

Article

M-Polar Fuzzy Graphs and Deep Learning for the Design of Analog Amplifiers

Malinka Ivanova ^{1,*}  and Mariana Durcheva ^{1,2}

¹ Department of Informatics, Faculty of Applied Mathematics and Informatics, Technical University of Sofia, 1797 Sofia, Bulgaria

² Department of Mathematics, Shamoon College of Engineering, Ashdod 77245, Israel

* Correspondence: m_ivanova@tu-sofia.bg

Abstract: The design of analog circuits is a complex and repetitive process aimed at finding the best design variant. It is characterized by uncertainty and multivariate approaches. The designer has to make different choices to satisfy a predefined specification with required parameters. This paper proposes a method for facilitating the design of analog amplifiers based on m-polar fuzzy graphs theory and deep learning. M-polar fuzzy graphs are used because of their flexibility and the possibility to model different real-life multi-attribute problems. Deep learning is applied to solve a regression task and to predict the membership functions of the m-polar fuzzy graph vertices (the solutions), taking on the role of domain experts. The performance of the learner is high since the obtained errors are very small: Root Mean Squared Error is from 0.0032 to 0.0187, Absolute Error is from 0.022 to 0.098 and Relative Error is between 0.27% and 1.57%. The proposed method is verified through the design of three amplifiers: summing amplifier, subtracting amplifier, and summing/subtracting amplifier. The method can be used for improving the design process of electronic circuits with the possibility of automating some tasks.

Keywords: m-polar fuzzy graph; machine learning; deep learning; analog amplifier; design process; automation

MSC: 05C72; 68T07; 94C15



Citation: Ivanova, M.; Durcheva, M. M-Polar Fuzzy Graphs and Deep Learning for the Design of Analog Amplifiers. *Mathematics* **2023**, *11*, 1001. <https://doi.org/10.3390/math11041001>

Academic Editor: Francisco Rodrigues Lima Junior

Received: 11 December 2022

Revised: 7 February 2023

Accepted: 11 February 2023

Published: 15 February 2023



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1. Introduction

The design process of analog circuits is characterized by its complexity and uncertainty, but also by its multivariate approaches. According to one predefined specification with initial parameters, it is possible to have multiple solutions that satisfy this assignment. It is obvious that the designer must find the best design by entering one iterative procedure, and making one or another decision. In support of the designer, a wide variety of methods and methodologies are proposed—from the use of standard matrix theory [1] to contemporary machine learning approaches [2].

The topic related to machine learning-driven design of analog circuits is under extensive investigation, because of the desire for facilitating the designer and automating as many engineering tasks as possible. This could reduce the design time, increase the design quality and decrease the effort of both a beginner and experienced engineer.

Some works show the applicability of evolutionary computing in analog circuit design and possibilities for time controlling [3], achieving efficient and flexible design [4], and design optimization [5]. Bayesian optimization techniques are also under investigation, pointing out another different possible approach for improving the circuit design process regarding design speed and accuracy [6,7].

Deep learning (DL) as a part of machine learning is based on architectures of artificial neural networks (ANNs) and is used for solving classification or regression tasks in different domains and industries, including electronic circuit design [8] and analysis [9]. This

popularity of supervised DL is due to its capability to perform predictions and classifications with high accuracy. This is possible because of the flexibility of ANNs construction and DL algorithm parameters adjustment.

Nowadays, the theory of fuzzy graphs gets its popularity due to its wide applications in many areas of real life. The concept of a fuzzy graph was initially introduced by Kauffman [10], but the development of the fuzzy graphs theory is due to Rosenfeld [11] and Yeh and Bang [12]. The fuzzy graphs theory has been developing in different directions, such as fuzzy tolerance graphs [13], fuzzy threshold graphs [14], interval-valued fuzzy graphs [15–17], fuzzy k-competition graphs and p-competition fuzzy graphs [18], m-step fuzzy competition graphs [19], and hesitant fuzzy graphs [20]. In 1994, Zhang [21,22] introduced the concept of bipolar fuzzy sets as a generalization of fuzzy sets. This theory was developed by Akram [23] to present bipolar fuzzy graphs. In 2014, Chen et al. [24] discussed the notion of m-polar fuzzy sets as a generalization of bipolar fuzzy sets. Ghorai and Pal [25] introduced the m-polar fuzzy graphs as a generalization of bipolar fuzzy graphs and defined different operations. The theory of m-polar fuzzy graphs was further developed in [26–29]. A detailed explanation could be found in two contemporary monographs [30,31].

This work is the first to use m-polar fuzzy graphs theory with the benefits of deep learning for the design of analog amplifiers, but we consider the idea to combine the advantages of fuzzy graphs and deep learning to be very promising. Graphs theory is used, taking into account its powerful capability for solving different real-life problems. Modelling of such problems often involves multi-polar information, including uncertainty and process limits, so attracting m-polar fuzzy graphs for describing such problems is very useful. Deep learning predictions are utilized in the role of the domain experts to point out the membership functions of m-polar fuzzy graph vertices. The regression task is solved with high performance.

The aim of the paper is to present a method for designing analog amplifiers by discussing some basic concepts regarding m-polar fuzzy graphs and considering the advantages of deep learning. The following is a summary of the research work's main contribution:

1. The activity for designing analog amplifiers is defined as a multi-attribute problem, which is solved here using the m-polar fuzzy graphs and deep learning.
2. For the verification of the proposed method, three electronic circuits are designed: an inverting summing amplifier, a subtracting amplifier (differential amplifier), and a summing/subtracting amplifier with an operational amplifier.
3. The predictive models are experimentally evaluated using data sets collected considering the functional and electrical behavior of the examined circuits.

The paper is organized as follows: In Section 2, a review of the basic concept of fuzzy graphs theory and m-polar fuzzy graphs as well as different types of product operations for m-polar fuzzy graphs are presented. Section 3 is dedicated to discussing the usage of ANNs in solving circuit design problems. Section 4 considers the proposed method. In Section 5, the presented method for analog circuit design is demonstrated and verified in the real-life application of analog amplifier design. The conclusion is driven in the last section.

2. Fuzzy Graphs Theory

2.1. Basic Concepts of Fuzzy Graphs

A graph is a pair $G = (V, E)$ of a nonempty set of vertices V (or nodes) and a set of edges E . Each edge has either one or two vertices associated with it, called its *endpoints*. An edge is said to connect its endpoints. A graph is called *simple* if it has no loops and no multiple edges.

Definition 1 [32]. A fuzzy graph $G = (V, \sigma, \mu)$ is a triple consisting of a nonempty set V together with a pair of functions $\sigma : V \rightarrow [0, 1]$ and $\mu : E \rightarrow [0, 1]$ such that for all $x, y \in V$, $\mu(xy) \leq \sigma(x) \wedge \sigma(y)$, where \wedge stands for the minimum.

Definition 2 [33]. A fuzzy graph $G = (V, \sigma, \mu)$ is complete if $\mu(xy) = \sigma(x) \wedge \sigma(y)$ for all $x, y \in V$.

Definition 3 [24]. An m -polar fuzzy set (or a $[0, 1]^m$ -set) on X is mapping $M : X \rightarrow [0, 1]^m$.

The $[0, 1]^m$ (m -power of $[0, 1]$) is considered to be a poset with point-wise order \leq (m is natural), where \leq is defined by $x \leq y \Leftrightarrow P_i(x) \leq P_i(y)$ for each $i = 1, 2, \dots, m$, $x, y \in [0, 1]^m$, and $P_i : [0, 1]^m \rightarrow [0, 1]$ is the i -th projection mapping ($i = 1, 2, \dots, m$). Here, $1 = (1, 1, \dots, 1)$ is the greatest value and $0 = (0, 0, \dots, 0)$ is the smallest value in $[0, 1]^m$.

Definition 4 [24]. Let σ be an m -polar fuzzy set on a set V . An m -polar fuzzy relation on σ is an m -polar fuzzy set μ of $V \times V$ such that $\mu(xy) \leq \sigma(x) \wedge \sigma(y)$ for all $x, y \in V$, i.e., for each $i = 1, 2, \dots, m$, for all $x, y \in V: P_i \circ \mu(xy) \leq P_i \circ \sigma(x) \wedge P_i \circ \sigma(y)$.

Definition 5 [30]. An m -polar fuzzy graph $G = (V, \sigma, \mu)$ is a triple consisting of a nonempty set V together with a pair of functions $\sigma : V \rightarrow [0, 1]^m$ and $\mu : E = V \times V \rightarrow [0, 1]^m$, where σ is an m -polar fuzzy set on the set of vertices V and μ is an m -polar fuzzy relation in V such that for all $x, y \in V, \mu(xy) \leq \sigma(x) \wedge \sigma(y)$, where \wedge stands for minimum.

It can be noted that $\mu(x, y) = 0$ for all $x, y \in V \times V - E$.

2.2. Products in m -Polar Fuzzy Graphs

2.2.1. Direct (Tensor) Product

Definition 6 [30]. Let $G_1 = (\sigma_1, \mu_1)$ of $G_1^* = (V_1, E_1)$ and $G_2 = (\sigma_2, \mu_2)$ of $G_2^* = (V_2, E_2)$ be two m -polar fuzzy graphs. The direct product of G_1 and G_2 is denoted by $G_1 \times G_2$ and is defined as a pair $G_1 \times G_2 = (\sigma_1 \times \sigma_2, \mu_1 \times \mu_2)$, such that for each $i = 1, 2, \dots, m$:

$$P_i \circ (\sigma_1 \wedge \sigma_2)(x_1, x_2) = P_i \circ \sigma_1(x_1) \wedge P_i \circ \sigma_2(x_2) \text{ for all } (x_1, x_2) \in V_1 \times V_2$$

$$P_i \circ (\mu_1 \wedge \mu_2)((x_1, x_2)(y_1, y_2)) = P_i \circ \mu_1(x_1y_1) \wedge P_i \circ \mu_2(x_2y_2)$$

for all $x_1y_1 \in E_1, x_2y_2 \in E_2$.

Proposition 1 [30]. The direct product of two m -polar fuzzy graphs is an m -polar fuzzy graph.

2.2.2. Semi-Strong Product

Definition 7 [31]. Let $G_1 = (\sigma_1, \mu_1)$ of $G_1^* = (V_1, E_1)$ and $G_2 = (\sigma_2, \mu_2)$ of $G_2^* = (V_2, E_2)$ be two m -polar fuzzy graphs. The semi-strong product of G_1 and G_2 (denoted by $G_1 \boxtimes G_2$) is defined as a graph $G_1 \boxtimes G_2 = (\sigma_1 \boxtimes \sigma_2, \mu_1 \boxtimes \mu_2)$ of $G^* = (V_1 \times V_2, E)$ (here $E = \{(x_1, y_1)(x_1, y_2) \mid x_1 \in V_1, y_1y_2 \in E_2\} \cup \{(x_1, y_1)(x_2, y_2) \mid x_1x_2 \in E_1, y_1y_2 \in E_2\}$) such that for each $i = 1, 2, \dots, m$:

$$P_i \circ (\sigma_1 \boxtimes \sigma_2)(x, y) = P_i \circ \sigma_1(x) \wedge P_i \circ \sigma_2(y) \text{ for all } (x, y) \in V_1 \times V_2;$$

$$P_i \circ (\mu_1 \boxtimes \mu_2)(x_1, x_2)(x_1, y_2) = P_i \circ \sigma_1(x) \wedge P_i \circ \mu_2(x_2y_2) \text{ for all } x_1 \in V_1 \text{ and } x_2y_2 \in E_2;$$

$$P_i \circ (\mu_1 \boxtimes \mu_2)(x_1, x_2)(y_1, y_2) = P_i \circ \mu_1(x_1y_1) \wedge P_i \circ \mu_2(x_2y_2) \text{ for all } x_1x_2 \in E_1 \text{ and } x_2y_2 \in E_2;$$

Proposition 2 [26]. The semi-strong product of two m -polar fuzzy graphs is an m -polar fuzzy graph.

2.2.3. Strong Product

Definition 8 [31]. Let $G_1 = (\sigma_1, \mu_1)$ of $G_1^* = (V_1, E_1)$ and $G_2 = (\sigma_2, \mu_2)$ of $G_2^* = (V_2, E_2)$ be two m -polar fuzzy graphs. The strong product of G_1 and G_2 (denoted by $G_1 \otimes G_2$) is defined as a graph $G_1 \otimes G_2 = (\sigma_1 \otimes \sigma_2, \mu_1 \otimes \mu_2)$ of $G^* = (V_1 \times V_2, E)$ where

$$E = \{(x_1, y_1)(x_1, y_2) \mid x_1 \in V_1, y_1y_2 \in E_2\} \cup \{(x_1, y)(x_2, y) \mid x_1x_2 \in E_1, y \in V_2\} \\ \cup \{(x_1, y_1)(x_2, y_2) \mid x_1x_2 \in E_1, y_1y_2 \in E_2\}$$

such that for each $i = 1, 2, \dots, m$:

$$P_i \circ (\sigma_1 \otimes \sigma_2)(x, y) = P_i \circ \sigma_1(x) \wedge P_i \circ \sigma_2(y) \text{ for all } (x, y) \in V_1 \times V_2;$$

$$P_i \circ (\mu_1 \otimes \mu_2)(x_1, x_2)(x_1, y_2) = P_i \circ \sigma_1(x_1) \wedge P_i \circ \mu_2(x_2y_2) \text{ for all } x_1 \in V_1 \text{ and } x_2y_2 \in E_2;$$

$$P_i \circ (\mu_1 \otimes \mu_2)(x_1, x_2)(y_1, x_2) = P_i \circ \mu_1(x_1y_1) \wedge P_i \circ \sigma_2(x_2) \text{ for all } x_1y_1 \in E_1 \text{ and } x_2 \in V_2;$$

$$P_i \circ (\mu_1 \otimes \mu_2)(x_1, x_2)(y_1, y_2) = P_i \circ \mu_1(x_1y_1) \wedge P_i \circ \mu_2(x_2y_2) \text{ for all } x_1y_1 \in E_1 \text{ and } x_2y_2 \in E_2.$$

Proposition 3 [31]. *The strong product of two m-polar fuzzy graphs is an m-polar fuzzy graph.*

2.2.4. Lexicographic Product

Definition 9 [30]. *Let $G_1 = (\sigma_1, \mu_1)$ of $G_1^* = (V_1, E_1)$ and $G_2 = (\sigma_2, \mu_2)$ of $G_2^* = (V_2, E_2)$ be two m-polar fuzzy graphs. The lexicographic product of G_1 and G_2 (denoted by $G_1 \bullet G_2$) is defined as a pair $G_1 \bullet G_2 = (\sigma_1 \bullet \sigma_2, \mu_1 \bullet \mu_2)$ such that for each $i = 1, 2, \dots, m$:*

$$P_i \circ (\sigma_1 \bullet \sigma_2)(x_1, x_2) = P_i \circ \sigma_1(x_1) \wedge P_i \circ \sigma_2(x_2) \text{ for all } (x_1, x_2) \in V_1 \times V_2;$$

$$P_i \circ (\mu_1 \bullet \mu_2)((x_1, x_2)(x_1, y_2)) = P_i \circ \sigma_1(x_1) \wedge P_i \circ \mu_2(x_2y_2) \text{ for all } x_1 \in V_1 \text{ and } x_2y_2 \in E_2;$$

$$P_i \circ (\mu_1 \bullet \mu_2)(x_1, x_2)(y_1, y_2) = P_i \circ \mu_1(x_1y_1) \wedge P_i \circ \mu_2(x_2y_2) \text{ for all } x_1y_1 \in E_1 \text{ and } x_2y_2 \in E_2;$$

Proposition 4 [30]. *The lexicographic product of two m-polar fuzzy graphs is an m-polar fuzzy graph.*

Some applications of m-polar fuzzy graphs for facilitating the decision-making process in a wide variety of studied domains are discussed in [31,34,35]. It is obvious that the presented methods and techniques have the potential to deal with multi-attribute, multi-criteria, and multi-objective problems in uncertain and fuzzy environments. This is the reason such an approach is chosen for application in the field of electronic circuit design.

3. Deep Learning and Applications in Electronic Circuit Design

Deep Learning (DL) comprises multiple methods and techniques, based on ANNs, which are utilized for different purposes—from studying a process, event, or facts to analyzing, predicting, or optimizing some parameters. Contemporary surveys summarize approaches, types, and architectures of deep learning algorithms, as well as discuss their applications in the context of supervised, unsupervised, and reinforcement learning [36,37]. The advantages of DL, such as universal usage, robustness, workability with different data types, and scalability are also described. The cases in which DL is suitable for usage are explained to show its usefulness in the unavailability of domain experts, the impossibility to gather the expertise and the complexity of the problem.

In the automation of electronic circuit design, a few papers are devoted to the advantages of ANNs utilization in support of the designers’ tasks.

Dieste-Velasco et al. propose a methodology for assisting the analytical design of analog circuits as it combines the statistical technique factorial design of experiments and ANNs [38]. Such an approach allows the behavioral modelling of circuits to be done with high accuracy.

The work of Devi et al. is focused on the automated design of analog circuits through the usage of ANN-based supervised learning [39]. The method is verified in the design of

two different analog circuits and the obtained results are characterized by high accuracy and small mean squared error.

Wang et al. applied deep learning to solve the sizing problem of analog circuits [40]. Two ANNs architectures are proposed, Recurrent Neural Network (RNN) and DL, which are proven to possess the capability to predict the transistor size with high accuracy.

More sophisticated solutions for automating the design process of complex analog systems [41] and for improving the circuit synthesis [42] point out the possibility of reducing the design process resources and decreasing the designer’s effort.

Budak et al. deal with the speed of the design of integrated analog circuits proposing an efficient method for sizing [43]. The method is verified in the design of analog amplifiers and a comparator, and the results show its benefit.

It seems that DL possesses a big potential for the implementation of a supportive, automated, and smart design process that facilitates decision-making and problem-solving. All these advantages and possibilities are considered when choosing the DL for creating predictive models and speeding up the circuit design.

4. Proposed Method

The design of analog amplifiers could be defined as a multi-attribute problem, which is solved here through the utilization of the m-polar fuzzy graphs theory and deep learning. The developed method for amplifier design is presented in Figure 1 and includes three stages:

- In the first stage, a dataset is prepared according to a predefined specification regarding the designed amplifier. All possible variants of the designed electronic circuit are found and membership functions of attributes are predicted through a deep learning algorithm.
- The second stage points out the suitable design solutions, considering the requested parameters, and after obtaining the membership values of vertices and edges, an m-polar fuzzy graph is constructed.
- In the third stage, the most suitable solutions are prioritized, finding the best one, according to the user’s specifications and certain requirements.

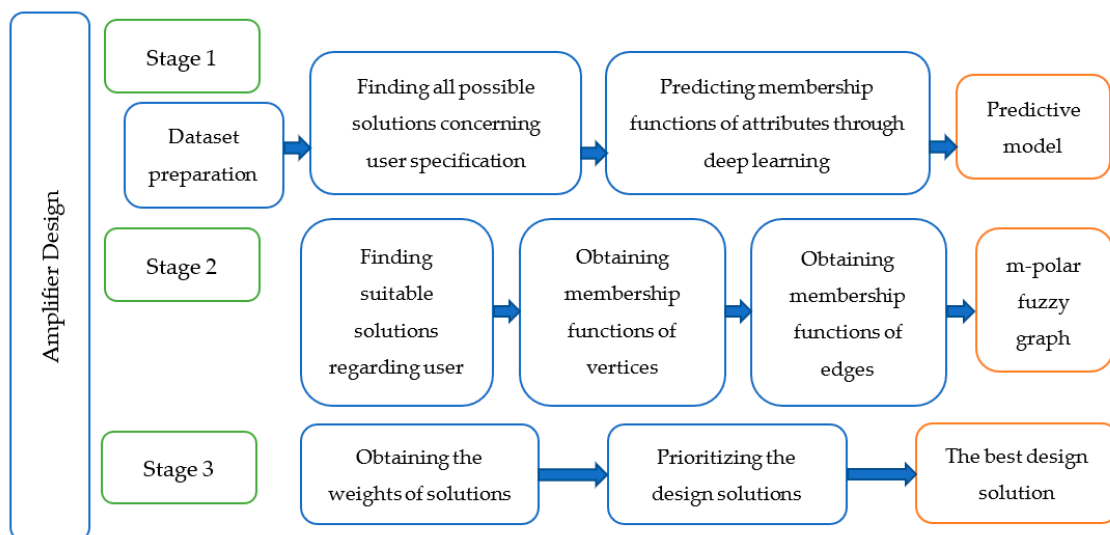


Figure 1. Proposed method.

5. Experimentation and Results

The problem related to finding the most useful designs of an electronic circuit is examined as a multi-attribute problem for solving. The typical attributes of a given circuit that better explain its electrical and functional behavior are chosen, as they have to be

adjusted considering an input specification. For the verification of the proposed method, comprising m-polar fuzzy graphs theory and DL, three electronic circuits are designed: an inverting summing amplifier, a subtracting amplifier (differential amplifier), and a summing/subtracting amplifier with an operational amplifier.

5.1. Design of Inverting Summing Amplifier

Summing amplifiers are very often implemented through operational amplifiers with negative feedback and topology shown in Figure 2. They are used for solving equations such as [44]:

$$v_{out} = -(k_1v_1 + k_2v_2 + \dots + k_nv_n) \tag{1}$$

where scaling coefficients k_1, k_2, \dots, k_n are defined as the ratio between the feedback resistor R_F and the input resistors R_1, R_2, \dots, R_n , v_1, v_2, \dots, v_n are input voltage signals, and v_{out} is the output voltage.

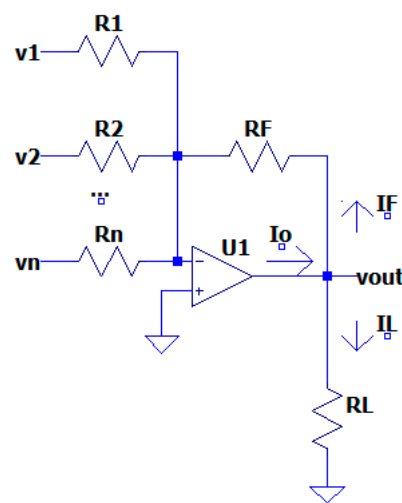


Figure 2. Inverting summing amplifier [44].

One of the challenges for designers is how to satisfy Equation (1), taking into account the possible ranges of input values v_1, v_2, \dots, v_n and output signal v_{out} and considering the allowed range of power dissipation P_D typical for the used operational amplifier for a given load R_L .

Power dissipation has to possess appropriate values for the normal functioning of the operational amplifier. Otherwise, the operational amplifier will be damaged and the realized analog circuit will not work as expected or will stop working. The amplifier power dissipation P_D is found by knowing the supply voltage V_S , maximal quiescent current I_Q , output current I_o , load current I_L , and output voltage v_{out} [45].

The design process of summing amplifiers is related to finding the values of input resistors R_1, R_2, \dots, R_n , taking into account the given scaling coefficients k_1, k_2, \dots, k_n in Equation (1), the output voltage v_{out} at given input voltages v_1, v_2, \dots, v_n , and at certain values of the feedback resistor R_F and the load resistor R_L .

Let us suppose that the used operational amplifier is OPA 322 [46] and the equation to be solved when designing an inverting summing amplifier, taking into consideration the allowed power dissipation is:

$$v_{out} = -(3v_1 + 6v_2) \tag{2}$$

During the first stage of the proposed method, a dataset in the form of Table 1 is prepared, considering a predefined specification. The investigated range of R_F is from 60 kΩ to 6 kΩ, v_1 and v_2 have values from 0.01V to 0.5V, and R_L takes values from 0.5 kΩ to 10 kΩ. During the design process multiple solutions S_1, S_2, \dots, S_n are possible, which must first be found. All solutions possess common attributes $A_1 = R_1, A_2 = R_2, A_3 = v_1, A_4 = v_2$,

$$A_5 = v_{out}, A_6 = R_F, A_7 = R_L, A_8 = P_D, \text{ e.g., } S_1 = f(A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8), \\ S_2 = f(A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8), \dots, S_n = f(A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8).$$

Table 1. Prepared dataset with possible solutions.

S	$R_F, k\Omega$	$R_1, k\Omega$	$R_2, k\Omega$	v_1, V	v_2, V	v_{out}, V	$R_L, k\Omega$	P_D, mW	$\sigma(P_D)$
S_1	60	20	10	0.01	0.01	0.09	10	10.511	0.999
S_2	60	20	10	0.05	0.01	0.21	10	10.590	0.992
S_3	60	20	10	0.1	0.01	0.36	10	10.684	0.983
S_4	60	20	10	0.15	0.01	0.51	10	10.772	0.975
S_5	60	20	10	0.2	0.01	0.66	10	10.855	0.968
...	

All obtained solutions are assigned membership values for each attribute $\sigma^{S_i}(A_j)$ corresponding to the membership functions. The created dataset is presented in Table 1 and the last column includes the membership values of power dissipation P_D . The membership values of other attributes are not shown, but they are predicted in a similar way through a deep learning algorithm with a regression task. The ANN includes two hidden layers with 50 neurons at each layer. The utilized activation function is Rectifier. The dataset contains 13,090 records, of which 70% are used for training and 30% for testing.

The achieved accuracy of the predictive model is assessed considering standard metrics such as Root mean squared error (RMSE) = 0.0098, Absolute error (AE) = 0.0053, and Relative error (RE) = 0.98%. Figure 3 presents the prediction chart and the linear regression result.

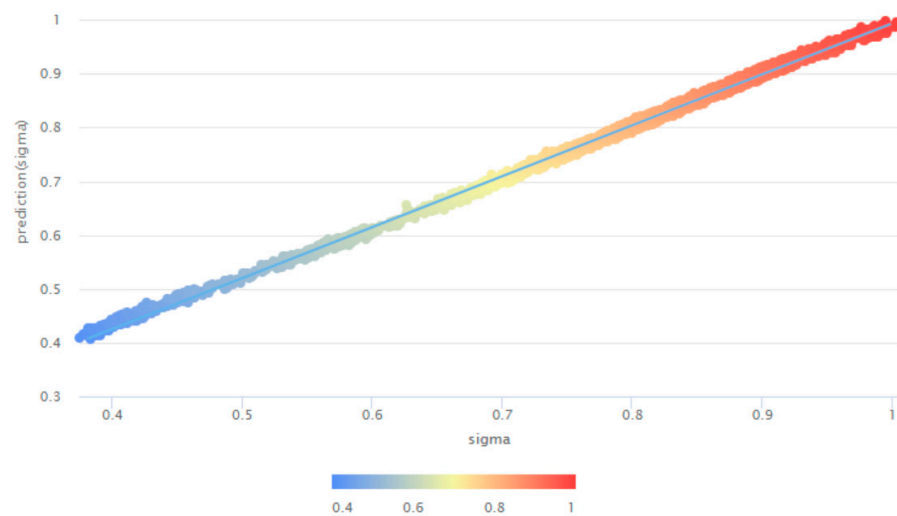


Figure 3. Prediction chart of the power dissipation P_D membership values (sigma) at inverting summing amplifier design.

In the second stage, the designer is looking for solutions with maximal membership function of power dissipation P_D , e.g., minimal values of power dissipation P_D and allowable values of the rest of the examined attributes. It is found that these requirements are satisfied by 51 of 13,090 designs: $SS_1, SS_2, \dots, SS_{51}$. The membership functions of vertices (solutions) of the fuzzy graph are predicted through deep learning as ten random solutions are shown in Figure 4. For further exploration, five solutions which are the most appropriate, are considered. Thus, effort and resources are reduced, selecting and examining the closest to the user specification solutions.

In the third stage, the membership values of the solutions considering the membership values of every attribute, are calculated through the weight function:

$$w(S_i) = \frac{\sum \sigma^{SS_i}(A_j)}{n} \tag{4}$$

where n is the number of attributes.

Figure 6 gives information on the calculated weights of every candidate for the best solution. It seems that the SS₃ is the best-found design considering the specified requirements. The best design could be used for amplifier prototyping with priorities in comparison to the other four designs.

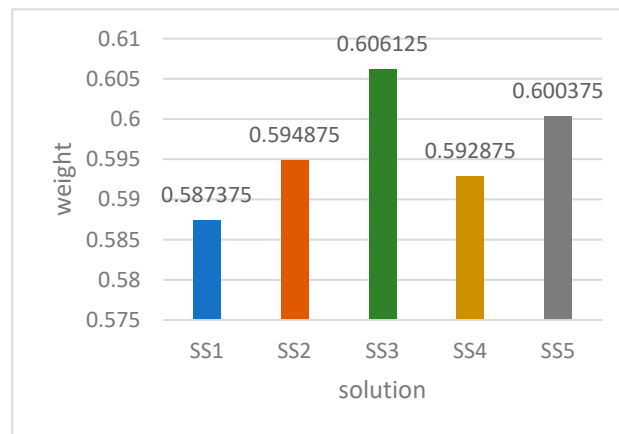


Figure 6. Calculated weights of the solutions at design of the inverting summing amplifier.

5.2. Design of Subtracting Amplifier (Differential Amplifier)

The function realized from the subtracting amplifier (differential input amplifier), presented in Figure 7, is described by the following equation [44]:

$$v_{out} = k(v_2 - v_1) \tag{5}$$

where $k = \frac{R_F}{R_1}$, e.g., the output voltage is proportional to or equal to the difference between the input voltages.

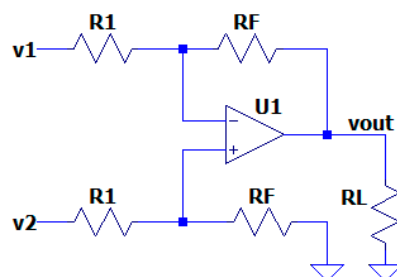


Figure 7. Subtracting amplifier [44].

Let us suppose that the specification requires the subtracting amplifier to realize the following function at minimal power dissipation P_D :

$$v_{out} = 3(v_2 - v_1) \tag{6}$$

The suggestion is that the used operational amplifier is OPA 322 [46] and the explored range of R_F is from 60 kΩ to 3 kΩ, v_1 and v_2 have values from 0.01V to 0.5V, R_L takes values from 0.5 kΩ to 10 kΩ. The design solutions are characterized by seven attributes:

$A_1 = R_1, A_2 = v_1, A_3 = v_2, A_4 = v_{out}, A_5 = R_F, A_6 = R_L, A_7 = P_D,$
 e.g., $S_1 = f(A_1, A_2, A_3, A_4, A_5, A_6, A_7), S_2 = f(A_1, A_2, A_3, A_4, A_5, A_6, A_7), \dots,$
 $S_n = f(A_1, A_2, A_3, A_4, A_5, A_6, A_7).$

The prepared dataset is similar to Table 1. The membership values of the rest of the attributes are also predicted through a deep-learning algorithm. The learning records are 5710 and the prediction chart of the membership values of power dissipation (sigma) is presented in Figure 8. The created predictive model is characterized by very small errors: RMSE = 0.0032, AE = 0.0022, RE = 0.27%.

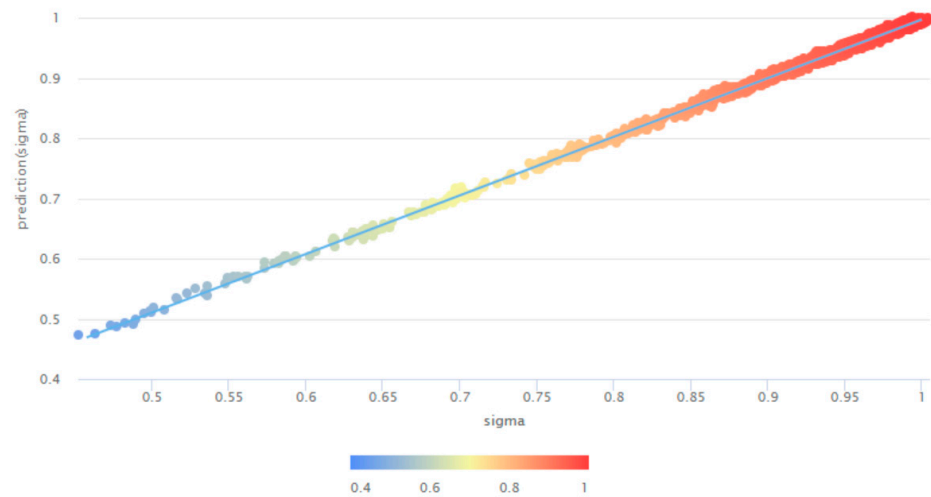


Figure 8. The prediction chart of the membership values of power dissipation (sigma) at subtracting amplifier design.

Figure 9 depicts the membership values of ten randomly chosen designs.

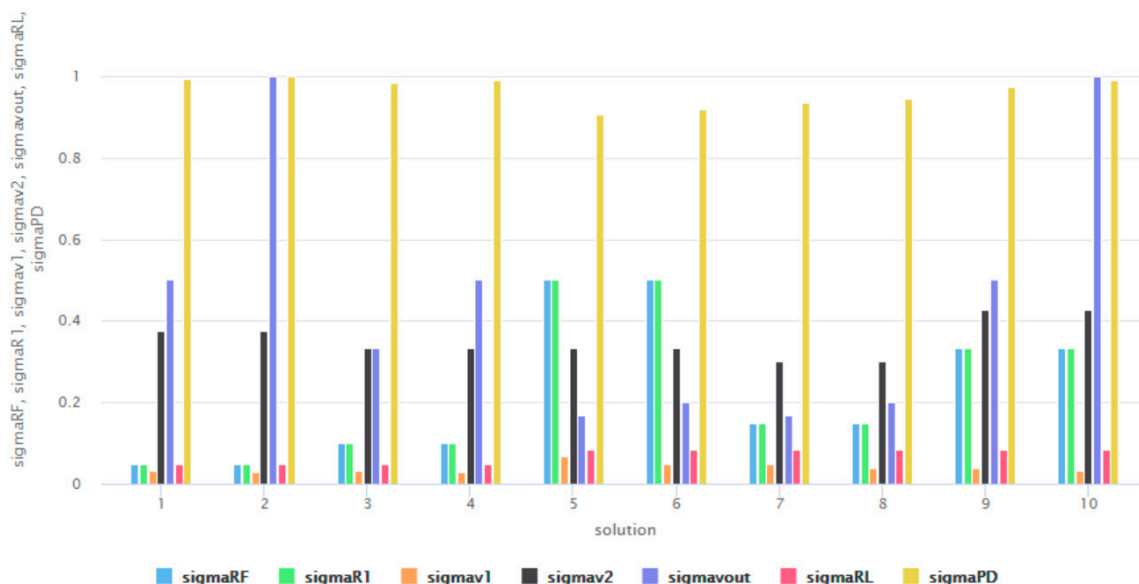


Figure 9. Membership values of randomly chosen solutions at design of subtracting amplifier.

At the second stage, out of all possible 5710 designs, eight are selected—these with the maximal membership values of power dissipation P_D .

For fuzzy graph construction, in addition to the membership values of vertices, it is necessary to know the membership values of the edges, which are calculated similarly to the previous demonstration and are presented in Table 3. The m-polar fuzzy graph itself

is shown in Figure 10. Its structure points out the most suitable designs found, which are eight $SS_1 \div SS_8$. They are connected because they share common attributes.

Table 3. Membership values of the edges.

$SS_i \otimes SS_k$	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8
$E_{12} = SS_1 \otimes SS_2$	1	1	0.222	0.3	1	1	1	1
$E_{23} = SS_2 \otimes SS_3$	1	1	0.25	0.333	1	1	1	1
$E_{34} = SS_3 \otimes SS_4$	1	1	0.285	0.375	1	1	1	1
$E_{45} = SS_4 \otimes SS_5$	1	1	0.333	0.428	1	1	1	1
$E_{56} = SS_5 \otimes SS_6$	1	1	0.4	0.5	1	1	1	1
$E_{67} = SS_6 \otimes SS_7$	1	1	0.5	0.6	1	1	1	1
$E_{78} = SS_7 \otimes SS_8$	1	1	0.666	0.75	1	1	1	1
...

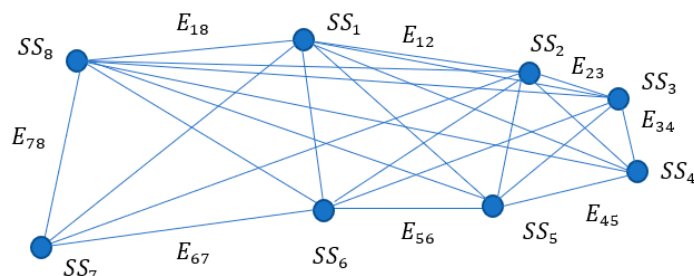


Figure 10. M-polar fuzzy graph at design of subtracting amplifier.

In the third stage, the best solution found is SS_8 , after calculating the values of the weighting function (ordinate axes) as it is shown in Figure 11. It seems that SS_8 is the design, which satisfies the user requirements in the best way. Moreover, the remaining seven also could be applied because of the closest weighting values to the most suitable design.

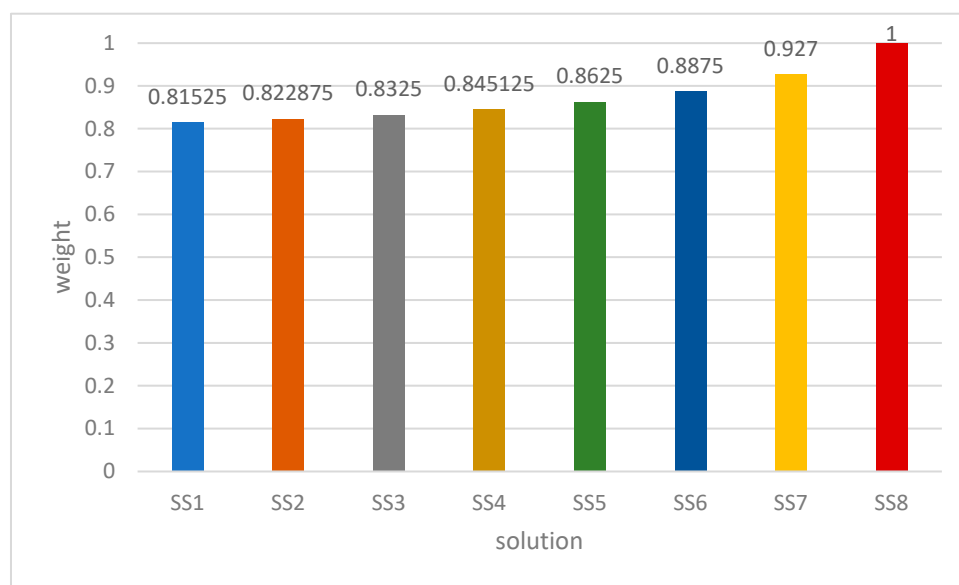


Figure 11. Prioritizing the suitable designs of the subtracting amplifier.

5.3. Summing and Subtracting Amplifier

The electronic circuit of the summing and subtracting amplifier is presented in Figure 12. The output voltage v_{out} is calculated as the sum of the input voltages applied to the non-inverting input $v_{11}, v_{12}, \dots, v_{1n}$, and the subtraction of the input voltages at the inverting input $v_{21}, v_{22}, \dots, v_{2n}$. So, the implemented function of this amplifier can be presented in the following form [44]:

$$v_{out} = (k_{11}v_{11} + k_{12}v_{12} + \dots + k_{1n}v_{1n}) - (k_{21}v_{21} + k_{22}v_{22} + \dots + k_{2n}v_{2n}) \quad (7)$$

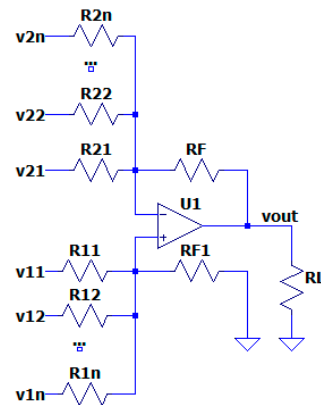


Figure 12. Summing and subtracting amplifier [44].

The sizing coefficients are defined as the ratio between the feedback resistor and the respective input resistor $k_{1j} = \frac{R_{F1}}{R_{1j}}$ and $k_{2j} = \frac{R_F}{R_{2j}}$.

If the specification says that the amplifier must satisfy the following equation:

$$v_{out} = (9v_{11} + 2v_{12}) - (6v_{21} + 3v_{22}) \quad (8)$$

then the first stage includes the dataset preparation, which is similar to the one shown in Table 1 and the aim is the design with minimal power dissipation P_D to be found considering a given load R_L . The examined design solutions are 144,414: $S_1 \div S_{144414}$ as they possess 11 common attributes: $A_1 = R_F, A_2 = R_1, A_3 = R_2, A_4 = R_3, A_5 = R_4, A_6 = v_1, A_7 = v_2, A_8 = v_3, A_9 = v_4, A_{10} = v_{out}, A_{11} = P_D$. All obtained solutions are assigned predicted membership values for each attribute $\sigma^{S_i}(A_j)$ through the usage of a deep learning algorithm. The prediction chart of membership values of power dissipation P_D is presented in Figure 13. The predictive model is evaluated, and it is characterized by very small errors: RMSE = 0.0187, AE = 0.0098, RE = 1.57%.

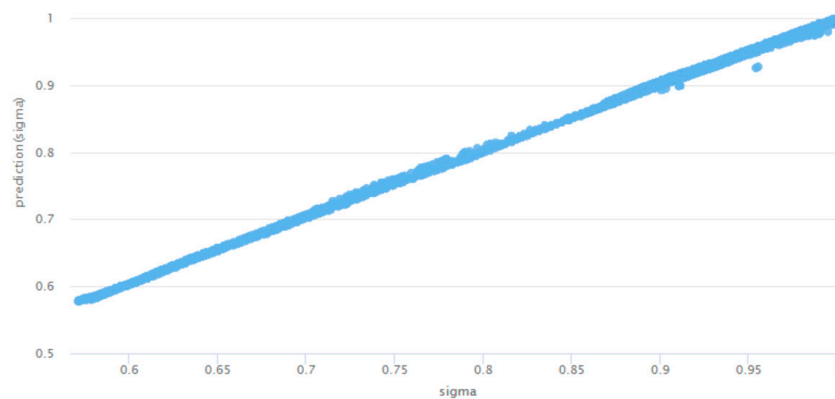


Figure 13. The prediction chart of the membership values of power dissipation (sigma) at design of summing and subtracting amplifier.

The membership values of ten random solutions out of 144,414 found solutions are presented in Figure 14.

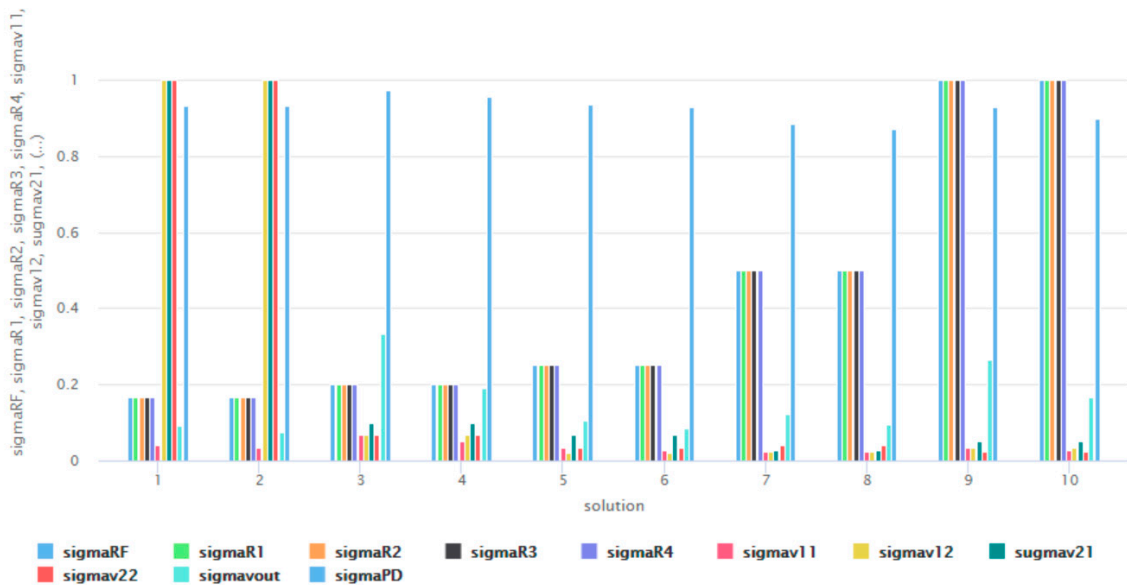


Figure 14. Sigma membership values of ten random designs of summing and subtracting amplifier.

Stage two includes selecting the most suitable solutions from 144,414 designs. The membership values of the most appropriate 39 designs $SS_1 \div SS_{39}$ are calculated, as well as the membership functions of the edges (SS_i, SS_k) taking into account the attributes $A_1 \div A_{11}$. Then, the m-polar fuzzy graph is constructed.

In the third stage, for every attribute, the membership values of the edges between the vertices are calculated and the weights of the solutions are presented in Figure 15. The best solution found is SS_1 according to the predefined user specification.

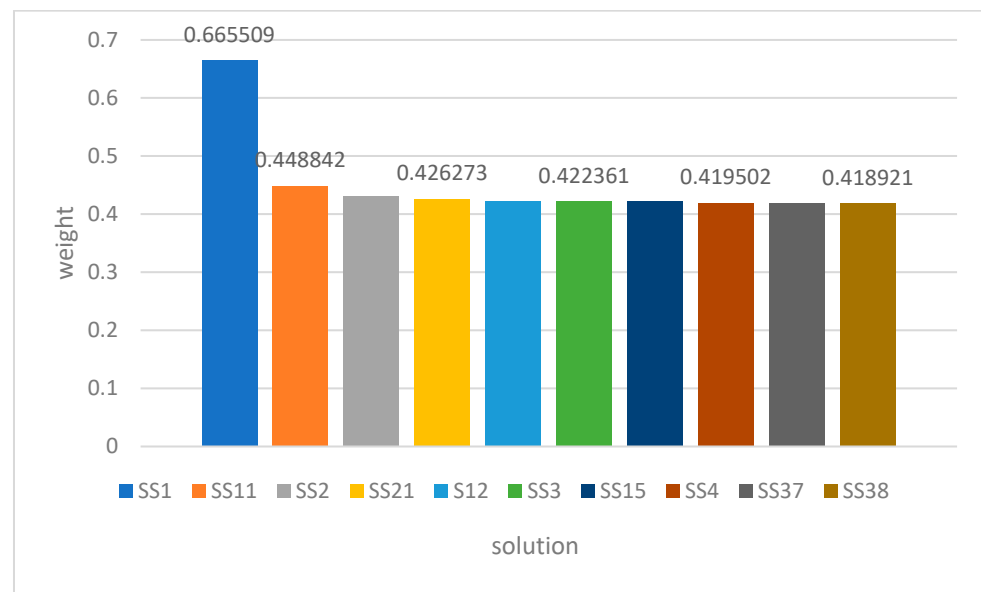


Figure 15. Prioritizing the suitable designs of the summing and subtracting amplifier.

6. Conclusions

The paper presents a novel method for supporting the design process of analog amplifiers based on the concepts of m-polar fuzzy graphs and deep learning techniques. For the

verification of the proposed method, three electronic circuits are designed: an inverting summing amplifier, a subtracting amplifier (differential amplifier), and a summing/subtracting amplifier with an operational amplifier as the solved problem is related to finding the designs with the smallest power dissipation and performing a certain mathematical function. The findings point out that:

- The synergetic combination of m-polar fuzzy graphs theory and DL leads to obtaining the most suitable solutions only in three stages, extremely reducing the number of repetitive tasks concerning the calculation of the values of designs' attributes, their comparison, and design selection.
- DL is a suitable approach when expert opinion could be predicted and used for further analysis. In this work, the membership functions of attributes are predicted instead of the expert votes to be gathered. The created predictive models are evaluated, and it is proved that they are characterized with high precision since the obtained errors are very small: RMSE is from 0.0032 to 0.0187, AE is from 0.022 to 0.098, and RE is between 0.27% and 1.57%.
- Fuzzy graph construction gives a possibility for very fast finding the eligible designs, proposes apparatus for their prioritization, and an opportunity for reaching the best design according to a given predefined user specification.

The method can be applied to the design of any electronic circuit to assist a designer in decision-making when the task is multivariate and the environment is complex and uncertain. A promising future work includes further exploration of the advantages and applicability of the m-polar fuzzy graphs theory and deep learning in support of circuit designers and analysts.

Author Contributions: Conceptualization, M.D. and M.I.; methodology, M.I.; software, M.I.; validation M.I. and M.D.; formal analysis M.D. and M.I.; investigation M.I. and M.D.; resources, M.D. and M.I.; writing—original draft preparation, M.D. and M.I.; writing—review and editing, M.D. and M.I. All authors have read and agreed to the published version of the manuscript.

Funding: This research is supported by Bulgarian National Science Fund in the scope of the project "Exploration of the application of statistics and machine learning in electronics" under contract number KII-06-H42/1.

Data Availability Statement: Not applicable.

Conflicts of Interest: The authors declare no conflict of interest.

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