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Article

Inelastic Collision Influencing the Rotational Dynamics of a Non-Rigid Asteroid (of Rubble Pile Type)

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Abstract: We have considered here a novel particular model for dynamics of a *non-rigid* asteroid rotation, assuming the added mass model instead of the concept of *Viscoelastic Oblate Rotators* to describe the physically reasonable response of a 'rubble pile' volumetric material of asteroid with respect to the action of a projectile impacting its surface. In such a model, the response is approximated as an inelastic collision in which the projectile pushes the 'rubble pile' parts of the asteroid together to form a mostly solidified plug in the crater during the sudden impact on the asteroid's surface. Afterwards, the aforementioned 'solidified plug' (having no sufficient adhesion inside the after-impact crater) will be pushed outside the asteroid's surface by centrifugal forces, forming a secondary rotating companion around the asteroid. Thus, according to the fundamental law of angular momentum conservation, the regime of the asteroid's rotation should be changed properly. Namely, changes in rotational dynamics stem from decreasing the asteroid's mass (due to the fundamental law of angular momentum conservation). As the main finding, we have presented a new solving procedure for a semi-analytical estimation of the total mass of the aforementioned 'solidified plug', considering the final spin state of rotation for the asteroid with minimal kinetic energy reduced during a long time period by the inelastic (mainly, tidal) dissipation. The asteroid is assumed to be rotating mainly along the maximal inertia axis with a proper spin state corresponding to minimal energy with a fixed angular momentum.

Keywords: inelastic collision; tidal dissipation; asteroid rotation; angular momentum

MSC: 70F15; 70F07

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1. Introduction, the Dynamical Model

One of the most important events in the fields of space science and celestial mechanics concerns exploring the recent phenomenon related to NASA's confirmation that the DART mission impact changed an asteroid's motion in space [?]. Meanwhile, specialists who were responsible at NASA for numerical support of the aforementioned project were very surprised that the resulting (after the impact) angular velocity of asteroid rotation turned out to differ from the one calculated by them previously, according to their numerical code. In this respect, the main aim and motivation of this case study is to suggest a physical model with a clear semi-analytical algorithm for calculation and estimation of inelastic collision influencing the rotational dynamics of a non-rigid asteroid (of rubble pile type) when it moves in an elliptic orbit, preferably outside the Hill sphere of any large celestial body. We explore the changes in purely rotational motion under the action of inelastic collision onto a surface of a rotating asteroid; in addition, such collision is considered to be preferably acting normally to the surface, not in a regime of ricocheting when the projectile hits a surface and moves away from it at an angle. Here, a rubble pile asteroid (Figure **??**) is not assumed to be rotating as a rigid body, which means that distances between various points inside the volume of the asteroid should not be constant and should not be elongated negligibly during rotation.

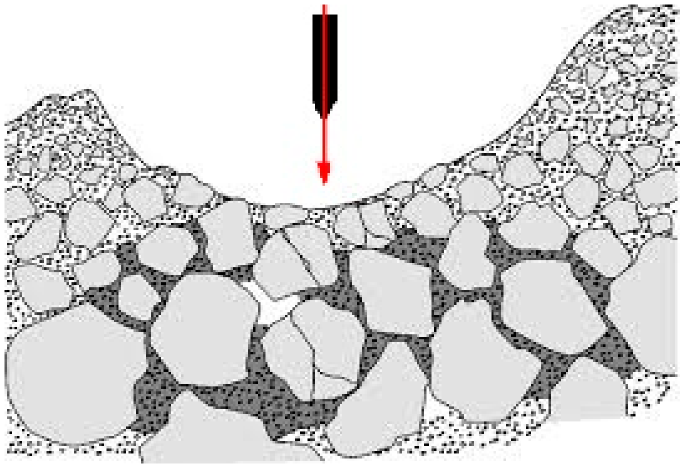

Figure 1. Schematic illustrating the surface of a rubble pile asteroid (and the projectile approaching to hit that surface).

It is worth noting that most of all the registered asteroids (NEO, near-Earth objects) correspond to rubble pile type [**? ? ? ? ?**].

Let us outline once again [**? ?**] that it is very important to create an adequate physical model along with a clear semi-analytical algorithm or mathematical code for calculating changes in the asteroid's spinning stage, taking into account the results of inelastic collision or impact onto its surface with the further aim of comparing such results with data of astrometric observations relating to the resulting regimes of the angular asteroid's rotation.

First, we will use the added mass model [**? ?**] instead of the concept of *Viscoelastic Oblate Rotators* [**? ?**] to describe the shear thickening response of a rubble pile material forming the asteroid with respect to an impacting projectile. In such a model, the response on the external stress is approximated as an inelastic collision in which the projectile pushes the 'rubble pile' parts of the asteroid together to form a mostly solidified plug. This plug then presses and pulls the surrounding medium downward like a 'snow plough' [**? ?**], adding over time the mass that the projectile needs to displace. As such, the idea here is to think of the impact as an inelastic collision with a growing mass [**?**] (with given initial conditions). Here in (1), M is the mass of the projectile attacking the asteroid in the normal direction Oz with respect to its surface; $z = z(t)$ is the coordinate corresponding to variable distance along the Oz-axis (with origin of Oz-axis on the asteroid's surface); m_{plug} is the growing mass of inelastic plug; and g is the acceleration due to gravity on the asteroid's surface (can be considered as sufficiently negligible). The growing mass (increasing with time) can be approximated in (2) with a cone-like region around the impacting projectile by a simple well-known formula for mass of (increasing with time) volume of the cone, with given density:

$$(M + m_{plug})\frac{d^2z(t)}{dt^2} = -\left(\frac{dm_{plug}}{dt}\frac{dz(t)}{dt}\right) - Mg \tag{1}$$

$$m_{plug} = \frac{1}{3}\pi\rho(CR_M + kz(t))^2 kz(t) \tag{2}$$

where ρ is the mean value of the surface and immediate subsurface density of the rubble pile asteroid's material; R_M is the radius of the impacting projectile with a circular surface area; and C and k are defined as the 'added-mass coefficient' and 'front coefficient', respectively (both dimensionless). C was introduced in [**? ?**] to describe that the impacted medium differs from a liquid or solid state, and C is a variable parameter in the model, e.g., for various mediums: $C \in \left(10^{-6}, 3 \times 10^{-3}\right)$ considered in [**?**] and $C \in (0.37, 1)$ considered in [?]. k is the dimensionless distance between the solidification front of the growing mass and the projectile's position (such dimensionless distance is proportional to the radius of the projectile; in [?], $k \in (0, 13)$; in [?], $k \cong 12.5$). We will consider here and below $k \cong (20, 25)$ (the main reason is that the diameter of the crater after the impact of a projectile or a small asteroid is estimated to be 20 times larger than the diameter of mean cross-section of a projectile; this is a common opinion known to be agreed upon by reasonable consensus between most members of the celestial mechanics community).

Afterwards, the aforementioned 'solidified plug' (being a non-natural formation of the material of a 'rubble pile' asteroid and having no sufficient adhesion between 'solidified plug' and surface of after-impact crater inside) will be pushed outside the asteroid surface by centrifugal forces, forming the secondary rotating companion around the asteroid. Thus, according to the fundamental law of angular momentum conservation, the regime of the asteroid's rotation should be changed accordingly [?]. Namely, if we assume the evolution of spin towards rotation along the maximal inertia axis [?] due to the process of nutation relaxation, such an assumption should mean the spinning up of angular velocity along the maximal inertia axis, where the spinning up of angular velocity will be (linearly) dependent on decreasing the mass of the asteroid (due to angular momentum conservation).

2. Semi-Analytical Exploration of the System of Equations (1)–(2)

Let us consider a first approximation for solutions of the system of Equations (1) and (2) (in the absence of gravity, $g \cong 0$; a thin rod as a projectile, $R_M << 1$); these simplifications yield approximate solutions of (1), taking into account (2) (in the text below, restriction for choosing the initial condition $p(0) = \left(\frac{dz(t)}{dt}\right)\Big|_{t=0} \neq 0$ is assumed to be valid):

$$\begin{gathered}\left(M + \tfrac{1}{3}\pi\rho(kz(t))^3\right)\frac{d^2z(t)}{dt^2} = -\pi\rho k(kz(t))^2\left(\frac{dz(t)}{dt}\right)^2 \left\{p(z) = \left(\frac{dz(t)}{dt}\right),\ \frac{d^2z(t)}{dt^2} = \left(\frac{dp}{dz}\right)p\right\} \\ \Rightarrow \frac{\left(M+\frac{1}{3}\pi\rho(kz)^3\right)}{2}\frac{d\left(p^2\right)}{dz} = -\pi\rho k(kz)^2p^2 \Rightarrow ln\left(\frac{p^2}{p^2(0)}\right) = -2\pi\rho\int_0^z\left(\frac{(kz)^2}{M+\frac{1}{3}\pi\rho(kz)^3}\right)d(kz) \\ \Rightarrow ln\left(\frac{p^2}{p^2(0)}\right) = 2\pi\rho\left(\frac{1}{3\left(\frac{1}{3}\pi\rho\right)}\right)ln\left(\frac{M}{\left|M+\frac{1}{3}\pi\rho(kz)^3\right|}\right),\ \Rightarrow\ p = \pm p(0)\left(\frac{M}{M+\frac{1}{3}\pi\rho(kz)^3}\right)\end{gathered} \tag{3}$$

and thus, we obtain from (3):

$$\left(Mkz + \frac{\pi\rho}{12}(kz)^4\right) = \pm kp(0)M\int dt \tag{4}$$

We need an additional simplifying assumption to solve (4) semi-analytically; let us suggest that the mass of the projectile M is much less than the mass of ejected 'rubble pile' parts of the asteroid. In this way, we obtain as follows:

$$z = \frac{\sqrt[4]{\left(\frac{12k|p(0)|}{\pi\rho}\right)Mt}}{k} \tag{5}$$

Approximate Solution (5) lets us estimate the total mass of 'rubble pile' material ejected from the asteroid (taking into account that $R_M << 1$):

$$m_{plug} = \frac{1}{3}\pi\rho\left[\left(\frac{12k|p(0)|}{\pi\rho}\right)Mt\right]^{\frac{3}{4}} \tag{6}$$

Thus, angular velocity ω of asteroid rotation will be (linearly) increasing as much as the mass of the asteroid is being decreased (6) (due to the fundamental law of angular momentum conservation) at ejecting 'rubble pile' material after the impact by the projectile outside the asteroid's surface (below, M_A is the mass of the asteroid; $p(0) = \left(\frac{dz(t)}{dt}\right)\Big|_{t=0} \neq 0$); we suggest no crucial changes in the shape of the asteroid (as such, the main contribution to the changes of the asteroid's principal moments of inertia stem from changes in the asteroid's mass, $\omega_0 = \omega|_{t=0} \neq 0$).

$$\omega = \frac{\omega_0}{1 - \frac{\pi\rho}{3M_A}\left[\left(\frac{12k|p(0)|}{\pi\rho}\right)Mt\right]^{\frac{3}{4}}} \cong \omega_0 \left(1 + \frac{\pi\rho}{3M_A}\left[\left(\frac{12k|p(0)|}{\pi\rho}\right)Mt\right]^{\frac{3}{4}}\right) \tag{7}$$

The abovementioned Scenario (7) of increasing angular velocity of asteroid rotation will be obviously actual during the limited time period; namely, until the moment when the entire mass of m_{plug} (of the inelastic 'after-impact' plug) is pushed out from the main body of the asteroid by centrifugal forces.

3. Discussion & Conclusions

We have considered here a novel model for the dynamics of a *non-rigid* asteroid rotation, assuming the added mass model [**? ?**] instead of the concept of *Viscoelastic Oblate Rotators* [**? ?**] to describe the physically reasonable response of a 'rubble pile' volumetric material of an asteroid with respect to the action of a projectile impacting its surface. In such a model, the response is approximated as an inelastic collision in which the projectile pushes the 'rubble pile' parts of the asteroid together to form a mostly solidified plug in the crater during the sudden impact on the asteroid's surface.

Afterwards, the aforementioned 'solidified plug' (having no sufficient adhesion inside the after-impact crater) will be pushed outside of the asteroid's surface by centrifugal forces, forming the secondary rotating companion around the asteroid or, highly likely, transforming into a tail of 'rubble pile' parts and particles behind the direction of the main motion of the asteroid in space. Thus, according to the fundamental law of angular momentum conservation, the regime of the asteroid's rotation should be changed properly [**?**]. Namely, changes in rotational dynamics stem from decreasing the asteroid's mass (due to the fundamental law of angular momentum conservation, even for the regimes of non-rigid rotation). As the main finding, we have presented a new solving procedure for semi-analytical estimation of the total mass of the aforementioned 'solidified plug', considering the final spin state of rotation for the asteroid with minimal kinetic energy reduced during a long time period by the inelastic (mainly, tidal) dissipation. The asteroid is assumed to be rotating mainly along the maximal inertia axis with the proper spin state corresponding to minimal energy with a fixed angular momentum.

It is of general interest to illuminate the solving procedure of Equations (1) and (2) without a lot of simplifying assumptions. The only assumption we should use (as the first approximation) is the absence of gravity, $g \cong 0$.

$$\begin{aligned}
&\left\{p(z) = \left(\frac{dz(t)}{dt}\right), m_{plug} = \frac{1}{3}\pi\rho(CR_M + kz(t))^2 kz(t)\right\} \Rightarrow \\
&\left\{\frac{dm_{plug}}{dt} = \frac{2}{3}\pi\rho(CR_M + kz)k^2 z \cdot p(z) + \frac{1}{3}\pi\rho(CR_M + kz)^2 k \cdot p(z)\right\} \Rightarrow \\
&\left(M + \frac{1}{3}\pi\rho(CR_M + kz)^2 kz\right) p\frac{dp(z)}{dz} = -\frac{1}{3}\pi\rho(CR_M + kz)k \cdot p^2(2kz + (CR_M + kz)) - Mg \\
&\Rightarrow \left(M + \frac{1}{3}\pi\rho(CR_M + kz)^2 kz\right)\frac{d\left(p^2\right)}{dz} \cong -\frac{2}{3}\pi\rho(CR_M + kz)k \cdot p^2(2kz + (CR_M + kz)) \\
&\Rightarrow ln\left(\frac{p^2}{p^2(0)}\right) = -\frac{2\pi\rho}{3}\int\left(\frac{(CR_M + y)(CR_M + 3y)}{M + \frac{\pi\rho}{3}y(CR_M + y)^2}\right)dy \; \{y = kz\}
\end{aligned} \tag{8}$$

where the right part of (8) can be semi-analytically approximated if we assume again that the mass of the projectile M is much less than the mass of ejected 'rubble pile' parts of the asteroid (below, $p(0) = \left(\frac{d\,z\,(t)}{d\,t}\right)\Big|_{t=0} \neq 0$ as previously).

$$\begin{aligned}
&ln\left(\frac{p^2}{p^2(0)}\right) \cong -2\int\left(\frac{CR_M + y}{y(CR_M + y)}\right)dy - 2\int\left(\frac{2y}{y(CR_M + y)}\right)dy \; \{y = kz\} \Rightarrow \\
&p \cong \pm\frac{p(0)}{kz}\left(\frac{CR_M}{CR_M + kz}\right)^2 \Rightarrow \int\left(kz(CR_M + kz)^2\right)d(kz) = \pm kp(0)(CR_M)^2 t
\end{aligned} \tag{9}$$

We can conclude from (9) that the approximated solution will be typical and the same as in (4) and (5); thus, we will have (6) and (7) as a final result.

As for the domain in which the ejection of the growing mass of inelastic plug occurs and regarding the initial conditions, let us consider only the Cauchy problem in a half-space (where mass is considered to be at rest at the initial moment in time).

It is worth noting that the abovementioned assumption regarding the absence of gravity, $g \cong 0$, is actual for asteroids less than 1 km in size. As remarked in [**?**], we presumably believe that most asteroids larger than ~1 km are gravitational aggregates.

Finalizing this section, let us note that useful articles regarding the problem under consideration should be cited [**? ?**], and let us also remark that we assumed a significant simplification regarding our model in a way that the impact would have to happen in an equatorial region of such a critical rotator, where mass shedding takes place (we ignore the influence of multiple immediate impact ejecta; we also ignore the influence of tidal effect [**?**] from large celestial bodies on the surface of the asteroid along with the effect of sudden change of regime [**?**] of ejecta flow immediately after impact along with relativistic effect [**?**]). We should especially note that the surface and immediate subsurface density of the rubble pile asteroid's material in Equation (2) is supposed to be substantially less than the 'mean' (bulk) density of an asteroid. As to the other applications for the semi-analytical method or solving procedure that has been developed in the frame of this work, one can find them useful for simplifying numerical experiments when testing other rubble pile media with the aim of preventing penetration inside them upon impact by a hypervelocity projectile (beyond the scope of our study) or when testing other non-Newtonian media (fluids or gels) for inventing active armor for defense of the human body (this does not relate to the subject or results of our research); see [**?**] for the case of non-Newtonian media.

To the best of our knowledge, there are no alternative semi-analytical methods that use a similar ansatz, approach, or final results to which we can compare the results of this work. This proves the novelty and the originality both of the solving algorithm and the results presented in our model with respect to the dynamics of a non-rigid asteroid rotation (after the projectile impacts the asteroid's surface) or even with respect to applying such results to the hydrodynamical reaction of surfaces of non-Newtonian media [**?**] under the direct shot (beyond the scope of our study), hitting or impacting them by a hypervelocity projectile in the form of a thin rod.

4. Highlighting Remarks

- The influence of inelastic collision on rotation of a rubble pile asteroid is studied.
- The approach stems from non-Newtonian media, applied for the first time for asteroid rotation.
- The added mass model is presented to describe the impact of the projectile on the asteroid.
- The impact activates solidifying a plug in a rubble pile surface via dynamic hit front.
- The projectile pushes 'rubble pile' parts together to form a solidified plug in the crater.
- Then, the 'solidified plug' is pushed outside of the asteroid by centrifugal forces.
- A new solving procedure for estimation of the mass of the 'solidified plug' is presented.
- Changes in rotational dynamics stem from decreasing the asteroid's mass.
- The asteroid rotates mainly along the maximal inertia axis with fixed angular momentum.

5. Clarifying Remarks Regarding the Steps of Derivation of Equation (3)

Let us clarify in (10) the steps of derivations of Equation (3), where change of variables: $p(z) = \left(\frac{dz(t)}{dt}\right) \Rightarrow \frac{d^2z(t)}{dt^2} = \frac{d\left(\frac{dz(t)}{dt}\right)}{dz}\left(\frac{dz(t)}{dt}\right) = \frac{d(p(z))}{dz}p(z)$ is used:

$$\begin{aligned}
&\left(M+\tfrac{1}{3}\pi\rho(kz(t))^3\right)\frac{d^2z(t)}{dt^2} = -\pi\rho k(kz(t))^2\left(\frac{dz(t)}{dt}\right)^2 \left\{p(z) = \left(\frac{dz(t)}{dt}\right),\ \frac{d^2z(t)}{dt^2} = \left(\frac{dp}{dz}\right)p\right\} \Rightarrow \\
&\Rightarrow \frac{(M+\frac{1}{3}\pi\rho(kz)^3)}{2}\left(2\left(p\frac{dp}{dz}\right)\right) = -\pi\rho k(kz)^2p^2 \Rightarrow \int_{p^2(0)}^{p^2}\frac{d(p^2)}{p^2} = -2\pi\rho\int_0^{kz}\left(\frac{(kz)^2}{M+\frac{1}{3}\pi\rho(kz)^3}\right)d(kz) \\
&\Rightarrow ln\left(\frac{p^2}{p^2(0)}\right) = -\frac{2\pi\rho}{3(\frac{1}{3}\pi\rho)}\int_M^{(M+\frac{1}{3}\pi\rho(kz)^3)}\left(\frac{1}{M+\frac{1}{3}\pi\rho(kz)^3}\right)d\left(M+\tfrac{1}{3}\pi\rho(kz)^3\right) \Rightarrow \\
&\Rightarrow ln\left(\frac{p^2}{p^2(0)}\right) = \frac{\pi\rho}{3(\frac{1}{3}\pi\rho)}ln\left(\left(\frac{M}{M+\frac{1}{3}\pi\rho(kz)^3}\right)^2\right), \Rightarrow p = \pm p(0)\left(\frac{M}{M+\frac{1}{3}\pi\rho(kz)^3}\right),
\end{aligned} \tag{10}$$

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