

Article

A Novel Interval-Valued Decision Theoretic Rough Set Model with Intuitionistic Fuzzy Numbers Based on Power Aggregation Operators and Their Application in Medical Diagnosis

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Abstract: Intuitionistic fuzzy information is a potent tool for medical diagnosis applications as it can represent imprecise and uncertain data. However, making decisions based on this information can be challenging due to its inherent ambiguity. To overcome this, power aggregation operators can effectively combine various sources of information, including expert opinions and patient data, to arrive at a more accurate diagnosis. The timely and accurate diagnosis of medical conditions is crucial for determining the appropriate treatment plans and improving patient outcomes. In this paper, we developed a novel approach for the three-way decision model by utilizing decision-theoretic rough sets and power aggregation operators. The decision-theoretic rough set approach is essential in medical diagnosis as it can manage vague and uncertain data. The redesign of the model using interval-valued classes for intuitionistic fuzzy information further improved the accuracy of the diagnoses. The intuitionistic fuzzy power weighted average (IFPWA) and intuitionistic fuzzy power weighted geometric (IFPWG) aggregation operators are used to aggregate the attribute values of the information system. The established operators are used to combine information within the intuitionistic fuzzy information system. The outcomes of various alternatives are then transformed into interval-valued classes through discretization. Bayesian decision rules, incorporating expected loss factors, are subsequently generated based on this foundation. This approach helps in effectively combining various sources of information to arrive at more accurate diagnoses. The proposed approach is validated through a medical case study where the participants are classified into three different regions based on their symptoms. In conclusion, the decision-theoretic rough set approach, along with power aggregation operators, can effectively manage vague and uncertain information in medical diagnosis applications. The proposed approach can lead to timely and accurate diagnoses, thereby improving patient outcomes.

Keywords: intuitionistic fuzzy sets; three-way decision; decision-theoretic rough sets; power aggregation operators; decision making; optimization; efficiency

MSC: 03E72; 94D05

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1. Introduction

1.1. Evaluation of Medical Diagnosis

The accurate diagnosis of medical diseases can be a complex process as they can manifest with a variety of symptoms. Common symptoms may include fever, coughing, fatigue, vomiting, diarrhea, and skin rashes. Physicians rely on a range of diagnostic procedures such as patient history, physical examination, laboratory tests, imaging studies, and other techniques to arrive at a diagnosis. Medical professionals need to consider several factors, including the patient's age, medical history, lifestyle, and genetic predisposition while making decisions during the diagnosis process [\[1\]](#page-16-0). In complex medical conditions, physicians may use diagnostic algorithms or decision trees to support the decision-making process. In recent years, medical technology advancements, including new imaging techniques, genetic testing, and AI-based decision-making tools, have significantly improved the accuracy and speed of medical diagnoses [\[2,](#page-16-1)[3\]](#page-16-2). These advancements help medical professionals to make more informed decisions and provide patients with better healthcare outcomes [\[4](#page-16-3)[,5\]](#page-16-4).

1.2. Three-Way Decision in the Medical Field

Rough set (RS) theory is a mathematical structure that deals with incomplete and uncertain data in a systematic way. The theory was established by Zdzislaw Pawlak [\[6\]](#page-16-5) in the early 1980s as a method for dealing with vague and uncertain information. In medical diagnosis, RS theory can be used to help detect the presence or absence of a particular disease or condition based on incomplete or uncertain information [\[7\]](#page-16-6). This can include symptoms, medical history, test results, and other relevant data [\[8\]](#page-16-7). The fundamental theory behind RS theory is to divide a set of data into subsets based on their attributes, such as symptoms or test results. This process can help to identify the prominent features or factors that are most closely associated with a particular disease or condition. Once the data have been divided into subsets, RS theory can be used to identify rules or patterns that can be used to make predictions about whether a particular patient has a particular disease or condition. Many researchers worked on this notion to identify novel algorithms for diagnosis of diseases [\[9\]](#page-16-8). El-Bably et al. [\[8,](#page-16-7)[10\]](#page-16-9) introduced the soft and rough approximation and applied it to diagnose the medical problem. Hosny et al. [\[11\]](#page-16-10) worked on the extension of RS using the maximal right neighborhood system and its application in the medical field. Al-Shami et al. [\[12\]](#page-16-11) defined maximal rough neighborhoods and applied this approach to medical diseases.

Attansove [\[13\]](#page-16-12) developed the idea of an intuitionistic fuzzy set (IFS) which is the generalization of a fuzzy set (FS). In IFS, there are two grades of membership and grades of non-membership of an element of universal set, respectively. Intuitionistic fuzzy sets played a very important part in the medical field to identify diseases and problems. The application of IFS in medical diagnosis has been studied in various contexts. One area of application is in the diagnosis of medical conditions where there is significant uncertainty and variability in symptoms and test results. IFS can help to capture this uncertainty and provide more nuanced diagnostic information. For example, in the diagnosis of a complex disease such as cancer, IFS can be used to represent the degree of certainty or uncertainty in the diagnosis based on various diagnostic criteria such as the results of blood tests, imaging studies, and biopsy findings. This can help to provide more accurate and reliable diagnoses, as well as more personalized treatment plans. Jiang et al. [\[14\]](#page-16-13) used IFS for medical image fusion using entropy measures. Recently, Mehmood et al. [\[15](#page-16-14)[,16\]](#page-16-15) generalized the intuitionistic fuzzy sets and applied these approaches to medical diagnosis. De et al. [\[17\]](#page-16-16) also analyzed an application of IFS in medical diagnosis and Davvaz et al. [\[18\]](#page-16-17) produced a similar technique. Szmidt et al. [\[19\]](#page-16-18) explored IFS in intelligent data analysis for medical diagnosis. During the decision making for IFS, the aggregation operators help a lot to calculate the values of the attributes. Therefore, experts proposed many aggregation operators; for example, Xu et al. [\[20\]](#page-16-19) designed power aggregation operators for IFS and applied them in MADM. In 2006, some geometric aggregation operators were produced

for IFS by Xu [\[21\]](#page-16-20). Wajid et al. [\[22\]](#page-17-0) presented a novel TWD approach for IHFS. Recently, Senapati and Garg [\[23](#page-17-1)[,24\]](#page-17-2) also explored some novel operators.

Three-way decision (TWD) is a very important generalization of RS theory introduced by Yao [\[25,](#page-17-3)[26\]](#page-17-4). A three-way decision for medical diagnosis involves considering three possible outcomes: positive, negative, or inconclusive. Positive: If the medical diagnosis is positive, it means that the patient has the condition or disease being evaluated for. In this case, the patient would need to receive treatment for the condition and the medical team would need to monitor their progress. Negative: If the medical diagnosis is negative, it means that the patient does not have the condition or disease being evaluated for. In this case, the patient may not require any treatment and the medical team may need to investigate other potential causes of the patient's symptoms. Inconclusive: If the medical diagnosis is inconclusive, it means that the test results are not clear enough to determine whether the patient has the condition or disease being evaluated for. In this case, the patient may need to undergo further testing or evaluation to arrive at a more definitive diagnosis. Recently, Li et al. [\[27](#page-17-5)[,28\]](#page-17-6) applied TWD techniques for hybrid decision making to diagnose medical problems. Hu et al. [\[29](#page-17-7)[,30\]](#page-17-8) presented the concept of a lattice model for medical diagnosis using TWD. Jia and Fan [\[31\]](#page-17-9) composed TWD models for multi-criteria environments. Ye et al. [\[32\]](#page-17-10) combined the TWD notion with the trending research area fuzzy information system. Similarly, many scholars explored this area and proposed novel approaches in different extensions of fuzzy sets [\[33](#page-17-11)[–35\]](#page-17-12).

1.3. Motivation for Proposed Work

In the literature, we found that three-way decision TWD models are very useful in diagnosing medical problems. By combining IFS and TWD [\[36\]](#page-17-13), a very powerful theory is produced to cope with the vagueness and unclear situation. It is noted that for aggregation, the results of many participants based on TWD is a very difficult problem. Researchers used the classical way to calculate the alternatives for TWD [\[37–](#page-17-14)[40\]](#page-17-15). In the existing TWD model [\[25](#page-17-3)[,37\]](#page-17-14), to determine the equivalence classes, an external concept is required. Moreover, the threshold is used to classify the alternatives into three regions.

The main purpose of composing this work is to design a novel algorithm for a TWD model-based decision-theoretic rough set using aggregation operators and an improved TWD decision approach based on interval-valued equivalence classes for IFS. The developed approach fulfils the lackness and resolves the computing problem for TWD. Below is a demonstration of this analysis' major contribution.

- i. Construct the concept of intervals for membership grades of IFS using the step size function;
- ii. Develop the equivalence classes based on intervals and called interval-valued classes;
- iii. To cope with the issues of computing and saving time, IFPWA and IFPWG aggregation operators are developed for the TWD model;
- iv. An algorithm is proposed to classify the different patients and to diagnose the disease on the basis of multiple symptoms.

The rest of the article is given as follows: In Section [2,](#page-3-0) we have overviewed the basic notion of IFS, power aggregation operators, and three-way decision (TWD). In Section [3,](#page-6-0) we have designed intervals for the membership grade using the step size function. Based on the intervals, equivalence classes are produced and remodel the TWD for IFS. In Section [4,](#page-9-0) we have designed a proper algorithm with a flow chart and explained the approach step by step. In Section [5,](#page-11-0) we have discussed a case study and utilized the proposed approach to diagnosis a medical problem to classify the alternatives with power aggregation operators for IFS. Some advantages and benefits of proposed models are discussed in detail. Section [6](#page-15-0) includes the conclusion and future plan of the authors.

2. Preliminaries 2. Preliminaries

In this section, we update models for IFSs and several concepts pertaining to power In this section, we update models for IFSs and several concepts pertaining to power aggregation operators. Table [1](#page-3-1) is added to describe the abbreviations of the symbols for a since of under the $\frac{1}{2}$ easiness of understanding. $\frac{1}{2}$, $\frac{1}{2}$, *Mathematics* **2023**, *11*, x FOR PEER REVIEW 4 of 19 *Mathematics* **2023**, *11*, x FOR PEER REVIEW 4 of 19

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Table 1. Symbols with their descriptions. **Table 1.** Symbols with their descriptions. Table 1 Symbols with their descriptions. easiness of understanding. Table 1. Symbols with their descriptions.

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Le 41 [0, 1]. *presented as:* set [0, 1]. As an extension of the FS model, Atanassov [13] proposed the IFS model. IFS simul-As all extension of the FS model, Attainassov $[13]$ proposed the IFS model. IFS simulation of the FS in and α As an extension of the FS model, Atanassov [13] proposed the IFS model. IFS simultaneously delivers MG and NMG while FS just delivers the MG of an element in a given set $[0, 1].$ that \mathbf{M}_0 and \mathbf{M}_0 while \mathbf{M}_0 is an element in a given in a giv *2.1. IFSs* ΔS all extension of the F3 model, Adamssov [15] proposed $[0, 1]$. $\Delta \varepsilon$ an extension of the FS model. At an escoy [13] proposed the IFS model. IF Ω_{E} poouely delivers MG and NMC while ES just delivers the MG of an element is $\begin{bmatrix} 0 & 1 \end{bmatrix}$ IFPOWA Intuitionistic Fuzzy Power Order Weighted averaging IFPOWG Intuitionistic Fuzzy Power Order Weighted Geometric $\begin{bmatrix} 0 & 1 \end{bmatrix}$ IFPOWA Intuitionistic Fuzzy Power Order Weighted averaging IFPOWG Intuitionistic Fuzzy Power Order Weighted Geometric As an extension of the F5 model, Atanassov $[15]$ proposed the H5 model. If 5 simula As an extension of the F5 model, Atanassov [15] proposed the H5 model. IF5 simula-
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Definition 1 ([13]). Let an IFS W on E be symbolized by $m(e)$ and $n(e)$. Mathematically, it is *where it is a multiplier is* $W = \langle a, m \rangle$ *(a)* $a = \langle a \rangle |a - \langle b \rangle|$ $\in E$ \setminus (1) *presented as: presented as:* taneously delivers MG and NMG while FS just delivers the MG of an element in a given *2.1. IFSs* IFPWA Intuitionistic Fuzzy Power Weighted Averaging IFPWG Intuitionistic Fuzzy Power Weighted Geometric IFPWA Intuitionistic Fuzzy Power Weighted Averaging IFPWG Intuitionistic Fuzzy Power Weighted Geometric TWD Three-Way Decision MG Membership Grade **Table 1.** Symbols with their descriptions. **Table 1.** Symbols with their descriptions. **Table 1.** Symbols with their descriptions. aggregation operators. Table 1 is added to describe the abbreviations of the symbols for *presented as:* **Definition 1** ([13]). Let an IFS W on E be symbolized by $m(e)$ and $n(e)$. Mathematically, it is *presented as:* **Definition 1** *Definition 1**Let an IFS symbolized by an interval* α *b**and* **()** *and* **()** *and* **() is** *it is a interval in the symbolized by an interval in the symbolized by an interval in the symbolized by an inter* **Definition 1** [13]: *Let an IFS* Ⱳ *on be symbolized by* () *and* ()*. Mathematically, it is* that are $\frac{d}{dt}$ and $\frac{d}{dt}$ in a given i **Definition 1** ([13]) Let an IES W on E be symbolized by $m(e)$ and $n(e)$. Mathematically it is *2.1. IFSs* P chillion 1 ([10]). Let up the P we fit D we fit D to D if D by D (c) what D (c). Fundamentally $\mathbf{E} \left(\begin{array}{cc} \mathbf{E} & \mathbf{E} \\ \mathbf{E} & \mathbf{E} \end{array} \right)$ $\mathbf{E} \left(\begin{array}{cc} \mathbf{E} & \mathbf{E} \\ \mathbf{E} & \mathbf{E} \end{array} \right)$ **Definition 1** ([13]) Let an IES W on E be sumbolized by $m(e)$ and $n(e)$. Mathematically it is $10₁$ is Fuzzy Power Order Weighted averaging IFPOWG Intuitionistic Functionistic Functio $10₁$ intervalstic Fuzzy Power Order Weighted averaging IFPOWG Intervalstic Fuzzy Power Order Weighted Geometric Fuzzy Power Order Weighted Geometric Fuzzy Power Order Weighted Geometric Fuzzy Power Order Weighted G **Definition 1** ([13]). Let an IFS W on E be symbolized by $m(e)$ and $n(e)$. Mathematically, it is \mathcal{L} Decision Rules International Rules International Rules International Rules \mathcal{L} (1) $presented$ as: p resented as: P **Collicution Symbol Description** $\sum_{i=1}^{n}$ $\sum_{i=1}^{n}$ $\sum_{i=1}^{n}$ aggregation operators. Table 1 is added to describe the abbreviations of the symbols for the $M_A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

presented as:
\n
$$
W = \langle e, m_W(e), n_W(e) \rangle \mid e \in E \rangle \tag{1}
$$

 $0 \leq m(e) + n(e) \leq 1$ for all $e \in E$. Generally, the pair (m,n) represents the IFN. where $m_W(e): E \to [0, 1]$ and $n_W(e): E \to [0, 1]$ signify the MG and NMG with condition *where* Ⱳ(): → [0, 1] *and* Ⱳ(): → [0, 1] *signify the MG and NMG with condition* 0 ≤ *where* Ⱳ(): → [0, 1] *and* Ⱳ(): → [0, 1] *signify the MG and NMG with condition* 0 ≤ $0 \leq m(e) + n(e) \leq 1$ for all $e \in E$. Generally, the pair (m, n) represents the IFN. *presented as: presented as:* (c), $F \rightarrow [0, 1]$ and \cdots (c), $F \rightarrow [0, 1]$ proposed the MC and NMC milk condition $\frac{1}{\sqrt{N}}$ and $\frac{1}{\sqrt{N}}$ and $\frac{1}{\sqrt{N}}$ and $\frac{1}{\sqrt{N}}$ is $\frac{1}{\sqrt{N}}$ is $\frac{1}{\sqrt{N}}$ in a given in a $\mathcal{L}(\mathcal{C})$. $\frac{1}{\sqrt{2}}$ of and $\frac{1}{\sqrt{6}}$ in a given $\frac{1}{\sqrt{6}}$ in a given $0 \leq m(e) + n(e) \leq 1$ for all $e \in E$. Generally, the pair (m,n) represents the IFN. where $m_W(e): E \to [0, 1]$ and $n_W(e): E \to [0, 1]$ signify the MG and NMG with condition $0 \le m(e) + n(e) \le 1$ for all $e \in E$. Generally, the pair (m, n) represents the IFN. $0 \leq m(e) + n(e) \leq 1$ for all $e \in E$. Generally, the pair (m,n) represents the IFN. \mathcal{L} decision \mathcal{L} DRs Decision Rules IFRS Intuitionistic Fuzzy Rough Set DRsDecision Rules IFRS Intuitionistic Fuzzy Rough Set DRs Decision Rules IFRS Intuitionistic Fuzzy Rough Set $\sum_{i=1}^{n}$ $\sum_{i=1}^{n}$ $\sum_{i=1}^{n}$ $\sum_{i=1}^{n}$ $\sum_{i=1}^{n}$ $\sum_{i=1}^{n}$ easiness of understanding. *Mathematics* **2023**, *11*, x FOR PEER REVIEW 4 of 19

Definition 2. For IFNs, $W = (m_W, n_W)$ and the score function and accuracy functions are denoted and defined as follows: denoted and defined as follows: *where* Ⱳ(): → [0, 1] *and* Ⱳ(): → [0, 1] *signify the MG and NMG with condition* 0 ≤ *where* Ⱳ(): → [0, 1] *and* Ⱳ(): → [0, 1] *signify the MG and NMG with condition* 0 ≤ taneously delivers $M_{\rm H}$ and $M_{\rm H}$ is an element in a given in \mathbf{D} em \mathbf{r} ition 2. For IEMs $W = (m_1, m_2)$ and the score function and accuracy functions are denoted and defined as follows: \mathbf{w} , \mathbf{w} , **Definition 2.** For IFNs, $W = (m_W, n_W)$ and the score function and accuracy functions are easiness of understanding. The unit of understanding $\mathbf{D}\mathbf{e}$

Definition 2: *For IFNs,* Ⱳ = (Ⱳ, Ⱳ) *and the score function and accuracy functions are de-*

$$
S(W) = m_W - n_W, \t S(W) \in [-1, 1]
$$
 (2)

$$
H(W) = m_W + n_W, \qquad H(W) \in [0, 1]
$$
 (3)

For comparing two IFNs, W_1 and W_2 , the score function and accuracy function provide the assistance as below. $\frac{1}{2}$ and $\frac{1}{2}$ a *assistance as below, , the score function and accuracy function provide the For comparing two IFNs,* Ⱳ¹ *and* Ⱳ² *noted and defined as follows: noted and defined as follows:* () + () ≤ 1 *for all* ∈ *. Generally, the pair (,) represents the IFN.* () + () ≤ 1 *for all* ∈ *. Generally, the pair (,) represents the IFN.* \mathcal{B} *Fance as below,* \mathcal{B} **Definition 2:** *For IFNs,* Ⱳ = (Ⱳ, Ⱳ) *and the score function and accuracy functions are de-*For comparing two IFNs, W_1 and W_2 , the score function and accuracy function provide the () + () ≤ 1 *for all* ∈ *. Generally, the pair (,) represents the IFN. where we below,* $\frac{1}{2}$ $\frac{$ *where* Ⱳ(): → [0, 1] *and* Ⱳ(): → [0, 1] *signify the MG and NMG with condition* 0 ≤ *presented as:* $\frac{1}{2}$ and $\frac{1}{2}$ and $\frac{1}{2}$ and $\frac{1}{2}$ are $\frac{1}{2}$ and $\frac{1}{2}$ a For comparing two IFNs, W_1 and W_2 , the score function and accuracy function provide the ϵ low, ϵ *2.1. IFSs* $\mathit{below},$ IFSs Intuitionistic Fuzzy Sets IFN Intuitionistic Fuzzy Number

(i) *If* $S(W_1) > S(W_1)$ then $W_1 > W_2$;) *then* Ⱳ¹ > Ⱳ² (i) If $S(W_1) > S(W_1)$ then $W_1 > W_2$; W_2 ; *presented as: presented as:* (i) *2.1. IFSs*

Definition 2: *For IFNs,* Ⱳ = (Ⱳ, Ⱳ) *and the score function and accuracy functions are de-*

- (ii) If $S(W_1) < S(W_1)$ then $W_1 < W_2$;
(iii) If $S(W_1) S(W_1)$ then (i) If $S(W_1) > S(W_1)$ then $W_1 > W_2$,

(ii) If $S(W_1) < S(W_1)$ then $W_1 < W_2$; $\overline{W_2}$; $(1 \leq \frac{n}{2})$ \mathcal{L} / (ii) If $S(W_1) < S(W_1)$ then $W_1 < W_2$;
(iii) If $S(W_1) < S(W_1)$ then $W_1 < W_2$; W_2 ;
- (ii) *If* $S(W_1) = S(W_1)$ *then* $W_1 \leq W_2$,
(iii) *If* $S(W_1) = S(W_1)$ *then*;) $\mathcal{L} = \{ \mathcal{L}_1, \ldots, \mathcal{L}_N \}$ $\begin{pmatrix} m \\ m \end{pmatrix}$ $\begin{pmatrix} n \\ n \end{pmatrix}$ $\int f \cdot f(y_1) \cdot g(y_1)$
i) If $S(W_1) - S(W_1)$ (iii) If $S(W_1) = S(W_1)$ then;
 $V_1 = V_2 + V_3 + V_4$ *presented as:* (iii) If $S(W_1) = S(W_1)$ then;

set [0, 1].

- *a*. If $H(W_1) > H(W_1)$ then $W_1 > W_2$; $\mathcal{L}^{(1)}(M) \geq \mathcal{L}^{(1)}(M) \geq \mathcal{L}^{(1)}(M)$) *then;* (*i*, *If* $\frac{1}{l}$ a. If $H(W_1) > H(W_1)$ then $W_1 > W_2$; *assistance as below,* $\begin{array}{c} (N_1 > W_2) \\ (N_2 > W_1) \end{array}$ $\binom{1}{2}$ W₂;
 $\binom{1}{2}$ M₂ W_2 ; *presented* as:
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 $\frac{1}{2}$ *presented* as: $\mathcal{L} \left(\begin{array}{c} 1 \ 1 \end{array} \right)$: $\mathcal{L} \left(\begin{array}{c} 1 \ 1 \end{array} \right)$ *and* (*)* **and** () **and** () **and** () it is *i*ndependent by *LAC* () in *presented* 2 a. If $H(W_1) > H(W_1)$ then $W_1 > W_2$: a. If $H(W_1) > H(W_1)$ then W_1 . \mathcal{I} If $H(W_{\epsilon}) > H(W_{\epsilon})$ then $W_{\epsilon} > W_{\epsilon}$.
- b. If $H(W_1) < H(W_1)$ then $W_1 < W_2$; *w*, $\mu_{\text{max}} = \frac{1}{2} \int \ln(W_1) \cdot \ln(W_1) \cdot \ln(W_1) \cdot \ln(W_2) \cdot \ln(W_3) \cdot \ln(W_4) \cdot \ln(W_5) \cdot \ln(W_6) \cdot \ln(W_7) \cdot \ln(W_7) \cdot \ln(W_8) \cdot \ln(W_7) \cdot \ln(W_8) \cdot \ln(W_9) \cdot \ln(W_9) \cdot \ln(W_1) \cdot \ln(W_1$ *w*, $\frac{1}{2}$ **f** $\frac{1}{2}$ $\$ *where if* $\mu_1(\mathbf{w}_1) \leq \mu_2(\mathbf{w}_1)$ *and* $\mathbf{w}_1 \leq \mathbf{w}_2$ *,* $\mu_3(\mathbf{w}_2) = \mu_4(\mathbf{w}_3) + \mu_5(\mathbf{w}_4) + \mu_6(\mathbf{w}_5)$ μ [1(W₁) \le 11(W₁) *inen* W₁ \le W₂,
 μ \le μ (W₁) \le μ \le μ ₀ W₁ \le W₁ W_2 ;
- c. If $H(W_1) = H(W_1)$ then $W_1 = W_2$. *then* $\frac{1}{2}$ *If* $H(W_1) = H(W_1)$ *then* $W_1 = W$ *c. If* $H(W_1) = H(W_1)$ *then* $W_1 = W$ c. If $H(W_1) = H(W_1)$ then $W_1 = W_2$. *assistance as below,* W_2 . $A = H(W_1)$ then $W_1 = W_2$. $(1 - w)^2$. c. If $H(W_1) = H(W_1)$ then $W_1 = W_2$. *noted and defined as follows:* $(v, \quad y \; \Pi(\mathbf{w}_1) - \Pi(\mathbf{w}_1)$ then $\mathbf{w}_1 - \mathbf{w}_2$. (i) $\int f(x) dx = \int f(x) f(x) dx$, $\int f(x) dx = \int f(x)$, $\int f(x) dx$, $(f_1 I_1(\mathbf{w}_1) - I_1(\mathbf{w}_1))$ then $\mathbf{w}_1 - \mathbf{w}_2$. c. If $H(W_1) = H(W_1)$ then $W_1 = W_2$. W_2 . *presented as:* c. If $H(W_1) = H(W_1)$ then $W_1 = W_2$.

Definition 3. Suppose $W_1 = (m_1, n_1)$ and $W_2 = (m_2, n_2)$ are IFSs, some basic operations are described as follows: described as follows: *described as follows:* $\sum_{i=1}^{n} a_i$ *) and* Ⱳ² *= (*² *,* ² *) are IFSs, some basic operations are* **Definition 3:** *Suppose* Ⱳ¹ *= (*¹ *,* ¹ *) and* Ⱳ² *= (*² **1011 5.** *Suppose* $w_1 = (m_1, n_1)$ *unu* w_2) < (Ⱳ¹) *then* Ⱳ¹ < Ⱳ² *; a. Suppose* $\mathbf{w}_1 = (m_1, n_1)$ and $\mathbf{w}_2 = (m_2, n_2)$ **b.If** $\frac{1}{2}$ $\frac{1$ **Definition 3** Suppose $W_1 = (m_1)$ **Definition** 5. Suppose $\mathbf{w}_1 = (m_1, n_1)$ and $\mathbf{w}_2 = (m_2, n_2)$ are 11 55, some basic operations) *then* Ⱳ¹ < Ⱳ² *;* **inition 3.** Suppose $W_1 = (m_1, n_1)$ and $W_2 = (m_2, n_2)$ are $\frac{1}{1}$ \cdots $\frac{1}{1}$, \cdots $\frac{1}{1}$, \cdots $\frac{1}{1}$, \cdots $\frac{1}{1}$ **Definition 3.** Suppose $W_1 = (m_1, n_1)$ and $W_2 = (m_2, n_2)$ are IFSs, some basic operations are $\mathbf{v}_1 - (m_1, n_1)$ and tion 3 Suppose $W_1 = (m_1, n_1)$ and $W_2 = (m_2, n_2)$ are IFSs, some basic operations are **Definition 3.** Suppose $W_1 = (m_1, n_1)$ and $W_2 = (m_2, n_2)$ are IFSs, some basic operations are described as follows: *noted and defined as follows:* () + () ≤ 1 *for all* ∈ *. Generally, the pair (,) represents the IFN.* () + () ≤ 1*for all* ∈ *. Generally, the pair (,) represents the IFN.* () +() ≤ 1 *for all* ∈ *. Generally, the pair (,) represents the IFN.* () + () ≤ 1 *for all* ∈ *. Generally, the pair (,) represents the IFN. where* Ⱳ(): → [0, 1]*and* Ⱳ(): → [0, 1] *signify the MG and NMG with condition* 0 ≤ **Definition 2:** *For IFNs,*Ⱳ = (Ⱳ, Ⱳ) *and the score function and accuracy functions are de-***DEFINITION 2: For** *For IFNs,* $\mathbf{F}(\mathbf{r}) = \mathbf{F}(\mathbf{r})$ and accuracy function and accuracy functions are de-**Definition 5.** Suppose $W_1 = (m_1, n_1)$ and $W_2 = (m_2, n_2)$ are *where* Ⱳ(): → [0, 1] *and* Ⱳ(): → [0, 1] *signify the MG and NMG with condition* 0 ≤ **Definition 3.** Suppose $W_1 = (m_1, n_1)$ and $W_2 = (m_2, n_2)$ are IFSS, some *where* Ⱳ(): → [0, 1]*and* Ⱳ(): → [0, 1] *signify the MG and NMG with condition* 0 ≤ $(10n\ 3. \quad \text{Suppose } w_1 = (m_1, n_1) \text{ and } w_2 = (m_2, n_2) \text{ are 11-33, some basic open.}$ *where* Ⱳ(): → [0,1]*and* Ⱳ(): → [0, 1] *signify the MG and NMG with condition* 0 ≤ **Definition 3.** Suppose $W_1 = (m_1, n_1)$ and $W_2 = (m_2, n_2)$ are IFSs, some basic operations are *west we as follows.*
 where the mass of light condition of MG and NMG with conditions of MG and NMG with conditions of the MG and NMG *where is a* $\frac{1}{2}$ *signify the MG* and NMG and NMG with conditions of MG and NMG with condition 0 ≤ ≤ ≤ $\frac{1}{2}$ θ *described as followers*: $A = \sum_{i=1}^{n} A_i$ and $A = \sum_{i=1}^{n} A_i$ is the ISS model. If $A = \sum_{i=1}^{n} A_i$ is simulated. If $A = \sum_{i=1}^{n} A_i$ is simulated. If $A = \sum_{i=1}^{n} A_i$ is not in A_i if A_i is not in A_i if A_i is not in A_i is not in A_i i **Example 3** is a given $\mathbf{M} = (m_1, m_1)$ and $\mathbf{W}_2 = (m_2, m_2)$ are in as

) and Ⱳ² *= (*²

Definition 3: *Suppose* Ⱳ¹ *= (*¹

c. If (Ⱳ¹

) *then* Ⱳ¹ = Ⱳ²

- (i) $W_1 \oplus W_2 = (\{m_1 + m_2 m_1m_2\}, \{n_1n_2\})$; (i) $W_1 \oplus W_2 = (\{m_1 + m_2 - m_1m_2\})$ $m_1 \oplus W_2 = (\{m_1 + m_2 - m_1m_2\}, \{n_1n_2\})$ $W_2 = (\{m_1 + m_2 - m_1m_2\}, \{n_1n_2\});$ (i) *II* (i) $W_1 \oplus W_2 = (\{m_1 + m_2 - m_1m_2\}, \{n_1, n_2\})$; 2 *f J t*
) **.** (i) $W_1 \oplus W_2 = (\{m_1 + m_2 - m_1m_2\} \{n_1, n_2\})$. $\mathbf{W}_1 \oplus \mathbf{W}_2 = (\{m_1 + m_2 - m_1m_2\}, \{n_1n_2\}),$ (i) $W_1 \oplus W_2 = (\{m_1 + m_2 - m_1m_2\}, \{n_1n_2\});$ (1) $W_1 \oplus W_2 = (\{m_1 + m_2 - m_1m_2\}, \{n_1n_2\})$ (i) $W_1 \oplus W_2 = (\{m_1 + m_2 - m_1m_2\}, \{n_1n_2\});$ (i)
- (ii) $W_1 \otimes W_2 = (\{m_1 + m_2 m_1m_2\}, \{n_1 + n_2\})$;

(ii) $W_1 \otimes W_2 = (\{m_1m_2\}, \{n_1 + n_2 n_1n_2\})$; (ii) $\mathbf{w}_1 \otimes \mathbf{w}_2 = (\begin{matrix} 1 \\ m_1 m_2 \end{matrix}, \begin{matrix} 1 \\ n_1 \end{matrix})$ $\begin{array}{ccc} \overline{1} & \overline{1} & \overline{1} & \overline{1} & \overline{1} \end{array}$ $(1 + \frac{1}{2})$ (ii) $W_1 \otimes W_2 = (\{m_1m_2\}, \{n_1 + n_2 - n_1n_2\})$ $\left(\frac{1}{2} \right)$ (i) $W_1 \oplus W_2 = (\{m_1 + m_2 - m_1m_2\}, \{n_1, n_2\})$
(ii) $W_1 \otimes W_2 = (\{m_1m_2\}, \{n_1 + n_2 - n_1n_2\})$ $\mathbf{w}_1 \otimes \mathbf{w}_2 = (\mathbf{w}_1)^T$ $W_1 \otimes W_2 = (\mu_1 m_2 f_1) \mu_1 + \mu_2$
 $W_2 = (1 - (\mu_1)^2 \mu_2) \tau > 0$ (ii) $W_1 \otimes W_2 = (\{m_1m_2\}, \{n_1+n_2-n_1n_2\})$ *, the score function and accuracy function provide the* (ii) $W_1 \otimes W_2 = (\{m_1m_2\}, \{n_1+n_2-n_1n_2\})$ $\binom{n+1}{2}$. (Ⱳ) = ^Ⱳ − Ⱳ, (Ⱳ) ∈ [−1, 1] (2) (Ⱳ) = ^Ⱳ − Ⱳ, (Ⱳ) ∈ [−1, 1] (2) $\frac{1}{2}$ $\frac{1}{2}$, $\frac{1}{2}$ $(\cdot_1 \cdot \cdot \cdot_2)$
- (iii) $\exists W_1 = \left(1 (1 m)^{\frac{1}{2}}, n^{\frac{1}{2}}\right), \exists > 0;$ $- (1 - m)^{7}$, n^{7}), $1 > 0$; $\left(1 - \frac{n}{1}\right)$, $\left(1 - \frac{n}{1}\right)$ \int_{0}^{∞} (iii) $\exists W_1 = (1 - (1 - n))$ (iii) $\mathbf{J}W_1 = (1 - (1 - m)^{\mathbf{J}}, n^{\mathbf{J}}), \mathbf{J} > 0;$ *described as follows: c. If* (Ⱳ¹) = (Ⱳ¹) *then* Ⱳ¹ = Ⱳ² *.* $A_1 = (1 - (1 - m)^{J} \cdot n^{J})$. $J > 0$. $f(x) = \frac{1}{x} \left(\frac{1}{x} + \frac{m}{x} \right)$ $\int_{-1}^{1} f \cdot d \cdot d \cdot$ *a*. $\mathbf{v}_1 - (\mathbf{1} - \mathbf{m}) \cdot \mathbf{n}$ / \mathbf{v}_2 *a. If* (Ⱳ¹) > (Ⱳ¹) *then* Ⱳ¹ > Ⱳ² *a. If* (Ⱳ¹) > (Ⱳ¹ \int_{0}^{1} $\frac{1}{2}$ $\frac{1}{2}$ (iii) $\mathbb{J}W_1 = (1 - (1 - m)^2, n^2)$, $\mathbb{J} > 0$; (iii) **If** (i (iii) *If* (iii) *If* (iii) **If** (i *assistance as below, assistance as below, assistance as below, For comparing two IFNs,* Ⱳ¹ *and* Ⱳ² *, the score function and accuracy function provide the For comparing two IFNs,* Ⱳ¹ *and* Ⱳ² *, the score function and accuracy function provide the* $(1 - m), n^{\circ}$, $d > 0$, $> 0,$ (iii) $\mathbf{JW}_1 = (1 - (1 - m)^{\mathsf{T}}, n^{\mathsf{T}}), \mathbf{J} > 0;$ $>$ 0;
- (iv) $W_1^{\mathsf{J}} = ((m)^{\mathsf{J}} \cdot 1 (1 n)^{\mathsf{J}} \cdot \mathsf{J} > 0$: (iv) $W_1^{\text{I}} = ((m)^1, 1 - (1 - n)^1), \text{J}$ =1 = 1 ῶ ∈ [0, 1]. *Thus, the operator is a* $= ((m)^2, 1 - (1 - i))$ (iv) $W_1^{\texttt{I}} = ((m)^{\texttt{J}}, 1 - (1 - n)^{\texttt{J}}), \texttt{J} >$ $\mathbf{u}_1 - \mathbf{v}_2$ $W_{\mathbf{r}}^{\mathbf{I}} = (\hat{i}m)^{\mathbf{I}} \quad 1 - (1 - n)^{\mathbf{I}} \quad \mathbf{I} > 0$ $\mathbf{w}_1 = (\mathbf{w}, \mathbf{u} \cdot \mathbf{u}) \quad (1 \quad \mathbf{w}) \quad (\mathbf{w}, \mathbf{v})$ (v) $w_1 = (m)$, $1 - (1 - n)$ \sum_{μ} $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$, $\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$, $\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$, $\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$, $\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$, $\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$, $\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$, $\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$, $\begin{pm$ \int_{0}^{1} $\sqrt{1}$ *d* $\sum_{i=1}^{1}$ $\sum_{i=1}^{1}$) *then* Ⱳ¹ = Ⱳ² δ *iv*) $W_1^{\perp} = ((m)^2, 1 - (1 - n)^2), \exists > 0;$ $= ((m)^{-}, 1 - (1 - n)^{-})$, $\Box > 0$; $\binom{d}{r}$, 1 – $(1-n)^d$, J > 0;) *then* Ⱳ¹ < Ⱳ² $W_1 = \left(\binom{m}{1}, 1 - \binom{m}{2}, \ldots\right)$ $\binom{m}{1 - (1 - n)}$, $\frac{1}{n} > 0$, \sim $\overrightarrow{M_1}$ $\overrightarrow{M_2}$ \overline{a} $\mathbf{w}_1 = (m)$ (iv) $W_1^{\text{J}} = ((m)^{\text{J}}, 1 - (1 \left(\begin{array}{cc} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{array}\right)$ (iv) $W_1 = \left(\binom{m}{r}, 1 - (1 - n)\right)$ \overrightarrow{M}_{1} \overrightarrow{M}_{2} \overrightarrow{M}_{3} \overrightarrow{M}_{4} \overrightarrow{M}_{5} \overrightarrow{M}_{6} (iv) $W_1 = \left(\frac{m}{n}, 1 - (1 - n)\right)$, $\rightarrow 0$; \mathcal{L} $\mathbf{w}_1 - (\mathbf{w})$, $\mathbf{r} - (\mathbf{r})$ (V) $W_1 - (m)$, $1 - (1 - n)$ $\mathbf{w}_1 - (\mathbf{w}_1, \mathbf{r}_1 - (\mathbf{r}_1 - \mathbf{r}_1))$, $\mathbf{r}_2 > 0$, (iv) $W_1^J = ((m)^J, 1 - (1 - n)^J), J > 0;$ *, the score function and accuracy function provide the* $\mathsf{I} > 0;$ $\left(\frac{1}{2}\right)^{3/2}$ $\begin{pmatrix} -1 & 1 \\ 0 & -1 \end{pmatrix}$ (iv) $W_1^{\mathsf{T}} = ((m)^{\mathsf{T}}, 1 - (1 - n)^{\mathsf{T}}), \mathsf{T} > 0;$
- $\begin{pmatrix} x \\ y \end{pmatrix}$ *MC* = (n, m) (v) $W_1^c = (n_1, m_1).$ **Definition 4** [20]: *Assume that* Ⱳ = (**Definition 4** [20]: *Assume that* Ⱳ = ((v) $W_1^c = (n_1, m_1).$ *described as follows:* $\overline{}$ (**iii**)^{*If*} (**i**)) *then;* F_1). (m, m) $\left(\frac{1}{2}, \frac{1}{2}\right)$ *noted and defined as follows:* $\mathbf{v} = \mathbf{w}_1 - (n_1, m_1).$ ${\bf W}_1 - (n_1, m_1)$ *.* (v) $W_1^c = (n_1, m_1).$ (v) $W_1^c =$

) *then* Ⱳ¹ = Ⱳ²

b. If (Ⱳ¹

Table 1. Symbols with their descriptions.

c. If (Ⱳ¹

) = (Ⱳ¹

easiness of understanding.

Definition 4 ([20]). Assume that $W_j = (m_j, n_j)$ is a collection of IFSs; the weights $\tilde{\omega}_j =$ $(\omega_1,\omega_2,\ \ldots,\ \omega_n)$ for ω_j is ω_j p *ing of IFPWA* $_{\omega}$: $W^{n} \rightarrow W$ where $(\tilde{\omega}_1, \tilde{\omega}_2, \ldots, \tilde{\omega}_n)^T$ for W_i with $\sum_{i=1}^n \tilde{\omega}_i = 1$ and $\tilde{\omega}_i \in [0, 1]$. Thus, the IFPWA_{ω} operator is a *mapping* of $IFPWA_{\omega}$: $W^{n} \rightarrow W^{n}wh$ $\left(\tilde{\omega}_1, \tilde{\alpha} \right)$ m appii $(\tilde{\omega}_1, \tilde{\omega}_2)$. $mapping$ *o* $(\tilde{\omega}_1, \tilde{\omega}_2, \ldots, \tilde{\omega}_n)^T$ $mapping$ of IFPWA σ_n)^T for $\text{$ *mapping of* I *FPWA_ω: W* \rightarrow *W where* $(\tilde{\omega}_1, \tilde{\omega}_2, \ldots, \tilde{\omega}_n)^T$ for W_j with $\sum_{j=1}^n \tilde{\omega}_j =$ *mapping of IFPWA* $_{\omega}$: $W^{n} \rightarrow W$ where $(\tilde{\omega}_1, \tilde{\omega}_2, \ldots, \tilde{\omega}_n)^T$ for W_j with $\sum_{j=1}^n \tilde{\omega}_j = 1$ and $\tilde{\omega}_j \in$ mapping of IFPWA_{ω}: $W^n \rightarrow W$ where *^j*∈ [0, 1]. *Thus, the IFPWA^ω operator is a mapping of IFPWAω: presented as:* $W^n \rightarrow W$ where *presented as:* **W** where $\frac{d}{dx}$ $t_0(\tilde{\omega}_1,\tilde{\omega}_2,\ldots,\tilde{\omega}_n)^T$ for W_i with $\sum_{i=1}^n\tilde{\omega}_i=1$ and $\tilde{\omega}_i\in[0, 1]$. Thus, the IFPWA_{ω} operator is mappin $t_1,\tilde{\omega}_2,\ldots,\tilde{\omega}_n$ ^T for W_i with $\sum_{i=1}^n\tilde{\omega}_i=1$ and $\tilde{\omega}_i\in[0, 1]$. Thus, the IFPWA_{ω} operator is a pping of $\tilde{\omega}_n$ ^T for W_i with $\sum_{i=1}^n \tilde{\omega}_i = 1$ and $\tilde{\omega}_i \in [0, 1]$. Thus, the IFPWA_w operator is a $F P W A_{\omega}$ (i) ∈1 ⊕ ∈1 ⊕ 2 = (i) ∈1 ∈ 12 = (i) ∈12 = $\frac{1}{2}$ ${\rm Definition~4~(}$ [20 $(\tilde{\omega}_1, \tilde{\omega}_2)$ mapping ($\mathcal{M} = (\omega, \omega)$ is a giller time of IFSs, the mojelite $\tilde{\omega}$ $\begin{array}{ccc} \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{array}$ **Definition 1** *Definition 1**and* **(***)***.** *Let**on**b**b**symbolized by <i>and* () *and* () *and* **()** *and* $\sum_{i=1}^{\infty}$ **Definition 1** *Definition 1**and* (*)*. *Let* \mathbf{p} $(\omega_1, \omega_2, \ldots)$ mapping of IFPW₁ $A = \binom{m}{j}$ by $\binom{n}{j}$ proposed the ISS model. If $s = 1$ $t_{\rm{eq}} = t$ and $\omega_j \in [0, 1]$. Thus, the TFP w A_ω operator is a wnere **Definition 4** ([20]). Assume that $W_j = (m_j, n_j)$ is a collection of IFSs; the weights for $\rm\,W\,$ with $\rm \sum_{j=1}^{n}\omega_{j} = 1$ and $\rm \omega_{j} \in [0,1].$ Thus, the IFPWA $_{\omega}$ operator is a PWA_{ω} : $W^{\prime\prime} \rightarrow W$ where χ_1 ^T for **I** mapping of IFPWA_{ω}: $W^n \rightarrow W$ where $\left| \begin{array}{cc} 1 & 2 \\ 1 & 2 \end{array} \right|$ A and $\omega_j \in [0, 1]$. Thus, the IFPWA_{ω} operator is a taneously delivers \mathcal{L} and \mathcal{L} is an element in a given in $(\tilde{\omega}_1, \tilde{\omega}_2, \ldots, \tilde{\omega}_n)^T$ for W_i with $\sum_{i=1}^n \tilde{\omega}_i = 1$ and $\tilde{\omega}_i \in [0, 1]$. Thus, the $\sum_{i=1}^{n}$ $\sum_{j=1}^{n}$ denotes $\sum_{i=1}^{n}$ $(\omega_1, \omega_2, \ldots, \omega_n)$ for w_j with $\sum_{j=1}^{\infty} \omega_j = 1$ and $\omega_j \in [0, 1]$. Thus, the IFFWA_{ω} opera mapping of TFPWA_{ω}: W \rightarrow W where $\left(\tilde{\omega}_1 \right)$ *map* $\sum_{n=0}^{\infty}$ Non-membership Grade DTRS $\binom{n}{r}$ by $\binom{n}{r}$ by $\binom{n}{r}$ by $\binom{n}{r}$ by $\binom{n}{r}$ $(\omega_1, \omega_2, \ldots, \omega_n)$ for w_j with $\sum_{j=1}^{\infty} \omega_j = 1$ and $\omega_j \in [0, 1]$. Thus, the IFFWA_w operator is a mapping of IFPWA_{ω}: W \rightarrow W where $\sum_{n=0}^{\infty}$ $\sum_{n=0}^{\in$ $(\omega_1, \omega_2, \ldots, \omega_n)$ for w_j with $\sum_{j=1}^{\infty} \omega_j = 1$ and $\omega_j \in [0, 1]$. Thus, the IFFWA_{ω} operator is a mapping of IFPWA_{ω}: W \rightarrow W where **Definition 4** ([20]). Assume that $W_j = (m_j, n_j)$ is a collection of IFSs; the $F(\omega_1,\omega_2,\ldots,\omega_n)$ for W_j with $\sum_{i=1}^n \omega_i = 1$ and $\omega_j \in [0, 1]$. Thus, the IFPV mapping of IFPW A_{ω} : $W^n \rightarrow W$ where $F(\omega_1,\omega_2,\ldots,\omega_n)$ for W_j with $\sum_{i=1}^n \omega_i = 1$ and $\omega_j \in [0, 1]$. Thus, the IFPWA_{ω} mapping of IFPW A_{ω} : $W^{n} \rightarrow W$ where $F(\omega_1,\omega_2,\ldots,\omega_n)$ for W_j with $\sum_{i=1}^n\omega_i=1$ and $\omega_j\in[0,\,1]$. Thus, the IFPWA_{ω} operator is a mapping of IFPW A_{ω} : $W^{n} \rightarrow W$ where $F(\omega_1,\omega_2,\ldots,\omega_n)$ for W_j with $\sum_{i=1}^n\omega_i=1$ and $\omega_j\in[0,1]$. Thus, the IFPWA_{ω} operator is a mapping of IFPWA_{ω}: $W^n \rightarrow W$ where $T_{j} = \begin{bmatrix} \omega_1, \omega_2, \dots, \omega_n \end{bmatrix}$ with $\sum_{j=1}^{n} \omega_j = 1$ and $\omega_j \in [0, 1]$. Thus, the ITT W_{j} operator is **Definition 4** ([20]). Assume that $W_j = (m_j, n_j)$ is a collection of IFSs; the weights of mapping of $11 \text{ F} \text{v} \lambda \omega$. $\text{w} \rightarrow \text{w}$ where $(\tilde{\omega}_1$ (μ_X, ν) = (μ_X, ν) = (μ_X, ν) = (μ_X, ν) $(\tilde{\omega}_1, \tilde{\omega}_2)$. (111 VVI_ω) . W \rightarrow W where $(\tilde{\omega}_1, \tilde{\omega}_2, \ldots,$ $(\mathbf{W} \times \mathbf{W})$ and $(\mathbf{W} \times \mathbf{W})$ $(\tilde{\omega}_1, \tilde{\omega}_2, \ldots, \tilde{\omega}_n)^T$ for \mathcal{L} $\mathcal{$ $(\tilde{\omega}_1, \tilde{\omega}_2, \ldots, \tilde{\omega}_n)^T$ for W_j with $\sum_{j=1}^n \tilde{\omega}_j = 1$ and $\tilde{\omega}_j \in$ () + ()≤ 1*for all* ∈ *. Generally, the pair (,) represents the IFN.* 2.3 2.4 mapping of IFPWA_{ω}: $W^n \rightarrow W$ where $\sum_{i=1}^{\infty} \frac{1}{i} \left(\frac{1}{i} \sum_{i=1}^{n} \sum_{j=1}^{n} \$ μ *mapping of IFPWA* $\omega: W^n \rightarrow W$ where () + () ≤1 *for all* ∈ *. Generally, the pair (,) represents the IFN.* $\frac{20}{\mu}$ is $\frac{20}{\mu}$ of $\frac{20}{\mu}$ if $\frac{20}{\mu}$, $\frac{20}{\mu}$ is $\frac{20}{\mu}$, $\frac{20}{\mu}$ is $\frac{20}{\mu}$ $(\tilde{\omega}_1, \tilde{\omega}_2, \tilde{\omega}_3)$ $\tilde{\omega}_3$ *for* $W: \text{with } \nabla^n_i, \tilde{\omega}_3$ $\begin{bmatrix} 1 & 2 \end{bmatrix}$ $\begin{bmatrix} u_1 & 1 \end{bmatrix}$ $\begin{bmatrix} 1 & 2 \end{bmatrix}$ $\begin{bmatrix} 1 & 2 \end{bmatrix}$ $(\tilde{\omega}_1, \tilde{\omega}_2) = (\tilde{\omega}_2)^T$ for W_1 with $\sum_{i=1}^n \tilde{\omega}_i = 1$ and $\tilde{\omega}_i \in [0, 1]$. Thus, the $(\tilde{\omega}_1, \tilde{\omega}_2, \ldots, \tilde{\omega}_n)^T$ for W_j with $\sum_{j=1}^n \tilde{\omega}_j = 1$ and $\tilde{\omega}_j \in [0, 1]$. Thus, the IF manning of LEDMA \cdots Mⁿ DRs Decision Rules IFRS Intuitionistic Fuzzy Rough Set IFPOWAIntuitionistic Fuzzy Power Order Weighted averaging IFPOWG Intuitionistic Fuzzy Power Order Weighted Geometric IFPWA Intuitionistic Fuzzy Power Weighted Averaging IFPWGIntuitionistic Fuzzy Power Weighted GeometricIFPOWA Intuitionistic Fuzzy Power Order Weighted averaging IFPOWG Intuitionistic Fuzzy Power Order Weighted Geometric mapping of $IFPWA_{\alpha}$: $W^n \rightarrow W$ where $(\tilde{\omega}_1, \tilde{\omega}_2, \ldots, \tilde{\omega}_n)^T$ for W_i with $\sum_{i=1}^n \tilde{\omega}_i$ $(\tilde{\omega}_1, \tilde{\omega}_2, \ldots, \tilde{\omega}_n)^T$ for W_j with mapping of IFPWA_{ω}: $W^n \rightarrow$ IFS International Functionistic Functionistic Functionistic Functionistic Functionistic Functionistic Function
The contract Function International Function International Function International Function International Funct $(\tilde{\omega}_1, \tilde{\omega}_2, \ldots, \tilde{\omega}_n)^T$ for W_i with $\sum_{i=1}^n \tilde{\omega}_i = 1$ and mannino of IFPWA \cdot Wⁿ \rightarrow W where TWD Three-Way Decision MG Membership Grade $(\tilde{\omega}_1, \tilde{\omega}_2, \ldots, \tilde{\omega}_n)^T$ for W ; with $\sum_i^n A_i \tilde{\omega}_i = 1$ and $\tilde{\omega}_i \in$

FSs Fuzzy Sets IHFSs Intuitionistic Hesitant Fuzzy Sets

In this section, we update models for \mathbb{R}^n and several concepts \mathbb{R}^n and several concepts \mathbb{R}^n

2.1. IFSs

2.1. IFSs

Ⱳ = 〈,Ⱳ(), Ⱳ())| ∈ 〉 (1)

For Γ -statistic Hesitant Fuzzy Sets Intuitionistic Hesitant Fuzzy Sets Intuitionistic Hesitant Fuzzy Sets International European Sets International European Sets International European Sets International European Sets

In this section, we update models for $\mathcal{L}_{\mathcal{S}}$ and several concepts pertaining to power and several concepts $\mathcal{L}_{\mathcal{S}}$

$$
\bigoplus_{j=1}^{n} (\widetilde{\omega}_{j}(1+T(W_{j})W_{j})
$$
\n
$$
= \left(1 - \prod_{j=1}^{n} (1-(m_{j})^{\frac{\widetilde{\omega}_{j}(1+T(W_{j}))}{\sum_{j=1}^{n} \omega_{j}(1+T(W_{j}))}}, \prod_{j=1}^{n} (n_{j})^{\frac{\widetilde{\omega}_{j}(1+T(W_{j}))}{\sum_{j=1}^{n} \omega_{j}(1+T(W_{j}))}}\right)
$$

 \mathbf{r} $\mathbf{$ *For comparing two IFNs,* Ⱳ¹ *and* Ⱳ² *assistance as below, noted and defined as follows: noted and defined as follows: where presented as:* 2.1 *bere* \mathcal{I} intervalstic Fuzzy Power Weighted Averaging Intervalstic Fuzzy Power Weighted Geometric Fuzzy Power Geometric Fuzzy Power Weighted Geometric Fuzzy Power Weighted Geometric Fuzzy Power Weighted Geometric Fuzzy Pow $\frac{1}{2}$) *then* Ⱳ¹ < Ⱳ² *a. If* (Ⱳ¹ $_{\textit{above}}$ *b. If* (Ⱳ¹ $\sum_{i=1}^{n}$ z_{1} *a* z_{2} \overline{z} *b. If* (Ⱳ¹ \mathcal{L} (iii) \mathcal{L} $\sum_{i=1}^{n}$ **Table 1.** Symbols with their descriptions. **Table 1.** Symbols with their descriptions. **Symbol Description Symbol Description** *where* Ⱳ(): → [0, 1] *and* Ⱳ(): → [0, 1] *signify the MG and NMG with condition* 0 ≤ *where* Ⱳ(): → [0, 1] *and* Ⱳ(): → [0, 1] *signify the MG and NMG with condition*0 ≤ *where* Ⱳ(): → [0, 1] *and* Ⱳ(): → [0, 1] *signify the MG and NMG with condition* 0 ≤ () + () ≤1 *for all* ∈ *. Generally, the pair (,) represents the IFN.* ν *here* α *L a Mathematics <i>2023***,** *2023 L a Mathematics <i>2023***,** *2023* ω and ω and ω in a given DRs Decision Rules IFRS Intuitionistic Fuzzy Rough Set

(iv) ₍

) *then* Ⱳ¹ = Ⱳ²

set [0, 1].
Set [0, 1].
Set [0, 1].

taneously delivers MG and NMG while FS just delivers the MG of an element in a given

) = (Ⱳ¹

 $\overline{}$

As an extension of the FS model, Atanassov $\mathcal{I}(\mathcal{I})$ proposed the IFS simulation $\mathcal{I}(\mathcal{I})$

= (¹

) *then* Ⱳ¹ = Ⱳ²

) *then* Ⱳ¹ < Ⱳ²

FSs Fuzzy Sets IHFSs Intuitionistic Hesitant Fuzzy Sets

taneously delivers MG and NMG while FS just delivers the MG of an element in a given

As an extension of the FS model, Atanassov \mathbb{R}^n proposed the IFS simulation \mathbb{R}^n

where
\n
$$
T(W_j) = \sum_{i=1}^{n} \tilde{\omega}_j Sup(W_j, W_i)
$$
\n
$$
i \neq j
$$

and () + () ≤ 1 *for all* ∈ *. Generally, the pair (,) represents the IFN. presented as:* set [0, 1]. set [0, 1]. **definition** θ *and* **and Definition** 3: *<u>Contract</u>* **and** *then* **then 1** $\frac{1}{2}$ \mathcal{A}) *then* Ⱳ¹ > Ⱳ² \mathcal{L}_{S} **Symbol Description Symbol Description** I **2. Preliminaries** d \mathcal{A} and \mathcal{A}

) and () and

$$
Sup(W_j, W_i) = 1 - d(W_j, W_i)
$$

$$
d(W_j, W_i) = \frac{1}{m} \sum_{\substack{i=1 \\ i \neq j}}^m (|m_i - m_j| + |n_i - n_j|)
$$

 \mathcal{C} : (i) \mathcal{C} = ([20]) \mathcal{C} = \mathcal{C} = **tinition 5** ([20]). For IFNs, $W_j = (m_j)$ $\langle t | \tilde{\omega}_j > 0, \tilde{\omega}_j \in [0, 1] \text{ and } \sum_{j=1}^n \tilde{\omega}_j = 1.$ follows: $\mathbf{Definition} \mathbf{S}$ ([20]) $\mathbf{For}\ \mathbf{IFN}\mathbf{S}$ M $\liminf_{n \to \infty} \frac{1}{n} = \frac{n}{n}$
that $\tilde{\omega}_1 > 0$, $\tilde{\omega}_2 \in [0, 1]$ and $\sum_{n=1}^n \tilde{\omega}_2 =$ $\lim_{j \to \infty} \omega_j > 0, \ \omega_j \in [0, 1]$ and *described as follows:* that $\tilde{\omega}_i > 0$, $\tilde{\omega}_i \in [0, 1]$ and \sum^n \int_0^1 \int_0^1 \int $\frac{1}{2}$ $\frac{1}{$ D_{α} G_{α} iti $\alpha \in (0.01)$ that $\tilde{\omega}_i > 0$, $\tilde{\omega}_i \in [0, 1]$ and $\sum_{i=1}^{n'} \tilde{\omega}_i =$ $\frac{1}{\sqrt{1 - \frac{1}{2}}}$ j = $(i \in \mathbb{N})$ *b***.** *If inition 5* ([20]). For IFNs, $W_i = (m_i, n_i)$ *it* $\tilde{\omega}_i > 0$, $\tilde{\omega}_i \in [0, 1]$ and $\sum_{i=1}^{n'} \tilde{\omega}_i = 1$. **Definition 3:** *Suppose* Ⱳ¹ *= (*¹ *ion 5* ([20]). For IFNs, $W_i = (m_i, n_i)$ u $\lambda > 0$, $\tilde{\omega}_i \in [0, 1]$ and $\sum_{i=1}^{n'} \tilde{\omega}_i = 1$. A ma *b.* For IFNs, $W_i = (m_i, n_i)$ with their α $\sum_{i=1}^{n} \tilde{\omega}_i = 1$. A mapping of 1 *c. If* (Ⱳ¹) = (Ⱳ¹) *then* Ⱳ¹ = Ⱳ² *.* $\omega_j = 1$. *A* mapping of IFPOWA_{ω}: **W** \rightarrow **W** is defined *) are IFSs, some basic operations are c. If* (Ⱳ¹) = (Ⱳ¹) *then* Ⱳ¹ = Ⱳ² *.* $\mathcal{D} = 1.$ *A mapping of <code>IFPOWA</code>* $_{\omega}$ *: w* $^{\circ}$ \rightarrow *W* $^{\circ}$ *<i>is defined as a. If* (Ⱳ¹) > (Ⱳ¹) *then* Ⱳ¹ > Ⱳ² *;* μ_j , n_j) with their weights $\omega_j = (\omega_1, \omega_2, ...)$ 1. *A mapping of IFPOWA* $_{\omega}$: **w**^{α} \rightarrow **w** . For IFNs, $W_j = (m_j, n_j)$ with their we [0, 1] and $\sum_{j=1}^{n} \tilde{\omega}_j = 1$. A mapping of 1*F* $[20]$). For IFNs, $W_i = (m_i, n_i)$), 1] and $\sum_{j=1}^n \tilde{\omega}_j = 1$. A mapping of IFF \mathbf{z}_j with their weights $\mathbf{\tilde{\omega}}_j = (\mathbf{\tilde{\omega}}_1, \mathbf{\tilde{\omega}}_2, \ldots, \mathbf{\tilde{\omega}}_n)$ *A* mapping of $IFPOWA_{\omega}: W^{n} \rightarrow W$ is (u_i, n_i) with their weights $\tilde{\omega}_i$ = *mapping of IFPOWA* $_{\omega}$: $W^{n} \rightarrow W$ *is a* **nition 5** ([20]). For IFNs, $W_j = (m_j, n_j)$ with their weights $\tilde{\omega}_j = (\tilde{\omega}_1, \tilde{\omega}_2, \ldots, \tilde{\omega}_j)^T$ such *that* $\tilde{\omega}_i > 0$, $\tilde{\omega}_i \in$ f_s , $W_j=(m_j,\ n_j)$ with their weights $\tilde\omega_j=(\tilde\omega_1,\tilde\omega_2,\ \ldots,\ \tilde\omega_j)^\mathsf{T}$ such *and* $\sum_{i=1}^{n} \tilde{\omega}_i = 1$. $\tilde{\mu}_j = (m_j, n_j)$ with their weights $\tilde{\omega}_j = (\tilde{\omega}_1, \tilde{\omega}_2, \, \dots, \, \tilde{\omega}_j)^T$ such $d\sum_{i=1}^n \tilde{\omega}_i = 1$. A n **Definition 5** ([20]). For IFNs, $W_j = (m_j, n_j)$ with their weights $\omega_j = (\omega_1, \omega_2, ..., \omega_j)$ such
that $\tilde{\omega}_i > 0$, $\tilde{\omega}_i \in [0, 1]$ and ∇^n , $\tilde{\omega}_i = 1$, A manning of LEDOWA, \mathcal{M}^{n} , \mathcal{M}^{n} , W is defined as $ln\omega_1 > 0, \omega_1 \in [0, 1]$ **Definition** 3 ([20]). For If Ns, $\mathbf{w}_j = (m_j, n_j)$ with their weights $\mathbf{w}_j = (\mathbf{w}_1, \mathbf{w}_2, \dots, \mathbf{w}_j)$ such that $\tilde{\alpha}_i > 0$, $\tilde{\alpha}_i \in [0, 1]$ and ∇^n , $\tilde{\alpha}_i = 1$, A manning of IEDOWA, ∂M^n , ∂M^n is defin $\omega_1 > 0, \omega_1 \in [0, 1]$ and ω_1 **Definition 5** ([20]). *For IFNs,* $W_j = (m_j, n_j)$ with their weights $\tilde{\omega}_j = (\tilde{\omega}_1, \tilde{\omega}_2, \ldots, \tilde{\omega}_j)^T$ su *where* $\omega_j > 0$, $\omega_j \in [0, 1]$ *and* $\Delta_{j=1}^{\infty} \omega_j = 1$. A mapping ω_j in 1 σ over ω_i , we \rightarrow we is acquied () + () ≤ 1*for all* ∈ *. Generally, the pair (,) represents the IFN.* **Definition 5** ([20]). *For IFNs,* $W_j = (m_j, n_j)$ *with their weights* $\tilde{\omega}_j =$ that $\tilde{\omega}_j > 0$, $\tilde{\omega}_j \in [0, 1]$ and $\sum_{j=1}^n \tilde{\omega}_j = 1$. A mapping of IFPOWA_{ω}: **Definition 5** ([20]). *For IFNs,* $W_j = (m_j, n_j)$ *with their weights* $\tilde{\omega}_j = (\tilde{\omega}_1, \tilde{\omega}_2)$ *that* $\tilde{\omega}_j > 0$, $\tilde{\omega}_j \in [0, 1]$ and $\sum_{j=1}^n \tilde{\omega}_j = 1$. A mapping of IFPOWA_{ω}: $W^n \to$ **Definition 5** ([20]). *For IFNs,* $W_j = (m_j, n_j)$ *with their weights* $\tilde{\omega}_j = (\tilde{\omega}_1, \tilde{\omega}_2, ...$ that $\tilde{\omega}_j > 0$, $\tilde{\omega}_j \in [0, 1]$ and $\sum_{j=1}^n \tilde{\omega}_j = 1$. A mapping of IFPOWA_{ω}: $W^n \to W$ **Definition 5** ([20]). *For IFNs,* $W_j = (m_j, n_j)$ with their weights $\tilde{\omega}_j = (\tilde{\omega}_1, \tilde{\omega}_2, \ldots, \tilde{\omega}_j)^T$ that $\tilde{\omega}_j > 0$, $\tilde{\omega}_j \in [0, 1]$ and $\sum_{j=1}^n \tilde{\omega}_j = 1$. A mapping of IFPOWA_{ω}: $W^n \to W$ is define *j T such* that $\tilde{\omega}_j > 0$, $\tilde{\omega}_j \in [0, 1]$ and $\sum_{j=1}^{n'} \tilde{\omega}_j = 1$. A mapping of IFPOWA_{ω}: 1 $follows:$ 10000 5000 5000 5 1 *follows:* $\frac{1}{2}$ *presented as:* $W^n \to W$ is defined as *presented as: W* is defined as **As an extending** \mathbf{A} \mathbf{A} and \mathbf{A} is \mathbf{A} and \mathbf{A} is \mathbf{A} and \mathbf{A} is \mathbf{A} and \mathbf{A} and \mathbf{A} is \mathbf{A} and \mathbf{A} is \mathbf{A} is \mathbf{A} and \mathbf{A} is \mathbf{A} and $\mathbf{A$ that $\tilde{\omega}_i > 0$, $\tilde{\omega}_i \in [0, 1]$ and $\sum_{i=1}^n \tilde{\omega}_i = 1$. A mapping of IFPOWA $\omega: W^n \to W$ is defined **As an extending The FS model in the FS model is model.** If $\tilde{\omega}_i = (\tilde{\omega}_1, \tilde{\omega}_2, \ldots, \tilde{\omega}_i)^T$ such tat $\tilde{\omega}_i > 0$, $\tilde{\omega}_i \in [0, 1]$ and $\sum_{i=1}^n \tilde{\omega}_i = 1$. A mapping of IFPOWA $\omega: W^n \to W$ is defined as (a) Eq. (FNs, $W_i = (m_i, n_i)$ with their weights $\tilde{\omega}_i = (\tilde{\omega}_1, \tilde{\omega}_2, \ldots, \tilde{\omega}_i)^T$ such $\tilde{\omega}_i \in [0, 1]$ and $\sum_{i=1}^n \tilde{\omega}_i = 1$. A mapping of IFPOWA $\omega : W^n \to W$ is defined as set [0, 1]. Definition 5 ($[20]$ that $\tilde{\omega}_j$ taneously delivers $M_{\rm G}$ is a given in a g $i = (m_j,$ taneously delivers M_G and N_G while \overline{S} is an element in a given in a n_j , n_j) u $that \tilde{\omega}_i > 0$ ℷ $(\alpha_j,\ n_j)$ with their weights $\omega_j=(\omega_1,\omega_2,\ \ldots,\ \omega_j)$ such $\omega_j =$ 1. A mapping of TFPOWA $_\omega$:W $^\circ$ \rightarrow W $^\circ$ is aefined as **Definition 5** ([20]), For IFNs, $W_i = (m_i, n_i)$ *As a matrice of the FS model* $\sum_{i=1}^{n} \tilde{\omega}_i = 1$. A mapping of IFPOWA_{ω}: $W^n \rightarrow W$ is defined as **Definition 5** ([20]) For IFNs, $W = (m - n)$ with their weights $\tilde{\omega} = 0$ $(1, 2, 3)$ $(1, 2, 3)$ *i. n*_i) wi that $\tilde{\omega}_j > 0$, $\tilde{\omega}_j \in [0, 1]$ and $\sum_{i=1}^n \tilde{\omega}_j = 1$. A mapping of IFPOWA_{ω}: $W^n \to W$ is defined as Two Two Three-Way Decision $\mathcal{L}_\mathcal{A}$ and $\mathcal{L}_\mathcal{A}$ and $\mathcal{L}_\mathcal{A}$ and $\mathcal{L}_\mathcal{A}$ and $\mathcal{L}_\mathcal{A}$ **Demittion 5** ([20]). For IFINS, $\mathbf{w}_j = (m_j, n_j)$ with their weights $\mathbf{w}_j = (\mathbf{w}_j, n_j)$ If the $\omega_j > 0$, $\omega_j \in [0, 1]$ and $\sum_{j=1}^{\infty} \omega_j = 1$. A mapping 0 TPPOVA ω , $w \to w$ is defined I_{SUSY} $\sum_{i=1}^{n}$ intuitionistic Functionistic Functionistic Functionistic Functionistic Functionistic Functionistic Function T_{tot} \approx $\frac{0}{\pi}$ \approx $\lim_{\epsilon \to 0}$ grade ϵ (b) ϵ and ϵ $\lim_{j=1}$ $\lim_{\epsilon \to 1}$ $\lim_{\epsilon \to 0}$ $\lim_{\epsilon \to 0}$ $\lim_{j \to \infty}$ I_{S}) *then* Ⱳ¹ > Ⱳ² **Definition**) *then;* \bigcup **Definition** \mathbf{F} ([20]). For IEMs $\mathbf{W}_{i} = (m_{i}, n_{i})$ suith their smights $\tilde{\mathbf{G}}_{i} = (\tilde{\mathbf{G}}_{i}, \tilde{\mathbf{G}}_{i})$ If $\tilde{\theta} > 0$, $\tilde{\theta} \in [0, 1]$ and $\tilde{\theta}^{n'}$, $\tilde{\theta} = 1$, A manning of IEDOMA, Number TWD Three-Way Decision MG Membership Grade \mathcal{L} **Definition** \mathbf{F} ([20]) For IEMs $W_{ij} = (m_i - n_j)$ suith their spaights $\tilde{\omega}_{ij} = (\tilde{\omega}_{ij} - \tilde{\omega}_{ij})$ H_1 $\tilde{E}_1 > 0$, $\tilde{E}_2 \subset [0, 1]$ and $\sum n^2 \tilde{E}_1 = 1$, A manning of LEDOWA, $W_1^{n^2}$, $W_2^{n^2}$ $F_{\text{N}}(x)$ \rightarrow $F_{\text{N}}(x)$ \rightarrow $F_{\text{N}}(x)$ \rightarrow $F_{\text{N}}(x)$ \rightarrow $F_{\text{N}}(x)$ \mathcal{L} Non-membership Grade DTRS Decision-Theoretic Rough Set \mathcal{L} $\mathbf{D} \in \text{G}(20)$ $\mathbf{E} \in \text{G}(20)$ $T_{j=1}^{max}$ $\omega_j > 0$, $\omega_j \in [0, 1]$ and $\omega_j = 0$ \mathcal{L} eitien E (1901). $\Gamma_{\alpha\mu}$ IFMe 1 $\Delta_{j} > 0$, $\omega_{j} \in [0, 1]$ with $\Delta_{j=1}$ $\omega_{j} = 1$ $E(G_0)$ $E_0 u I F N c M = \frac{1}{2} a u$ $\omega_j \subset \left[\begin{smallmatrix}0 & 1\end{smallmatrix}\right]$ with $\omega_j = 1$ or $j = 1$. It imapped *For comparing the scale if the score function and* $\sum_{i=1}^{n}$ $\alpha_{i}^{(n)}$ *,* $\alpha_{i}^{(n)}$ *and* $\sum_{i=1}^{n}$ *A manning of IFPOI*) *then* Ⱳ¹ > Ⱳ² $\mathbf{y} = (m_i, n_i)$ with their weights $\tilde{\mathbf{\omega}}_i = (\tilde{\mathbf{\omega}}_i)$ $\mathcal{L}(\omega_j > 0, \omega_j \in [0, 1]$ and $\mathcal{L}_{j=1}(\omega_j = 1, 2, 1)$ mapping by 111 OVV21 ω , $\mathbf{w} \rightarrow \mathbf{w}$ is action $\frac{1}{100}$ m_i , n_i) with their weights $\tilde{\omega}_i=(\tilde{\omega}_1,\tilde{\omega})$ \mathcal{S} b, $\mathbf{w}_j \in [0, 1]$ and $\mathcal{L}_{j=1}^{\mathcal{S}} \mathbf{w}_j = 1$. It mapping by 11 I \mathcal{O} **v** \mathcal{S} **v** \mathcal{S} is defined FSs Fuzzy Sets IHFSs Intuitionistic Hesitant Fuzzy Sets that $\tilde{\omega}_j > 0$, $\tilde{\omega}_j \in [0,1]$ and $\sum_{j=1}^n \tilde{\omega}_j = 1$. A mapping of IFPOV *For comparing two IFNs,* Ⱳ¹ *and* Ⱳ² $=(m_i, n_i)$ with their weight **Table 1.** Symbols with their descriptions. **Definition 5** ([20]). For IFNs, $W_j = (m_j, n_j)$ with their weights $\tilde{\omega}_j = (\tilde{\omega}_1, \tilde{\omega}_2, \ldots, \tilde{\omega}_j)^T$ such that $\tilde{\omega}_i > 0$, $\tilde{\omega}_i \in [0, 1]$ and $\sum_{i=1}^{n'} \tilde{\omega}_i$ **5** ([20]). For IFNs, $W_i = (m_i, n_i)$ with their weights $\tilde{\omega}_i = (\tilde{\omega}_1, \tilde{\omega}_2, \ldots, \tilde{\omega}_i)^T$ such $\alpha_0 \geq 0$, $\tilde{\omega}_i \in [0, 1]$ and $\sum_{i=1}^n \tilde{\omega}_i = 1$. A mapping of IFPOWA ω_i , $W^n \to W$ is defined as $\stackrel{\sim}{s}$: **efinition 5** ([20]). For IFNs, $W_j = (m_j, n_j)$ with their weights $\tilde{\omega}_j$ $lows:$ $a(m_i, n_i)$ with their weights $\tilde{\omega}_i = (\tilde{\omega}_1, \tilde{\omega}_2, \ldots, \tilde{\omega}_i)^T$ such $=$ 1. A mapping of IFPOW i) with their weights $\tilde{\omega}_i = (\tilde{\omega}_1, \tilde{\omega}_2, \ldots, \tilde{\omega}_i)^T$ such algregation 1. A mapping of $\text{IFPOW}\xspace^{\mathcal{A}}$ is $\text{W}^n \rightarrow \text{W}$ is defined as **Definition 5** ([20]). For IFNs, $W_i = (m_i, n_i)$ with their weights $\tilde{\omega}_i = (\tilde{\omega}_1, \tilde{\omega}_2, \ldots, \tilde{\omega}_i)^T$ such $that \tilde{\omega}_i > 0, \tilde{\omega}_i \in [0, 1]$ and $\sum_{i=1}^{n}$. **Definition 2:** *For IFNs,* Ⱳ = (Ⱳ, Ⱳ) *and the score function and accuracy functions are de-***Mathematics 2023**, **11**, **11**, **11**, **11**, **4 1**, **11**, **11 11**, **11 11**, **11 11 11 11 11 11 11 11 11 11 11 11 11 11 11 11 11 11 11**

where
\n
$$
T(W_j) = \sum_{i=1}^{n} \tilde{\omega}_j \text{Sup}(W_j, W_i)
$$
\nand
\n
$$
\text{Sup}(W_j, W_i) = 1 - d(W_j, W_i)
$$
\n
$$
d(W_j, W_i) = \frac{1}{m} \sum_{\substack{i=1 \\ i \neq j}}^{m} (\vert m_i - m_j \vert + \vert n_i - n_j \vert)
$$
\n
$$
\text{Definition 5 (120)}. \text{ For IFNs, } W_j = (m_j, n_j) \text{ with their weights } \tilde{\omega}_j = (\tilde{\omega}_1, \tilde{\omega}_2, ..., \tilde{\omega}_j)^T \text{ such that } \tilde{\omega}_j > 0, \tilde{\omega}_j \in [0, 1] \text{ and } \sum_{j=1}^{n} \tilde{\omega}_j = 1. \text{ A mapping of IFPOWA}_{\omega}: W^{\eta} \to W \text{ is defined as follows:}
$$
\n
$$
\begin{array}{c}\n\text{If } W_{\sigma(j)}(W_1, W_2, ..., W_n) = \frac{j-1}{\sum_{j=1}^{n} \tilde{\omega}_j (1 + T(W_{\sigma(j)}))} \\
\text{If } W_{\sigma(j)}(W_1, W_2, ..., W_n) = \frac{j-1}{\sum_{j=1}^{n} \tilde{\omega}_j (1 + T(W_{\sigma(j)}))} \\
\text{If } W_{\sigma(j)}(W_1, W_2, ..., W_n) = \frac{j-1}{\sum_{j=1}^{n} \tilde{\omega}_j (1 + T(W_{\sigma(j)}))} \\
\text{Definition 6 (121).} \text{ Assume that } W_j = (m_j, n_j) \text{ is a collection of IFNs and the weights} \\
\tilde{\omega}_j = (\tilde{\omega}_1, \tilde{\omega}_2, ..., \tilde{\omega}_n)^T \text{ for } W_i \text{ and } \sum_{i=1}^{n} \tilde{\omega}_j = 1 \text{ where } \tilde{\omega}_j \in [0, 1]. \text{ Then, the IFPWG}_{\omega}\n\end{array}
$$

Definition 6 ([21]). Assume that $W_i = (m_i, n_i)$ is a collection of IFNs and the weights **Definition 6** ([21]). Assume that $W_j = (m_j, n_j)$ is a collection of IFNs and the weights $\tilde{\omega}_j = (\tilde{\omega}_1, \tilde{\omega}_2, \dots, \tilde{\omega}_n)^T$ for W_j and $\sum_{i=1}^n \tilde{\omega}_j = 1$ where $\tilde{\omega}_j \in [0, 1]$. Then, the IFPWG_{ω} operator is a mapping of IFPW G_{ω} : $W^{n} \rightarrow W$ where $\omega_j = (\omega_1, \omega_2, \ldots, \omega_n)$ for W_j and $\sum_{i=1}^n$ Assume that $W_j = (m_j, n_j)$ is a collection of $\sum_{i=1}^{n} f_i$ $\sum_{i=1}^{n} f_i$ $\sum_{i=1}^{n} f_i$ $\sum_{i=1}^{n} f_i$ \int for W_j and $\sum_{j=1}^n \omega_j = 1$ where ω_j **Definition 6** ([21]). Assume that $W_j = (m_j, n_j)$ is a collection of IFNs and the weights $\sum_{i=1}^{n} \sum_{j=1}^{n} x_j$ $\tilde{\omega}_j = (\tilde{\omega}_1, \tilde{\omega}_2, \ldots, \tilde{\omega}_n)^T$ for W_j and $\sum_{j=1}^n \tilde{\omega}_j = 1$ where $\tilde{\omega}_j \in [$ $\omega_j - (\omega_1, \omega_2, \ldots, \omega_n)$) *then;* $\frac{1}{2}$ $\int \nabla^n \cdot \tilde{f} = 1$ *zykove* $\tilde{G} \subset [0, 1]$ *The* $\sum_{j=1}^{l} \frac{L_j}{M} =$ $\mathbf{u}_j = [\mathbf{v}_j, \mathbf{v}_j, \mathbf{v}_k]$ operator is a mapping of IFPW G_{ω} : $W^{n} \rightarrow W$ where $\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$ is $\begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$ Then the I \int_{I}^{1} = 1 where $\omega_j \in [0, 1]$. Then, the 1 **Definition 6** ([21]). Assume that $W_j = (m_j, n_j)$ is a collection of IFNs and the weights $\mathcal{L} \subset [0, 1]$ Then the IEDIMC $\tilde{\omega}_j = (\tilde{\omega}_1, \tilde{\omega}_2, \dots, \tilde{\omega}_n)^T$ for W_j and $\sum_{j=1}^n \tilde{\omega}_j = 1$ where $\tilde{\omega}_j \in [0, 1]$. Then, the IFPWG_w =1 = 1 ῶ ∈ [0, 1]. *Thus, the operator is a* **Definition 4**[20]: *Assume that* Ⱳ = ((i) ∈1 ⊕ ∈1 ⊕ 2 = (i) ∈1 ∈ 12 = (i) ∈12 = $\frac{1}{2}$ ${\bf Definition}$ 6 ([21 $t_{M_{\text{max}}}$ (ϵ) is a gillection of IFNs and the majobia $\ddot{\omega}_j = (\ddot{\omega}_1, \ddot{\omega}_2)$ $A = \langle m_j, n_j \rangle$ is a concentrated the $\langle m_i, n_i \rangle$ $\tilde{\omega}_j = (\tilde{\omega}_1, \tilde{\omega}_2, ..., \tilde{\omega}_n)^T$ for W_j and $\sum_{j=1}^n \tilde{\omega}_j = 1$ where $\tilde{\omega}_j \in [0, 1]$. Then, the IFPWG_{ω} **Definition 6** ([21]). Assume that $W_j = (m_j, n_j)$ is a collection of IFNs and the w $(1, 2, 3)$ $(1, 2, 3)$ $\frac{1}{2}$, $\frac{1}{2}$ $\tilde{\omega}_j = (\tilde{\omega}_1, \tilde{\omega}_2, \ldots, \tilde{\omega}_n)^T$ for W_j and $\sum_{j=1}^n \tilde{\omega}_j = 1$ where $\tilde{\omega}_j \in [0, 1]$. Then, the IF. $\sum_{i=1}^{n}$ $\omega_j = (\omega_1, \omega_2, \dots, \omega_n)$ for w_j and $\sum_{j=1}^{\infty} \omega_j = 1$ where $\omega_j \in [0, 1]$. Then, the IIT we ω $\sum_{i=1}^{n}$ $\omega_j = (\omega_1, \omega_2, \dots, \omega_n)$ for w_j and $\Delta_{j=1}^{\infty} \omega_j = 1$ where $\omega_j \in [0, 1]$. Then, the II P $w \in \omega$

$$
\begin{aligned}\n &\stackrel{\mathcal{B}}{\otimes} (\widetilde{\omega}_{j}(1+T(\boldsymbol{W}_{j})\boldsymbol{W}_{j}) \\
 &IFPWG_{\omega}(\boldsymbol{W}_{1}, \boldsymbol{W}_{2}, \ldots, \boldsymbol{W}_{n}) = \frac{j-1}{\sum_{j=1}^{n} \widetilde{\omega}_{j}(1+T(\boldsymbol{W}_{j}))} \\
 &= \left(\prod_{j=1}^{n} (m_{j})^{\frac{\widetilde{\omega}_{j}(1+T(\boldsymbol{W}_{j}))}{\sum_{j=1}^{n} \widetilde{\omega}_{j}(1+T(\boldsymbol{W}_{j}))}}, 1 - \prod_{j=1}^{n} (1 - (n_{j})^{\frac{\widetilde{\omega}_{j}(1+T(\boldsymbol{W}_{j}))}{\sum_{j=1}^{n} \omega_{j}(1+T(\boldsymbol{W}_{j}))}} \right)\n \end{aligned}
$$

, ῶ² , … , ῶ) (ῶ¹ , ῶ² (iv) Ⱳ¹ *assistance as below, noted and defined as follows: noted and defined as follows: where presented as:* (i) *If* (Ⱳ¹

where
\n
$$
T(W_j) = \sum_{i=1}^{n} \tilde{\omega}_j Sup(W_j, W_i)
$$
\n
$$
i \neq j
$$

.

;

b. If (Ⱳ¹

) *then* Ⱳ¹ = Ⱳ²

) *then* Ⱳ¹ < Ⱳ²

) < (Ⱳ¹

(iv)
1910 - Paris Barbara
1910 - Paris Barbara

)
1940 – Charles Barnett, amerikansk politik
1940 – Charles Barnett, amerikansk politik

) – († 1852)
1910 – († 1852)
1910 – († 1852)

) *then* Ⱳ¹ = Ⱳ²

) *then* Ⱳ¹ < Ⱳ²

.

;

Definition 7 ([21]). *For IFNs,* $W_j = (m_j, n_j)$ with their weights $\tilde{\omega}_j = (\tilde{\omega}_1, \tilde{\omega}_2, \ldots, \tilde{\omega}_j)^T$ such *where* $\omega_j > 0$ *,* $\omega_j \in [0, 1]$ *and* $\sum_{j=1}^{\infty} \omega_j = 1$. A *mapping of ITTOWG* with \rightarrow **w** is acjinear. () + () ≤ 1 *for all* ∈ *. Generally, the pair (,) represents the IFN.* **Definition 7** ([21]). *For IFNs,* $W_j = (m_j, n_j)$ *with their weights* $\tilde{\omega}_j =$ *for* Ⱳ that $\tilde{\omega}_j > 0$, $\tilde{\omega}_j \in [0, 1]$ and $\sum_{j=1}^n \tilde{\omega}_j = 1$. A mapping of IFPOWG_{ω}: $follows:$ **Definition 7** ([21]). *For IFNs,* $W_j = (m_j, n_j)$ with their weights $\tilde{\omega}_j = (\tilde{\omega}_1, \tilde{\omega}_2)$ *that* $\tilde{\omega}_j > 0$, $\tilde{\omega}_j \in [0, 1]$ and $\sum_{j=1}^n \tilde{\omega}_j = 1$. A mapping of IFPOW $G_{\omega}: W^n \to$ **Definition** 7 ([21]). *For IFNs,* $W_j = (m_j, n_j)$ with their weights $\tilde{\omega}_j = (\tilde{\omega}_1, \tilde{\omega}_2, ...$ that $\tilde{\omega}_j > 0$, $\tilde{\omega}_j \in [0, 1]$ and $\sum_{j=1}^n \tilde{\omega}_j = 1$. A mapping of IFPOWG_w: $W^n \to W$ **Definition 7** ([21]). For IFNs, $W_j = (m_j, n_j)$ with their weights $\tilde{\omega}_j = (\tilde{\omega}_1, \tilde{\omega}_2, \ldots, \tilde{\omega}_j)^T$ that $\tilde{\omega}_j > 0$, $\tilde{\omega}_j \in [0, 1]$ and $\sum_{j=1}^n \tilde{\omega}_j = 1$. A mapping of IFPOW $G_{\omega}: W^n \to W$ is define *j*) *T such* that $\tilde{\omega}_j > 0$, $\tilde{\omega}_j \in [0, 1]$ and $\sum_{j=1}^n \tilde{\omega}_j = 1$. A mapping of IFPOW G_{ω} : 1 $follows:$ f ollows: 1 $follows:$ *presented as:* $W^n \to W$ is defined as *presented as: W* is defined as **As an extending of the FS model in the IFS model is modelled** to $\tilde{\mathbf{p}}$ and $\tilde{\mathbf{p}}$ and $\tilde{\mathbf{p}}$ simulated the IFS sim that $\tilde{\omega}_i > 0$, $\tilde{\omega}_i \in [0, 1]$ and $\sum_{i=1}^n \tilde{\omega}_i = 1$. A manning of IFPOW $G_{\omega_i}: W^n \to W$ is defined **As an extending Terms** $K_i = (m_i, n_i)$ with their weights $\tilde{\omega}_i = (\tilde{\omega}_1, \tilde{\omega}_2, \ldots, \tilde{\omega}_i)^T$ such tat $\tilde{\omega}_i > 0$, $\tilde{\omega}_i \in [0, 1]$ and $\sum_{i=1}^n \tilde{\omega}_i = 1$. A mapping of IFPOW $G_{\omega_i}: W^n \to W$ is defined as 1). For IFNs, $W_i = (m_i, n_i)$ with their weights $\tilde{\omega}_i = (\tilde{\omega}_1, \tilde{\omega}_2, \ldots, \tilde{\omega}_i)^T$ such α , $\tilde{\omega}_i \in [0, 1]$ and $\sum_{i=1}^n A_i$ is defined as α is defined as set [0, 1]. Definition $7([2]$ that $\tilde{\omega}$, τ $j = (m_j,$ τ n_j , n_j) u $\int_{i}^{\pi} f(x) \, dx \, d\Omega$ μ_j , n_j) wun ineir weights $\omega_j = (\omega_1, \omega_2, \ldots, \omega_j)$ such $\omega_j =$ 1. A mapping of IFPOWG_{ω}: $\mathbf{w} \rightarrow \mathbf{w}$ is aefined as *Definition 7 ([21]). For IFNs*, W that $\tilde{\omega}_j > 0$, $\tilde{\omega}_j \in [0, 1]$ and $\sum_{j=1}^n \tilde{\omega}_j = 1$. A mapping of IFPOW G_{ω} : $W^n \to W$ is defined as $\frac{1}{100}$ and $\frac{1}{100}$ while $\frac{1}{100}$ while $\frac{1}{100}$ and $\frac{1}{100$ **Definition 7** ([21]). For IFNs, $W_j = (m_j, n_j)$ with their weights $\tilde{\omega}_j = (m_j, n_j)$ that $\tilde{\omega}_j > 0$, $\tilde{\omega}_j \in [0, 1]$ and $\sum_{j=1}^{n'} \tilde{\omega}_j = 1$. A mapping of IFPOW $G_{\omega}: W^n \to W$ is defined as

$$
\begin{aligned}\n&\text{IFPOWG}_{\widetilde{\omega}}(W_1, W_2, \ldots, W_n) = \frac{\sum\limits_{j=1}^{n} (\widetilde{\omega}_j (1+T(W_{\sigma(j)}) W_{\sigma(j)})}{\sum_{j=1}^{n} \widetilde{\omega}_j (1+T(W_{\sigma(j)}))} \\
&= \left(\prod\limits_{j=1}^{n} (m_{\sigma(j)})^{\sum_{j=1}^{n} \widetilde{\omega}_j (1+T(W_{\sigma(j)}))}, 1-\prod\limits_{j=1}^{n} (1-(n_{\sigma(j)})^{\sum_{j=1}^{n} \widetilde{\omega}_j (1+T(W_{\sigma(j)}))}\right)\n\end{aligned}
$$

L.L. A Review of Decision-Theoretic Rough Set Model 2.2. A Review of Decision-Theoretic Rough Set Model

The DTRS theory is a framework that involves a collection of states, X and X' , indicational ing the presence or absence of components in X. The theory employs a series of actions, $a = \frac{1}{2}$, $a \frac{1}{2}$ α distinct zones namely the positive region $\mathcal{P}oc(Y)$ boundary region $\mathcal{R}ud(Y)$ and ence district zones, namely the positive region b $\sigma_0(x)$, boundary region $\mathcal{S}_m(x)$, and
negative region $\mathcal{N}eg(X)$. Additionally, a matrix Table 2, $\mathcal{M} = {\{\mathbb{J}_{\sigma\tau}\}}_{3\times2}$ ($\sigma = \mathcal{P}, \mathcal{B}, \mathcal{N}$, \mathcal{A}_{N} , when an object becomes X, is represented by $\mathbf{J}_{\mathcal{P}N}$, $\mathbf{J}_{\mathcal{B}N}$, and \mathbf{J}_{N} . Conversely, when $\mathcal{L}_{\mathcal{N}}$, when an object becomes $\mathcal{L}_{\mathcal{N}}$, is represented by $\mathcal{L}_{\mathcal{N}}$, $\mathcal{L}_{\mathcal{N}}$, and $\mathcal{L}_{\mathcal{N}}$, $\mathcal{L}_{\mathcal{N}}$. Conversely, when by $\exists \infty \pi$, $\exists \infty \pi$, and $\exists \pi \pi$. The classifica $i \in \mathbb{Z}$ are stated as: $\mathcal{A}ct = {\mathcal{A}_{\mathcal{P}}, \mathcal{A}_{\mathcal{B}}, \mathcal{A}_{\mathcal{N}}}$, where $\mathcal{A}_{\mathcal{P}}, \mathcal{A}_{\mathcal{B}},$ and $\mathcal{A}_{\mathcal{N}}$ act for the decisions to accept, defer, or and $\tau = \mathcal{P}, \mathcal{N}$ delivers the cost parameters. The cost related with actions $\mathcal{A}_{\mathcal{P}}, \mathcal{A}_{\mathcal{B}}$, and by $\mathbf{I}_{\mathscr{P},N}$, $\mathbf{I}_{\mathscr{B},N}$, and $\mathbf{I}_{N,N}$. The classification losses $\mathscr{E}(\mathscr{A}_{\mathscr{P}}|[A])$ related with the three actions $(1, 2, 2)$ $(1, 2, 2)$ reject an object A based on its classification, respectively. The objects are classified into three distinct zones, namely the positive region $\mathcal{P}os(X)$, boundary region $\mathcal{B}nd(X)$, and an item does not belong to X , the corresponding expenses for the three actions are denoted \mathcal{F}^{\prime} $\left| \begin{array}{c} 1 \end{array} \right|$) *then* Ⱳ¹ > Ⱳ² (iii) *If* (Ⱳ¹ $\sqrt{1 - \frac{1}{2}}$) *then;*

are stated as:
\n
$$
\mathcal{E}(\mathscr{A}_{\mathscr{P}}|[A]) = \mathbf{1}_{\mathscr{B}\mathscr{P}}P(X|[A]) + \mathbf{1}_{\mathscr{P}\mathscr{N}}P(X^{\prime}|[A])
$$
\n
$$
\mathcal{E}(\mathscr{A}_{\mathscr{B}}|[A]) = \mathbf{1}_{\mathscr{B}\mathscr{P}}P(X|[A]) + \mathbf{1}_{\mathscr{B}\mathscr{N}}P(X^{\prime}|[A])
$$
\n
$$
\mathcal{E}(\mathscr{A}_{\mathscr{N}}|[A]) = \mathbf{1}_{\mathscr{N}\mathscr{P}}P(X|[A]) + \mathbf{1}_{\mathscr{N}\mathscr{N}}P(X^{\prime}|[A])
$$

Bayesian decision theory provides the principles for the minimum-loss decision. $\frac{1}{2}$ Ravesian decision theory provides the $\frac{1}{2}$ ian decision theory provides the princir Bayesian decision theory provides the principles for the minimum-loss decision.

- 1. If $\mathcal{E}(\mathcal{A}_{\mathcal{P}}|[A]) \leq \mathcal{E}(\mathcal{A}_{\mathcal{B}}|[A])$ and $\mathcal{E}(\mathcal{A}_{\mathcal{P}}|[A]) \leq \mathcal{E}(\mathcal{A}_{\mathcal{N}}|[A])$, then $A \in POS(X)$;
	- 1. If $\mathcal{E}(\mathcal{A}_{\mathcal{B}}|[A]) \leq \mathcal{E}(\mathcal{A}_{\mathcal{B}}|[A])$ and $\mathcal{E}(\mathcal{A}_{\mathcal{B}}|[A]) \leq \mathcal{E}(\mathcal{A}_{\mathcal{B}}|[A])$
2. If $\mathcal{E}(\mathcal{A}_{\mathcal{B}}|[A]) \leq \mathcal{E}(\mathcal{A}_{\mathcal{B}}|[A])$ and $\mathcal{E}(\mathcal{A}_{\mathcal{B}}|[A]) \leq \mathcal{E}(\mathcal{A}_{\mathcal{N}}|[A])$, then $A \$ $2.$ and $\mathcal{E}(\mathcal{A}_{\mathcal{B}}([A]) \leq \mathcal{E}(\mathcal{A}_{\mathcal{B}}([A]))$, then A If $\mathcal{E}(\mathcal{A}_k[[A]) \leq \mathcal{E}(\mathcal{A}_k[[A])$ and $\mathcal{E}(\mathcal{A}_{\kappa} | [A]) \leq \mathcal{E}(\mathcal{A}_{\varnothing} | [A])$, then $A \in$
	- $\overline{3}$. If $\mathcal{E}(\mathcal{A}_{\mathcal{N}}|[A]) \leq \mathcal{E}(\mathcal{A}_{\mathcal{P}}|[A])$ and $\mathcal{E}(\mathcal{A}_{\mathcal{N}}|[A]) \leq \mathcal{E}(\mathcal{A}_{\mathcal{B}}|[A]),$ then $A \in NEG$ $\mathbf{u} \mathbf{L} \mathbf{G}(\mathbf{A}) \cdot$ Λ *j*. $(3, 0)$ $(1, -1)$ $(2, 0)$ $(1, -1)$ $(3, 0)$ $(4, 0)$ $(5, 0)$ $(1, -1)$ $\frac{\partial f}{\partial y} = 0$ $\left(\mathscr{A}^{\mathscr{N}}\right)[1] \geq 0$ $\left(\mathscr{A}^{\mathscr{N}}\right)[2]$ and \mathscr{O} If $\mathcal{E}(\mathscr{A}_{\mathscr{N}}|[A]) \leq \mathcal{E}(\mathscr{A}_{\mathscr{P}}|[A])$ and $\mathcal{E}(\mathscr{A}_{\mathscr{N}}|[A])$ $\frac{1}{2}$ $\frac{1}{2}$ (iv) *i*
) = $\left[\left(\frac{x_1}{x_2} \right) \right] \leq 0 \left(\frac{x_2}{x_2} \right) \left[\frac{x_1}{x_2} \right]$, and $\left(\frac{x_2}{x_2} \right)$ 3. If $\mathcal{E}(\mathscr{A}_{\mathscr{N}}|[A]) \leq \mathcal{E}(\mathscr{A}_{\mathscr{P}}|[A])$ and $\mathcal{E}(\mathscr{A}_{\mathscr{N}}|[A]) \leq \mathcal{E}(\mathscr{A}_{\mathscr{B}}|[A]),$ then $A \in NEG(X)$. $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

 $D(y^{\prime} | A) = 1$ the decision weight A , A and B are helitary is a follower. $f(X|X|) = 1$, the decision rules 1, 2, and 3 can be updated as ionows. $\mathbf{y}_0 = \mathbf{y}_0 - \mathbf{y}_0 = \mathbf{y}_0 - \mathbf{y}_0 = \mathbf{y}_0$
 $\mathbf{y}_0 = \mathbf{y}_0 - \mathbf{y}_0$
 $\mathbf{y}_0 = \mathbf{y}_0 - \mathbf{y}_0$ $\left[\alpha \mid \alpha\right]$ = 1, the decision rules 1, 2, and 5 can be updated as follows. $\begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$ $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ $\exists_{\mathscr{P}} \mathscr{P} \leq \exists_{\mathscr{R}} \mathscr{P} \leq \exists_{\mathscr{N}} \mathscr{P}, \exists_{\mathscr{N}} \mathscr{N} \leq \exists_{\mathscr{R}} \mathscr{N} \leq \exists_{\mathscr{P}} \mathscr{N} \text{ and } P(X|[A]) +$ $P(X^{\prime} \big| [A] \big) = 1$, the decision rules 1, 2, and 3 can be updated as follows: Given the prereq Given the prerequisites of $\exists_{\mathscr{P}} \mathscr{P} \leq \exists_{\mathscr{G}}$ $\leq \mathsf{J}_{\mathscr{B}\mathscr{P}} \leq \mathsf{J}_{\mathscr{N}\mathscr{P}}$, $\mathsf{J}_{\mathscr{N}\mathscr{N}}$ V): Given the prerequisites of $\mathbf{I}_{\mathscr{P}\mathscr{P}} \leq \mathbf{I}_{\mathscr{B}\mathscr{P}} \leq \mathbf{I}_{\mathscr{N}\mathscr{P}}$, $\mathbf{I}_{\mathscr{N}\mathscr{N}} \leq \mathbf{I}_{\mathscr{B}\mathscr{N}} \leq \mathbf{I}_{\mathscr{P}\mathscr{N}}$ and $P(X|[A])$ +

- 4. If $P(X|[A]) \geq \alpha$, then $A \in POS(X)$; **4.** If $P(X|[A]) \ge \alpha$, then $A \in POS(X);$
5. If $\beta \ge P(Y|[A]) \le \alpha$, then $A \subseteq PND(Y).$
	- $\inf_{\mathbf{A}} \{X \mid [A] \} \leq \alpha$, then $A \in \text{RND}(X)$.
 $\text{If } \beta \leq P(X | [A]) \leq \alpha$ then $A \in \text{RND}(X)$. $\bigcup_{i \in \mathcal{B}} N(n_i, k)$. *for* Ⱳ *with* ∑ ῶ (ῶ¹ ,ῶ² $\frac{1}{n}$ *DND*(*A*), *the operator is a <i>operator is a operator*. \overline{Y} $\overline{$ *Definition* $\mathbf{D}(\mathbf{Y})$ 5. If $\beta < P(X|[A]) < \alpha$, then $A \in BND(X)$;
 β if $P(X|[A]) < \beta$ then $A \subset NEC(Y)$
	- $\inf P \leq I(X|[A]) \leq \alpha$, then $A \in \text{NEC}(X)$ *map* β *f* $P(X|[A]) \leq \beta$ *, then* $A \in NEG(X)$. *mapping of :* Ⱳ → Ⱳ *where*

where

$$
\alpha = \frac{\gimel_{\mathscr{P}\mathscr{N}} - \gimel_{\mathscr{B}\mathscr{N}}}{(\gimel_{\mathscr{P}\mathscr{N}} - \gimel_{\mathscr{B}\mathscr{P}}) - (\gimel_{\mathscr{B}\mathscr{P}} - \gimel_{\mathscr{P}\mathscr{P}})} \text{ and } \beta = \frac{\gimel_{\mathscr{B}\mathscr{N}} - \gimel_{\mathscr{B}\mathscr{N}} - \gimel_{\mathscr{P}\mathscr{N}}}{(\gimel_{\mathscr{B}\mathscr{N}} - \gimel_{\mathscr{P}\mathscr{P}}) + (\gimel_{\mathscr{N}\mathscr{P}} - \gimel_{\mathscr{B}\mathscr{P}})}
$$

Table 2. Cost matrix.

3. A Novel Decision-Theoretic Rough Set Model Based on Interval-Valued Classes for Intuitionistic Fuzzy Sets

The following section presents a novel approach to modeling DTRS using intuitionistic fuzzy environments and interval design. This novel technique produces interval-valued equivalence classes which can be used to partition the universe into three distinct areas, including $\mathcal{P}os(X)$, $\mathcal{N}eg(X)$, and $\mathcal{B}nd(X)$ regions, for participant classification. To discretize the information system, we have developed interval-valued equivalence classes instead of traditional equivalence classes. This has been achieved with the help of a step-size function that aids in the partitioning of alternatives into intervals. The step-size function is defined as follows:

Definition 8. We define and denote the intervals \mathcal{F}_N for approximation classes based on MGs for a *collection of IFNs* $A_i = (m_i n_i)$ *where, i* = 1, 2, . . . *n*,

$$
\mathcal{F}_N = [Min(m_i), Min(m_i) + h] \tag{4}
$$

where step size function (*h*) *is defined for the membership grades of IFNs as*

$$
h = \frac{Max(m_i) - Min(m_i)}{N}
$$

where N is the number of intervals \mathcal{I}_N *which we require.*

As per Yao's parental concept [\[25\]](#page-17-3)*, equivalence classes can be used to determine the approximation classes. Additionally, by defining intervals* \mathcal{I}_N *according to Equation (4), the Nth* interval-valued equivalence classes $\left[A\right]_I$ can be developed for the alternatives, as follows:

Definition 9. The interval-valued equivalence classes $[A]_I$ for $I \subseteq A$ t for the alternatives A_i are *designed as*

$$
[A]_I = \{A : A_i \in \mathcal{I}_N\}
$$

Definition 10. *The membership function for interval-valued classes* $[A_k]$, $k \in \mathbb{N}$ *for all IFNs is defined as*

$$
P(X|[A_k]_I) = \frac{|X \cap [A_k]|}{|[A_k]|} \tag{5}
$$

Interval-Valued Decision-Theoretic Rough Set Model

We have employed Definition [\[8\]](#page-16-7) to create a new model for three-way decision making under an IF environment. We have established the cost parameter matrix \mathcal{M} based on intuitionistic fuzzy cost values which is shown in Table [3.](#page-7-0) Using Bayesian theory, we have described the expected losses $\mathcal{E}(\mathcal{A}_{\sigma} | [A])$, $\sigma = \mathcal{P}, \mathcal{B}, \mathcal{N}$ for taking actions with the given set of states for the interval-valued equivalence class [*A*] as follows:

$$
\mathcal{E}(\mathscr{A}_{\mathscr{P}}|[A]) = \mathbb{J}_{\mathscr{P}\mathscr{P}}P(X|[A]) \oplus \mathbb{J}_{\mathscr{P}\mathscr{N}}P\left(X'\big|\big[A\big]\right)
$$

$$
\mathcal{E}(\mathscr{A}_{\mathscr{B}}|[A]) = \mathbb{J}_{\mathscr{B}\mathscr{P}}P(X|[A]) \oplus \mathbb{J}_{\mathscr{B}\mathscr{N}}P\left(X'\big|\big[A\big]\right)
$$

$$
\mathcal{E}(\mathscr{A}_{\mathscr{N}}|[A]) = \mathbb{J}_{\mathscr{N}\mathscr{P}}P(X|[A]) \oplus \mathbb{J}_{\mathscr{N}\mathscr{N}}P\left(X'\big|\big[A\big]\right)
$$

 $P(X|[A]) + P(X^{\prime}|[A]) + \Delta(A) = 1$ where $\Delta(A)$ is an error function. We have

$$
\mathcal{E}(\mathcal{A}_{\mathcal{P}}|[A]) = \mathbf{I}_{\mathcal{P}\mathcal{P}}P(X|[A]) \oplus \mathbf{I}_{\mathcal{P}\mathcal{N}}[1 - P(X|[A]) - \Delta(A)] \n\mathcal{E}(\mathcal{A}_{\mathcal{B}}|[A]) = \mathbf{I}_{\mathcal{B}\mathcal{P}}P(X|[A]) \oplus \mathbf{I}_{\mathcal{B}\mathcal{N}}[1 - P(X|[A]) - \Delta(A)] \n\mathcal{E}(\mathcal{A}_{\mathcal{N}}|[A]) = \mathbf{I}_{\mathcal{N}\mathcal{P}}P(X|[A]) \oplus \mathbf{I}_{\mathcal{N}\mathcal{N}}[1 - P(X|[A]) - \Delta(A)]
$$
\n(6)

Table 3. Intuitionistic fuzzy cost matrix.

Bayesian decision theory offers the decision guidelines for the minimum-loss decision.

- 7. If $\mathcal{E}(\mathcal{A}_{\mathcal{P}}|[A]) \leq \mathcal{E}(\mathcal{A}_{\mathcal{B}}|[A])$ and $\mathcal{E}(\mathcal{A}_{\mathcal{P}}|[A]) \leq \mathcal{E}(\mathcal{A}_{\mathcal{N}}|[A]),$ then $A \in POS(X);$
- 8. If $\mathcal{E}(\mathcal{A}_{\mathcal{B}}|[A]) \leq \mathcal{E}(\mathcal{A}_{\mathcal{B}}|[A])$ and $\mathcal{E}(\mathcal{A}_{\mathcal{B}}|[A]) \leq \mathcal{E}(\mathcal{A}_{\mathcal{N}}|[A]),$ then $A \in BND(X)$;
- 9. If $\mathcal{E}(\mathcal{A}_{\mathcal{N}}|[A]) \leq \mathcal{E}(\mathcal{A}_{\mathcal{P}}|[A])$ and $\mathcal{E}(\mathcal{A}_{\mathcal{N}}|[A]) \leq \mathcal{E}(\mathcal{A}_{\mathcal{B}}|[A])$, then $A \in NEG(X)$.

The expected losses are evaluated based on the intuitionistic cost parametric values in Table [3](#page-7-0) for interval-valued classes for identifying the thresholds which are designed as follows:

$$
\mathcal{E}(\mathcal{A}_{\mathcal{P}}|[A]) = \left[1 - (1 - m_{\mathcal{P}\mathcal{P}})^{P(X|[A])}(1 - m_{\mathcal{P}\mathcal{N}})^{1 - P(X|[A]) - \Delta(A)}, (n_{\mathcal{P}\mathcal{P}})^{P(X|[A])}, (n_{\mathcal{P}\mathcal{N}})^{1 - P(X|[A]) - \Delta(A)}\right]
$$

\n
$$
\mathcal{E}(\mathcal{A}_{\mathcal{B}}|[A]) = \left[1 - (1 - m_{\mathcal{B}\mathcal{P}})^{P(X|[A])}(1 - m_{\mathcal{B}\mathcal{N}})^{1 - P(X|[A]) - \Delta(A)}, (n_{\mathcal{B}\mathcal{P}})^{P(X|[A])}, (n_{\mathcal{B}\mathcal{N}})^{1 - P(X|[A]) - \Delta(A)}\right]
$$

\n
$$
\mathcal{E}(\mathcal{A}_{\mathcal{N}}|[A]) = \left[1 - (1 - m_{\mathcal{N}\mathcal{P}})^{P(X|[A])}(1 - m_{\mathcal{N}\mathcal{N}})^{1 - P(X|[A]) - \Delta(A)}, (n_{\mathcal{N}\mathcal{P}})^{P(X|[A])}, (n_{\mathcal{N}\mathcal{N}})^{1 - P(X|[A]) - \Delta(A)}\right]
$$

\nLet $m(\mathcal{E})_{\sigma} = 1 - (1 - m_{\sigma\mathcal{P}})^{P(X|[A])}(1 - m_{\sigma\mathcal{N}})^{1 - P(X|[A]) - \Delta(A)}$ and $n(\mathcal{E})_{\sigma} = 1 - n_{\mathcal{N}\mathcal{N}} \mathcal{E}(\mathcal{A}_{\mathcal{P}\mathcal{P}}[X|X|) - \Delta(A)$

Let
$$
m(\epsilon)_{\sigma} = 1 - (1 - m_{\sigma} \varphi)^{1 - m_{\sigma}} \cdot (1 - m_{\sigma} \varphi)^{1 - m_{\sigma}}
$$
 and $n(\epsilon)_{\sigma} = (n_{\sigma} \varphi)^{P(X|[A])} (n_{\sigma} \varphi)^{1 - P(X|[A]) - \Delta(A)}$; here, $\sigma = \mathcal{P}, \mathcal{B}, \mathcal{N}$.

The membership and non-membership grades of the cost parameter are used to calculate the categorization losses, $m(\mathscr{E})_{\sigma}$ and $n(\mathscr{E})_{\sigma}$, respectively. Furthermore, according to Bayesian decision theory, the new DRs are clearly examined by $m(\mathcal{E})_{\sigma}$ for the minimum-loss categorization. The categorization losses $m(\mathcal{E})_{\sigma}$ and $n(\mathcal{E})_{\sigma}$ are determined by the MG and NMG of the cost parameter, respectively. Furthermore, the new decision rules are examined based on Bayesian decision theory using $m(\mathcal{E})_{\sigma}$ for the minimum-loss categorizations.

- 10. If $m(\mathcal{E})_{\mathcal{P}} \leq m(\mathcal{E})_{\mathcal{B}}$ and $m(\mathcal{E})_{\mathcal{P}} \leq m(\mathcal{E})_{\mathcal{N}}$, then $A \in \mathcal{P}os(X)$;
- 11. If $m(\mathcal{E})_{\mathscr{B}} \le m(\mathcal{E})_{\mathscr{P}}$ and $m(\mathcal{E})_{\mathscr{B}} \le m(\mathcal{E})_{\mathscr{N}}$ then $X \in \mathscr{B}nd(X)$;
12. If $m(\mathcal{E})_{\mathscr{N}} < m(\mathcal{E})_{\mathscr{B}}$ and $m(\mathcal{E})_{\mathscr{N}} < m(\mathcal{E})_{\mathscr{B}}$ then $X \in \mathscr{N}eg(X)$.
- If $m(\mathcal{E})_{\mathcal{N}} \leq m(\mathcal{E})_{\mathcal{P}}$ and $m(\mathcal{E})_{\mathcal{N}} \leq m(\mathcal{E})_{\mathcal{B}}$ then $X \in \mathcal{N}eg(X)$.

Based on categorization losses, if $m(\mathcal{E})_{\mathcal{P}} \leq m(\mathcal{E})_{\mathcal{P}}$, then

$$
\ln \left[1 - (1 - m_{\mathcal{B}\mathcal{B}})^{P(X|[A])}(1 - m_{\mathcal{B}\mathcal{N}})^{1 - P(X|[A]) - \Delta(A)}\right] \geq \ln \left[1 - (1 - m_{\mathcal{B}\mathcal{B}})^{P(X|[A])}(1 - m_{\mathcal{B}\mathcal{N}})^{1 - P(X|[A]) - \Delta(A)}\right]
$$

By (10)

$$
P(X|[A]) \ge (1 - \Delta(A)) \frac{\ln\left[\frac{1 - m_{\mathcal{B}A}}{1 - m_{\mathcal{B}\mathcal{P}}}\right]}{\ln\left(\frac{1 - m_{\mathcal{B}\mathcal{P}}}{1 - m_{\mathcal{B}\mathcal{P}}}\right)} \frac{1 - m_{\mathcal{B}A}}{1 - m_{\mathcal{B}A}}}
$$

Similarly

$$
m(\mathcal{E})_{\mathcal{N}} \leq m(\mathcal{E})_{\mathcal{N}} \Rightarrow P(X|[A]) \geq (1 - \Delta(A)) \frac{\ln\left[\frac{1 - m_{\mathcal{N}}\mathcal{N}}{1 - m_{\mathcal{P}}\mathcal{N}}\right]}{\ln\left(\frac{1 - m_{\mathcal{P}}\mathcal{N}}{1 - m_{\mathcal{P}}\mathcal{N}}\right)} \frac{\ln\left[\frac{1 - m_{\mathcal{N}}\mathcal{N}}{1 - m_{\mathcal{P}}\mathcal{N}}\right]}{\ln\left(\frac{1 - m_{\mathcal{P}}\mathcal{N}}{1 - m_{\mathcal{P}}\mathcal{N}}\right)} \frac{\ln\left[\frac{1 - m_{\mathcal{N}}\mathcal{N}}{1 - m_{\mathcal{P}}\mathcal{N}}\right]}{\ln\left(\frac{1 - m_{\mathcal{P}}\mathcal{N}}{1 - m_{\mathcal{P}}\mathcal{N}}\right)} \frac{\ln\left[\frac{1 - m_{\mathcal{P}}\mathcal{N}}{1 - m_{\mathcal{P}}\mathcal{N}}\right]}{\ln\left(\frac{1 - m_{\mathcal{P}}\mathcal{N}}{1 - m_{\mathcal{P}}\mathcal{N}}\right)} \frac{\ln\left[\frac{1 - m_{\mathcal{P}}\mathcal{N}}{1 - m_{\mathcal{P}}\mathcal{N}}\right]}{\ln\left(\frac{1 - m_{\mathcal{P}}\mathcal{N}}{1 - m_{\mathcal{P}}\mathcal{N}}\right)} \frac{\ln\left[\frac{1 - m_{\mathcal{P}}\mathcal{N}}{1 - m_{\mathcal{P}}\mathcal{N}}\right]}{\ln\left(\frac{1 - m_{\mathcal{P}}\mathcal{N}}{1 - m_{\mathcal{P}}\mathcal{N}}\right)} \frac{\ln\left[\frac{1 - m_{\mathcal{N}}\mathcal{N}}{1 - m_{\mathcal{P}}\mathcal{N}}\right]}{\ln\left(\frac{1 - m_{\mathcal{P}}\mathcal{N}}{1 - m_{\mathcal{P}}\mathcal{N}}\right)} \frac{\ln\left[\frac{1 - m_{\mathcal{N}}\mathcal{N}}{1 - m_{\mathcal{P}}\mathcal{N}}\right]}{\ln\left(\frac{1 - m_{\mathcal{P}}\mathcal{N}}{1 - m_{\mathcal{P}}\mathcal{N}}\right)} \frac{\ln\left[\
$$

The decision rules are constructed as (P) , (B) , and (N) by using the thresholds are given as,

 (P) If $P(X|[A]) \geq \chi_1$ and $P(X|[A]) \geq \psi_1$, then $A \in \mathcal{P}os(X)$ (*B*) If $P(X|[A]) \leq \chi_1$ and $P(X|[A]) \geq \omega_1$, then $A \in \mathcal{B}nd(X)$ (N) If $P(X|[A]) \leq \omega_1$ and $P(X|[A]) \leq \psi_1$, then $A \in \mathcal{N}eg(X)$, here

$$
\chi_1 = (1 - \Delta(A)) \frac{\ln\left[\frac{1 - m_{\mathcal{B}\mathcal{N}}}{1 - m_{\mathcal{B}\mathcal{P}}}\right]}{\ln\left(\frac{1 - m_{\mathcal{B}\mathcal{P}}}{1 - m_{\mathcal{B}\mathcal{P}}}\times \frac{1 - m_{\mathcal{B}\mathcal{N}}}{1 - m_{\mathcal{P}\mathcal{N}}}\right)}
$$
(7)

$$
\psi_1 = (1 - \Delta(A)) \frac{\ln\left[\frac{1 - m_{\mathcal{N},\mathcal{N}}}{1 - m_{\mathcal{P},\mathcal{N}}}\right]}{\ln\left(\frac{1 - m_{\mathcal{P},\mathcal{P}}}{1 - m_{\mathcal{N},\mathcal{P}}}\times \frac{1 - m_{\mathcal{N},\mathcal{N}}}{1 - m_{\mathcal{P},\mathcal{N}}}\right)}
$$
(8)

$$
\omega_1 = (1 - \Delta(A)) \frac{\ln\left[\frac{1 - m_{\mathcal{N}\mathcal{N}}}{1 - m_{\mathcal{R}\mathcal{N}}}\right]}{\ln\left(\frac{1 - m_{\mathcal{R}\mathcal{R}}}{1 - m_{\mathcal{N}\mathcal{P}}}\right)} \tag{9}
$$

The rules for deciding the elements $(P) - (N)$ have so far been characterized by utilizing three thresholds $χ_1(e)$, $ψ_1(e)$, and $ω_1(e)$ from the membership grade perspective. Furthermore, DRs (13)–(15) from the viewpoint of NMG are conferred.

- 13. If $n(\mathcal{E})_{\mathcal{P}} \leq n(\mathcal{E})_{\mathcal{B}}$ and $n(\mathcal{E})_{\mathcal{P}} \leq n(\mathcal{E})_{\mathcal{N}}$, then $A \in \mathcal{P}os(X)$;
- 14. If $n(\mathcal{E})_{\mathscr{B}} \le n(\mathcal{E})_{\mathscr{P}}$ and $n(\mathcal{E})_{\mathscr{B}} \le n(\mathcal{E})_{\mathscr{N}}$ then $A \in \mathscr{B}nd(X)$;
- 15. If $n(\mathcal{E})_{\mathcal{N}} \leq n(\mathcal{E})_{\mathcal{P}}$ and $n(\mathcal{E})_{\mathcal{N}} \leq n(\mathcal{E})_{\mathcal{B}}$ then $A \in \mathcal{N}eg(X)$.

Given that $P(X|[A]) = 1 - P(X^{\prime}||[A]) - \Delta(A)$, DRs are expressed based on the complement of conditional probability as below.

If $n(\mathcal{E})_{\mathcal{P}} \leq n(\mathcal{E})_{\mathcal{B}}$, then

$$
P(X|[A])\ln\left(\frac{n_{\mathscr{P}\mathscr{P}}}{n_{\mathscr{B}\mathscr{P}}}\times\frac{n_{\mathscr{B}\mathscr{N}}}{n_{\mathscr{P}\mathscr{N}}}\right) \geq (1-\Delta(A))\ln\frac{\beta_N(n_{\mathscr{B}\mathscr{N}})}{\beta_N(n_{\mathscr{B}\mathscr{P}})}
$$

Thus

$$
P(X|[A]) \ge (1 - \Delta(A)) \frac{\ln \frac{n_{\mathcal{BM}}}{n_{\mathcal{B}\mathcal{P}}}}{\ln \left(\frac{n_{\mathcal{BM}}}{n_{\mathcal{B}\mathcal{P}}} \times \frac{n_{\mathcal{BM}}}{n_{\mathcal{P}\mathcal{N}}} \right)}
$$

Similarly

$$
n(\mathcal{E})_{\mathcal{P}} \geq n(\mathcal{E})_{\mathcal{N}} \Longrightarrow P(X|[A])
$$

\n
$$
\geq (1 - \Delta(A)) \frac{\ln \frac{n_{\mathcal{A}}}{n_{\mathcal{P}} \mathcal{N}}}{\ln(\frac{n_{\mathcal{P}} \mathcal{P}} \times \frac{n_{\mathcal{A}} \mathcal{N}}{n_{\mathcal{P}} \mathcal{N}})})
$$

\n
$$
n(\mathcal{E})_{\mathcal{B}} \geq n(\mathcal{E})_{\mathcal{P}} \Longrightarrow P(X|[A])
$$

\n
$$
\leq (1 - \Delta(A)) \frac{\ln \frac{n_{\mathcal{B}} \mathcal{N}}{n_{\mathcal{B}} \mathcal{P}} \times \frac{n_{\mathcal{A}} \mathcal{N}}{n_{\mathcal{P}} \mathcal{N}}})}{\ln(\mathcal{E})_{\mathcal{B}} \geq n(\mathcal{E})_{\mathcal{N}} \Longrightarrow P(X|[A])
$$

\n
$$
\geq (1 - \Delta(A)) \frac{\ln \frac{n_{\mathcal{A}} \mathcal{N}}{n_{\mathcal{B}} \mathcal{P}} \times \frac{n_{\mathcal{A}} \mathcal{N}}{n_{\mathcal{P}} \mathcal{N}}})}{\ln(\mathcal{E})_{\mathcal{N}} \geq n(\mathcal{E})_{\mathcal{P}} \Longrightarrow P(X|[A])
$$

\n
$$
\leq (1 - \Delta(A)) \frac{\ln \frac{n_{\mathcal{A}} \mathcal{N}}{n_{\mathcal{B}} \mathcal{P}} \times \frac{n_{\mathcal{A}} \mathcal{N}}{n_{\mathcal{B}} \mathcal{N}}}}{n(\mathcal{E})_{\mathcal{N}} \geq n(\mathcal{E})_{\mathcal{B}} \Longrightarrow P(X|[A])
$$

\n
$$
\leq (1 - \Delta(A)) \frac{\ln \frac{n_{\mathcal{A}} \mathcal{N}}{n_{\mathcal{P}} \mathcal{P}} \times \frac{n_{\mathcal{A}} \mathcal{N}}{n_{\mathcal{B}} \mathcal{N}}}}{\ln(\frac{n_{\mathcal{B}} \mathcal{P}} \times \frac{n_{\mathcal{A}} \mathcal{N}}{n_{\mathcal{B}} \mathcal{N}})}
$$

\n
$$
\leq (1 - \Delta(A)) \frac{\ln \frac{n_{\mathcal{
$$

Obviously, the decision rules are easily revised as (*P*2) − (*N*2).

where

$$
\chi_2 = (1 - \Delta(A)) \frac{\ln \frac{n_{\mathcal{BM}}}{n_{\mathcal{B}\mathcal{P}}}}{\ln \left(\frac{n_{\mathcal{BM}}}{n_{\mathcal{B}\mathcal{P}}} \times \frac{n_{\mathcal{BM}}}{n_{\mathcal{PM}}} \right)}
$$
(10)

$$
\psi_2 = (1 - \Delta(A)) \frac{\ln \frac{n_{\mathcal{N}}}{n_{\mathcal{P}\mathcal{N}}}}{\ln \left(\frac{n_{\mathcal{P}\mathcal{P}}}{n_{\mathcal{N}}\mathcal{P}} \times \frac{n_{\mathcal{N}}}{n_{\mathcal{P}\mathcal{N}}} \right)}
$$
(11)

$$
\omega_2 = (1 - \Delta(A)) \frac{\ln \frac{n_{\mathcal{N}}}{n_{\mathcal{B}}}}{\ln \left(\frac{n_{\mathcal{B}}}{n_{\mathcal{N}} \mathcal{P}} \times \frac{n_{\mathcal{N}}}{n_{\mathcal{B}} \mathcal{N}} \right)}
$$
(12)

Noticeably, providing the IFN cost parameters are also fulfilled,

and

$$
\frac{\ln \frac{n_{\mathscr{P}\mathscr{P}}}{n_{\mathscr{B}\mathscr{P}}}}{\ln \frac{n_{\mathscr{B}\mathscr{P}}}{n_{\mathscr{P}\mathscr{N}}}} < \frac{\ln \frac{n_{\mathscr{B}\mathscr{P}}}{n_{\mathscr{N}\mathscr{P}}}}{\ln \frac{n_{\mathscr{N}\mathscr{N}}}{n_{\mathscr{B}\mathscr{N}}}}
$$

ω^{*j*}(*A*) < *ψ*^{*j*}(*A*) < *χ*_{*j*}(*A*) is obtained where $χ$ ^{*j*}(*A*) ∈ (0,1], *ω*^{*j*}(*A*) ∈ (0,1], and $\psi_i(A) \in (0,1]$, $(i=1,2)$. Therefore, the general DRs $(P3) - (N3)$ can be described as below:

 $(P3)$ If $P(X|[A]) \geq \chi_j$, then $A \in \mathcal{P}os(U)$ $(B3)$ If $\omega_j < P(X|[A]) < \chi_j$, then $A \in \mathcal{B}nd(U)$ $(N3)$ If $P(X|[A]) \leq \omega_j$, then $A \in \mathcal{N}eg(U)$,

From the above obtained results, the TWDM-IFNs are described according to the Bayesian DRs, as below.

- 16. If $P(X|[A]) \geq \chi_j$, then take $\mathscr{A}_{\mathscr{P}}$;
- 17. If $\omega_j < P(X|[A]) < \chi_j$, then take $\mathscr{A}_{\mathscr{B}}$;
- 18. If $P(X|[A]) \leq \omega_j$, then take $\mathscr{A}_{\mathscr{N}}$.

In TWD, two threshold pairs $(\chi_1(A), \omega_1(A))$ and $(\chi_2(A), \omega_2(A))$ are obtained from various viewpoints in (16)–(18). Thus, actions are taken while $P(X|[A])$ is at the corresponding positive, boundary, and negative region thresholds.

4. Proposing an Algorithm to Apply the Interval-Valued Decision-Theoretic Rough Set Model to an Intuitionistic Fuzzy Environment

This section discusses the detailed application of $IFPWA_ω$ and $IFPWG_ω$ aggregation operators under IF information for decision-theoretic rough set models. We have outlined five steps for selecting the three-way DRs for various participants.

Let $E = \{A_1, A_2, \ldots, A_n\}$ be the collection of alternatives and $X = \{Yes, No\}$ be a set that indicates the decision for alternatives, where *X* is a subset of *E*. The flow chart of the three-way decision model is displayed in Figure [1.](#page-10-0)

Step 1. Evaluate the intuitionistic fuzzy information system with conditional and decision attributes.

For comparing two IFNs, Ⱳ¹ *and* Ⱳ²

assistance as below,

Definition 3: *Suppose* Ⱳ¹ *= (*¹

c. If (Ⱳ¹

 $\mathcal{L}^{\text{max}}_{\text{max}}$ $\mathcal{L}^{\text{max}}_{\text{max}}$ $\mathcal{L}^{\text{max}}_{\text{max}}$

Step 2. For alternatives, A_i ($i = 1, 2, ..., n$) aggregate all the IF attributes $A_{ij}(j=1, 2, ..., m)$ into a general solution A_i applying IFPWA_{$\tilde{\omega}$} and IFPWG_w operators $\frac{1}{2}$ *with* ∑ ῶ =1 = 1 ῶ ∈ [0, 1]. *Thus, the operator is a* Step 2. Step 2. For alternatives, $A_i(i = 1, 2, ..., n)$ aggregate all the IF attributes $\frac{1}{2}$ as: },{¹ + ² − 1² **Definition 3:** *Suppose* Ⱳ¹ *= (*¹ $\overline{}$ as: $\sum_{i=1}^{\infty}$ as: $\sum_{i=1}^{\infty}$ $\sum_{i=1}^{\infty}$ $\frac{1}{2}$ $\frac{1}{2}$ $A_{ij}(j = 1, 2, ...)$) *then* Ⱳ¹ = Ⱳ² $I = 1, 2, \dots$ m into a general solution A applying $IEBWA$ and $IEBWG$ operators If $\sum_{i=1}^n$ is $\sum_{i=1}^n$ Power Order Weighted averaging IFPOWG intuitionistic $\sum_{i=1}^n$ Power Order Weighted Geometric Fuzzy Power Order Weighted Geometric Geometric Geometric Geometric Geometric Geometric Geometri n $\frac{1}{1}$ $\frac{1}{2}$ $\frac{1}{2}$ w) into a general solution A, applying $\frac{1}{10}$ $\frac{1}{10}$ $A_{ij}(j = 1, 2, ..., m)$ into a general solution A_i applying IFPW $A_{\tilde{\omega}}$ and IFPW G_{ω} operators n) > (Ⱳ¹) *then* Ⱳ¹ > Ⱳ² ω and ω and ω and ω and ω ω \sim Theoretic Rough Set ω η IFSs Intuitionistic Fuzzy Sets IFN Intuitionistic Fuzzy Number $A_{ij}(j = 1, 2, ..., m)$ into a general solution A_i applying IFPW $A_{\tilde{\omega}}$ and IFPW G_{ω} operators \mathcal{L} non-membership \mathcal{L} Decision-Theoretic Rough Set \mathcal{L} \sup 2. For antenanyls, $\lim_{t \to \infty} \frac{f(t-1)}{t-1}$ $\lim_{t \to \infty} \frac{f(t)}{t-1}$ ω . FSs Fuzzy Sets IHFSs Intuitionistic Hesitant Fuzzy Sets **Symbol Description Symbol Description** T , Z , ..., m and a general solution M_1 applying H is M_0 and H if M_0 operators η $\frac{F}{\epsilon}$ Fuzzy Sets Internatives, $\frac{F}{\epsilon}$ $\frac{F}{\epsilon}$ $\frac{F}{\epsilon}$ is $\frac{F}{\epsilon}$ in $\frac{F}{\epsilon}$ in $\frac{F}{\epsilon}$ in $\frac{F}{\epsilon}$ \mathbf{W}

) are IFSs, some basic operations are

Symbol Description Symbol Description

, the score function and accuracy function provide the

DRs Decision Rules IFRS Intuitionistic Fuzzy Rough Set

IFSs Intuitionistic Fuzzy Sets IFN Intuitionistic Fuzzy Number

IFSs Intuitionistic Fuzzy Sets IFN Intuitionistic Fuzzy Number

$$
\begin{aligned}\n&\text{HFPWA}_{\omega}(W_1, W_2, \ldots, W_n) = \frac{j=1}{\sum_{j=1}^n \tilde{\omega}_j (1+T(W_j))} \\
&= \left(1 - \prod_{j=1}^n \left(1 - (m_j)\right)^{\frac{\tilde{\omega}_j (1+T(W_j))}{\sum_{j=1}^n \omega_j (1+T(W_j))}}, \prod_{j=1}^n \left(n_j\right)^{\frac{\tilde{\omega}_j (1+T(W_j))}{\sum_{j=1}^n \tilde{\omega}_j (1+T(W_j))}}\right)\n\end{aligned}
$$

 and *noted and defined as follows:* $\lim_{x\to a}$ ${\bf d}$

,

2.1. IFSs

, the score function and accuracy function provide the

, ¹

2.1. IFSs

) *then* Ⱳ¹ > Ⱳ²

) = (Ⱳ¹

For comparing two IFNs, Ⱳ¹ *and* Ⱳ²

 $\overline{}$

) *then* Ⱳ¹ = Ⱳ²

2.1. IFSs

.

, ²

DRs Decision Rules IFRS Intuitionistic Fuzzy Rough Set

) and Ⱳ² *= (*²

$$
IHPPWG_{\omega}(W_1, W_2, ..., W_n) = \frac{j=1}{\sum_{j=1}^n \tilde{\omega}_j (1+T(W_j))}
$$

$$
= \left(\prod_{j=1}^n (m_j)^{\frac{\tilde{\omega}_j (1+T(W_j))}{\sum_{j=1}^n \tilde{\omega}_j (1+T(W_j))}}, 1 - \prod_{j=1}^n (1-(n_j)^{\frac{\tilde{\omega}_j ((1+T(W_j))}{\sum_{j=1}^n \omega_j (1+T(W_j))}})\right)
$$

Step 3. Compute the interval-valued equivalence classes **i** as per Definition 9. requivalence classes using the sugges *i* quivalence classes using the suggested Step 3. Compute the interval-valued equivalence classes using the suggested intervals (iii) *If* (Ⱳ¹) = (Ⱳ¹) *then; noted and defined as follows:* **Definition 2:** *For IFNs,* Ⱳ = (Ⱳ, Ⱳ) *and the score function and accuracy functions are de-*() + () ≤ 1 *for all* ∈ *. Generally, the pair (,) represents the IFN.* **Definition 2:** *For IFNs,* Ⱳ= (Ⱳ, Ⱳ) *and the score function and accuracy functions are denoted and defined as follows:* **Definition 2:** *For IFNs,* Ⱳ = (Ⱳ, Ⱳ) *and the score function and accuracy functions are de-*() + () ≤ 1 *for all* ∈ *. Generally, the pair (,) represents the IFN.* Step 3. Compute the interval-valued equivalence classes using the suggested intervals

Step 4. Fix the set of states (X, X') and compute the membership fu Sup 4. The first of states (X, X) and compute the fitting-
mbership function, and error functions for the participants. *themocronap tu* and compute the membership function *theoring rance* Step 4. Fix the set of states (X, X') and compute the membership function, non-
phorehip function, and error functions for the participants membership function, and error functions for the participants.
Star 5. Calculation of avenated lagase and thresholds based on the seat parameter.) *then* Ⱳ¹ > Ⱳ² *; nep* 4. The die set of state Step 4. Fix the set of states (X, X') and compute the membership function, non-

Step 5. Calculation of expected losses and thresholds based on the cost parameter
trix referenced in Table 8 using Equation (6) *) are IFSs, some basic operations are* his ion the participants.
resec and thresholds based on the cos *c. If* (Ⱳ¹ ilation of expected losses and thresholds based on the cos
in Table 8 using Equation (6) *firstion of the elements depending when* matrix referenced in Table 8 using Equation (6) .
Street Classification of the state of th Step 5. Calculation of expected losses and thresholds based on the cost parameter *x*, *donanding unan their membership value ation (6).* The score function θ function provide the *score function provide the score function* θ function θ (6) .

Step 6. Classification of the elements depending upon their membership values using thresholds given in decision rules 16–18. *a defining apon area inemeeting values asing) are IFSs, some basic operations are described as follows: derivier* make 16.18 **Definition** 3: *Suppose* **3:** *Suppose*repending upon their membership value *assistance as below, assistance as below,* Classification of the elements depending upon their membership values using Step 6. Classification of the elements depending upon their membership values using
holds given in decision rules 16–18 *assistance as below, For comparing two IFNs,* Ⱳ¹ *and* Ⱳ² thresholds given in decision rules 16–18. $\frac{1}{2}$ *the score function provide the accuracy function provide the accuracy function provide the states the states states assistance as below,*

Figure 1. Flow chart of the interval-valued decision-theoretic rough set model. **Figure 1.** Flow chart of the interval-valued decision-theoretic rough set model.

5. A Case Study

This section includes an illustrative example aimed at determining whether or not a patient has a medical condition through a diagnostic investigation process. The objective is to approve or rule out the existence of the disease.

5.1. Explanation of the Problem

Medical diagnosis is an incredibly crucial task that involves determining which disease or condition a person is suffering from based on their symptoms. Achieving a correct diagnosis is crucial and medical professionals rely on their expertise and experience to make the right decision. With the aid of Intuitionistic Fuzzy Rough Sets (IFRS), healthcare practitioners can enhance their diagnostic accuracy while managing complex linguistic concepts. The use of IFRS has been incredibly successful in medical diagnoses, as demonstrated in numerous studies, including references [\[17](#page-16-16)[,28\]](#page-17-6). Figure [2](#page-11-1) offers a graphical depiction of the medical diagnosis process that highlights the utility of IFRS in this context and Figure [3](#page-15-1) shows the graphical representation of decided elements based on the $IFPWA_{\omega}$ and *IFPWG^ω* operators.

Figure 2. Medical diagnosis diagram. **Figure 2.** Medical diagnosis diagram.

 $\frac{1}{2}$ is the given Table 4 shows the alternatives $\frac{1}{2}$ for all $\frac{1}{2}$ for all $\frac{1}{2}$ and $\frac{1}{2}$ are alternatives I), Assuming there are 15 alternatives (A_i) participating in the diagnosis of the "Coron-

$\frac{1}{2}$ in $I = \{I_1(Chestpain), I_2(Fever), I_3(Fatique), I_4(Cough)\}$

is considered. Moreover, the decision attribute is represented by the sets as follows, dicate a positive decision for the existence of the disease. The diagnosis of the disease is $X = \{A_1, A_2, A_4, A_{15}, A_{11}\}\$ and $X' = \{A_3, A_5, A_6, A_7, A_8, A_9, A_{10}, A_{12}, A_{13}, A_{14}\}\$ which inmade by experts based on the participants' input and the resulting decisions are weighted using a weight vector $\omega = \{0.2, 0.3, 0.4, 0.1\}$. Next, we present a stepwise algorithm to elaborate on the diagnosis of this disease.

Step 1: The given Table [4](#page-12-0) shows the IF information of all the alternatives which participated.

easiness of understanding.

Alternatives	I_1	I ₂	I_3	I_4	${\bf D}$
A_1	(0.1, 0.3)	(0.4, 0.5)	(0.1, 0.5)	(0.1, 0.5)	Yes
A_2	(0.4, 0.5)	(0.5, 0.4)	(0.5, 0.3)	(0.2, 0.6)	Yes
A_3	(0.2, 0.3)	(0.2, 0.4)	(0.6, 0.2)	(0.4, 0.5)	No
A_4	(0.4, 0.2)	(0.1, 0.2)	(0.7, 0.4)	(0.3, 0.1)	Yes
A_5	(0.5, 0.3)	(0.5, 0.2)	(0.3, 0.2)	(0.4, 0.2)	No
A_6	(0.6, 0.2)	(0.7, 0.1)	(0.4, 0.1)	(0.4, 0.4)	No
A_7	(0.7, 0.1)	(0.2, 0.2)	(0.5, 0.2)	(0.5, 0.2)	$\rm No$
A_8	(0.3, 0.4)	(0.3, 0.3)	(0.6, 0.2)	(0.2, 0.3)	No
A_9	(0.4, 0.2)	(0.5, 0.2)	(0.7, 0.2)	(0.3, 0.5)	No
A_{10}	(0.5, 0.2)	(0.8, 0.1)	(0.2, 0.3)	(0.4, 0.3)	$\rm No$
A_{11}	(0.6, 0.2)	(0.9, 0.1)	(0.5, 0.3)	(0.5, 0.4)	Yes
A_{12}	(0.8, 0.1)	(0.0, 0.9)	(0.6, 0.4)	(0.2, 0.2)	No
A_{13}	(0.9, 0.1)	(0.3, 0.2)	(0.4, 0.3)	(0.4, 0.3)	$\rm No$
A_{14}	(0.1, 0.2)	(0.2, 0.2)	(0.6, 0.3)	(0.3, 0.4)	$\rm No$
A_{15}	(0.8, 0.1)	(0.1, 0.3)	(0.3, 0.4)	(0.4, 0.2)	Yes

Table 4. An IF information table of alternatives. **Definition 1** *Definition 1 a ble 4. An IF information table of alternatives* Lable 4. An Table 4. An IF information table of alternatives

Definition 1 [13]: *Let an IFS* Ⱳ *on be symbolized by* () *and* ()*. Mathematically, it is*

easiness of understanding.

Step 2. For alternatives $A_i(i = 1, 2, ..., 15)$, determine all the conditional attributes numbers utilizing $IFPWA_{\tilde{\omega}}$ or $IFPWG_{\tilde{\omega}}$ operators in the following: | Step 2 As an extension of the FS model, Atanassov \mathbb{R} model, Atanassov \mathbb{R} model. If \mathbb{R} is simulated. If \mathbb{R} **Step 2.** For alternatives $A_i(i = 1, 2, ..., 15)$, determine all the conditional attributes ${1 \over 2}$, ${1 \over 2}$, ${1 \over 2}$, ${1 \over 2}$ $\rm {NG}_{\tilde{G}}$ operators in the following: numbers utilizing $IFPWA_{\tilde{\omega}}$ or $IFPWG_{\tilde{\omega}}$ operators in the following: **Step 2.** For alternatives A_i ($i = 1, 2, ..., 15$), deter IFSs Intuitionistic Fuzzy Sets IFN Intuitionistic Fuzzy Number **Step 2.** For alternatives A_i ($i = 1, 2, ..., 15$), determine IFSs Intuitionistic Fuzzy Sets IFN Intuitionistic Fuzzy Number **Step 2.** For alternatives A_i ($i = 1, 2, ..., 15$), determine all the conditional attributes 5tep 2. 1°C
1¹¹ : *then are stiller <u>theories</u>* dume mbers utilizing $IFDMA \approx \text{or } IFPWC \approx \text{operators}$ in the following: $\begin{array}{ccc} \circ & \omega & \omega & \cdot \end{array}$ $\text{Pr}\left[\text{IIFW}(A \times \text{IIFBW}_{\text{C}}\right]$ por intuition in the following: $\sum_{i=1}^{\infty}$ averaging IFPOWG Intuitionistic Fuzzy Power Order Weighted Geometric Fuzzy Power Order Weighted) *then* Ⱳ¹ > Ⱳ² **Step 2.** For alternatives $A_i (i = 1, 2, ..., 15)$, determine all the conditional attributes
numbers utilizing LEPWA \sim or LEPWC \sim operators in the following: **Step 2.** For alternatives $A_i(i = 1, 2, ..., 15)$, determine all the conditional attributes \sim $\frac{m}{m}$ \sim $\frac{m}{m}$ $I = \frac{I}{I}$ is the set of $I = \frac{I}{I}$ intuition $I = \frac{I}{I}$ in $I = \frac{I}{I}$ and $I = \frac{I}{I}$ an The σ_0 defined σ_1 and σ_0 or σ_1 and σ_0 defined in the following $\sum_{i=1}^{n} \sum_{i=1}^{n} \sum_{i=1}^{n} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{j=1}^{n} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{i=1}^{n} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{i=1}^{n} \sum_{i=1}^{n} \sum_{j=1}^{n}$ numbers utilizing $IFPWA_{\tilde{\omega}}$ or $IFPWG_{\tilde{\omega}}$ operators in the following: σ Way Decision MG Membership σ $F_S = \frac{F_S}{F_S}$ Fig. $F_S = \frac{F_S}{F_S}$ $F_S = \frac{F_S}{F_S}$ is the following ω If $\sum_{i=1}^n$ is $\sum_{i=1}^n$ if $\sum_{i=1}^n$ if $\sum_{i=1}^n$ if $\sum_{i=1}^n$ if $\sum_{i=1}^n$ if $\sum_{i=1}^n$ if $\sum_{i=1}^n$ *noted and defined as follows:*

$$
\bigoplus_{i=1}^{n} (\tilde{\omega}_{j}(1+T(W_{i})W_{i})
$$
\n
$$
= \left(1 - \prod_{i=1}^{n} (1 - (m_{i})^{\sum_{i=1}^{n} \omega_{j}(1+T(W_{i}))}, \prod_{i=1}^{n} (m_{i})^{\sum_{i=1}^{n} \omega_{j}(1+T(W_{i}))}, \prod_{i=1}^{n} (n_{i})^{\sum_{i=1}^{n} \omega_{j}(1+T(W_{i}))}\right)
$$

or $\overline{\text{or}}$) *then* Ⱳ¹ > Ⱳ² \mathbf{u}

or
\n
$$
\begin{aligned}\n &\stackrel{n}{\otimes} (\tilde{\omega}_{j}(1+T(\mathbf{W}_{i})\mathbf{W}_{i})) \\
 &IFPWG_{\omega}(\mathbf{W}_{1}, \mathbf{W}_{2}, \dots, \mathbf{W}_{n}) = \frac{i=1}{\sum_{i=1}^{n} \tilde{\omega}_{j}(1+T(\mathbf{W}_{i}))} \\
 &= \left(\prod_{i=1}^{n} (m_{i})^{\sum_{j=1}^{n} \tilde{\omega}_{j}(1+T(\mathbf{W}_{i}))}, 1 - \prod_{i=1}^{n} (1 - (n_{i})^{\sum_{i=1}^{n} \omega_{j}(1+T(\mathbf{W}_{i}))}) \right)\n \end{aligned}
$$
\n\nThe outcomes are presented in Table 5.

 \checkmark
The outcomes are presented in Table 5. *a.* $\frac{1}{2}$ *a. If* (Ⱳ¹ The outcomes are presented in Table 5 . *noted and defined as follows:* The outcomes are presented in Table 5. $\frac{1}{2}$ **in** Table 5 () $\overrightarrow{ }$ *for all the outcomes are presented in Table 5.* $\underline{P}(x) = \frac{P(x)}{P(x)}$ *and* $\underline{P}(x) = \frac{P(x)}{P(x)}$ *and* $\underline{P}(x) = \frac{P(x)}{P(x)}$ The outcomes are prese

ble 5. Aggregated outcomes of attributes of all alternatives. ϵ all alternatives (iii) *If* (Ⱳ¹) = (Ⱳ¹ gated outcomes of attributes of all alternativ e 5. Aggregated outcomes of attributes of all alternatives. *noted and defined as follows: noted and defined as follows:* **Definition 2:** *For IFNs,* Ⱳ = (Ⱳ, Ⱳ) *and the score function and accuracy functions are de-*Table 5. Aggregated outcomes of attributes of all alternatives. (v) Ⱳ¹

<i>Alternatives</i>	IFPWA _{$\tilde{\omega}$}	IFPW $G_{\tilde{\omega}}$
A_1	(0.0203, 0.458)	(0.151, 0.470)
A_2	(0.466, 0.374)	(0.450, 0.393)
A_3	(0.429, 0.281)	(0.347, 0.306)
A_4	(0.500, 0.261)	(0.333, 0.292)
A_5	(0.409, 0.214)	(0.389, 0.217)

() + () ≤ 1 *for all* ∈ *. Generally, the pair (,) represents the IFN.*

IFPWA Intuitionistic Fuzzy Power Weighted Averaging IFPWG Intuitionistic Fuzzy Power Weighted Geometric

ℷ ℷ

Table 5. *Cont.* (v) Ⱳ¹ $Table 5⁶$

 $\mathcal{L}^{\text{max}}_{\text{max}}$ $\mathcal{L}^{\text{max}}_{\text{max}}$

IFPWA Intuitionistic Fuzzy Power Weighted Averaging IFPWG Intuitionistic Fuzzy Power Weighted Geometric

Step 3. Calculate the interval-valued equivalence classes based on the proposed approach and for step size $n = 5$ represented in Table [6.](#page-13-0)

> **Table 6.** Interval-valued equivalence classes. *, the score function and accuracy function provide the*

) are IFSs, some basic operations are

 ${1 \over 2}$, ${1 \over 2}$,

ℷ ℷ

Step 4. The set of states for Yes is $X = \{A_1, A_2, A_4, A_{15}, A_{11}\}\$ and for No is $X' = \{A_3, A_5, A_6, A_7, A_8, A_9, A_{10}, A_{12}, A_{13}, A_{14}\}.$ Now calculate the membership val-ues, non-membership values, and error values in the following Table [7.](#page-13-1)

Table 7. Membership values, non-membership values, and error values.)*.*

Step 5. The cost parameter matrix is given in Table [8](#page-14-0) and the aggregation of the thresholds by Equations (4)–(6) is represented in Table [9.](#page-14-1)) = (Ⱳ¹) *then;* $\lim_{t \to \infty}$ *by* Equations (*x*) (*b*) is represented in Tac IFPWA Intuitionistic Fuzzy Power Weighted Averaging IFPWG Intuitionistic Fuzzy Power Weighted Geometric OCP *S*. The cost parameter OCP is OCP intervalsed OCP $\ln \text{C}$ $\sigma_{\ell} = m$ Intuitionistic Fuzzy Power Weighted Averaging Intuitionistic Fuzzy Power Weighted Geometric Fuzzy Power Weighted Geometric Fuzzy Power Weighted Geometric Fuzzy Power Weighted Geometric Fuzzy Power Weighted Geom IFPOWA Intuitionistic Fuzzy Power Order Weighted averaging IFPOWG Intuitionistic Fuzzy Power Order Weighted Geometric

Table 8. Intuitionistic fuzzy cost parameter matrix.

;

) *then* Ⱳ¹ > Ⱳ²

Table 9. Thresholds for all elements. $\overline{}$ Ⱳ = 〈,Ⱳ(), Ⱳ())| ∈ 〉 (1)

) > (Ⱳ¹

Step 6. Finally, the classification of the elements based on the decision rules presented in Equations (16)–(18) for *POS*, *NEG*, and *BND* regions is shown in Table [10,](#page-14-2) \mathbf{a} , 1 − (1 −) $\left\langle \right\rangle$

Table 10. Classification of alternatives accordingly.

The results show that the alternatives in the *POS* zone have confirmed the presence of coronavirus disease; in the *NEG* region alternatives are safe and in the *BND* region alternatives are not confirmed. In addition, for new alternatives, we can classify them based on the descriptions of the already evaluated alternatives. Figure [3](#page-15-1) shows the effects on the alternative due to the $IFPWA_ω$ and $IFPWG_ω$ operators.

5.2. Benefits of the Proposed Model 5.2. Benefits of the Proposed Model

In the proposed approach, there are benefits which are disclosed in the following: In the proposed approach, there are benefits which are disclosed in the following:

- (1) The most attractive and significant role of this approach is that it is a more generalized $\overline{}$ form. This approach is a generalized form of IFSs. If the NMGs are reduced to zero then the IFSs are converted into fuzzy sets;
The
- (2) The power aggregation operators are very suitable and simple operators to cope with (2) The power aggregation operators are very suitable and simple operators to cope with the problem of decision making under a fuzzy environment especially; these operators the to conclude the attribute's values of elements. To consider the importance, these operators are designed for novel data and used to aggregate the information;
The printing connection in the literature for TAD consisted the theories of YouTa help to conclude the attribute's values of elements. To consider the importance, these
- and are very traditional. In this approach, we used some new steps for TWD, such as $\frac{1}{2}$ and are very traditional. In this approach, we used some new step for TWD, such as power aggregation operators which are designed. Moreover, interval-valued classes power aggregation operators which are designed. Moreover, interval-valued classes (3) The existing approaches in the literature for TWD consist of the theories of Yao [\[37\]](#page-17-14) are developed to classify the participants;
- are developed to classify the participants; (4) In this medical case, diagnosing the disease is a very big issue for experts as well as patients. To cope with this challenge, we created a model made up of many patients with their disease's attributes. Finally, the experts calculated the decisions.

6. Conclusions and Future Work

In the article, we firstly reviewed the basic idea of intuitionistic fuzzy sets and power aggregation operators. Moreover, we revised the model of three-way decision based on the Bayesian theory introduced by Yao [\[25\]](#page-17-3). In classical TWD models, equivalence classes play a vital role in discretizing the information system. In this paper, we developed a novel approach to discretize the information table. To classify the participants, interval-valued classes are used and three zones on the bases of those classes. The Bayesian model for minimizing risk is also revised for decision taking. Aggregation operators are used to aggregate the results and compose the attributes values into single values. Considering the importance of operators, we utilized power aggregation operators. Moreover, an algorithm to identify the disease using the proposed approach was produced. We disclosed the benefits of the approach: this approach is more general than the existing TWD model. Next, the findings of this study will be enlarged to the extension of the fuzzy and rough data

and some new aggregation operators to cope with real-life problems will be developed. Moreover, we will utilize the established approach towards the existing literature [\[41](#page-17-16)[–45\]](#page-17-17).

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