

Article

# Selection of Optimal Approach for Cardiovascular Disease Diagnosis under Complex Intuitionistic Fuzzy Dynamic Environment

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**Abstract:** Cardiovascular disease (CVD) is a leading global health concern. There is a critical need for accurate and reliable decision-making tools to select the optimal approach for diagnosing cardiovascular disease (CVD). In this study, we have addressed this pressing issue. Complex intuitionistic fuzzy set (CIFS) theory is adept at encapsulating vagueness due to its capability to encompass comprehensive problem specifications characterized by both intuitionistic uncertainty and periodicity. Within the scope of this article, we present two novel aggregation operators: the complex intuitionistic fuzzy dynamic weighted averaging (CIFDWA) operator and the complex intuitionistic fuzzy dynamic weighted geometric (CIFDWG) operator. Some intriguing characteristics of these operators are elucidated, and important special cases are also defined in detail. We devise an enhanced score function to rectify the deficiencies observed in the existing score function under complex intuitionistic fuzzy knowledge. Furthermore, these operators are employed in the development of a systematic approach for the handling of multiple attribute decision-making (MADM) scenarios involving complex intuitionistic fuzzy data. Moreover, we undertake the resolution of an MADM problem, wherein we ascertain the optimal approach for diagnosing cardiovascular disease (CVD) through the utilization of the proposed operators, thereby substantiating their utility in decision-making processes. Finally, we conduct a comprehensive comparative analysis, pitting the presented operators against an array of existing counterparts, in order to demonstrate the reliability and stability inherent in the derived methodologies.

**Keywords:** complex intuitionistic fuzzy sets; dynamic aggregation operators; decision-making methods; cardiovascular disease

**MSC:** 03E72; 94D05



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## 1. Introduction

### 1.1. Background

Algorithmic decision-making systems have greatly improved the management of large healthcare datasets, particularly in prediction and diagnosis [1,2]. In various fields, such as computer science, business administration, medical diagnostics, and enterprise management, strategic decision-making tools play a crucial role, especially in dynamic market product selection. Each attribute involved in decision-making plays a unique and influential role, often leading to different attribute weights in the framework of MADM [3,4]. The

utilization of fuzzy sets (FS) is imperative in addressing the complexities and uncertainties inherent in real-world decision-making. Zadeh [5] introduced the theory of fuzzy sets, which has since proven to be valuable in evaluating uncertain and ambiguous data. The central concept of a fuzzy set revolves around membership degrees, bounded within the range  $[0, 1]$ . The challenge of aggregating attributes and quantifying dissimilarities among them is prevalent across various domains, leading to an increasing reliance on fuzzy sets, as documented in the existing literature [6–8]. Over time, the scope of FS has evolved beyond its initial focus on membership degrees, overlooking the non-membership aspect. In response, Atanassov [9] introduced the concept of non-membership degrees within the domain of intuitionistic fuzzy sets (IFS). The mathematical formulation designates the membership degree as  $\mu(z)$ , while the non-membership degree is represented as  $\nu(z)$ , subject to the constraint:  $0 \leq \mu(z) + \nu(z) \leq 1$ . Fuzzy set theory is a specific case of IFS, achieved by setting  $\nu(z) = 0$ . Addressing the challenge of aggregating and quantifying the distance between multiple attributes, researchers have proposed various IFS-based approaches in different disciplines. For example, Liu et al. [10] developed a hybrid approach incorporating variable weighting for interval-valued IFS. Thao [11] investigated entropies and divergence measures for IFS, considering Archimedean norms. Gohain et al. [12] extended this research by examining the similarity and distance measures associated with IFSs. Garg and Rani [13] identified and studied similarity measures tailored to IFSs. Hayat et al. [14] explored new aggregation operators for effectively representing information within an IFS framework. For further advances in the field of IFSs, readers are recommended to read references [15–22].

The practical effectiveness of the information provided in the previous paragraph is restricted due to its reliance on FS and IFS, which only process one dimension of data at a time. Consequently, it is possible that experts have incurred significant data losses due to these factors. In lieu of FS and IFS, a need has arisen for a technique that can effectively handle two-dimensional information. After substantial research, Ramot et al. [23] addressed this issue by developing a unique theory of complex fuzzy sets (CFSs), which incorporates a periodic term in the membership degree, referred to as the “phase term”, that assumes a pivotal and significant function within the context of the decision-making process. Furthermore, the task of aggregating and determining the distance between multiple attributes remains challenging for everyone. Given these challenging scenarios, certain considerations have arisen regarding the utilization of CFSs in various fields [24–27]. The concept of a complex-valued non-membership degree is introduced in [28] to develop the idea of CIFS, a generalization of CFS.

The challenge of aggregating and measuring the distance between multiple attributes remains a complex task. In response to these challenges, various considerations have emerged regarding the application of CIFS in different domains. For instance, Garg and Rani [29] explored novel aggregation operators within the realm of CIFS. Reference [30] delved into robust aggregation information within CIFS, while [31] investigated generalized geometric aggregation information in this context. Moreover, in [32], a comprehensive theory was developed for prioritized aggregation operators concerning complex intuitionistic fuzzy soft information, with a subsequent focus on their practical use in decision-making. Masmali et al. [33] introduced a technique for finding an optimal water purification procedure using a complex intuitionistic fuzzy Dombi environment.

### *1.2. Novelty, Objectives, and Main Outcomes of the Study*

The above-mentioned research efforts primarily address decision-making scenarios where all initial decision data are presented simultaneously. However, in various decision-making contexts, such as dynamic medical diagnostics, multi-period investment decision-making, the dynamic assessment of military system efficiency, and personnel dynamic evaluation, it is common for the primary decision-related data to be gathered at disparate time intervals. Thus, it becomes important to conduct research dealing with the problems associated with dynamic fuzzy MADM [34,35]. Dynamic aggregation operators offer

adaptability and precision in decision-making by accommodating changing data with regard to time. They enable real-time decisions, reduce risks, optimize resource allocation, aid in strategic planning, and can lead to cost savings.

The primary objective of this study is to enhance the field of MADM by addressing the unique challenges posed by dynamic decision data. We aim to develop novel dynamic aggregation operators and associated methodologies to effectively handle complex intuitionistic fuzzy information. The study proposes two novel dynamic aggregation operators, namely, the CIFDWA operator and the CIFDWG operator, for the purpose of aggregating complex intuitionistic fuzzy information in the context of MADM problems. Additionally, based on the CIFDWA and CIFDWG operators, we developed respective step-by-step mathematical mechanisms for solving complex intuitionistic fuzzy dynamic MADM problems in which all attribute values are expressed as complex intuitionistic fuzzy numbers collected in different time periods. Our research endeavors revolve around several key theoretical framework objectives.

- i. A novel score function that enhances the complex intuitionistic fuzzy system while mitigating the limitations of the previous score function is formulated. The task is achieved through the utilization of advanced mathematical and statistical methods. This improves the accuracy and precision of the grading system.
- ii. Two novel aggregation operators, namely, the CIFDWA operator and the CIFDWG operator, are proposed for the purpose of aggregating complex intuitionistic fuzzy dynamic information in the context of MADM problems.
- iii. A comprehensive imperative description is provided to elucidate the fundamental characteristics of the operators under consideration, specifically their idempotency, monotonicity, and boundedness.
- iv. These operators are employed in the development of a systematic approach for the handling of MADM scenarios involving complex intuitionistic fuzzy data.
- v. The practical application of the CIFDWA and CIFDWG operators is demonstrated through their implementation for an MADM problem that involves identifying the most effective strategy for diagnosing cardiovascular disease. This practical application serves to demonstrate the effectiveness of our operators in enhancing decision-making processes.
- vi. The stability and efficacy of the proposed approach is validated by conducting comparative analyses with various existing studies.

The remaining part of this work is organized in the following manner: In Section 2, we provide a comprehensive exposition of fundamental definitions. We also address the shortcomings in the existing score function and introduce a novel score function tailored to rectify this limitation within the context of the complex intuitionistic fuzzy environment. Section 3 expounds upon dynamic aggregation operators designed for CIFS and delves into their foundational properties. In Section 4, we elucidate a method for addressing the intricate problem of multiple attribute decision making (MADM) with complex intuitionistic fuzzy information by employing complex intuitionistic fuzzy dynamic weighted aggregation operators. In Section 5, these newly formulated operators are employed to ascertain the most efficacious approach for diagnosing cardiovascular disease. Additionally, we present a comparative analysis, aiming to elucidate the effectiveness and viability of this innovative strategy in contrast to established methodologies. Finally, the paper culminates by furnishing a conclusion of the principal findings and deliberating on potential implications.

## 2. Preliminaries

This section elucidates fundamental definitions essential for comprehending the subject matter discussed in this study. Additionally, we introduce a novel score function that enhances the complex intuitionistic fuzzy system and addresses the limitations of the previous score function.

The subsequent definition provides an overview of the IFS concept, which arises as an expansion of traditional fuzzy sets by accounting for both membership and non-membership degrees of an element.

**Definition 1 ([9]).** An IFS, denoted as  $A$  within the universal set  $Z$ , is formally characterized as  $A = \{(z, \mu_A(z), \nu_A(z)) : z \in Z\}$ . In this representation,  $\mu_A$  and  $\nu_A$  are functions mapping elements of  $Z$  to the closed interval  $[0, 1]$  and known as membership and non-membership functions, respectively. These functions adhere to the constraint  $0 \leq \mu_A(z) + \nu_A(z) \leq 1$ . Moreover, the hesitancy margin of the IFS, denoted as  $\pi_A(z)$ , is described by  $\pi_A(z) = 1 - \mu_A(z) - \nu_A(z)$ .

In the following definition, we expound upon the conceptual framework of CFS, a significant advancement of classical fuzzy sets that adeptly addresses both phase and periodicity aspects.

**Definition 2 ([23]).** A CFS denoted as  $C$  over  $Z$  is formally characterized as:  $C = \{(z, \mu_C(z)) : z \in Z\}$ . Here,  $\mu_C$  represents the complex-valued membership degree function, which maps elements of  $Z$  to a subset of complex numbers  $\{a : a \in \mathbb{C}, |a| \leq 1\}$  and is defined as:  $\mu_C(z) = r_C(z)e^{i2\pi\theta_C(z)}$ . In this expression,  $i = \sqrt{-1}$  and the parameters  $r_C(z)$  and  $\theta_C(z)$  satisfy the constraints  $0 \leq r_C(z), \theta_C(z) \leq 1$ .

Now, we provide the definition of CIFS, an extension of both IFS and CFS.

**Definition 3 ([28]).** A CIFS denoted as  $C$  over  $Z$  is formally described as follows:  $C = \{(z, \mu_C(z), \nu_C(z)) : z \in Z\}$ . Here,  $\mu_C$  and  $\nu_C$  represent the complex-valued membership and non-membership functions, respectively, that assign to each element a complex number in the unit closed disk, and are defined as:  $\mu_C(z) = r_C(z)e^{i2\pi\theta_C(z)}$  and  $\nu_C(z) = K_C(z)e^{i2\pi\varphi_C(z)}$

Moreover, in these expressions, the parameters  $r_C(z), K_C(z), \theta_C(z)$ , and  $\varphi_C(z)$  satisfy the constraints  $0 \leq r_C(z), K_C(z), \theta_C(z), \varphi_C(z), r_C(z) + K_C(z), \theta_C(z) + \varphi_C(z) \leq 1$ .

For the sake of clarity and convenience, we denote the membership and non-membership degrees of an element  $z \in Z$  as  $C = ((\Gamma, \theta), (K, \varphi))$ , referring to it as a complex intuitionistic fuzzy number (CIFN). In this CIFN framework, the following conditions are met:  $0 \leq \Gamma, K, \Gamma + K \leq 1$ , and  $0 \leq \theta, \varphi, \theta + \varphi \leq 1$ .

Aggregation operators serve as fundamental mathematical tools for combining multiple inputs into a singular, distinct output. In the following discourse, we elucidate arithmetic aggregation operators tailored for the synthesis of complex intuitionistic fuzzy (CIF) information.

**Definition 4 ([30]).** Let  $\alpha_\gamma = ((\Gamma_\gamma, \theta_\gamma), (K_\gamma, \varphi_\gamma))$  for  $\gamma = 1, 2, \dots, n$  be a collection of CIFNs. Furthermore, let  $w = [w_1, w_2, \dots, w_n]^T$  denotes the weight vector associated with  $\alpha_\gamma$ , such that  $w_\gamma \in [0, 1]$ , satisfying the constraint  $\sum_{\gamma=1}^n w_\gamma = 1$ . For a set of  $n$  CIFNs,  $\alpha_1, \alpha_2, \dots, \alpha_n$ , the mapping complex intuitionistic fuzzy weighted averaging operator (CIFWA), denoted as CIFWA:  $\psi^p \rightarrow \psi$ , is defined as follows:

$$CIFWA(\alpha_1, \alpha_2, \dots, \alpha_n) = \bigoplus_{\gamma=1}^n w_\gamma \alpha_\gamma = \left( \left( 1 - \prod_{\gamma=1}^n (1 - \Gamma_\gamma)^{w_\gamma}, 1 - \prod_{\gamma=1}^n (1 - \theta_\gamma)^{w_\gamma} \right), \left( \prod_{\gamma=1}^n (K_\gamma)^{w_\gamma}, \prod_{\gamma=1}^n (\varphi_\gamma)^{w_\gamma} \right) \right) \tag{1}$$

**Definition 5 ([30]).** Let  $\alpha_\gamma = ((\Gamma_\gamma, \theta_\gamma), (K_\gamma, \varphi_\gamma))$  for  $\gamma = 1, 2, \dots, n$  be a collection of CIFNs. Furthermore, let  $w = [w_1, w_2, \dots, w_n]^T$  denotes the weight vector associated with  $\alpha_\gamma$ , such that  $w_\gamma \in [0, 1]$ , satisfying the constraint  $\sum_{\gamma=1}^n w_\gamma = 1$ . For a set of  $n$  CIFNs,  $\alpha_1, \alpha_2, \dots, \alpha_n$ , the mapping complex intuitionistic fuzzy weighted geometric operator (CIFWG), denoted as CIFWG :  $\psi^p \rightarrow \psi$ , is defined as follows:

$$CIFWG(\alpha_1, \alpha_2, \dots, \alpha_n) = \otimes_{\gamma=1}^n \alpha_{\gamma}^{w_{\gamma}} = \left( \begin{matrix} \left( \prod_{\gamma=1}^n (\Gamma_{\gamma})^{w_{\gamma}}, \prod_{\gamma=1}^n (\theta_{\gamma})^{w_{\gamma}} \right), \\ \left( 1 - \prod_{\gamma=1}^n (1 - K_{\gamma})^{w_{\gamma}}, 1 - \prod_{\gamma=1}^n (1 - \varphi_{\gamma})^{w_{\gamma}} \right) \end{matrix} \right) \tag{2}$$

In the subsequent definition, we rewrite the notion of an intuitionistic fuzzy variable.

**Definition 6 ([35]).** Let  $t$  be a time variable, we define  $\alpha_t$  as an intuitionistic fuzzy variable, where  $\alpha_t = (\mu_t, \nu_t)$ , characterized by  $\mu_t$  and  $\nu_t$ , both of which are constrained within the interval  $[0, 1]$  and subject to the condition  $\mu_t + \nu_t \leq 1$ . For intuitionistic fuzzy variable  $\alpha_t = (\mu_t, \nu_t)$ , if  $t = t_1, t_2, \dots, t_p$  then  $\alpha_{t_1}, \alpha_{t_2}, \dots, \alpha_{t_p}$  indicate  $p$  intuitionistic fuzzy numbers collected at  $p$  different periods.

The next definition provides the concept of CIFN which is used in evaluation and ranking procedures based on designated score and accuracy functions within the established framework.

**Definition 7 ([28]).** Let CIFN  $\alpha_0$  be represented as  $((\Gamma_0, \theta_0), (K_0, \varphi_0))$ , where the score function is defined as:  $S(\alpha_0) = \frac{1}{2}(\Gamma_0 - K_0 + \theta_0 - \varphi_0)$ ,  $S(\alpha_0) \in [-1, 1]$ .

Additionally, the accuracy function is specified as:  $H(\alpha_0) = \frac{1}{2}(\Gamma_0 + K_0 + \theta_0 + \varphi_0)$ ,  $H(\alpha_0) \in [0, 1]$ . Furthermore, it is worth noting that any pair of CIFNs,  $\alpha_1$  and  $\alpha_2$ , adhere to the following comparative principles:

- i. If  $S(\alpha_1) > S(\alpha_2)$ , then  $\alpha_1 > \alpha_2$
- ii. If  $S(\alpha_1) < S(\alpha_2)$ , then  $\alpha_1 < \alpha_2$
- iii. If  $S(\alpha_1) = S(\alpha_2)$ , then  $H(\alpha_1) > H(\alpha_2) \implies \alpha_1 > \alpha_2$ ,  $H(\alpha_1) < H(\alpha_2) \implies \alpha_1 < \alpha_2$  and  $H(\alpha_1) = H(\alpha_2) \implies \alpha_1 = \alpha_2$

*Improved Score Function*

Here, we endeavor to illuminate the constraints inherent in the score function employed for CIFNs, as previously articulated in the literature [28]. Subsequent to this exposition, our discourse centers upon the refinement of the score function in order to ameliorate this deficiency.

**Example 1.** Let us consider two arbitrary CIFNs, denoted as  $\alpha_1 = ((0.4, 0.6), (0.3, 0.2))$  and  $\alpha_2 = ((0.7, 0.3), (0.1, 0.4))$ . The application of Definition 7 to CIFNs  $\alpha_1$  and  $\alpha_2$  yields  $S(\alpha_1) = S(\alpha_2) = 0.25$  and  $H(\alpha_1) = H(\alpha_2) = 0.75$ . It is evident, based on property 3(c) in Definition 7, that CIFNs  $\alpha_1$  and  $\alpha_2$  cannot be compared.

The aforementioned example serves to underscore the deficiency intrinsic to the extant score function within our scope of study. Consequently, this prompts our pursuit of refining the score function, resulting in the introduction of an updated score function expounded upon in the subsequent definition.

**Definition 8.** Let  $\alpha_0 = ((\Gamma_0, \theta_0), (K_0, \varphi_0))$  signify a CIFN. The improved score function denoted as  $C(\alpha_0)$  for CIFNs is formulated as follows:

$$C(\alpha_0) = \frac{1}{2}(\Gamma_0 - K_0 + \theta_0 - \varphi_0 + \Gamma_0 K_0)$$

Herein,  $C(\alpha_0) \in [-1, 1]$ .

Moreover, it is imperative to highlight that the aforementioned proposed score function adheres to the comparison law for any pair of CIFNs  $\alpha_1$  and  $\alpha_2$ , that is,  $C(\alpha_1) > C(\alpha_2) \implies \alpha_1 > \alpha_2$ ,  $C(\alpha_1) < C(\alpha_2) \implies \alpha_1 < \alpha_2$ , and  $C(\alpha_1) = C(\alpha_2) \implies \alpha_1 = \alpha_2$ .

To illuminate the precision and utility of this proposed score function tailored for CIFNs, let us delve into the following illustrative example.

**Example 2.** Consider two CIFNs,  $\alpha_1 = ((0.4, 0.6), (0.3, 0.2))$  and  $\alpha_2 = ((0.7, 0.3), (0.1, 0.4))$ , which are arbitrarily selected. Notably, Example 1 has previously demonstrated the limitations of the extant score function when applied to these CIFNs. Applying the framework delineated in Definition 7 to these CIFNs yields  $C(\alpha_1) = 0.51$  and  $C(\alpha_2) = 0.68$ . Consequently, in accordance with the precept elucidated in Property 2 of Definition 7, it becomes evident that  $\alpha_1 < \alpha_2$ . This compelling evidence leads to the inference that  $\alpha_2$  is indeed superior to  $\alpha_1$ .

### 3. Dynamic Operations on CIFNs

The process of aggregating information is a basic and important topic of research within the field of information fusion. The operators CIFWA and CIFWG are restricted to the aggregation of complex intuitionistic fuzzy information that involves time-independent arguments. When considering the element of time, it is important to note that complex intuitionistic fuzzy information can be acquired at various intervals. In such cases, it is necessary to ensure that the aggregation operators and their corresponding weights are not held constant. As a consequence, in the subsequent sections based on [36], we initially establish the conceptual framework of a complex intuitionistic fuzzy variable.

#### 3.1. Operational Laws of Dynamic CIFNs

In this subsection, we introduce the concept of a complex intuitionistic fuzzy dynamic variable and delineate its fundamental laws.

In the following definition, we develop the concept of a complex intuitionistic fuzzy variable.

**Definition 9.** In the realm of mathematical discourse, we introduce the concept of a “complex intuitionistic fuzzy variable,” denoted as  $\alpha_t$ , where  $t$  is a time variable. This variable is characterized by the following components:  $((\Gamma_t, \theta_t), (K_t, \varphi_t))$ , such that  $\Gamma_t, K_t, \theta_t, \varphi_t \in [0, 1]$ . Additionally, these parameters must adhere to the constraint  $\Gamma_t + K_t \leq 1$  and  $\theta_t + \varphi_t \leq 1$ .

In a broader perspective, if we consider a sequence of time instances,  $t_1, t_2, \dots, t_p$ , then  $\alpha_{t_1}, \alpha_{t_2}, \dots, \alpha_{t_p}$  represent  $p$  distinct CIFNs, each associated with a specific time period.

In the domain of CIFNs, we articulate the fundamental principles governing their interrelationship in Definitions 10 and 11.

**Definition 10.** Consider two CIFNs,  $\alpha_{t_1} = ((\Gamma_{t_1}, \theta_{t_1}), (K_{t_1}, \varphi_{t_1}))$  and  $\alpha_{t_2} = ((\Gamma_{t_2}, \theta_{t_2}), (K_{t_2}, \varphi_{t_2}))$ . The essential operational laws governing their interaction are as follows:

- i.  $\alpha_{t_1} \leq \alpha_{t_2}$  if  $\Gamma_{t_1} \leq \Gamma_{t_2}, K_{t_1} \geq K_{t_2}$  and  $\theta_{t_1} \leq \theta_{t_2}, \varphi_{t_1} \geq \varphi_{t_2}$
- ii.  $\alpha_{t_1} = \alpha_{t_2}$  if and only if  $\alpha_{t_1} \subseteq \alpha_{t_2}$  and  $\alpha_{t_2} \subseteq \alpha_{t_1}$
- iii.  $\alpha_{t_1}^c = ((K_{t_1}, \varphi_{t_1}), (\Gamma_{t_1}, \theta_{t_1}))$

**Definition 11.** For two CIFNs,  $\alpha_{t_1} = ((\Gamma_{t_1}, \theta_{t_1}), (K_{t_1}, \varphi_{t_1}))$  and  $\alpha_{t_2} = ((\Gamma_{t_2}, \theta_{t_2}), (K_{t_2}, \varphi_{t_2}))$ , in tandem with a positive real scalar factor  $\lambda$ , the general operations are succinctly articulated as follows:

- i.  $\alpha_{t_1} \oplus \alpha_{t_2} = \left( \left( 1 - \prod_{k=1}^2 (1 - \Gamma_{t_k}), 1 - \prod_{k=1}^2 (1 - \theta_{t_k}) \right), \left( \prod_{k=1}^2 K_{t_k}, \prod_{k=1}^2 \varphi_{t_k} \right) \right)$
- ii.  $\alpha_{t_1} \otimes \alpha_{t_2} = \left( \left( \prod_{k=1}^2 \Gamma_{t_k}, \prod_{k=1}^2 \theta_{t_k} \right), \left( 1 - \prod_{k=1}^2 (1 - K_{t_k}), 1 - \prod_{k=1}^2 (1 - \varphi_{t_k}) \right) \right)$
- iii.  $\lambda \alpha_{t_1} = \left( 1 - (1 - \Gamma_{t_1})^\lambda, 1 - (1 - \theta_{t_1})^\lambda \right), \left( (K_{t_1})^\lambda, (\varphi_{t_1})^\lambda \right)$
- iv.  $\alpha_{t_1}^\lambda = \left( (\Gamma_{t_1})^\lambda, (\theta_{t_1})^\lambda \right), \left( 1 - (1 - K_{t_1})^\lambda, 1 - (1 - \varphi_{t_1})^\lambda \right)$

#### 3.2. Structural Properties of CIFDWA Operator

In this subsection, we introduce the concept of a CIFDWA operator and establish its fundamental structural properties.

**Definition 12.** We consider a set CIFNs, denoted as  $\alpha_{t_k} = ((\Gamma_{t_k}, \theta_{t_k}), (K_{t_k}, \varphi_{t_k}))$ , with respect to different time periods  $t_k$  ( $k = 1$  to  $p$ ). We also have weight vector  $\lambda_t = [\lambda_{t_1}, \lambda_{t_2}, \dots, \lambda_{t_p}]^T$  associated with time periods  $t_k$ , where  $\lambda_{t_k} \in [0, 1]$  and  $\sum_{k=1}^p \lambda_{t_k} = 1$ . The CIFDWA operator is a mapping CIFDWA :  $\psi^p \rightarrow \psi$ , defined as follows;

$$\text{CIFDWA}(\alpha_{t_1}, \alpha_{t_2}, \dots, \alpha_{t_p}) = \bigoplus_{k=1}^p \lambda_{t_k} \alpha_{t_k} \tag{3}$$

This operator combines these CIFNs using their associated weights in the framework of complex intuitionistic fuzzy logic.

The following theorem shows that the aggregated value of any number of CIFNs within the framework of the CIFDWA operator yields another CIFN.

**Theorem 1.** Consider a collection of CIFNs represented as  $\alpha_{t_k} = ((\Gamma_{t_k}, \theta_{t_k}), (K_{t_k}, \varphi_{t_k}))$ , observed at  $p$  distinct time periods denoted by  $t_k$ , where  $k$  ranges from 1 to  $p$ . Let  $\lambda_t = [\lambda_{t_1}, \lambda_{t_2}, \dots, \lambda_{t_p}]^T$  be the associated weight vector of time periods  $t_k$ , such that  $\lambda_{t_k} \in [0, 1]$ , satisfying the constraint  $\sum_{k=1}^p \lambda_{t_k} = 1$ . Therefore, the aggregated value of these CIFNs, as determined within the framework of the CIFDWA, is also a CIFN. This aggregated CIFN is computed as follows:

$$\text{CIFDWA}(\alpha_{t_1}, \alpha_{t_2}, \dots, \alpha_{t_p}) = \left( \left( 1 - \prod_{k=1}^p (1 - \Gamma_{t_k})^{\lambda_{t_k}}, 1 - \prod_{k=1}^p (1 - \theta_{t_k})^{\lambda_{t_k}} \right), \left( \prod_{k=1}^p (K_{t_k})^{\lambda_{t_k}}, \prod_{k=1}^p (\varphi_{t_k})^{\lambda_{t_k}} \right) \right).$$

**Proof of Theorem 1.** We establish the validity of the theorem through the application of the mathematical induction technique.

For the base case when  $p = 2$ , we have two CIFNs, namely,  $\alpha_{t_1} = ((\Gamma_{t_1}, \theta_{t_1}), (K_{t_1}, \varphi_{t_1}))$  and  $\alpha_{t_2} = ((\Gamma_{t_2}, \theta_{t_2}), (K_{t_2}, \varphi_{t_2}))$ . Employing the operations defined for CIFNs, we obtain the following expressions:

$$\lambda_{t_1} \alpha_{t_1} = \left( \left( 1 - (1 - \Gamma_{t_1})^{\lambda_{t_1}}, 1 - (1 - \theta_{t_1})^{\lambda_{t_1}} \right), \left( (K_{t_1})^{\lambda_{t_1}}, (\varphi_{t_1})^{\lambda_{t_1}} \right) \right) \text{ and}$$

$$\lambda_{t_2} \alpha_{t_2} = \left( \left( 1 - (1 - \Gamma_{t_2})^{\lambda_{t_2}}, 1 - (1 - \theta_{t_2})^{\lambda_{t_2}} \right), \left( (K_{t_2})^{\lambda_{t_2}}, (\varphi_{t_2})^{\lambda_{t_2}} \right) \right)$$

Applying the CIFDWA operator to  $\alpha_{t_1}$  and  $\alpha_{t_2}$ , we combine these two CIFNs as follows:

$$\begin{aligned} \text{CIFDWA}(\alpha_{t_1}, \alpha_{t_2}) &= \lambda_{t_1} \alpha_{t_1} \oplus \lambda_{t_2} \alpha_{t_2} \\ &= \left( \left( 1 - (1 - \Gamma_{t_1})^{\lambda_{t_1}}, 1 - (1 - \theta_{t_1})^{\lambda_{t_1}} \right), \left( (K_{t_1})^{\lambda_{t_1}}, (\varphi_{t_1})^{\lambda_{t_1}} \right) \right) \\ &\oplus \left( \left( 1 - (1 - \Gamma_{t_2})^{\lambda_{t_2}}, 1 - (1 - \theta_{t_2})^{\lambda_{t_2}} \right), \left( (K_{t_2})^{\lambda_{t_2}}, (\varphi_{t_2})^{\lambda_{t_2}} \right) \right) \end{aligned}$$

It follows that:

$$\begin{aligned} \text{CIFDWA}(\alpha_{t_1}, \alpha_{t_2}) &= \left( \left( 1 - \prod_{k=1}^2 (1 - \Gamma_{t_k})^{\lambda_{t_k}}, 1 - \prod_{k=1}^2 (1 - \theta_{t_k})^{\lambda_{t_k}} \right), \left( \prod_{k=1}^2 (K_{t_k})^{\lambda_{t_k}}, \prod_{k=1}^2 (\varphi_{t_k})^{\lambda_{t_k}} \right) \right) \end{aligned}$$

Hence, we have established the correctness of the theorem for the base case when  $p = 2$ .

Now, we proceed with the induction step. Let us assume that the result holds for  $p = m$ , where  $m$  is a positive integer. This means that:

$$CIFDWA(\alpha_{t_1}, \alpha_{t_2}, \dots, \alpha_{t_m}) = \left( \begin{matrix} \left(1 - \prod_{k=1}^m (1 - \Gamma_{t_k})\right)^{\lambda_{t_k}}, 1 - \prod_{k=1}^m (1 - \theta_{t_k})^{\lambda_{t_k}} \\ \left(\prod_{k=1}^m (K_{t_k})^{\lambda_{t_k}}, \prod_{k=1}^m (\varphi_{t_k})^{\lambda_{t_k}}\right) \end{matrix} \right)$$

Now, for the case when  $p = m + 1$ , we have:

$$\begin{aligned} & CIFDWA(\alpha_{t_1}, \alpha_{t_2}, \dots, \alpha_{t_m}, \alpha_{t_{m+1}}) \\ &= (\lambda_{t_1} \alpha_{t_1}) \oplus (\lambda_{t_2} \alpha_{t_2}) \oplus \dots \oplus (\lambda_{t_m} \alpha_{t_m}) \oplus (\lambda_{t_{m+1}} \alpha_{t_{m+1}}) \\ &= \left( \left(1 - \prod_{k=1}^m (1 - \Gamma_{t_k})\right)^{\lambda_{t_k}}, 1 - \prod_{k=1}^m (1 - \theta_{t_k})^{\lambda_{t_k}} \right), \left( \prod_{k=1}^m (K_{t_k})^{\lambda_{t_k}}, \prod_{k=1}^m (\varphi_{t_k})^{\lambda_{t_k}} \right) \\ &\oplus \left( \left(1 - (1 - \Gamma_{t_{m+1}})\right)^{\lambda_{t_{m+1}}}, 1 - (1 - \theta_{t_{m+1}})^{\lambda_{t_{m+1}}} \right), \left( (K_{t_{m+1}})^{\lambda_{t_{m+1}}}, (\varphi_{t_{m+1}})^{\lambda_{t_{m+1}}} \right) \end{aligned}$$

This shows that:

$$\begin{aligned} & CIFDWA(\alpha_{t_1}, \alpha_{t_2}, \dots, \alpha_{t_m}, \alpha_{t_{m+1}}) \\ &= \left( \left(1 - \prod_{k=1}^{m+1} (1 - \Gamma_{t_k})\right)^{\lambda_{t_k}}, 1 - \prod_{k=1}^{m+1} (1 - \theta_{t_k})^{\lambda_{t_k}} \right), \left( \prod_{k=1}^{m+1} (K_{t_k})^{\lambda_{t_k}}, \prod_{k=1}^{m+1} (\varphi_{t_k})^{\lambda_{t_k}} \right) \end{aligned}$$

This expression is consistent with the formula provided in the theorem statement. Therefore, by the principle of mathematical induction, we have demonstrated that the result holds for all positive integers of  $p$ . □

The following example serves to elucidate the aforementioned assertion.

**Example 3.** Suppose  $\alpha_{t_1} = ((0.6, 0.5), (0.3, 0.5))$ ,  $\alpha_{t_2} = ((0.4, 0.8), (0.2, 0.1))$ ,  $\alpha_{t_3} = ((0.3, 0.3), (0.4, 0.7))$ , and  $\alpha_{t_4} = ((0.9, 0.4), (0.1, 0.4))$  are any four CIFNs and  $\lambda_t = [0.35, 0.15, 0.3, 0.2]^T$  denotes the weight vectors of the time periods  $t_k$  where,  $k = 1, 2, 3, 4$ . Then, we have  $\prod_{k=1}^4 (1 - \Gamma_{t_k})^{\lambda_{t_k}} = 0.381$ ,  $\prod_{k=1}^4 (1 - \theta_{t_k})^{\lambda_{t_k}} = 0.499$ ,  $\prod_{k=1}^4 (K_{t_k})^{\lambda_{t_k}} = 0.247$  and  $\prod_{k=1}^4 (\varphi_{t_k})^{\lambda_{t_k}} = 0.146$ .

This implies that

$$\begin{aligned} CIFDWA(\alpha_{t_1}, \alpha_{t_2}, \alpha_{t_3}, \alpha_{t_4}) &= \bigoplus_{k=1}^4 \lambda_{t_k} \alpha_{t_k} \\ &= ((0.619, 0.501), (0.247, 0.416)) \end{aligned}$$

Therefore, we deduce that the outcome derived from the preceding discourse also constitutes a CIFN.

The following result delineates that the collection of any number of CIFNs manifests the property of idempotency within the context of the CIFDWA operator.

**Theorem 2. (Idempotency Property):** A set of CIFNs  $\alpha_{t_k} = ((\Gamma_{t_k}, \theta_{t_k}), (K_{t_k}, \varphi_{t_k}))$ , for  $k$  ranging from 1 to  $p$ , satisfies the condition  $\alpha_{t_k} = \alpha_{t_0}$  for all  $t_k$ , where  $\alpha_{t_0} = ((\Gamma_{t_0}, \theta_{t_0}), (K_{t_0}, \varphi_{t_0}))$  is itself a CIFN. We also have weight vector  $\lambda_t = [\lambda_{t_1}, \lambda_{t_2}, \dots, \lambda_{t_p}]^T$  associated with time periods  $t_k$ , where  $\lambda_{t_k} \in [0, 1]$  and  $\sum_{k=1}^p \lambda_{t_k} = 1$ . Therefore,  $CIFDWA(\alpha_{t_1}, \alpha_{t_2}, \dots, \alpha_{t_p}) = \alpha_{t_0}$ .



**Proof of Theorem 2.** Given that  $\alpha_{t_k} = \alpha_{t_0}$  for all  $k$ , it follows from Definition 10 that  $\Gamma_{t_k} = \Gamma_{t_0}, \theta_{t_k} = \theta_{t_0}, K_{t_k} = K_{t_0}$ , and  $\varphi_{t_k} = \varphi_{t_0}$  for all  $k$ . Substituting these relations into Theorem 1, we obtain the following:

$$\begin{aligned} CIFDWA(\alpha_{t_1}, \alpha_{t_2}, \dots, \alpha_{t_p}) &= \left( \left( 1 - \prod_{k=1}^p (1 - \Gamma_{t_0})^{\lambda_{t_k}}, 1 - \prod_{k=1}^p (1 - \theta_{t_0})^{\lambda_{t_k}} \right), \right. \\ &\quad \left. \left( \prod_{k=1}^p (K_{t_0})^{\lambda_{t_k}}, \prod_{k=1}^p (\varphi_{t_0})^{\lambda_{t_k}} \right) \right) \\ &= \left( \left( 1 - (1 - \Gamma_{t_0})^{\sum_{k=1}^p \lambda_{t_k}}, 1 - (1 - \theta_{t_0})^{\sum_{k=1}^p \lambda_{t_k}} \right), \right. \\ &\quad \left. \left( (K_{t_0})^{\sum_{k=1}^p \lambda_{t_k}}, (\varphi_{t_0})^{\sum_{k=1}^p \lambda_{t_k}} \right) \right) \\ &= ((\Gamma_{t_0}, \theta_{t_0}), (K_{t_0}, \varphi_{t_0})) \end{aligned}$$

Hence, we conclude that  $CIFDWA(\alpha_{t_1}, \alpha_{t_2}, \dots, \alpha_{t_p}) = \alpha_{t_0}$ .  $\square$

The following result delineates that the collection of any number of CIFNs manifests the monotonicity property within the framework governed by the CIFDWA operator.

**Theorem 3. (Monotonicity Property):** Consider two collections of CIFNs, denoted as  $\alpha_{t_k} = ((\Gamma_{t_k}, \theta_{t_k}), (K_{t_k}, \varphi_{t_k}))$  and  $\alpha'_{t_k} = ((\Gamma'_{t_k}, \theta'_{t_k}), (K'_{t_k}, \varphi'_{t_k}))$  for all  $k = 1, 2, 3, \dots, p$ . Let  $\lambda_t = [\lambda_{t_1}, \lambda_{t_2}, \dots, \lambda_{t_p}]^T$  denotes the weight vector associated with time periods  $t_k$ , such that  $\lambda_{t_k} \in [0, 1]$  and  $\sum_{k=1}^p \lambda_{t_k} = 1$ . If the following conditions hold for each  $k$ :  $\Gamma_{t_k} \leq \Gamma'_{t_k}, K_{t_k} \geq K'_{t_k}, \theta_{t_k} \leq \theta'_{t_k}$ , and  $\varphi_{t_k} \geq \varphi'_{t_k}$ , then we can establish that:  $CIFDWA(\alpha_{t_1}, \alpha_{t_2}, \dots, \alpha_{t_p}) \leq CIFDWA(\alpha'_{t_1}, \alpha'_{t_2}, \dots, \alpha'_{t_p})$ .

**Proof of Theorem 3.** In view of the given presentations of  $\alpha_{t_k}$  and  $\alpha'_{t_k}$ , the corresponding CIFDWA's outcomes are as follows:

$$CIFDWA(\alpha_{t_1}, \alpha_{t_2}, \dots, \alpha_{t_p}) = ((\Gamma_t, \theta_t), (K_t, \varphi_t)) \text{ and } CIFDWA(\alpha'_{t_1}, \alpha'_{t_2}, \dots, \alpha'_{t_p}) = ((\Gamma'_t, \theta'_t), (K'_t, \varphi'_t)).$$

By observing that  $\Gamma_{t_k} \leq \Gamma'_{t_k}$ , we can deduce that  $1 - \Gamma_{t_k} \geq 1 - \Gamma'_{t_k}$ , which in turn implies  $\prod_{k=1}^p (1 - \Gamma_{t_k})^{\lambda_{t_k}} \geq \prod_{k=1}^p (1 - \Gamma'_{t_k})^{\lambda_{t_k}} \implies 1 - \prod_{k=1}^p (1 - \Gamma_{t_k})^{\lambda_{t_k}} \leq 1 - \prod_{k=1}^p (1 - \Gamma'_{t_k})^{\lambda_{t_k}}$ . Hence, we can conclude that  $\Gamma_t \leq \Gamma'_t$ .

Similarly, by considering  $K_{t_k} \geq K'_{t_k}$ , we derive  $\prod_{k=1}^p (K_{t_k})^{\lambda_{t_k}} \geq \prod_{k=1}^p (K'_{t_k})^{\lambda_{t_k}}$ , which implies  $K_t \geq K'_t$ .

Therefore, utilizing Definition 10, we establish the desired result.  $\square$

The subsequent result elucidates that the collection of any number of CIFNs possesses the boundedness property within the domain of CIFDWA operator.

**Theorem 4. (Boundedness Property)** Let  $\alpha_t^- = \left( \left( \min_{t_k} \{ \Gamma_{t_k} \}, \min_{t_k} \{ \theta_{t_k} \} \right), \left( \max_{t_k} \{ K_{t_k} \}, \max_{t_k} \{ \varphi_{t_k} \} \right) \right)$  and  $\alpha_t^+ = \left( \left( \max_{t_k} \{ \Gamma_{t_k} \}, \max_{t_k} \{ \theta_{t_k} \} \right), \left( \min_{t_k} \{ K_{t_k} \}, \min_{t_k} \{ \varphi_{t_k} \} \right) \right)$  be the lower and upper bounds of the CIFNs  $\alpha_{t_k} = ((\Gamma_{t_k}, \theta_{t_k}), (K_{t_k}, \varphi_{t_k}))$ . We also have weight vector  $\lambda_t = [\lambda_{t_1}, \lambda_{t_2}, \dots, \lambda_{t_p}]^T$  associated with time periods  $t_k$ , where  $\lambda_{t_k} \in [0, 1]$  and  $\sum_{k=1}^p \lambda_{t_k} = 1$ . Therefore,  $\alpha_t^- \leq CIFDWA(\alpha_{t_1}, \alpha_{t_2}, \dots, \alpha_{t_p}) \leq \alpha_t^+$ .

**Proof of Theorem 4.** Consider the result of applying the CIFDWA operator to the collection of CIFNs, denoted as  $CIFDWA(\alpha_{t_1}, \alpha_{t_2}, \dots, \alpha_{t_p}) = ((\Gamma_t, \theta_t), (K_t, \varphi_t))$ .

For each CIFN  $\alpha_{t_k}$ ,  $\min_{t_k}\{\Gamma_{t_k}\} \leq \Gamma_{t_k} \leq \max_{t_k}\{\Gamma_{t_k}\} \implies 1 - \max_{t_k}\{\Gamma_{t_k}\} \leq 1 - \Gamma_{t_k} \leq 1 - \min_{t_k}\{\Gamma_{t_k}\} \implies \prod_{k=1}^p \left(1 - \max_{t_k}\{\Gamma_{t_k}\}\right)^{\lambda_{t_k}} \leq \prod_{k=1}^p (1 - \Gamma_{t_k})^{\lambda_{t_k}} \leq \prod_{k=1}^p \left(1 - \min_{t_k}\{\Gamma_{t_k}\}\right)^{\lambda_{t_k}} \implies \left(1 - \max_{t_k}\{\Gamma_{t_k}\}\right)^{\sum_{k=1}^p \lambda_{t_k}} \leq \prod_{k=1}^p (1 - \Gamma_{t_k})^{\lambda_{t_k}} \leq \left(1 - \min_{t_k}\{\Gamma_{t_k}\}\right)^{\sum_{k=1}^p \lambda_{t_k}} \implies 1 - \max_{t_k}\{\Gamma_{t_k}\} \leq \prod_{k=1}^p (1 - \Gamma_{t_k})^{\lambda_{t_k}} \leq 1 - \min_{t_k}\{\Gamma_{t_k}\} \implies \min_{t_k}\{\Gamma_{t_k}\} \leq 1 - \prod_{k=1}^p (1 - \Gamma_{t_k})^{\lambda_{t_k}} \leq \max_{t_k}\{\Gamma_{t_k}\}$ . Hence,  $\min_{t_k}\{\Gamma_{t_k}\} \leq \Gamma_t \leq \max_{t_k}\{\Gamma_{t_k}\}$ .

Additionally,  $\min_{t_k}\{K_{t_k}\} \leq K_{t_k} \leq \max_{t_k}\{K_{t_k}\} \implies \prod_{k=1}^p \left(\min_{t_k}\{K_{t_k}\}\right)^{\lambda_{t_k}} \leq \prod_{k=1}^p \{K_{t_k}\}^{\lambda_{t_k}} \leq \prod_{k=1}^p \left(\max_{t_k}\{K_{t_k}\}\right)^{\lambda_{t_k}} \implies \left(\min_{t_k}\{K_{t_k}\}\right)^{\sum_{k=1}^p \lambda_{t_k}} \leq \prod_{k=1}^p (K_{t_k})^{\lambda_{t_k}} \leq \left(\max_{t_k}\{K_{t_k}\}\right)^{\sum_{k=1}^p \lambda_{t_k}} \implies \min_{t_k}\{K_{t_k}\} \leq K_t \leq \max_{t_k}\{K_{t_k}\}$ .

Similarly, we can determine that  $\min_{t_k}\{\theta_{t_k}\} \leq \theta_t \leq \max_{t_k}\{\theta_{t_k}\}$  and  $\min_{t_k}\{\varphi_{t_k}\} \leq \varphi_t \leq \max_{t_k}\{\varphi_{t_k}\}$ . Hence, by employing Definition 10, we determine that

$$\alpha_t^- \leq \text{CIFDWA}(\alpha_{t_1}, \alpha_{t_2}, \dots, \alpha_{t_p}) \leq \alpha_t^+$$

□

### 3.3. Structural Properties of CIFDWG Operator

In this section, we introduce the notion of complex intuitionistic fuzzy dynamic weighted geometric operator and establish its fundamental structural properties.

**Definition 13.** Let  $\alpha_{t_k} = ((\Gamma_{t_k}, \theta_{t_k}), (K_{t_k}, \varphi_{t_k}))$  be a collection of CIFNs at  $p$  different periods  $t_k$ , where  $k = 1, 2, \dots, p$ . Furthermore, let  $\lambda_t = [\lambda_{t_1}, \lambda_{t_2}, \dots, \lambda_{t_p}]^T$  denotes the weight vector associated with time periods  $t_k$  and  $\lambda_{t_k} \in [0, 1]$ , satisfying the constraint  $\sum_{k=1}^p \lambda_{t_k} = 1$ . The CIFDWG operator is a mapping  $\text{CIFDWG} : \psi^p \rightarrow \psi$ , is defined as follows:

$$\text{CIFDWG}(\alpha_{t_1}, \alpha_{t_2}, \dots, \alpha_{t_p}) = \otimes_{k=1}^p \alpha_{t_k}^{\lambda_{t_k}} \tag{4}$$

The following theorem elucidates that the combined value resulting from the aggregation of any number of CIFNs conforms to the CIFN structure within the context of CIFDWG operator.

**Theorem 5.** Let  $\alpha_{t_k} = ((\Gamma_{t_k}, \theta_{t_k}), (K_{t_k}, \varphi_{t_k}))$  be a collection of CIFNs at  $p$  different periods  $t_k$  ( $k = 1, 2, \dots, p$ ). Let  $\lambda_t = [\lambda_{t_1}, \lambda_{t_2}, \dots, \lambda_{t_p}]^T$  denotes the weight vector associated with time periods  $t_k$ , and  $\lambda_{t_k} \in [0, 1]$ , satisfying the constraint  $\sum_{k=1}^p \lambda_{t_k} = 1$ . Therefore, the aggregated value of these CIFNs in the framework of the CIFDWG operator is also a CIFN and is determined as follows:

$$\text{CIFDWG}(\alpha_{t_1}, \alpha_{t_2}, \dots, \alpha_{t_p}) = \left( \left( \prod_{k=1}^p (\Gamma_{t_k})^{\lambda_{t_k}}, \prod_{k=1}^p (\theta_{t_k})^{\lambda_{t_k}} \right), \left( 1 - \prod_{k=1}^p (1 - K_{t_k})^{\lambda_{t_k}}, 1 - \prod_{k=1}^p (1 - \varphi_{t_k})^{\lambda_{t_k}} \right) \right)$$

**Proof of Theorem 5.** We establish the validity of the theorem through the application of the mathematical induction technique.

For the base case when  $p = 2$ , we have two CIFNs, namely,  $\alpha_{t_1} = ((\Gamma_{t_1}, \theta_{t_1}), (K_{t_1}, \varphi_{t_1}))$  and  $\alpha_{t_2} = ((\Gamma_{t_2}, \theta_{t_2}), (K_{t_2}, \varphi_{t_2}))$ . Employing the operations defined for CIFNs, we obtain the following expressions:

$$\alpha_{t_1}^{\lambda_{t_1}} = \left( \left( (\Gamma_{t_1})^{\lambda_{t_1}}, (\theta_{t_1})^{\lambda_{t_1}} \right), \left( 1 - (1 - K_{t_1})^{\lambda_{t_1}}, 1 - (1 - \varphi_{t_1})^{\lambda_{t_1}} \right) \right) \tag{5}$$

and

$$\alpha_{t_2}^{\lambda_{t_2}} = \left( \left( (\Gamma_{t_2})^{\lambda_{t_2}}, (\theta_{t_2})^{\lambda_{t_2}} \right), \left( 1 - (1 - K_{t_2})^{\lambda_{t_2}}, 1 - (1 - \varphi_{t_2})^{\lambda_{t_2}} \right) \right) \tag{6}$$

Now, applying the CIFDWG operator to  $\alpha_{t_1}$  and  $\alpha_{t_2}$ , we combine these two CIFNs as follows:

$$\begin{aligned} \text{CIFDWG}(\alpha_{t_1}, \alpha_{t_2}) &= \alpha_{t_1}^{\lambda_{t_1}} \otimes \alpha_{t_2}^{\lambda_{t_2}} \\ &= \left( \left( (\Gamma_{t_1})^{\lambda_{t_1}}, (\theta_{t_1})^{\lambda_{t_1}} \right), \left( 1 - (1 - K_{t_1})^{\lambda_{t_1}}, 1 - (1 - \varphi_{t_1})^{\lambda_{t_1}} \right) \right) \\ &\quad \otimes \left( \left( (\Gamma_{t_2})^{\lambda_{t_2}}, (\theta_{t_2})^{\lambda_{t_2}} \right), \left( 1 - (1 - K_{t_2})^{\lambda_{t_2}}, 1 - (1 - \varphi_{t_2})^{\lambda_{t_2}} \right) \right) \end{aligned}$$

It follows that:

$$\begin{aligned} \text{CIFDWG}(\alpha_{t_1}, \alpha_{t_2}) &= \left( \prod_{k=1}^2 (\Gamma_{t_k})^{\lambda_{t_k}}, \prod_{k=1}^2 (\theta_{t_k})^{\lambda_{t_k}} \right), \\ &\quad \left( 1 - \prod_{k=1}^2 (1 - K_{t_k})^{\lambda_{t_k}}, 1 - \prod_{k=1}^2 (1 - \varphi_{t_k})^{\lambda_{t_k}} \right) \end{aligned}$$

Hence, we have established the correctness of the theorem for the base case when  $p = 2$ .

Now, we proceed with the induction step. Let us assume that the result holds for  $p = m$ , where  $m$  is a positive integer. This means that:

$$\text{CIFDWG}(\alpha_{t_1}, \alpha_{t_2}, \dots, \alpha_{t_p}) = \left( \left( \prod_{k=1}^m (\Gamma_{t_k})^{\lambda_{t_k}}, \prod_{k=1}^m (\theta_{t_k})^{\lambda_{t_k}} \right), \left( 1 - \prod_{k=1}^m (1 - K_{t_k})^{\lambda_{t_k}}, 1 - \prod_{k=1}^m (1 - \varphi_{t_k})^{\lambda_{t_k}} \right) \right) \tag{7}$$

Now, for the case when  $p = m + 1$ , we have:

$$\begin{aligned} \text{CIFDWG}(\alpha_{t_1}, \alpha_{t_2}, \dots, \alpha_{t_m}, \alpha_{t_{m+1}}) &= \alpha_{t_1}^{\lambda_{t_1}} \otimes \alpha_{t_2}^{\lambda_{t_2}} \otimes \dots \otimes \alpha_{t_m}^{\lambda_{t_m}} \otimes \alpha_{t_{m+1}}^{\lambda_{t_{m+1}}} \\ &= \left( \left( \prod_{k=1}^m (\Gamma_{t_k})^{\lambda_{t_k}}, \prod_{k=1}^m (\theta_{t_k})^{\lambda_{t_k}} \right), \left( 1 - \prod_{k=1}^m (1 - K_{t_k})^{\lambda_{t_k}}, 1 - \prod_{k=1}^m (1 - \varphi_{t_k})^{\lambda_{t_k}} \right) \right) \\ &\quad \otimes \left( \left( (\Gamma_{t_{m+1}})^{\lambda_{t_{m+1}}}, (\theta_{t_{m+1}})^{\lambda_{t_{m+1}}} \right), \left( 1 - (1 - K_{t_{m+1}})^{\lambda_{t_{m+1}}}, 1 - (1 - \varphi_{t_{m+1}})^{\lambda_{t_{m+1}}} \right) \right) \end{aligned}$$

It follows that:

$$\begin{aligned} \text{CIFDWG}(\alpha_{t_1}, \alpha_{t_2}, \dots, \alpha_{t_m}, \alpha_{t_{m+1}}) &= \left( \left( \prod_{k=1}^{m+1} (\Gamma_{t_k})^{\lambda_{t_k}}, \prod_{k=1}^{m+1} (\theta_{t_k})^{\lambda_{t_k}} \right), \left( 1 - \prod_{k=1}^{m+1} (1 - K_{t_k})^{\lambda_{t_k}}, 1 - \prod_{k=1}^{m+1} (1 - \varphi_{t_k})^{\lambda_{t_k}} \right) \right) \end{aligned}$$

This expression is consistent with the formula provided in the theorem statement. Therefore, by the principle of mathematical induction, we have demonstrated that the result holds for all positive integers of  $p$ .  $\square$

The following example serves to elucidate the previously asserted fact.

**Example 4.** Suppose  $\alpha_{t_1} = ((0.4, 0.5), (0.4, 0.3))$ ,  $\alpha_{t_2} = ((0.5, 0.8), (0.3, 0.1))$ ,  $\alpha_{t_3} = ((0.7, 0.3), (0.2, 0.6))$ , and  $\alpha_{t_4} = ((0.9, 0.4), (0.1, 0.4))$  are any four CIFNs and  $\lambda_t = [0.35, 0.15, 0.3, 0.2]^T$  denotes the weight vector of the periods  $t_k$  where  $k = 1, 2, 3, 4$ . Then, we have

$$\prod_{k=1}^4 (\Gamma_{t_k})^{\lambda_{t_k}} = 0.575, \prod_{k=1}^4 (\theta_{t_k})^{\lambda_{t_k}} = 0.440$$

$$\prod_{k=1}^4 (1 - K_{t_k})^{\lambda_{t_k}} = 0.726, \prod_{k=1}^4 (1 - \varphi_{t_k})^{\lambda_{t_k}} = 0.596$$

This implies that

$$\begin{aligned} \text{CIFDWG}(\alpha_{t_1}, \alpha_{t_2}, \alpha_{t_3}, \alpha_{t_4}) &= \otimes_{k=1}^p \alpha_{t_k}^{\lambda_{t_k}} \\ &= ((0.575, 0.440), (0.274, 0.404)) \end{aligned}$$

Therefore, we conclude that the result of the preceding discussion is also a CIFN.

The forthcoming result delineates that the aggregation of any number of CIFNs within the context of a CIFDWG operator exhibits the idempotency property.

**Theorem 6 (Idempotency Property).** A set of CIFNs  $\alpha_{t_k} = ((\Gamma_{t_k}, \theta_{t_k}), (K_{t_k}, \varphi_{t_k}))$ , for  $k$  ranging from 1 to  $p$ , satisfies the condition  $\alpha_{t_k} = \alpha_{t_0}$  for all  $t_k$ , where  $\alpha_{t_0} = ((\Gamma_{t_0}, \theta_{t_0}), (K_{t_0}, \varphi_{t_0}))$  is itself a CIFN. We also have weight vector  $\lambda_t = [\lambda_{t_1}, \lambda_{t_2}, \dots, \lambda_{t_p}]^T$  associated with time periods  $t_k$ , where  $\lambda_{t_k} \in [0, 1]$  and  $\sum_{k=1}^p \lambda_{t_k} = 1$ . Therefore,  $\text{CIFDWG}(\alpha_{t_1}, \alpha_{t_2}, \dots, \alpha_{t_p}) = \alpha_{t_0}$ .

**Proof of Theorem 6.** The proof of this theorem follows a similar logic to that of Theorem 2.  $\square$

The following result elucidates that the collection of any number of CIFNs exhibits the characteristic of monotonicity within the domain of CIFDWG operator.

**Theorem 7 (Monotonicity Property).** Consider two collections of CIFNs, denoted as  $\alpha_{t_k} = ((\Gamma_{t_k}, \theta_{t_k}), (K_{t_k}, \varphi_{t_k}))$  and  $\alpha'_{t_k} = ((\Gamma'_{t_k}, \theta'_{t_k}), (K'_{t_k}, \varphi'_{t_k}))$  for all  $t_k = 1, 2, 3, \dots, p$ . Let  $\lambda_t = [\lambda_{t_1}, \lambda_{t_2}, \dots, \lambda_{t_p}]^T$  be the associated weight vector of time periods  $t_k$ , such that  $\lambda_{t_k} \in [0, 1]$ , satisfying the constraint  $\sum_{k=1}^p \lambda_{t_k} = 1$ . If the following conditions hold for each  $t_k$ :  $\Gamma_{t_k} \leq \Gamma'_{t_k}$ ,  $K_{t_k} \geq K'_{t_k}$ ,  $\theta_{t_k} \leq \theta'_{t_k}$ , and  $\varphi_{t_k} \geq \varphi'_{t_k}$ , then we can establish that:  $\text{CIFDWG}(\alpha_{t_1}, \alpha_{t_2}, \dots, \alpha_{t_p}) \leq \text{CIFDWG}(\alpha'_{t_1}, \alpha'_{t_2}, \dots, \alpha'_{t_p})$ .

**Proof of Theorem 7.** The proof of this theorem follows a similar logic to that of Theorem 3.  $\square$

The subsequent result expounds that the collection of any number of CIFNs conforms to the boundedness property within the framework of CIFDWG operator.

**Theorem 8 (Boundedness Property).** Let  $\alpha_t^- = \left( \left( \min_{t_k} \{ \Gamma_{t_k} \}, \min_{t_k} \{ \theta_{t_k} \} \right), \left( \max_{t_k} \{ K_{t_k} \}, \max_{t_k} \{ \varphi_{t_k} \} \right) \right)$  and  $\alpha_t^+ = \left( \left( \max_{t_k} \{ \Gamma_{t_k} \}, \max_{t_k} \{ \theta_{t_k} \} \right), \left( \min_{t_k} \{ K_{t_k} \}, \min_{t_k} \{ \varphi_{t_k} \} \right) \right)$  be the lower and upper bounds of the CIFNs  $\alpha_{t_k} = ((\Gamma_{t_k}, \theta_{t_k}), (K_{t_k}, \varphi_{t_k}))$ .

We also have weight vector  $\lambda_t = [\lambda_{t_1}, \lambda_{t_2}, \dots, \lambda_{t_p}]^T$  associated with time periods  $t_k$ , where  $\lambda_{t_k} \in [0, 1]$  and  $\sum_{k=1}^p \lambda_{t_k} = 1$ . Therefore,  $\alpha_t^- \leq CIFDWG(\alpha_{t_1}, \alpha_{t_2}, \dots, \alpha_{t_p}) \leq \alpha_t^+$ .

**Proof of Theorem 8.** The proof of this theorem follows a similar logic to that of Theorem 4.  $\square$

#### 4. Algorithm to Solve MADM Problems by Complex Intuitionistic Fuzzy Dynamic Weighted Aggregation Operators

In this section, we elucidate a method for addressing the intricate problem of multiple attribute decision making (MADM) with complex intuitionistic fuzzy information by employing complex intuitionistic fuzzy dynamic weighted aggregation operators.

Let us denote a discrete set of alternatives as  $A = \{A_1, A_2, \dots, A_m\}$ . Additionally, we consider a set of attributes denoted as  $C = \{C_1, C_2, \dots, C_n\}$ , where the corresponding weight vector is represented as  $w = (w_1, w_2, \dots, w_n)^T$ , where  $w_\gamma \geq 0$  for  $\gamma = 1, 2, \dots, n$  and  $\sum_{\gamma=1}^n w_\gamma = 1$ . Furthermore, for  $p$  discrete time periods denoted as  $t_k$ , where  $k = 1, 2, \dots, p$ , each associated with a weight vector  $[\lambda_{t_1}, \lambda_{t_2}, \dots, \lambda_{t_p}]^T$ , with  $\lambda_{t_k} \in [0, 1]$  and  $\sum_{k=1}^p \lambda_{t_k} = 1$ .

Let  $R_{t_k} = (\delta_{ij(t_k)})_{m \times n} = ((\Gamma_{ij(t_k)}, \theta_{ij(t_k)}), (K_{ij(t_k)}, \varphi_{ij(t_k)}))$  be the complex intuitionistic fuzzy decision matrix at  $p$  distinct time periods  $t_k$ , where  $k = 1, 2, \dots, p$ . In this matrix,  $\Gamma_{ij(t_k)}$  and  $\theta_{ij(t_k)}$  represent the degrees to which the alternative  $A_i$  satisfies the attribute  $C_j$  at time periods  $t_k$ , whereas  $K_{ij(t_k)}$  and  $\varphi_{ij(t_k)}$  signify the degrees to which the alternative  $A_i$  does not satisfy the attribute  $C_j$  at time periods  $t_k$ . Importantly, these values are bounded within the range  $[0, 1]$  and they adhere to the conditions  $0 \leq \Gamma_{ij(t_k)} + K_{ij(t_k)}, \theta_{ij(t_k)} + \varphi_{ij(t_k)} \leq 1$ , where  $i = 1, 2, \dots, m$  and  $j = 1, 2, \dots, n$ .

To tackle the MADM problem, the following algorithms are formulated:

##### 4.1. Algorithm for CIFDWA

Step 1. Employ the CIFDWA operator to aggregate the complex intuitionistic fuzzy decision matrices as follows:

$$\begin{aligned} (\delta_{ij(t_k)}) &= ((\Gamma_{ij(t_k)}, \theta_{ij(t_k)}), (K_{ij(t_k)}, \varphi_{ij(t_k)})) = CIFDWA(\delta_{ij(t_1)}, \delta_{ij(t_2)}, \dots, \delta_{ij(t_p)}) \\ &= ((1 - \prod_{k=1}^p (1 - \Gamma_{ij(t_k)})^{\lambda_{t_k}}, 1 - \prod_{k=1}^p (1 - \theta_{ij(t_k)})^{\lambda_{t_k}}), \\ &\quad (\prod_{k=1}^p (K_{ij(t_k)})^{\lambda_{t_k}}, \prod_{k=1}^p (\varphi_{ij(t_k)})^{\lambda_{t_k}})) \end{aligned}$$

This operation aggregates all the complex intuitionistic fuzzy decision matrices  $R_{t_k} = (\delta_{ij(t_k)})_{m \times n} = ((\Gamma_{ij(t_k)}, \theta_{ij(t_k)}), (K_{ij(t_k)}, \varphi_{ij(t_k)}))_{m \times n}$  where  $k = 1, 2, \dots, p$  into a collective complex intuitionistic fuzzy decision matrix  $R = (\delta_{ij})_{m \times n} = ((\Gamma_{ij}, \theta_{ij}), (K_{ij}, \varphi_{ij}))_{m \times n}$ .

Step 2. Utilize the CIFWA operator to determine the overall values  $(\delta_i)$  for each alternative  $A_i$ , where  $i = 1, 2, \dots, m$ :

$$\begin{aligned} (\delta_i) &= ((\Gamma_i, \theta_i), (K_i, \varphi_i)) = CIFWA(\delta_{i1}, \delta_{i2}, \dots, \delta_{in}) \\ &= ((1 - \prod_{\gamma=1}^n (1 - \Gamma_{i\gamma})^{w_\gamma}, 1 - \prod_{\gamma=1}^n (1 - \theta_{i\gamma})^{w_\gamma}), \\ &\quad (\prod_{\gamma=1}^n (K_{i\gamma})^{w_\gamma}, \prod_{\gamma=1}^n (\varphi_{i\gamma})^{w_\gamma})) \end{aligned} \tag{8}$$

Step 3. Compute the score values  $C(\delta_i)$  corresponding to each alternative  $A_i$  by using Definition 8.

Step 4. Rank all the alternatives  $A_i$  ( $i = 1, 2, \dots, m$ ) and select the best one(s) according to their  $C(\delta_i)$ .

#### 4.2. Algorithm for CIFDWG

Step 1. Utilize the CIFDWG operator:

$$\begin{aligned}
 (\delta_{ij(t_k)}) &= \left( \left( \Gamma_{ij(t_k)}, \theta_{ij(t_k)} \right), \left( K_{ij(t_k)}, \varphi_{ij(t_k)} \right) \right) = CIFDWG \left( \delta_{ij(t_1)}, \delta_{ij(t_2)}, \dots, \delta_{ij(t_p)} \right) \\
 &= \left( \left( \prod_{k=1}^p \left( \Gamma_{ij(t_k)} \right)^{\lambda_{t_k}}, \prod_{k=1}^p \left( \theta_{ij(t_k)} \right)^{\lambda_{t_k}} \right), \right. \\
 &\quad \left. \left( \left( 1 - \prod_{k=1}^p \left( 1 - K_{ij(t_k)} \right)^{\lambda_{t_k}}, 1 - \prod_{k=1}^p \left( 1 - \varphi_{ij(t_k)} \right)^{\lambda_{t_k}} \right) \right) \right) \tag{9}
 \end{aligned}$$

This operation aggregates all the complex intuitionistic fuzzy decision matrices  $R_{t_k} = \left( \delta_{ij(t_k)} \right)_{m \times n} = \left( \left( \Gamma_{ij(t_k)}, \theta_{ij(t_k)} \right), \left( K_{ij(t_k)}, \varphi_{ij(t_k)} \right) \right)_{m \times n}$  where  $k = 1, 2, \dots, p$  into a collective complex intuitionistic fuzzy decision matrix  $= (\delta_{ij})_{m \times n} = \left( \left( \Gamma_{ij}, \theta_{ij} \right), \left( K_{ij}, \varphi_{ij} \right) \right)_{m \times n}$ .

Step 2. Employ the CIFWG operator to determine the overall values  $(\delta_i)$  for each alternative  $A_i$ , where  $i = 1, 2, \dots, m$  :

$$(\delta_i) = \left( \left( \Gamma_i, \theta_i \right), \left( K_i, \varphi_i \right) \right) = CIFWG(\delta_{i1}, \delta_{i2}, \dots, \delta_{in}) = \left( \left( \prod_{\gamma=1}^n \left( \Gamma_{i\gamma} \right)^{w_\gamma}, \prod_{\gamma=1}^n \left( \theta_{i\gamma} \right)^{w_\gamma} \right), \right. \tag{10}$$

$$\left. \left( \left( 1 - \prod_{\gamma=1}^n \left( 1 - K_{i\gamma} \right)^{w_\gamma}, 1 - \prod_{\gamma=1}^n \left( 1 - \varphi_{i\gamma} \right)^{w_\gamma} \right) \right) \right)$$

Step 3. Calculate the score values  $C(\delta_i)$  corresponding to each alternative  $A_i$  by utilizing Definition 8.

Step 4. Arrange all the alternatives  $A_i$  ( $i = 1, 2, \dots, m$ ) in order of preference and designate the best-performing one(s) in accordance with their  $C(\delta_i)$ .

### 5. Application of Proposed Complex Intuitionistic Fuzzy Dynamic Aggregation Operators in an MADM Problem

In this section, we further elaborate on the previously discussed technique by providing a numerical example. We then compare the outcomes of this example with those of the existing research.

#### 5.1. Case Study

Cardiovascular diseases (CVDs) encompass a cluster of chronic conditions that have detrimental effects on the cardiovascular system. A substantial proportion of the populace within the United States harbors perilous levels of saturated fat, a condition manifesting in multiple manifestations. In order to manage CVD, individuals have the option to implement lifestyle modifications or seek medical intervention through prescribed medication under the guidance of a healthcare professional. Early detection of CVD facilitates a more favorable outcome for successful treatment. CVD is the primary contributor to mortality rates on a global scale as well as within the United States. Annually, a staggering number of 655,000 individuals in the United States suffer deaths due to heart disease. Approximately 50% of the population in the United States experiences some form of CVD. Both genders are affected. In reality, CVD is responsible for the mortality of one in every three women. This phenomenon has an impact on individuals across diverse age groups, races, and socioeconomic statuses. The primary behavioral risk factors associated with CVD and stroke include inadequate diet, physical inactivity, tobacco smoking, and excessive alcohol consumption. Behavioral risk factors can lead to elevated cholesterol levels, higher blood glucose levels, increased serum lipids, and overweight or obesity in adults. These intermediary risk factors serve as salient indicators of an elevated propensity for the occurrence of adverse cardiovascular events such as myocardial infarction, hemorrhage, cardiac arrhythmia, and associated sequelae. It is plausible to subject these risk factors to scrutiny and evaluation within primary care settings. Initiatives encompassing smoking cessation, reduction in sodium consumption, augmentation of fruit and vegetable intake, engagement in routine physical exercise, and the moderation of alcohol consumption constitute viable strategies for mitigating the risk of cardiovascular diseases. To foster and sustain healthful behaviors among individuals, it becomes imperative to institute health policies

that facilitate the affordability and accessibility of salubrious alternatives. Furthermore, it is imperative to acknowledge the existence of other contributory factors that underlie the etiology of cardiovascular disorders. The primary factors that have an influence on social, economic, and cultural development are globalization, urbanization, and population aging. Genetic predispositions, psychological stress, and socioeconomic hardship are additional risk factors for CVD. Medical therapies aimed at addressing hypertension, hyperglycemia, and elevated serum lipid profiles are necessary to reduce the risk of cardiovascular events such as myocardial infarctions and cerebrovascular events in these patients. According to the World Health Organization (WHO), approximately 35% of deaths in Pakistan are attributed to CVD. Due to an aging population and increasing rates of risk factors such as hypertension, hypercholesterolemia, and diabetes, the prevalence of CVD in Pakistan is on the rise. Lifestyle modifications have the potential to mitigate the risk of cardiovascular disease. The risk factors outlined in this study include:

- i. Hypertension, a medical ailment, induces the stiffening and constriction of arterial walls as a consequence of heightened blood pressure levels. This elevation in blood pressure significantly elevates the susceptibility to heart attacks, strokes, and kidney failure. The etiological factors contributing to high blood pressure encompass sedentary living, psychological stress, dietary habits, and genetic predisposition.
- ii. Lipoproteins circulating in the bloodstream may have an elevated concentration of cholesterol, which is characterized as a lipidaceous, waxy compound. Elevated levels of cholesterol within the circulatory system can precipitate atherogenesis, a pathological process entailing the deposition of cholesterol within the walls of arteries. Atherosclerosis-induced vascular narrowing significantly augments the susceptibility to myocardial infarctions and cerebrovascular events. Factors contributing to heightened cholesterol levels encompass suboptimal dietary patterns, sedentary lifestyle choices, and a familial predisposition to the ailment.
- iii. Smoking has a chance to cause damage to the lining of the end of the arteries, increasing the chance of having a heart attack or stroke. Moreover, the consequences of other risk factors, such as high blood pressure and cholesterol levels, may be further intensified due to the presence of this illness.
- iv. Diabetes is a chronic medical disorder that causes poor consumption and storage of glucose, a form of sugar, within the body. High levels of blood glucose can have a negative impact on vascular health, hence increasing one's risk of CVD.

There are non-modifiable risk factors for cardiovascular disease, including age, gender, and family medical history. The previously mentioned risk factors are subject to modification. In general, the male population has a higher susceptibility to cardiovascular illness in comparison to their female counterparts, with this risk increasing as individuals advance in age. The existence of a family history of CVD increases an individual's susceptibility to developing the condition.

In Pakistan, there are multiple strategies for the prevention and management of CVD. These approaches include:

- a. Promoting cardiovascular health and mitigating the risk of disease can be achieved through a range of beneficial lifestyle choices. These include maintaining a healthy weight through balanced nutrition and regular exercise while also refraining from smoking. Engaging in regular physical activity not only enhances cardiovascular well-being but also aids in weight management. A diet rich in fruits, vegetables, whole grains, and lean proteins can effectively lower blood pressure and cholesterol levels, offering multiple health benefits. Finally, abstaining from tobacco use significantly reduces the risk of cardiovascular disease.
- b. Pharmacological interventions such as antihypertensive agents, statins, and antiplatelet medications play a pivotal role in the management and mitigation of CVD risks. These therapeutic modalities are often prescribed by healthcare practitioners and necessitate strict adherence to prescribed regimens to ensure their efficacy.

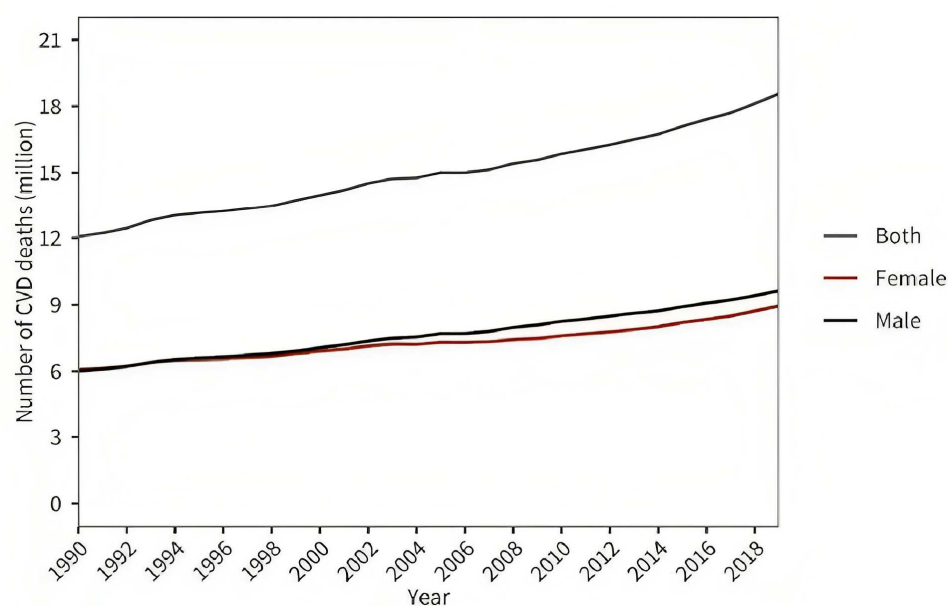
- c. Primary prevention entails a proactive approach aimed at averting the onset of CVD in asymptomatic individuals. This multifaceted strategy encompasses the adoption of a healthy dietary regimen, regular physical exercise, and systematic screening for predisposing risk factors, including but not limited to elevated blood pressure and cholesterol levels.
- d. Secondary prevention constitutes a pivotal facet of the comprehensive strategy against CVD. This imperative entails the attenuation of CVD progression in individuals already afflicted by the ailment. Such an endeavor necessitates the implementation of judicious lifestyle modifications, the judicious administration of pharmacotherapies, and the diligent oversight of healthcare professionals to monitor advancements and ameliorate potential side effects.
- e. The meticulous management of risk factors assumes paramount significance in the realm of CVD prevention and control. By effectively addressing conditions such as hypertension, hypercholesterolemia, and diabetes, individuals can substantially curtail their vulnerability to CVD. The orchestration of this preventive symphony encompasses lifestyle modifications, pharmaceutical interventions, and vigilant medical supervision.
- f. Community-based initiatives comprise a vital stratum within the multifaceted approach to combating CVD. These initiatives encompass a gamut of endeavors, including public health crusades, educational programs, and support networks, all geared towards fostering awareness regarding CVD and advocating for healthier lifestyles among the populace.

According to available data from 2019, CVD emerged as the primary cause of mortality in Asia, resulting in over 10.8 million deaths. This comprised nearly 35% of the total number of fatalities recorded in the region [37]. Approximately 39% of these fatalities related to CVD were deemed avoidable. The number of premature fatalities exceeded that of premature CVD deaths in the United States (23%), Europe (22%), and globally (34%). Ischemic heart disease (IHD), commonly known as a stroke, accounts for a staggering 87% of CVD fatalities. On a global scale, the approximate fatality count attributable to CVDs exhibited a noteworthy rise, ascending from approximately 12.1 million in 1990, with an equitable distribution between both genders, to a figure of 18.6 million in 2019. This later statistic featured a breakdown of 9.6 million male fatalities and 8.9 million female fatalities (Figure 1). The source of Figure 1 is the Institute for Health Metrics and Evaluation (IHME), GBD Compare Data Visualization. Seattle, WA: IHME, University of Washington, 2020. This is freely available from <http://vizhub.healthdata.org/gbdcompare> (accessed on 18 March 2023).

The increase in patient treatment costs and challenges in Pakistan's pharmaceutical sector over the past few decades can be attributed to various factors, including the uneven allocation of medical resources, low effectiveness within the medical industry, and an inadequate health system. The severity of these disorders is increasing. Due to the phenomenon of socioeconomic globalization in the twenty-first century, there has been a notable and persistent rise in individuals' living standards. There is an increasing conflict between human growth and environmental concerns. Various urban areas in Pakistan have encountered severe climatic conditions. The medical sector in Pakistan is currently facing challenges due to increasingly apparent environmental issues. Currently, there is a rapid increase in the number of CVD patients. Due to the better treatment services and care environments offered by large hospitals in comparison to small clinics, people exhibit a greater inclination towards seeking medical advice and assistance at these institutions. The largest hospital in Pakistan, Lahore Hospital, is outfitted with sophisticated medical apparatus and resources. In a nutshell, the Lahore Hospital has experienced an immense increase in burden over the past several decades, rendering it incapable of meeting the increased demands. The implementation of a hierarchical medical system in the context of Lahore Hospital is seen as a viable solution aimed at alleviating the stress associated with patient load. The aim is to classify the level of treatment complexity based on the



nature of the medical issue. The diverse range of medical degrees offered by various institutes enables professionals to effectively address a wide array of illnesses. One basic issue associated with the hierarchical medical system refers to the delineation of various categories of illness severity. Instead of a scenario where all patients converge on a grade III or class A facility, individuals with diverse medical conditions have the option to choose from different levels of hospitals inside the hierarchical medical care system. The initial step in building the hierarchical structure involves the identification of different levels of illness severity, which is an essential step in the whole process. The fundamental objective of this case study is to classify the different degrees of CVD in order to support the hierarchical structure of the medical system.



**Figure 1.** Evolution of global cardiovascular disease-related mortality, 1990–2019.

### 5.2. Illustrated Example

The process of selecting a cadre of consultants with a profound understanding of the problem's significance is pivotal. Given that the focal point of our paper revolves around medical diagnosis, our selection has primarily focused on physicians possessing expertise in this domain. These consultants have been tasked with diagnosing cardiovascular disease (CVD) using a variety of alternative diagnostic measures and attributes considered essential to the diagnostic process.

Let  $\{A_1, A_2, A_3, A_4\}$  be the set of alternatives to diagnose CVD;

- i.  $A_1$ : Clinical symptoms;
- ii.  $A_2$ : Patient history;
- iii.  $A_3$ : Medical history;
- iv.  $A_4$ : Diagnostic test;

Let  $\{C_1, C_2, C_3, C_4, C_5\}$  be the set of attributes each contributing to the diagnosis of CVD.

- i.  $C_1$ : Accuracy;
- ii.  $C_2$ : Efficiency;
- iii.  $C_3$ : Reliability;
- iv.  $C_4$ : Expertise required;
- v.  $C_5$ : Sensitivity;

To create a CIFN, these factors are subsequently classified into two distinct characteristics as delineated below:

- a. Accurate diagnosis ensures appropriate treatment planning and monitoring progress.
- b. Efficiency guarantees a streamlined diagnostic procedure and reduces result retrieval delays.
- c. The reliability of the diagnostic process promotes consistency and trustworthiness.
- d. Expertise is crucial in determining the most appropriate diagnostic tests depending on the patient’s symptoms and concerns.
- e. The sensitivity factor plays a critical role in facilitating timely detection and enabling rapid response.

The four possible alternative  $A_i$  values where,  $i = 1, 2, 3, 4$  are to be evaluated using the complex intuitionistic fuzzy information by the decision maker under the above five attributes of  $C_j$ , where,  $j = 1, 2, 3, 4, 5$  at the periods  $t_1, t_2$ , and  $t_3$ , where  $t_1$  spans from 1990–1999,  $t_2$  represents the time period 2000–2009, and  $t_3$  describes the development from 2010–2019, respectively. The weight vector of the periods assigned by the group of consultants is  $\lambda_t = [0.2, 0.3, 0.5]^T$ , where  $\sum_{k=1}^3 \lambda_{t_k} = 1$  and the weight vector of the attributes is  $w = [0.15, 0.10, 0.25, 0.30, 0.20]^T$ , where  $\sum_{\gamma=1}^5 w_{\gamma} = 1$ . The expert opinions of the group of consultants to evaluate the credibility of each alternative  $A_i$  corresponding to each attribute  $C_j$  with respect to the time periods  $t_1, t_2$  and  $t_3$  are summarized in the following assessment matrices:  $R_{t_1}$  in Table 1,  $R_{t_2}$  in Table 2, and  $R_{t_3}$  in Table 3, respectively, having entries as CIFNs.

**Table 1.** Assessment matrix acquired from  $R_{t_1}$ .

	$C_1$	$C_2$	$C_3$	$C_4$	$C_5$
$A_1$	((0.6,0.9),(0.1,0.1))	((0.8,0.1),(0.1,0.4))	((0.6, 0.6), (0.2, 0.2))	((0.8,0.7),(0.1,0.2))	((0.5,0.5),(0.4,0.3))
$A_2$	((0.7,0.6),(0.3,0.3))	((0.4,0.9),(0.2,0.1))	((0.7,0.7),(0.2,0.3))	((0.4,0.6),(0.3,0.1))	((0.6,0.6),(0.4,0.3))
$A_3$	((0.6,0.6),(0.2,0.2))	((0.6,0.6),(0.3,0.1))	((0.5,0.8),(0.3,0.1))	((0.7,0.7),(0.1,0.2))	((0.6,0.5),(0.4,0.4))
$A_4$	((0.3,0.4),(0.6,0.4))	((0.6,0.6),(0.3,0.4))	((0.3,0.4),(0.5,0.6))	((0.7,0.7),(0.1,0.1))	((0.5,0.7),(0.3,0.3))

**Table 2.** Assessment matrix acquired from  $R_{t_2}$ .

	$C_1$	$C_2$	$C_3$	$C_4$	$C_5$
$A_1$	((0.2,0.8),(0.5,0.1))	((0.7,0.3),(0.3,0.3))	((0.6,0.5),(0.1,0.1))	((0.6,0.5),(0.3,0.4))	((0.3,0.8),(0.3,0.2))
$A_2$	((0.5,0.3),(0.4,0.6))	((0.3,0.1),(0.6,0.3))	((0.7,0.3),(0.1,0.5))	((0.6,0.3),(0.3,0.5))	((0.3,0.5),(0.4,0.2))
$A_3$	((0.5,0.5),(0.3,0.4))	((0.4,0.3),(0.2,0.5))	((0.6,0.4),(0.4,0.4))	((0.7,0.6),(0.2,0.3))	((0.6,0.3),(0.3,0.6))
$A_4$	((0.4,0.8),(0.5,0.1))	((0.7,0.9),(0.1,0.1))	((0.6,0.5),(0.1,0.3))	((0.8,0.5),(0.1,0.4))	((0.6,0.8),(0.2,0.2))

**Table 3.** Assessment matrix acquired from  $R_{t_3}$ .

	$C_1$	$C_2$	$C_3$	$C_4$	$C_5$
$A_1$	((0.6,0.4),(0.1,0.5))	((0.4,0.9),(0.5,0.1))	((0.5,0.5),(0.3,0.3))	((0.4,0.9),(0.5,0.1))	((0.8,0.6),(0.2,0.3))
$A_2$	((0.3,0.8),(0.3,0.1))	((0.8,0.3),(0.1,0.6))	((0.7,0.6),(0.2,0.2))	((0.2,0.7),(0.8,0.2))	((0.7,0.7),(0.2,0.2))
$A_3$	((0.5,0.3),(0.4,0.6))	((0.3,0.1),(0.6,0.3))	((0.8,0.3),(0.1,0.5))	((0.1,0.3),(0.6,0.5))	((0.5,0.4),(0.3,0.1))
$A_4$	((0.9,0.6),(0.1,0.2))	((0.5,0.5),(0.2,0.1))	((0.6,0.6),(0.3,0.2))	((0.7,0.7),(0.3,0.2))	((0.7,0.4),(0.2,0.5))

Step 1. Utilize the CIFDWA operator to aggregate all the complex intuitionistic fuzzy decision matrices  $R_{t_k}$  into a collective complex intuitionistic fuzzy decision matrix  $\tilde{R}$  presented in the Table 4:

**Table 4.** Collective complex intuitionistic fuzzy decision matrix  $\tilde{R}$  using CIFDWA.

	$C_1$	$C_2$	$C_3$	$C_4$	$C_4$
$A_1$	$((0.508, 0.698), (0.162, 0.224))$	$((0.609, 0.722), (0.311, 0.183))$	$((0.553, 0.522), (0.199, 0.199))$	$((0.574, 0.798), (0.311, 0.174))$	$((0.650, 0.660), (0.259, 0.266))$
$A_2$	$((0.466, 0.666), (0.327, 0.213))$	$((0.637, 0.489), (0.197, 0.341))$	$((0.700, 0.553), (0.162, 0.286))$	$((0.387, 0.591), (0.489, 0.229))$	$((0.590, 0.629), (0.282, 0.217))$
$A_3$	$((0.522, 0.434), (0.319, 0.426))$	$((0.402, 0.291), (0.376, 0.281))$	$((0.704, 0.480), (0.189, 0.339))$	$((0.481, 0.501), (0.302, 0.357))$	$((0.553, 0.394), (0.318, 0.226))$
$A_4$	$((0.747, 0.647), (0.232, 0.187))$	$((0.589, 0.704), (0.176, 0.131))$	$((0.552, 0.536), (0.239, 0.281))$	$((0.734, 0.650), (0.173, 0.214))$	$((0.638, 0.624), (0.217, 0.342))$

Step 2. Utilize the CIFWA operator to compute the overall values  $\delta_i = ((\Gamma_i, \theta_i), (K_i, \varphi_i))$  of the alternative  $A_i$  values, where  $i = 1, 2, 3, 4$ . Computed values are presented in Table 5.

**Table 5.** Aggregated values of alternatives using CIFWA.

	$\delta_i$
$A_1$	$((0.580, 0.699), (0.243, 0.204))$
$A_2$	$((0.560, 0.593), (0.286, 0.247))$
$A_3$	$((0.561, 0.447), (0.279, 0.322))$
$A_4$	$((0.666, 0.625), (0.205, 0.235))$

Step 3. Calculate the scores  $C(A_i)$ , where  $i = 1, 2, 3, 4$  of the overall complex intuitionistic fuzzy preference values  $\delta_i$ , to rank all the alternatives  $A_i$ .

$$C(A_1) = 0.486$$

$$C(A_2) = 0.390$$

$$C(A_3) = 0.282$$

$$C(A_4) = 0.494$$

Step 4. Since  $C(A_4) > C(A_1) > C(A_2) > C(A_3)$ , the alternatives are ranked as follows:  $A_4 > A_1 > A_2 > A_3$ .

The above discussion shows that the diagnostic test is the best approach to diagnosing CVD.

Similarly, the above MADM problem in the framework of the CIFDWG operator is solved as follows:

Step 1: Utilize the CIFDWG operator to aggregate all the CIF decision matrices  $R_{t_k}$  into a collective CIF decision matrix  $\tilde{R}$ . The outcomes of these calculations are displayed in Table 6.

Step 2. Utilize the CIFWG operator to obtain the overall values  $\delta_i = ((\Gamma_i, \theta_i), (K_i, \varphi_i))$  of the alternatives  $A_i$ , where  $i = 1, 2, 4, 4$ . The outcomes of these calculations are provided in Table 7.

**Table 6.** Collective complex intuitionistic fuzzy decision matrix  $\tilde{R}$  using CIFDWG.

	$C_1$	$C_2$	$C_3$	$C_4$	$C_5$
$A_1$	$((0.432, 0.579), (0.246, 0.329))$	$((0.543, 0.417), (0.378, 0.231))$	$((0.548, 0.518), (0.225, 0.225))$	$((0.519, 0.718), (0.378, 0.221))$	$((0.543, 0.631), (0.274, 0.2))$
$A_2$	$((0.414, 0.563), (0.332, 0.329))$	$((0.519, 0.269), (0.311, 0.444))$	$((0.700, 0.503), (0.172, 0.154))$	$((0.319, 0.526), (0.626, 0.289))$	$((0.526, 0.614), (0.307, 0.221))$
$A_3$	$((0.519, 0.402), (0.334, 0.481))$	$((0.376, 0.199), (0.449, 0.335))$	$((0.668, 0.398), (0.242, 0.406))$	$((0.265, 0.437), (0.421, 0.392))$	$((0.548, 0.384), (0.321, 0.349))$
$A_4$	$((0.566, 0.603), (0.358, 0.218))$	$((0.574, 0.619), (0.193, 0.170))$	$((0.522, 0.524), (0.294, 0.331))$	$((0.729, 0.633), (0.206, 0.249))$	$((0.625, 0.511), (0.221, 0.384))$

**Table 7.** Aggregated values of alternatives using CIFWG.

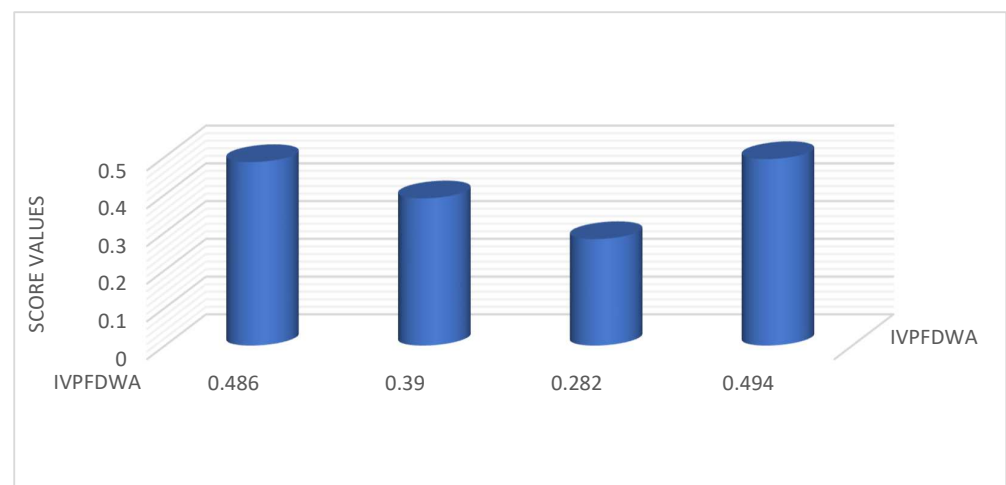
	$\delta_i$
$A_1$	$((0.519, 0.591), (0.302, 0.250))$
$A_2$	$((0.468, 0.507), (0.402, 0.268))$
$A_3$	$((0.442, 0.379), (0.350, 0.396))$
$A_4$	$((0.611, 0.573), (0.255, 0.287))$

Step 3. Calculate the scores  $C(A_i)$ , where  $i = 1, 2, 3, 4$  of the overall complex intuitionistic fuzzy preference values  $\delta_i$ , to rank all the alternative  $A_i$  values.

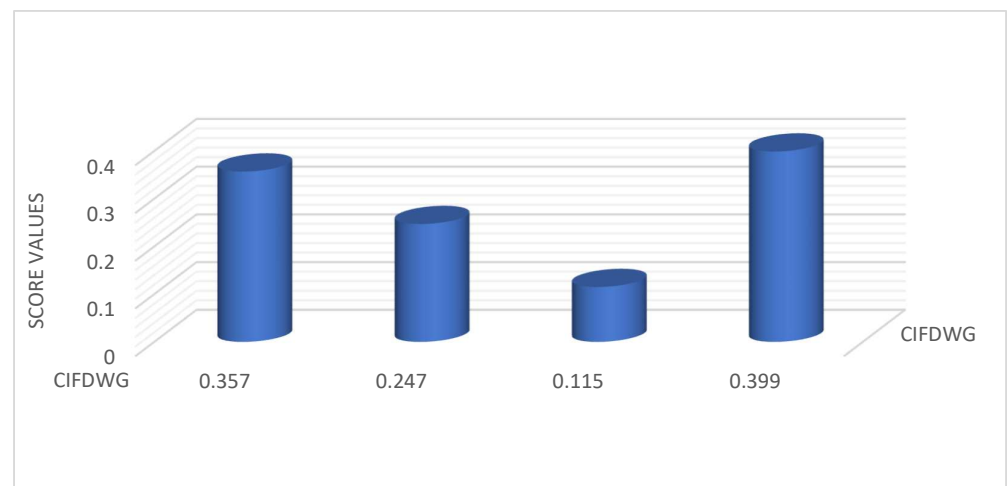
$$C(A_1) = 0.357C(A_2) = 0.247C(A_3) = 0.115C(A_4) = 0.399$$

Step 4. Since  $C(A_4) > C(A_1) > C(A_2) > C(A_3)$ , the alternatives are ranked as follows:  $A_4 > A_1 > A_2 > A_3$ .

The above discussion shows that the diagnostic test is the best approach to diagnosing CVD. Figures 2 and 3 show the graphical presentation of the rankings of alternatives obtain through CIFDWA and CIFDWG operators, respectively.



**Figure 2.** Ranking of alternatives using CIFDWA.



**Figure 3.** Ranking of alternatives using CIFDWG.

### 5.3. Comparative Analysis

In this discussion, we conduct a comparative analysis to evaluate the validity of the proposed techniques compared to the existing strategies developed in [22,29–31].

**Comparison 1.** Alcantud et al.'s [22] IFS-based aggregation operators have a structural flaw: they do not have the phase term, which means that they cannot handle the complex intuitionistic fuzzy data presented in Tables 1–3. Our operators are complex intuitionistic fuzzy dynamic aggregation operators involving both phase and amplitude terms and, as a result, have been successfully applied to complex intuitionistic fuzzy data (See Tables 1–7).

**Comparison 2.** Garg and Rani's [29] CIFS-based aggregation operators do not involve time periods, resulting in a notable loss of information. The proposed aggregation operators are dynamic, and the data is collected from three different time intervals. Complex intuitionistic fuzzy dynamic aggregation operators have the capability to handle this type of data, while CIFS-based aggregation operators lack this property. The CIFS-based aggregation operators proposed in [29] cannot handle the data presented in Tables 1–3.

**Comparison 3.** Garg and Rani's [30] CIFS-based averaging geometric operator is not dynamic, meaning it cannot be applied when the initial decision data is collected from three different time intervals, as is the case in our study. Thus, the proposed complex intuitionistic fuzzy dynamic aggregation operators can handle such cases when others fail.

**Comparison 4.** Garg and Rani's [31] generalized geometric operator within CIFS is also independent of time, leading to a substantial loss of information. Our study, in the dynamic complex intuitionistic fuzzy context with varying time periods, enhances accuracy. The geometric operators in [31] cannot evaluate decision data (Tables 1–3) collected from three different time intervals.

#### Key Points of Comparative Analysis

Comparison 1 unequivocally establishes the superiority of complex intuitionistic fuzzy dynamic aggregation operators over IFS-based aggregation operators for handling complex intuitionistic fuzzy data, enhancing the ability to rank the alternatives accurately.

Comparisons 2–4 clearly demonstrate that the proposed complex intuitionistic fuzzy dynamic aggregation operators are capable of addressing scenarios where the operators introduced in [29–31] fall short. Therefore, CIFS dynamic operators accommodate data collected from various time intervals, enhancing their versatility compared to static CIFS-based methods.

## 6. Conclusions

In this article, we endeavor to introduce innovative approaches to address decision-making challenges within dynamic complex intuitionistic fuzzy settings. While the litera-

ture has already developed several valuable operators, none of them explicitly consider the time period factor in complex intuitionistic fuzzy environments. Consequently, employing a dynamic complex intuitionistic fuzzy model proves to be a more effective means of representing information related to time-dependent issues, as it can effectively handle two-dimensional data within a unified framework. Taking these considerations into account, we introduce a novel set of operators, namely, CIFDWA and CIFDWG within the CIFS framework. We investigate the properties of these operators and propose an updated novel score function for the purpose of evaluating and selecting the optimal alternative. Within this investigation, we have presented a novel methodology to tackle dynamic CIF- MADM problems. This approach leverages the CIFDWA and CIFDWG operators to manage decision-related information concerning attribute values called CIFNs. Importantly, our approach takes into account data collected at different time periods during the decision-making process within this dynamic context.

Furthermore, we demonstrate the practical application of these recently developed techniques in selecting the most suitable approach for diagnosing CVD in a patient. Lastly, we conduct a comparative analysis to underscore the significance and validity of these innovative methodologies when compared to existing techniques.

Since we have worked in a complex intuitionistic fuzzy environment, the proposed scheme cannot solve decision data with a sum of membership and non-membership values greater than 1. This is a limitation of the study. In future research endeavors, we plan to explore more extended operators, such as the dynamic ordered weighted averaging/geometric operator and dynamic Dombi aggregation operators. We also intend to investigate the application of dynamic complex intuitionistic fuzzy MADM in various domains, including dynamic investment analysis, dynamic personnel assessment, and the dynamic evaluation of military system efficiency.

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## References

1. Grote, T.; Berens, P. On the ethics of algorithmic decision-making in healthcare. *J. Med. Ethics* **2020**, *46*, 205–211. [[PubMed](#)]
2. Mustapha, M.T.; Ozsahin, D.U.; Ozsahin, I.; Uzun, B. Breast cancer screening based on supervised learning and multi-criteria decision-making. *Diagnostics* **2022**, *12*, 1326.
3. Kong, Z.; Ai, J.; Wang, L.; Li, P.; Ma, L.; Lu, F. New normal parameter reduction method in fuzzy soft set theory. *IEEE Access* **2018**, *7*, 2986–2998.
4. Zhao, M.; Qin, S.S.; Li, Q.W.; Lu, F.Q.; Shen, Z. The likelihood ranking methods for interval type-2 fuzzy sets considering risk preferences. *Math. Probl. Eng.* **2015**, *2015*, 680635.
5. Zadeh, L.A. Fuzzy sets. *Inf. Control* **1965**, *8*, 338–353.
6. Fatimah, F.; Rosadi, D.; Hakim, R.F.; Alcantud, J.C.R. N-soft sets and their decision making algorithms. *Soft Comput.* **2018**, *22*, 3829–3842.
7. Akram, M.; Adeel, A.; Alcantud, J.C.R. Group decision-making methods based on hesitant N-soft sets. *Expert Syst. Appl.* **2019**, *115*, 95–105.
8. Akram, M.; Ali, G.; Alcantud, J.C.; Fatimah, F. Parameter reductions in N-soft sets and their applications in decision-making. *Expert Syst.* **2021**, *38*, e12601.
9. Atanassov, K.T.; Stoeva, S. Intuitionistic fuzzy sets. *Fuzzy Sets Syst.* **1986**, *20*, 87–96.
10. Liu, S.; Yu, W.; Chan, F.T.; Niu, B. A variable weight-based hybrid approach for multi-attribute group decision making under interval-valued intuitionistic fuzzy sets. *Int. J. Intell. Syst.* **2021**, *36*, 1015–1052.

11. Thao, N.X. Some new entropies and divergence measures of intuitionistic fuzzy sets based on Archimedean t-conorm and application in supplier selection. *Soft Comput.* **2021**, *25*, 5791–5805.
12. Gohain, B.; Dutta, P.; Gogoi, S.; Chutia, R. Construction and generation of distance and similarity measures for intuitionistic fuzzy sets and various applications. *Int. J. Intell. Syst.* **2021**, *36*, 7805–7838.
13. Garg, H.; Rani, D. Novel similarity measure based on the transformed right-angled triangles between intuitionistic fuzzy sets and its applications. *Cogn. Comput.* **2021**, *13*, 447–465.
14. Hayat, K.; Tariq, Z.; Lughofer, E.; Aslam, M.F. New aggregation operators on group-based generalized intuitionistic fuzzy soft sets. *Soft Comput.* **2021**, *25*, 13353–13364. [[PubMed](#)]
15. Ecer, F.; Pamucar, D. MARCOS technique under intuitionistic fuzzy environment for determining the COVID-19 pandemic performance of insurance companies in terms of healthcare services. *Appl. Soft Comput.* **2021**, *104*, 107199.
16. Wu, X.; Song, Y.; Wang, Y. Distance-based knowledge measure for intuitionistic fuzzy sets with its application in decision making. *Entropy* **2021**, *23*, 1119.
17. Augustine, E.P. Novel correlation coefficient for intuitionistic fuzzy sets and its application to multi-criteria decision-making problems. *Int. J. Fuzzy Syst. Appl. IJFSA* **2021**, *10*, 39–58.
18. Yang, J.; Yao, Y. A three-way decision based construction of shadowed sets from Atanassov intuitionistic fuzzy sets. *Inf. Sci.* **2021**, *577*, 1–21.
19. Mahmood, T.; Ali, W.; Ali, Z.; Chinram, R. Power aggregation operators and similarity measures based on improved intuitionistic hesitant fuzzy sets and their applications to multiple attribute decision making. *Comput. Model. Eng. Sci.* **2021**, *126*, 1165–1187.
20. Ocampo, L.; Tanaid, R.A.; Tiu, A.M.; Selerio, E., Jr.; Yamagishi, K. Classifying the degree of exposure of customers to COVID-19 in the restaurant industry: A novel intuitionistic fuzzy set extension of the TOPSIS-Sort. *Appl. Soft Comput.* **2021**, *113*, 107906.
21. Dymova, L.; Kaczmarek, K.; Sevastjanov, P.; Sułkowski, Ł.; Przybyszewski, K. An approach to generalization of the intuitionistic fuzzy TOPSIS method in the framework of evidence theory. *J. Artif. Intell. Soft Comput. Res.* **2021**, *11*, 157–175. [[CrossRef](#)]
22. Alcantud, J.C.R.; Khameneh, A.Z.; Kilicman, A. Aggregation of infinite chains of intuitionistic fuzzy sets and their application to choices with temporal intuitionistic fuzzy information. *Inf. Sci.* **2020**, *514*, 106–117. [[CrossRef](#)]
23. Ramot, D.; Milo, R.; Friedman, M.; Kandel, A. Complex fuzzy sets. *IEEE Trans. Fuzzy Syst.* **2002**, *10*, 171–186. [[CrossRef](#)]
24. Liu, P.; Ali, Z.; Mahmood, T. The distance measures and cross-entropy based on complex fuzzy sets and their application in decision making. *J. Intell. Fuzzy Syst.* **2020**, *39*, 3351–3374. [[CrossRef](#)]
25. Al-Qudah, Y.; Hassan, N. Operations on complex multi-fuzzy sets. *J. Intell. Fuzzy Syst.* **2017**, *33*, 1527–1540. [[CrossRef](#)]
26. Alkouri, A.U.M.; Salleh, A.R. Linguistic variable, hedges and several distances on complex fuzzy sets. *J. Intell. Fuzzy Syst.* **2014**, *26*, 2527–2535. [[CrossRef](#)]
27. Li, C.; Chiang, T.W. Function approximation with complex neuro-fuzzy system using complex fuzzy sets—a new approach. *New Gener. Comput.* **2011**, *29*, 261–276. [[CrossRef](#)]
28. Alkouri, A.S.; Salleh, A.R. Complex intuitionistic fuzzy sets. *AIP Conf. Proc.* **2012**, *1482*, 464–470.
29. Garg, H.; Rani, D. Novel aggregation operators and ranking method for complex intuitionistic fuzzy sets and their applications to decision-making process. *Artif. Intell. Rev.* **2020**, *53*, 3595–3620. [[CrossRef](#)]
30. Garg, H.; Rani, D. Robust averaging–geometric aggregation operators for complex intuitionistic fuzzy sets and their applications to MCDM process. *Arab. J. Sci. Eng.* **2020**, *45*, 2017–2033. [[CrossRef](#)]
31. Garg, H.; Rani, D. Generalized geometric aggregation operators based on t-norm operations for complex intuitionistic fuzzy sets and their application to decision-making. *Cogn. Comput.* **2020**, *12*, 679–698. [[CrossRef](#)]
32. Ali, Z.; Mahmood, T.; Aslam, M.; Chinram, R. Another view of complex intuitionistic fuzzy soft sets based on prioritized aggregation operators and their applications to multiattribute decision making. *Mathematics* **2021**, *9*, 1922. [[CrossRef](#)]
33. Masmali, I.; Khalid, A.; Shuaib, U.; Razaq, A.; Garg, H.; Razzaque, A. On Selection of the Efficient Water Purification Strategy at Commercial Scale Using Complex Intuitionistic Fuzzy Dombi Environment. *Water* **2023**, *15*, 1907. [[CrossRef](#)]
34. Xu, Z.; Yager, R.R. Dynamic intuitionistic fuzzy multi-attribute decision making. *Int. J. Approx. Reason.* **2008**, *48*, 246–262. [[CrossRef](#)]
35. Wei, G.W. Some geometric aggregation functions and their application to dynamic multiple attribute decision making in the intuitionistic fuzzy setting. *Int. J. Uncertain. Fuzziness Knowl. Based Syst.* **2009**, *17*, 179–196. [[CrossRef](#)]
36. Chen, S.M.; Tan, J.M. Handling multicriteria fuzzy decision-making problems based on vague set theory. *Fuzzy Sets Syst.* **1994**, *67*, 163–172. [[CrossRef](#)]
37. Farid HM, A.; Riaz, M.; Khan, Z.A. T-spherical fuzzy aggregation operators for dynamic decision-making with its application. *Alex. Eng. J.* **2023**, *72*, 97–115. [[CrossRef](#)]

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