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On Cyclic Contractive Mappings of Kannan and Chatterjea Type in Generalized Metric Spaces

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Abstract: Novel cyclic contractions of the Kannan and Chatterjea type are presented in this study. With the aid of these brand-new contractions, new results for the existence and uniqueness of fixed points in the setting of complete generalized metric space have been established. Importantly, the results are generalizations and extensions of fixed point theorems by Chatterjea and Kannan and their cyclical expansions that are found in the literature. Additionally, several of the existing results on fixed points in generalized metric space will be generalized by the results presented in this work. Interestingly, the findings have a variety of applications in engineering and sciences. Examples have been given at the end to show the reliability of the demonstrated results.

Keywords: fixed point; Kannan contraction; Chatterjea contraction; $\mathcal{G}$-metric; nonlinear cyclic mapping

MSC: 54H25; 46T99; 47H10

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1. Introduction

Over the years, a large number of researchers have attempted to generalize the usual metric space concept, e.g., the studies in [1–3]. However, many of these generalizations were refuted by other studies, e.g., [4–7], due to the fundamental flaws they contained. A solid generalization known as $\mathcal{G}$-metric space was introduced in 2006 [8], in an appropriate structure, which corrected all of the shortcomings of earlier generalizations. The so-called $\mathcal{G}$-metric space, as introduced in [8], is given below.

Definition 1 ([8])**.** *Let $\mathcal{A}$ be a nonempty set and let the mapping $\mathcal{G} : \mathcal{A} \times \mathcal{A} \times \mathcal{A} \to \mathbb{R}^+$ satisfy*

- $\mathcal{G}(\zeta_1, \zeta_2, \zeta_3) = 0$ *if* $\zeta_1 = \zeta_2 = \zeta_3$,
- $0 < \mathcal{G}(\zeta_1, \zeta_1, \zeta_2)$ *whenever* $\zeta_1 \neq \zeta_2$, *for all* $\zeta_1, \zeta_2 \in \mathcal{A}$,
- $\mathcal{G}(\zeta_1, \zeta_1, \zeta_2) \leq \mathcal{G}(\zeta_1, \zeta_2, \zeta_3)$ *whenever* $\zeta_2 \neq \zeta_3$, *for all* $\zeta_1, \zeta_2, \zeta_3 \in \mathcal{A}$,
- $\mathcal{G}(\zeta_1, \zeta_2, \zeta_3) = \mathcal{G}(\zeta_1, \zeta_3, \zeta_2) = \mathcal{G}(\zeta_2, \zeta_1, \zeta_3) = \dots,$
- $\mathcal{G}(\zeta_1, \zeta_2, \zeta_3) \leq \mathcal{G}(\zeta_1, \zeta, \zeta) + \mathcal{G}(\zeta, \zeta_2, \zeta_3)$, *for all* $\zeta_1, \zeta_2, \zeta_3, \zeta \in \mathcal{A}$.

Then, the mapping $\mathcal{G}$ is called a generalized metric and is denoted by the $\mathcal{G}$-metric on $\mathcal{A}$. In addition, $(\mathcal{A}, \mathcal{G})$ is called a generalized metric space and is denoted by $\mathcal{G}$-metric space.

In what follows, examples of the presented $\mathcal{G}$-metric space are given.

Example 1 ([8])**.** *Let $(\mathcal{A}, h)$ be any metric space and let the mappings $\mathcal{G}_r : \mathcal{A} \times \mathcal{A} \times \mathcal{A} \to \mathbb{R}^+$ and $\mathcal{G}_t : \mathcal{A} \times \mathcal{A} \times \mathcal{A} \to \mathbb{R}^+$ be defined as*

$$\mathcal{G}_r(\zeta_1, \zeta_2, \zeta_3) = h(\zeta_1, \zeta_2) + h(\zeta_2, \zeta_3) + h(\zeta_1, \zeta_3),$$
$$\mathcal{G}_t(\zeta_1, \zeta_2, \zeta_3) = \max\{h(\zeta_1, \zeta_2), h(\zeta_2, \zeta_3), h(\zeta_1, \zeta_3)\}, \forall \zeta_1, \zeta_2, \zeta_3 \in \mathcal{A}.$$

Then, $(\mathcal{A}, \mathcal{G}_r)$ and $(\mathcal{A}, \mathcal{G}_t)$ are generalized metric spaces.

Definition 2 ([8]). *Let $(\mathcal{A}, \mathcal{G})$ be a generalized metric space and let $\{a_n\}$ be a sequence of points in $\mathcal{A}$. Then,*

- *If $\lim_{n,m\to\infty} \mathcal{G}(a, a_n, a_m) = 0$, i.e., for any $\epsilon > 0$, $\exists$ an integer $N \in \mathbb{N}$ such that $\mathcal{G}(a, a_n, a_m) < \epsilon$, for all $n, m \geq N$, then the point $a \in \mathcal{A}$ is called the limit of the sequence $\{a_n\}$, and $\{a_n\}$ is said to be $\mathcal{G}$-convergent to a;*
- *If $\lim_{n,m,k\to\infty} \mathcal{G}(a_n, a_m, a_k) = 0$, i.e., for any given $\epsilon > 0$, $\exists$ an integer $N \in \mathbb{N}$ such that $\mathcal{G}(a_n, a_m, a_k) < \epsilon$, for all $n, m, k \geq N$, then the sequence $\{a_n\}$ is called $\mathcal{G}$-Cauchy;*
- *The space $(\mathcal{A}, \mathcal{G})$ is said to be a complete $\mathcal{G}$-metric space if every $\mathcal{G}$-Cauchy sequence $\{a_n\}$ in $\mathcal{A}$ is $\mathcal{G}$-convergent in $\mathcal{A}$.*

Proposition 1 ([8]). *Let $(\mathcal{A}, \mathcal{G})$ be a generalized metric, $\mathcal{G}$-metric, space. Then, the sequence $\{a_m\}$ is $\mathcal{G}$-convergent to a if and only if $\lim_{m\to\infty} \mathcal{G}(a_m, a_m, a) = 0$ if and only if $\lim_{m\to\infty} \mathcal{G}(a_m, a, a) = 0$ if and only if $\lim_{m,n\to\infty} \mathcal{G}(a_m, a_n, a) = 0$.*

Proposition 2 ([8]). *Let $(\mathcal{A}, \mathcal{G})$ be a generalized metric, $\mathcal{G}$-metric, space. Then, the sequence $\{a_m\}$ is $\mathcal{G}$-Cauchy in $\mathcal{A}$ if and only if $\lim_{m,n\to\infty} \mathcal{G}(a_m, a_n, a_n) = 0$.*

Recalling that a point a^* is called a fixed point for a function f whenever $f(a^*) = a^*$, impressively, several theorems on the existence and uniqueness of fixed points and other conclusions were obtained in the aforementioned generalization of the usual metric space; for instance, one can refer to the studies in [9–12] and references therein.

Interestingly, throughout the past years, there have also been various attempts to expand and generalize Banach's contraction mapping principle [13], which is a fundamental concept that is applied to many problems in science and engineering. It needs to be affirmed that one of the key findings in analysis is the fixed point theorem of Banach, which is very well-known and has been applied in numerous mathematical areas. Kannan [14] successfully extended the Banach contraction principle as described below.

Definition 3 ([14]). *A mapping $\mathcal{K} : \mathcal{A} \to \mathcal{A}$, where $(\mathcal{A}, h)$ is a usual metric space, is called Kannan contraction if $\exists\ v \in \left[0, \frac{1}{2}\right)$ such that $\forall\ \zeta_1, \zeta_2 \in \mathcal{A}$, the inequality*

$$h(\mathcal{K}\zeta_1, \mathcal{K}\zeta_2) \leq v[h(\zeta_1, \mathcal{K}\zeta_1) + h(\zeta_2, \mathcal{K}\zeta_2)],$$

holds.

Kannan was able to prove that if $\mathcal{K}$ is a Kannan contraction mapping, then it has a unique fixed point provided $\mathcal{A}$ is complete. Another extension of Banach contraction was introduced by Chatterjea [15] and is given below.

Definition 4 ([15]). *A mapping $\mathcal{K} : \mathcal{A} \to \mathcal{A}$, where $(\mathcal{A}, h)$ is a usual metric space, is called a Chatterjea contraction if $\exists\ v \in \left[0, \frac{1}{2}\right)$ such that $\forall\ \zeta_1, \zeta_2 \in \mathcal{A}$, the inequality*

$$h(\mathcal{K}\zeta_1, \mathcal{K}\zeta_2) \leq v[h(\zeta_1, \mathcal{K}\zeta_2) + h(\zeta_2, \mathcal{K}\zeta_1)],$$

holds.

Similar to Kannan, Chatterjea [15], using his new definition, managed to prove that Chatterjea contraction mapping has a unique fixed point provided $\mathcal{A}$ is complete. Interestingly, Zamfirescu [16] in 1972 presented a fixed point result that combines the contractions of Chatterjea, Kannan, and Banach, which is stated below.

Theorem 1 ([16])**.** *Let $(\mathcal{A}, h)$ be a complete metric space and let $\mathcal{K} : \mathcal{A} \to \mathcal{A}$ be a mapping for which $\exists$ scalars v_1, v_2, and v_3 that satisfy $0 \leq v_1 < 1$, $0 \leq v_2, v_3 < \frac{1}{2}$, such that for any $\zeta_1, \zeta_2 \in \mathcal{A}$ at least one of the following is satisfied.*

- $h(\mathcal{K}\zeta_1, \mathcal{K}\zeta_2) \leq v_1 h(\zeta_1, \zeta_2)$;
- $h(\mathcal{K}\zeta_1, \mathcal{K}\zeta_2) \leq v_2[h(\zeta_1, \mathcal{K}\zeta_1) + h(\zeta_2, \mathcal{K}\zeta_2)]$;
- $h(\mathcal{K}\zeta_1, \mathcal{K}\zeta_2) \leq v_3[h(\zeta_1, \mathcal{K}\zeta_2) + h(\zeta_2, \mathcal{K}\zeta_1)]$.

Then, $\mathcal{K}$ has a unique fixed point a^*. Moreover, the Picard iteration, $\{a_n\}_{n=0}^{\infty}$, which is given by $a_{n+1} = \mathcal{K}a_n$, $n = 0, 1, 2, \ldots$ converges to a^* for any $a_0 \in \mathcal{A}$.

By taking into consideration non-empty closed subsets $\{B_j\}_{j=1}^{q}$ of a complete metric space and a cyclical operator $\mathcal{K} : \bigcup_{j=1}^{q} B_j \to \bigcup_{j=1}^{q} B_j$, i.e., satisfies $\mathcal{K}(B_j) \subseteq B_{j+1} \forall j \in \{1, 2, \ldots, q\}$, the cyclical extensions for the above fixed point results were discovered later by researchers. With the use of fixed point structure arguments, Rus gave a cyclical extension for Kannan's result in his work [17], while Petric gave cyclical extensions for Zamfirescu and Chatterjea results in [18].

Khan et al. [19] addressed the idea of a control function in light of altering distances that led to a new class of fixed point problems. Numerous publications on metric fixed point theory have employed altering distances, for instance, see [20–24] and references therein.

Here, in this work, we consider the generalization of the usual metric space that was introduced in [8] and present new extensions and generalizations of Banach, Kannan, and Chatterjea contractions and their cyclical expansions. In addition, some of the fixed point theorems that are found in the literature in the setting of $\mathcal{G}$-metric spaces are generalized here in this study. The presented results are obtained with the help of the continuous function $\Theta : [0, \infty)^3 \to [0, \infty)$ that satisfies $\Theta(\zeta_1, \zeta_2, \zeta_3) = 0$ if and only if $\zeta_1 = \zeta_2 = \zeta_3 = 0$, and the altering distance function Π that is defined in the sequel. In the end, examples have been given to show the reliability of the demonstrated results, and we conclude with a section of conclusions.

Definition 5. *Let $\Pi : [0, \infty) \to [0, \infty)$ be a function that is continuous, non-decreasing, and satisfies $\Pi(s) = 0$ if and only if $s = 0$. Then, Π shall be called an altering distance function.*

2. Main New Results in $\mathcal{G}$-Metric Spaces

We start this section by presenting what shall be called a $\mathcal{G}$-$(\Pi - \Theta)$-cyclic Kannan contraction and a $\mathcal{G}$-$(\Pi - \Theta)$-cyclic Chatterjea contraction. Then, we give our main work and results.

Definition 6. *Let $\mathcal{K} : \bigcup_{j=1}^{q} B_j \to \bigcup_{j=1}^{q} B_j$ be a cyclical operator, where $\{B_i\}_{j=1}^{q}$ are non-empty closed subsets of a $\mathcal{G}$-metric space $(\mathcal{A}, \mathcal{G})$. Then $\mathcal{K}$ is called a $\mathcal{G}$-$(\Pi - \Theta)$-cyclic Kannan contraction if $\exists$ scalars α, γ with $0 \leq \beta < 1$ and $0 < \alpha + \beta < 1$, such that for any $\zeta_1 \in B_j, \zeta_2, \zeta_3 \in B_{j+1}, j = 1, 2, \ldots, q$, we have*

$$\begin{aligned}\Pi(\mathcal{G}(\mathcal{K}\zeta_1, \mathcal{K}\zeta_2, \mathcal{K}\zeta_3)) \leq{}& \Pi(\alpha\mathcal{G}(\zeta_1, \mathcal{K}\zeta_1, \mathcal{K}\zeta_1) + \beta(\mathcal{G}(\zeta_2, \mathcal{K}\zeta_2, \mathcal{K}\zeta_2) + \mathcal{G}(\zeta_3, \mathcal{K}\zeta_3, \mathcal{K}\zeta_3))) \\ &- \Theta(\mathcal{G}(\zeta_1, \mathcal{K}\zeta_1, \mathcal{K}\zeta_1), \mathcal{G}(\zeta_2, \mathcal{K}\zeta_2, \mathcal{K}\zeta_2), \mathcal{G}(\zeta_3, \mathcal{K}\zeta_3, \mathcal{K}\zeta_3)),\end{aligned}$$

where Π and Θ are the two functions given earlier.

Definition 7. *Consider the same assumptions given in Definition 6. Then, $\mathcal{K}$ is called a $\mathcal{G}$-$(\Pi - \Theta)$-cyclic Chatterjea contraction if $\exists$ scalars α, β with $0 \le \alpha \le \frac{1}{2}$ and $0 < \alpha + \beta < 1$, such that for any $a \in B_j, b, c \in B_{j+1}, j = 1, 2, \ldots, q$, we have*

$$\begin{aligned}\Pi(\mathcal{G}(\mathcal{K}a, \mathcal{K}b, \mathcal{K}c)) \le \Pi(&\alpha\mathcal{G}(a, \mathcal{K}b, \mathcal{K}c) + \beta\mathcal{G}(b, c, \mathcal{K}a)) \\ &-\Theta(\mathcal{G}(a, \mathcal{K}b, \mathcal{K}c), \mathcal{G}(b, c, \mathcal{K}a), \mathcal{G}(c, b, \mathcal{K}a)),\end{aligned}$$

where again, Π and Θ are the two functions given earlier.

Theorem 2. *Let $\{B_i\}_{j=1}^{q}$ be non-empty closed subsets of a complete $\mathcal{G}$-metric space $(\mathcal{A}, \mathcal{G})$ and $\mathcal{K} : \bigcup_{j=1}^{q} B_j \to \bigcup_{j=1}^{q} B_j$ be a cyclical operator. Assume $\mathcal{K}$ satisfies at least one of the following statements:*

S1. *$\exists$ real numbers α, γ with $0 \le \gamma < 1$ and $0 < \alpha + \gamma < 1$, such that for any $a \in B_j, b \in B_{j+1}, j = 1, 2, \ldots, q$, we have*

$$\begin{aligned}\Pi(\mathcal{G}(\mathcal{K}a, \mathcal{K}b, \mathcal{K}b)) \le \Pi(&\alpha\mathcal{G}(a, \mathcal{K}a, \mathcal{K}a) + \gamma\mathcal{G}(b, \mathcal{K}b, \mathcal{K}b)) \\ &-\Theta(\mathcal{G}(a, \mathcal{K}a, \mathcal{K}a), \mathcal{G}(b, \mathcal{K}b, \mathcal{K}b), \mathcal{G}(b, \mathcal{K}b, \mathcal{K}b)).\end{aligned}$$

S2. *$\exists$ real numbers α, δ with $0 \le \alpha \le \frac{1}{2}$ and $0 < \alpha + \delta < 1$, such that for any $a \in B_j, b \in B_{j+1}, j = 1, 2, \ldots, q$, we have*

$$\begin{aligned}\Pi(\mathcal{G}(\mathcal{K}a, \mathcal{K}b, \mathcal{K}b)) \le \Pi(&\alpha\mathcal{G}(a, \mathcal{K}b, \mathcal{K}b) + \delta\mathcal{G}(b, b, \mathcal{K}a)) \\ &-\Theta(\mathcal{G}(a, \mathcal{K}b, \mathcal{K}b), \mathcal{G}(b, b, \mathcal{K}a), \mathcal{G}(b, b, \mathcal{K}b)).\end{aligned}$$

Then, $\mathcal{K}$ has a unique fixed point $a^* \in \bigcap_{j=1}^{q} B_j$.

Proof. Consider the recursive sequence $a_{n+1} = \mathcal{K}a_n, n \ge 0$ with an arbitrary initial starting value $a_0 \in \bigcup_{j=1}^{q} B_j$. If $\exists$ a value $n_0 \in \mathbb{N}$ such that $a_{n_0+1} = a_{n_0}$, then the existence of the fixed point is achieved. Hence, we assume $a_{n+1} \ne a_n$, for all the values $n = 0, 1, \ldots$. Due to this assumption, one shall be sure that $\exists\ j_n \in \{1, \ldots, q\}$ such that $a_{n-1} \in B_{j_n}$ and $a_n \in B_{j_{n+1}}$. Now, let first $\mathcal{K}$ satisfy the first statement, i.e., S1. Then, we have

$$\begin{aligned}\Pi(\mathcal{G}(a_n, a_{n+1}, a_{n+1})) &= \Pi(\mathcal{G}(\mathcal{K}a_{n-1}, \mathcal{K}a_n, \mathcal{K}a_n)) \\ &\le \Pi(\alpha\mathcal{G}(a_{n-1}, \mathcal{K}a_{n-1}, \mathcal{K}a_{n-1}) + \gamma\mathcal{G}(a_n, \mathcal{K}a_n, \mathcal{K}a_n)) \\ &- \Theta(\mathcal{G}(a_{n-1}, \mathcal{K}a_{n-1}, \mathcal{K}a_{n-1}), \mathcal{G}(a_n, \mathcal{K}a_n, \mathcal{K}a_n), \mathcal{G}(a_n, \mathcal{K}a_n, \mathcal{K}a_n)) \\ &= \Pi(\alpha\mathcal{G}(a_{n-1}, a_n, a_n) + \gamma\mathcal{G}(a_n, a_{n+1}, a_{n+1})) \\ &- \Theta(\mathcal{G}(a_{n-1}, a_n, a_n), \mathcal{G}(a_n, a_{n+1}, a_{n+1}), \mathcal{G}(a_n, a_{n+1}, a_{n+1})) \\ &\le \Pi(\alpha\mathcal{G}(a_{n-1}, a_n, a_n) + \gamma\mathcal{G}(a_n, a_{n+1}, a_{n+1})).\end{aligned}$$

Due to the fact that Π is non-decreasing, one gets

$$\mathcal{G}(a_n, a_{n+1}, a_{n+1}) \le \alpha\mathcal{G}(a_{n-1}, a_n, a_n) + \gamma\mathcal{G}(a_n, a_{n+1}, a_{n+1}),$$

which leads to

$$\mathcal{G}(a_n, a_{n+1}, a_{n+1}) \le \frac{\alpha}{1-\gamma}\mathcal{G}(a_{n-1}, a_n, a_n), \forall n. \tag{1}$$

Since $0 < \alpha + \gamma < 1$, one gets $\mathcal{G}(a_n, a_{n+1}, a_{n+1})$ is a non-increasing sequence of non-negative real numbers. Therefore, $\exists\ l \ge 0$ such that

$$\lim_{n\to\infty} \mathcal{G}(a_n, a_{n+1}, a_{n+1}) = l.$$

Exploiting the continuity of the functions Π and Θ, one gets

$$\begin{aligned} \Pi(l) &\leq \Pi((\alpha+\gamma)l) - \Theta(l,l,l) \\ &\leq \Pi(r) - \Theta(l,l,l), \end{aligned}$$

which leads to $\Theta(l,l,l) = 0$, and as a result, $l = 0$.
In the same way, if $\mathcal{K}$ satisfies the second statement, i.e., S2, then we get

$$\begin{aligned} \Pi(\mathcal{G}(a_n, a_{n+1}, a_{n+1})) &= \Pi(\mathcal{G}(\mathcal{K}a_{n-1}, \mathcal{K}a_n, \mathcal{K}a_n)) \\ &\leq \Pi(\alpha\mathcal{G}(a_{n-1}, \mathcal{K}a_n, \mathcal{K}a_n) + \gamma\mathcal{G}(a_n, a_n, \mathcal{K}a_{n-1})) \\ &\quad -\Theta(\mathcal{G}(a_{n-1}, \mathcal{K}a_n, \mathcal{K}a_n), \mathcal{G}(a_n, a_n, \mathcal{K}a_{n-1}), \mathcal{G}(a_n, a_n, \mathcal{K}a_{n-1})) \\ &= \Pi(\alpha\mathcal{G}(a_{n-1}, a_{n+1}, a_{n+1}) + \gamma\mathcal{G}(a_n, a_n, a_n)) \\ &\quad -\Theta(\mathcal{G}(a_{n-1}, a_{n+1}, a_{n+1}), \mathcal{G}(a_n, a_n, a_n), \mathcal{G}(a_n, a_n, a_n)) \\ &\leq \Pi(\alpha G(a_{n-1}, a_{n+1}, a_{n+1})). \end{aligned}$$

Now, as Π is non-decreasing, one gets

$$\mathcal{G}(a_n, a_{n+1}, a_{n+1}) \leq \alpha\mathcal{G}(a_{n-1}, a_{n+1}, a_{n+1}). \tag{2}$$

Using the rectangular inequality implies

$$\begin{aligned} \mathcal{G}(a_n, a_{n+1}, a_{n+1}) &\leq \alpha\mathcal{G}(a_{n-1}, a_{n+1}, a_{n+1}) \\ &\leq \alpha[\mathcal{G}(a_{n-1}, a_n, a_n) + \mathcal{G}(a_n, a_{n+1}, a_{n+1})], \end{aligned}$$

which leads to

$$\mathcal{G}(a_n, a_{n+1}, a_{n+1}) \leq \frac{\alpha}{1-\alpha}\mathcal{G}(a_{n-1}, a_n, a_n). \tag{3}$$

Due to the fact that $0 \leq \alpha \leq \frac{1}{2}$, we have $\{\mathcal{G}(a_n, a_{n+1}, a_{n+1})\}$ is a non-increasing sequence of non-negative real numbers. Therefore, $\exists\, l \geq 0$ such that

$$\lim_{n\to\infty} \mathcal{G}(a_n, a_{n+1}, a_{n+1}) = l.$$

For the case $\alpha = 0$, one clearly gets, $l = 0$, and, for $0 < \alpha < \frac{1}{2}$, one gets $\frac{\alpha}{1-\alpha} < 1$, and hence by induction, one gets

$$\mathcal{G}(a_n, a_{n+1}, a_{n+1}) \leq \left(\frac{\alpha}{1-\alpha}\right)^n \mathcal{G}(a_0, a_1, a_1),$$

and therefore, $l = 0$.
Lastly, for $\alpha = \frac{1}{2}$, from (2), one gets

$$\mathcal{G}(a_{n-1}, a_{n+1}, a_{n+1}) \geq 2\mathcal{G}(a_n, a_{n+1}, a_{n+1}),$$

and therefore,

$$\lim_{n\to\infty} \mathcal{G}(a_{n-1}, a_{n+1}, a_{n+1}) \geq 2l;$$

however,

$$\mathcal{G}(a_{n-1}, a_{n+1}, a_{n+1}) \leq \mathcal{G}(a_{n-1}, a_n, a_n) + \mathcal{G}(a_n, a_{n+1}, a_{n+1}),$$

which leads, as $n \to \infty$, to

$$\lim_{n\to\infty} \mathcal{G}(a_{n-1}, a_{n+1}, a_{n+1}) \leq 2l.$$

Hence, $\lim_{n\to\infty} \mathcal{G}(a_{n-1}, a_{n+1}, a_{n+1}) = 2l$.
Now, with the use of the continuity of the functions Π and Θ, and as $\alpha = \frac{1}{2}$, one gets

$$\begin{aligned}\Pi(l) &\leq \Pi\left(\frac{1}{2}\cdot 2l\right) - \Theta(2l,0,0) \\ &= \Pi(l) - \Theta(2l,0,0),\end{aligned}$$

which leads to $\Theta(2l,0,0) = 0$, and therefore, $l = 0$.
Next, we show that for every $\epsilon > 0$, $\exists\, n \in \mathbb{N}$ such that if $r, s \geq n$ with $r - s \equiv 1(m)$, then $\mathcal{G}(a_r, a_s, a_s) < \epsilon$ which is needed in order to prove that $\{a_n\}$ is indeed a $\mathcal{G}$-Cauchy sequence in $\mathcal{A}$.
We use the proof by contradiction, and hence we assume that $\exists\, \epsilon > 0$ such that for any $n \in \mathbb{N}$, we can find $r_n > s_n \geq n$ with $r_n - s_n \equiv 1(m)$ that satisfy $\mathcal{G}(a_{r_n}, a_{s_n}, a_{s_n}) \geq \epsilon$.
Taking $n > 2m$, one then can choose r_n corresponding to $s_n \geq n$ in such a way that it is the smallest integer with $r_n > s_n$ satisfying $r_n - s_n \equiv 1(m)$ and $\mathcal{G}(a_{r_n}, a_{s_n}, a_{s_n}) \geq \epsilon$. Hence, $\mathcal{G}(a_{s_n}, a_{s_n}, a_{r_{n-m}}) < \epsilon$.
Applying the rectangular inequality, one gets

$$\begin{aligned}\epsilon \leq \mathcal{G}(a_{r_n}, a_{s_n}, a_{s_n}) &\leq \mathcal{G}\left(a_{r_{n-1}}, a_{s_n}, a_{s_n}\right) + \mathcal{G}\left(a_{r_{n-1}}, a_{r_{n-1}}, a_{r_n}\right) \\ &\leq \mathcal{G}\left(a_{r_{n-2}}, a_{s_n}, a_{s_n}\right) + \mathcal{G}\left(a_{r_{n-2}}, a_{r_{n-2}}, a_{r_{n-1}}\right) + \mathcal{G}\left(a_{r_{n-1}}, a_{r_{n-1}}, a_{r_n}\right) \\ &\;\;\vdots \\ &\leq \mathcal{G}(a_{s_n}, a_{s_n}, a_{r_{n-m}}) + \sum_{j=1}^{m} \mathcal{G}\left(a_{r_{n-j}}, a_{r_{n-j}}, a_{r_{n-j+1}}\right) \\ &< \epsilon + \sum_{j=1}^{m} \mathcal{G}\left(a_{r_{n-j}}, a_{r_{n-j}}, a_{r_{n-j+1}}\right).\end{aligned}$$

Taking the limit as n goes to infinity, and considering

$$\lim_{n\to\infty} \mathcal{G}(a_n, a_{n+1}, a_{n+1}) = 0,$$

lead to

$$\epsilon \leq \lim_{n\to\infty} \mathcal{G}(a_{r_n}, a_{s_n}, a_{s_n}) < \epsilon + 0 = \epsilon,$$

and hence, $\lim_{n\to\infty} \mathcal{G}(a_{r_n}, a_{s_n}, a_{s_n}) = \epsilon$.
Using the rectangle inequality implies

$$\begin{aligned}\mathcal{G}(a_{s_n}, a_{s_n}, a_{r_n}) &\leq \mathcal{G}(a_{r_n}, a_{r_{n+1}}, a_{r_{n+1}}) + \mathcal{G}(a_{r_{n+1}}, a_{s_n}, a_{s_n}) \\ &\leq \mathcal{G}(a_{r_n}, a_{r_{n+1}}, a_{r_{n+1}}) + \mathcal{G}(a_{r_{n+1}}, a_{s_{n+1}}, a_{s_{n+1}}) + \mathcal{G}(a_{s_{n+1}}, a_{s_n}, a_{s_n}) \\ &\leq \mathcal{G}(a_{r_n}, a_{r_{n+1}}, a_{r_{n+1}}) + \mathcal{G}(a_{r_{n+1}}, a_{s_{n+1}}, a_{s_{n+1}}) + \mathcal{G}(a_{s_n}, a_{s_{n+1}}, a_{s_{n+1}}) \\ &\quad + \mathcal{G}(a_{s_{n+1}}, a_{s_{n+1}}, a_{s_n}).\end{aligned}$$

Additionally,

$$\begin{aligned}\mathcal{G}(a_{r_{n+1}}, a_{s_{n+1}}, a_{s_{n+1}}) &\leq \mathcal{G}(a_{r_{n+1}}, a_{s_n}, a_{s_n}) + \mathcal{G}(a_{s_n}, a_{s_{n+1}}, a_{s_{n+1}}) \\ &\leq \mathcal{G}(a_{r_{n+1}}, a_{r_n}, a_{r_n}) + \mathcal{G}(a_{r_n}, a_{s_n}, a_{r_n}) + \mathcal{G}(a_{s_n}, a_{s_{n+1}}, a_{s_{n+1}}) \\ &\leq \mathcal{G}(a_{r_n}, a_{r_{n+1}}, a_{r_{n+1}}) + \mathcal{G}(a_{r_{n+1}}, a_{r_{n+1}}, a_{r_n}) + \mathcal{G}(a_{r_n}, a_{s_n}, a_{s_n}) \\ &\quad + \mathcal{G}(a_{s_n}, a_{s_{n+1}}, a_{s_{n+1}}).\end{aligned}$$

Letting n go to infinity and considering $\lim_{n\to\infty} \mathcal{G}(a_n, a_{n+1}, a_{n+1}) = 0$ implies
$\epsilon \leq \lim_{n\to\infty} \mathcal{G}\left(a_{r_{n+1}}, a_{s_{n+1}}, a_{s_{n+1}}\right) \leq \epsilon$, which leads to $\lim_{n\to\infty} \mathcal{G}\left(a_{r_{n+1}}, a_{s_{n+1}}, a_{s_{n+1}}\right) = \epsilon$.

Now, let $\mathcal{K}$ satisfy the first statement. Then, since a_{r_n} and a_{s_n} are in distinct consecutively labeled sets B_j and B_{j+1}, for a particular $1 \leq j \leq m$, one gets

$$\begin{aligned} \Pi\big(\mathcal{G}\big(a_{s_{n+1}}, a_{s_{n+1}}, a_{r_{n+1}}\big)\big) &= \Pi(\mathcal{G}(\mathcal{K}a_{s_n}, \mathcal{K}a_{s_n}, \mathcal{K}a_{r_n})) \\ &\leq \Pi(\alpha\mathcal{G}(a_{s_n}, \mathcal{K}a_{s_n}, \mathcal{K}a_{s_n}) + \gamma\mathcal{G}(a_{r_n}, \mathcal{K}a_{r_n}, \mathcal{K}a_{r_n})) \\ &- \Theta(\mathcal{G}(a_{r_n}, \mathcal{K}a_{r_n}, \mathcal{K}a_{r_n}), \mathcal{G}(a_{s_n}, \mathcal{K}a_{s_n}, \mathcal{K}a_{s_n}), \mathcal{G}(a_{s_n}, \mathcal{K}a_{s_n}, \mathcal{K}a_{s_n})). \end{aligned}$$

Taking the limit as n goes to infinity in the last inequality, one obtains

$$\Pi(\epsilon) \leq \Pi(0) - \Theta(0,0,0) = 0.$$

Hence, $\epsilon = 0$ which leads to a contradiction.
Similarly, if $\mathcal{K}$ satisfies the second statement, then one gets

$$\begin{aligned} \Pi\big(\mathcal{G}\big(a_{s_{n+1}}, a_{s_{n+1}}, a_{r_{n+1}}\big)\big) &= \Pi(\mathcal{G}(\mathcal{K}a_{s_n}, \mathcal{K}a_{s_n}, \mathcal{K}a_{r_n})) \\ &\leq \Pi(\alpha\mathcal{G}(a_{r_n}, \mathcal{K}a_{s_n}, \mathcal{K}a_{s_n}) + \gamma\mathcal{G}(a_{s_n}, a_{s_n}, \mathcal{K}a_{r_n})) \\ &\quad -\Theta(\mathcal{G}(a_{r_n}, \mathcal{K}a_{s_n}, \mathcal{K}a_{s_n}), \mathcal{G}(a_{s_n}, a_{s_n}, \mathcal{K}a_{r_n}), \mathcal{G}(a_{s_n}, a_{s_n}, \mathcal{K}a_{r_n})). \end{aligned}$$

Again, taking the limit as n goes to infinity in the last inequality, one gets

$$\Pi(\epsilon) \leq \Pi((\alpha + \gamma)\epsilon) - \Theta(\epsilon, \epsilon, \epsilon).$$

Since $0 < \alpha + \gamma < 1$, we get $\Theta(\epsilon, \epsilon, \epsilon) = 0$, and therefore, $\epsilon = 0$, which is again a contradiction.
As a consequence, one can find for $\epsilon > 0$, an integer $n_0 \in \mathbb{N}$ such that if $r, s > n_0$ with $r - s = 1(m)$, then $\mathcal{G}(a_r, a_s, a_s) < \epsilon$.
Using the fact that $\lim\limits_{n\to\infty} \mathcal{G}(a_n, a_{n+1}, a_{n+1}) = 0$, one can find an integer $n_1 \in \mathbb{N}$ such that

$$\mathcal{G}(a_n, a_{n+1}, a_{n+1}) \leq \frac{\epsilon}{m}, \text{ for } n > n_1.$$

In addition, for some integers $p, q > \max\{n_0, n_1\}$ and $q > p$, $\exists\, \ell \in \{1, 2, \ldots, m\}$ such that $q - p = \ell(m)$. Hence, $q - p + i = 1(m)$ for $i = m - \ell + 1$. Therefore, one gets

$$\mathcal{G}(a_p, a_p, a_q) \leq \mathcal{G}(a_p, a_p, a_{q+i}) + \mathcal{G}(a_{q+i}, a_{q+i}, a_{q+i-1}) + \ldots + \mathcal{G}(a_{q+1}, a_{q+1}, a_q),$$

which leads to

$$\mathcal{G}(a_p, a_p, a_q) \leq \epsilon + \frac{\epsilon}{m} \sum_{i=1}^{m} 1 = 2\epsilon.$$

Hence, $\{a_n\}$ is a $\mathcal{G}$-Cauchy sequence in $\bigcup\limits_{j=1}^{q} B_j$, and consequently converges to some $a^* \in \bigcup\limits_{j=1}^{q} B_j$. However, in view of the cyclical condition, the sequence $\{a_n\}$ has an infinite number of terms in each B_j, for $j = 1, 2, \ldots, q$. Therefore, $a^* \in \bigcap\limits_{j=1}^{q} B_j$.
In order to show that a^* is a fixed point of $\mathcal{K}$, we assume $a^* \in B_j$, and $\mathcal{K}a^* \in B_{j+1}$, and we consider a sub-sequence a_{n_ℓ} of $\{a_n\}$ where $a_{n_\ell} \in B_{j-1}$. Now, if $\mathcal{K}$ satisfies the first statement, then

$$\begin{aligned} \Pi\big(\mathcal{G}\big(a_{n_{\ell+1}}, \mathcal{K}a^*, \mathcal{K}a^*\big)\big) &= \Pi(\mathcal{G}(\mathcal{K}a_{n_\ell}, \mathcal{K}a^*, \mathcal{K}a^*)) \\ &\leq \Pi(\alpha\mathcal{G}(a_{n_\ell}, \mathcal{K}a_{n_\ell}, \mathcal{K}a_{n_\ell}) + \gamma\mathcal{G}(a^*, \mathcal{K}a^*, \mathcal{K}a^*)) \\ &\quad -\Theta(\mathcal{G}(a_{n_\ell}, \mathcal{K}a_{n_\ell}, \mathcal{K}a_{n_\ell}), \mathcal{G}(a^*, \mathcal{K}a^*, \mathcal{K}a^*), \mathcal{G}(a^*, \mathcal{K}a^*, \mathcal{K}a^*)) \\ &\leq \Pi(\alpha\mathcal{G}(a_{n_\ell}, \mathcal{K}a_{n_\ell}, \mathcal{K}a_{n_\ell}) + \gamma\mathcal{G}(a^*, \mathcal{K}a^*, \mathcal{K}a^*)). \end{aligned}$$

Taking the limit as ℓ goes to infinity, one gets

$$\Pi(\mathcal{G}(a^*, \mathcal{K}a^*, \mathcal{K}a^*)) \leq \Pi(\alpha\mathcal{G}(a^*, a^*, a^*) + \gamma\mathcal{G}(a^*, \mathcal{K}a^*, \mathcal{K}a^*)).$$

Knowing that the function Π is non-decreasing, one gets

$$\mathcal{G}(a^*, \mathcal{K}a^*, \mathcal{K}a^*) \leq \gamma\mathcal{G}(a^*, \mathcal{K}a^*, \mathcal{K}a^*).$$

Now, using $0 \leq \gamma < 1$, one gets $\mathcal{G}(a^*, \mathcal{K}a^*, \mathcal{K}a^*) = 0$, and therefore, $a^* = \mathcal{K}a^*$.
In a similar way, if $\mathcal{K}$ satisfies the second statement, then

$$\begin{aligned}
\Pi\big(\mathcal{G}\big(a_{n_{\ell+1}}, \mathcal{K}a^*, \mathcal{K}a^*\big)\big) &= \Pi(\mathcal{G}(\mathcal{K}a_{n_\ell}, \mathcal{K}a^*, \mathcal{K}a^*)) \\
&\leq \Pi(\alpha\mathcal{G}(a_{n_\ell}, \mathcal{K}a^*, \mathcal{K}a^*) + \gamma\mathcal{G}(a^*, a^*, \mathcal{K}a_{n_\ell})) \\
&\quad -\Theta(\mathcal{G}(a_{n_\ell}, \mathcal{K}a^*, \mathcal{K}a^*), \mathcal{G}(a^*, a^*, \mathcal{K}a_{n_\ell}), \mathcal{G}(a^*, a^*, \mathcal{K}a_{n_\ell})) \\
&\leq \Pi(\alpha\mathcal{G}(a_{n_\ell}, \mathcal{K}a^*, \mathcal{K}a^*) + \gamma\mathcal{G}(a^*, a^*, \mathcal{K}a_{n_\ell})).
\end{aligned}$$

Taking again the limit as ℓ goes to infinity, one gets

$$\Pi(\mathcal{G}(a^*, \mathcal{K}a^*, \mathcal{K}a^*)) \leq \Pi(\alpha\mathcal{G}(a^*, \mathcal{K}a^*, \mathcal{K}a^*) + \gamma\mathcal{G}(a^*, a^*, a^*)).$$

Again, exploiting that the function Π is non-decreasing, one obtains

$$\mathcal{G}(a^*, \mathcal{K}a^*, \mathcal{K}a^*) \leq \alpha\mathcal{G}(a^*, \mathcal{K}a^*, \mathcal{K}a^*).$$

Now, since $0 \leq \alpha \leq \frac{1}{2}$, one gets $\mathcal{G}(a^*, \mathcal{K}a^*, \mathcal{K}a^*) = 0$, and therefore, $a^* = \mathcal{K}a^*$. □

Theorem 3. *Let $\{B_j\}_{j=1}^{q}$ be non-empty closed subsets of a complete $\mathcal{G}$-metric space $(\mathcal{A}, \mathcal{G})$ and $\mathcal{K} : \bigcup_{j=1}^{q} B_j \to \bigcup_{j=1}^{q} B_j$ be a cyclical operator. Further, assume $\mathcal{K}$ is either a $\mathcal{G}$-$(\Pi - \Theta)$-cyclic Kannan contraction, Definition 6, or a $\mathcal{G}$-$(\Pi - \Theta)$-cyclic Chatterjea contraction, Definition 7. Then, $\mathcal{K}$ has a unique fixed point $a^* \in \bigcap_{j=1}^{q} B_j$.*

Proof. Taking $\zeta_3 = \zeta_2$ in Definition 6 and $c = b$ in Definition 7, the proof follows directly from the proof of Theorem 2 with $\gamma = 2\beta$ for the first statement and $\delta = \beta$ for the second statement. □

3. Applications and Examples

In this section, applications of the results are given in order to show the reliability of the demonstrated results.

Example 2. *Consider the complete $\mathcal{G}$-metric space, $(\mathcal{A}, \mathcal{G})$ and the mapping $\mathcal{K} : \mathcal{A} \to \mathcal{A}$ that is a cyclical operator, where $\mathcal{A} = \bigcup_{j=1}^{n} B_j$ and $\{B_j\}_{j=1}^{n}$ are non-empty closed subsets of $(\mathcal{A}, \mathcal{G})$. If, for any $\zeta_1 \in B_j$, $\zeta_2 \in B_{j+1}$, $j = 1, 2, \ldots, n$, with $B_{n+1} = B_1$, at least one of the following holds:*

$$\int_0^{\mathcal{G}(\mathcal{K}\zeta_1, \mathcal{K}\zeta_2, \mathcal{K}\zeta_2)} \omega(u)\, du \leq \int_0^{\alpha\mathcal{G}(\zeta_1, \mathcal{K}\zeta_1, \mathcal{K}\zeta_1) + \gamma\mathcal{G}(\zeta_2, \mathcal{K}\zeta_2, \mathcal{K}\zeta_2)} \omega(u)\, du,$$

or

$$\int_0^{\mathcal{G}(\mathcal{K}\zeta_1, \mathcal{K}\zeta_2, \mathcal{K}\zeta_2)} \omega(u)\, du \leq \int_0^{\alpha\mathcal{G}(\zeta_1, \mathcal{K}\zeta_2, \mathcal{K}\zeta_2) + \gamma\mathcal{G}(\mathcal{K}\zeta_1, \zeta_2, \zeta_2)} \omega(u)\, du,$$

where $\omega : [0, \infty) \to [0, \infty)$ is a Lebesgue integrable mapping that satisfies $\int_0^u \omega(\tau)\, d\tau > 0$, for $u > 0$, then $\mathcal{K}$ has a unique fixed point $a^* \in \bigcap_{j=1}^{n} B_j$.

This is a straightforward conclusion that one can easily obtain. To that end, let $\Pi : [0, \infty) \to [0, \infty)$ *be defined as* $\Pi(u) = \int_0^u \omega(\tau)\, d\tau > 0$. *Then,* Π *is an altering distance function, and by choosing* $\Theta(\zeta_1, \zeta_2, \zeta_3) = 0$, *one gets the result.*

Example 3. *Let* $\mathcal{G}(\zeta_1, \zeta_2, \zeta_3) = |\zeta_1 - \zeta_2| + |\zeta_2 - \zeta_3| + |\zeta_1 - \zeta_3|$ *and* $\mathcal{A} = [-1, 1] \subseteq \mathbb{R}$. *Moreover, consider the mapping* $\mathcal{K} : [-1, 0] \cup [0, 1] \to [-1, 0] \cup [0, 1]$ *which is defined by*

$$\mathcal{K}(t) = \begin{cases} -\frac{1}{3} t e^{-\frac{1}{|t|}}, & t \in [-1, 0), \\ 0, & t = 0, \\ -\frac{1}{2} t e^{-\frac{1}{|t|}}, & t \in (0, 1]. \end{cases}$$

By taking $\Theta(s, t, u) = 0$, $\Pi(s) = s$, *and* $a \in [0, 1]$, $b \in [-1, 0]$, *one obtains*

$$\begin{aligned} \mathcal{G}(\mathcal{K}\zeta_1, \mathcal{K}\zeta_2, \mathcal{K}\zeta_2) &= |\mathcal{K}\zeta_1 - \mathcal{K}\zeta_2| + |\mathcal{K}\zeta_1 - \mathcal{K}\zeta_2| + |\mathcal{K}\zeta_2 - \mathcal{K}\zeta_2| \\ &= |\mathcal{K}\zeta_1 - \mathcal{K}\zeta_2| + |\mathcal{K}\zeta_1 - \mathcal{K}\zeta_2| \\ &= \left| -\frac{1}{2}\zeta_1 e^{-\frac{1}{|\zeta_1|}} + \frac{1}{3}\zeta_2 e^{-\frac{1}{|\zeta_2|}} \right| + \left| -\frac{1}{2}\zeta_1 e^{-\frac{1}{|\zeta_1|}} + \frac{1}{3}\zeta_2 e^{-\frac{1}{|\zeta_2|}} \right| \\ &\leq \frac{1}{2}|\zeta_1| + \frac{1}{3}|\zeta_2| + \frac{1}{2}|\zeta_1| + \frac{1}{3}|\zeta_2| \\ &\leq \frac{1}{2}\left|\zeta_1 + \frac{1}{2}\zeta_1 e^{-\frac{1}{|\zeta_1|}}\right| + \frac{1}{3}\left|\zeta_2 + \frac{1}{3}\zeta_2 e^{-\frac{1}{|\zeta_2|}}\right| + \frac{1}{2}\left|\zeta_1 + \frac{1}{2}\zeta_1 e^{-\frac{1}{|\zeta_1|}}\right| \\ &\quad + \frac{1}{3}\left|\zeta_2 + \frac{1}{3}\zeta_2 e^{-\frac{1}{|\zeta\zeta_2|}}\right| \\ &= \frac{1}{2}|\mathcal{K}\zeta_1 - \zeta_1| + \frac{1}{3}|\mathcal{K}\zeta_2 - \zeta_2| + \frac{1}{2}|\mathcal{K}\zeta_1 - \zeta_1| + \frac{1}{3}|\mathcal{K}\zeta_2 - \zeta_2| \\ &= \frac{1}{2}(|\mathcal{K}\zeta_1 - \zeta_1| + |\mathcal{K}\zeta_1 - \zeta_1|) + \frac{1}{3}(|\mathcal{K}\zeta_2 - \zeta_2| + |\mathcal{K}\zeta_2 - \zeta_2|) \\ &= \frac{1}{2}\mathcal{G}(\zeta_1, \mathcal{K}\zeta_1, \mathcal{K}\zeta_1) + \frac{1}{3}\mathcal{G}(\zeta_2, \mathcal{K}\zeta_2, \mathcal{K}\zeta_2), \end{aligned}$$

and hence, $\mathcal{K}$ *has a unique fixed point in the intersection of* $[-1, 0]$ *and* $[0, 1]$ *which is* a^* *equals zero.*

4. Conclusions

New results on the existence and uniqueness of fixed points in the context of complete generalized metric space have been proved using the novel cyclic contractions of Kannan and Chatterjea type that have been introduced in this study. Importantly, the findings are expansions and generalizations of existing fixed point theorems by Kannan and Chatterjea and their cyclical extensions. Moreover, the results given in this paper will also extend number of previous results on fixed points in generalized metric spaces.

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