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Forecasting BDI Sea Freight Shipment Cost, VIX Investor Sentiment and MSCI Global Stock Market Indicator Indices: LSTAR-GARCH and LSTAR-APGARCH Models

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Abstract: Prediction of the economy in global markets is of crucial importance for individuals, decisionmakers, and policies. To this end, effectiveness in modeling and forecasting the directions of such leading indicators is of crucial importance. For this purpose, we analyzed the Baltic Dry Index (BDI), Investor Sentiment Index (VIX), and Global Stock Market Indicator (MSCI) for their distributional characteristics leading to proposed econometric methods. Among these, the BDI is an economic indicator based on shipment of dry cargo costs, the VIX is a measure of investor fear, and the MSCI represents an emerging and developed county stock market indicator. By utilizing daily data for a sample covering 1 November 2007–30 May 2022, the BDI, VIX, and MSCI indices are investigated with various methods for nonlinearity, chaos, and regime-switching volatility. The BDS independence test confirmed dependence and nonlinearity in all three series; Lyapunov exponent, Shannon, and Kolmogorov entropy tests suggest that series follow chaotic processes. Smooth transition autoregressive (STAR) type nonlinearity tests favored two-regime GARCH and Asymmetric Power GARCH (APGARCH) nonlinear conditional volatility models where regime changes are governed by smooth logistic transitions. Nonlinear LSTAR-GARCH and LSTAR-APGARCH models, in addition to their single-regime variants, are estimated and evaluated for in-sample and out-of-sample forecasts. The findings determined significant prediction and forecast improvement of LSTAR-APGARCH, closely followed by LSTAR-GARCH models. Overall results confirm the necessity of models integrating nonlinearity and volatility dynamics to utilize the BDI, VIX, and MSCI indices as effective leading economic indicators for investors and policymakers to predict the direction of the global economy.

Keywords: BDI; VIX; MSCI; volatility; LSTAR-GARCH; LSTAR-APGARCH; nonlinear time series

MSC: 62P05; 11K31; 62F10; 62M10

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1. Introduction

This study aims to examine the Baltic Dry Index (BDI), the Investor Sentiment Index (VIX), and the MSCI index, which are considered important indicators regarding providing early signals to predict down-turns and crises in the global economy. Among the three indices, the VIX and MSCI are the more commonly known indices in contrast to the BDI. Though the prediction of future fluctuations in markets is highly relevant, the modeling of indices that are considered leading indicators is decidedly challenging. To overwhelm the uncertainty of the future, some researchers debated indicators such as the VIX and MSCI in addition to the BDI and questioned whether the so-called leading indicators provide significant insight, especially for the indices reported in daily frequency.

The VIX is based on future contracts and options to achieve information regarding future behavior in financial markets. The MSCI is calculated through a weighting scheme through the use of developed and emerging stock markets. However, though relatively less attended, the BDI is based on worldwide shipment costs of goods and materials through major shipment routes, and the index is directly related to supply and demand conditions, a characteristic that relates the index directly to economic production. On the other hand, all three indices have a common characteristic: their highly volatile and nonlinear nature, which leads to a loss in efficiency in empirical models which aim at predicting and forecasting these series to utilize them as leading indicators. To this end, the paper aims at the utilization of Logistic Smooth Transition Autoregressive (LSTAR) augmented variants of traditional GARCH (LSTAR-GARCH) in addition to LSTAR augmented Asymmetric Power GARCH (LSTAR-APGARCH) models to capture the regime-dependent nonlinear and volatility dynamics to achieve important tools to predict and forecast the movements in the BDI, VIX, and MSCI indices. The general framework of these indices is evaluated, which will also provide important insights.

Though the index is not well explored in the literature, the BDI has a long historical background dating back to 1744. The BDI was originated by the first committee of 23 transport merchants. This development led to the establishment and development of a company in 1857. The Baltic Dry was developed in 1985 as the Baltic Dry Index (BDI) [1]. The BDI index is calculated by the Baltic Exchange, which is a London-based organization that provides vital information on maritime trade and shipment rates. In its calculation, Baltic Exchange utilizes major routes and shipment rates for various different types of vessel sizes. In the calculation, the shipment costs for the four major vessel sizes, Capesize, Panamax, Supramax, and Handysize, are added to create the BDI composite index. In the weighting scheme, the typical number of ships in each vessel type and their relative daily shipping rates are used in the calculation of the BDI. The Handysize vessels are the smallest and cheapest, whilst the Capesize vessels are the biggest and most expensive. The weighted rates are then determined by the average number of ships in each category, taking into account the size of each of the four vessels. The BDI is measured in points, with higher values corresponding to higher shipping costs and, thus, increased demand for raw materials. Investors, analysts, and traders can use the index to track international trade activities and evaluate the state of the world economy. To provide a more complete view of economic trends, the BDI should be used in conjunction with other data sources since it is only one indication of economic activity. With these characteristics, the BDI has been considered an indicator that effectively represents shipping costs in addition to being taken as keeping and determining the pulse of trade and production activities around the Globe [2].

Among the three indices subject to the study, the BDI index is a relatively less explored index in the literature though the BDI is directly linked to global economic conditions. Since the index is based on the movement of sea transportation, it is also an important indicator for crisis prediction. Nowadays, in addition to being an important source of information for shipping costs of commercial goods, the BDI is also a tool used in economic forecasts.

The BDI is a different index relative to the other indices subject to our research. The BDI is an important index based on shipment costs, and the index is highly linked to global economic demand and supply conditions in the goods markets. While the VIX is based on future options and contracts in financial markets, and the MSCI is directly linked to global stock market conditions, the BDI, as an index linked to the conditions in the international shipments' costs, reacts differently to economic fluctuations. The BDI is under the influence of port and dock costs and placement fees, in addition to transportation costs and is also linked to fluctuations in oil prices. Though the BDI is dependent on these by its structure, these factors also make it exposed to global demand and supply of manufactured products. Due to various shipment routes and different cost structures, the BDI is formed as a daily collection of average prices of 23 major shipment routes worldwide. In this respect, the BDI is not influenced by economic indicators such as the unemployment rate, inflation and oil prices which could be affected or manipulated by states and speculators. However, the BDI is difficult to manipulate or influence. The conduct of the BDI being affected by factors such as supply and demand relationships can be shown as a reason for this. Another reason is that the number of ships available on earth that can affect or manipulate demands is limited. Building more ships will result in more costs [3].

The BDI is a vehicle and an indicator with five different sub-indices for the investors [3]. After the economic recession and during economic growth, the increase in demand for raw materials increases investments in this area, and consequently, growth in shipping volumes is seen [4]. In general, when combined with changes in global demand for raw materials used in production [5], inflexible vessel supply has strong effects on the conditional average of the BDI index and sharp fluctuations in its variant to cope with the short-term increase in demand due to its lack of flexibility [6].

The Cboe Volatility Index (VIX), introduced by the Chicago Board Options Exchange (CBOE) in 1993, is the second index to be explored in the study. The VIX is a real-time index that indicates the market's expectations for the relative strength of the S&P 500 Index's near-term price fluctuations (SPX). The index generates a 30-day forward estimate of the implied value of option prices in the calculation. Therefore, the index represents implied volatility in the prices of SPX index options with close expiration dates. The VIX is frequently considered a good indicator of market emotions and expectations and, particularly, the level of fear among traders measured by the VIX volatility index. The level of volatility or risk rises in direct proportion to how dramatically the index's price swings occur. In the mathematical calculation of the VIX index, the implied value of option prices is the probability that the price of a particular stock will increase or decrease by an amount sufficient to reach the strike price, also known as the exercise price, which determines the price of options. Only SPX options with expiration dates that are between 23 and 37 days are taken in the calculation of the index. Further, investors are known to utilize financial derivatives, including ETFs, VIX futures and options on fluctuations in the VIX. Without presenting the mathematical formula here, the formula theoretically operates as follows: the weighted values of numerous SPX puts and calls over a wide range of strike prices are combined to calculate the anticipated volatility of the S&P 500. All such qualified options must have legitimate, non-zero bid and ask prices that reflect the opinion of the market regarding which options' strike prices will be reached by the underlying equities in the time left before expiration [7].

The VIX has gained popularity among investors and has been modeled by empirical methods as a subject of a lot of research in literature. The VIX has been commonly known as the investor sentiment or fear index and reacts sharply to changes in trends, especially to high levels of market turmoil. Past experiences have shown that sharp inclines in the VIX effectively point to harsh drops in global financial market indices. An example is the 2008 sharp incline in the VIX before the 2009 Global Crisis. Furthermore, though the VIX is among the largest indices in terms of volume and liquidity among all other financially volatile indices, its efficiency in the predictability of financial crises is questioned in the literature [8]. The third index, MSCI, is an index generally considered an important barometer or indicator of the health of the global economy in addition to being considered as representing the performance in the global stock markets. If the history of the MSCI is investigated, it is observed that the MSCI dates back to 1969, the introduction by Capital International of several stock indices to reflect international markets outside the United States, making it among the first global stock indices. Morgan Stanley then purchased the data license rights of Capital International in 1986 and started to use the MSCI acronym instead. As of 2007, the New York Stock Exchange (NYSE) has become an independent public company. Though the MSCI world index has been assumed to be composed of stocks from different countries since then, our investigation shows that the index, and the included sectors in the MSCI world index (MSCI), is, in fact, an index that constitutes around 85% of total capitalization of stock markets in the world with a major share belonging to USA stocks. Since the MSCI captures large and mid-cap representation across 23 stock markets of developed countries (DC), with 1508 constituents, the index covers approximately 85% of the free float-adjusted market capitalization. As a result, though the included stocks are not from all over the world, the MSCI index is widely accepted as a benchmark for international stock portfolio positions. In addition to the MSCI, Morgan Stanley publishes the MSCI All Country World Index (MSCI ACWI). Comparatively, while the MSCI's coverage corresponds to 100% of indices from DC countries, the MSCI ACWI is 88% DC and 12% major emerging market stocks of the MSCI Emerging Market Index (MSCI EMI). In terms of market capitalization, large-cap stocks take the largest share in both the MSCI World and MSCI ACWI, and both indices have very close weighting schemes. The shares of sectors in both indices are very close in terms of share and the included sectors. In both indices, included sectors with the largest shares are financial, information tech, consumer durables, industrial and health care sectors with 14.6%, 20.8%, 11.6%, 9.5%, and 12.9% weights in the MSCI ACWI and 13.5%, 21.1%, 11.3%, 9.95%, and 14.02% for the MSCI World. Further, though indices are reported for the world, the majority of stocks are USA oriented. The share of shares of stocks from USA markets is 69.7% in the MSCI World and 61.9% in MSCI ACWI. A major difference is Chinese stocks. While no Asian stocks are included in the MSCI World, the share of Chinese stocks is 3.49% in MSCI ACWI. In both indices, stocks with large capitalizations are included to both indices. The mean cap of stocks is 30 and 18 billion US dollars for MSCI World and MSCI ACWI indices. A third index is the MSCI Emerging Markets Index. However, its coverage is largely for the Asian markets instead of the major emerging markets of the world. China's share is 33.4%, Taiwan's is 14.4%, in addition to South Korea with a share of 11.8%, India with 13% and Brazil with 5% only. Hence, the sectoral coverage of the index is largely high-tech and blue-chip stocks. Under these factors, all three indices are indices that are drivers of the world stock market fluctuations.

MSCI indices are considered indicators of investors' behavior and expectations. Inclines in the MSCI index suggest that the investors are optimistic about the global economy and the good performance of the stock markets around the globe. Conversely, falls in the index are attributed to poor performance in the global economy and stock markets in addition to being linked to investors with negative expectations about future performances. Because all MSCI indices are reviewed every quarter and re-balanced on the basis of the number and reflectivity of shares twice a year, all ETF and investment funds follow these indices as a benchmark in addition to making similar and also identical changes to the shares of assets in their portfolio. For this reason, the MSCI index and related indices have significant influence and power in affecting and changing the financial market, and their influence on diverting the markets cannot be diminished. Moreover, it is common to observe that the recently added stocks to the index increase their share price, whereas the opposite, the removal of a stock from the index, generally reduces the share price. Therefore, it is also subject to discussions and criticism whether the index is open to manipulation or not.

This study also hypothesized that the BDI MSCI and VIX indices are pioneering real and financial indicators, respectively, with good predictive performances. However, the predictability and forecastability of the three indices are challenging. For the reasons stated, the sample covers the period of 1 November 2007–30 May 2022, which includes the Global Crisis and, recently, the COVID-19 economic shutdown. The three indices are explored with a family of non-linear volatility models, which aim at their regime-dependent asymmetric behavior in conditional volatilities in these indices that make the predictive power of econometric models for these series challenging. As will be seen, all series are subject to nonlinear and leptokurtic distribution with heavy tails with smooth changes between distinct regimes. As a result, the BDI, VIX and MSCI indicators are modeled with smooth logistic transition autoregressive—generalized autoregressive conditional heteroskedasticity (LSTAR-GARCH) and its generalization to nonlinear LSTAR type asymmetric power GARCH (LSTAR-APGARCH) model which are shown to produce effective results in forecasting relative to their single-regime variants. With this respect, we propose that to utilize the three indices as leading economic indicators; they should be efficiently forecasted in order to achieve effective investment strategies along with achieving effective economic policy recommendations. Therefore, the paper focuses on the examination of whether the BDI, MSCI and VIX indices retain forward-looking properties. The paper has three central contributions. First is the modeling of the BDI, VIX and MSCI indices concurrently for comparative purposes by integrating the nonlinear characteristics of these indices. The second is to recommend a hybrid modeling method that benefits from the logistic transition auto-regressive model (LSTAR) and the STAR-GARCH model proposed in the literature to be further extended to the asymmetric power GARCH model (APGARCH) to achieve the LSTAR-APGARCH model. The third is in terms of sample size. The sample covers one of the largest in the literature, especially for the BDI, which corresponds to 1 November 2007–30 May 2022 with daily series. The sample has the potential to provide important information given the existence of the recent COVID-19 and 2008 Great Recession periods in addition to recessionary and expansionary business cycle periods. As a result, the period is expected to cover quite diverse data-generating processes, possibly to be modeled with nonlinear regime-dependent models instead of traditional linear time series approaches.

The paper is organized as follows. The literature review is given in Part 2. The data, unit root tests, nonlinearity tests and chaos tests are in Part 3. Methods are presented in Part 4, where the econometric methodology of LSTAR-GARCH and LSTAR-APGARCH models are presented. The results section is given in Part 5, which also includes a discussion section. The conclusion is given in the last section.

2. Literature Review

If an overlook to the empirical papers with econometric approaches is evaluated, among the indices to be analyzed in this study, papers focusing on modeling the BDI is rather limited. Especially lack of studies focusing on the utilization of daily BDI is noteworthy. In contrast, the body of research with empirical models of forecasting the VIX and MSCI is relatively less scarce. In the first stage of this section selected research focusing on the BDI will be provided for their distinguished contributions.

By studying the predictability of the futures of the dry load market on the main roads and by utilizing monthly changes in the BDI, Chen et al. showed that spot prices and trade routes do not have long-term relationships with the size of all three shipment routes [9]. Leonov and Nikolov used wavelength and nonlinear artificial neural network models to estimate dry load transport fluctuations and transport rates of the Baltic Panamax 2A route and Baltic Panamax 3A route [10]. Duru et al. exploited the Fuzzy Integrated Logical Forecasting model with an integrated logic estimation model to investigate the performance of the effect of dry cargo transport's limited vessel rental contract rates over time [11]. Chen and Wang explored the freight market indices, and their findings with the EGARCH model underlined leverage effects in their volatility, and their findings emphasized the asymmetric volatility response of freight indices amid negative and positive shocks [12]. Geman and Smith explored different shipping markets in financial literature and put forth the rates of the freight that drive the indicator and their relationship with the BDI in the global economy [4]. For the BDI, Geman and Smith's findings emphasized the existence of structural breaks between 1988 and 2003 in mid-2003 with their Constant Elasticity of Variance model; therefore, their findings underlined the necessity of utilization of models before and after the break periods separately [4]. Oomen's study reviewed the impact of BDI returns on the futures trading returns and the potential impact of the BDI on the stock market indices of 11 developed countries [3]. The study was pivotal in displaying the association between the BDI and stock market returns as well as showing the BDI as an important predictor of these markets [3]. An important finding of Oomen that we should emphasize is the determination of the predictive power of the BDI in modeling financial markets in the world [3]. Further, the findings also underlined the existence of structural changes that hamper the efficiency of the models, and Oomen's suggested division of the sample into four sub-periods proved significant results [3].

The existence of nonlinearity in the BDI is a central characteristic of the BDI. The index is directly related to shipment costs that are under the influence of business cycles in the world. Various studies have hinted at the nonlinear behavior in the BDI. Among these, Papaillias and Thomakos stated that the annual growth of the BDI is subject to cyclical behavior, which became disturbed during the 2007 crisis, and the cyclical component of the BDI series could be used for forecasting with the use of spectral analysis and trigonometric regressions [13]. The findings of [13], in this respect, pointed at not only nonlinearity but also structural changes in time in the distributional properties of the BDI. Lin and Sim's research investigated the effects of international trade on GDP growth in less developed nations, and showed that the BDI is an effective measure of the costs of trade transport [2]. To investigate the volatility of BDI, ref. [14] proposed a method based on the EMD approach, which parsed the original freight price series into several independent internal modes to improve the BDI prediction of neural networks models. The proposed method undertakes nonlinearity in these series more effectively than the traditional linear econometric approaches [14]. Further, [6] debated the challenges of the BDI's predictability due to nonlinearity in addition to putting forth the ability of the BDI to model economic growth. Regarding nonlinearity, ref. [8] put forth the threshold effects in the daily BDI and VIX, and their findings underline regime-specific mean and volatility behavior in daily BDI price changes with TAR-TR-GARCH models. To this end, ref. [8] underlined the necessity to augment the GARCH models with TAR-TR-GARCH models to capture nonlinearity in the BDI.

A set of literature has focused on multivariate models to investigate the relations of the BDI with various variables. The traditional ARIMA time series, fractional ARIMA and various machine learning methods are evaluated in forecasting BDI and it is shown that no significant improvement could be achieved and results improve only if model forecasts are combined [15]. Ruan et al. investigated cross-correlations between the BDI and crude oil spot prices using the cross-correlation analysis (MF-DCCA) model, which benefits from multifractal trending [16]. Through the MF-DCCA, the cross-correlation between each pair of BDI and crude oil prices is demonstrated, which is strong in the short term, but the cross-correlations in the long term are weakly permanent [16]. By utilizing Markov-Switching VAR, the nonlinear relations between the BDI, economic growth and gold prices are determined in addition to providing the BDI as an important tool to model economic series and precious metals [17]. The impact of the BDI on commodity futures and foreign exchange and stock markets was investigated using the BEKK-GARCH-X model for a sample covering 1 October 2007 to 31 October 2018, and findings confirmed spillovers between the BDI and financial markets [18]. Kamal et al. examine short and long-term forecasts of the BDI with the proposed DERN deep learning model [19]. They showed that forecasts improve with the BDI, and modeling nonlinearity with neural networks provided tools to overcome uncertainty for market participants and shipowners [19]. Their findings also underlined the need to utilize nonlinear models for short and long-term maritime business decisions and to avoid market risk [19]. The decline in the trend of the BDI before an economic crisis is an important phenomenon [20]. The decline in the trend of the BDI became evident during the 2008 global crises and afterward, the commodity and stock markets took a hard hit [20]. The study also emphasized existence of a similar pattern before and during COVID-19, which underlined the predictive power of the BDI as a leading indicator for markets [20]. Yang et al. [21] proposed the necessity of nonlinear modeling of the Baltic Panama Index with support vector machines and wavelet transformations, the former for augmenting forecasts and the latter for the denoisification of data. Han et al. examined the predictive potential of BDI and various sub-indices in forecasting exchange rates, and they emphasized the efficiency of the BDI in in-sample and out-of-sample forecasts in modeling economic variables [22]. Zhang et al. inspected the BDI's forecasting potential with various nonlinear models [23]. They emphasized the importance of neural network models in capturing nonlinearity in addition to suggesting a combination of methods, i.e., dynamic fluctuation nets and AI, to increase the effectiveness in modeling and forecasting the BDI [23]. Chen et al. investigated the effects of BDI between crude oil and commodity markets with the Copula-VAR-BEKK-GARCH-X model [24]. Accordingly, dependence and spillover behavior between the BDI, iron, and brent oil are time-varying. Hence the results revealed the necessity of nonlinearity modeling [24].

A recent set of studies have associated freight transport costs with COVID-19. Michail and Melas explored COVID-19's impacts on the sea transport industry. Their findings emphasized that freight prices are highly affected caused from conditions for dry load, clean and dirty transfer tankers through VAR models and GARCH models [25]. The findings of [25] further confirmed pandemics' effects on the freight industry and information obtained through volatility modeling could be used as a hedge for risk in stock markets. Ref. [26] showed that COVID-19 had strong effects on the container transport operations of firms in addition to effects on their financial structure and employment in the sector. Ref. [27] investigated the effects of COVID-19 on the world stock market, proxied with the World-MSCI, world energy market, proxied with the MSCI-Energy index and freight costs measured with the BDI, by using structural VAR models for 21 January 2020–26 February 2021 period. Though their findings revealed negative effects of COVID-19 on all three variables, among these, the MSCI and MSCI-Energy were most badly hit by COVID-19 relative to a lessened effect on the BDI [27]. However, the lessened effect could also be due to the unexpectedly sharp decline in oil prices during the COVID-19 shutdown, which counteracted the sharp inclines in shipment costs [27]. Chen et al. showed the significance of spillovers of volatility between oil, iron-ore prices and BDI during COVID-19 [24]. Ref. [28] explored the effects of COVID-19 on the S&P index in the US in addition to global economic activity proxied by the BDI, and the spread among 10-year treasury and federal funds rates with SVAR models. His findings underlined the negative effects of COVID-19 [28]. The lockdown policies had strong effects on shipping during COVID-19 [29]. Zhao et al. showed that the global dry carriage was largely affected by quarantine policies during the second month of COVID-19 when the BDI dropped about 35.5% compared with the same month a year before [29]. Ref. [30] stress the impact of COVID-19 on BDI volatility during COVID-19. With GARCH-MIDAS models, their findings signified that as the number of infected inclined, so did the BDI volatility in addition to similar results obtained for sub-variables [30]. The infections also influenced the volatility of shipment cost variables such as crude oil price, freight rate, container idle rate, global port calls, and port congestion levels [30]. Hence COVID-19 infections significantly influenced the volatility of the BDI and other shipment variables [30]. In order to achieve the goal of cost or risk control, the shipping industry is suggested to benefit from the development of the epidemic in numerous nations as a reference in addition to the utilization of models aiming at modeling the trend of and volatility of the BDI, which is found to be under the influence of changes in the confirmed Covid cases [30]. Jeris and Nath, by using the wavelet approach and daily data for 21 January 2020–30 October 2020, showed that COVID-19 inclined volatility of the BDI in addition to its similar effects on economic policy uncertainty, crude oil price volatility and banking stock market volatility in the USA [31].

The VIX is among one of the indices studied extensively in the literature. Copeland and Copeland (1999) explored the impacts of the VIX on large and small-capitalization stocks and inclines in the daily VIX exhibits effects in the following days on portfolios containing large-capitalization stocks and the VIX was shown to help on investor decisions once used effectively in their portfolios [32]. Ref. [33] debated the return and volatility relationship in the implied volatility indices. According to [33], sharp inclines in the VIX point at oversold markets and future returns are positive (negative) after the periods with very high (low) levels of VIX. Ref. [34] advocated TAR-type threshold responses of the S&P stock market to low investment fear and high investment fear regimes measured with the VIX index. These results confirmed that the best performance is achieved if the VIX determines the threshold variable compared to the other four candidate indicators [34]. By using a sample covering 1995–2022, a recent study [35] demonstrated that the VIX successfully estimates future stock market volatility in addition to the VIX being confirmed as a tool for portfolio insurance prices [7]. Consequently, due to the fact that the S&P 500 index options market is affected by hedgers that buy index put options, risk inclines hence, leads to investor sentiment increases with anticipations of a potential drop in the market [7]. By the use of linear VAR models, Ref. [36] investigated the VIX and stock market ETF relation and emphasized the negative effects of VIX shocks on S&P stock returns in addition to European ETF markets. Nevertheless, in addition to putting forth the correlation between the markets and VIX, in the spirit of the seminal studies of [32,33], high and rising levels of VIX were linked with a future rebound in the stock markets for the investors [36]. The lead-lag relation between the stock markets and VIX was noted to co-move significantly [36]. Further, if the VIX is in a high-volatility regime, the negative impact of the VIX could be comparatively more severe [33]. One of the early signs of such a finding was Guo and Wohar, who employed econometric techniques that put forth the mean shifts and, therefore, nonlinearity in the selected implied volatility indices in addition to shifts of the standard deviation of the VXO and VIX indices [37]. With this respect, their identified subperiods due to shifts are pre-1992, 1992–1997, and post-1997 periods [37].

Regarding the market risk and VIX relationship, Durand et al. (2011) showed that changes in the VIX affect market risk premium. An additional factor to the VIX is shown as the value premium by the use of the three-factor model within the Fama and French factors approach [38]. The VIX is also calculated and examined for India [39], and the study displayed that the VIX is a gauge of investor fear when markets are declining and when stock market volatility is rising [39]. Furthermore, the expected volatility is influenced by the actual return volatility signifying a direct association between VIX inclines and market turmoil [39].

A selected set of literature will be highlighted to evaluate the links between the MSCI and the world stock market performance in addition to forecast performance for the MSCI. Recent empirical research [40] on the MSCI world stock index utilized VAR and EGARCH models and demonstrated the efficiency of the MSCI in capturing global economic crisis by employing daily dataset covering 3 June 2002–22 March 2013 [40]. Further, Ref. [41] applied four asymmetric-GARCH models, including NA-GARCH, TGARCH, GJR-GARCH, and AV-GARCH, to model and forecast the MSCI's volatility. Within a risk exposure framework in the context of value-at-risk, their findings emphasized nonlinearity and asymmetry in the MSCI, in addition to confirming these factors improving the performances of GARCH models in forecasting MSCI [41]. The MSCI and MSCI-Energy sector indices were modeled for the effects of COVID-19 on these series in addition to the BDI as another leading indicator [27]. By using SVAR models, their study confirmed the negative impacts of COVID-19 on investigated leading indicators [27]. The literature above indicates that the modeling of the MSCI, in addition to the BDI and VIX, also necessitates taking nonlinearity and volatility in addition to leverage effects into consideration.

Though considered nonlinear in nature, the traditional Autoregressive Conditional Heteroskedasticity (ARCH) model [42] and Generalized ARCH (GARCH) model [43] do not take regime-dependency into account. For this purpose, the models in our study benefit from STAR-GARCH models [44] and their extensions. The STAR-GARCH model assumes STAR-type nonlinearity in the conditional mean process, and the conditional variance follows the GARCH process and under outliers, robustness of QMLE conditions were shown by [44]. Our study follows the LSTAR-GARCH and LSTAR-APGARCH modeling of [45]. STAR-GARCH models are employed to investigateSTAR-type nonlinearity in Bank of Finland's Banking and Finance Indices [46]. Various studies of Bildirici and Ersin highlight nonlinear volatility in crude oil prices, which is also an important cost factor for BDI dry cargo freight shipment costs index. Among these, Ref. [47] showed that modeling and forecasting crude oil prices requires LSTAR-type nonlinearity in the conditional mean and variance processes. The findings of Bildirici and Ersin in [48] confirmed the integration of fractional integration into the LSTAR-LST-GARCH family of models, including LSTAR-LST-FIGARCH and LSTAR-LST-FIAPGARCH, which benefit from STAR-type nonlinearity [47]. For forecasting gains in future crude oil prices, Ref. [49] proposed a new set of STAR-GARCH models with exponential AR components, and the results confirmed better forecasting capabilities of the STAR-GARCH, LSTAR-GARCH and ESTAR-GARCH models over single-regime GARCH models in modeling and forecasting exchange rates of the British pound and Botswana Pula against the US dollar [49]. Bildirici et al. [8] examined the regime-dependent volatility of the daily VIX and BDI with TAR-TR-GARCH models. LSTAR-GARCH and LSTAR-APGARCH models will be evaluated in the next section.

3. Data

3.1. Data Sources

BDI, VIX, and MSCI data were collected from the Bloomberg system. The sample covered daily working days covering the 1 November 2007–30 May 2022 period. The variables were subject to logarithmic transformation and were denoted as $lbdi_t$, $lmsci_t$ and $lvix_t$ for logarithmic BDI, VIX and MSCI, respectively. To obtain daily percentage (%) returns, the logarithmic series were first differenced, and these were denoted as $\Delta lbdi_t$, $\Delta lvix_t$ and $\Delta lmsci_t$ where Δ refers to the first difference operator. Therefore, the analyses in this study were conducted with $\Delta lbdi_t$, $\Delta lvix_t$ and $\Delta lmsci_t$ series representing the daily % returns of BDI, VIX and MSCI, respectively.

3.2. Descriptive Statistics

The descriptive statistics of the dataset and the results are included in Table 1. Skewness and kurtosis statistics reveal that series are skewed and in addition, subject to leptokurtic distribution with heavy-tails. As a typical, skewness = 0.67 and kurtosis = 7.36 for $\Delta lvix_t$ daily series. This also translates into non-normality of their distribution in all series analyzed. The Jarque–Bera test of normality tests null hypothesis of H_0 normally distributed time series against non-normality. The JB test statistic follows χ^2 (q) distribution with degrees of freedom q = 2. For $\Delta lvix_t$, $\Delta lbdi_t$ and $\Delta lmsci_t$, the JB = 6003.86, 3816.86 and 36,310.69, larger than the critical value 5.99 at 5% significance level, H_0 is strongly rejected for all series evaluated. Once the source is evaluated, not only the skewness but the excess kurtosis in the variables lead to the finding of leptokurtic distribution with heavy tails, as indications of heteroscedasticity, leading to the necessity of modeling the series with volatility models.

Table 1. Descriptive Statistics for VIX, BDI and MSCI Daily % Returns.

	Sd.	**Sk.**	**Kr.**	**JB (p)**
$\Delta lvix_t$	0.0635	0.6735	7.3591	6003.86 (0.000)
$\Delta lbdi_t$	0.0578	−0.3961	10.9020	3816.86 (0.000)
$\Delta lmsci_t$	0.0720	−0.7457	18.3185	36,310.69 (0.000)

Notes. Sd, Sk and Kr represent standard deviation, skewness and kurtosis statistics. JB is the Jarque–Bera test statistic. JB > 5.99, the critical Chi-square value, leads to the rejection of H_0: series is normal distribution.

3.3. Unit Root Test Results

$lbdi_t$, $lvix_t$ and $lmsci_t$ series were examined with linear and traditional Phillips–Peron (PP), Augmented Dickey–Fuller (ADF) unit root tests in addition to non-parametric KPSS, the Kapetanios–Shin–Snell (KSS) nonlinear unit root test and Engle–Lee (Fourier–ADF) unit root test. In the literature, ADF and PP are known to have size distortions for nonlinear series, and the KPSS test is known to perform better under such cases. In contrast to ADF and PP tests assuming unit root under the null, the KPSS test assumes stationarity under the null hypothesis. The KSS test extended the ADF test to STAR-type nonlinearity to circumvent smooth forms of nonlinear forms captured with an exponential transition function. The Fourier–ADF test was also performed for confirmatory purposes. The test extended the ADF test to Fourier transformation at various k dimensions to control various forms of smooth structural changes with unknown break dates. The results are reported in Table 2.

Table 2. Results of Linear and Nonlinear Unit Roots and Stationarity Tests.

	ADF	PP	KPSS	KSS	Fourier–ADF
$lvix_t$	−1.78	−2.0019	1.7499	−1.3184	−1.29
$\Delta lvix_t$	−6.27 ***	−7.2489 ***	0.0465 ***	−7.5493 ***	−6.81 ***
$lbdi_t$	−1.34	−2.0027	1.4868	−1.4072	−1.44
$\Delta lbdi_t$	−5.37 ***	−6.5803 ***	0.0323 ***	−5.0719 ***	−5.12 ***
$lmsci_t$	−1.65	−1.7836	1.3591	−1.5168	−1.44
$\Delta lmsci_t$	−4.69 ***	−5.8227 ***	0.0105 ***	−5.0626 ***	−4.98 ***

Notes: The optimal lag length in ADF and KSS tests is obtained by Schwarz information criterion (SIC). For the PP and KPSS tests, Newey–West bandwidth and Bartlett kernel methods are utilized. Critical values for the ADF and PP tests are −2.58 for 10%, −2.89 for 5%, and −3.49 for 1%. For KPSS, 0.347 for 10%, 0.463 for 5%, and 0.739 for 1% significance levels. For the KSS test, critical values are −3.48 for 1%, −2.93 for 5%, and −2.66 for 10%. For all tests, the trend is found to be insignificant, and the tests are analyzed under the assumption of intercept-only. *** shows statistical significance at 1% significance level.

According to the ADF and PP unit root tests, $lbdi_t$, $lvix_t$ and $lmsci_t$ series contained unit roots at levels and became stationary once first differenced at conventional significance levels. The KPSS test confirmed the stationarity of the first differenced series. The KSS and Fourier ADF test results confirmed that the series were integrated of order one. As a result, the further analyses were conducted with first differenced with $\Delta lbdi_t$, $\Delta lvix_t$ and $\Delta lmsci_t$ series. The KSS and Fourier–ADF test findings also gave information regarding the series following nonlinear processes. The Fourier terms in the Fourier–ADF test were statistically significant. The STAR form estimation of the ADF testing model also led the gamma parameter to be significant.

3.4. BDS Nonlinearity Test Results

The BDS test benefits from chaos literature and tests independence under the null hypothesis at various correlation dimension and the test is accepted as a test of nonlinearity. The test results are reported in Table 3. The BDS test results favored the rejection of the null hypothesis of independence at 1% significance level for all series investigated. Therefore, the BDS test results favor nonlinearity in the examined BDI, VIX and MSCI series.

Table 3. BDS Test Results.

Variable:	$\Delta lvix_t$		$\Delta lbdi_t$		$\Delta lmsci_t$	
Dimensions	z-Statistic	p	z-Statistic	p	z-Statistic	p
2	13.76719	0.00	17.25177	0.00	20.69067	0.00
3	18.25974	0.00	18.47797	0.00	24.44491	0.00
4	20.65601	0.00	19.91302	0.00	27.75032	0.00
5	22.40098	0.00	21.95053	0.00	30.71721	0.00
6	23.83381	0.00	24.70978	0.00	33.66314	0.00

Notes. z is the z statistic of the BDS test and p is the p-value.

3.5. Chaotic Behavior Test Results

In this section, the BDI, VIX and MSCI indicators are evaluated for the largest Lyapunov exponent (LE), Shannon entropy (SE) and Kolmogorov entropy (KE) tests. The test results are given in Table 4 below.

Table 4. Chaotic Behavior Test Results.

	$\Delta lvix_t$	$\Delta lbdi_t$	$\Delta lmsci_t$
Largest LE	0.297	0.416	0.359
Shannon entropy (SE)	0.038	0.053	0.035
Kolmogorov entropy (KE)	0.041	0.074	0.033
Existence of chaotic behavior			
Decision:	Yes	Yes	Yes
Uncertainty			
Decision:	Yes	Yes	Yes
Eckmann–Ruelle condition			
Decision:	Yes	Yes	Yes

The largest LE was significantly positive for all series analyzed and reported in Table 4. As a result, the relevant LE exponents favor chaotic structure. However, given that for all series LE < 1, the form of chaotic structure in the series is not deterministic chaos. Under these conditions, the predictability of the analyzed series is expected to be very low. The findings emphasize that the series follow chaotic structures and is also subject to nonlinear stochastic processes. SE and KE results are reported in Table 4, and give significant information regarding the random processes, uncertainty and complexity of the series analyzed. In the case of entropy statistic = 1, the finding suggests that the series follows random processes or uncertainty. In case of the entropy measures reach 0, this finding would suggest the opposite, perfect certainty. Given that all SE and KE statistics for the series investigated are not zero, findings do not favor certainty, in addition to pointing at the existence of randomness and uncertainty.

The overall findings led to the necessity of modeling the investigated variables with nonlinear techniques, which also capture nonlinear volatility dynamics. In this respect, the series was modeled with LSTAR-GARCH and LSTAR-APGARCH models, which allow regime changes between two or more distinct regimes of GARCH and asymmetric power GARCH processes to be governed with logistic smooth transition functions.

4. Methods

The traditional time series models assume linearity in parameters and variables of econometric models. Further, though nonlinear in nature, the traditional ARCH model and GARCH (GARCH) model achieve nonlinearity through variables. The models are considered generally nonlinear; however they assume linearity in parameters, therefore, they will be considered as single-regime and linear due to their parametric structure. Further, a set of GARCH models link parameters of a GARCH family models to nonlinear functions to achieve nonlinear-in-parameters models which allow regime-dependent parameters.

Among these models, the study utilized Smooth Transition Autoregressive (STAR) type nonlinearity in the parameters of the volatility models. As will be seen, the models utilized in this study also included estimation of thresholds in BDI, VIX and MSCI % daily returns while allowing smooth transitions in contrast to [8] and logistic transitions in light of [48]. In the next section, their single-regime variants, the APGARCH and GARCH models are presented. Afterward, following the evaluation of the STAR-based volatility models, STAR, ST-ARCH, ST-GARCH, LSTAR-GARCH and LSTAR-APGARCH models will be evaluated.

4.1. Single-Regime GARCH and APGARCH Models

Following the discussion above, in this study, the BDI, VIX and MSCI indices were modeled with LSTAR-GARCH and LSTAR-APGARCH models. The LSTAR-GARCH and LSTAR-APGARCH models are obtained through three steps.

In the first step, we present single-regime APGARCH and GARCH models [43]. I APGARCH(p,q) model of orders p and q are defined as [47],

$$\xi_{t,\gamma} = \frac{c}{1-\sum_{j=1}^{q}\alpha_j} + \sum_{i=1}^{p} b_i(|X_{t-i}| - d_i X_{t-i})^{\gamma} + \sum_{i=1}^{p} b_i \sum_{k=1}^{\infty}\sum_{j_1=1}^{q}\ldots\sum_{j_k=1}^{q}\alpha_{j1}\ldots\alpha_{jk}\left(|X_{t-i-j_1-\ldots-j_k}| - d_i X_{t-i-j_1-\ldots-j_k}\right)^{\gamma} \quad (1)$$

The model relaxes the power term γ in the conditional volatility processes σ_t^{γ} as a parameter to be estimated. $\xi_{t,\gamma}$ is the conditional variance and X_t is a financial time series. For the APGARCH(p,q) model in Equation (1), the stability conditions are

$$\sum_{i=1}^{p} b_i E\{(|\varepsilon_t| - d_i\varepsilon_t)^{\gamma}\} + \sum_{j=1}^{q}\alpha_j < 1 \ and \ \sum_{j=1}^{q}\alpha_{j_{1\le j\le q}} < 1 \quad (2)$$

where b_i and α_j are ARCH and GARCH parameters for i = 1,2, ... ,p & j= 1,2, ... q. The model could be estimated with the Quasi Maximum Likelihood Estimator (QMLE). The general form of the APGARCH model is also written as [8],

$$\varepsilon_t = h_t v_t, v_t \sim N(0,1) \quad (3)$$

$$h_t^{\delta} = \omega + \sum_{i=1}^{n}\alpha_i\left(|\varepsilon_{t-i}| - \theta_i\varepsilon_{t-i}\right)^{\delta} + \sum_{i=1}^{r}\beta_i\sigma_{t-i}^{\delta} \quad (4)$$

Here, h_t^{δ} is the conditional variance with the non-negative power term $\delta > 0$ for the standard deviation h_t. Note that h_t is subject to Box–Cox transformation. θ_i represent the leverage effect imposing the model to differentiate the impacts of negative and positive shocks asymmetrically: for positive (negative) values of θ_i, past negative (positive) shocks have a stronger impact on the current conditional volatility compared to the past shocks [50]. The shocks are also interpreted as good news $\varepsilon_{t-i} > 0$ and bad news $\varepsilon_{t-i} < 0$ for future volatility in forecasting. The reason is conditional variance not only depends on the magnitude but also the sign of ε_t [50].

It should be noted that the model nests several models, including EGARCH, NA-GARCH and GARCH. Under certain conditions, the APGARCH model reduces to the GARCH model. In the case of a power term equal to δ = 2 and restricting the model to no leverage effects with θ_i = 0, the APGARCH(p,q) model reduces to GARCH(p,q) in Equation (4) [50].

4.2. STAR Models

The study primarily benefits from the STAR model [51] given as

$$y_t = (\Phi_{11} + \sum_{i=2}^{r}\Phi_{1i}y_{t-i+1})(1 - G(s_t;\gamma,c)) + (\Phi_{21} + \sum_{i=2}^{r}\Phi_{2i}y_{t-i+1})G(s_t;\gamma,c) + \varepsilon_t \quad (5)$$

where y_t is a time series $G(s_t;\gamma,c)$ is a twice-differentiable transition function which is bounded between [0, 1]. There are three main transition functions in STAR models, the Gaussian, logistic and exponential. Generally, applications utilize logistic and exponential functions following [51,52]. The first-order logistic transition function is

$$G(s_t;\gamma,c) = \frac{1}{1+\exp(-\gamma(s_t - c))} \quad (6)$$

By substituting Equation (5) into Equation (4), STAR model becomes the LSTAR model. In Equation (5), the distance between the optimum transition variable s_t and the threshold parameter c plays a crucial role in determining the sigmoid-type regime transition between two distinct regimes. The second form of $G(s_t;\gamma,c)$ is the exponential function,

$$G(s_t;\gamma,c) = 1 - exp\left(-\gamma(s_t - c)^2\right),\ \gamma > 0 \quad (7)$$

and if $G(s_t; \gamma, c)$ given in Equation (6) is utilized in the STAR model in Equation (4), the model becomes exponential STAR, i.e., ESTAR model [51]. For both LSTAR and ESTAR models, γ is the transition variable defining the speed of transition between regimes, and $\gamma > 0$ is non-negative [52]. The ESTAR model allows the modeling of a middle regime in addition to two symmetric outer regimes. However, while the LSTAR model behaves similarly to a threshold AR (TAR) model due to large positive values of γ, the ESTAR model loses its nonlinear properties for large values of γ, behaving such as a linear AR. Therefore, researchers should be warned if large gamma parameter estimates are obtained. The appropriate transition function is selected following the remaining STAR-type nonlinearity test based on the third-order Taylor expansion around $\gamma = 0$ [52]. This approach yields a sequence of F tests that allow distinction between ESTAR and LSTAR type transition functions [51].

The exponential and logistic functions are first-order transition functions. The literature for higher-order functions in STAR models exists. As a typical example, second-order or higher-order logistic functions are also suggested in addition to multiple transition functions in a model to capture more than two regimes [53]. Other transition functions include the Gaussian function and multiple regime transition functions [53]. STAR models with multiple transition functions are known as Additive-STAR models [54]. Though this paper is limited to STAR-type nonlinearity with logistic and exponential functions in Equations (5) and (6) following the general approach in the literature.

As shown in [51], the type of transition function is determined individually for each modeled time series through a set of F-type remaining nonlinearity tests based on the third order Taylor approximation of the STAR process.

In estimation, initial starting values for γ and c are achieved through grid-search to ease the nonlinear least squares estimation (NLS) [51] where the nonlinear model is

$$y_t = F(x_t, \Phi) + \varepsilon_t \tag{8}$$

and the NLS estimator aims at achieving the optimum parameter set Φ which minimizes the squared error function,

$$\Phi = argmin_\Phi \sum_{t=1}^{T} (y_t - F(x_i; \Phi))^2 = argmin_\Phi \sum_{t=1}^{T} {\varepsilon_t}^2. \tag{9}$$

Assuming ε_t is normally distributed, the NLS and Maximum Likelihood Estimation (MLE) are equivalent. Otherwise, the NLS is interpreted with QMLE. The STAR-GARCH model ensures regular conditions for the moments. It also provides the same conditions and the statistical properties of the QMLE. Lastly, it is convenient to write the LSTAR model by putting logistic transition function into the STAR model as

$$\begin{aligned} y_t = \Phi_1' y_{t-j} \Big\{ 1 - \big(\big(1 + exp - \big(\gamma\big(y_{t-j} - c\big)\big)\big)\big)^{-1} \Big\} \\ + \Big\{ \Phi_2' y_{t-j} \big(\big(1 + exp - \big(\gamma\big(y_{t-j} - c\big)\big)\big)\big)^{-1} \Big\} + \varepsilon_t \end{aligned} \tag{10}$$

which could also be presented in matrix form,

$$y_t = \Phi_1' y_{t-j} \Big(1 - G_t^L\Big) + \Phi_2' y_{t-j} G_t^L + \varepsilon_t. \tag{11}$$

Based on the discussions above, in this paper, the STAR model provides a basis to improve the performance of the GARCH models' prediction and forecasting possibilities for nonlinear BDI, MSCI and VIX daily % return time series. For this purpose, the APGARCH and the GARCH models will be integrated with the LSTAR model in the sections below.

4.3. ST-ARCH and ST-GARCH Models

4.3.1. ST-ARCH

Hagerud [55] presents the smooth transition-ARCH(q) model (ST-ARCH) as an ARCH process with smooth transitions as

$$\sigma_t^2 = w + \left(\sum_{i=1}^{q} \alpha_{1i}\varepsilon_{t-i}^2\right)(1 - F(\varepsilon_{t-1}, \theta)) + \left(\sum_{i=1}^{q} \alpha_{2i}\varepsilon_{t-i}^2\right) F(\varepsilon_{t-1}, \theta) \quad (12)$$

In Equation (12), α_{1i} and α_{2i} are regime-specific ARCH parameters of order q, $F(.)$ is a transition function, and w is the unconditional volatility. In addition, the σ_t^2 conditional variance follows smooth transitions for negative and positive shocks, the ε_{t-1} first lagged residual. Further, the model assumes the threshold as c = 0, therefore, dividing the regression space for negative and positive shocks.

4.3.2. Transition Functions

Following Hagerud [55], two transition functions for the ST-ARCH model are

$$F(\varepsilon_{t-1}, \theta) = \left(1 + e^{-\theta(\varepsilon_{t-1})}\right)^{-1} \quad (13)$$

$$F(\varepsilon_{t-1}, \theta) = \left(1 - e^{-\theta(\varepsilon_{t-1})^2}\right) \quad (14)$$

logistic and exponential functions, respectively. The speed of transition coefficient is non-negative, $\theta > 0$. Transition functions given in Equations (12) and (13) generate different dynamics for the conditional variance. The logistic form in (12) leads to a transition in the conditional variance process and specifies the dynamics of conditional variance differences depending on the types of shocks [55]. For very negative values of $\varepsilon_{t-1} \to -\infty$, the logistic function approaches $-1/2$, and for very positive $\varepsilon_{t-1} \to +\infty$, the function reaches $1/2$. One can also add $+\frac{1}{2}$ to the functional form in Equation (13) to achieve a function in the range of 0,1 instead of the $-1/2$,$+1/2$ range.

The exponential function in Equation (14), on the other hand, is symmetric for negative and positive values of error terms. As a consequence, it generates conditional variance dynamics depending on the severity of innovations. Further, the function defines two symmetric outer regimes as $\varepsilon_{t-1} \to +\infty$ and $\varepsilon_{t-1} \to -\infty$ in addition to a middle regime $\varepsilon_{t-1} \to 0$. For large values of θ, the logistic function approaches the step function (or identity function). As a result, the Logistic ST-ARCH model approaches Threshold-ARCH.

In case of large values being estimated for θ, the Exponential ST-ARCH model's effectiveness in modeling nonlinearity greatly reduces since the exponential function loses the middle regime that acts just as capturing an outlier. Hence, the researcher should question the estimated model with large transition parameters after estimation.

4.3.3. Positivity Constraints to achieve Positive Conditional Variance

In the logistic ST-ARCH model, the positivity of conditional variance is achieved with

$$\alpha_i \geq \frac{1}{2}|\delta_i| \quad (15)$$

and the stationarity for ε_t innovations is achieved for

$$\sum_{i=1}^{q} \left[\alpha_i - \frac{1}{2}|\delta_i| + max(\delta_i, 0)\right] < 1 \quad (16)$$

In the exponential ST-ARCH model, positivity of the conditional variance is obtained with

$$(\alpha_i + \beta)_i \geq 0 \quad (17)$$

in addition, the stationarity of ε_t innovations is realized if

$$\sum_{i=1}^{q}[\alpha_i + max(\delta_i, 0)] < 1 \tag{18}$$

4.3.4. ST-GARCH

The generalized form of the model is called the Smooth Transition GARCH (q,r). The model is presented as [56],

$$\sigma_t^2 = w + \left(\sum_{i=1}^{q} \alpha_{1i}\varepsilon_{t-i}^2\right)\left(1 - F\left(\varepsilon_{t-1}^2, \theta\right)\right) + \left(\sum_{i=1}^{q} \alpha_{2i}\varepsilon_{t-i}^2\right) F\left(\varepsilon_{t-1}^2, \theta\right) + \sum_{i=1}^{r} \beta_i \sigma_{t-i}^2 \tag{19}$$

For the model in Equation (18), the logistic and exponential transition functions in Equations (12) and (13) are updated with the transition variable ${\varepsilon^2}_{t-1}$, a proxy for realized volatility. Therefore, the distinction between negative and positive shocks is omitted. The most distinguishing factor of the ST-GARCH model is the inclusion of σ_{t-i}^2, the GARCH terms. However, the GARCH terms do not switch between regimes.

4.3.5. Positivity of Variance and Stationarity Conditions of ST-GARCH

In the Logistic version of the ST-GARCH model above, to achieve positive-defined conditional variance, the stability condition of GARCH (p,r) is generalized to the ST-GARCH model. Further, the necessary assumptions for the logistic ST-GARCH are [55],

$$\alpha_i \geq \frac{1}{2}|\delta_i| \tag{20}$$

for stationarity of ε_t innovations,

$$\sum_{i=1}^{q}\left[\alpha_i - \frac{1}{2}|\delta_i| + max(\delta_i, 0)\right] + \sum_{j=1}^{r}\beta_j < 1 \tag{21}$$

In the Exponential ST-GARCH model, the positivity of conditional variance necessitates [56],

$$\alpha_i + \beta_i \geq 0 \tag{22}$$

and stationarity condition is [56],

$$\sum_{i=1}^{q}[\alpha_i + max(\delta_i, 0)] < 1 \tag{23}$$

In line with ST-GARCH (p,q) model above, the generalization of GARCH models to STAR-type nonlinearity is suggested [57],

$$\begin{aligned}\sigma_t^2 = &\left(w_{10} + \sum_{i=1}^{p}\alpha_{1i}\varepsilon_{t-i}^2 + \sum_{j=1}^{q}\beta_{1j}\sigma_{t-j}^2\right)\left(1 - F\left(\varepsilon_{t-1}^2\right)\right) + \\ &\left(w_{20} + \sum_{i=1}^{p}\alpha_{2i}\varepsilon_{t-i}^2 + \sum_{j=1}^{q}\beta_{2j}\sigma_{t-j}^2\right)F\left(\varepsilon_{t-1}^2\right)\end{aligned} \tag{24}$$

Comparatively, the ST-GARCH model given in Equation (23) differs from the model in Equation (18) in terms of assuming the transition variable selected as ε_{t-1} instead of ${\varepsilon^2}_{t-1}$. Further, a more recent treatment of the ST-GARCH model with is the ANST-GARCH model of [58]. The model includes asymmetry as well as the GARCH-in-mean specification in the conditional variances [58].

4.3.6. LSTAR-GARCH

The LSTAR-GARCH model assumes regime changes subject to smooth transitions defined with the logistic transition function both in the conditional mean and in the conditional variance of a time series y_t. The LSTAR-GARCH(p,r) model is given as

$$y_t = \phi_1' y_{t-1} \times (1 - G(\varepsilon_{t-1}; \gamma, c)) + \phi_2' y_{t-1} \times G(\varepsilon_{t-1}; \gamma, c) \tag{25}$$

$$\varepsilon_t = \delta_t v_t, v_t \sim N(0,1) \tag{26}$$

$$\begin{aligned} \sigma_t^2 = &\left(\omega_1 + \sum_{i=1}^{p} \alpha_{1i}\varepsilon_{t-i}^2 + \sum_{j=1}^{r} \beta_{1j}\sigma_{t-j}^2\right) \times (1 - G(\varepsilon_{t-1}; \gamma, c)) + \\ &\left(\omega_2 + \sum_{i=1}^{p} \alpha_{2i}\varepsilon_{t-i}^2 + \sum_{j=1}^{r} \beta_{2j}\sigma_{t-j}^2\right) \times G(\varepsilon_{t-1}; \gamma, c) \end{aligned} \tag{27}$$

$$G(\varepsilon_{t-1}; \gamma, c) = \frac{1}{1 + e^{-\gamma(\varepsilon_{t-1} - c)}} \tag{28}$$

In the LSTAR-GARCH model given in Equations (25)–(28), the conditional mean of y_t time series is modeled as an LSTAR process in Equation (25), the remainder ε_t is decomposed into normally distributed v_t and heteroskedastic δ_t in Equation (26) which follows a two-regime conditional variance process given in Equation (27). Further, the regime transitions are subject to logistic $G(\varepsilon_{t-1}; \gamma, c)$ in Equation (28), which is a function of the width and sign of the spread between ε_{t-1} and c. In addition, γ the parameter that characterizes the speed of transition between regimes. In the model above, we assumed the approach followed in the ST-GARCH model in terms of including the first lag of shocks ε_{t-1} as the transition variable to capture the effects of negative and positive shocks on regime transitions. For simplicity, the threshold could be assumed as $c = 0$, or it could be estimated through data-driven methods.

4.3.7. LSTAR-APGARCH

The LSTAR-GARCH(p,r) model is extended to LSTAR-APGARCH(p,r). The single-regime APGARCH(p,r) process for the conditional variance is defined as

$$\sigma_t^{\delta} = \omega + \sum_{k=1}^{p} \alpha_k (|\varepsilon_{t-k}| - \lambda_k \varepsilon_{t-k})^{\delta} + \sum_{l=1}^{r} \beta_l \sigma_{t-l}^{\delta} + \varepsilon_t \tag{29}$$

By extending Equation (29) to LSTAR-type nonlinearity for two regimes, LSTAR-APGARCH is obtained,

$$\begin{aligned} \sigma_t^{\delta_{1,2}} = &(\omega_1 + \textstyle\sum_{k=1}^{p} \alpha_{1k}\left(|\varepsilon_{1t-k}| - \lambda_{1k}\varepsilon_{1t-k})^{\delta_1} + \sum_{l=1}^{r} \beta_{1l}\sigma_{t-l}^{\delta_1}\right)(1 - G(\varepsilon_{t-1}; \gamma, c) \\ &+(\omega_2 + \textstyle\sum_{k=1}^{p} \alpha_{2k}\left(|\varepsilon_{2t-k}| - \lambda_{2k}\varepsilon_{t-k})^{\delta_2} + \sum_{l=1}^{r} \beta_{2l}\sigma_{t-l}^{\delta_2}\right)(G(\varepsilon_{t-1}; \gamma, c) + \varepsilon_t \end{aligned} \tag{30}$$

the LSTAR-APGARCH(p,r) model is obtained by replacing the LSTAR-GARCH(p,r) process in Equation (27) with Equation (30). The model integrates regime-specific leverage effects with λ parameters into two regimes. In addition, similar to the LSTAR-GARCH model, the conditional mean process follows LSTAR process in Equation (24). The study assumes logistic transition functions in both LSTAR-GARCH(p,r) and LSTAR-APGARCH(p,r) models given in Equations (25)–(30). Further, if the orders of ARCH and GARCH terms in LSTAR-GARCH(p,r) is one, by p = 1 and r = 1, Equation (27) becomes LSTAR-GARCH(1,1) as,

$$\delta_t^2 = \left(\omega_1 + \alpha_1\varepsilon_{t-1}^2 + \beta_1\delta_{t-1}^2\right)(1 - G(\varepsilon_{t-1}; \gamma, c)) + \left(\omega_2 + \alpha_2\varepsilon_{t-1}^2 + \beta_2\delta_{t-1}^2\right)G(\varepsilon_{t-1}; \gamma, c) \tag{31}$$

Similarly, the LSTAR-APGARCH(1,1) representation of Equation (30) is,

$$\begin{aligned}\sigma_t^{\delta_{1,2}} = {} & \left(\omega_1 + \alpha_1\left(|\varepsilon_{1t-1}| - \lambda_1\varepsilon_{1t-1}\right)^{\delta_1} + \beta_1\sigma_{t-1}^{\delta_1}\right)(1 - G(\varepsilon_{t-1};\gamma,c)) \\ & +\left(\omega_2 + \alpha_2\left(|\varepsilon_{1t-1}| - \lambda_2\varepsilon_{2t-1}\right)^{\delta_2} + \beta_2\sigma_{t-1}^{\delta_2}\right)(G(\varepsilon_{t-1};\gamma,c) + \varepsilon_t\end{aligned} \tag{32}$$

4.3.8. ESTAR-GARCH and ESTAR-APGARCH

The LSTAR-GARCH and LSTAR-APGARCH models could also be extended to ESTAR-GARCH and ESTAR-APGARCH models. The exponential transition function is,

$$G_E(\varepsilon_{t-1};\gamma,c) = 1 - e^{-\gamma(\varepsilon_{t-1}-c)^2} \tag{33}$$

and by substituting $G(\varepsilon_{t-1};\gamma,c)$ with $G_E(\varepsilon_{t-1};\gamma,c)$ in Equation (33), the models in Equations (31) and (32) become ESTAR-GARCH(1,1) and ESTAR-APGARCH(1,1) models, respectively.

5. Results

In this section, the BDI, VIX and MSCI indices are modeled with nonlinear LSTAR-GARCH and LSTAR-APGARCH models. As shown in Section 4, both models utilize logistic smooth transition functions to capture transition dynamics between sub-regression spaces in which $\Delta lbdi_t$, $\Delta lvix_t$ and $\Delta lmsci_t$ series follow regime-dependent and asymmetric conditional volatility processes.

5.1. STAR-GARCH Type Nonlinearity Tests and Model Selection

As an initial step, following the methodology, all three series were modeled with single-regime GARCH and APGARCH models. The estimated models were tested for STAR-type nonlinearity following [51,52]'s remaining STAR-type nonlinearity tests. The STAR-type nonlinearity testing was based on Taylor expansions and also helped distinguish between logistic and exponential transition functions in addition to testing linearity (For a discussion for multiple STAR modeling that allows more than two regimes, readers are referred to [54]. The testing process necessitated testing the single-regime model under the null to be tested for a two-regime model alternative. In case of rejecting the null hypothesis of the single-regime model, a second step involved testing the remaining STAR-type nonlinearity with two regimes under the null to be tested against a three-regime STAR-type nonlinearity which assumes two logistic transition functions. Therefore, the above-mentioned linearity testing methodology not only allowed for testing single-regime GARCH against two-regime STAR-GARCH models, but also allowed the determination of the number of regimes in the STAR-GARCH model. To save space, the test results are not reported but are available upon request. The overall test results favored two-regime LSTAR-GARCH and two-regime LSTAR-APGARCH against their single-regime variants). The test results are given in Table 5.

Due to model specification, all tests assumed the transition variable as ε_{t-1}. Accordingly, F test results favored nonlinearity for all modeled series, and F4, F3, and F2 tests resulted in the selection of logistic transition functions for all models at a 5% significance level. Results also confirmed STAR-GARCH and STAR-APGARCH models against their single-regime counterparts.

In the previous section, in the determination of orders of stationarity, KPSS and KSS tests were conducted to obtain results robust to certain forms of nonlinearity [59,60], The existence of nonlinearity of series were further confirmed with the BDS test [61]. At this section, single-regime GARCH and APGARCH models are tested against their two-regime nonlinear LSTAR-GARCH variants with Lagrange-Multiplier tests (LR) under nuisance parameters [62]. After estimation, LR tests confirmed the acceptance of two-regime variants in all single-regime models. We also estimated three regime models however the estimation results led to inconsistent results if more than one transition function. Ref. [44] discussed testing for and modeling of STAR-GARCH models. In the next stage, the models were estimated with QMLE (Quasi-Maximum Likelihood Estimator). QMLE is susceptible to the initial values. According to [63], the statistical and structural characteristics of STAR-GARCH models generally limit these models to two regimes, though m-regime extensions are possible [63]. In addition to two-regime models, we estimated three-regime models. However, these led to inconsistent results. As a result, we maintained two-regime models similar to the general tendency in the literature.

Table 5. STAR-type Nonlinearity and Model Architecture Selection Tests.

Baseline Model:	Modeled Series:	F	F4	F3	F2	Selected Trans. f.	Selected Model:
GARCH	$\Delta lbdi_t$	0.0012	0.0310	0.1431	0.0051	logistic	LSTAR-GARCH
APGARCH	$\Delta lbdi_t$	0.0314	0.0403	0.0914	0.0037	logistic	LSTAR-APGARCH
GARCH	$\Delta lvix_t$	0.0071	0.0470	0.1752	0.0038	logistic	LSTAR-GARCH
APGARCH	$\Delta lvix_t$	0.0296	0.0348	0.1108	0.0044	logistic	LSTAR-APGARCH
GARCH	$\Delta lmsci_t$	0.0099	0.0465	0.1073	0.0049	logistic	LSTAR-GARCH
APGARCH	$\Delta lmsci_t$	0.0015	0.0397	0.0874	0.0028	logistic	LSTAR-APGARCH

Notes: p-values are reported for F tests of STAR-type nonlinearity and model selection. F is the test statistic for testing single-regime GARCH against two-regime, STAR-GARCH. F4, F3 and F2 are sub-tests used to distinguish between logistic and exponential transition functions. Due to model specification, the transition variable is assumed as ε_{t-1}.

5.2. Model Estimation Results

The model estimation results are reported in Table 6 in three subsections; (a), (b) and (c), which correspond to parameter estimates and diagnostic testsfor BDI, VIX and MSCI daily % returns, respectively. In all estimated models, the stability condition for GARCH models, the sum of ARCH and GARCH coefficients being less than 1, is satisfied. The calculated t-test statistics show that the majority of parameters are statistically significant with minor exceptions. Among these, interesting results have been found for the BDI and MSCI indices.

Table 6. (a) Model Estimation Results for BDI Daily % Returns; (b) Model Estimation Results for VIX Daily % Returns; (c) Model Estimation Results for MSCI Daily % Returns.

Model:	LSTAR-GARCH		LSTAR-APGARCH	
Regime:	**1**	**2**	**1**	**2**
		(a)		
Cst(M)	0.006 ***	−0.007 ***	−0.004 ***	−0.007 ***
	(16.23)	(−27.84)	(−3.22)	(−3.36)
Cst(V)	0.326 ***	0.734 ***	1.245 ***	0.557 **
	(8.406)	(2.65)	(2.335)	(2.075)
ARCH	0.629 ***	0.047 ***	0.137	0.154 ***
	(5.795)	(5.239)	(0.5845)	(3.045)
GARCH	0.005 ***	0.547 ***	0.742 ***	0.508 ***
	(3.328)	(5.478)	(8.617)	(3.61)
APARCH (Gamma1)	-	-	−0.431	0.760 **
	-	-	(−0.4233)	(2.205)
APARCH (Delta)	-	-	0.595 ***	0.683
			(3.666)	(1.206)

Table 6. *Cont.*

Model:	**LSTAR-GARCH**		**LSTAR-APGARCH**	
Regime:	**1**	**2**	**1**	**2**
		Diagnostics:		
LogL	6.091	6.309	5.443	6.327
AIC	−7.08	−7.25	−6.26	−7.27
SIC	−7.07	−7.24	−6.24	−7.25
Q (5)	2.97	4.47	1.88	4.61
Q (10)	3.26	4.92	2.26	5.24
ARCH (1–2)	0.78	0.19	1.46	1.58
ARCH (1–5)	1.49	1.27	0.77	1.21
		(b)		
Cst(M)	−0.022 ***	0.002 ***	−0.022 ***	0.016 ***
	(−9.0)	(39.8)	(−6.13)	(3.64)
Cst(V)	0.692 ***	0.847 ***	−0.001 **	1.064 ***
	(3.932)	(3.048)	(−2.120)	(2.512)
ARCH	0.148 ***	0.1460 ***	0.006 *	0.1463 ***
	(3.952)	(2.9)	(1.797)	(2.349)
GARCH	0.613 ***	0.712 ***	0.985 ***	0.745 ***
	(8.046)	(9.284)	(20.06)	(10.12)
APARCH (Gamma1)	-	-	−0.169	−0.137 **
	-	-	(−0.438)	(−2.502)
APARCH (Delta)	-	-	0.93 ***	1.305 **
	-	-	(2.809)	(2.489)
		Diagnostics:		
LogL	10.006	11.478	9.958	9.292
AIC	−5.40	−4.88	−5.37	−4.78
SIC	−5.39	−4.87	−5.36	−4.77
Q (5)	1.16	1.08	1.8	1.08
Q (10)	1.38	1.52	2.12	1.54
ARCH (1–2)	0.02	0.08	1.03	0.03
ARCH (1–5)	0.55	0.34	0.66	0.29
		(c)		
Cst(M)	0.001 ***	0.015 ***	0.059 ***	−0.033 ***
	(3.882)	(6.14)	(3.442)	(−4.712)
Cst(V)	0.125 **	0.261 ***	−0.773 ***	−0.066 ***
	(2.114)	(3.441)	(−4.334)	(−8.444)
ARCH	0.041 *	0.858 ***	0.006	0.147 *
	(1.706)	(35.72)	(0.7354)	(1.653)
GARCH	0.954 ***	0.151 ***	0.858 ***	0.821 ***
	(39.32)	(6.337)	(8.628)	(7.515)

Table 6. *Cont.*

Model:	LSTAR-GARCH		LSTAR-APGARCH	
Regime:	**1**	**2**	**1**	**2**
APARCH (Gamma1)	-	-	0.246 **	0.048
	-	-	(2.027)	(1.209)
APARCH (Delta)	-	-	0.290 **	0.838 ***
	-	-	(2.065)	(3.807)
Diagnostics:				
LogL	6.865	7.652	6.954	7.748
AIC	−9.09	−8.80	−8.26	−9.23
SIC	−9.08	−8.79	−8.25	−9.21
Q (5)	6.47	1.17	2.43	3.35
Q (10)	1.29	1.63	3.32	6.06
ARCH (1–2)	1.35	0.50	1.35	0.52
ARCH (1–5)	1.53	0.21	1.16	0.32

Notes: t statistics are given in parentheses. *, **, *** denote significance at 10%, 5%, 1% significance levels, respectively. LogL is the log-likelihood, AIC and SIC are Akaike and Schwarz information criteria. Q (5) and Q (10) are Ljung-Box autocorrelations tests at order 5 and 10. ARCH(1–2) and (1–5) are ARCH-LM test statistics for testing ARCH-type heteroskedasticity at orders 1–2 and 1–5.

5.2.1. Comparison of Regime-Dependent ARCH and GARCH Parameters

There were major changes in the coefficient of ARCH and GARCH between regimes 1 and 2 for the BDI and MSCI. The GARCH coefficients were higher than the ARCH parameters as expected for all regimes and models, with an exception.

For the BDI, the GARCH coefficient for (regime) 1 for the LSTAR-GARCH was too low, estimated as 0.005, close to zero. In the second regime, the coefficient was calculated as 0.547. For the LSTAR-APGARCH, this problem was solved, with GARCH parameters estimated as 0.742 and 0.508 in the respectful regimes. For the MSCI, the GARCH parameter in regime 1 was very close to 0.954, while the second regime had a value of 0.151. ARCH coefficients were inevitably the opposite for these two regimes. For the LSTAR-APGARCH estimated for the MSCI, this problem was solved again since the GARCH coefficients was determined as 0.858 and 0.821 for regimes 1 and 2. The results show that the LSTAR-APGARCH models are better in capturing the expectations for the ARCH and GARCH parameters with these respects.

5.2.2. Comparisons of Asymmetric Power Effects

The asymmetric power terms were estimated as being regime-dependent in all LSTAR-APGARCH models. For the BDI, the asymmetric power parameter was estimated as 0.595 for regime 1 and as 0.683 for regime 2 in the LSTAR-APGARCH model. The LogL statistic was highest in regime 2 for the BDI's LSTAR-APGARCH model. For the BDI, the LogL values for the LSTAR-GARCH model were relatively closer and in addition, if evaluated together, the fit of LSTAR-GARCH was higher for the BDI in this respect. However, one should also keep in mind that if regime 2 is attained, the overall fit is the highest for the BDI series modeled with LSTAR-APGARCH. LSTAR-APGARCH in regime 2 is more meaningful in modeling the BDI data than the LSTAR-GARCH for the BDI however, if regime 1 is attained, the fit of the model would be worsened relatively.

5.2.3. Evaluation of In-Sample Fit

When the AIC and SIC values were evaluated, similar results were obtained: in both models, fit was relatively higher in regime 2. Interestingly, the APARCH (Gamma1) parameter was statistically no different than zero. This parameter is similar to the leverage parameter of the traditional GJR model and the parameter introduces asymmetry between negative and positive shocks. As a result, in regime 1 of the LSTAR-APGARCH model, the model uses this leverage effect, therefore, reduces to LSTAR-PGARCH model.

Though fit of all models is evaluated in this section, note that these results are for in-sample fit of the models only. The out-of-sample forecast evaluation will be crucial, for which the results will be given in the following section. If an overlook is presented, since the total sample covers 1 November 2007–30 May 2022, the out-of-sample forecast results are obtained for the 10 days ahead, corresponding to the last two weeks of the dataset since the data is for working days. As a result, we decided not to reject models in terms of in-sample results, and we maintained the models to be analyzed for their out-of-sample forecast performances in the next section. So far, the models provided interesting characteristics which distinguish the volatility dynamics in two different regimes for all models estimated for the BDI daily % returns.

In section (b) of Table 6, the models estimated for the VIX led to important findings. The stability condition is satisfied for both models for the VIX. For in-sample fit, the use of LSTAR-GARCH models for the VIX is clearer. The AIC and SIC and in addition LogL statistics show better fit for the LSTAR-GARCH relative to the LSTAR-APGARCH if two regimes are evaluated for both models. However, again it should be stated that out-of-sample performances would provide decisive information with this respect. The ARCH term in regime 1 of LSTAR-APGARCH is statistically significant at 10% significance level. However, the parameter is significant at 5% in regime 2 in addition to significant GARCH parameters in both regimes.

5.2.4. APARCH Parameters: Comparative Analysis

Similar to the BDI's LSTAR-APGARCH model, the APARCH(Gamma1) parameter is not significant. Therefore, it is zero in regime 1. As a result, the process that the conditional variance follows in regime 1 reduces to an LSTAR-PGARCH model since the leverage effect is insignificant for this regime due to the insignificant gamma parameter. This interpretation for both the BDI and VIX should be made with caution since the model is still asymmetric as a whole, especially the power terms being statistically different in each regime. As a typical example, the APARCH(delta) is estimated as 0.93 in regime 1 and as 1.305 in regime 2, both significant, leading to higher levels of power terms and more drastic levels of volatility in regime 2. Further, the Q statistics and ARCH tests in diagnostics suggest no autocorrelation and no remaining ARCH effects.

The results for the MSCI index daily % returns are reported in section (c) of Table 6. For the MSCI, the use of LSTAR-APGARCH models in both regimes is more meaningful considering the AIC and SIC results in addition to LogL statistics. With asymmetric power terms, the explanatory power increases for the MSCI. The stability condition is satisfied for both models. However, the ARCH term is statistically insignificant in regime 1 of the LSTAR-APGARCH model.

Further, similar to the model for the VIX, the APARCH(Gamma1) is statistically insignificant in regime 1. Therefore, the process in this regime becomes an LSTAR-PGARCH. However, again, the asymmetric power terms are still statistically different than zero, and for both regimes of the LSTAR-APGARCH model, the power terms are regime-dependent. Note that though certain minor insignificant results occur for the model parameters, it should also be noted that the models are selected over their single-regime counterparts, and the out-of-sample forecast performances will provide important insights in the next section.

5.3. Out-of-Sample Forecast Results

As noted in the previous section, the daily sample covers 1 November 2007–30 May 2022. The last 10 days are not utilized in the estimation of models and are kept out for forecast evaluation. As a result, this section aims at providing a comparative analysis on forecast performances of the GARCH, APGARCH, LSTAR-GARCH and LSTAR-APGARCH models for 10-working-days-ahead, which corresponds to a two-week period of future forecasts. The models are evaluated for their out-of-sample performances and the results are reported in Table 7 where the median squared error (MedSE) statistics are reported. The MedSE criteria is selected due to its non-sampling estimation capabilities and the statistic is robust to outliers.

Table 7. Out-of-Sample Forecast Evaluation.

Median Squared Error (MedSE)						
	GARCH	**APGARCH**	**LSTAR-GARCH**		**LSTAR-APGARCH**	
Regime/ Variable:	-	-	Regime 1	Regime 2	Regime 1	Regime 2
$\Delta lbdi_t$	0.21×10^{-1}	0.31×10^{-1}	7.00×10^{-5}	4.08×10^{-8}	3.82×10^{-3}	3.82×10^{-3}
$\Delta lvix_t$	0.35×10^{-1}	0.24×10^{-1}	3.81×10^{-3}	3.81×10^{-5}	3.81×10^{-5}	1.59×10^{-7}
$\Delta lmsci_t$	0.10×10^{-1}	0.66×10^{-1}	8.52×10^{-9}	1.64×10^{-10}	3.83×10^{-6}	7.87×10^{-6}

If the single-regime GARCH and APGARCH models given in columns 1 and 2 are evaluated, the APGARCH model is observed to lead to lower MedSE, suggesting better forecast performance for all series. If their nonlinear counterparts are evaluated, there is a significant improvement in forecast performance in terms of the drastic decline in MedSE statistics for LSTAR-GARCH and LSTAR-APGARCH models. The results are obtained by assuming that the conditional volatility process is either in regime 1 or regime 2 instead of reporting a single MedSE for the model. With this approach, we stress that uncertainty is likely, and the analyzed variable could shift to another regime during the forecast period due to an external shock. Though the model assumes a smooth transition between regimes, such a forecast practice could lead to important insights. The overall result shows that, for the LSTAR-GARCH model, regime 2 led to the lowest MedSE in all regimes for all variables forecasted. In addition, whatever the regime is, both regimes of the LSTAR-GARCH perform better than the GARCH and APGARCH, the single-regime models. It is observed that, in the first regime, the MedSE value for the LSTAR-GARCH model for the BDI is 0.00007, and this value becomes 0.00000004083 in regime 2 for the LSTAR-GARCH model. The difference between the MedSE in regimes 1 and 2 should be interpreted as the difference in the volatility dynamics. A similar reduction in MedSE is also observed for all estimated LSTAR-GARCH models including those for the VIX and MSCI. Hence, the results confirm higher uncertainty in the first regimes of the LSTAR-GARCH models. However, even in the high-forecast-uncertainty regime, the MedSE statistic is still lower for the LSTAR-GARCH compared to the single-regime models, signifying forecast performance for the LSTAR-GARCH specification.

For the LSTAR-APGARCH models, forecast improvement is achieved for all series forecasted over the single-regime GARCH and APGARCH models. However, if regime results are evaluated, differences should be taken into consideration. For the BDI, both regimes provided similar forecast performance since MedSE = 0.00382 in both regimes of the LSTAR-APGARCH. The MedSE statistics are lower in both regimes of LSTAR-GARCH for the BDI. Therefore, LSTAR-GARCH performs better in forecasting compared to LSTAR-APGARCH for this series. For the MSCI, a similar finding holds. The LSTAR-APGARCH model provided better forecast accuracy for the MSCI in both regimes relative to single-regime GARCH and APGARCH. The MedSE statistics are calculated as 0.00000383 and 0.00000787 for regimes 1 and 2. Though the model provides an improvement, it falls behind if compared to the LSTAR-GARCH specification for which the MedSE statistics are strikingly less, 0.00000000852 and 0.000000000164, hence, for the MSCI, LSTAR-GARCH provided best forecast performance over all models analyzed. For the VIX, on the other hand, it is clear that the LSTAR-APGARCH model performs best in regime 2 and regime 1.

However, looking closely, LSTAR-GARCH regime 2 performs similar to LSTAR-APGARCH regime 1. For the VIX, it is clear that LSTAR-APGARCH specification leads to significant performance improvement for regime 2. Therefore, the overall forecast performances show that both LSTAR-GARCH and LSTAR-APGARCH models provide important forecast improvement over the traditional GARCH and the asymmetric power upgraded APGARCH model. Further results also suggest that regime-dependence is an important phenomenon in forecast practices. Therefore, following the findings given above, policymakers interested in utilizing the BDI, VIX and MSCI indices as economic indicators should keep their regime-dependent characteristics and most importantly, ignoring nonlinearity would lead to drastic reduction in forecast performances.

5.4. Discussion

The study reveals the significant consequences of the predecessor models. The empiric results obtained in the study have important outcomes. The key results are summarized in Table 8.

Table 8. Summary of Key Results.

	LSTAR-GARCH and APGARCH for BDI	**LSTAR-GARCH and APGARCH for VIX**	**LSTAR-GARCH and APGARCH for MSCI**
Summary of characteristics of the econometric model:			
Regime effect on conditional mean	YES	YES	YES
Regime effect conditional variance	YES	YES	YES
Regime-specific leverage effects	YES	YES	YES
Summary of Empirical Signs:			
The ability to capture the nonlinearity and variance that remains in the residuals *	YES	YES	YES
Improvement in modeling variance that is not captured with traditional models *	YES	YES	YES
Goodness-of-fit improvement over traditional models **	YES	YES	YES
In-sample prediction improvement over traditional models **	YES (AIC)/ YES (SIC)	YES (AIC)/ NO (SIC)	YES (AIC)/ NO (SIC)
improvement in out-of-sample performance ***	YES	YES	YES

Notes: * Based on BDS, STAR type remaining nonlinearity and ARCH-LM tests. ** Based on in-sample performances measured with SIC, AIC and LogL statistics. Traditional model estimation results are not reported to save space. They are available upon request from the corresponding author. *** Based on MedSE statistics obtained for out-of-sample forecasts upto 10 days ahead.

The following results are obtained through the models estimated regarding the BDI, VIX and MSCI indices. i. The nonlinearity and regime-dependency in the conditional variance of the daily BDI, MSCI and VIX returns cannot be rejected. ii. To model the BDI, MSCI, and VIX indices, which are the leading indicator candidates examined in the study, regime-dependent leverage effects must be included in the conditional variance regimes. iii. In addition to the regime effects captured by LSTAR-GARCH parameters, modeling regime changes governed with smooth changes provide significant improvement in the accuracy of in-sample predictions for the VIX, MSCI and BDI series investigated. Given that all three series are considered as leading economic indicators, investors and policymakers should consider the utilization of nonlinearity in forecasting practices. iv. Important policy recommendations are generated from the findings of the study. It is necessary to improve prediction and forecast accuracy by implying smooth transition into single-regime models in order to consider the VIX, BDI and MSCI as leading economic indicators. Otherwise, due to nonlinear volatility dynamics, the traditional GARCH and APGARCH models suffer in producing effectiveness in forecasts. Given the nature of the series analyzed, careful investigation of the distributional properties of these series are necessary. Investors and policymakers should consider the asymmetry, nonlinearity and regime-dependent asymmetric power dynamics to hinder success in assessing future economic conditions.

In these respects, in addition to the VIX and MSCI economic indicators variables, the volatility of the Baltic Dry Load Index (BDI), is shown be an important economic indicator that can be considered a leading indicator of financial conditions. Moreover, changes in the calculation methods made from time to time in financial indicators such as the VIX or MSCI, which have volatility in their nature, or changing the weights of the criteria that reweighted the index, such as a certain share or stock or sectors, are sometimes the subject of discussions in the literature. To this end, the forecasting and modeling performances of the VIX and MSCI indices are criticized as not reflecting reality. As a matter of fact, a set of research studies criticize the performance of the MSCI index in terms of its failure to predict the 2008 Global Financial Crisis in the USA.

It is very difficult to manipulate the BDI because the BDI is an index representing shipping costs with more than 20 shipping routes worldwide, and the index provides a measure of raw material costs in the world. The BDI is considered an economic indicator that is independent of political influences and directly impacted by supply/demand economic conditions. aims to explore financial crises with modeling techniques. The fact is that investors and policymakers are interested in practical economic indicators that they can use in policy and investment decisions. The Baltic Dry Load Index (BDI) is considered a candidate index, which can be taken by many researchers as an economic indicator on a global scale. The BDI aims to provide an index based on the cost of raw materials such as iron, coal, cement, and grain worldwide. In addition, the BDI Index includes both the volatility inherited by the rigidity of crude oil prices and the docking fees, such as the port and load download and fill. Therefore, given the cost of the materials used in production and the demand conditions that encourage manufacturers to change production decisions, ship berthing, loading, filling charges and freight volumes are largely affected by global supply and demand conditions. As a result, the BDI is highly responsive to the global demand-supply conditions for raw materials used for manufactured products, in addition to the cost of oil prices and freight prices affected by the volatility. However, although the BDI is an important indicator, leptokurtic distribution and its nature creates difficulties in modeling and predicting the index as a result of the nonlinear state of the BDI time series.

While the literature focused on modeling, estimating and estimating the BDI, VIX and MSCI indices is limited, studies aimed at modeling these variables with various econometric techniques are still in progress, and studies are increasing. With the accuracy motivation to explain the above-mentioned financial conditions, these series are important to review the asymmetric power (APGARCH) amplified conditions of the extended conditional mean and conditional variance with the non-linear threshold autoregressive model (TAR), Smooth Transition Autoregressive (STAR) model family, Logistic Smooth Transition Autoregressive (LSTAR) and the GARCH model family.

6. Conclusions

BDI, MSCI and VIX predictive variable data were analyzed for the periods of 1 November 2007 through 30 May 2022 with LSTAR-GARCH and LSTAR-APGARCH models. According to the results of the models, investors and policy makers should consider the threshold effects and nonlinearity of the MSCI, VIX and BDI series in assessing the three series as pioneering indicators of future economic activity. In addition, as the literature research revealed, both series were successful in the global recession forecast, which began in late 2008. Efficiency in the estimation of the off-sampling path followed by the two series is significant at this time. When assessed in terms of the compatibility of the predicted models with the series, nonlinear models with threshold characteristics have provided high performance for improved in-sample modeling. In addition, it has been observed that the family of STAR type nonlinear volatility models (LSTAR-GARCH and LSTAR-APGARCH) provides significant gains in sampling accuracy, significantly improving the model's descriptive effectiveness. In addition, given the results of other applied models, the generalized GARCH, LSTAR-GARCH and LSTAR-APGARCH models have been compared with their predictive accuracy for 10 business days in different conditional volatility models with APGARCH architecture, and the out-of-sample estimation of LSTAR-GARCH and LSTAR-APGARCH models has proven to be extremely strong.

Given the successful estimation capabilities of the proposed models, asymmetric power cannot be disallowed by the power of incorporating regime transitions and threshold effects into the analysis at the same time. Therefore, in a non-linear threshold-type environment, when the model is extended with a smooth transient asymmetric force for each regime, it is proof that the prediction challenges in the MSCI and BDI can be overcome and that the MSCI, VIX and BDI indices can be used as an economic indicator. For the BDI, it is recommended to use predictive model with LSTAR-APGARCH method for all fluctuations, where predictive modeling results are meaningful; The MSCI and VIX financial pioneering indicators, which are discussed as more open to manipulations due to the changing calculation method in the literature, have been observed to be interpreted with LSTAR type models, especially in wavy periods, as they cause deviations on basic GARCH and APGARCH models.

As a result, the volatility and non-linearity of the BDI, VIX and MSCI indices must be kept in mind in order to make them more important today, such as the VIX and MSCI indices, and to be used as leading indicators and even considered as benchmarks.

It is necessary to select the disposing models. In addition, in future studies to achieve predictive improvements, focusing on models that aim to model different types of nonlinear GARCH models and non-linear series that allow not only conditional averages to change but also conditional variance processes between different regimes is recommended for policymakers and investors. The BDI is a leading economic indicator and is becoming a useful and important predictive for economic crises. Because the BDI's volatility is susceptible to volatility in the world's economies and the BDI Index has the ability to reflect the volatility of the financial conditions of the global economy with more than 20 shipments worldwide, our analysis results suggest that it can predict the direction of the global economy for investors and policymakers in advance.

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