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Regional Consensus Control for Multi-Agent Systems with Actuator Saturation

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Abstract: This paper considers the regional consensus problem for multi-agent systems with actuator saturation. By utilizing the theory of convex set, a novel multiple nonlinear feedback control protocol is presented, which can effectively reduce the conservatism in dealing with saturated nonlinear input. In order to obtain a larger estimate on the domain of consensus, the composite Laplacian quadratics function is constructed to derive sufficient conditions for the consensus of multi-agent systems. In addition, an alternative convex hull representation is employed to further enlarge the above-mentioned domain of consensus. Finally, a numerical simulation case study illustrates the validity as well as the superiority of the proposed approaches.

Keywords: regional consensus; multiple nonlinear feedback; composite Laplacian quadratics function; alternative convex hull representation

MSC: 93D05

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1. Introduction

Multi-agent systems (MASs) research started at the outset of the 21st century. The concept of multi-agent systems was discovered from the group behavior of animals in nature. Through the interaction between individuals, these groups eventually emerge with a formal and orderly group behavior. Since some states of MASs need to be agreed in many applications, consensus of MASs is often considered one of the most important issues in multi-agent cooperative control. Multi-agent cooperative control aims to design a distributed control protocol to make all agents achieve a predefined specified target, and initially Yamaguchi et al. in [1] implemented robot formations through a control protocol for MASs. The investigation of consensus control of MASs has been widely considered by the research community in recent years. The work in [2] gave the theoretical framework of the typical consensus model as well as the concept of control protocol for MASs. The research conducted in [3] proposed a consensus control protocol based on coupling weights of adjacent agents, and the consensus problem was transformed into the stability analysis of a group of low-dimensional matrices. Based on Riccati inequality and algebraic graph theory, it was reported in [4] that the consensus of MASs can be achieved in fixed and switching topologies. Moreover, similar research has been carried out as follows: the consensus of time-delay MASs [5,6], the consensus of MASs with switching topologies [7,8], and the consensus control protocol for heterogeneous MASs [9,10].

Actuator saturation is an ubiquitous phenomenon existing in engineering applications and it has typical characteristics of nonlinearity [11,12], so the treatment of saturation nonlinearity becomes an popular but difficult subject. In [13], the saturation term was converted into a locally sector-bounded function for the first time, and sector-bounded constraints were then applied to derive sufficient conditions of stability based on linear matrix inequalities. The subsequent work in [14] presented an adaptive event-triggered consensus protocol for MASs with actuator saturation. Meanwhile, the work in [15] embedded the network-based consensus of MASs with bounded inputs into its control design. However, it is relatively conservative to deal with saturation nonlinearity by using sector-bounded constraints. Thus, the study carried out in [16] proposed the convex combination technique to handle the saturation phenomenon. Using the convex combination technique, the consensus of saturated MASs with disturbances was achieved in [17]. Meanwhile, the exponential consensus control protocol was designed in [18] for Markov jump MASs with input saturation, and the regional consensus of differential inclusions MASs was studied in [19].

It is introduced in [20] that the global stabilization is impossible for an exponentially unstable system with bounded input. As a result, the unstable MASs involving saturated actuators can only achieve regional consensus; in other words, the consensus can be reached if and only if the initial state of every agent is in the domain of consensus (DC). In practical engineering, a larger DC can provide greater freedom for the initial state of systems. Therefore, how to determine a larger DC is one of the critical problems in the research of regional consensus. In the aforementioned works [17–19], the DC was estimated by the level set of Laplacian quadratics function (LQF), but this method was also somehow conservative in the sense of the estimate of DC. Thus, as an extension of composite Lyapunov function in [21], the composite Laplacian quadratics function (CLQF) was proposed in [22]. The advantage of CLQF mainly lies in that its level set is the convex hull of several ellipsoids, which would be useful to indicate what practical engineering applications the solution proposed in the article will be used for. To the author's knowledge, most results on the consensus for saturated MASs were achieved by linear feedback control protocol and the DC was estimated by an LQF approach. However, the traditional linear feedback control method is still conservative in dealing with saturated nonlinearity, and the level set of LQF also has some limitation in estimating the DC.

Based on the above discussions, this paper achieves the regional consensus of saturated MASs by a novel nonlinear feedback approach, and the estimate of DC is enlarged by constructing non-LQF. The contributions are mainly listed in the following three points:

- A novel multiple nonlinear feedback control protocol is proposed for saturated MASs and sufficient conditions are presented that guarantee the consensus of MASs, which is more general than the traditional linear control protocol considered in [17–19].
- The level set included in CLQF is established to estimate DC for saturated MASs, which larger than estimated by the LQF approach.
- An improvement of the consensus control and the estimation of DC is achieved by utilizing an alternative convex hull representation—enlarged.

The remaining sections of this paper are as follows. Section 2 provides the system description and preliminaries. In Section 3, three different solutions are presented for the consensus problem of saturated MASs, and the corresponding optimization problems of maximizing the DC estimate are also derived. In Section 4, a numerical simulation case is given to demonstrate the obtained results. Section 5 is about the conclusion of this paper.

Notations. $I[1,\ N]$ indicates the integer set $\{1, 2, \cdots, N\}$ and $sign(\cdot)$ stands for the symbol function. $\otimes$ means the Kronecker product symbol. Let $V(x) = x^{\mathrm{T}}Px$ be the Lyapunov function. $L_V = \{x : V(x) \leq 1\}$ is the 1-level set of $V(x)$ and $\varepsilon(P) = \{x : x^{\mathrm{T}}Px \leq 1\}$ is an ellipsoid. For a square matrix A, $He(A) = A + A^{\mathrm{T}}$, $s.t$ is the abbreviation of subject to, and $\sup\limits_{B>0} \alpha$ denotes the supremum of the optimization α under the constraint $B > 0$.

2. System Description and Preparations

The typical multi-agent model consisting of N identical agents with actuator saturation is as follows:

$$\dot{x}_i = Ax_i + B\mathscr{N}_{sat}(u_i),\ i \in I[1, N], \tag{1}$$

where $x_i \in R^n$ is the state and $u_i \in R^m$ is the input for $i \in I[1,N]$. $A \in R^{n\times n}$ and $B \in R^{n\times m}$ are given matrices. The standard saturation function $\mathscr{N}_{sat}(u_i) \in R^m$ is defined by $\mathscr{N}_{sat}(u_i) = [\mathscr{N}_{sat}(u_{i1}), \mathscr{N}_{sat}(u_{i2}), \cdots, \mathscr{N}_{sat}(u_{im})]^{\mathrm{T}}$, where $\mathscr{N}_{sat}(u_{ih}) = sign(u_{ih})min\{1, |u_{ih}|\}$, $h \in I[1,m]$.

In MASs, the communication topology of the agent network is essential to their cooperation and it can be represented by the graph $\mathscr{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{v_1, v_2, \cdots, v_N\}$ is the set of nodes and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ represents the set of edges. The specific reference can be shown in Figure 1. Denote $\mathcal{A} = [a_{il}] \in R^{N\times N}$ as the adjacency matrix of $\mathscr{G}$, where $a_{il} = 1$ if $i \neq l$ and $a_{il} = 0$ if $i = l$. The Laplacian matrix $\mathcal{L}$ of $\mathscr{G}$ is defined by $\mathcal{L} = \mathcal{L}_{il} \in R^{N\times N}$, where $\mathcal{L}_{il(i\neq l)} = -a_{il}$ and $\mathcal{L}_{ii} = \sum_{i=1}^{N} a_{il}$. In this paper, we consider the MAS whose graph $\mathscr{G}$ is undirected, that is, $(v_1, v_2) \in \mathcal{E}$ is equivalent to $(v_2, v_1) \in \mathcal{E}$ for all $v_i \in \mathcal{V}$. In addition, the graph $\mathscr{G}$ is assumed to be connected, in other words, there exists a path from any node to any other node in this graph.

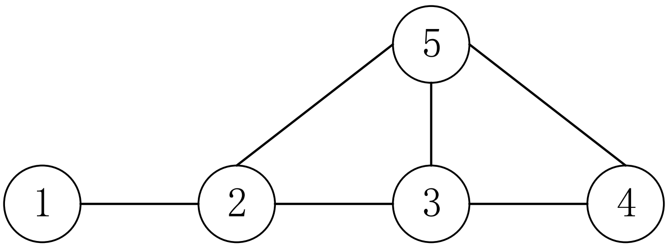

Figure 1. The communication topology graph of five agents.

Let $x = [x_1^{\mathrm{T}}, \cdots, x_N^{\mathrm{T}}]^{\mathrm{T}} \in R^{Nn}$ and $u = [u_1^{\mathrm{T}}, \cdots, u_N^{\mathrm{T}}]^{\mathrm{T}} \in R^{Nm}$, system (1) can be transformed into the following congregated form:

$$\dot{x} = (I_N \otimes A)x + (I_N \otimes B)\mathscr{N}_{sat}(u), \tag{2}$$

where I_N is an identity matrix.

Definition 1 ([19])**.** *For initial conditions $x_i(0) \in \mathcal{X}$, $x_l(0) \in \mathcal{X}$ for $\mathcal{X} \in R^n$, if the following equality holds*

$$\lim_{t\to\infty} \|x_i(t) - x_l(t)\| = 0, i, l \in I[1,N],$$

then MAS (2) is said to achieve the consensus and the set $\mathcal{X}$ is called DC.

Definition 2 ([20])**.** *Let $Q_k \in R^{n\times n} > 0$ for $k \in I[1,K]$, and $\Gamma = \{\gamma = (\gamma_1 \cdots \gamma_K)^{\mathrm{T}} \in R^K : \gamma_k \geq 0,\ \sum_{k=1}^{K} \gamma_k = 1\}$. The CLQF is defined by*

$$V(x) = \min_{\gamma\in\Gamma} x^{\mathrm{T}}[\mathcal{L} \otimes (\sum_{k=1}^{K} \gamma_k Q_k)^{-1}]x.$$

Meanwhile, the optimal $\gamma(x)$ is determined by

$$\gamma^*(x) = \arg\min_{\gamma\in\Gamma} x^{\mathrm{T}}[\mathcal{L} \otimes (\sum_{k=1}^{K} \gamma_k Q_k)^{-1}]x.$$

Remark 1. *Here, we recall some properties of CLQF in [22]. The CLQF is an extension of the composite Lyapunov function in [21] and it has similar properties as the composite Lyapunov function. Denote $L_V = \{x : V(x) \leq 1\}$ as the 1-level set of $V(x)$. It is obvious that $V(x)$ reduces to an LQF $x^{\mathrm{T}}(\mathcal{L} \otimes Q_k^{-1})x$ when $K = 1$. The advantage of CLQF is that its level sets are the convex hull of the level sets of LQF $x^{\mathrm{T}}(\mathcal{L} \otimes Q_k^{-1})x$, i.e., $L_V = co\{\varepsilon(\mathcal{L} \otimes Q_k^{-1}), k \in I[1,K]\}$.*

For the Laplacian matrix $\mathcal{L}$ of graph $\mathcal{G}$, denote its ith eigenvalue as λ_i for $i \in I[1,N]$. Since $\mathcal{G}$ is connected, 0 is an eigenvalue with the corresponding eigenvector $l = [1,1,\cdots,1]^{\mathrm{T}}$ and other eigenvalues are all positive. Thus, without loss of generality, it is further assumed that $0 = \lambda_1 < \lambda_2 \leq \cdots \leq \lambda_N$. Since the graph $\mathcal{G}$ is undirected, the Laplacian matrix $\mathcal{L}$ is symmetric, i.e., $a_{il} = a_{li}$, which can be diagonalized with $\mathcal{L} = U^{\mathrm{T}}\Lambda U$, where $U \in R^{N\times N}$ is a unitary matrix satisfying $U^{\mathrm{T}}U = UU^{\mathrm{T}} = I$ and $\Lambda = diag([\lambda_1,\cdots,\lambda_N])$. By removing the first row of U and Λ, denote the resulting matrices as $\widetilde{U} \in R^{(N-1)\times N}$ and $\widetilde{\Lambda} = diag([\lambda_2,\cdots,\lambda_N])$. It follows from $\lambda_1 = 0$ that $\mathcal{L} = \widetilde{U}^{\mathrm{T}}\widetilde{\Lambda}\widetilde{U}$. Let $y = (U \otimes I_N)x = [y_1^{\mathrm{T}},\cdots,y_N^{\mathrm{T}}]^{\mathrm{T}} \in R^{Nn}$ for $y_i \in R^n$, then $\widetilde{y}$ is defined by $\widetilde{y} = (\widetilde{U} \otimes I_N)x = [y_2^{\mathrm{T}},\cdots,y_N^{\mathrm{T}}]^{\mathrm{T}} \in R^{(N-1)n}$. Thus, the following holds

$$\begin{aligned} V(\widetilde{y}) &= \min_{\gamma\in\Gamma} \widetilde{y}^{\mathrm{T}}[\widetilde{\Lambda} \otimes (\sum_{k=1}^{K} \gamma_k Q_k)^{-1}]\widetilde{y} \\ &= \min_{\gamma\in\Gamma} x^{\mathrm{T}}[\mathcal{L} \otimes (\sum_{k=1}^{K} \gamma_k Q_k)^{-1}]x = V(x). \end{aligned} \tag{3}$$

Lemma 1 ([23])**.** *The boundary of L_V is defined by ∂L_V. Let $\mathcal{O}_j = \partial L_V \cap \varepsilon(\mathcal{L} \otimes Q_j^{-1}) = \{x : V(x) = x^{\mathrm{T}}(\mathcal{L} \otimes Q_j^{-1})x = 1\}$ for $j \in I[1,K]$, we have*

$$\mathcal{O}_j = \{x \in \partial L_V : \sum_{i=2}^{N} \lambda_i y_i^{\mathrm{T}} Q_j^{-1}(Q_k - Q_j)Q_j^{-1} y_i \leq 0,\ k \in I[1,K]\}.$$

Lemma 2 ([21])**.** *It is assumed that $\gamma_j^* > 0$ for $j \in I[1,K_0]$ and $\gamma_j^* = 0$ for $j \in I[K_0+1,K]$. Let $Q(\gamma^*) = \sum_{j=1}^{K_0} \gamma_j^* Q_j$ and $x_j = Q_j Q(\gamma^*)^{-1}x$ for $j \in I[1,K_0]$, then we have $V(x) = V(x_j) = x_j^{\mathrm{T}}(\mathcal{L} \otimes Q_j^{-1})x_j$ and $V_x(x) = V_x(x_j) = 2[\mathcal{L} \otimes Q^{-1}(\gamma^*)]x$ for $x_j \in (V(x))^{1/2}\mathcal{O}_i$, $j \in I[1,K_0]$, where $V_x(x)$ stands for the gradient of $V(x)$ at x. Furthermore, $V_{\widetilde{y}}(\widetilde{y}) = 2[\widetilde{\Lambda} \otimes Q^{-1}(\widetilde{\gamma}^*)]\widetilde{y}$, $\forall\, \widetilde{y} \in R^{(N-1)n}$.*

Denote a group of matrices by $E = \{E_1,\cdots,E_{2^m}\}$ with $E_r^- = I - E_r$ for $r \in I[1,2^m]$, where E_r is a diagonal matrix whose diagonal elements are either 0 or 1. Thus, the following lemma is given:

Lemma 3 ([16])**.** *If there exist $F, H \in R^{m\times n}$ and $x \in \mathscr{L}(H)$, then*

$$\mathcal{N}_{sat}(u) \in co\{E_r F x + E_r^- H x : r \in I[1,2^m]\},$$

where $co\{\cdot\}$ is the convex hull of a set and $\mathscr{L}(H) = \{x \in R^n, |Hx|_\infty \leq 1\}$.

3. The Design of Consensus Protocol

In this section, the improved consensus control protocol is designed for MASs subject to actuator saturation, and the CLQF is constructed to derive sufficient conditions for the consensus of MASs. Furthermore, the maximal estimate of DC is determined by solving an optimization problem. In order to compare with the traditional linear feedback method in [17–19], Theorem 2 is given. In addition, by employing an alternative convex hull representation, Theorem 3 provides less conservative results.

3.1. Multiple Nonlinear Feedback for Regional Consensus

In order to achieve the consensus of MAS (2), a novel nonlinear consensus control protocol is presented as

$$u_i = \sum_{l=1}^{N} a_{il} F(\gamma^*) Q(\gamma^*)^{-1}(x_i - x_l),\ i \in I[1,N]. \tag{4}$$

In view of Lemma 3, it follows from (4) that the saturated consensus control protocol is as follows:

$$\mathcal{N}_{sat}(u) \in co\{\mathcal{L} \otimes [E_r F(\gamma^*) Q(\gamma^*)^{-1} + E_r H(\gamma^*)]x,\ r \in I[1, 2^m]\}, \tag{5}$$

where

$$F(\gamma^*) = \sum_{j=1}^{K} \gamma_j^* F_j,\ \ H(\gamma^*) = \sum_{j=1}^{K} \gamma_j^* H_j Q_j Q(\gamma^*)^{-1},\ \ Q(\gamma^*) = \sum_{j=1}^{K} \gamma_j^* Q_j. \tag{6}$$

Remark 2. *The protocol* (5) *is an extension of saturated linear consensus control protocol in [17–19]. By constructing the nonlinear consensus control protocol* (5) *combining an optimal function γ^*, the group of 2^m possible combinations of feedback gain matrices F and H are generalized to $K2^m$ possible combinations of F_j and H_j. Meanwhile, sufficient conditions of the consensus are also relaxed.*

Theorem 1. *Let $Q_j \in R^{n \times n} > 0$ for $j \in I[1, K]$. If there exist $F_j, H_j \in R^{m \times n}$, $\sigma_{rjk} \geq 0$ for $r \in I[1, 2^m]$, $j, k \in I[1, K]$, such that*

$$He[AQ_j + \lambda_i B(E_r F_j + E_r^- H_j Q_j)] - \sum_{k=1}^{K} \sigma_{rjk}(Q_k - Q_j) \leq 0, \tag{7}$$

$$\varepsilon(\mathcal{L} \otimes Q_j^{-1}) \subset \mathscr{L}(\mathcal{L} \otimes H_j). \tag{8}$$

Then, the consensus of MAS (2) *can be achieved under the protocol* (5) *and the invariant set L_V is contained in DC.*

Proof. Under the consensus control protocol (5), the closed-loop system (2) becomes

$$\dot{x} \in co\{(I_N \otimes A)x + (I_N \otimes B)[\mathcal{L} \otimes (E_r F(\gamma^*) Q(\gamma^*)^{-1} + E_r^- H(\gamma^*))x],\ r \in I[1, 2^m]\}. \tag{9}$$

According to the structure of CLQF $V(x)$, its derivative can be calculated by $\dot{V}(x) = V_x(x)\dot{x}$, where $V_x(x) = (\frac{\partial V(x)}{\partial x_1}, \cdots, \frac{\partial V(x)}{\partial x_n})$. Based on (9), $\dot{x}$ is a multi-valued mapping function, that is, $\dot{x}$ may be a set for each $x \in R^{Nn}$. Let $\dot{V}_r(x)$ be an element in $\dot{V}(x)$ for $r \in I[1, 2^m]$, and it is defined by

$$\dot{V}_r(x) = V_x(x)\{(I_N \otimes A)x + (I_N \otimes B)[\mathcal{L} \otimes (E_r F(\gamma^*) Q(\gamma^*)^{-1} + E_r^- H(\gamma^*))x]\}. \tag{10}$$

Multiplying (7) from both sides by Q_j^{-1}, one can get

$$\Pi = He[Q_j^{-1} A + \lambda_i Q_j^{-1} B(E_r F_j Q_j^{-1} + E_r^- H_j)] - \sum_{k=1}^{K} \sigma_{rjk} Q_j^{-1}(Q_k - Q_j) Q_j^{-1} \leq 0. \tag{11}$$

The consensus of system (9) is analyzed in two cases. Firstly, let us consider $x \in \mathcal{O}_j$ for $j \in I[1, K]$. Then $V(x) = V(\tilde{y}) = \tilde{y}^{\mathrm{T}}(\Lambda \otimes Q_j^{-1})\tilde{y}$ and γ^* is a vector whose jth element is 1 and the rest are zeros. Then, we have $F(\gamma^*) Q(\gamma^*)^{-1} = F_j Q_j^{-1}$, $H(\gamma^*) = H_j$ and $V_x(x) = V_{\tilde{y}}(\tilde{y}) = 2(\widetilde{\Lambda} \otimes Q_j^{-1})\tilde{y}$. The following equality holds for any $r \in I[1, 2^m]$:

$$\begin{aligned}\dot{V}_r(x) =& \dot{V}_r(\widetilde{y}) \\ =& 2\widetilde{y}^{\mathrm{T}}(\widetilde{\Lambda} \otimes Q_j^{-1})[(I_{N-1} \otimes A) + (I_{N-1} \otimes B)(\widetilde{\Lambda} \otimes (E_r F_j Q_j^{-1} + E_r^- H_j))]\widetilde{y} \\ =& 2\widetilde{y}^{\mathrm{T}}[\widetilde{\Lambda} \otimes Q_j^{-1}A + \widetilde{\Lambda}^2 \otimes Q_j^{-1}B(E_r F_j Q_j^{-1} + E_r^- H_j)]\widetilde{y} \\ =& \sum_{i=2}^{N} \lambda_i y_i^{\mathrm{T}} He[Q_j^{-1}A + \lambda_i Q_j^{-1}B(E_r F_j Q_j^{-1} + E_r^- H_j)]y_i.\end{aligned} \tag{12}$$

It follows from (11) and (12) that

$$\begin{aligned}\dot{V}_r(x) =& \sum_{i=2}^{N} \lambda_i y_i^{\mathrm{T}} He[Q_j^{-1}A + \lambda_i Q_j^{-1}B(E_r F_j Q_j^{-1} + E_r^- H_j)]y_i \\ \leq& \sum_{i=2}^{N}\sum_{k=1}^{K} \lambda_i \sigma_{rjk} y_i^{\mathrm{T}} Q_j^{-1}(Q_k - Q_j)Q_j^{-1} y_i.\end{aligned} \tag{13}$$

According to Lemma 1, $\sum_{i=2}^{N} \lambda_i y_i^{\mathrm{T}} Q_j^{-1}(Q_k - Q_j)Q_j^{-1} y_i \leq 0$, it can be obtained that $\dot{V}_r(x) \leq 0$. Since $r \in I[1, 2^m]$, it means that $\dot{V}(x) \leq \underset{r \in I[1,2^m]}{max} \dot{V}_r(x) \leq 0$.

In the following part, the case where $x \in \partial L_V$ will be considered. $\widetilde{\gamma}^*(\widetilde{y})$ is defined as the optimal parameters of $V(\widetilde{y})$ and $\gamma^*(x) = \widetilde{\gamma}^*(\widetilde{y})$. Based on Lemma 2, $\gamma_j^* > 0$ for $j \in I[1, K_0]$ and $\gamma_j^* = 0$ for $j \in I[K_0+1, K]$, we have $V_x(x) = V_{\widetilde{y}}(\widetilde{y}) = 2(\widetilde{\Lambda} \otimes Q(\widetilde{\gamma}^*(\widetilde{y}))^{-1})\widetilde{y}$, $F(\widetilde{\gamma}^*(\widetilde{y}))Q(\widetilde{\gamma}^*(\widetilde{y}))^{-1} = \sum_{j=1}^{K_0} \gamma_j^* F_j Q_j^{-1}$, $H(\widetilde{\gamma}^*(\widetilde{y})) = \sum_{j=1}^{K_0} \gamma_j^* H_j$, and $\widetilde{y} = \sum_{i=2}^{N}\sum_{j=1}^{K_0} \lambda_i \gamma_j^* \widetilde{y}_{ij}$ for $\widetilde{y}_{ij} \in \mathscr{O}_j$. The operator $\dot{V}_r(x)$ is computed by

$$\begin{aligned}\dot{V}_r(x) =& \dot{V}_r(\widetilde{y}) \\ =& 2\sum_{j=1}^{K_0} \gamma_j^* \widetilde{y}_j^{\mathrm{T}}(\widetilde{\Lambda} \otimes Q_j^{-1})[(I_{N-1} \otimes A) + (I_{N-1} \otimes B)(\widetilde{\Lambda} \otimes (E_r F_j Q_j^{-1} + E_r^- H_j))]\widetilde{y}_j \\ =& 2\sum_{j=1}^{K_0} \gamma_j^* \widetilde{y}_j^{\mathrm{T}}[\widetilde{\Lambda} \otimes Q_j^{-1}A + \widetilde{\Lambda}^2 \otimes Q_j^{-1}B(\widetilde{\Lambda} \otimes (E_r F_j Q_j^{-1} + E_r^- H_j))]\widetilde{y}_j \\ =& \sum_{i=2}^{N}\sum_{j=1}^{K_0} \lambda_i \gamma_j^* y_{ij}^{\mathrm{T}} He[Q_j^{-1}A + \lambda_i Q_j^{-1}B(E_r F_j Q_j^{-1} + E_r^- H_j)]y_{ij}.\end{aligned} \tag{14}$$

It follows from (11) and (14) that

$$\begin{aligned}\dot{V}_r(x) =& \sum_{i=2}^{N}\sum_{j=1}^{K_0} \lambda_i \gamma_j^* y_{ij}^{\mathrm{T}} He[Q_j^{-1}A + \lambda_i Q_j^{-1}B(E_r F_j Q_j^{-1} + E_r^- H_j)]y_{ij} \\ \leq& \sum_{i=2}^{N}\sum_{j=1}^{K_0}\sum_{k=1}^{K} \sigma_{ijk} \lambda_i \gamma_j^* y_{ij}^{\mathrm{T}} Q_j^{-1}(Q_k - Q_j)Q_j^{-1} y_{ij}.\end{aligned} \tag{15}$$

Since $\sigma_{ijk} > 0$, $\gamma_j^* > 0$ and $\sum_{i=2}^{N} \lambda_i y_i^{\mathrm{T}} Q_j^{-1}(Q_k - Q_j)Q_j^{-1} y_i \leq 0$, then $\dot{V}_r(x) \leq 0$. Thus,$\dot{V}(x) \leq \max_{r \in I[1,2^m]} \dot{V}_r(x) \leq 0$. Therefore, the proof is completed. □

In the sequel, we will determine the maximal L_V from all the ellipsoids satisfying the condition of set invariance, and take the maximal L_V as the estimate of DC. According to the definition of L_V, the range of invariant set L_V is related to $\varepsilon(\mathcal{L} \otimes Q_j^{-1})$ with $\varepsilon(\mathcal{L} \otimes Q_j^{-1}) = \{x \in R^n : x^{\mathrm{T}}(\mathcal{L} \otimes Q_j^{-1})x \leq 1\}$. Thus,

$$\varepsilon_{\widetilde{y}}(\widetilde{\Lambda} \otimes Q_j^{-1}) = \{\widetilde{y} \in R^n : \widetilde{y}^{\mathrm{T}}(\widetilde{\Lambda} \otimes Q_j^{-1})\widetilde{y} \leq 1\} = \varepsilon(\mathcal{L} \otimes Q_j^{-1}). \tag{16}$$

In what follows, we take the size of $\varepsilon_{\tilde{y}}(\tilde{\Lambda} \otimes Q_j^{-1})$ into consideration. In order to maximize the ellipsoid $\varepsilon_{\tilde{y}}(\tilde{\Lambda} \otimes Q_j^{-1})$, a reference set $R = co\{r_1, r_2, \cdots, r_q\}$ with $r_p \in R^{(N-1)n}$, $p \in I[1, q]$ is given. Let $\varepsilon_{\tilde{y}}(\tilde{\Lambda} \otimes Q_j^{-1})$ contain αR with the largest α possible, the maximal estimate of DC can be obtained. Thus, the optimal problem is formulated as follows:

$$\begin{aligned} &\sup_{Q_j>0, \sigma_{rjk}>0, F_j, H_j} \alpha, \\ &s.t\ (a)\ Conditions\ (7)\ and\ (8), \\ &(b)\ \alpha R \subseteq \varepsilon_{\tilde{y}}(\tilde{\Lambda} \otimes Q_j^{-1}). \end{aligned} \tag{17}$$

Using the result in [19], the condition (b) can be transformed into

$$\begin{bmatrix} 1 & \alpha r_p^{\mathrm{T}} \\ \alpha r_p & \tilde{\Lambda}^{-1} \otimes (\sum_{k=1}^{K} \gamma_k Q_k) \end{bmatrix} \geq 0. \tag{18}$$

While the condition (8) is equivalent to $l_{ii} h_{jq} Q_j h_{jq}^{\mathrm{T}} \leq 1, i \in I[1, N], j \in I[1, K], q \in I[1, m]$, where l_{ii} is the ith diagonal element of $\mathcal{L}$ and h_{jq} is the qth row of H_j. Let $l_{max} = max\{l_{ii} : i \in I[1, N]\}$, it is obvious that $l_{ii} h_{jq} Q_j h_{jq}^{\mathrm{T}} \leq l_{max} h_{jq} Q_j h_{jq}^{\mathrm{T}} \leq 1$, and it can be rewritten as

$$\begin{bmatrix} l_{max}^{-1} & z_q \\ z_q^{\mathrm{T}} & Q_j \end{bmatrix} \geq 0,\ j \in I[1, K], q \in I[1, m], \tag{19}$$

where z_q is the qth row of $H_j Q_j$. Then the optimal problem (17) can be reformulated as

$$\begin{aligned} &\sup_{Q_j>0, \sigma_{rjk}>0, F_j, H_j} \alpha, \\ &s.t.\ Inequalities\ (7),\ (18)\ and\ (19). \end{aligned} \tag{20}$$

3.2. Linear Feedback for Regional Consensus

If the consensus control protocol is in the form of linear feedback in [17–19], i.e.,

$$u_i = \sum_{l=1}^{N} a_{il} F(x_i - x_l),\ i \in I[1, N], \tag{21}$$

the following theorem is given. In view of Lemma 3, the saturated control protocol is designed by

$$\mathcal{N}_{sat}(u) \in co\{\mathcal{L} \otimes (E_r F + E_r H)x,\ r \in I[1, 2^m]\}. \tag{22}$$

Theorem 2. *Let $Q_j \in R^{n \times n} > 0$ for $j \in I[1, K]$. If there exist $F, H \in R^{m \times n}$, $\sigma_{rjk} \geq 0$ for $r \in I[1, 2^m]$, $j, k \in I[1, K]$, such that*

$$He[AQ_j + \lambda_i B(E_r F + E_r^- H)Q_j] - \sum_{k=1}^{K} \sigma_{rjk}(Q_k - Q_j) \leq 0, \tag{23}$$

$$\varepsilon(\mathcal{L} \otimes Q_j^{-1}) \subset \mathscr{L}(\mathcal{L} \otimes H). \tag{24}$$

Then, the consensus of MAS (2) can be achieved under the protocol (22) and the invariant set L_V is contained in DC.

Proof. Theorem 2 is a special case of Theorem 1, and its proof is omitted here. □

Next, similar to the technique in Section 3.1, the optimization problem for maximizing the estimate of DC can be formulated as follows:

$$\begin{aligned} &\sup_{Q_j>0,\sigma_{rjk}>0,F,H} \quad \alpha, \\ &s.t\ (a)\ Conditions\ (23)\ and\ (24), \\ &(b)\ \alpha R \subseteq \varepsilon_{\widetilde{y}}(\widetilde{\Lambda} \otimes Q_j^{-1}). \end{aligned} \tag{25}$$

Meanwhile, let z_q be the qth row of HQ_j and $l_{max} = max\{l_{ii} : i \in I[1, N]\}$, the optimal problem (25) has the following form:

$$\begin{aligned} &\sup_{Q_j>0,\sigma_{rjk}>0,F,H} \quad \alpha, \\ &s.t.\ (a)\ Inequality\ (23), \\ &(b) \begin{bmatrix} 1 & \alpha r_p^{\mathrm{T}} \\ \alpha r_p & \widetilde{\Lambda}^{-1} \otimes (\sum_{k=1}^{K} \gamma_k Q_k) \end{bmatrix} \geq 0, \\ &(c) \begin{bmatrix} l_{max}^{-1} & z_q \\ z_q^{\mathrm{T}} & Q_j \end{bmatrix} \geq 0. \end{aligned} \tag{26}$$

3.3. The Improved Results

In this subsection, an alternative convex hull representation is used to obtain less conservative estimate of DC than that resulting from Theorem 1.

Lemma 4 ([24])**.** *If there exist $F, H_r \in R^{m\times n}$, and $x \in \mathscr{L}(H_r)$ for $r \in I[1, 2^m]$, then*

$$\mathscr{N}_{sat}(u) \in co\{E_r F x + E_r^- H_r x : r \in I[1, 2^m]\},$$

where $\mathscr{L}(H_r) = \{x \in R^n, |H_r x|_\infty \leq 1\}$.

In view of Lemma 4, it follows from the control protocol (4) that

$$\mathscr{N}_{sat}(u) \in co\{\mathcal{L} \otimes [(E_r F(\gamma^*) Q(\gamma^*)^{-1} + E_r^- H_r(\gamma^*)]x),\ r \in I[1, 2^m]\}. \tag{27}$$

Remark 3. *Protocol (27) is a more general form of protocol (5). By applying the alternative convex hull representation, each vertex of the convex hull in protocol (27) is configured with an independent auxiliary matrix, H_r. Therefore, each subfunction of CLQF contains different auxiliary matrices, and a set of less conservative consensus conditions can be obtained.*

Theorem 3. *Let $Q_j \in R^{n\times n} > 0$ for $j \in I[1, K]$. If there exist $F_j, H_{rj} \in R^{m\times n}$, $\sigma_{rjk} \geq 0$ for $r \in I[1, 2^m]$, $j, k \in I[1, K]$, such that*

$$He[AQ_j + \lambda_i B(E_r F_j + E_r^- H_{rj} Q_j)] - \sum_{k=1}^{K} \sigma_{rjk}(Q_k - Q_j) \leq 0, \tag{28}$$

$$\varepsilon(\mathcal{L} \otimes Q_j^{-1}) \subset \mathscr{L}(\mathcal{L} \otimes H_{rj}). \tag{29}$$

Then, the consensus of MAS (2) can be achieved under protocol (27) and the invariant set L_V is contained in DC.

Proof. Theorem 3 is a general case of Theorem 1, and its proof is omitted here. □

Similarly, the optimization problem for maximizing the estimate of DC is formulated as follows:

$$\begin{aligned} &\sup_{Q_j>0,\sigma_{rjk}>0,F,H} \quad \alpha, \\ &s.t\ (a)\ Conditions\ (28)\ and\ (29), \\ &(b)\ \alpha R \subseteq \varepsilon_{\widetilde{y}}(\widetilde{\Lambda} \otimes Q_j^{-1}). \end{aligned} \tag{30}$$

Let z_q be the qth row of $H_{rj}Q_j$ and $l_{max} = max\{l_{ii} : i \in I[1, N]\}$, the optimization problem (30) is equivalent to

$$\begin{aligned} &\sup_{Q_j>0,\sigma_{rjk}>0,F_j,H_{rj}} \quad \alpha, \\ &s.t.\ (a)\ Inequality\ (28), \\ &(b) \begin{bmatrix} 1 & \alpha r_p^{\mathrm{T}} \\ \alpha r_p & \widetilde{\Lambda}^{-1} \otimes (\sum_{k=1}^{K} \gamma_k Q_k) \end{bmatrix} \geq 0, \\ &(c) \begin{bmatrix} l_{max}^{-1} & z_q \\ z_q^{\mathrm{T}} & Q_j \end{bmatrix} \geq 0. \end{aligned} \tag{31}$$

4. Case Simulation

Consider the mechanical model of MAS (1) proposed in [19,25], and the system matrices are given as follows:

$$A = \begin{bmatrix} 0 & 2 \\ -3 & 0.4 \end{bmatrix}, \ B = \begin{bmatrix} 3 & 1 \\ -0.7 & -1 \end{bmatrix}.$$

The communication topology graph of MAS is shown in Figure 1, and the corresponding adjacency matrix $\mathcal{A}$ and Laplacian matrix $\mathcal{L}$ are computed as

$$\mathcal{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix}, \ \mathcal{L} = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ -1 & 3 & -1 & 0 & -1 \\ 0 & -1 & 3 & -1 & -1 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & -1 & -1 & -1 & 3 \end{bmatrix}.$$

The reference set is selected as $r_p = [0.05,\ 0.1,\ 0.05,\ 0.1,\ 0.05,\ 0.1,\ 0.05,\ 0.1]^{\mathrm{T}}$. The optimization problem (20) is not a convex optimization problem. It is difficult to obtain the global optimal solution of the BMI optimization problem. A second way is to solve the BMI through an iterative LMI. Here, we will use the path-following method in [26] to solve the problem. The core idea of the path-following method is as follows: given the known well-dimensioned matrices A and B, if the matrix norm of the unknown well-dimensioned matrices A_1 and B_1 is far less than the corresponding matrix norm of A and B, then $(A + A_1)(B + B_1) \approx AB + AB_1 + A_1B$. The left side of the approximately equal sign can be seen as the product of two unknown matrices, and the right side is the sum of several matrix products containing at least one unknown matrix, thus solving the problem of the product of two unknown matrices. The original BMI optimization problem can be reduced to the LMI problem by using the approximate form on the right. Then, we can obtain a level set with $\alpha = 7.2463$ and

$$Q_1 = \begin{bmatrix} 6.2244 & 1.0806 \\ 1.0806 & 7.7985 \end{bmatrix}, \ Q_2 = \begin{bmatrix} 6.2697 & 1.1890 \\ 1.1890 & 8.1780 \end{bmatrix}.$$

In order to compare with the linear feedback method of Theorem 3.2, we solve the optimal problem (26) and obtain that $\alpha = 6.8296$ and

$$Q_1 = \begin{bmatrix} 5.7701 & 1.0591 \\ 1.0591 & 7.2342 \end{bmatrix}, \quad Q_2 = \begin{bmatrix} 5.8205 & 1.1505 \\ 1.1505 & 7.5647 \end{bmatrix}.$$

As a further improvement of Theorem 3.1, Theorem 3.3 gives a level set with $\alpha = 7.6325$ and

$$Q_1 = \begin{bmatrix} 7.0771 & 1.2472 \\ 1.2472 & 8.8677 \end{bmatrix}, \quad Q_2 = \begin{bmatrix} 7.1310 & 1.3673 \\ 1.3673 & 9.2886 \end{bmatrix}.$$

If the LQF (the method in [19,25]) is applied, we obtain an ellipsoid with $\alpha = 5.4374$ and

$$Q = \begin{bmatrix} 3.6565 & 0.7246 \\ 0.7246 & 4.6370 \end{bmatrix}.$$

To compare these results, the level set of agent 1 is simulated and the rest of agents are similar to agent 1. Figure 2 shows four level sets obtained by different approaches. The result obtained by CLQF (Theorem 2) has a larger DC estimate than the results in [19,25]. Moreover, the nonlinear control protocol presented in this paper (Theorem 1) provides a larger ellipsoidal estimate. As a result of improvement, the estimate of DC obtained by the alternative convex hull representation (Theorem 3) is the largest. In order to verify the designed control protocol, the initial state $x(0) = [-0.1, -0.2, 0.3, 0.35, -0.15, 0.15, -0.4, 0.55, -0.1, -1]^{\mathrm{T}}$ is taken into account. Solving the optimal problem (20), the control gain matrices F_j and auxiliary gain matrices H_j are obtained as follows:

$$F_1 = \begin{bmatrix} -0.2257 & 0.0130 \\ -0.0788 & 0.2700 \end{bmatrix},$$
$$F_2 = \begin{bmatrix} -0.2279 & 0.0048 \\ -0.0963 & 0.2734 \end{bmatrix},$$

$$H_1 = \begin{bmatrix} -0.1934 & 0.0198 \\ -0.0470 & 0.1812 \end{bmatrix},$$
$$H_2 = \begin{bmatrix} -0.2002 & 0.0072 \\ -0.0655 & 0.1661 \end{bmatrix}.$$

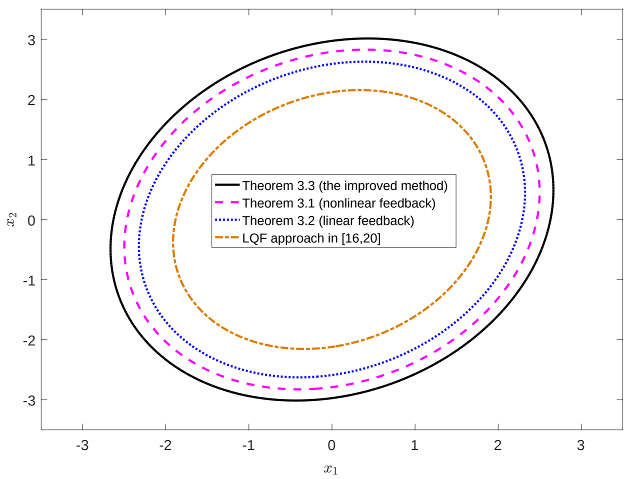

Figure 2. The invariant sets of the CLQF (derived by Theorems 1–3) and the LQF (derived by the method in [25,26]).

The state response errors between agent 1 and agent i are defined by $[e_{1i1}, e_{1i2}]^T$, and these simulation results are presented in Figures 3 and 4. Meanwhile, the responses of saturated input u_{i1} and u_{i2} are depicted in Figures 5 and 6. It is obvious that the state errors of agents and the saturated inputs gradually converge to zero.

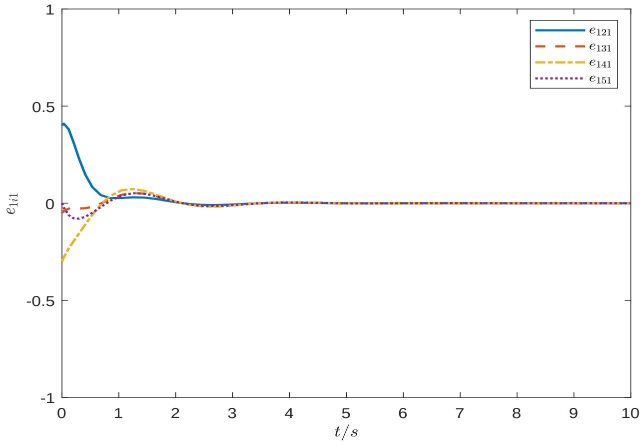

Figure 3. The state response errors e_{1i1} under the initial state x_0 and the control input u_i.

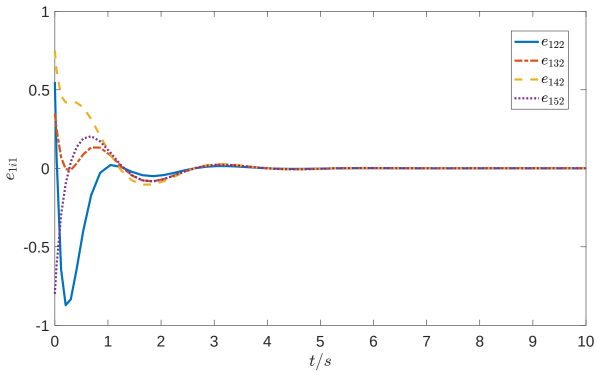

Figure 4. The state response errors e_{1i2} under the initial state x_0 and the control input u_i.

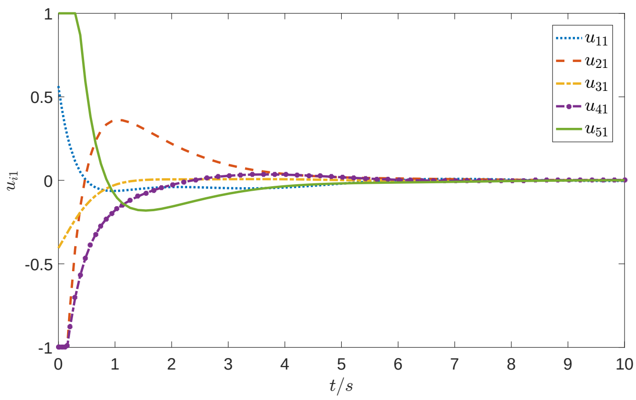

Figure 5. The responses of saturated input u_{i1} on the boundary of $\varepsilon(\mathcal{L} \otimes Q_j^{-1})$.

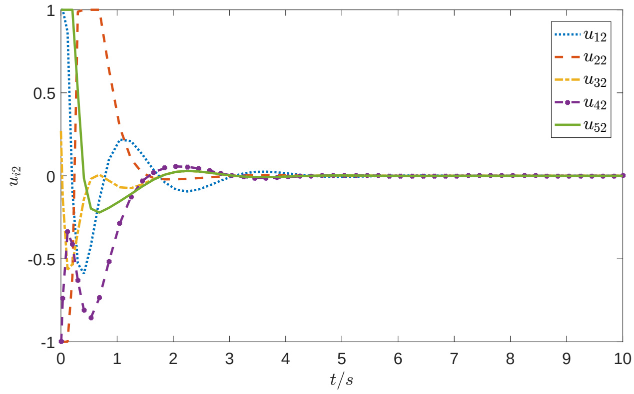

Figure 6. The responses of saturated input u_{i2} on the boundary of $\varepsilon(\mathcal{L} \otimes Q_j^{-1})$.

5. Conclusions

In this paper, the authors address the consensus problem of MASs with actuator saturation. Firstly, a novel multiple nonlinear control protocol is designed, and the CLQF is constructed to guarantee the consensus of MASs. Then, the maximal DC estimate is derived by an optimization problem in terms of bilinear matrix inequalities. To compare with the traditional consensus control protocol in [19,25], Theorem 2 is presented. Moreover, an alternative convex hull representation is applied to further improve the designed consensus control protocol. Finally, the simulation results reveal that the proposed control protocol makes all agents achieve consensus and provides a larger estimate of DC than the approaches in [19,25].

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