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Review

A Literature Review of Railway Pricing Based on Revenue Management

Xueyi Guan [1], Jin Qin [1,2,*], Chenghui Mao [1] and Wenliang Zhou [1,2]

[1] School of Traffic & Transportation Engineering, Central South University, Changsha 410075, China
[2] Rail Data Research and Application Key Laboratory of Hunan Province, Changsha 410075, China
* Correspondence: qinjin@csu.edu.cn

Abstract: In recent decades, railway passenger transport enterprises have been exploring numerous operation and management strategies to improve service quality and market competitiveness of railway passenger transport so as to ensure that the interests of railway passenger transport enterprises are maximized when taking social welfare into account. However, there are still shortcomings in the current research with respect to determining the pricing mechanism and formulating a reasonable price. This paper systematically reviews the scientific literature related to railway pricing, focusing on the application of basic price methods, mathematical programming methods, and data-driven methods in railway pricing, with the hope of proposing an innovative direction to solve existing problems. The main subjects involved in the formulation of railway pricing are passenger groups and transportation companies. The research can be conducted from four broad aspects: passenger demand, passenger time value, market segmentation, and the equilibrium relationship between rail service supply and passenger demand. On the basis of absorbing and summarizing the strengths and weaknesses of previous studies, this paper puts forward suggestions for improvement and innovative directions which will help promote railway passenger transport services from the perspective of pricing, thereby enhancing the sustainability of railway transport.

Keywords: revenue management; railway pricing; dynamic pricing; differentiated pricing; data-driven

MSC: 49M99; 90B50; 90C90

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1. Introduction

Since human society entered the age of industrialization, the research of railway pricing in railway operation management has received extensive attention and has achieved great results. In the current era of information and digitalization, the research on railway pricing has also gradually exposed some urgent problems, such as digital dynamic pricing. Therefore, this paper systematically sorts out, summarizes, and reviews the research status of railway pricing in the past 30 years in order to provide reference and inspiration for subsequent researchers.

Passenger fare revenue is the main source of economic income for railway operating departments. Considering the complexity of railway pricing, the formulation of railway prices must take into account the interests of all aspects of society. The pricing rule based on revenue management is a profit maximization method for enterprises to improve their own and social benefits, which has been widely used in hotels (e.g., Bitran and Mondschein (1995) [1] and Bitran and Gilbert (1996) [2]), electricity (e.g., Schweppe et al. (1987) [3] and Oren and Smith (1993) [4]), retail (e.g., Bitran and Mondschein (1997) [5] and Subrahmanyan and Shoemaker (1996) [6]), transportation (e.g., Ciancimino et al. (1999) [7] and Kasilingam (1997) [8]), and other fields. The earliest pricing rules were fixed pricing by operators based on a basic price system, but with the continual development of the market, the simple pricing mechanism could no longer meet the needs of the market and the country.

For this reason, Cournot [9], in 1983, used mathematical functions to establish a product price–demand model and calculated the optimal pricing in order to expose the internal evolution law of pricing; however, his method was considered static pricing. Since then, in order to deal with time-varying demand and complex network issues, researchers have proposed dynamic pricing strategies, differentiated pricing strategies, and collaborative optimization strategies in consideration of product availability and timeliness. The goal was to improve the railway price formation mechanism and build a diversified pricing system, and this system has made a huge contribution.

Recently, traffic engineers and traffic economists have paid increasingly more attention to the research on railway pricing. Traffic engineers have continually improved the construction of railway passenger transport service facilities and mastered the operation rules of the traffic system through modeling. In addition, transportation economists build models mainly through mathematical theoretical knowledge in order to obtain comprehensive and optimal railway operating enterprise benefits and social welfare. After reviewing the literature on railway pricing in recent years, we found that revenue management has become one of the main research methods. As a classic method in economics, revenue management is a management mode that implements various price standards to customers by establishing a real-time forecasting model and analyzing demand behavior on the basis of market segmentation. It can translate the right data into clear and tangible actionable recommended decisions that enable clients to price, forecast, and report quickly and with confidence, leading to improved business performance. Revenue management is widely used in coordination and pricing problems, and the key to the pricing problem lies in the construction of participants, influencing factors, and models. This paper uses content analysis and bibliometric methods to review the railway pricing on the basis of revenue management and summarizes and evaluates the literature from three aspects: methodology, pricing strategy, and influencing factors. The aim is to determine the decision-making and pricing model construction of railway pricing through a literature review so as to provide scientific and practical methods for subsequent railway pricing researchers, to help them provide new ideas for railway pricing innovation, and to create new mathematical models.

The writing framework of the article is as follows. In the second section, we briefly introduce the methods of the literature review, mainly including research questions, keywords, and literature selection criteria. In Section 3, we categorize in detail the research methods on the basis of revenue management, including the basic price system, the mathematical programming method, and the data-driven method. The mathematical programming method concentrates on the detailed discussion of the pricing considering the characteristics of dynamic pricing, time–space differentiation, and market segmentation. Finally, some conclusions and suggestions for future railway pricing research are presented.

2. Methodology

In order to ensure the objectivity of the research results and the repeatability of the research, this paper systematically reviews the research on railway pricing on the basis of revenue management in the past 30 years by referring to the method of a structured literature review [10]. The research methodology of the paper includes three main stages: (1) research planning; (2) examination paper identification and analysis; and (3) research evaluation and synthesis. In the first stage, the research questions and the scope of the literature review are determined, and then the literature search plan is designed to determine the inclusion and exclusion criteria of the literature. The second stage conducts preliminary quantitative research and descriptive analysis on the selected literature and evaluates and classifies the literature according to the research questions. In the third stage, a comprehensive review of the literature is performed. The literature is evaluated and analyzed using content analysis and bibliometric methods to identify gaps in previous research and to identify future research trends. As mentioned above, railway pricing is a key issue affecting the sustainability of rail services. After identifying the need for such a review and research gaps, we set three main research questions (RQ 1, RQ 2, and RQ 3):

RQ 1: What are the main players and key research scenarios of railway pricing?
RQ 2: Which pricing management strategies and methods are optimal?
RQ 3: What are the main and most important factors affecting railway prices?

To answer the posed research questions, we conducted a comprehensive review of scientific papers in the field of railway pricing. Utilization of clear literature selection criteria minimized investigator inclusion and exclusion bias and increased data heterogeneity. The literature was mainly from Web of Science, SCOPUS, and X-mol databases; these three databases are leading and extensive citation databases, covering most of the railway pricing literature. We conducted a subject search in the database using keywords such as "railway", "revenue management", and "pricing", and we identified a total of 397 papers. Papers were evaluated and selected through a two-step screening process. First, we further reviewed the titles, abstracts, and keywords of the papers according to the inclusion and exclusion criteria, and a total of 121 papers were considered relevant to the topic. Second, we performed a full-text read, followed by a snowball search of their references, applying the same inclusion and exclusion criteria. Ultimately, 57 papers were retained for review in this study. The literature selection process is shown in Figure 1. The inclusion and exclusion criteria of the literature are listed in Table 1.

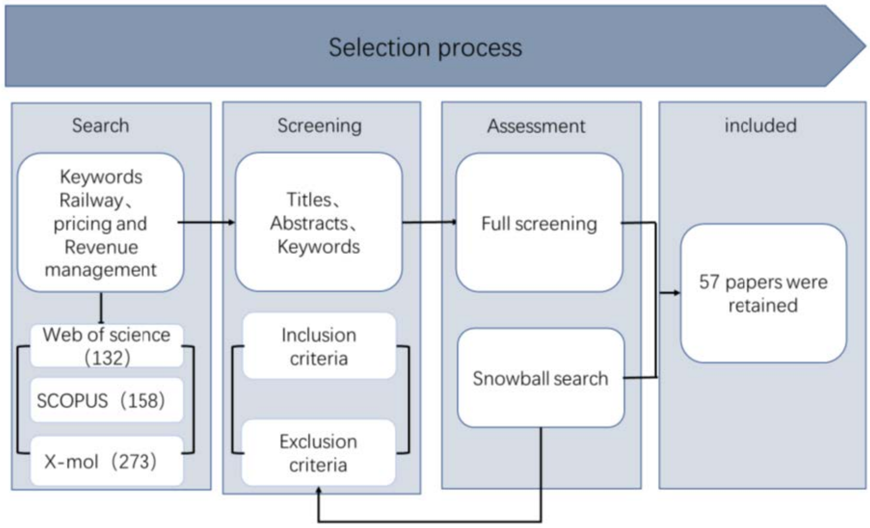

Figure 1. Literature selection process.

Table 1. The inclusion and exclusion criteria.

Inclusion Criteria	Exclusion Criteria
Full journals and conference proceedings	Lectures, grey literature, and presentations
English language	Non-English language
Peer-reviewed	Not peer-reviewed
The method of revenue management is used to study railway pricing	The paper only mentions railway, revenue management, or prices as one of the important aspects, but does not carry out specific analysis and research
Railway price is a decision variable which is not endogenous to the system	Railway price is an exogenous variable

Figure 2 presents the literature publication and citation report in Web of Science searched by using the railway pricing and revenue management. As we can see, from 2000 to 2022, the number corresponding to the height of the histogram in the figure represents the number of publications in that year. In total, there were 72 articles published, and the number of articles published after 2016 increased sharply, indicating that research on railway pricing was becoming increasingly favored by researchers. The blue line represents the number of citations per year, which also shows a trend of increasing year by year, with a sharp increase after 2016, which further shows that railway pricing is receiving increasingly more attention.

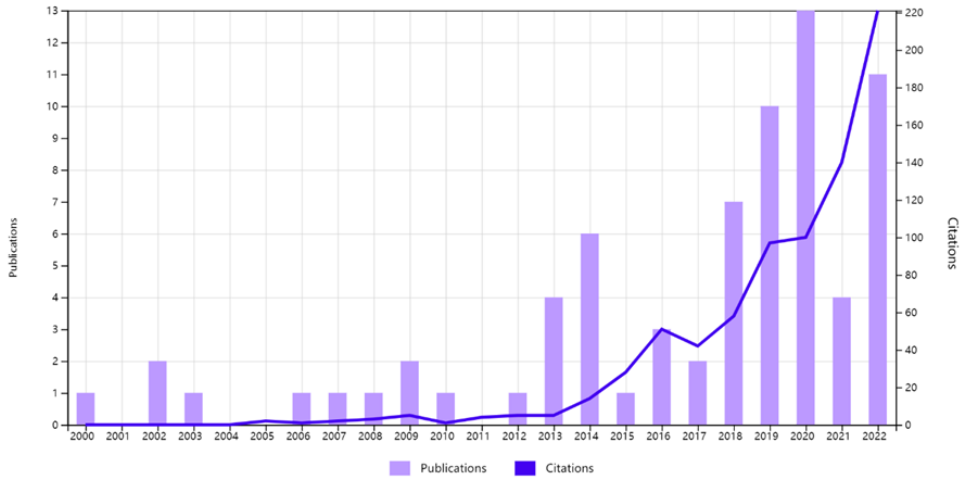

Figure 2. Literature publication and citation report in Web of Science.

3. Literature Classification and Analysis

The essence of the revenue management (RM) pricing method is that operating companies regard revenue as the operating goal. The cost is introduced into the mathematical model as a variable to provide accurate forecasts, improve pricing decisions, and provide in-depth business insights for operating companies to help them achieve revenue improvement, including improving revenue management operations and fostering sustainable profitability. Revenue management theory was initially applied in the aviation field and was then gradually introduced into other transportation fields. Revenue management pricing is a dynamic pricing method that considers the impact of current passenger flow on the next period's pricing; hence, it is also called a real-time pricing scheme. The dynamic adjustment of railway pricing based on the revenue management method includes three complete steps: first, predicting short-term passenger flow; second, allocating tickets during the pre-sale period and establishing a passenger ticket allocation model; and finally, adding a dynamic ticket adjustment mechanism to adjust the price of previously allocated tickets.

Revenue management theory can provide industry-leading revenue management solutions for businesses of all types and sizes in the global hotel and tourism industry. In 1999, when revenue management theory was first introduced into the research of railway passenger transport pricing, it was widely accepted and formed a complete mathematical formula [7]. It involves not only ticket pricing but also capacity allocation, route optimization, timetable optimization, and other considerations. Revenue management was applied primarily in two price structures: fixed pricing and dynamic pricing. Under fixed pricing, revenue management methods consider mainly how capacity allocation balances limited supply and fluctuating demand. However, it considers only pricing linked to seat control not the possibility of demand shifting between similar time slots or between different classes of fares. Under dynamic pricing, revenue management methods focus on how to use differentiated pricing schemes to meet different passenger needs. In other words, the dynamic pricing model (i.e., the model pricing of products according to the market demand for products and the purchasing power of customers) is based on the idea that revenue management can solve the problem of demand transfer. Two models within the area of dynamic pricing are (1) multi-layer ticket price and multi-segment-trip model and (2) the single-trip dynamic pricing model [11]. In other words, it can construct a differentiated itinerary model, formulating differentiated fares according to the differentiated needs of passengers so as to improve the flexibility of railway pricing.

3.1. Basic Price Method

Early studies on revenue management focused more on capacity management and overbooking; railway pricing was usually based on a basic price, with managers typically maximizing revenue by opening and closing different price levels. In order to cope with long-distance passenger transport and to address overbooking, seat allocation, and network flow management, Amtrak developed the ARROW revenue management system and achieved an average additional revenue of 3–5% [12]. French National Railways (SNCF), in the early 1990s, cooperated with SABER to design, develop, and implement the railway ticket booking, distribution system, and comprehensive decision support system, which included the revenue management system (RailRev), timetable preparation system (RailPlus), and seat management system (RailCap) [13]. These revenue management systems were then utilized to increase revenue by increasing attendance. Deutsche Bahn (DBAG) established a new revenue management system (PET) for railway transportation on the basis of fundamental price and other services [12]. This system was comparable to low-cost air transportation, whereby they provided preferential tickets and multi-level discount tickets for passengers who booked in advance. These revenue management systems based on a basic price system have greatly improved the revenue of railway operators. They have, however, ignored the principle that price is one of the most effective means to adjust demand in a short period of time. Traditional research focuses, such as capacity optimization, stock management, and other issues, are inseparable from price decisions.

Table 2 shows the application of existing revenue management systems in the railway transportation industry and revenue management systems in different countries.

Table 2. The existing revenue management systems.

	United States	**France**	**Germany**
Operator	Amtrak	SNCF	DBAG
Development time	1991	1993	2002
Development purpose	Increase the attendance rate of long-distance lines and increase the income of busy lines	Improve off-peak occupancy and increase revenue on busy lines	Alleviate peak hour congestion
Present situation	Expanded	Simplified	Simplified

3.2. Mathematical Programming Method

The mathematical programming method establishes a mathematical model through the basic information provided by the investigation. It reflects the relationship between pricing activities and other economic factors and obtains alternative solutions by means of computer technology. The model then reveals the impact of pricing activities on various policies, thus providing several options. The development of many factors in the pricing system is restricted by both objective factors and subjective factors of traffic participants. To determine a scientific pricing mechanism is to specifically determine the optimal subjective control variables in the pricing structure system so as to optimize the overall goal. Among them, dynamic pricing strategy, differentiated pricing strategy, and collaborative optimization strategy are the most widely studied methods.

3.2.1. Dynamic Pricing Strategy

The dynamic pricing strategy is a pricing method that is derived from the static pricing strategy to overcome the shortcomings of that method. Dynamic pricing means that the seller dynamically adjusts the commodity price over time on the basis of information, such as sales time, demand information, and commodity inventory. Dynamic pricing is also the most common pricing strategy for civil aviation and railway transportation, and its theoretical basis is dynamic programming theory. The advantage of dynamic programming is that, as the supply and demand change, it can calculate the optimal ticket price under any state (that is, the combination of the number of remaining seats and the remaining pre-sale time); thus, it is also known as real-time pricing (RTP) [14]. Although applied to other fields, Kincaid and Darling [15] were the first to study the problem of dynamic pricing of perishable products in continuous time. As the research on the related theories and methods of dynamic pricing became more mature, many scholars began to introduce the dynamic pricing method into the railway field to improve the pricing system of railway passenger transport.

Whelan et al. [16], Chang [17], and Sibdari et al. [18] proposed a pricing model for congestion and travel time on railways in the early peak hours, and they used real data to verify the fare difference between peak and off-peak hours. The rationality and applicability of the value show that the adopted pricing method can indeed alleviate the congestion during peak hours. Following is the pricing model proposed by Sibdari et al. [18]; the model successfully addresses issues such as peak hour pricing and ticket limits without reducing passenger demand.

$$Max \quad V\left(t, \bar{t}_i, p_i, p, T, R, \Delta_i^{early}, \Delta_i^{late}\right) \tag{1a}$$

$$subject\ to \;\; px + \sum_{i=1}^{r} d_i p_i = R[\lambda] \tag{1b}$$

$$t + \sum_{i=1}^{r} d_i t_i = T[\mu] \tag{1c}$$

$$t_i \geq \bar{t}_i[k_i] \tag{1d}$$

$$e_i(t_{oi} - t_i) = \Delta_i^{early}[\theta_i] \tag{1e}$$

$$l_i(-t_{oi} + t_i) = \Delta_i^{late}[\vartheta_i] \tag{1f}$$

$$e_i \Delta_i^{early} \leq \Delta_{early}^{*}[\varpi_i] \tag{1g}$$

$$l_i \Delta_i^{late} \leq \Delta_{late}^{*}[\psi_i] \tag{1h}$$

$$\{d_i, e_i, l_i\} = (0,1), \left\{t, \bar{t}_i, p_i, p, T, R, \Delta_i^{early}, \Delta_i^{late}\right\} \geq 0 \tag{1i}$$

where

i—travel choice index ($i = 1 \cdots r$)
V—utility
R—income level (in thousands of dollars)
t—time spent in activities other than travel time (in hours)
t_i—travel time for selected travel choice i (in hours)
$\overline{t_i}$—minimum time requirement to travel on travel choice i (in hours)
t_{oi}—(desired arrival time)-(departure time to travel on travel choice i) (in hours)
T—total available time (in hours)
p_i—cost of travel choice i (in dollars)
p—cost of goods other than travel (in dollars)
x—consumption of goods other than travel
d_i—1 if travel choice i is selected, 0 otherwise
e_i—1 if early arrival is observed when travel choice i is selected, 0 otherwise
l_i—1 if late arrival is observed when travel choice i is selected, 0 otherwise
Δ_{early}^{*}—maximum available early arrival flexibility (in hours)
Δ_{late}^{*}—maximum available late arrival flexibility (in hours)
Δ_i^{early}—early arrival time when travel choice i is selected (in hours)
Δ_i^{late}—late arrival time when travel choice i is selected (in hours)
λ—the Lagrangian multiplier of income constraint

μ—the Lagrangian multiplier of time constraint
$k_i, \theta_i, \vartheta_i, \varpi_i$—the Lagrangian multipliers of the marginal utilities of decreasing the early/late arrival amount by reducing the departure time

Equation (1a) represents the object function considering travel costs, benefits, and early/late arrival times during peak hours. Constraint Equation (1b) indicates that the total cost consists of travel costs and consumption costs other than travel. Similarly, the second constraint (Equation (1c)) indicates that the total time includes travel time, work time, and leisure time, while (1d) ensure that passengers will choose the optimal option in terms of travel time selection. The remaining constraints (Equations (1e)–(1h)) state that the passenger will arrive at the destination early or late within the optimal early/delay range interval. Finally, the individual parameters in Equation (1i) represent the Lagrangian multipliers.

In order to explore the characteristics of the scenarios of travel differences, Ozbay et al. [19] optimized the fare model by taking into account the destination of passengers and the choice of arrival and departure times of trains and used New Jersey railway data to validate the model. Hetrakul [20] further explored the travel choices of passengers and quantified the influence of different travel factors on the travel choices of railway passengers on the basis of the online booking data. Sato and Sawakil [21] used a dynamic programming method to study the dynamic pricing problem of high-speed rail on the basis of the travel choice behavior of passengers. However, the conclusion is still far from solving the actual problem due to limited data. Gama [22] improved the pricing model of the railway system by evaluating the cross-elasticity between American airline and rail fares, and Vuuren [23] used the Ramsey pricing model to prove the relationship between marginal cost and demand price elasticity. Bharill and Rangaraj [24] proposed a pricing model considering overbooking and cancellations on the basis of analyzing demand price elasticity. Cirillo, Hetrakul, and Toobaie [25] used the multinomial logit model (MNL) of the Amtrak Acela Express train to construct a model for passenger choice of booking time. They determined the response of passenger demand to price through linear regression and used a nonlinear model to maximize predicted revenue. Zhang, Lang, and Jin [26] proposed a dynamic pricing model for Chinese train passenger groups. They assumed that booking requests follow a Gaussian distribution and described the size of each group by a Poisson distribution. Then, the authors considered that the effect of fare on the probability of purchase was in line with the logit model.

3.2.2. Differentiated Pricing Strategy

Differentiated pricing originates from price discrimination in economics. Basically, it sets different prices for different commodity attributes and characteristics of substitutable products. With the continual improvement of this pricing method, it has been gradually applied in many fields. The diversified development of the railway industry was impeded by the structure of its ticket prices. Inspired by the differentiated pricing problems of other industries, the research on the difference of railway fares has gradually attracted widespread attention of scholars. Si-Ming Li [27] was one of the first researchers to develop differentiated pricing in railway transportation. Through that research, it was found that it is feasible to apply differentiated pricing, a traffic management method, to a highly automated railway system.

Demand Differentiation

When the supply exceeds the demand in the passenger transport market, the pricing discourse power will shift from the supply-side to the demand-side. Especially when the "inertia" and "asymmetry" of passenger preference are discovered, the research begins to pay attention to the demand elasticity of passenger travel. Demand elasticity describes the relationship between ticket price and passenger volume. Inertia is the cost of searching for other modes of travel, so maintaining the status quo until larger expenditures occur is the best decision. "Asymmetry" is due to the ratchet effect, in which passengers react differently to price increases and decreases of the same magnitude [28].

The application of demand elasticity in railway ticket pricing is more common, which is generally shown as base price plus flexible adjustment price. This type of pricing scheme is actually a price discrimination scheme. Its difficulty lies in the measurement of elasticity of demand. Some scholars use survey data [29] or regression methods to carry out research in this area. Through the demand elasticity analysis of different transport distances, it is found that the longer the distance is, the greater the elasticity is, and the low-price strategy should be adopted for long-distance tickets [30]. Through the demand elasticity analysis of different seasons, passenger demand is flexible with regard to the high season ticket price floating and flexible with regard to the low season ticket price floating; the high season should not raise the price, and the low season should reduce the price [31]. The demand elasticity of middle- and high-income groups is small, whereas that of the low-income group is larger [32]. For example, on the basis of accurately forecasting traffic volume, the demand function is determined, and the demand elasticity of railway passengers is obtained by using the least squares method [33]. Most of the above methods consider linear demand elasticity, yet nonlinear demand elasticity is also involved in the pricing model of high-speed railway passenger transport [34]. Due to the different demand elasticity of different routes, transport distances, short and high seasons, different groups, different seats, and different economic regions, the appropriate ticket prices should be set. Although the elasticity coefficient of passenger demand can be obtained by the above methods, the data are limited by time and space and do not have real-time and representative.

In addition, there is literature combining demand elasticity with other pricing methods. Examples include using price elasticity on the basis of market segmentation to establish a pricing model for high-speed rail tickets [35] or combining the demand elasticity with the marginal cost pricing method to ensure that the purpose of maximizing consumer surplus is achieved under the premise of balance between revenue and expenditure.

Market Segmentation

Market segmentation means that railway operators divide the passenger transport market into several sub-markets according to certain standards. This division standard has two main directions: consumer-oriented and product-oriented [36]. Product-oriented segmentation refers to the subdivision of services in different spaces at the same time, such as the subdivision of business seats, first-class seats, and second-class seats, as well as the subdivision of different time stages in the same space, namely peak and off-peak. The consumer-oriented type of segmentation is to segment the market according to the characteristics of passenger travel and apply a differentiated pricing strategy, including differentiated pricing based on elements such as travel distance, travel time, site selection, route, or service selection [37]. After combing the relevant literature, it can be seen that the research on railway ticket pricing based on market segmentation is classified into two types: those divided by time factors (ticket purchase time and travel time) and those divided by passenger type (annual ticket, monthly ticket, or round-trip discount).

1. Time factor

The time factor is viewed in three ways: first, based on passenger travel value; second, based on the difference in travel time; and third, segmented according to travel time and pre-sale time.

As a generalized cost, the time value of passenger travel directly determines the willingness of passengers to pay for services and will affect travel choices of passengers. Han et al. [38] calculated the time value of passenger travel, such as calculating the time value of passengers according to the fatigue recovery time model and labor market salary level [34] or by collecting passenger survey data to establish a railway pricing model on the basis of time value [19]. This calculation uses the time value as the basis for market segmentation and flexibly adjusts the ticket price on the basis of the time value of the existing ticket price, which is a highly operable differential pricing method.

The second differential pricing method distinguishes between peak and valley periods. Considering the double peaks (Spring Festival and Summer Transport) of China's railway passenger transport as an example, the pricing goal of the inelastic peak period is welfare maximization; the pricing goal of the elastic off-peak market is profit maximization [23,39]. For the low and peak seasons of high-speed rail travel, the rising and falling ratios, respectively, can be set, and the elastic formula is applied to describe the relationship between volume and price [40]. There is also an article that divides the peak-to-valley ratio of China's high-speed rail passenger transport into 1/3 and 2/3 according to the proportion of holidays; this methodology uses the Ramsey pricing method to determine the fare strategy for maximizing welfare [41].

The third differential pricing method sets differential fares for trains with different travel times to maximize the revenue of high-speed rail tickets [42]. With the change of income level, passengers pay more attention to travel time, and differential fares can increase high-speed rail attendance., Thus, the fare should be reduced on weekdays and increased on holidays [43].

2. Passenger Type

There are five common classifications of passenger types: (1) Time and fare are the two main factors affecting passenger travel, and thus become the basis for passenger type classification. For example, passengers are divided into departure-time-sensitive passengers and price-sensitive passengers [44]. (2) The market can be divided according to the income, consumption level, and consumption structure of passengers. For example, passengers are divided into four types: efficiency type, economy type, leisure type, and high-end type, and a personalized service fare strategy is proposed for each type of passenger [45]. (3) Passenger types can be classified according to loyalty. For example, the demand for high-speed passenger transport is divided into three categories: loyal airline passengers, high-speed rail loyal passengers, and potential passengers, and the choice of potential passengers is described by using probability [46]. (4) Passengers can be divided by travel purpose, such as migrant workers, non-economic travel, and business travel [47]. (5) On the basis of the number of tickets purchased at one time, passengers can be divided into group passengers and individual passengers. For example, 20 tickets purchased at one time can be considered as the standard to distinguish between group passengers and individual passengers, and this cut-off value can be used to design group and individual ticket prices [26]. In addition, there are also studies in the literature that combine multiple indicators to classify passenger groups, such as age, gender, travel date, travel distance, ticket purchase method, and advance ticket purchase time as classification indicators to segment the high-speed rail passenger transport market [48]. Targeting passenger demand is an important revenue growth point for high-speed rail. Short-term discounts are implemented for groups with high demand elasticity, and long-term discounts are implemented for passenger groups with inelastic demand.

Different fare structures for different travel groups not only have appeared in academic research, but also have traces in practice. For example, in the current high-speed rail fare structure, the first-class and second-class fares are also comparable to those of business travelers and leisure travelers. Accurate passenger type classification is the premise of setting ticket price classification, and the ticket price design based on passenger heterogeneity can guide passengers to arrange travel scientifically and rationally.

3.2.3. Collaborative Optimization Pricing Strategy

At present, the optimization of railway pricing has received great attention, but the joint optimization of existing fares and seat allocation, vehicle operation plan, ticket allocation, etc., to make full use of train seat capacity is still an urgent problem to be solved in the current market-oriented reform of railway transportation enterprises.

In the literature, the joint optimization problem was first considered by Kuyumcu and Garcia-Diaz [49], who considered seat waste in the air transportation network and thus established a 0–1 integer programming model to jointly optimize the ticket price and seat allocation. Then, Ongprasert [50] introduced this theory into railway revenue management in 2006, which gradually aroused the interest of traffic researchers in the joint optimization of railway fares. However, when Bertsimas and de Boer [51] studied the joint pricing and seat allocation problems of air transport, they found that the optimization problem is not always concave, so the iterative nonlinear optimization algorithm used does not always guarantee the optimal solution. This is also the shortcoming of current research. In subsequent studies, scholars are also continually improving the joint optimization of railway fare and seat allocation (Xu et al., [52] and B; Xu et al., [53]). In order to overcome the defects of iterative nonlinear algorithm, Xu et al. [54] established a non-concave nonlinear mixed integer optimization model considering the sensitivity of demand to fare and applied linearization technology and relaxation technology to calculate the global optimal solution. In addition, the deterministic and stochastic models proposed by Cizaire [55] are also worthy of research for the joint optimization problem of ticket price and seat allocation for solving two products and two time-frames in railways.

On the basis of revenue management theory, we see that the overall optimization of the railway network is affected not only by seat allocation, but also reasonable railway passenger train operation plans and ticket allocation are important factors affecting railway fares. In order to maximize the comprehensive income of the railway network system, Zhang et al. [56] established an integrated model that considers train frequency, parking mode, and ticket allocation and considered multiple related factors to achieve global optimization. Through the case study of this model, it is easy to realize that joint optimization of railway planning can significantly increase revenue and reduce passenger waiting time. This optimization follows a specific mathematical expression:

$$\begin{aligned} Max\ R = &\sum_{t}^{T}\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} P_{ij}(t)\times\tau_i(t)\times x_{ij}(t) \\ &-\left(\sum_{t=1}^{T}\sum_{i=1}^{n}\tau_i(t)(F_i(t))+\sum_{t=1}^{T}\alpha(t)Y(t)\right) \\ &-\left(K\times S\sum_{t}^{T}\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\tau_i(t)\times x_{ij}(t)\right) \\ &-\sum_{t}^{T}\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\alpha(t)\times\tau_i(t)\times x_{ij}(t)\times W_i \\ &-\sum_{t}^{T}\sum_{i=1}^{n-1}\left(((1-\alpha(t))+\alpha(t)(1-\tau_i(t)))\times d_i\times\left(A_i^{t-1}\times\beta+A_i^t\right)\right) \end{aligned} \tag{2a}$$

$$s.t.\quad \left(\sum_{k=1}^{i}\sum_{j=i+1}^{n} x_{kj}(t)\right)\leq C,\ \ \forall i,j\in N \tag{2b}$$

$$[1-\alpha(t)]x_{kj}(t)=0,\ \ \forall i,j\in N \tag{2c}$$

$$[1-\tau_i(t)]x_{kj}(t)=0,\ \ \forall i,j\in N \tag{2d}$$

$$[1-\alpha(t)]\tau_i(t)=0,\ \ \forall i\in N \tag{2e}$$

$$\tau_n(t)=\alpha(t) \tag{2f}$$

$$A_i(t)\leq A_i^{Max},\ \forall i\in N \tag{2g}$$

$$\alpha(t),\tau_i(t)\in\{0,1\} \tag{2h}$$

where

n—Number of stations
N—The set of station node $N=[1,2,\ldots,n]$
t—Operation period $t\in[1,2,\ldots,T]$
P_{ij}—The price between the station i and station j
$x_{ij}(t)$—The number of passengers onboard from station i to station j in period t Y—Fixed operation costs per train
F_i—Fixed stopping costs at the station i

K—Coefficient of time values
S—Fixed stopping time at stations
W_i—Fixed waiting costs per passenger at the station i
d^t—Fixed delayed cost per passenger if the train does not operate/stop in period t
β—Passenger transfer rate from one period to next period
A_i^t—Passenger arrival rate for station i in period t
A_i^{Max}—The maximization capacity of station i
$\tau_i(t)$—If a train stops at the station i in period t, then $\tau_i(t) = 1$, otherwise $\tau_i(t) = 0$
$\alpha(t)$—If a train operates in period t, then $\alpha(t) = 1$, otherwise $\alpha(t) = 0$

Equations (2a) and (2b) represent the optimal total revenue and capacity of trains, respectively. Equation (2c) ensures that passengers have a certain correspondence with service decision $\alpha(t)$ at time t. Similarly, Equation (2d) defines the correspondence between the stopping pattern $\tau_i(t)$ and the onboard passengers at time t; in other words, passengers cannot board the train if the train does not stop at station i. Equation (2e) means that there is no train at the station during the train rest time t, and Equation (2f) ensures that trains will stop at the terminal. Finally, Equations (2g) and (2h) reveal the capacity of each station and the meaning of the variable, respectively, and $\tau_i(t)$ and $\alpha(t)$ are both variables whose values are 0 or 1.

In order to improve the adaptability of the joint optimization model, taking into consideration the complex vehicle operating environment of the railway network, Deng et al. [57] established a multi-train ticket system that considers the ticket purchase process and passenger demand during the pre-sale period. This is system is constructed on the basis of Han's [58] research on the collaborative optimization of high-speed rail parking planning and ticket allocation. The joint allocation model greatly improves the adaptability of the joint optimization model of fare and ticket allocation. In addition, Wang et al. [59] and Qin et al. [35], taking into account the differences in passenger demand, proposed a joint optimization method for multi-level fare and ticket allocation for high demand, which improved the service quality of railway trains.

To sum up, the existing research on collaborative optimization of passenger ticket pricing in the field of railway transportation is relatively small, most of the research methods are unable to solve large-scale problems, and there is a certain gap between them and practical applications. Regarding pricing strategies, the research primarily considers only the differentiated pricing strategy or the dynamic pricing strategy but does not combine the two strategies.

3.3. Data-Driven Approach

Most scholars have ignored the importance of historical booking data and railway network information for road network optimization when studying dynamic pricing in spite of extensive studies of existing dynamic pricing methods. Reasonable use of road network information and data can grasp all business characteristics and factors affecting demand patterns, which is of great significance for optimizing comprehensive income under variable road network conditions.

In contrast to other recent scholars who consider the relationship between supply and demand and other influencing factors to establish a mathematical model to determine the optimal price, establishing a data-driven optimization program is more suitable for the railway service system and can improve the comprehensive income of the railway. The current research on data-driven railway passenger fare optimization is still in its infancy. Talluri and Van Ryzin [60] first applied sales data to railway passenger revenue management (RPRM) in 2004, and it was widely accepted by later generations. In subsequent studies, scholars continue to mine and use data. Dutta and Ghosh [61] used data forecasting technology, optimization technology, and numerical simulation technology to improve the comprehensive benefits of national railways in emerging Asian economies (NREAE) as well as to consider passenger demand and expected marginal seats in their mathematical model income. Sun et al. [62] exploited booking data to mine passengers' choice preferences and used two machine learning methods to quantify the attractiveness of different types of transportation products. Their model expression is as follows:

$$f(X) = \langle W, X \rangle + b \ \omega \in R^m, b \in R^l \tag{3a}$$

$$while \min_{(W,b,\xi_i,\xi_i^*)} \frac{1}{2}\|W\|^2 + C\sum_{i=1}^{n}(\xi_i + \xi_i^*) \tag{3b}$$

$$k_{RBF}(X, X') = \exp\left(-\gamma\|X - X'\|^2\right) \tag{3c}$$

$$s.t. \quad y_i\langle W, X_1 \rangle - b \le \varepsilon + \xi_i, \forall i \tag{3d}$$

$$\langle W, X_1 \rangle + b - y_i \le \varepsilon + \xi_i^*, \forall i \tag{3e}$$

$$\xi_i, \xi_i^* \ge 0, \forall i \tag{3f}$$

where

$\langle \cdot, \cdot \rangle$—dot product
ξ_i, ξ_i^*—slack variables
ε—insensitive
C—penalty parameter
f—function
X_i—feature vector of $i - th$ set of training data
$\|W\|^2$—the Euclidean norm
y_i—the target output of the $i - th$ set of training data
n—the size of training dataset
γ—kernel parameter

Equation (3a) is the objective function that can obtain the target output y_i as flat as possible for all training sets. In order to find the function that satisfies the requirement of accuracy ε, the convex optimization problem is utilized and the objective function (Equation (3b)) is obtained. Equation (3c) is the most widely used kernel function currently, and the remaining formulation (Constraints (3d), (3e), and (3f)) are used to solve constrained convex optimization.

With the continual development of data in the application of railway passenger pricing, Salehi et al. [63] comprehensively considered the train supply capacity and demand capacity on the basis of Sun et al. [62]. Facing the problem of station congestion, Yin et al. [64], Mo et al. [65], and Kamandanipour et al. [66] modeled railway networks considering demand over time, and their method was evaluated using historical data. Hetrakul and Cirillo [67] focused more on the heterogeneity of passengers, predicted the purchase information of heterogeneous passengers through historical data, and proposed a comprehensive pricing method. In addition to studying factors such as supply and demand uncertainties, recent research will also mine the inherent distribution laws of data and apply them. Pratikto [68] developed a general model for RPRM problems, including demand forecasting, air ticket pricing, and seat allocation, during which he applied hierarchical Bayesian estimation, stochastic preference simulation, and cubic spline interpolation methods to estimate travel demand, solving the model through enumeration rules using the expected marginal seat revenue (EMSR) heuristic method. Yan et al. [69] proposed a nonlinear programming model for high-speed rail passenger transport network with probabilistic requirements. They considered the revenue optimization of seat inventory control decisions. Kankanit and Moryadee [70] introduced a dynamic pricing scheme for Thai high-speed trains in response to changes in service specification and purchase timing. They applied a linear demand function to find the optimal price on the basis of historical data. In order to improve the accuracy and availability of data, Kaushik [71] introduced the expectation maximization (EM) theory to the historical data of railway fares and designed corresponding algorithms to modify the incorrect data and eliminate invalid assumptions.

With the continual development and maturity of computer technology, increasingly more scholars are actively applying these new methods to the optimization of railway fares, among which the most widely used is the machine learning method. Alamdari, Anjos, and Savard [72] introduced machine learning methods to fast rule generation models and applied them to a major European railway service provider to test their proposed machine learning techniques. Kamandanipour et al. [71] proposed a data-driven RPRM approach with the aim of determining the optimal payoff of combined dynamic fare and capacity allocation in a single-train, multi-service mode. In the subsequent study, Kamandanipour et al. [73] further improved the approach and proposed a data-driven dynamic pricing method in 2022, which used a three-step process of machine learning and optimization tools to maximize the train capacity constraint The results of numerical research using Fadak's five-star train booking data also prove this point.

In order to highlight the guidance of the three research questions in Section 2 of this paper, the literature is roughly classified according to these three questions shown in Table 3, highlighting the main logic of the paper.

Table 3. Reference category corresponds to the three research questions.

Classification Criteria	Paper References
Main player	Besanko et al. [32], Worcester et al. [36], Cervero [37]
Scenarios	Sibdari et al. [18], Ozbay et al. [19], Zhang, Lang, and Jin [26]
Management strategies	Qin et al. [31], Qin et al. [35]
Methods	Sato and Sawakil [21], Cirillo et al. [25], Sun et al. [62]
Influencing factors	Xu et al. [52], Xu et al. [54]

4. Discussion and Conclusions

As one of the fundamental industries of the national economy, the reform of railway pricing involves many interests and will have a tremendous impact on the development of the national economy. Therefore, it is important to discuss and explore the deficiencies of the railway pricing mechanism so as to provide theoretical support for the further railway pricing reform. Through the review of railway pricing research in this paper, we found that many revenue management methods are used to solve railway pricing problems in complex railway networks under the combined influence of various influencing factors. In addition, a comprehensive examination of the relevant literature was carried out using content analysis methods and bibliometric methods, which will contribute to the development of innovative scientific methods and models. The following discussion provides some suggestions and opinions on future research directions.

First of all, emerging research methods (such as machine learning, which uses data or past experience to optimize performance criteria for computer programs) have considerable advantages for explaining and computing current problems and models. Applying these emerging research methods to the existing complex networks of multi-level, multi-train railway pricing and other joint optimization models of influencing factors will greatly promote the research innovation of railway pricing. Furthermore, these emerging research methods can also be used to analyze the impact of different pricing or sales strategies on the comprehensive revenue of railway operators. In addition, combined with the current data on the impact of travel demand in society since the coronavirus disease 2019 (COVID-19) pandemic, exploring the development direction of railway pricing under special circumstances will increase the flexibility and applicability of the pricing mechanism.

Secondly, the existing joint optimization research considers mainly single factors, such as pricing, seat allocation, travel plan, and ticket allocation. When addressing the complex railway network systems, joint optimization considering a single influencing factor may have insufficient applicability. Thus, the joint optimization model of railway pricing, which considers the integration of multiple factors comprehensively, may increase the computational difficulty. Therefore, an algorithm exploring the multi-factor joint optimization model of pricing, seat allocation, route planning, and travel plan urgently needs to be solved.

Finally, with the continual development of the application of information and digital technology, the complexity of the railway transportation network environment continues to increase, which also brings greater challenges to the differentiated railway pricing. When formulating a differentiated railway pricing scheme, regional differences, passenger product differences, and passenger differences will all lead to the uncertainty in demand. Therefore, it is also a breakthrough point of the current research to comprehensively consider the different perceptions of different types of passengers on fare, time, and comfort so as to formulate a multi-level price, optimize resource allocation, and maximize the benefits of both parties. Furthermore, in addition to improving operational efficiency and optimizing services from the above three aspects, improving operators' revenue can also improve the competitiveness of railway passenger transport compared with air and road transport from the perspective of service quality.

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