-_{\mathfrak{K}}(\hbar) \geq \mathcal{I}\mathbf{m}^-_{\mathfrak{L}}(\hbar), \mathcal{I}\mathbf{m}^+_{\mathfrak{K}}(\hbar) \geq \mathcal{I}\mathbf{m}^+_{\mathfrak{L}}(\hbar),\\ \mathcal{F}\mathbf{a}^-_{\mathfrak{K}}(\hbar) \geq \mathcal{F}\mathbf{a}^-_{\mathfrak{L}}(\hbar), \mathcal{F}\mathbf{a}^+_{\mathfrak{K}}(\hbar) \geq \mathcal{F}\mathbf{a}^+_{\mathfrak{L}}(\hbar)\end{gathered}$$

ii. *The union of* $\mathfrak{K}$ *and* $\mathfrak{L}$ *is the* $IVFN_{set}\mathfrak{D}$, *defined by*

$$\begin{gathered}\mathfrak{D} = \mathfrak{K} \cup \mathfrak{L} = \{\langle \hbar, \mathcal{T}\mathbf{r}_{\mathfrak{D}}(\hbar), \mathcal{I}\mathbf{m}_{\mathfrak{D}}(\hbar), \mathcal{F}\mathbf{a}_{\mathfrak{D}}(\hbar)\rangle | \hbar \in \mathfrak{A}\}\text{where}\\ \mathcal{T}\mathbf{r}_{\mathfrak{D}}(\hbar) = \left[\mathcal{T}\mathbf{r}^-_{\mathfrak{D}}(\hbar), \mathcal{T}\mathbf{r}^+_{\mathfrak{D}}(\hbar)\right], \mathcal{I}\mathbf{m}_{\mathfrak{D}}(\hbar) = \left[\mathcal{I}\mathbf{m}^-_{\mathfrak{D}}(\hbar), \mathcal{I}\mathbf{m}^+_{\mathfrak{D}}(\hbar)\right], \mathcal{F}\mathbf{a}_{\mathfrak{D}}(\hbar) =\\ \left[\mathcal{F}\mathbf{a}^-_{\mathfrak{D}}(\hbar), \mathcal{F}\mathbf{a}^+_{\mathfrak{D}}(\hbar)\right]\text{and}\\ \mathcal{I}\mathbf{m}^-_{\mathfrak{D}}(\hbar) = \max\left(\mathcal{I}\mathbf{m}^-_{\mathfrak{K}}(\hbar), \mathcal{I}\mathbf{m}^-_{\mathfrak{L}}(\hbar)\right),\ \mathcal{I}\mathbf{m}^+_{\mathfrak{D}}(\hbar) = \max\left(\mathcal{I}\mathbf{m}^+_{\mathfrak{K}}(\hbar), \mathcal{I}\mathbf{m}^+_{\mathfrak{L}}(\hbar)\right),\\ \mathcal{I}\mathbf{m}^-_{\mathfrak{D}}(\hbar) = \min\left(\mathcal{I}\mathbf{m}^-_{\mathfrak{K}}(\hbar), \mathcal{I}\mathbf{m}^-_{\mathfrak{L}}(\hbar)\right),\ \mathcal{I}\mathbf{m}^+_{\mathfrak{D}}(\hbar) = \min\left(\mathcal{I}\mathbf{m}^+_{\mathfrak{K}}(\hbar), \mathcal{I}\mathbf{m}^+_{\mathfrak{L}}(\hbar)\right),\\ \mathcal{F}\mathbf{a}^-_{\mathfrak{D}}(\hbar) = \min\left(\mathcal{F}\mathbf{a}^-_{\mathfrak{K}}(\hbar), \mathcal{F}\mathbf{a}^-_{\mathfrak{L}}(\hbar)\right),\ \mathcal{F}\mathbf{a}^+_{\mathfrak{D}}(\hbar) = \min\left(\mathcal{F}\mathbf{a}^+_{\mathfrak{K}}(\hbar), \mathcal{F}\mathbf{a}^+_{\mathfrak{L}}(\hbar)\right).\end{gathered}$$

or simply we can write,

$$\begin{gathered}\mathfrak{K} \cap \mathfrak{L} = \left\{\hbar, \left[\max\left(\mathcal{T}\mathbf{r}^-_{\mathfrak{K}}(\hbar), \mathcal{T}\mathbf{r}^-_{\mathfrak{L}}(\hbar)\right), \max\left(\mathcal{T}\mathbf{r}^+_{\mathfrak{K}}(\hbar), \mathcal{T}\mathbf{r}^+_{\mathfrak{L}}(\hbar)\right)\right],\right.\\ \left[\min\left(\mathcal{I}\mathbf{m}^-_{\mathfrak{K}}(\hbar), \mathcal{I}\mathbf{m}^-_{\mathfrak{L}}(\hbar)\right), \min\left(\mathcal{I}\mathbf{m}^+_{\mathfrak{K}}(\hbar), \mathcal{I}\mathbf{m}^+_{\mathfrak{L}}(\hbar)\right)\right],\\ \left.\left[\min\left(\mathcal{F}\mathbf{a}^-_{\mathfrak{K}}(\hbar), \mathcal{F}\mathbf{a}^-_{\mathfrak{L}}(\hbar)\right), \min\left(\mathcal{F}\mathbf{a}^+_{\mathfrak{K}}(\hbar), \mathcal{F}\mathbf{a}^+_{\mathfrak{L}}(\hbar)\right)\right]\right|, \hbar \in \mathfrak{A}\}\end{gathered}$$

iii. *The intersection of* $\mathfrak{K}$ *and* $\mathfrak{L}$ *is the* $IVFN_{set}\mathfrak{D}$, *defined by*

$$\begin{gathered}\mathfrak{D} = \mathfrak{K} \cap \mathfrak{L} = \{\langle \hbar, \mathcal{T}\mathbf{r}_{\mathfrak{D}}(\hbar), \mathcal{I}\mathbf{m}_{\mathfrak{D}}(\hbar), \mathcal{F}\mathbf{a}_{\mathfrak{D}}(\hbar)\rangle | \hbar \in \mathfrak{A}\}\\ \mathcal{T}\mathbf{r}_{\mathfrak{D}}(\hbar) = \left[\mathcal{T}\mathbf{r}^-_{\mathfrak{D}}(\hbar), \mathcal{T}\mathbf{r}^+_{\mathfrak{D}}(\hbar)\right], \mathcal{I}\mathbf{m}_{\mathfrak{D}}(\hbar) = \left[\mathcal{I}\mathbf{m}^-_{\mathfrak{D}}(\hbar), \mathcal{I}\mathbf{m}^+_{\mathfrak{D}}(\hbar)\right],\\ \mathcal{F}\mathbf{a}_{\mathfrak{D}}(\hbar) = \left[\mathcal{F}\mathbf{a}^-_{\mathfrak{D}}(\hbar), \mathcal{F}\mathbf{a}^+_{\mathfrak{D}}(\hbar)\right]\text{ and}\\ \mathcal{T}\mathbf{r}^-_{\mathfrak{D}}(\hbar) = \min\left(\mathcal{T}\mathbf{r}^-_{\mathfrak{K}}(\hbar), \mathcal{T}\mathbf{r}^-_{\mathfrak{L}}(\hbar)\right),\ \mathcal{T}\mathbf{r}^+_{\mathfrak{D}}(\hbar) = \min\left(\mathcal{T}\mathbf{r}^+_{\mathfrak{K}}(\hbar), \mathcal{T}\mathbf{r}^+_{\mathfrak{L}}(\hbar)\right),\\ \mathcal{I}\mathbf{m}^-_{\mathfrak{D}}(\hbar) = \max\left(\mathcal{I}\mathbf{m}^-_{\mathfrak{K}}(\hbar), \mathcal{I}\mathbf{m}^-_{\mathfrak{L}}(\hbar)\right),\ \mathcal{I}\mathbf{m}^+_{\mathfrak{D}}(\hbar) = \max\left(\mathcal{I}\mathbf{m}^+_{\mathfrak{K}}(\hbar), \mathcal{I}\mathbf{m}^+_{\mathfrak{L}}(\hbar)\right),\\ \mathcal{F}\mathbf{a}^-_{\mathfrak{D}}(\hbar) = \max\left(\mathcal{F}\mathbf{a}^-_{\mathfrak{K}}(\hbar), \mathcal{F}\mathbf{a}^-_{\mathfrak{L}}(\hbar)\right),\ \mathcal{F}\mathbf{a}^+_{\mathfrak{D}}(\hbar) = \max\left(\mathcal{F}\mathbf{a}^+_{\mathfrak{K}}(\hbar), \mathcal{F}\mathbf{a}^+_{\mathfrak{L}}(\hbar)\right).\end{gathered}$$

or simply we can write.

$$\begin{gathered}\mathfrak{K} \cap \mathfrak{L} = \left\{\hbar, \left[\min\left(\mathcal{T}\mathbf{r}^-_{\mathfrak{K}}(\hbar), \mathcal{T}\mathbf{r}^-_{\mathfrak{L}}(\hbar)\right), \min\left(\mathcal{T}\mathbf{r}^+_{\mathfrak{K}}(\hbar), \mathcal{T}\mathbf{r}^+_{\mathfrak{L}}(\hbar)\right)\right],\right.\\ \left[\max\left(\mathcal{I}\mathbf{m}^-_{\mathfrak{K}}(\hbar), \mathcal{I}\mathbf{m}^-_{\mathfrak{L}}(\hbar)\right), \max\left(\mathcal{I}\mathbf{m}^+_{\mathfrak{K}}(\hbar), \mathcal{I}\mathbf{m}^+_{\mathfrak{L}}(\hbar)\right)\right],\\ \left.\left[\max\left(\mathcal{F}\mathbf{a}^-_{\mathfrak{K}}(\hbar), \mathcal{F}\mathbf{a}^-_{\mathfrak{L}}(\hbar)\right), \max\left(\mathcal{F}\mathbf{a}^+_{\mathfrak{K}}(\hbar), \mathcal{F}\mathbf{a}^+_{\mathfrak{L}}(\hbar)\right)\right]\right|, \hbar \in \mathfrak{A}\}\end{gathered}$$

iv. *The complement of* $\mathfrak{K}$ *is the* $IVFN_{set}\mathfrak{K}^c$, *defined by*

$$\begin{gathered}\mathfrak{K}^c = \{\langle \hbar, \mathcal{T}\mathbf{r}_{\mathfrak{K}^c}(\hbar), \mathcal{I}\mathbf{m}_{\mathfrak{K}^c}(\hbar), \mathcal{F}\mathbf{a}_{\mathfrak{K}^c}(\hbar)\rangle | \hbar \in \mathfrak{A}\}\text{where}\\ \mathcal{T}\mathbf{r}_{\mathfrak{K}^c}(\hbar) = \mathcal{F}\mathbf{a}_{\mathfrak{K}}(\hbar) = \left[\mathcal{F}\mathbf{a}^-_{\mathfrak{K}}(\hbar), \mathcal{F}\mathbf{a}^+_{\mathfrak{K}}(\hbar)\right]\\ \mathcal{I}\mathbf{m}^-_{\mathfrak{K}^c}(\hbar) = 1 - \mathcal{I}\mathbf{m}^+_{\mathfrak{K}}(\hbar), \mathcal{I}\mathbf{m}^+_{\mathfrak{K}^c}(\hbar) = 1 - \mathcal{I}\mathbf{m}^-_{\mathfrak{K}}(\hbar)\\ \mathcal{F}\mathbf{a}_{\mathfrak{K}^c}(\hbar) = \mathcal{T}\mathbf{r}_{\mathfrak{K}}(\hbar) = \left[\mathcal{T}\mathbf{r}^-_{\mathfrak{K}}(\hbar), \mathcal{T}\mathbf{r}^+_{\mathfrak{K}}(\hbar)\right]\end{gathered}$$

or simply we can write.

$$\mathfrak{K}^c = \left\{\left\langle \hbar, \left[\mathcal{F}\mathbf{a}^-_{\mathfrak{K}}(\hbar), \mathcal{F}\mathbf{a}^+_{\mathfrak{K}}(\hbar)\right], \left[1 - \mathcal{I}\mathbf{m}^+_{\mathfrak{K}}(\hbar), 1 - \mathcal{I}\mathbf{m}^-_{\mathfrak{K}}(\hbar)\right], \left[\mathcal{T}\mathbf{r}^-_{\mathfrak{K}}(\hbar), \mathcal{T}\mathbf{r}^+_{\mathfrak{K}}(\hbar)\right]\right\rangle \middle| \hbar \in \mathfrak{A}\right\}.$$

Definition 4. *The* $IVFN_{set}$ *is known as an absolute* $IVFN_{set}$, *denoted by* $1_{\mathfrak{A}}$, $\Leftrightarrow$ *its membership values are defined as*

$$\begin{gathered}\left[\mathcal{T}\mathbf{r}^-_{\mathfrak{K}}(\hbar), \mathcal{T}\mathbf{r}^+_{\mathfrak{K}}(\hbar)\right] = [1,\ 1]\ ;\\ \left[\mathcal{I}\mathbf{m}^-_{\mathfrak{K}}(\hbar),\ \mathcal{I}\mathbf{m}^+_{\mathfrak{K}}(\hbar)\right] = [0,\ 0]\\ \left[\mathcal{F}\mathbf{a}^-_{\mathfrak{K}}(\hbar), \mathcal{F}\mathbf{a}^+_{\mathfrak{K}}(\hbar)\right] = [0,\ 0].\end{gathered}$$

Definition 5. *The empty* $IVFN_{set}$ *is denoted by* $0_{\mathfrak{A}}$, *if its membership values are defined as*

$$\begin{gathered}\left[\mathcal{T}r^-_{\mathfrak{K}}(\hbar), \mathcal{T}r^+_{\mathfrak{K}}(\hbar)\right] = [0,0];\\ \left[\mathcal{I}m^-_{\mathfrak{K}}(\hbar),\ \mathcal{I}m^+_{\mathfrak{K}}(\hbar)\right] = [1,1];\\ \left[\mathcal{F}a^-_{\mathfrak{K}}(\hbar), \mathcal{F}a^+_{\mathfrak{K}}(\hbar)\right] = [0,0].\end{gathered}$$

Example 1. *Consider two* $IVFN_{set}$, *defined over* $\mathfrak{A}$ *as*

$$\mathfrak{K}=\left\{\begin{array}{c}\langle p_1,[0.85,0.90],[0.80,0.85],[0.80,0.90]\rangle,\langle p_2,[0.85,0.85],[0.80,0.80],[0.80,0.90]\rangle,\\ \langle p_3,[0.90,0.95],[0.83,0.86],[0.82,0.81]\rangle\end{array}\right\}$$
$$\mathfrak{L}=\left\{\begin{array}{c}\langle p_1,[0.80,0.90],[0.80,0.80],[0.80,0.90]\rangle,\langle p_2,[0.81,0.85],[0.82,0.82],[0.84,0.91]\rangle,\\ \langle p_3,[0.92,0.95],[0.85,0.87],[0.83,0.85]\rangle\end{array}\right\}$$

then

$$\mathfrak{K}\cap\mathfrak{L}=\left\{\begin{array}{c}\langle p_1,[0.80,0.90],[0.80,0.85],[0.80,0.90]\rangle,\langle p_2,[0.81,0.85],[0.82,0.82],[0.84,0.91]\rangle,\\ \langle p_3,[0.90,0.95],[0.85,0.87],[0.83,0.85]\rangle\end{array}\right\}$$
$$\mathfrak{K}\cup\mathfrak{L}=\left\{\begin{array}{c}\langle p_1,[0.85,0.90],[0.80,0.80],[0.80,0.90]\rangle,\langle p_2,[0.85,0.85],[0.80,0.80],[0.80,0.90]\rangle,\\ \langle p_3,[0.92,0.95],[0.83,0.86],[0.82,0.81]\rangle\end{array}\right\}$$
$$\mathfrak{K}^c=\left\{\begin{array}{c}\langle p_1,[0.80,0.90],[0.15,0.20],[0.85,0.90]\rangle,\langle p_2,[0.80,0.90],[0.20,0.20],[0.85,0.85]\rangle,\\ \langle p_3,[0.82,0.81],[0.14,0.17],[0.83,0.86]\rangle\end{array}\right\}$$

Theorem 1. *For any $IVFN_{set}$, $\mathfrak{K}$ is defined on the absolute $IVFN_{set}\mathfrak{A}$.*

i. $\left.\begin{array}{l}\mathfrak{K}\cup 0_{\mathfrak{A}}=\mathfrak{K}\\ \mathfrak{K}\cap 1_{\mathfrak{A}}=\mathfrak{K}\end{array}\right\}$*(Identity Law)*

ii. $\left.\begin{array}{l}\mathfrak{K}\cap 0_{\mathfrak{A}}=0_{\mathfrak{A}}\\ \mathfrak{K}\cup 1_{\mathfrak{A}}=1_{\mathfrak{A}}\end{array}\right\}$*(Domination Law)*

Proof.

(i) Let $\mathfrak{K}$ and $0_{\mathfrak{A}}$ be two $IVFN_{set}$ on $\mathfrak{A}$, defined by

$$\mathfrak{K}=\left\{\left\langle \hbar,\left[\boldsymbol{\mathcal{T}}\mathbf{r}_{\mathfrak{K}}^-(\hbar),\boldsymbol{\mathcal{T}}\mathbf{r}_{\mathfrak{K}}^+(\hbar)\right],\left[\boldsymbol{\mathcal{I}}\mathbf{m}_{\mathfrak{K}}^-(\hbar),\boldsymbol{\mathcal{I}}\mathbf{m}_{\mathfrak{K}}^+(\hbar)\right],\left[\boldsymbol{\mathcal{F}}\mathbf{a}_{\mathfrak{K}}^-(\hbar),\boldsymbol{\mathcal{F}}\mathbf{a}_{\mathfrak{K}}^+(\hbar)\right]\right\rangle\middle|\hbar\in\mathfrak{A}\right\}$$

$0_{\mathfrak{A}}$, is defined as follows: $0_{\mathfrak{A}}=\{\langle\hbar,[0,\ 0],[1,\ 1],[1,\ 1]\rangle|\hbar\in\mathfrak{A}\}$
So, $\mathfrak{K}\cup 0_{\mathfrak{A}}=\left\{\hbar,\left[\max\left(\boldsymbol{\mathcal{T}}\mathbf{r}_{\mathfrak{K}}^-(\hbar),0\right),\max\left(\boldsymbol{\mathcal{T}}\mathbf{r}_{\mathfrak{K}}^+(\hbar),0\right)\right],\right.$

$$\left[\min\left(\boldsymbol{\mathcal{I}}\mathbf{m}_{\mathfrak{K}}^-(\hbar),1\right),\min\left(\boldsymbol{\mathcal{I}}\mathbf{m}_{\mathfrak{K}}^+(\hbar),1\right)\right],$$
$$\left.\left[\min\left(\boldsymbol{\mathcal{F}}\mathbf{a}_{\mathfrak{K}}^-(\hbar),1\right),\min\left(\boldsymbol{\mathcal{F}}\mathbf{a}_{\mathfrak{K}}^+(\hbar),1\right)\right]\middle|\hbar\in\mathfrak{A}\right\}$$

Therefore, $\mathfrak{K}\cup 0_{\mathfrak{A}}=\left\{\left\langle\hbar,\left[\boldsymbol{\mathcal{T}}\mathbf{r}_{\mathfrak{K}}^-(\hbar),\boldsymbol{\mathcal{T}}\mathbf{r}_{\mathfrak{K}}^+(\hbar)\right],\left[\boldsymbol{\mathcal{I}}\mathbf{m}_{\mathfrak{K}}^-(\hbar),\boldsymbol{\mathcal{I}}\mathbf{m}_{\mathfrak{K}}^+(\hbar)\right],\left[\boldsymbol{\mathcal{F}}\mathbf{a}_{\mathfrak{K}}^-(\hbar),\boldsymbol{\mathcal{F}}\mathbf{a}_{\mathfrak{K}}^+(\hbar)\right]\right\rangle\middle|\ \hbar\in\mathfrak{A}\right\}$

$$\mathfrak{K}\cup 0_{\mathfrak{A}}=\mathfrak{K}$$

In a similar way, we can prove $\mathfrak{K}\cap 1_{\mathfrak{A}}=\mathfrak{K}$

(ii) Let $\mathfrak{K}$ and $0_{\mathfrak{A}}$ be two $IVFN_{set}$ on $\mathfrak{A}$, defined by

$$\mathfrak{K}=\left\{\left\langle\hbar,\left[\boldsymbol{\mathcal{T}}\mathbf{r}_{\mathfrak{K}}^-(\hbar),\boldsymbol{\mathcal{T}}\mathbf{r}_{\mathfrak{K}}^+(\hbar)\right],\left[\boldsymbol{\mathcal{I}}\mathbf{m}_{\mathfrak{K}}^-(\hbar),\ \boldsymbol{\mathcal{I}}\mathbf{m}_{\mathfrak{K}}^+(\hbar)\right],\left[\left[\boldsymbol{\mathcal{F}}\mathbf{a}_{\mathfrak{K}}^-(\hbar),\boldsymbol{\mathcal{F}}\mathbf{a}_{\mathfrak{K}}^+(\hbar)\right\rangle\middle|\hbar\in\mathfrak{A}\right\}\right.\right.$$

$0_{\mathfrak{A}}$, is defined as follows: $0_{\mathfrak{A}}=\{\langle\hbar,[0,\ 0],[1,\ 1],[1,\ 1]\rangle|\hbar\in\mathfrak{A}\}$
So, $\mathfrak{K}\cap 0_{\mathfrak{A}}=\left\{\hbar,\left[\min\left(\boldsymbol{\mathcal{T}}\mathbf{r}_{\mathfrak{K}}^-(\hbar),0\right),\min\left(\boldsymbol{\mathcal{T}}\mathbf{r}_{\mathfrak{K}}^+(\hbar),0\right)\right],\right.$

$$\left[\max\left(\boldsymbol{\mathcal{I}}\mathbf{m}_{\mathfrak{K}}^-(\hbar),1\right),\max\left(\boldsymbol{\mathcal{I}}\mathbf{m}_{\mathfrak{K}}^+(\hbar),1\right)\right],$$
$$\left.\left[\max\left(\boldsymbol{\mathcal{F}}\mathbf{a}_{\mathfrak{K}}^-(\hbar),1\right),\max\left(\boldsymbol{\mathcal{F}}\mathbf{a}_{\mathfrak{K}}^+(\hbar),1\right)\right]\middle|\hbar\in\mathfrak{A}\right\}$$

Therefore, $\mathfrak{K}\cap 0_{\mathfrak{A}}=\{\langle\hbar,[0,\ 0],[1,\ 1],[1,\ 1]\rangle|\hbar\in\mathfrak{A}\}$

$$\mathfrak{K}\cap 0_{\mathfrak{A}}=0_{\mathfrak{A}}$$

In the similar way, we can prove $\mathfrak{K}\cup 1_{\mathfrak{A}}=1_{\mathfrak{A}}$

□

Definition 6. *Suppose*

$$\mathfrak{K} = \left\{ \left\langle \left[\mathcal{T}\mathbf{r}^{-}_{\mathfrak{K}}(\hbar), \mathcal{T}\mathbf{r}^{+}_{\mathfrak{K}}(\hbar)\right], \left[\mathcal{I}\mathbf{m}^{-}_{\mathfrak{K}}(\hbar), \mathcal{I}\mathbf{m}^{+}_{\mathfrak{K}}(\hbar)\right], \left[\left[\mathcal{F}\mathbf{a}^{-}_{\mathfrak{K}}(\hbar), \mathcal{F}\mathbf{a}^{+}_{\mathfrak{K}}(\hbar)\right]\right\rangle : \hbar \in \mathfrak{A} \right\}$$

and

$$\mathfrak{L} = \left\{ \left\langle \left[\mathcal{T}\mathbf{r}^{-}_{\mathfrak{L}}(\hbar), \mathcal{T}\mathbf{r}^{+}_{\mathfrak{L}}(\hbar)\right], \left[\mathcal{I}\mathbf{m}^{-}_{\mathfrak{L}}(\hbar), \mathcal{I}\mathbf{m}^{+}_{\mathfrak{L}}(\hbar)\right], \left[\mathcal{F}\mathbf{a}^{-}_{\mathfrak{L}}(\hbar), \mathcal{F}\mathbf{a}^{+}_{\mathfrak{L}}(\hbar)\right]\right\rangle : \hbar \in \mathfrak{A} \right\}$$

be two $IVFN_{set}$, then

$$A_{\mathrm{IVFN_{set}}} + B_{\mathrm{IVFN_{set}}} = \left\{ \left\langle \begin{array}{c} \hbar_1 + \hbar_2, \left[\mathcal{T}\mathbf{r}^{-}_{\mathfrak{K}}(\hbar) + \mathcal{T}\mathbf{r}^{-}_{\mathfrak{L}}(\hbar) - \mathcal{T}\mathbf{r}^{-}_{\mathfrak{K}}(\hbar)\mathcal{T}\mathbf{r}^{-}_{\mathfrak{L}}(\hbar), \mathcal{T}\mathbf{r}^{+}_{\mathfrak{K}}(\hbar) + \mathcal{T}\mathbf{r}^{-}_{\mathfrak{L}}(\hbar) - \mathcal{T}\mathbf{r}^{+}_{\mathfrak{K}}(\hbar)\mathcal{T}\mathbf{r}^{+}_{\mathfrak{L}}(\hbar)\right], \\ \left[\mathcal{I}\mathbf{m}^{-}_{\mathfrak{K}}(\hbar)\mathcal{I}\mathbf{m}^{-}_{\mathfrak{L}}(\hbar), \mathcal{I}\mathbf{m}^{+}_{\mathfrak{K}}(\hbar)\mathcal{I}\mathbf{m}^{+}_{\mathfrak{L}}(\hbar)\right], \\ \left[\mathcal{F}\mathbf{a}^{-}_{\mathfrak{K}}(\hbar)\mathcal{F}\mathbf{a}^{-}_{\mathfrak{L}}(\hbar), \mathcal{F}\mathbf{a}^{+}_{\mathfrak{K}}(\hbar)\mathcal{F}\mathbf{a}^{+}_{\mathfrak{L}}(\hbar)\right] \end{array} \right\rangle \right\}, \hbar \in \mathfrak{A}$$

$$A_{\mathrm{IVFN_{set}}} \cdot B_{\mathrm{IVFN_{set}}} = \left\{ \left\langle \begin{array}{c} \hbar_1 \hbar_2, \left[\mathcal{T}\mathbf{r}^{-}_{\mathfrak{K}}(\hbar)\mathcal{T}\mathbf{r}^{-}_{\mathfrak{L}}(\hbar), \mathcal{T}\mathbf{r}^{+}_{\mathfrak{K}}(\hbar)\mathcal{T}\mathbf{r}^{+}_{\mathfrak{L}}(\hbar)\right], \\ \left[\mathcal{I}\mathbf{m}^{-}_{\mathfrak{K}}(\hbar) + \mathcal{I}\mathbf{m}^{-}_{\mathfrak{L}}(\hbar) - \mathcal{I}\mathbf{m}^{-}_{\mathfrak{K}}(\hbar)\mathcal{I}\mathbf{m}^{-}_{\mathfrak{L}}(\hbar), \mathcal{I}\mathbf{m}^{+}_{\mathfrak{K}}(\hbar) + \mathcal{I}\mathbf{m}^{+}_{\mathfrak{L}}(\hbar) - \mathcal{I}\mathbf{m}^{+}_{\mathfrak{K}}(\hbar)\mathcal{I}\mathbf{m}^{+}_{\mathfrak{L}}(\hbar)\right], \\ \left[\mathcal{F}\mathbf{a}^{-}_{\mathfrak{K}}(\hbar) + \mathcal{F}\mathbf{a}^{-}_{\mathfrak{L}}(\hbar) - \mathcal{F}\mathbf{a}^{-}_{\mathfrak{K}}(\hbar)\mathcal{F}\mathbf{a}^{-}_{\mathfrak{L}}(\hbar), \mathcal{F}\mathbf{a}^{+}_{\mathfrak{K}}(\hbar) + \mathcal{F}\mathbf{a}^{+}_{\mathfrak{L}}(\hbar) - \mathcal{F}\mathbf{a}^{+}_{\mathfrak{K}}(\hbar)\mathcal{F}\mathbf{a}^{+}_{\mathfrak{L}}(\hbar)\right] \end{array} \right\rangle \right\}, \hbar \in \mathfrak{A}$$

Definition 7. *Let* $\alpha = \left(\left[\mathcal{T}r^{A-}_{\alpha}, \mathcal{T}r^{A+}_{\alpha}\right], \left[\mathcal{I}m^{A-}_{\alpha}, \mathcal{I}m^{A+}_{\alpha}\right], \left[\mathcal{F}a^{A-}_{\alpha}, \mathcal{F}a^{A+}_{\alpha}\right]\right)$, $\alpha_1 = \left(\left[\mathcal{T}r^{A-}_{\alpha_1}, \mathcal{T}^{A+}_{\alpha_1}\right], \left[\mathcal{I}m^{A-}_{\alpha_1}, \mathcal{I}^{A+}_{\alpha_1}\right], \left[\mathcal{F}a^{A-}_{\alpha_1}, \mathcal{F}a^{A+}_{\alpha_1}\right]\right)$ *and* $\alpha_2 = \left(\left[\mathcal{T}r^{A-}_{\alpha_2}, \mathcal{T}r^{A+}_{\alpha_2}\right], \left[\mathcal{I}m^{A-}_{\alpha_2}, \mathcal{I}m^{A+}_{\alpha_2}\right], \left[\mathcal{F}a^{A-}_{\alpha_2}, \mathcal{F}a^{A+}_{\alpha_2}\right]\right)$ *be three $IVFN_{number}$. Then*

(i) $\alpha_1 = \alpha_2$ *if and only if* $\mathcal{T}\mathbf{r}^{A-}_{\alpha_1} = \mathcal{T}\mathbf{r}^{A-}_{\alpha_2}, \mathcal{I}\mathbf{m}^{A-}_{\alpha_1} = \mathcal{I}\mathbf{m}^{A-}_{\alpha_2}, \mathcal{F}\mathbf{a}^{A-}_{\alpha_1} = \mathcal{F}\mathbf{a}^{A-}_{\alpha_2}$

(ii) $\alpha_1 \prec \alpha_2$ *if and only if*

(iii) $\mathcal{T}\mathbf{r}^{A-}_{\alpha_1} \leq \mathcal{T}\mathbf{r}^{A-}_{\alpha_2}, \mathcal{T}\mathbf{r}^{A+}_{\alpha_1} \leq \mathcal{T}\mathbf{r}^{A+}_{\alpha_2}; \mathcal{I}\mathbf{m}_{\alpha_1}^{A-} \geq \mathcal{I}\mathbf{m}^{A-}_{\alpha_2}, \mathcal{I}\mathbf{m}^{A-}_{\alpha_1} \geq \mathcal{I}\mathbf{m}^{A-}_{\alpha_2}; \mathcal{F}\mathbf{a}_{\alpha_1}^{A-} \geq \mathcal{F}\mathbf{a}^{A-}_{\alpha_2}; \mathcal{F}\mathbf{a}^{A+}_{\alpha_2} \geq \mathcal{F}\mathbf{a}^{A+}_{\alpha_2}$

(iv) $\alpha_1 \bigoplus \alpha_2 =$
$$\left[\begin{array}{c} \sqrt[3]{\left(\mathcal{T}\mathbf{r}^{A-}_{\alpha_1}\right)^3 + \left(\mathcal{T}\mathbf{r}^{A-}_{\alpha_2}\right)^3 - \left(\mathcal{T}\mathbf{r}^{A-}_{\alpha_1}\right)^3 . \left(\mathcal{T}\mathbf{r}^{A-}_{\alpha_2}\right)^3}, \sqrt[3]{\left(\mathcal{T}\mathbf{r}^{A+}_{\alpha_1}\right)^3 + \left(\mathcal{T}\mathbf{r}^{A+}_{\alpha_2}\right)^3 - \left(\mathcal{T}\mathbf{r}^{A+}_{\alpha_1}\right)^3 . \left(\mathcal{T}\mathbf{r}^{A+}_{\alpha_2}\right)^3}, \\ \left[\mathcal{I}\mathbf{m}^{A-}_{\alpha_1}\mathcal{I}\mathbf{m}^{A-}_{\alpha_2}, \mathcal{I}\mathbf{m}^{A+}_{\alpha_1}\mathcal{I}\mathbf{m}^{A+}_{\alpha_2}\right], \left[\mathcal{F}\mathbf{a}^{A-}_{\alpha_1}\mathcal{F}\mathbf{a}^{A-}_{\alpha_2}, \mathcal{F}\mathbf{a}^{A+}_{\alpha_1}\mathcal{F}\mathbf{a}^{A+}_{\alpha_2}\right] \end{array}\right]$$

(v) $\alpha_1 \bigotimes \alpha_2 =$
$$\left[\begin{array}{c} \left[\mathcal{T}\mathbf{r}^{A-}_{\alpha_1} . \mathcal{T}\mathbf{r}^{A-}_{\alpha_2}, \mathcal{T}\mathbf{r}^{A+}_{\alpha_1} . \mathcal{T}\mathbf{r}^{A+}_{\alpha_2}\right] \\ \left[\sqrt[3]{\left(\mathcal{I}\mathbf{m}^{A-}_{\alpha_1}\right)^3 + \left(\mathcal{I}\mathbf{m}^{A-}_{\alpha_2}\right)^3 - \left(\mathcal{I}\mathbf{m}^{A-}_{\alpha_1}\right)^3 . \left(\mathcal{I}\mathbf{m}^{A-}_{\alpha_2}\right)^3}, \sqrt[3]{\left(\mathcal{I}\mathbf{m}^{A+}_{\alpha_1}\right)^3 + \left(\mathcal{I}\mathbf{m}^{A+}_{\alpha_2}\right)^3 - \left(\mathcal{I}\mathbf{m}^{A+}_{\alpha_1}\right)^3 . \left(\mathcal{I}\mathbf{m}^{A+}_{\alpha_2}\right)^3}\right] \\ \left[\sqrt[3]{\left(\mathcal{F}\mathbf{a}^{A-}_{\alpha_1}\right)^3 + \left(\mathcal{F}\mathbf{a}^{A-}_{\alpha_2}\right)^3 - \left(\mathcal{F}\mathbf{a}^{A-}_{\alpha_1}\right)^3 . \left(\mathcal{F}\mathbf{a}^{A-}_{\alpha_2}\right)^3}, \sqrt[3]{\left(\mathcal{F}\mathbf{a}^{A+}_{\alpha_1}\right)^3 + \left(\mathcal{F}\mathbf{a}^{A+}_{\alpha_2}\right)^3 - \left(\mathcal{F}\mathbf{a}^{A+}_{\alpha_1}\right)^3 . \left(\mathcal{F}\mathbf{a}^{A+}_{\alpha_2}\right)^3}\right] \end{array}\right]$$

(vi) $k\alpha =$
$$\left(\left[\sqrt[3]{1 - \left(1 - \left(\left(\mathcal{T}\mathbf{r}^{A-}_{\alpha}\right)^3\right)^k\right)}, \sqrt[3]{1 - \left(1 - \left(\left(\mathcal{T}\mathbf{r}^{A+}_{\alpha}\right)^3\right)^k\right)}\right], \left[\left(\mathcal{I}\mathbf{m}^{A-}_{\alpha}\right)^k, \left(\mathcal{I}\mathbf{m}^{A+}_{\alpha}\right)^k\right], \left[\left(\mathcal{F}\mathbf{a}^{A-}_{\alpha}\right)^{\boldsymbol{k}}, \left(\mathcal{F}\mathbf{a}^{A+}_{\alpha}\right)^{\boldsymbol{k}}\right]\right)$$

(vii) $\alpha^k =$
$$\left(\left[\left(\mathcal{T}\mathbf{r}^{A-}_{\alpha}\right)^k, \left(\mathcal{T}\mathbf{r}^{A+}_{\alpha}\right)^k\right], \left[\sqrt[3]{1 - \left(1 - \left(\left(\mathcal{I}\mathbf{m}^{A-}_{\alpha}\right)^3\right)^k\right)}, \sqrt[3]{1 - \left(1 - \left(\left(\mathcal{I}\mathbf{m}^{A+}_{\alpha}\right)^3\right)^k\right)}\right], \left[\sqrt[3]{1 - \left(1 - \left(\left(\mathcal{F}\mathbf{a}^{A-}_{\alpha_1}\right)^3\right)^k\right)}, \sqrt[3]{1 - \left(1 - \left(\left(\mathcal{F}\mathbf{a}^{A-}_{\alpha_2}\right)^3\right)^k\right)}\right]\right).$$

Remark 2.

i. *If* $\alpha = ([a,b],[c,d],[e,f]) = ([1,1],[0,0],[0,0])$ *and* $k > 0$ *then,*

$$k\alpha = \left(\left[\sqrt[3]{1 - \left(1 - (a^3)^k\right)}, \sqrt[3]{1 - \left(1 - (b^3)^k\right)}\right], \left[(c)^k, (d)^k\right], \left[(e)^k, (f)^k\right]\right) = ([1,1],[0,0],[0,0])\alpha^k$$
$$= \left(\left[(a)^k, (b)^k\right], \left[\sqrt[3]{1 - \left(1 - \left(c\right)^3\right)^k}, \sqrt[3]{1 - \left(1 - \left(d\right)^3\right)^k}\right], \left[\sqrt[3]{1 - \left(1 - \left((e)^3\right)^k\right)}, \sqrt[3]{1 - \left(1 - \left((f)^3\right)^k\right)}\right]\right)$$
$$= ([1,1],[0,0],[0,0])$$

ii. *If* $\alpha = ([a,b],[c,d],[e,f]) = ([0,0],[1,1],[1,1])$ *and* $k > 0$ *then,*

$$k\alpha = \left(\left[\sqrt[3]{1-\left(1-(a^3)^k\right)}, \sqrt[3]{1-\left(1-(b^3)^k\right)}\right], \left[(c)^k, (d)^k\right], \left[(e)^k, (f)^k\right]\right) = ([0,0],[1,1],[1,1])\alpha^k$$
$$= \left(\left[(a)^k, (b)^k\right], \left[\sqrt[3]{1-\left(1-\left(c\right)^3\right)^k\Big)}, \sqrt[3]{1-\left(1-\left(d\right)^3\right)^k\Big)}\right], \left[\sqrt[3]{1-\left(1-\left((e)^3\right)^k\right)}, \sqrt[3]{1-\left(1-\left((f)^3\right)^k\right)}\right]\right)$$
$$= ([0,0],[1,1],[1,1])$$

iii. *If* $k = 1$ *then* $k\alpha = \alpha; \alpha^k = \alpha$

Definition 8. *Consider* $\alpha_j = \left(\left[\mathcal{T}r_{\alpha_j}^{A-}, \mathcal{T}r_{\alpha_j}^{A+}\right], \left[\mathcal{I}m_{\alpha_j}^{A-}, \mathcal{I}m_{\alpha_j}^{A+}\right], \left[\mathcal{F}a_{\alpha_j}^{A-}, \mathcal{F}a_{\alpha_j}^{A+}\right]\right)$ *is a set of the* $IVFN_{number}$ *where* $j = 1, 2, \ldots, r$. *Then, the* $IVFNW_{average}$ *operator is as follows:*

$IVFNWA(\alpha_1, \alpha_2, ..\alpha_r) = \ominus_{j=1}^{r} w_j \alpha_j$ where w_j is weight value with $w_j \in [0,1]$ and $\sum_{j=1}^{r} w_j = 1$

$$IVFNWA(\alpha_1, \alpha_2, ..\alpha_r) = \left[\left(1-\prod_{j=1}^{r}\left(1-\left(\mathcal{T}\mathbf{r}_{\alpha_j}^{A-}\right)^3\right)^{w_j}\right)^{\frac{1}{3}}, \left(1-\prod_{j=1}^{r}\left(1-\left(\mathcal{T}\mathbf{r}_{\alpha_j}^{A+}\right)^3\right)^{w_j}\right)^{\frac{1}{3}}\right],$$
$$\left[\prod_{j=1}^{r}\left(\mathcal{I}\mathbf{m}_{\alpha_j}^{A-}\right)^{w_j}, \prod_{j=1}^{r}\left(\mathcal{I}\mathbf{m}_{\alpha_j}^{A+}\right)^{w_j}\right], \left[\prod_{j=1}^{r}\left(\mathcal{F}\mathbf{a}_{\alpha_j}^{A-}\right)^{w_j}, \prod_{j=1}^{r}\left(\mathcal{F}\mathbf{a}_{\alpha_j}^{A+}\right)^{w_j}\right]$$

Definition 9. *Consider* $\alpha_j = \left(\left[\mathcal{T}r_{\alpha_j}^{A-}, \mathcal{T}r_{\alpha_j}^{A+}\right], \left[\mathcal{I}m_{\alpha_j}^{A-}, \mathcal{I}m_{\alpha_j}^{A+}\right], \left[\mathcal{F}a_{\alpha_j}^{A-}, \mathcal{F}a_{\alpha_j}^{A+}\right]\right)$ *is a set of the* $IVFN_{number}$ *where* $j = 1, 2, \ldots, r$. *Then, the* $IVFNW_{graph}$ *operator is as follows:* $IVFNWG(\alpha_1, \alpha_2, ..\alpha_r) = \ominus_{j=1}^{r} \alpha_j^{w_j}$ *where* w_j *is weight value with* $w_j \in [0,1]$ *and* $\sum_{j=1}^{r} w_j = 1$

$$IVFNW_{graph}(\alpha_1, \alpha_2, ..\alpha_r)$$
$$= \left(\left[\prod_{j=1}^{r}\left(\mathcal{T}\mathbf{r}_{\alpha_j}^{A-}\right)^{w_j}, \prod_{j=1}^{r}\left(\mathcal{T}\mathbf{r}_{\alpha_j}^{A+}\right)^{w_j}\right], \left[\left(1-\prod_{j=1}^{r}\left(1-\left(\mathcal{I}\mathbf{m}_{\alpha_j}^{A-}\right)^3\right)^{w_j}\right)^{\frac{1}{3}}, \left(1-\prod_{j=1}^{r}\left(1-\left(\mathcal{I}\mathbf{m}_{\alpha_j}^{A+}\right)^3\right)^{w_j}\right)^{\frac{1}{3}}\right],\right.$$
$$\left.\left[\left(1-\prod_{j=1}^{r}\left(1-\left(\mathcal{F}\mathbf{a}_{\alpha_j}^{A-}\right)^3\right)^{w_j}\right)^{\frac{1}{3}}, \left(1-\prod_{j=1}^{r}\left(1-\left(\mathcal{F}\mathbf{a}_{\alpha_j}^{A+}\right)^3\right)^{w_j}\right)^{\frac{1}{3}}\right]\right)$$

4. Score and Accuracy Function for the *$IVFN_{number}$*

Finding the solutions to Multi-Criteria Decision-Making (MCDM) problems in an uncertainty situation is a challenging task in today's world. In real-time situations, the membership values of $\mathcal{T}r$, $\mathcal{F}a$ and $\mathcal{I}n$ for a certain problem cannot be an exact value but are defined by possible interval values. So, researchers introduced the IVF_{set}, $IVIF_{set}$ and IVN_{set}. There are many studies available in the literature about grouping operators and determination methods in Table 2. In the Decision-Making (DM) process, one can find the best alternative among a set of feasible ones by using MCDM techniques. HWang and Yoon [26] introduced TOPSIS, which is another well-known MCDM approach to finding the best alternative. To date, the IF_{set}, $IVIF_{set}$, and IVN_{set} are widely used in DM problems. Additionally, the SN_{set} and IVN_{set} are extensions of the N_{set}.

Singh et al. [24] defined score and accuracy functions using N_{set} to solve problems in MCDM for ranking the SVN_{set} and IVN_{set}.

Table 2. Literature Survey.

Authors	Topic	Year
Nayagam et al. [27,28]	Ranking method on the $IVIF_{set}$.	2007 and 2008
Xu, Liu, and Xie [29,30]	Weighted SF and AF to rank the $IVIF_{set.}$	2007 and 2009
Ye [31]	Novel AF for ranking the different alternatives under the IF_{set} and $IVIF_{set}$.	2009
Yager [6]	Aggregation operators in the Pythagorean environment.	2013
Ye [32]	Correlation coefficient under SVN_{set}.	2013
Yager [7]	PF_{set}.	2014
Garg [9]	AF for the $IVPF_{set}$.	2016
Garg [25]	$IPFW_{average}$ and $IPFW_{graph}$ operators in the DM problem in the $IVPF_{set}$ environment.	2016
Garg [25]	Correlation coefficient between the PF_{set}.	2016

Definition 10. *Score functions of the $SVN_{set.}$ Let $A = \left\{\left\langle \hbar,\ \left[\mathcal{T}r_A^-(\hbar),\ \mathcal{T}r_A^+(\hbar)\right], \left[\mathcal{I}m_A^-(\hbar),\ \mathcal{I}m_A^+(\hbar)\right],\ \left[\mathcal{F}a_A^-(\hbar),\ \mathcal{F}a_A^+(\hbar)\right]\right\rangle \middle|, \hbar \in \mathfrak{A}\right\}$. That is, $\tilde{A} = ([a,\ b], [c,\ d], [e,\ f])$ be an IN_{number}. The Score function (SF) of the SVN_{set} is interpreted as $S(\alpha) = \frac{2+a+b-2c-2d-e-f}{4}$, where $S(\alpha) \in\ [0,1]$ (Şahin and Nancy [33–35])*

$$S(\alpha) = \frac{4 + (a+b-2c-2d-e-f)(4-a-b-e-f)}{8}$$

where $S(\alpha) \in\ [0,1]$ (Singh et al. [2])

$$S(\alpha) = (2+a+b-2c-2d-e-f)(2(4-a-b-c-d)),$$

where $S(\alpha) \in\ [0,1]$ and $a+b+c+d \neq\ 4$ as $0 \leq\ a \leq\ b\ \leq\ 1,\ 0 \leq\ c \leq\ d \leq\ 1.$

$$H(\alpha) = \frac{(a+b-d(1-b)-c(1-a)-f\ (1-c)-e(1-d)}{2},$$

$H(\alpha) \in\ [0,1]$ (Sahin [34]).

Definition 11. *An Accuracy functions of the $SVN_{set.}$ Let $A = \{\langle \hbar,\ \mathcal{T}r_A(\hbar),\ \mathcal{I}m_A(\hbar),\ \mathcal{F}a_A(\hbar)\rangle | \hbar \in \mathfrak{A}\}$ be the SVN_{set}. For convenience, the $N_{set}A = \langle\ a,b,c\ \rangle$, (ahin and Nancy* [34,35] is defined as $S(\alpha) = \frac{1+a-2b-c}{2},\ S(\alpha) \in\ [0,1]$

$$S(\alpha) = \frac{1+(a-2b-c)(2-a-c)}{2},\ S(\alpha) \in\ [0,1]$$

The Accuracy function (AF) of $SVN_{set}f$ (Nancy and Şahin [34,35]) is interpreted as

$$H(\alpha) = a - b(1-a) - c(1-b), H(\alpha) \in\ [0,1]$$
$$H(\alpha) = a - 2b - c,\ H(\alpha) \in\ [0,1].$$

Definition 12. *Score and accuracy functions of the $IVPF_{set}$ and $IVPF_{number}$. Score function of the* $\boldsymbol{PF_{set}}$ *A on S is given by Zhang et al. [21], who introduced the (SF) as $S(\alpha) = (\mathcal{T}r_A)^2 - \mathcal{F}a_A$, where $\alpha = (\mathcal{T}r_A, \mathcal{F}a_A)$ and $S(\alpha) \in\ [-1,\ 1]$. The AF is $(\alpha) = (\mathcal{T}r_A)^2 + (\mathcal{F}a_A)^2$, where $H(\alpha) \in\ [0,1]$.*

Score and Accuracy functions of the $IVPF_{set}$

$$S(\alpha) = \langle [a,b],\ [c,d]\rangle \text{ where} [a,b] \subseteq\ [0,\ 1],\ [c,d] \subseteq\ [0,1] \text{ and } b^2 + d^2 \leq\ 1.$$

The score function of the $IVPF_{number}$ is $S(\alpha) = \frac{a^2+b^2-c^2-d^2}{2}$, *where $S(\alpha) \in\ [-1,\ 1]$*

The accuracy function for the $IVPF_{number}$ *is* $(\alpha) = \frac{a^2+b^2+c^2+d^2}{2}$, *where* $H(\alpha) \in [0,\ 1]$. *Garg [31] observed that the above SF and AF for the* $IVPF_{number}$ *are suitable for certain cases; for example,* $\alpha_1 = \langle\ [0,\ 0.5],\ [0.1, 0.7] \rangle$ *and* $\alpha_2 = \langle\ [0.3,\ 0.4],\ [0.5, 0.5]\ \rangle$ *are the two* $IVPF_{set}$, *then we obtain,* $S(\alpha_1) = S(\alpha_2) = -0.1250$ *and* $H(\alpha_1)) = H(\alpha_2) = 0.3750$. *Hence, he proposed an improved score function as follows:*

$$L(\alpha) = \frac{\left(a^2 - c^2\right)\left(1 + \left(\sqrt{(1 - a^2 - c^2)}\right)\right) + \left(b^2 - d^2\right)\left(1 + \left(\sqrt{(1 - b^2 - d^2)}\right)\right)}{2},$$

where $L(\alpha) \in [-1, 1]$

Based on the improved score function, he gave the following comparison law for the DM process by the $IVPF_{number}$

if $L(\alpha) < L\ (\beta)$, *then* $\alpha < \beta$, $L(\alpha) > L\ (\beta)$, *then* $\alpha > \beta$, $L(\alpha) = L(\beta)$, *then* $\alpha = \beta$. *He also verified this with the above two examples,*

$$\begin{gathered}\mathrm{L}(\alpha_1) = -0.1912 \text{ and } \mathrm{L}(\alpha_2) = -0.2246, \text{ the alternative } \alpha_1 \text{ is better than } \alpha_2 \text{ and} \\ L(\alpha_1) = -0.3368 and L(\alpha_2) = -0.3233,\ the\ alternative\ \alpha_2\ is\ better\ than\ \alpha_1.\end{gathered}$$

Definition 13. *Score and accuracy functions of the* FF_{set} *and* $IVFF_{number}$. *Senapati and Yager [10] proposed the* FF_{set} *in 2019. They have also compared it to other kinds of* F_{set}. *Complement operator and set of operations for the* FF_{set} *were found. They defined SF and AF for the* FF_{set} *ranking and applied it to the DM problem. Score function of the* FF_{set} *is* $S(\alpha) = (\boldsymbol{\mathcal{T}r}_A)^3 - (\boldsymbol{\mathcal{F}a}_A)^3$ *where* $\alpha = (\boldsymbol{\mathcal{T}r}_A, \boldsymbol{\mathcal{F}a}_A)$ *and* $S(\alpha) \in [-1, 1]$. *The accuracy function of the* FF_{set} *is* $H(\alpha) = (\boldsymbol{\mathcal{T}r}_A)^3 + (\boldsymbol{\mathcal{F}a}_A)^3$ *where* $H(\alpha) \in [0, 1]$. *Senapati and Yager [10] explained the SF and values lie between [−1,1]. Later, Laxminarayan Sahoo [36] observed that* $SF(F) \in [-1,\ 1]$ *and the function are positive when* $SF(F) \in [0,\ 1]$ *and negative when* $SF(F) \in [-1,\ 0)$. *To score functions when score values lie in the interval between 0 and 1, he has also introduced the following formulae.*

$$\begin{gathered}(\text{Type } 1)\ S_{1F}(\widetilde{F}) = \frac{1+\mathcal{T}\mathbf{r}_A^3 - \mathcal{F}\mathbf{a}_A^3}{2} \\ (\text{Type } 2)\ S_{2F}(\widetilde{F}) = \frac{1+2\mathcal{T}\mathbf{r}_A^3 - \mathcal{F}\mathbf{a}_A^3}{3} \\ (\text{Type } 3)\ S_{3F}(\widetilde{F}) = \frac{\left(1+\mathcal{T}\mathbf{r}_A^3 - \mathcal{F}\mathbf{a}_A^3\right)\left(|\mathcal{T}\mathbf{r}_\mathbf{A} - \mathcal{F}\mathbf{a}_A|\right)}{2}\end{gathered}$$

Rani et al. [15] introduced the following:

The score function of the $IVFF_{number}$ $\lambda = \langle [a, b],\ [c, d] \rangle$ *where* $[a, b] \subseteq [0,\ 1]$, $[c, d] \subseteq [0, 1]$ *and* $b^3 + d^3 \le 1$. $S(\alpha) = \frac{a^3+b^3-c^3-d^3}{2}$ *where* $S(\alpha) \in [-1,\ 1]$

The accuracy function for the $IVFF_{number}$ *is* $H(\alpha) = \frac{a^3+b^3+c^3+d^3}{2}$ *where* $H(\alpha) \in [0,\ 1]$

Jeevaraj [16] introduced a new score function for comparing such types of $IVFF_{number}$, *as follows:*

$$S(\alpha) = \frac{-a^3 + b^3 + c^3 - d^3}{2} \text{ where } S(\alpha) \in [-0.5,\ 0.5]$$

The accuracy function for the $IVFF_{number}$ *is* $H(\alpha) = \frac{-a^3+b^3-c^3+d^3}{2}$ *where* $H(\alpha) \in$ [- 0.5, 0.5]

Rani et al. [15] introduced a new score function for comparing such types of $IVFF_{number}$, *as follows:*

$$S(\alpha) = \frac{\left(a^3 - c^3\right)\left(1 + \sqrt{(1 - a^3 - c^3)}\right) + \left(b^3 - d^3\right)\left(1 + \sqrt{(1 - b^3 - d^3)}\right)}{2}, where\ S(\alpha) \in [-1,\ 1].$$

Definition 14. *Proposed Score Functions of the* ***IVFN$_{set}$(IVFN$_{set}$)}*** $\langle [\mathcal{T}r_A^-(\hbar), \mathcal{T}r_A^+(\hbar)], [\mathcal{I}m_A^-(\hbar), \mathcal{I}m_A^+(\hbar)], [\mathcal{F}a_A^-(\hbar), \mathcal{F}a_A^+(\hbar)] \rangle = \alpha = \langle [a,b], [c,d], [e, f] \rangle$ *where* $[a,b] \subseteq [0, 1], [c,d] \subseteq [0, 1], [e,f] \subseteq [0, 1]$ *and* $b^3 + d^3 + f^3 \leq 1$.

The score functions of the $IVFN_{set}$

$$S_1(\alpha) = \frac{\left(a^3+b^3-c^3-d^3-e^3-f^3\right)}{2} \text{ where } S_1(\alpha) \in [-1, 1]$$
$$S_2(\alpha) = \frac{\left(a^3+b^3+c^3+d^3+e^3+f^3\right)}{2} \text{ where } S_2(\alpha) \in [0, 1]$$
$$S_3(\alpha) = \frac{4+\left(a^3+b^3-2c^3-2d^3-e^3-f^3\right)\left(4-a^3-b^3-c^3-d^3-e^3-f^3\right)}{8} \text{ where } S_3(\alpha) \in [0, 1]$$

For maximum property, $\langle [0,0], [0,0], [1, 1] \rangle$ *and minimum property,* $\langle [1, 1], [0, 0], [0, 0.] \rangle$

See in Table 3.

Table 3. Values of score functions for different membership values.

a	**b**	**c**	**d**	**e**	**f**	S_1	S_2	S_3
0.8	0.85	0.85	0.8	0.9	0.92	−0.753833	0.46999225	0.420965198
0.8	0.85	0.85	0.85	0.9	0.91	−0.792348	0.47961825	0.442667238
0.8	0.8	0.85	0.85	0.9	0.95	−0.8953125	0.479828125	0.439103631
0.8	0.85	0.82	0.85	0.9	0.91	−0.7609695	0.471773625	0.4243537
0.8	0.85	0.85	0.85	0.85	0.91	−0.7349105	0.465258875	0.406265978
0.85	0.9	0.8	0.8	0.8	0.85	−0.4545625	0.449421875	0.397060871

5. Applications of Interval-Valued Fermatean Neutrosophic Numbers

MCDM techniques are used to solve real-world problems in the context of uncertainty. There are two famous methods that help determine the solution to MCDM problems. The Analytical Hierarchy Process (AHP) is one of these two methods that can be used to analyze such problems by branching techniques to identify the best solution through the weight of the criterion. TOPSIS is another of the most popular MCDM models that helps select the best solutions. But in the AHP model, the number of criteria does not give clear information, whereas TOPSIS determines the ranking based on several criteria. In this technique, ideal values are either positive or negative based on the shortest and farthest distances.

In this section, we study the lecturer evaluation along with the $IVFN_{set}$. This study presents a ranking of the six different lecturers who work at one of the leading institutions in Tamil Nadu based on weighted performance evaluation criteria.

Anh Duc Do et al. [37] divided the criteria for evaluating the efficiency and talent of lecturers in an educational institution into four main groups: self-evaluation, manager valuation, peer evaluation, and student-based evaluation (Wu et al. [38]), as shown in the below Figure 3.

It is noted that the above-listed criteria may differ with respect to the infrastructure, level of students, salary given to the faculty, and workload of each institution. So, we have modified the above list of criteria and sub-criteria. We follow the following criteria structure for the lecturer evaluation in Figure 4 and Table 4:

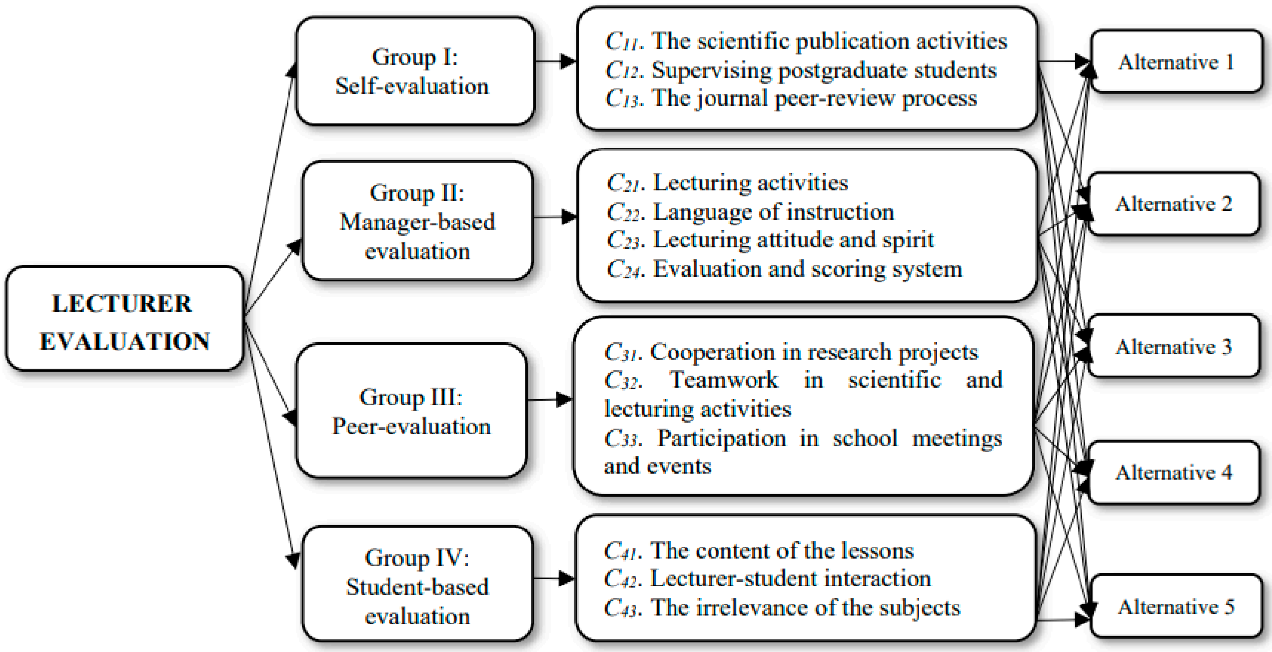


Figure 3. Lecturer evaluation—criteria.

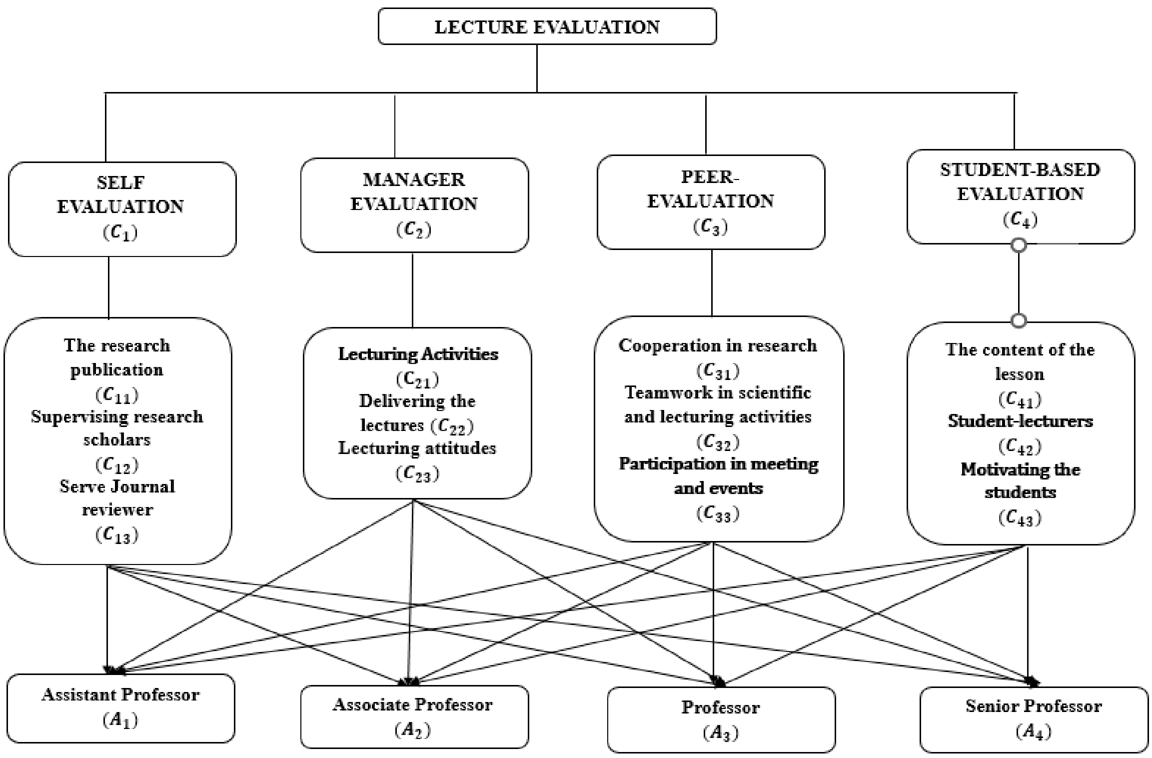


Figure 4. Modified criteria used in lecturer evaluation.

Table 4. Criteria and sub-criteria—Lecturer evaluation framework.

Name and Number Criteria	Description
Scientific publication (C_{11})	Faculty research publication in reputed journals is an important factor in the academic community in developing countries. The proportionality of the number of published articles in the last two years with the total in a year is measured by this criterion.
Supervising research scholars (C_{12})	Guiding the scholar is an essential criterion for the evaluation of a lecturer in any academic institution. Scholars expect their supervisors to be more efficient people and should have updated knowledge to perform their research activities. The number of scholars under a lecturer is also one of the key factors for this criterion. If the supervisor needs the minimum time to publish an article with their scholars, this reflects the capability of the lecturer.
Serve Journal reviewer (C_{13})	Becoming a peer reviewer in an indexed journal is one of the most important steps to reaching academic success as a lecturer. This will happen only when a lecturer has more knowledge in their research field. Lecturers' feedback about the submitted articles, suggestions to improve the article, and recommendations to the editors of the journal about the status of the articles.
Lecturing activities (C_{21})	It represents the number of hours a lecturer has spent teaching the subject to their students according to the allotted slot. In general, at the institutions, the duration of each lecture is about 50 mins to 1h. The number of publications can be considered with lecture time.
Language of instruction (C_{22})	This criterion measures the use of the lecturer's non-native language.
Lecturing attitude (C_{23})	Lecturing attitude includes coming to class late, canceling classes without a reason, etc.
Cooperation in research (internally funded) projects (C_{31})	Under this criterion, the lecturer gives importance to maintaining research records and involvement to reduce the time spent complementing the projects while also exchanging their knowledge.
Teamwork in scientific and teaching activities(C_{32})	This criterion includes the lecturer's contribution in various fields and the number of resources to increase the count of research publications as a team. Inter-department research publications contribute more to the institutional ranking system.
Participation in institutional meetings and events (C_{33})	Organizing institutional events like convocation, college day, scholarship day, and sports day will be successful only when lecturers build good relationships with their co-workers and students.
The content of the lessons (C_{41})	The teaching and learning process involves the quality of the teaching materials given, and lecturers should elaborate on real-world problems during their lectures.
Student–Lecturer (C_{42})	A healthy relationship between Lecturer–student includes the expectation that the lecturer should share their experience and knowledge among all the students without any special treatment or discrimination. Lecturers should maintain a decent relationship with their students and respect the decorum of their institution.
Motivating the students (C_{43})	Each lecturer acts as a mentor for at least a small group of students. Lecturers should advise the students regarding internal problems like the interactions of each student with their classmates, family issues, and disciplinary actions.

Using the TOPSIS method, the solution of the MCDM problem concludes the relationship between the shortest distance from the positive ideal solution and the farthest distance from the negative ideal solution. The ideal classical TOPSIS method can be presented using the following five levels:

***Level 1*:** Construct the DM matrix $\left(C = \left[c_{ij}\right]_{m\times n}\right)$.

***Level 2*:** Find the Normalized DM $\left(R = \left[r_{ij}\right]_{m\times n}\right)$.

Level 3: Find the $+ve$ and $-ve$ ideal solutions (PIS and NIS).

Level 4: Calculate the separation measures for both ideal solutions.

Level 5: Finalize the best alternative.

Any educational institution needs to evaluate the quality of the faculty members in the four different positions $A = (A_1, A_2, A_3, A_4)$ according to four criteria: self-evaluation (C_1), managerevaluation (C_2), peer evaluation (C_3), and student-based evaluation (C_4). In every appraisal of the institution, we must measure the quality and quantity of the work performed by different designations of the faculty members. This is mandatory for the gradual growth of the institution. Based on the past five years of data in an educational institution, we construct a decision matrix in terms of the Interval-valued Fermatean Neutrosophic values. Since measuring the faculty's strength is not based on an exact single value and these values fail under the uncertainty environment, we use $IVFF_{set}$. The past five years data was obtained through a questionnaire prepared and circulated among all faculty members at a leading education institute in south India.

Level 1: For a multiple attribute decision-making problem, let $C = (C_1, C_2, C_3, C_4)$ be a discrete set of alternatives. $A = (A_1, A_2, A_3, A_4)$ be the set of attributes. $W = (w_1, w_2, w_3, w_4)^T$ be the weighting vector of the attributes, and $\sum_{j=1}^{4} w_{j=1}$ where $\omega = (0.30, 0.30, 0.20)^{\mathrm{T}}$ be unknown.

In ***Level 1***, the construct decision matrix, $C = \left[c_{ij}\right]_{m\times n}$ is the decision matrix, where $\left\langle \left[\boldsymbol{\mathcal{T}}\mathbf{r}_A^-(\hbar), \boldsymbol{\mathcal{T}}\mathbf{r}_A^+(\hbar)\right], \left[\boldsymbol{\mathcal{I}}\mathbf{m}_A^-(\hbar), \boldsymbol{\mathcal{I}}\mathbf{m}_A^+(\hbar)\right], \left[\boldsymbol{\mathcal{F}}\mathbf{a}_A^-(\hbar), \boldsymbol{\mathcal{F}}\mathbf{a}_A^+(\hbar)\right] \right\rangle$ and is in the form of the $IVFF_{set}$.

See in Table 5.

Table 5. Criteria and alternatives with interval-valued Fermatean Neutrosophic values.

Alternatives\ Criteria	A_1	A_2	A_3	A_4
C_1	([0.85, 0.90], [0.85, 0.85], [0.80, 0.80])	([0.85, 0.85], [0.85, 0.87], [0.83, 0.85])	([0.81, 0.91], [0.86, 0.89], [0.82, 0.86])	([0.87, 0.92], [0.85, 0.85], [0.80, 0.80])
C_2	([0.85, 0.91], [0.85, 0.86], [0.80, 0.84])	([0.82, 0.90], [0.80, 0.85], [0.80, 0.80])	([0.85, 0.92], [0.85, 0.85], [0.80, 0.83])	([0.85, 0.91], [0.80, 0.85], [0.81, 0.85])
C_3	([0.85, 0.93], [0.85, 0.87], [0.80, 0.81])	([0.85, 0.91], [0.81, 0.85], [0.80, 0.80])	([0.85, 0.91], [0.85, 0.87], [0.80, 0.82])	([0.80, 0.90], [0.85, 0.85], [0.80, 0.80])
C_4	([0.85, 0.94], [0.85, 0.88], [0.80, 0.82])	([0.85, 0.92], [0.82, 0.85], [0.80, 0.80])	([0.82, 0.92], [0.85, 0.85], [0.82, 0.82])	([0.80, 0.94], [0.83, 0.85], [0.80, 0.80])

That is, the DM matrix

$$\mathrm{C} = \begin{pmatrix} ([0.85,0.90],[0.85,0.85],[0.80,0.80]) & ([0.85,0.85],[0.85,0.87],[0.83,0.85]) & ([0.81,0.91],[0.86,0.89],[0.82,0.86]) & ([0.87,0.92],[0.85,0.85],[0.80,0.80]) \\ ([0.85,0.91],[0.85,0.86],[0.80,0.84]) & ([0.82,0.90],[0.80,0.85],[0.80,0.80]) & ([0.85,0.92],[0.85,0.85],[0.80,0.83]) & ([0.85,0.91],[0.80,0.85],[0.81,0.85]) \\ ([0.85,0.93],[0.85,0.87],[0.80,0.81]) & ([0.85,0.91],[0.81,0.85],[0.80,0.80]) & ([0.85,0.91],[0.85,0.87],[0.80,0.82]) & ([0.80,0.90],[0.85,0.85],[0.80,0.80]) \\ ([0.85,0.94],[0.85,0.88],[0.80,0.82]) & ([0.85,0.92],[0.82,0.85],[0.80,0.80]) & ([0.82,0.92],[0.85,0.85],[0.82,0.82]) & ([0.80,0.94],[0.83,0.85],[0.80,0.80]) \end{pmatrix}$$

The numbers $([0.85,\ 0.90],\ [0.85,\ 0.85],\ [0.80,\ 0.80])$, corresponding to A_1 and C_1, represent that the degree of A_1 supports C_1, which lies in [0.85, 0.90], but the degree of A_1 does not support C_1, which lies in $[0.85,\ 0.85]$. Also, the degree of A_1 neutral to C_1, which lies in $[0.80,\ 0.80]$. All other degrees of alternativehave the same meaning.

In general, benefit and cost fall into these two categories. Normalize these values into a dimensionless matrix through which criteria can be compared easily. The construction of a Normalized Decision Matrix (NDM) is obtained at the next level by using the rule below:

$$R = \left[r_{ij}\right]_{m\times n} \text{ is the NDM, where } R = \begin{cases} d_{ij}, & if\ criterion\ C_{ij}\ is\ of\ the\ benefit\ type \\ \overline{d_{ij}}, & if\ criterion\ C_{ij}\ is\ of\ the\ cost\ type \end{cases}$$
$$\overline{d_{ij}} = \left\langle \left[\boldsymbol{\mathcal{F}}_A^-(p), \boldsymbol{\mathcal{F}}_A^+(p)\right], \left[1 - \boldsymbol{\mathcal{I}}_A^-(p), 1 - \boldsymbol{\mathcal{I}}_A^+(p)\right], \left[\boldsymbol{\mathcal{T}}_A^-(p), \boldsymbol{\mathcal{T}}_A^+(p)\right] \right\rangle$$

Level 2: As the criteria of C_2 and C_4 are the cost criteria and C_1 and C_3 are the benefit criteria, the NFM-DM of R is given by

$$R = \begin{pmatrix} ([0.85, 0.90], [0.85, 0.85], [0.80, 0.80]) & ([0.83, 0.85], [0.15, 0.13], [0.85, 0.85]) & ([0.81, 0.91], [0.86, 0.89], [0.82, 0.86]) & ([0.80, 0.80], [0.15, 0.15], [0.87, 0.92]) \\ ([0.85, 0.91], [0.85, 0.86], [0.80, 0.84]) & ([0.80, 0.80], [0.20, 0.15], [0.82, 0.90]) & ([0.85, 0.92], [0.85, 0.85], [0.80, 0.83]) & ([0.81, 0.85], [0.20, 0.15], [0.85, 0.91]) \\ ([0.85, 0.93], [0.85, 0.87], [0.80, 0.81]) & ([0.80, 0.80], [0.19, 0.15], [0.85, 0.91]) & ([0.85, 0.91], [0.85, 0.87], [0.80, 0.82]) & ([0.80, 0.80], [0.15, 0.15], [0.80, 0.90]) \\ ([0.85, 0.94], [0.85, 0.88], [0.80, 0.82]) & ([0.80, 0.90], [0.18, 0.15], [0.85, 0.92]) & ([0.82, 0.92], [0.85, 0.85], [0.82, 0.82]) & ([0.80, 0.80], [0.17, 0.15], [0.80, 0.94]) \end{pmatrix}$$

Level 3: Converting R into their collective score matrix—using $S_3(\alpha)$

$$M = \begin{pmatrix} 0.3919 & 0.4892 & 0.4397 & 0.4183 \\ 0.4224 & 0.4412 & 0.4208 & 0.4548 \\ 0.4280 & 0.4273 & 0.4170 & 0.4505 \\ 0.4458 & 0.4665 & 0.4056 & 0.4319 \end{pmatrix}$$

Level 4: In this level, ideal solutions consist of selecting the best values for each attribute from all alternatives.

Generally, the values of l^+ are complements of l^- and vice versa. The degree of l^+ to 1 and 0 is fixed, but the decision-maker may vary it. Hence, we consider $IVFNPI_{set} l^+$ and $IVFNNI_{set} l^-$ as follows:

$$l^+ = \left\langle \left[\max\left(a_{ij}\right), \max\left(b_{ij}\right)\right]; \left[\min\left(c_{ij}\right), \min\left(d_{ij}\right)\right]; \left[\min\left(e_{ij}\right), \min\left(f_{ij}\right)\right] \right\rangle$$
$$l^- = \left\langle \left[\min\left(a_{ij}\right), \min\left(b_{ij}\right)\right]; \left[\max\left(c_{ij}\right), \max\left(d_{ij}\right)\right]; \left[\max\left(e_{ij}\right), \max\left(f_{ij}\right)\right] \right\rangle \text{among all attributes.}$$

The PIS and NIS of two alternatives are found as

$$\begin{aligned} l^+ = \{ & \langle [0.85, 0.94]; [0.82, 0.85]; [0.80, 0.80] \rangle, \ \langle [0.85, 0.92]; [0.80, 0.85]; [0.80, 0.80] \rangle, \\ & \langle [0.85, 0.92]; [0.85, 0.85]; [0.80, 0.82] \rangle, \langle [0.87, 0.94]; [0.80, 0.85]; [0.80, 0.80] \rangle \} \\ l^- = \{ & \langle [0.85, 0.90]; [0.85, 0.88]; [0.80, 0.84] \rangle, \ \langle [0.82, 0.85]; [0.85, 0.87]; [0.83, 0.85] \rangle, \\ & \langle [0.81, 0.91]; [0.86, 0.89]; [0.83, 0.86] \rangle, \langle [0.80, 0.90]; [0.85, 0.85]; [0.81, 0.85] \rangle \\ S_3(l^+) &= \begin{pmatrix} 0.4127 & 0.3537 & 0.4161 & 0.4194 \end{pmatrix} \\ S_3(l^-) &= \begin{pmatrix} 0.4245 & 0.3593 & 0.4459 & 0.3863 \end{pmatrix} \end{aligned}$$

The distance between A_i and the ideal solution is calculated in ***level 5***

$$M_i^+ = \sum_{j=1}^{n} d\left(A_{ij},\ A_j^+\right) = \sqrt{\sum_{j=1}^{n} \left[w_j\left(S_3(l^+) - S_3\left(r_{ij}\right)\right)^2\right]^2}$$
$$M_i^- = \sum_{j=1}^{n} d\left(A_{ij},\ A_j^-\right) = \sqrt{\sum_{j=1}^{n} \left[w_j\left(S_3\left(r_{ij}\right) - S_3(l^-)\right)^2\right]^2}$$

Level 5: To compute the closeness coefficient (CC):

$$\mathrm{CC_K} = \frac{\mathrm{M_K^-}}{\mathrm{M_K^-} + \mathrm{M_K^+}}, \mathrm{K} = 1, 2, 3, 4$$

See in Table 6.

Table 6. Closeness coefficient for each alternative.

B_i	M_i^+	M_i^-	CC_i
A_1	0.0055	0.0051	0.4793
A_2	0.0023	0.0022	0.4902
A_3	0.0016	0.0071	0.8134
A_4	0.0038	0.0597	0.9396

Level 6: Based on the values of C_i, we rank the alternatives and select the best alternative(s). Therefore, the final and optimized ranking of the four major alternatives is $A_4 \succ A_3 \succ A_2 \succ A_1$, and thus, the best alternative is A_4.

6. Results and Discussion

In this approach, we describe a combination of quantitative assessment and multi-criteria decision-making models to evaluate lecturers' performances from various perspectives: self-assessment, peer assessment, managerial assessment, and student-based evaluation. This approach aims to overcome the challenge of differentiating between lecturers' potential capacities and their actual teaching effectiveness. In our article, we have introduced a new variant of the $\boldsymbol{N_{set}}$ called Interval-Valued Fermatean Neutrosophic Set ($\boldsymbol{IVFN_{set}}$). This new variant specifically deals with situations where there is partial ignorance, leading to uncertainty about whether something is true, false, or exists in an uncertain region. This concept is applied independently to a multi-decision process. This study expands upon the concept of Fermatean Neutrosophic Set ($\boldsymbol{FN_{set}}$), presenting an extension in the form of the $\boldsymbol{IVFN_{set}}$. The article highlights the algebraic properties and set theoretical aspects of the $\boldsymbol{IVFN_{set}}$, likely discussing how this new variant handles and represents partial ignorance in more detail. This research appears to be addressing a crucial challenge in education by proposing an innovative approach that considers various assessment perspectives and handles uncertainty effectively through the $\boldsymbol{IVFN_{set}}$. The presented results highlight the practical application and effectiveness of our methodology in making informed decisions about lecturers' performances.

Faculty evaluation is a crucial component of higher education institutions and plays a significant role in shaping educational goals and national development strategies. Evaluating faculty performance is essential for maintaining teaching competency, promoting scientific research, and creating a conducive learning environment. The importance of evaluating faculty performance in terms of teaching competency as a tool for decision-making, including employment and dismissal in this assessment, is seen as a means to ensure the quality of education and contribute to the overall development of the country's education system. Higher educational institutions should function as scientific research centers and encourage faculty to engage in research activities. This dual role of teaching and research contributes to the institution's credibility and the advancement of knowledge. Faculty evaluation is seen as a way to create an equal environment that fosters cooperative strategies among faculty members and nurtures the learning spirit of each student. This suggests that a well-structured evaluation system can positively impact the overall educational atmosphere. Assessing faculty performance provides a comprehensive perspective on the institution's achievements, including improving learning outcomes, identifying and nurturing young talents, and indirectly contributing to the country's wealth. Such assessments also establish the institution's reputation at both global and local levels. The evaluation process involves various complex factors such as personal interests, development strategies, and fairness in assessment. It is acknowledged that fair and accurate assessment is challenging and requires a multi-dimensional approach, including input from principals/managers, students, and peer reviews. The absence of appropriate standards and tools can lead to inaccuracies and subjectivity in evaluating faculty competence. We suggest that a well-rounded, multi-dimensional assessment process can enhance faculty knowledge, teaching capabilities, and professional development. As a whole, the multi-faceted nature of faculty evaluation, its significance in the educational landscape, and the challenges associated with implementing a fair and effective assessment system place an emphasis on considering local context, fostering research, and promoting a cooperative learning environment. This underscores the holistic approach required to evaluate and enhance faculty performance in higher education institutions.

The criteria and methods used in a Multi-Criteria Decision-Making (MCDM) process assess the performance and relative importance of lecturers. We have mentioned two popular MCDM models—the Analytical Hierarchy Process (AHP) and the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS)—that are commonly used to handle such assessments. The assessment process involves evaluating lecturers based on standards related to research capacity, teaching capacity, and service activities. These criteria are likely important aspects in determining the overall performance of lecturers.

MCDM involves making decisions based on multiple criteria that might be conflicting or competing. It is a way to handle complex decision scenarios that cannot be addressed using single criteria. AHP is a widely used MCDM method that breaks down complex problems into a hierarchical structure of criteria and sub-criteria. It allows assigning weights to these criteria based on their relative importance and then comparing alternatives based on these weighted criteria. AHP is particularly useful for dealing with structured problems and hierarchical decision contexts. The application of Neutrosophic Sets and related concepts in the context of lecturer evaluation uses Multi-Criteria Decision-Making (MCDM) techniques. Smarandache [2] introduced the concept of a Neutrosophic Set, which is characterized by three membership degrees: truth membership (T), indeterminacy membership (I), and falsity membership (F). These membership degrees are defined within the real standard or nonstandard unit interval. This concept allows for dealing with uncertainty and imprecision in various domains, including education. Neutrosophic Sets can be applied to educational problems when dealing with ranges that fall within the defined interval. This approach can help address issues related to imprecision and uncertainty in educational contexts. Wang et al. [3] introduced the concepts of a single-valued Neutrosophic Set and an interval-valued Neutrosophic Set. The interval-valued Neutrosophic Set extends the concept of the Neutrosophic Set by incorporating interval values for the membership degrees. This approach has been used in various fields, including decision-making sciences, social sciences, and the humanities, to handle problems involving vague, indeterminate, and inconsistent information. Ye [31] introduced the interval Neutrosophic Linguistic Set, which involves new aggregation operators for interval Neutrosophic linguistic information. This concept contributes to handling uncertain linguistic information. Broumi et al. [39] extended the TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) method to accommodate uncertain linguistic information within interval Neutrosophic Sets. This extension allows for decision-making when dealing with complex and uncertain data. The passage highlights that there is a lack of research integrating hierarchical TOPSIS with interval Fermatean Neutrosophic Sets, especially in the context of lecturer evaluation. This integration could address the limitations of traditional approaches to evaluating lecturers, which often face complexity and uncertainty. The study presented in the passage focuses on evaluating lecturers using MCDM models. The goal is to combine the hierarchical Neutrosophic TOPSIS technique, and the interval-valued complex set in a Neutrosophic environment to improve lecturer evaluation. The application of Neutrosophic Sets and related concepts to address the challenges of uncertainty and imprecision in lecturer evaluation uses MCDM techniques. By combining these innovative approaches, this study aims to provide a more effective and robust framework for assessing and ranking lecturers' performances.

Comparing with other models: The following table lists the results of the comparison. The proposed method and the classic TOPSIS method can solve problems in uncertain environments. However, the TOPSIS and AHP techniques have some disadvantages in terms of calculation methods and results. Moreover, the extent of the interval-valued Neutrosophic TOPSIS does not consider the capacity of each lecturer in the specific time period.

Method	Ranking
Interval neutrosophic TOPSIS (Chi and Liu [40]). Chi, P., and Liu, P. (2013). An extended TOPSIS method for the multiple attribute decision making problems based on interval neutrosophic set. Neutrosophic Sets and Systems, 1, 1–8.	$A_2 > A_3 > A_5 > A_1 > A_4$
AHP (Saaty [41]) Saaty, T. L. (1980). The analytic hierarchy process. New York, NY: McGraw-Hill Inc, 17–34.	$A_3 > A_2 > A_5 > A_4 > A_1$

Method	Ranking
TOPSIS (Hwang and Yoon [26]) Hwang, C.-L., and Yoon, K. (1981). Multiple Attribute Decision Making: Methods and Applications A State-of-the-Art Survey. Berlin Heidelberg: Springer-Verlag.	$A_3 > A_2 > A_5 > A_4 > A_1$
Interval complex Neutrosophic set (Anh Duc Doet al. [36]) Anh Duc Doa, Minh Tam Pham, Thi Hang Dinh, The Chi Ngo, Quoc Dat Luue, Ngoc Thach Phamf, Dieu Linh Hag, and Hong Nhat Vuong, Evaluation of lecturers' performance using a novel hierarchical multi-criteria model based on an interval complex Neutrosophic set, Decision Science Letters 9 (2020) 119–14.	$A_1 > A_3 > A_2 > A_5 > A_4$
The present work evaluates the quality of the faculty members in the four different positions $\boldsymbol{A} = (\boldsymbol{A}_1, \boldsymbol{A}_2, \boldsymbol{A}_3, \boldsymbol{A}_4)$ according to four criteria, namely self-evaluation $(\boldsymbol{C}_1)$, managerevaluation $(\boldsymbol{C}_2)$, peer evaluation $(\boldsymbol{C}_3)$, and student-based evaluation $(\boldsymbol{C}_4)$.	$A_4 \succ A_3 \succ A_2 \succ A_1$

7. Conclusions

In this article, our study developed a comprehensive assessment methodology using a hierarchical structure and a hierarchical TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution) approach, incorporating interval-valued complex Neutrosophic Sets for evaluating lecturer capability. The goals of this study seem to include addressing the potential competition between lecturers that can arise from such evaluations and ensuring fairness and transparency in the process. This study's methodology is designed to handle the complexity of assessment and decision-making in education and management systems. The hierarchical approach is then compared with other related methods to highlight its advantages and practicality. The results indicate that the proposed approach is effective and not limited to just lecturer evaluation; it can potentially be applied to other decision-making problems as well. However, as mentioned, there are certain limitations to this study. Unfortunately, you have not specified what those limitations are. Nonetheless, you also mentioned that future work is proposed to enhance the accuracy of lecturer evaluation. This improvement could be valuable for supporting real-world, dynamic decision-making in educational contexts. In summary, this study appears to contribute a novel approach to lecturer evaluation using a hierarchical structure and TOPSIS methodology, with an emphasis on fairness and transparency. The results suggest its efficiency and broader applicability, though there are acknowledged limitations that may guide future research.

This paper introduces the concept of the $\boldsymbol{IVFN_{set}}$ and its algebraic properties with an example. Also, we introduce a new set of score functions for the $\boldsymbol{IVFN_{set}}$ and use these functions to evaluate the lectures' performances that were studied.

8. Further Study

1. To define interval-valued Fermatean Neutrosophic Numbers.
2. To study the interval-valued Triangular Fermatean Neutrosophic Linear Programming Problem.
3. To study the interval-valued Fermatean trapezoidal and Fermatean triangular Neutrosophic numbers.

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