mathematics

Article

Study on the Selection of Pharmaceutical E-Commerce Platform Considering Bounded Rationality under Probabilistic Hesitant Fuzzy Environment

Zixue Guo [1,2] and Sijia Liu [1,*]

1 School of Management, Hebei University, Baoding 071002, China
2 Center for Common Prosperity Research, Hebei University, Baoding 071002, China
* Correspondence: liu_sj@stumail.hbu.edu.cn

Abstract: The selection of a pharmaceutical e-commerce platform is a typical multi-attribute group decision-making (MAGDM) problem. MAGDM is a common problem in the field of decision-making, which is full of uncertainty and fuzziness. A probabilistic hesitant fuzzy multi-attribute group decision-making method based on generalized TODIM is proposed for the selection of pharmaceutical e-commerce under an uncertain environment. Firstly, the credibility of a probabilistic hesitant fuzzy element is defined, and a credibility-based method for adjusting the weights of decision-makers and determining attribute weights is proposed, which fully considers the reliability of information provided by the decision-makers. Secondly, the power average (PA) operator is extended to the probabilistic hesitant fuzzy environment. The probabilistic hesitant fuzzy power average (PHFPA) operator and the probabilistic hesitant fuzzy power weighted average (PHFPWA) operator are defined, and their properties are discussed. Thirdly, considering the usual information expression of decision-makers in real life and the different risk attitudes towards gain and loss, the generalized TODIM method is extended to the probabilistic hesitant fuzzy environment to construct a prospect theory-based group decision-making method in the probabilistic hesitant fuzzy environment. Finally, the feasibility of the method in this paper is proved through the case of pharmaceutical e-commerce platform selection, and the stability of the method in this paper is verified by sensitivity analysis.

Keywords: multi-attribute group decision-making; probabilistic hesitant fuzzy sets; selection of pharmaceutical e-commerce platform; TODIM; credibility

MSC: 90B50

check for updates

Citation: Guo, Z.; Liu, S. Study on the Selection of Pharmaceutical E-Commerce Platform Considering Bounded Rationality under Probabilistic Hesitant Fuzzy Environment. *Mathematics* **2023**, *11*, 1859. https://doi.org/10.3390/math11081859

Academic Editors: Jorge de Andres Sanchez and Laura González-Vila Puchades

Received: 7 March 2023
Revised: 4 April 2023
Accepted: 11 April 2023
Published: 13 April 2023

1. Introduction

With the development of China's Internet technology and e-commerce, online shopping has become an important channel for consumer spending. In 2020, under the influence of COVID-19, the demand for pharmaceutical e-commerce platforms increased significantly, and the demand for online drug sales became prominent. According to the 49th Statistical Report on the Development of the Internet in China [1] released by the Internet Information Center, the number of online medical users in China reached 298 million by December 2021, which is an increase of 83.08 million compared to December 2020, accounting for 28.9% of the total Internet users. Online pharmacies provide people with more choices and practical convenience. By purchasing drugs online, consumers can greatly save time and transportation costs. At the same time, online medical care and online pharmacies have effectively distributed outpatient services, alleviating the current situation of difficulty in getting medical service. Consumers have realized the advantages of e-commerce in many aspects, and the public has a greater demand and expectation for pharmaceutical e-commerce platforms. For pharmaceutical enterprises, if they can choose an appropriate high-quality pharmaceutical e-commerce platform and carry out long-term cooperation, they can fully seize the opportunity, expand the coverage of drugs to achieve digital marketing, and improve the development speed of enterprises. Therefore, it is of great practical significance to study the selection of pharmaceutical e-commerce platform.

Since the evaluation of pharmaceutical e-commerce platforms needs to consider multiple indicators, the selection of a pharmaceutical e-commerce platform belongs to a multi-attribute decision-making problem. A single decision-maker may make mistakes in decision-making due to insufficient understanding of pharmaceutical e-commerce platforms or personal preferences. In order to reduce the risk of decision-making, several decision-makers are selected to evaluate and choose pharmaceutical e-commerce platform, and multi-attribute group decision-making method is used to study the selection of pharmaceutical e-commerce platform. There are two important difficulties in the multi-attribute group decision-making process, one of which is the representation of uncertain information. Professor Zadeh proposed the fuzzy set theory in 1965 [2]. Subsequently, the theory of fuzzy sets is extended by some scholars, such as trapezoidal fuzzy sets [3–6], intuitionistic fuzzy sets [7], interval intuitionistic fuzzy sets [8–10], Pythagorean fuzzy sets [11], hesitant fuzzy sets (HFS) [12], probabilistic hesitant fuzzy sets (PHFS) [13], etc. Among them, PHFS, as one of the extended forms of fuzzy sets, adds probability information to each membership degree, which can well express the different importance levels between different membership degrees and effectively describe the preferences of decision-makers. Therefore, PHFS can retain more original information and can express the uncertain preference information of decision-makers in a more comprehensive and detailed way, which can well enhance the rationality and credibility of decision results and be more in line with objective reality and the human way of thinking. In the multi-attribute group decision-making problem of selecting pharmaceutical e-commerce platforms, it is inaccurate to use accurate numbers to evaluate pharmaceutical e-commerce platforms due to the subjective evaluation factors involved in the evaluation indicators. In order to make the expression of decision information more accurate, this article chooses to use probability hesitant fuzzy sets to evaluate pharmaceutical e-commerce platforms.

Another important difficulty in multi-attribute group decision-making is the procedure of aggregating the decision-making information provided by decision-makers. Considering the correlation between indicators, many scholars choose to use aggregation operators such as Heronian mean operator, Bonferroni mean operator, Maclaurin symmetric mean operator, and other operators to aggregate information, but these operators cannot deal with outliers in the evaluation information. In the real multi-attribute group decision-making problem, the decision-maker's evaluation of the alternative is often subjective due to the influence of the decision-maker's personal preference and the fuzziness of the decision environment. However, from an objective point of view, the evaluation of multiple decision-makers on the same alternative under various attributes should be similar. Therefore, when decision-makers have bias or wrong information collection, abnormal data may appear, which may easily lead to unreasonable decision results. To solve this problem, Yager [14] proposed the Power Average (PA) operator in 2001. The PA operator can not only express the relationship between a single evaluation value and other evaluation values through the degree of support, but also obtain the corresponding harmonic weight through the degree of support so as to reduce the influence of unreasonable data on the final decision result. Subsequently, Xu and Yager [15] further developed the PA operator and defined the power ordered weighted average (POWA) operator. Since then, many scholars have studied the extension of the PA operator, resulting in the PA operator becoming rapidly developed and widely used [16–20].

After reviewing the literature, we find that there are some problems in the current research on multi-attribute group decision-making: (1) Compared with the classical hesitant fuzzy set, the probabilistic hesitant fuzzy set has better performance in expressing the hesitation of decision-makers, but there are few studies on the multi-attribute group decision-making problem with the evaluation information of a probabilistic hesitant fuzzy set. (2) As one of the classical aggregation operators, the PA operator has not been extended to the probabilistic hesitant fuzzy set; (3) In the process of multi-attribute group decision-making, decision-makers are not completely rational people, and their behaviors in the decision-making process will have an impact on the decision-making results.

To address the above problems, this study aims to propose a multi-attribute group decision-making method considering bounded rationality in a probabilistic hesitant fuzzy environment. The main work of this study is as follows: (1) In order to improve the reliability of group opinions, the credibility of probabilistic hesitant fuzzy element is defined, and the weights of decision-makers are adjusted according to the credibility. (2) The PA operator is extended to the probabilistic hesitant fuzzy environment, and the probabilistic hesitant fuzzy power average operator is defined for information aggregation of the decision-makers. (3) Taking into account the bounded rationality of decision-makers, the generalized TODIM method developed from prospect theory is extended to the probabilistic hesitant fuzzy environment. (4) The effectiveness and feasibility of the method in this paper are illustrated through a case study of pharmaceutical e-commerce selection.

The rest of this paper is organized as follows. Some basic concepts of PHFS are introduced in Section 2. The credibility of probabilistic hesitant fuzzy elements and the method for adjusting weights of decision-makers based on credibility are described in Section 3. The probabilistic hesitant fuzzy power average operator and its weighted form are defined, and their properties are explored in Section 4. A generalized TODIM multi-attribute group decision-making method in a probabilistic hesitant fuzzy environment is proposed in Section 5. A numerical example of pharmaceutical e-commerce platform selection is analyzed by the proposed method in Section 6 to demonstrate its validity and applicability. Finally, this paper is concluded in Section 7.

2. Preliminaries

In this section, some basic definitions and concepts associated with PHFS will be overviewed briefly.

2.1. Concept of PHFS

PHFS, as one of the important extended forms of fuzzy sets, take into full consideration both the different membership degrees and the probability of each membership degree, effectively taking into account the preferences of the decision-makers. The basic concept of PHFS is defined as follows:

Definition 1 ([21]). *Let X be a fixed set, then a PHFS on X is expressed by a mathematical symbol:*

$$H = \{ \langle x, h(p) \rangle | x \in X \} \tag{1}$$

where the set $h(p) = \{\gamma^\lambda(p^\lambda), \lambda = 1, 2, \ldots, l\}$ *is the basic tool to describe PHFS H, which is usually called probabilistic hesitant fuzzy element (PHFE). Membership degree* $\gamma^\lambda \in [0,1]$ *represents the possibility that the element* $x \in X$ *belongs to the PHFS H.* $p^\lambda \in [0,1]$ *indicates the possibility of* γ^λ*, and meets* $\sum_{\lambda=1}^{l} p^\lambda = 1$*. In particular, when* $p^\lambda = \frac{1}{l}$*, the PHFS degenerates into HFS.*

2.2. The Ranking Method of PHFE

In order to compare the size of PHFE, the score function, deviation function and comparison law of PHFE are defined as follows:

Definition 2 ([21]). *Generally, the elements in all PHFE* $h(p)$ *are sorted from small to large according to the membership degree. For a PHFE* $h(p) = \{\gamma^\lambda(p^\lambda), \lambda = 1, 2, \ldots, l\}$*, then its score function is defined as follows:*

$$s(h(p)) = \sum_{\lambda=1}^{l} \gamma^\lambda \cdot p^\lambda \tag{2}$$

where γ^λ *denotes the smallest value of* λ *in the PHFE.*

Definition 3 ([21]). *Let $h(p) = \{\gamma^\lambda(p^\lambda), \lambda = 1, 2, \ldots, l\}$ be the PHFE, then the deviation function is defined as follows:*

$$D(h(p)) = \sum_{\lambda=1}^{l}\left(\gamma^\lambda p^\lambda - s(h(p))\right)^2 \tag{3}$$

Definition 4 ([21]). *According to the score function and the deviation function, the comparison rules of the two PHFE $h_1(p), h_2(p)$ can be presented as follows:*
If $s(h_1(p)) > s(h_2(p))$, then $h_1(p)$ is greater than $h_2(p)$ which is denoted by $h_1(p) \succ h_2(p)$;
If $s(h_1(p)) = s(h_2(p))$, then:
if $D(h_1(p)) > D(h_2(p))$, then $h_2(p)$ is greater than $h_1(p)$ which is denoted by $h_2(p) \succ h_1(p)$;
if $D(h_1(p)) = D(h_2(p))$, then $h_1(p)$ and $h_2(p)$ represent the same information which is denoted by $h_1(p) \sim h_2(p)$.

Since the element dimension will increase when PHFE is operated, which leads to a geometric growth in calculation, a new type of PHFE operation rule is proposed in the literature.

Definition 5 ([22]). *Let $h(p), h_1(p), h_2(p)$ be three PHFE, $\lambda > 0$, then:*

$$h^C(p) = \left\{\left[1 - \gamma^\lambda\right]\left(p^\lambda\right), \lambda = 1, 2, \cdots, l\right\} \tag{4}$$

$$\alpha h(p) = \left\{\left[1 - \left(1 - \gamma^\lambda\right)^\alpha\right]\left(p^\lambda\right), \lambda = 1, 2, \cdots, l\right\}, \alpha > 0 \tag{5}$$

$$(h(p))^\alpha = \left\{\left[\left(\gamma^\lambda\right)^\alpha\right]\left(p^\lambda\right), \lambda = 1, 2, \cdots, l\right\}, \alpha > 0 \tag{6}$$

$$h_1(p) \oplus h_2(p) = \left\{\left[\gamma_1^\lambda + \gamma_2^\lambda - \gamma_1^\lambda\gamma_2^\lambda\right]\left(\overline{p_1^\lambda + p_2^\lambda}\right), \lambda = 1, 2, \cdots, l\right\} \tag{7}$$

$$h_1(p) \otimes h_2(p) = \left\{\left[\gamma_1^\lambda\gamma_2^\lambda\right]\left(\overline{p_1^\lambda + p_2^\lambda}\right), \lambda = 1, 2, \cdots, l\right\} \tag{8}$$

where the normalized probability $\overline{p_1^\lambda + p_2^\lambda} = \frac{p_1^\lambda + p_2^\lambda}{\sum_{\lambda=1}^{l}(p_1^\lambda + p_2^\lambda)}, \lambda = 1, 2, \cdots, l$, therefore $\sum_{\lambda=1}^{l}\left(p_1^\lambda + p_2^\lambda\right) = 1$. The calculation rules defined in this way allow the number of elements contained in the integrated result to remain unchanged, avoiding an increase in computational effort due to the increase in the number of dimensions during the PHFE operation.

2.3. The Distance Measure of PHFE

In the probabilistic hesitant fuzzy environment, a new probabilistic hesitant fuzzy distance measure is defined as follows:

Definition 6 ([23]). *Let $h_1 = \{\gamma_1^\lambda(p_1^\lambda), \lambda = 1, 2, \ldots, l\}$ and $h_2 = \{\gamma_2^\lambda(p_2^\lambda), \lambda = 1, 2, \ldots, l\}$ be two PHFEs elements that are equal. The probabilistic hesitant fuzzy distance measure can be defined as:*

$$d(h_1, h_2) = |f(h_1) - f(h_2)| + q(h_1, h_2) \tag{9}$$

where $f(h_i)$ is the hesitancy degree of PHFE h_i, $q(h_1, h_2)$ is the difference measure between the PHFEs h_1, h_2, and the hesitancy and difference measure of PHFE are defined as follows:

$$f(h_i) = \frac{1}{2}\left(\frac{1}{l}\sum_{\lambda=1}^{l}\left[\gamma_i^\lambda p_i^\lambda - \left(\frac{1}{l}\sum_{\lambda=1}^{l}\gamma_i^\lambda p_i^\lambda\right)\right]^2 + \sqrt{\left(1 - \frac{1}{1 + ln\, l}\right)}\right) \tag{10}$$

$$q(h_1, h_2) = \frac{1}{2l}\sum_{\lambda=1}^{l}\left(\left|\gamma_1^\lambda p_1^\lambda - \gamma_2^\lambda p_2^\lambda\right| + \left|\gamma_1^\lambda - \gamma_2^\lambda\right| \cdot \left|p_1^\lambda - p_2^\lambda\right|\right) \tag{11}$$

3. Credibility of PHFE

In this section, the concept of credibility of PHFE is defined, the method of adjusting the weights of the decision-makers based on credibility, and the method of determining attribute weights based on credibility are proposed. Credibility is applied to the selection of pharmaceutical e-commerce platforms to eliminate the influence of decision-makers on decision-making results due to personal bias or insufficient access to information.

3.1. Concept of Credibility of PHFE

With the gradual refinement of the social division of labor, in order to ensure the rationality and accuracy of the evaluation provided by decision-makers in the multi-attribute decision-making process, the level of knowledge of decision-makers with the domain being decided needs to be taken into account. In order to measure the reliability of evaluation information, the concept of credibility of PHFE is defined in this section, which indicates the familiarity of decision-makers with the domain to be decided. The higher the hesitancy degree of an alternative under a particular attribute, the more uncertain the evaluation of the decision-maker under this attribute and the lower the credibility of this decision-maker is. Therefore, the credibility of PHFE is defined as follows.

Definition 7. *Let $h(p) = \left\{\gamma^\lambda\left(p^\lambda\right), \lambda = 1, 2, \ldots, l\right\}$ be a PHFE, then the credibility of PHFE is defined as:*

$$R(h(p)) = 1 - f(h(p)) \tag{12}$$

where $f(h(p))$ is the hesitancy degree of $h(p)$.

Property 1. *According to the properties of hesitancy degree, credibility has the following properties:*

(1) $0 \leq R(h(p)) \leq 1$;

(2) *When there is only one membership degree in the PHFE, $R(h(p)) = 1$.*

Proof. (1) Since $l \geq 1$, so $1 - \frac{1}{1+ln\,l} \leq 1$ and therefore $\sqrt{\left(1 - \frac{1}{1+ln\,l}\right)} \leq 1$. Because $\gamma^\lambda \in [0,1], p^\lambda \in [0,1]$, then $\gamma_i^\lambda p_i^\lambda \in [0,1]$, therefore $\frac{1}{l}\sum_{\lambda=1}^{l} \gamma_i^\lambda p_i^\lambda \in [0,1]$, we can get $\frac{1}{l}\sum_{\lambda=1}^{l}\left[\gamma_i^\lambda p_i^\lambda - \left(\frac{1}{l}\sum_{\lambda=1}^{l} \gamma_i^\lambda p_i^\lambda\right)\right]^2 \in [0,1]$. Then the hesitancy degree is $f(h_i) = \frac{1}{2}\left(\frac{1}{l}\sum_{\lambda=1}^{l}\left[\gamma_i^\lambda p_i^\lambda - \left(\frac{1}{l}\sum_{\lambda=1}^{l} \gamma_i^\lambda p_i^\lambda\right)\right]^2 + \sqrt{\left(1 - \frac{1}{1+ln\,l}\right)}\right) \in [0,1]$, and according to Equation (14) we can get $R(h(p)) \in [0,1]$.

(2) When there is only one membership degree in the PHFE, it is obtained that:

$$\begin{cases} \gamma_i^\lambda p_i^\lambda = \frac{1}{l}\sum\limits_{\lambda=1}^{l} \gamma_i^\lambda p_i^\lambda \\ 1 - \frac{1}{1+ln\,l} = 0 \end{cases}$$

Then $f(h_i) = 0$, $R(h(p)) = 1$.

Thus, we complete the proof of properties of credibility of PHFE. □

3.2. Adjustment for Weights of the Decision-Makers

Due to the different knowledge level and expertise of each decision-maker, there will be some bias in the evaluation of pharmaceutical e-commerce platforms. Excessive trust in the evaluation of one decision-maker will lead to one-sided decision results. Therefore, the weights of decision-makers need to be adjusted in the multi-attribute group decision-making process. Generally, decision-maker weights are subjectively determined, representing the importance of the decision information provided by that decision-maker in the decision-making process. How to determine the decision-maker weights effectively has a huge impact on the decision result. Therefore, a method to adjust the decision-maker's weight based on the decision-maker's credibility in the probabilistic hesitant fuzzy environment is proposed in this paper. On the basis of considering the subjective weight of decision-makers, the original weight of the decision-makers is adjusted according to the credibility of the decision-makers. A higher weight is assigned to decision-makers with high credibility and a lower weight is assigned to decision-makers with low credibility, so as to obtain decision-maker weights that are more in line with the actual situation. Considering the above ideas, the formula for adjusting decision-maker weights based on credibility is defined as:

$$\eta_k' = \frac{\eta_k}{1 + \eta_k - \frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n} R\left(h_{ij}^k\right)} \tag{13}$$

where η_k' is the adjusted weight of the decision-maker, which can be regarded as the degree of support provided by the decision-maker, and meets $\eta_k' \in [0,1]$; $\frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n} R\left(h_{ij}^k\right)$ is the average credibility of the decision-maker.

The final weight of decision-maker d_k is obtained by normalizing η_k':

$$\eta_k'' = \frac{\eta_k'}{\sum_{k=1}^{t} \eta_k'} \tag{14}$$

Equation (13) can adjust the weight of the decision-maker according to the credibility of the decision-maker. When the credibility of decision-maker d_k is high, but the weight assigned to it is low, the decision-maker weight will be adjusted. The denominator of Equation (13) is less than 1, so the adjusted decision-maker weight is higher than the original decision-maker weight ($\eta_k' > \eta_k$). Thus, this decision-maker will receive more attention in the process of decision-making aggregation, which is more conducive to obtaining scientific and objective decision-making opinions. Similarly, when the credibility of a decision-maker is low, but the original weight is high, the above formula can also reduce the influence of his preference in the decision-making process.

3.3. Attribute Weight Determination Model Based on Credibility

According to the credibility defined in this paper, the higher the credibility, the better the real situation of the alternative can be reflected. Therefore, in order to ensure the rationality of the results, this paper determines the attribute weight of each pharmaceutical e-commerce platform according to the credibility of the evaluation of the decision-maker. That is, the higher the credibility of the evaluation of the alternative under attribute C_j, the more important the attribute is, and the higher the weight assigned to it. On the contrary, the lower the credibility, the lower the weight assigned to the attribute. According to the above ideas, the maximum deviation method is used to determine the attribute weights, and the PHFS is combined with the TODIM method to construct a selection method of pharmaceutical e-commerce platform. By extending the maximum deviation method to the probabilistic hesitant fuzzy environment to determine attribute weight, the objective function can be constructed as follows:

$$\begin{cases} max \sum\limits_{i=1}^{m}\sum\limits_{j=1}^{n} \omega_j R\left(h_{ij}\right) \\ \sum\limits_{j=1}^{n} \omega_j^2 = 1, 0 \le \omega_j \le 1 \end{cases} \tag{15}$$

where ω_j are the attribute weights, and $R\left(h_{ij}\right)$ is the credibility of h_{ij}.

In order to solve the above maximization deviation model, a Lagrangian function needs to be constructed.

$$L(\omega_j,\xi) = \sum_{i=1}^{m}\sum_{j=1}^{n}\omega_j R(h_{ij}) + \frac{1}{2}\xi\left(\sum_{j=1}^{n}\omega_j^2 - 1\right) \tag{16}$$

where ξ is the Lagrangian multiplier.

Let the partial derivatives of both ω_j and ξ be 0, then we can get:

$$\begin{cases} \frac{\partial L(\omega_j,\xi)}{\partial \omega_j} = \sum\limits_{i=1}^{m} R(h_{ij}) + \xi\omega_j = 0 \\ \frac{\partial L(\omega_j,\xi)}{\partial \xi} = \sum\limits_{j=1}^{n} \omega_j^2 - 1 = 0 \end{cases} \tag{17}$$

The optimal solution of attribute weight can be obtained as follows:

$$\omega_j^* = \frac{\sum_{i=1}^{m} R(h_{ij})}{\sqrt{\sum_{j=1}^{n}\left(\sum_{i=1}^{m} R(h_{ij})\right)^2}} \tag{18}$$

By normalizing the above equation, the final weight of the evaluation index C_j can be written as follows:

$$\omega_j = \frac{\omega_j^*}{\sum_{j=1}^{n}\omega_j^*} = \frac{\sum_{i=1}^{m} R(h_{ij})}{\sum_{i=1}^{m}\sum_{j=1}^{n} R(h_{ij})}, j = 1,2,\ldots,n \tag{19}$$

4. Probabilistic Hesitant Fuzzy Information Aggregation Operators

In this section, the PA operator is introduced into the probabilistic hesitant fuzzy environment, the probabilistic hesitant fuzzy power average (PHFPA) operator and the probabilistic hesitant fuzzy power weighted average (PHFPWA) operator are defined, and their properties including idempotence, boundedness, and permutation invariance are discussed. The PHFPWA operator is applied to the selection of pharmaceutical e-commerce platforms to eliminate the impact of extreme values on the decision results in the process of information aggregation.

4.1. PHFPA Operator

Firstly, the degree of support of PHFEs is defined.

Definition 8. *Let $h_i(p) = \left\{\gamma_i^\lambda(p_i^\lambda), \lambda = 1,2,\ldots,l\right\}(i = 1,2)$ be two PHFEs and the degree of support between them is defined as:*

$$sup(h_1(p),h_2(p)) = 1 - d(h_1(p),h_2(p)) \tag{20}$$

where $d(h_1(p),h_2(p))$ represents the distance between PHFE $h_1(p)$ and $h_2(p)$.

Property 2. *The degree of support meets following three properties:*

(1) $sup(h_i(p),h_j(p)) \in [0,1]$;
(2) $sup(h_i(p),h_j(p)) = sup(h_j(p),h_i(p))$;
(3) *If* $d(h_i(p),h_j(p)) < d(h_j(p),h_i(p))$, *then* $sup(h_i(p),h_j(p)) \geq sup(h_j(p),h_i(p))$.

According to the distance measure in Definition 6, Property 2 is easy to prove and will not be discussed here.

Based on Definition 8, the PHFPA operator is defined as follows:

Definition 9. *Let $h_i(p) = \{\gamma_i^\lambda(p_i^\lambda), \lambda = 1,2,\ldots,l\}$ be a set of PHFEs, then the probabilistic hesitant fuzzy power average (PHFPA) operator can be defined as:*

$$PHFPA(h_1(p), h_2(p), \ldots, h_n(p)) = \frac{\oplus_{i=1}^{n}(1 + T(h_i(p)))h_i(p)}{\sum_{i=1}^{n}(1 + T(h_i(p)))} \tag{21}$$

where $T(h_i(p)) = \sum_{j=1,j\neq i}^{n} sup\big(h_i(p), h_j(p)\big)$, and $sup\big(h_i(p), h_j(p)\big)$ denotes the degree of support of $h_i(p)$ and $h_j(p)$.

From Definitions 5 and 9, we can get the following result by using mathematical in-duction.

Theorem 1. *Let $h_i(p) = \{\gamma_i^\lambda(p_i^\lambda), \lambda = 1,2,\ldots,l\}$ be a set of PHFEs, then the aggregated value by the PHFPA operator is also a PHFE, and*

$$\begin{aligned} & PHFPA(h_1(p), h_2(p), \ldots, h_n(p)) \\ = & \frac{\oplus_{i=1}^{n}(1+T(h_i(p)))h_i(p)}{\sum_{i=1}^{n}(1+T(h_i(p)))} \\ = & \left\{\left[1 - \prod_{i=1}^{n}\left(1-\gamma_i^\lambda\right)^{\frac{1+T(h_i(p))}{\sum_{i=1}^{n}(1+T(h_i(p)))}}\right]\left(\overline{-\sum_{i=1}^{n} p_i^\lambda}\right), \lambda = 1,2,\cdots,l\right\} \end{aligned} \tag{22}$$

Proof. Firstly, prove the following formula using mathematical induction:

$$\begin{aligned} & \bigoplus_{i=1}^{n}(1 + T(h_i(p)))h_i(p) \\ & = \left\{\left[1 - \prod_{i=1}^{n}\left(1-\gamma_i^\lambda\right)^{1+T(h_i(p))}\right]\left(\overline{\sum_{i=1}^{n} p_i^\lambda}\right), \lambda = 1,2,\cdots,l\right\}, \end{aligned}$$

When $n = 2$, then

$$\begin{aligned} & \bigoplus_{i=1}^{2}(1 + T(h_i(p)))h_i(p) \\ = & (1 + T(h_1(p)))h_1(p) \bigoplus (1 + T(h_2(p)))h_2(p) \\ = & \left\{\left[1 - \left(1-\gamma_1^\lambda\right)^{1+T(h_1(p))}\right]\left(p_1^\lambda\right)\right\} \bigoplus \left\{\left[1 - \left(1-\gamma_2^\lambda\right)^{1+T(h_2(p))}\right]\left(p_2^\lambda\right)\right\} \\ = & \left\{\left[1 - \left(1-\gamma_1^\lambda\right)^{1+T(h_1(p))}\left(1-\gamma_2^\lambda\right)^{1+T(h_2(p))}\right]\left(\overline{p_1^\lambda + p_2^\lambda}\right), \lambda = 1,2,\cdots,l\right\}, \end{aligned}$$

Assume that when $n = k$, the following equation is established:

$$\bigoplus_{i=1}^{k}(1 + T(h_i(p)))h_i(p) = \left\{\left[1 - \prod_{i=1}^{k}\left(1-\gamma_i^\lambda\right)^{1+T(h_i(p))}\right]\left(\overline{\sum_{i=1}^{k} p_i^\lambda}\right), \lambda = 1,2,\cdots,l\right\}$$

Then, when $n = k + 1$,

$$\begin{aligned} & \bigoplus_{i=1}^{k+1}(1 + T(h_i(p)))h_i(p) \\ = & \bigoplus_{i=1}^{k}(1 + T(h_i(p)))h_i(p) \bigoplus (1 + T(h_{k+1}(p)))h_{k+1}(p) \\ = & \left\{\left[1 - \prod_{i=1}^{k}\left(1-\gamma_i^\lambda\right)^{1+T(h_i(p))}\right]\left(\overline{\sum_{i=1}^{k} p_i^\lambda}\right)\right\} \bigoplus \left\{\left[1 - \left(1-\gamma_{k+1}^\lambda\right)^{1+T(h_{k+1}(p))}\right]\left(p_{k+1}^\lambda\right)\right\} \\ = & \left\{\left[1 - \prod_{i=1}^{k+1}\left(1-\gamma_i^\lambda\right)^{1+T(h_i(p))}\right]\left(\overline{\sum_{i=1}^{k+1} p_i^\lambda}\right), \lambda = 1,2,\cdots,l\right\}, \end{aligned}$$

Secondly, according to the Definition 4 on PHFE operation rule, the result can be obtained as:

$$
\begin{aligned}
& \frac{\bigoplus_{i=1}^{n}(1+T(h_i(p)))h_i(p)}{\sum_{i=1}^{n}(1+T(h_i(p)))} \\
= \; & \frac{1}{\sum_{i=1}^{n}(1+T(h_i(p)))}\left\{\left[1-\prod_{i=1}^{n}\left(1-\gamma_i^{\lambda}\right)^{1+T(h_i(p))}\right]\left(\overline{\sum_{i=1}^{n} p_i^{\lambda}}\right), \lambda=1,2,\cdots,l\right\} \\
= \; & \left\{\left[1-\prod_{i=1}^{n}\left(\left(1-\gamma_i^{\lambda}\right)^{1+T(h_i(p))}\right)^{\frac{1}{\sum_{i=1}^{n}(1+T(h_i(p)))}}\right]\left(\overline{\sum_{i=1}^{n} p_i^{\lambda}}\right), \lambda=1,2,\cdots,l\right\} \\
= \; & \left\{\left[1-\prod_{i=1}^{n}\left(1-\gamma_i^{\lambda}\right)^{\frac{1+T(h_i(p))}{\sum_{i=1}^{n}(1+T(h_i(p)))}}\right]\left(\overline{\sum_{i=1}^{n} p_i^{\lambda}}\right), \lambda=1,2,\cdots,l\right\},
\end{aligned}
$$

where $\gamma_i^{\lambda} \in [0,1]$, $\frac{1+T(h_i(p))}{\sum_{i=1}^{n}(1+T(h_i(p)))} \in [0,1]$, then $\prod_{i=1}^{n}\left(1-\gamma_i^{\lambda}\right)^{\frac{1+T(h_i(p))}{\sum_{i=1}^{n}(1+T(h_i(p)))}} \in [0,1]$. Therefore, the aggregation result of membership degree is $1-\prod_{i=1}^{n}\left(1-\gamma_i^{\lambda}\right)^{\frac{1+T(h_i(p))}{\sum_{i=1}^{n}(1+T(h_i(p)))}} \in [0,1]$, which meets the property of membership degree of PHFE.

Furthermore, because of the definition of standardized probability in Definition 4, we can obtain that $\overline{\sum_{i=1}^{n} p_i^{\lambda}} \in [0,1]$, and then $\sum_{\lambda=1}^{l} \overline{\sum_{i=1}^{n} p_i^{\lambda}} = 1$, which meets the requirement of probability of PHFE.

Therefore, the result after aggregation by using $PHFPA$ is still a PHFE.

Thus, we complete the proof of Theorem 1. □

Based on Theorem 1, the basic properties of the HPFPA operator are as follows:

Property 3 (Idempotence). *Let $h_i(p) = \left\{\gamma_i^{\lambda}\left(p_i^{\lambda}\right), \lambda = 1,2,\ldots,l\right\}$ be a set of PHFEs, if for all $i = 1,2,\cdots,n$, we have $h_i(p) = h(p) = \left\{\gamma^{\lambda}\left(p^{\lambda}\right), \lambda = 1,2,\ldots,l\right\}$, then:*

$$PHFPA(h_1(p), h_2(p), \ldots, h_n(p)) = h(p) \tag{23}$$

Proof.

$$
\begin{aligned}
& PHFPA(h_1(p), h_2(p), \ldots, h_n(p)) \\
& = PHFPA(h(p), h(p), \ldots, h(p)) \\
& = \left\{\left[1-\prod_{i=1}^{n}\left(1-\gamma^{\lambda}\right)^{\frac{1+T(h_i(p))}{\sum_{i=1}^{n}(1+T(h_i(p)))}}\right]\left(\sum_{i=1}^{n} p_i^{\lambda} / \sum_{\lambda=1}^{l}\sum_{i=1}^{n} p_i^{\lambda}\right), \lambda=1,2,\cdots,l\right\} \\
& = \left\{\left[1-\left(1-\gamma^{\lambda}\right)^{\sum_{i=1}^{n}\frac{1+T(h_i(p))}{\sum_{i=1}^{n}(1+T(h_i(p)))}}\right]\left(\sum_{i=1}^{n} p_i^{\lambda} / \sum_{i=1}^{n}\sum_{\lambda=1}^{l} p_i^{\lambda}\right), \lambda=1,2,\cdots,l\right\} \\
& = \left\{\left[1-\left(1-\gamma^{\lambda}\right)^{1}\right]\left(np^{\lambda} / \sum_{i=1}^{n} 1\right), \lambda=1,2,\cdots,l\right\} \\
& = \left\{\gamma^{\lambda}\left(p^{\lambda}\right), \lambda=1,2,\cdots,l\right\} \\
& = h(p)
\end{aligned}
$$

Thus, we complete the proof of idempotence. □

Property 4 (Boundedness). *Let $h_i(p) = \left\{\gamma_i^{\lambda}\left(p_i^{\lambda}\right), \lambda = 1,2,\ldots,l\right\}$ be a set of PHFEs, if $h^{-}(p) = \left\{\min_i \gamma_i^{\lambda}\left(\min_i p_i^{\lambda}\right), \lambda = 1,2,\ldots,l\right\}$, $h^{+}(p) = \left\{max_i \gamma_i^{\lambda}\left(max_i p_i^{\lambda}\right), \lambda = 1,2,\ldots,l\right\}$, then:*

$$h^{-}(p) \le PHFPA(h_1(p), h_2(p), \ldots, h_n(p)) \le h^{+}(p) \tag{24}$$

Proof. Let $h(p) = PHFPA(h_1(p), h_2(p), \ldots, h_n(p))$, and $\omega_i = \frac{1+T(h_i(p))}{\sum\limits_{i=1}^{n}(1+T(h_i(p)))}$, then

$$s(h(p)) = \sum_{\lambda=1}^{l}\left\{\left[1-\prod_{i=1}^{n}\left(1-\gamma_i^\lambda\right)^{\omega_i}\right]\cdot\overline{\sum_{i=1}^{n}p_i^\lambda}\right\},$$

According to Definition 2 of the score function, it can be obtained that:

$$s_{h^-(p)} = \sum_{\lambda=1}^{l}\min_i \gamma_i^\lambda \cdot \min_i p_i^\lambda, s_{h^+(p)} = \sum_{\lambda=1}^{l}\max_i \gamma_i^\lambda \cdot \max_i p_i^\lambda,$$

Since $0 \leq \min\limits_i \gamma_i^\lambda \leq \gamma_i^\lambda \leq \max\limits_i \gamma_i^\lambda \leq 1$, it can be obtained that:

$$1-\min_i \gamma_i^\lambda \geq 1-\gamma_i^\lambda \geq 1-\max_i \gamma_i^\lambda,$$

Thus,

$$\prod_{i=1}^{n}\left(1-\min_i \gamma_i^\lambda\right)^{\omega_i} \geq \prod_{i=1}^{n}\left(1-\gamma_i^\lambda\right)^{\omega_i} \geq \prod_{i=1}^{n}\left(1-\max_i \gamma_i^\lambda\right)^{\omega_i},$$

Hence,

$$1-\prod_{i=1}^{n}\left(1-\min_i \gamma_i^\lambda\right)^{\omega_i} \leq 1-\prod_{i=1}^{n}\left(1-\gamma_i^\lambda\right)^{\omega_i} \leq 1-\prod_{i=1}^{n}\left(1-\max_i \gamma_i^\lambda\right)^{\omega_i},$$

Therefore,

$$\min_i \gamma_i^\lambda \leq 1-\prod_{i=1}^{n}\left(1-\gamma_i^\lambda\right)^{\omega_i} \leq \max_i \gamma_i^\lambda,$$

Furthermore, because of $0 \leq \min\limits_i p_i^\lambda \leq p_i^\lambda \leq \max\limits_i p_i^\lambda \leq 1$, it can be obtained that:

$$\overline{\sum_{i=1}^{n}\min_i p_i^\lambda} \leq \overline{\sum_{i=1}^{n}p_i^\lambda} \leq \overline{\sum_{i=1}^{n}\max_i p_i^\lambda}$$

Thus,

$$\min_i p_i^\lambda \leq \overline{\sum_{i=1}^{n}p_i^\lambda} \leq \max_i p_i^\lambda$$

Hence,

$$\sum_{\lambda=1}^{l}\min_i \gamma_i^\lambda \cdot \min_i p_i^\lambda \leq \sum_{\lambda=1}^{l}\left\{\left[1-\prod_{i=1}^{n}\left(1-\gamma_i^\lambda\right)^{\omega_i}\right]\cdot\overline{\sum_{i=1}^{n}p_i^\lambda}\right\} \leq \sum_{\lambda=1}^{l}\max_i \gamma_i^\lambda \cdot \max_i p_i^\lambda$$

Therefore,

$$h^-(p) \leq PHFPA(h_1(p), h_2(p), \ldots, h_n(p)) \leq h^+(p)$$

Thus, we complete the proof of boundedness. □

Property 5 (Permutation invariance)**.** *Let $h_i(p) = \left\{\gamma_i^\lambda\left(p_i^\lambda\right), \lambda = 1, 2, \ldots, l\right\}$ be a set of PHFEs, if $\left(h'_1(p), h'_2(p), \cdots, h'_n(p)\right)$ is any permutation of $\left(h_1(p), h_2(p), \cdots, h_n(p)\right)$, then:*

$$PHFPA(h_1(p), h_2(p), \ldots, h_n(p)) = PHFPA\left(h'_1(p), h'_2(p), \cdots, h'_n(p)\right) \quad (25)$$

Proof. Since $\left(h'_1(p), h'_2(p), \cdots, h'_n(p)\right)$ is the permutation of $(h_1(p), h_2(p), \cdots, h_n(p))$, there must exist a unique $h'_j(p)$ for each $h_i(p)$, then $h'_j(p) = h_i(p)$ and vice versa. Thus, $T\left(h'_j(p)\right) = T(h_i(p))$. It can be obtained that:

$$\begin{aligned} PHFPA(h_1(p), h_2(p), \ldots, h_n(p)) &= \frac{\bigoplus_{i=1}^{n}(1+T(h_i(p)))h_i(p)}{\sum\limits_{i=1}^{n}(1+T(h_i(p)))} \\ &= \frac{\bigoplus_{j=1}^{n}\left(1+T\left(h'_j(p)\right)\right)h'_j(p)}{\sum_{j=1}^{n}\left(1+T\left(h'_j(p)\right)\right)} \\ &= PHFPA\left(h'_1(p), h'_2(p), \ldots, h'_n(p)\right) \end{aligned}$$

Thus, we complete the proof of permutation invariance. □

4.2. PHFPWA Operator

According to the definition of PHFPA operator, the probabilistic hesitant fuzzy power weighted average operator is defined as below.

Definition 10. *Let $h_i(p) = \left\{\gamma_i^\lambda\left(p_i^\lambda\right), \lambda = 1, 2, \ldots, l\right\}$ be a set of PHFEs, and $\omega = (\omega_1, \omega_2, \cdots, \omega_n)^T$ be a vector of weights, which meets $\sum\limits_{i=1}^{n}\omega_i = 1, \omega_i \geq 0 (i = 1, 2, \cdots, n)$, then the probabilistic hesitant fuzzy power weighted average (PHFPWA) operator can be defined as:*

$$PHFPWA(h_1(p), h_2(p), \ldots, h_n(p)) = \frac{\bigoplus_{i=1}^{n}\omega_i(1+T(h_i(p)))h_i(p)}{\sum_{i=1}^{n}\omega_i(1+T(h_i(p)))} \tag{26}$$

Theorem 2. *Let $h_i(p) = \left\{\gamma_i^\lambda\left(p_i^\lambda\right), \lambda = 1, 2, \ldots, l\right\}$ be a set of PHFEs, then the aggregated value by the PHFPWA operator is also a PHFE, and*

$$\begin{aligned} &PHFPWA(h_1(p), h_2(p), \ldots, h_n(p)) \\ =\ & \frac{\bigoplus_{i=1}^{n}\omega_i(1+T(h_i(p)))h_i(p)}{\sum_{i=1}^{n}\omega_i(1+T(h_i(p)))} \\ =\ & \left\{\left[1 - \prod_{i=1}^{n}\left(1-\gamma_i^\lambda\right)^{\frac{\omega_i(1+T(h_i(p)))}{\sum_{i=1}^{n}\omega_i(1+T(h_i(p)))}}\right]\left(\prod_{i=1}^{n}p_i^\lambda\right)\right\} \end{aligned} \tag{27}$$

The proof of Theorem 2 is similar to Theorem 1 and it is therefore omitted.

Let $h_i(p) = \left\{\gamma_i^\lambda\left(p_i^\lambda\right), \lambda = 1, 2, \ldots, l\right\}$ be a set of PHFEs, then the $PHFPWA$ operator satisfies the following properties:

Property 6 (Idempotence). *If $h_i(p) = h(p) = \left\{\gamma^\lambda\left(p^\lambda\right), \lambda = 1, 2, \ldots, l\right\}(i = 1, 2, \cdots, n)$, then:*

$$PHFPWA(h_1(p), h_2(p), \ldots, h_n(p)) = h(p) \tag{28}$$

Property 7 (Boundedness). *If we make $h^-(p) = \left\{\min\limits_i \gamma_i^\lambda\left(\min\limits_i p_i^\lambda\right), \lambda = 1, 2, \ldots, l\right\}$, $h^+(p) = \left\{\max\limits_i \gamma_i^\lambda\left(\max\limits_i p_i^\lambda\right), \lambda = 1, 2, \ldots, l\right\}$, then:*

$$h^-(p) \leq PHFPWA(h_1(p), h_2(p), \ldots, h_n(p)) \leq h^+(p) \tag{29}$$

Property 8 (Permutation invariance). *If* $\left(h_1'(p), h_2'(p), \cdots, h_n'(p)\right)$ *is any permutation of* $(h_1(p), h_2(p), \cdots, h_n(p))$*, then:*

$$PHFPWA(h_1(p), h_2(p), \ldots, h_n(p)) = PHFPWA\left(h_1'(p), h_2'(p), \cdots, h_n'(p)\right) \tag{30}$$

The above three properties are proved in a similar way to Properties 3–5 and they are therefore omitted.

5. A Method of Pharmaceutical E-Commerce Platform Selection Based on Generalized TODIM under Probabilistic Hesitant Fuzzy Environment

5.1. Description of the Problem

For the selection of a pharmaceutical e-commerce platform, it is assumed that there are m pharmaceutical e-commerce platforms, and then the pharmaceutical e-commerce platform set is expressed as $A = \{A_1, A_2, \ldots, A_m\}(i = 1, 2, \ldots, m)$. There are n evaluation indexes in the evaluation index system of the pharmaceutical e-commerce platform, then the evaluation index set is $C = \{C_1, C_2, \ldots, C_n\}(j = 1, 2, \ldots, n)$, and the attribute weight vector is $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$, which satisfies $\sum_{j=1}^{n} \omega_j = 1, \omega_j \geq 0(i = 1, 2, \cdots, n)$. Let $D = \{d_1, d_2, \cdots, d_t\}$ be the set of decision-makers, and the weights of the decision-makers satisfies $\sum_{k=1}^{t} \eta_k = 1, \eta_k \geq 0(k = 1, 2, \cdots, t)$. The decision-maker evaluates the pharmaceutical e-commerce platforms using PHFE, which is denoted as $h(p) = \{\gamma^\lambda(p^\lambda), \lambda = 1, 2, \ldots, l\}$. For the k-th decision-maker d_k, the evaluation value of the pharmaceutical e-commerce platform A_i on the evaluation index C_j is represented by the PHFE $h_{ij}'^k(p_{ij}) = \left\{\gamma_{ij}'^{k\lambda}\left(p_{ij}'^{k\lambda}\right), i = 1, 2, \ldots, m; j = 1, 2, \ldots, n; k = 1, 2, \cdots, t; \lambda = 1, 2, \ldots, l\right\}$, then the k-th decision-maker's probabilistic hesitant fuzzy multi-attribute decision-making matrix $H'^k(P) = \left(h_{ij}'^k(p_{ij})\right)_{m \times n}$ can be expressed as:

$$H'^k(P) = \begin{bmatrix} h_{11}'^k(p_{11}) & h_{12}'^k(p_{12}) & \cdots & h_{1n}'^k(p_{1n}) \\ h_{21}'^k(p_{21}) & h_{22}'^k(p_{22}) & \cdots & h_{2n}'^k(p_{2n}) \\ \vdots & \vdots & \ddots & \vdots \\ h_{m1}'^k(p_{m1}) & h_{m2}'^k(p_{m2}) & \cdots & h_{mn}'^k(p_{mn}) \end{bmatrix} \tag{31}$$

When dealing with practical problems, different attributes have different physical scales. The evaluation attributes of pharmaceutical e-commerce platforms are generally classified as two types: benefit-type J_1 and cost-type J_2. For the benefit-type attributes, the corresponding probability hesitant fuzzy information remains unchanged. For the cost-type attributes, the membership degree in the probabilistic hesitant fuzzy information is complemented and the corresponding probability remains unchanged. Then the standardized matrix $E^k(P) = \left(e_{ij}^k(p_{ij})\right)_{m \times n}$ can be shown as:

$$e_{ij}^k(p_{ij}) = \begin{cases} \left\{\gamma_{ij}^{k,\lambda}\left(p_{ij}^{k,\lambda}\right) \middle| \lambda = 1, 2, \cdots, l\right\}, C_j \in J_1 \\ \left\{\left(1 - \gamma_{ij}^{k,\lambda}\right)\left(p_{ij}^{k,\lambda}\right) \middle| \lambda = 1, 2, \cdots, l\right\}, C_j \in J_2 \end{cases} \tag{32}$$

5.2. Probabilistic Hesitant Fuzzy Generalized TODIM Method

The TODIM (Tomada de decisao interativae multicritévio) method, as a multi-attribute decision-making method proposed on the basis of prospect theory, can take into account the different psychological behaviors of decision-makers on losses and gains, affecting the decision-making results. In the traditional TODIM method, the results may be contrary to the facts in some cases. Therefore, a simplified model of the TODIM method was proposed by Llamazares [24], and two examples were given to illustrate the results contrary to the expectations caused by the change of attribute weights, and then the concepts of weight consistency and weight monotonicity were proposed. By using the non-decreasing function in the definition of dominance, a generalized TODIM method is proposed, which avoids the violation of the results and facts caused by weight changes in some cases.

In the generalized TODIM method, the dominance of pharmaceutical e-commerce platform A_i to pharmaceutical e-commerce platform A_k under attribute C_j is defined as:

$$\Phi_j(A_i, A_k) = \begin{cases} g_1(\omega_k) f_1\left(y_{ij} - y_{kj}\right), & \left(y_{ij} - y_{kj}\right) \geq 0 \\ -g_2(\omega_k) f_2\left(y_{kj} - y_{ij}\right), & \left(y_{ij} - y_{kj}\right) < 0 \end{cases} \tag{33}$$

Among them, $g_1, g_2 : (0,1) \to (0,+\infty), f_1, f_2 : [0,1] \to [0,+\infty), f_1(0) = f_2(0) = 0$.

In this paper, the generalized TODIM method is extended to the probabilistic hesitant fuzzy environment, and the probabilistic hesitant fuzzy generalized TODIM method is constructed based on credibility. Then, the dominance of pharmaceutical e-commerce platform A_i to pharmaceutical e-commerce platform A_k under attribute C_j is defined as:

$$\Phi_j(A_i, A_k) = \begin{cases} g_1(\omega_j) f_1\left(R_{ij} - R_{kj}\right), & R_{ij} > R_{kj} \\ -g_2(\omega_j) f_2\left(R_{ij} - R_{kj}\right), & R_{ij} < R_{kj} \end{cases} \tag{34}$$

where R_{ij} is the credibility of pharmaceutical e-commerce platform A_i under attribute C_j, $g_1, g_2 : (0,1) \to (0,+\infty), f_1, f_2 : [0,1] \to [0,+\infty)$ and $f_1(0) = f_2(0) = 0$.

The total dominance degree of pharmaceutical e-commerce platform A_i is calculated according to the relative dominance degree, and the pharmaceutical e-commerce platforms are sorted according to the value of $\Phi(A_i)$. The total dominance degree of pharmaceutical e-commerce platform A_i is defined as:

$$\Phi(A_i) = \sum_{k=1}^{m} \sum_{j=1}^{n} \Phi_j(A_i, A_k) \tag{35}$$

The basic principle of the generalized TODIM method is consistent with that of the traditional TODIM method, but the calculation steps are simplified in the process of processing, making the calculation more concise. This paper assumes $g_1(x) = f_1(x) = x^\alpha$, $g_2(x) = \frac{1}{\theta} x^\beta$, $f_2(x) = x^\beta$, $\alpha = \beta = 0.5$, $\theta = 2.25$, which is consistent with the traditional TODIM method.

5.3. Decision-Making Process

Based on the above analysis, we give the specific steps of the selection method of a pharmaceutical e-commerce platform under a probabilistic hesitant fuzzy environment:

Step 1: The performance of each pharmaceutical e-commerce platform under each attribute is evaluated by each decision-maker using PHFE, and t probabilistic hesitant fuzzy decision-making matrices $H^k (k = 1, 2, \cdots, t)$ are obtained.

Step 2: Considering the cost attributes, the initial probabilistic hesitant fuzzy matrix is normalized according to Equation (32) to obtain the matrix $E^k(P)$.

Step 3: The hesitancy degree $f\left(h_{ij}^k\right)$ and credibility $R\left(h_{ij}^k\right)$ of PHFE h_{ij}^k are calculated by Equations (10) and (12), respectively, and adjust the subjective weights of decision-makers by Equations (13) and (14) to obtain the adjusted weights of decision-makers η_k''.

Step 4: Integrate the information of decision-makers using the $PHFPWA$ operator to obtain the comprehensive decision matrix $H = \left[h_{ij}\right]_{m \times n}$.

Step 5: The credibility of the comprehensive decision matrix is calculated, and the attribute weight determination model is built based on the credibility. The attribute weight vector $W = (\omega_1, \omega_2, \cdots, \omega_n)^T$ is obtained by Equations (18) and (19).

Step 6: Calculate the dominance $\Phi_j(A_i, A_s)$ of each pharmaceutical e-commerce platform for attribute C_j according to Equation (34), and the dominance matrix is obtained.

Step 7: Calculate the total dominance $\Phi(A_i)$ of pharmaceutical e-commerce platform A_i by Equation (35), and sort the pharmaceutical e-commerce platforms according to the size of $\Phi(A_i)$.

6. Numerical Example

6.1. Background

With the continuous promotion of the "Healthy China" strategy, the "Internet + medical health" model represented by pharmaceutical e-commerce platform is becoming a new trend. On 6 September 2022, the Ministry of Commerce released the 2021 Statistical Report on Pharmaceutical Circulation Industry. The report pointed out that in 2021, the total sales volume of seven categories of medical commodities in China reached CNY 2.6 trillion, and the total sales volume of pharmaceutical e-commerce platform direct reporting enterprises in 2021 reached CNY 216.2 billion (including the transaction volume of third-party trading service platform), accounting for 8.3% of the total scale of the national pharmaceutical market in the same period [25]. Since the outbreak of the COVID-19, the pharmaceutical e-commerce platform industry has developed rapidly, and the recognition of consumers of the pharmaceutical e-commerce platform has gradually increased. S Online consultation, self-testing, and drug sales have brought a lot of convenience to residents, and the demand for online drug sales has grown significantly. According to relevant data, during the epidemic, the number of daily online active users of pharmaceutical e-commerce platforms such as JD and Alibaba increased significantly from the previous month, and the peak could even rise to 1.48 million, with a maximum growth rate of 10% in the same period in 2019. According to data, based on the development of digital medical industry, China's pharmaceutical e-commerce platform and online consultation will enter a stage of rapid growth, and the market size is expected to reach CNY 1.2 trillion and CNY 407 billion, respectively, in 2030.

After a period of exploration and cultivation, China's pharmaceutical e-commerce platform industry has been basically formed, and a number of representative enterprises have emerged in their fields. Since 2018, the government has issued a series of policies to support the development of Internet hospitals, and has gradually liberalized the online sales of some prescription medicine, bringing a bright light to the development of pharmaceutical e-commerce platforms. In the future, compliant pharmaceutical e-commerce platforms are expected to establish a matching licensed pharmacist remote prescription examination system and prescription drug distribution system. The vigorous development of pharmaceutical e-commerce platforms has also brought new opportunities for pharmaceutical enterprises. The new Internet channels have broadened the sales scope of pharmaceutical enterprises, and the area covered by consumers is not limited by time and space. At the same time, through the management of the intelligent supply chain system and the unified deployment of medical storage and transportation, pharmaceutical enterprises have developed into low-cost modern green logistics to reduce the circulation cost. Therefore, it is of great practical significance for pharmaceutical enterprises to select an appropriate pharmaceutical e-commerce platform.

6.2. Research Hypothesis

Take pharmaceutical company A as an example. The enterprise has developed a new drug, and the management of the company plans to cooperate with a pharmaceutical e-commerce platform to promote the new drug. At present, there are four pharmaceutical e-commerce platforms $A = \{A_1, A_2, A_3, A_4\}$ for a company to choose from. We will evaluate these four platforms in five aspects, including type of drugs (C_1), price (C_2), response speed (C_3), level of medical personnel (C_4), and company qualification (C_5). The evaluation committee is composed of three decision-makers $d_i (i = 1, 2, 3)$, and the weight vector of the decision-makers is $\omega = (0.5, 0.3, 0.2)$. The evaluation value given by invited decision-makers is expressed by PHFE, and the initial evaluation results are shown in Tables 1–3.

Table 1. Original evaluation matrix of Expert 1 (d_1).

	A_1	A_2
c_1	$\{0.2(0.3), 0.8(0.7)\}$	$\{0.3(0.2), 0.7(0.8)\}$
c_2	$\{0.3(0.8), 0.8(0.2)\}$	$\{0.2(0.7), 0.9(0.3)\}$
c_3	$\{0.4(0.4), 0.5(0.6)\}$	$\{0.3(0.2), 0.6(0.8)\}$
c_4	$\{0.1(0.3), 0.4(0.7)\}$	$\{0.3(0.4), 0.6(0.6)\}$
c_5	$\{0.2(0.4), 0.6(0.6)\}$	$\{0.4(0.3), 0.5(0.7)\}$
	A_3	A_4
c_1	$\{0.6(0.9), 0.9(0.1)\}$	$\{0.4(0.4), 0.6(0.6)\}$
c_2	$\{0.3(0.4), 0.6(0.6)\}$	$\{0.6(0.4), 0.8(0.6)\}$
c_3	$\{0.5(0.7), 0.8(0.3)\}$	$\{0.7(0.4), 0.8(0.6)\}$
c_4	$\{0.3(0.4), 0.9(0.6)\}$	$\{0.2(0.6), 0.7(0.4)\}$
c_5	$\{0.2(0.4), 0.6(0.6)\}$	$\{0.2(0.4), 0.5(0.6)\}$

Table 2. Original evaluation matrix of Expert 2 (d_2).

	A_1	A_2
c_1	$\{0.5(0.3), 0.8(0.7)\}$	$\{0.3(0.2), 0.6(0.8)\}$
c_2	$\{0.3(0.8), 0.8(0.2)\}$	$\{0.4(0.7), 0.9(0.3)\}$
c_3	$\{0.4(0.3), 0.5(0.7)\}$	$\{0.3(0.4), 0.7(0.6)\}$
c_4	$\{0.2(0.4), 0.5(0.6)\}$	$\{0.4(0.3), 0.6(0.7)\}$
c_5	$\{0.3(0.5), 0.6(0.5)\}$	$\{0.4(0.3), 0.5(0.7)\}$
	A_3	A_4
c_1	$\{0.6(0.7), 0.8(0.3)\}$	$\{0.3(0.3), 0.6(0.7)\}$
c_2	$\{0.2(0.3), 0.3(0.7)\}$	$\{0.5(0.4), 0.8(0.6)\}$
c_3	$\{0.4(0.4), 0.7(0.6)\}$	$\{0.5(0.7), 0.8(0.3)\}$
c_4	$\{0.4(0.4), 0.9(0.6)\}$	$\{0.2(0.6), 0.7(0.4)\}$
c_5	$\{0.5(0.4), 0.8(0.6)\}$	$\{0.2(0.6), 0.6(0.4)\}$

Table 3. Original evaluation matrix of Expert 3 (d_3).

	A_1	A_2
c_1	$\{0.4(0.3), 0.8(0.7)\}$	$\{0.3(0.2), 0.6(0.8)\}$
c_2	$\{0.3(0.8), 0.9(0.2)\}$	$\{0.4(0.7), 0.9(0.3)\}$
c_3	$\{0.4(0.4), 0.5(0.6)\}$	$\{0.4(0.4), 0.6(0.6)\}$
c_4	$\{0.1(0.4), 0.4(0.6)\}$	$\{0.3(0.6), 0.7(0.4)\}$
c_5	$\{0.2(0.4), 0.6(0.6)\}$	$\{0.4(0.4), 0.5(0.6)\}$
	A_3	A_4
c_1	$\{0.5(0.6), 0.8(0.4)\}$	$\{0.4(0.3), 0.6(0.7)\}$
c_2	$\{0.5(0.2), 0.8(0.8)\}$	$\{0.2(0.4), 0.8(0.6)\}$
c_3	$\{0.3(0.4), 0.6(0.6)\}$	$\{0.7(0.4), 0.8(0.6)\}$
c_4	$\{0.5(0.4), 0.6(0.6)\}$	$\{0.4(0.6), 0.7(0.4)\}$
c_5	$\{0.3(0.4), 0.5(0.6)\}$	$\{0.3(0.3), 0.5(0.7)\}$

6.3. Data Processing and Alternative Ranking

Step 1: Data normalization processing. According to Equation (32), the decision matrix of the three decision-makers $H'^{k}(P)$ were transformed into $E^k(P)$, and the transformed results are shown in Tables 4–6.

Table 4. Standardized evaluation matrix of Expert 1 (d_1).

	A_1	A_2
c_1	$\{0.2(0.3), 0.8(0.7)\}$	$\{0.3(0.2), 0.7(0.8)\}$
c_2	$\{0.2(0.2), 0.7(0.8)\}$	$\{0.1(0.3), 0.8(0.7)\}$
c_3	$\{0.4(0.4), 0.5(0.6)\}$	$\{0.3(0.2), 0.6(0.8)\}$
c_4	$\{0.1(0.3), 0.4(0.7)\}$	$\{0.3(0.4), 0.6(0.6)\}$
c_5	$\{0.2(0.4), 0.6(0.6)\}$	$\{0.4(0.3), 0.5(0.7)\}$
	A_3	A_4
c_1	$\{0.6(0.9), 0.9(0.1)\}$	$\{0.4(0.4), 0.6(0.6)\}$
c_2	$\{0.4(0.6), 0.7(0.4)\}$	$\{0.2(0.6), 0.4(0.4)\}$
c_3	$\{0.5(0.7), 0.8(0.3)\}$	$\{0.7(0.4), 0.8(0.6)\}$
c_4	$\{0.3(0.4), 0.9(0.6)\}$	$\{0.2(0.6), 0.7(0.4)\}$
c_5	$\{0.2(0.4), 0.6(0.6)\}$	$\{0.2(0.4), 0.5(0.6)\}$

Table 5. Standardized evaluation matrix of Expert 2 (d_2).

	A_1	A_2
c_1	$\{0.5(0.3), 0.8(0.7)\}$	$\{0.3(0.2), 0.6(0.8)\}$
c_2	$\{0.2(0.2), 0.7(0.8)\}$	$\{0.1(0.3), 0.6(0.7)\}$
c_3	$\{0.4(0.3), 0.5(0.7)\}$	$\{0.3(0.4), 0.7(0.6)\}$
c_4	$\{0.2(0.4), 0.5(0.6)\}$	$\{0.4(0.3), 0.6(0.7)\}$
c_5	$\{0.3(0.5), 0.6(0.5)\}$	$\{0.4(0.3), 0.5(0.7)\}$
	A_3	A_4
c_1	$\{0.6(0.7), 0.8(0.3)\}$	$\{0.3(0.3), 0.6(0.7)\}$
c_2	$\{0.7(0.7), 0.8(0.3)\}$	$\{0.2(0.6), 0.5(0.4)\}$
c_3	$\{0.4(0.4), 0.7(0.6)\}$	$\{0.5(0.7), 0.8(0.3)\}$
c_4	$\{0.4(0.4), 0.9(0.6)\}$	$\{0.2(0.6), 0.7(0.4)\}$
c_5	$\{0.5(0.4), 0.8(0.6)\}$	$\{0.2(0.6), 0.6(0.4)\}$

Table 6. Standardized evaluation matrix of Expert 3 (d_3).

	A_1	A_2
c_1	$\{0.4(0.3), 0.8(0.7)\}$	$\{0.3(0.2), 0.6(0.8)\}$
c_2	$\{0.1(0.2), 0.7(0.8)\}$	$\{0.1(0.3), 0.6(0.7)\}$
c_3	$\{0.4(0.4), 0.5(0.6)\}$	$\{0.4(0.4), 0.6(0.6)\}$
c_4	$\{0.1(0.4), 0.4(0.6)\}$	$\{0.3(0.6), 0.7(0.4)\}$
c_5	$\{0.2(0.4), 0.6(0.6)\}$	$\{0.4(0.4), 0.5(0.6)\}$
	A_3	A_4
c_1	$\{0.5(0.6), 0.8(0.4)\}$	$\{0.4(0.3), 0.6(0.7)\}$
c_2	$\{0.2(0.8), 0.5(0.2)\}$	$\{0.2(0.6), 0.8(0.4)\}$
c_3	$\{0.3(0.4), 0.6(0.6)\}$	$\{0.7(0.4), 0.8(0.6)\}$
c_4	$\{0.5(0.4), 0.6(0.6)\}$	$\{0.4(0.6), 0.7(0.4)\}$
c_5	$\{0.3(0.4), 0.5(0.6)\}$	$\{0.3(0.3), 0.5(0.7)\}$

Step 2: Firstly, according to Equations (10) and (12), the hesitancy and credibility of decision-maker d_k about pharmaceutical e-commerce platform A_i under attribute C_j are obtained. Then, the weight of decision-maker is adjusted by Equations (13) and (14). The adjusted weight of decision-maker is as follows:

$$\eta_k'' = (0.4126, 0.3272, 0.2602)^T$$

Step 3: Integrate the information of decision-makers using the $PHFPWA$ operator to obtain the comprehensive decision matrix $H = [h_{ij}]_{m\times n}$ (Table 7):

Table 7. Evaluation matrix H.

	A_1	A_2
c_1	$\{0.364(0.300), 0.800(0.700)\}$	$\{0.300(0.200), 0.645(0.800)\}$
c_2	$\{0.175(0.200), 0.700(0.800)\}$	$\{0.100(0.300), 0.699(0.700)\}$
c_3	$\{0.400(0.367), 0.500(0.633)\}$	$\{0.327(0.333), 0.636(0.667)\}$
c_4	$\{0.134(0.367), 0.435(0.633)\}$	$\{0.334(0.433), 0.629(0.567)\}$
c_5	$\{0.234(0.433), 0.600(0.567)\}$	$\{0.400(0.333), 0.500(0.667)\}$
	A_3	A_4
c_1	$\{0.567(0.733), 0.849(0.267)\}$	$\{0.369(0.333), 0.600(0.667)\}$
c_2	$\{0.484(0.700), 0.700(0.300)\}$	$\{0.200(0.600), 0.575(0.400)\}$
c_3	$\{0.420(0.500), 0.725(0.500)\}$	$\{0.647(0.500), 0.800(0.500)\}$
c_4	$\{0.390(0.400), 0.857(0.600)\}$	$\{0.257(0.600), 0.700(0.400)\}$
c_5	$\{0.337(0.400), 0.661(0.600)\}$	$\{0.227(0.433), 0.535(0.567)\}$

Step 4: The credibility of the comprehensive decision matrix is calculated according to Equations (10) and (12), and then the reliability-based attribute weight determination model is built. The attribute weight is calculated according to Equations (18) and (19):

$$\omega_j = (0.1982, 0.1983, 0.2016, 0.2008, 0.2011)^T$$

Step 5: According to Equation (34), the dominance matrix Φ_j under attribute C_j can be obtained.

$$\Phi_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0.0103 & -0.2019 & 0.0421 & -0.0283 & -0.0456 \\ -0.1433 & -0.4034 & -0.0260 & 0.0441 & 0.0172 \\ -0.1256 & -0.4069 & -0.0531 & -0.0659 & -0.0430 \end{bmatrix}$$
$$\Phi_2 = \begin{bmatrix} -0.0232 & 0.0401 & -0.0928 & 0.0128 & 0.0206 \\ 0 & 0 & 0 & 0 & 0 \\ -0.1452 & -0.1555 & -0.0964 & 0.0459 & 0.0269 \\ -0.1277 & -0.1573 & -0.1069 & -0.0596 & 0.0069 \end{bmatrix}$$
$$\Phi_3 = \begin{bmatrix} 0.0639 & 0.0801 & 0.0118 & -0.0976 & -0.0381 \\ 0.0647 & 0.0694 & 0.0437 & -0.1016 & -0.0594 \\ 0 & 0 & 0 & 0 & 0 \\ 0.0308 & -0.0237 & -0.0463 & -0.1178 & -0.0574 \end{bmatrix}$$
$$\Phi_4 = \begin{bmatrix} 0.0560 & 0.0808 & 0.0241 & 0.0298 & 0.0195 \\ 0.0569 & 0.0702 & 0.0485 & 0.0269 & -0.0152 \\ -0.0691 & 0.0106 & 0.0210 & 0.0532 & 0.0260 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Step 6: The total dominance degree of pharmaceutical e-commerce platform A_i is calculated by Equation (35) as follows:

$$\Phi(A_1) = -1.4292, \Phi(A_2) = -0.8113$$
$$\Phi(A_3) = -0.1773, \Phi(A_4) = 0.4391$$

Finally, according to the size of $\Phi(A_i)$, the order of pharmaceutical e-commerce platforms can be obtained: $A_4 \succ A_3 \succ A_2 \succ A_1$, and A_4 should be selected as the cooperative pharmaceutical e-commerce platform.

6.4. Analysis of Sensitivity

Three different parameters are involved in this paper, which are risk preference coefficient θ and sensitivity coefficient α, β. In order to verify the robustness of the method proposed in this paper, different parameter values are calculated in this section.

(1) When $\alpha = \beta = 0.5$, by selecting different risk preference coefficients θ, the total dominance of each pharmaceutical e-commerce platform was calculated and ranked, and the result was shown in Figure 1.

(2) When $\theta = 2.25, \beta = 0.5$, by selecting different parameters α, the total dominance of each pharmaceutical e-commerce platform was calculated and ranked, and the result was shown in Figure 2a.

(3) When $\theta = 2.25, \alpha = 0.5$, by selecting different parameters β, the total dominance of each pharmaceutical e-commerce platform was calculated and ranked, and the result was shown in Figure 2b.

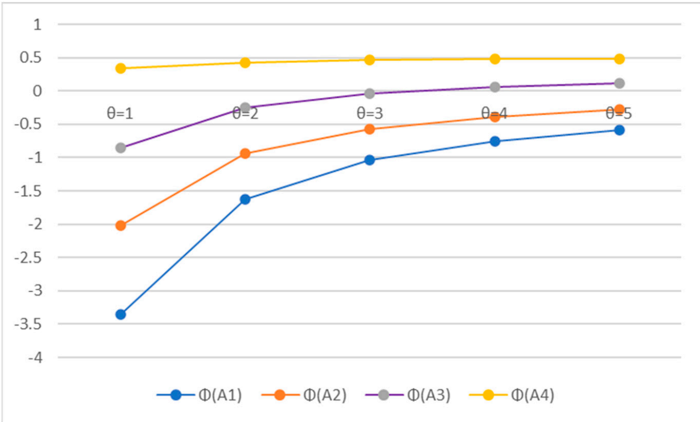
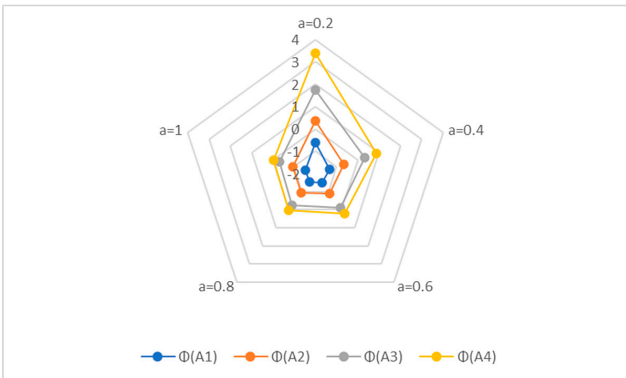
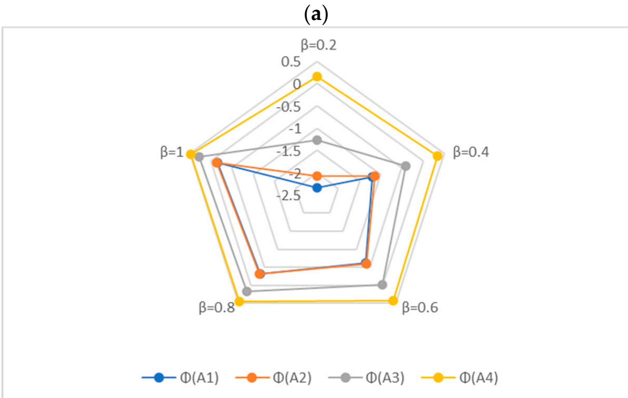


Figure 1. Influence of different values of θ on pharmaceutical e-commerce platform ranking.



(**a**)



(**b**)

Figure 2. Influence of different values of α and β on pharmaceutical e-commerce platform ranking. (**a**) Influence of different values of α on pharmaceutical e-commerce platform ranking. (**b**) Influence of different values of β on pharmaceutical e-commerce platform ranking.

It can be clearly seen from Figures 1 and 2 that:

(1) In the case study of this paper, when parameters θ, α, and β are changed, respectively, the optimal pharmaceutical e-commerce platform is always A_4.

(2) The difference between different parameter values is that the total dominance degree of pharmaceutical e-commerce platforms is different with different parameters, and the degree of differentiation between pharmaceutical e-commerce platforms is also different. According to Figure 1, the total dominance of each pharmaceutical e-commerce platform increases as θ increases. θ reflects the degree of sensitivity of the decision-maker to risk. The greater θ is, the lower the degree of risk aversion of the decision-maker is. That is, when facing risk, the smaller the impact of loss of the decision-makers and the smaller the gap of dominance degree between pharmaceutical e-commerce platforms will be. On the contrary, the smaller θ is, the higher the degree of risk aversion of decision-maker and the greater the gap of dominance degree between pharmaceutical e-commerce platforms. According to Figure 2a, with the increase of α, the total dominance of pharmaceutical e-commerce platforms gradually decreases, and the gap between pharmaceutical e-commerce platforms is also gradually narrowed. Similarly, according to Figure 2b, the total dominance of pharmaceutical e-commerce platforms increases with the increase of β, and the gap between pharmaceutical e-commerce platforms is also narrowed.

From the above analysis, it can be seen that the changes of θ, α and β will affect the total dominance degree, but the impact on the final ranking results of the pharmaceutical e-commerce platform is not obvious, which proves that the method proposed in this paper has stability within a certain range.

6.5. Comparative Analysis

In order to illustrate the reliability and rationality of the method proposed in this paper, a comparative analysis will be conducted with the PHFWBM operator and PHFWGBM operator proposed in Ref. [22], the PHFWMGSM operator proposed in Ref. [26], and the PHFOWA operator and PHFOWG operator proposed in Ref. [27]. The results are shown in Table 8.

Table 8. The combined score and sorting results obtained by using different operators.

Operator	A_1	A_2	A_3	A_4	Ranking Results
PHFPWA	−1.4292	−0.8113	−0.1773	0.4391	$A_4 \succ A_3 \succ A_2 \succ A_1$
PHFWBM [22]	0.218	0.202	0.232	0.251	$A_4 \succ A_3 \succ A_1 \succ A_2$
PHFWGBM [22]	0.372	0.357	0.394	0.425	$A_4 \succ A_3 \succ A_1 \succ A_2$
PHFWMGSM [26]	0.956	0.947	0.961	0.967	$A_4 \succ A_3 \succ A_1 \succ A_2$
PHFOWA [27]	0.404	0.433	0.419	0.453	$A_4 \succ A_2 \succ A_3 \succ A_1$
PHFOWG [27]	0.331	0.364	0.349	0.380	$A_4 \succ A_2 \succ A_3 \succ A_1$

It can be seen from Table 8 that the method proposed in this paper and the optimal pharmaceutical e-commerce platform obtained in Refs. [22,26,27] are all A_4, that is, the pharmaceutical company A should cooperate with pharmaceutical e-commerce platform A_4, indicating that the multi-attribute group decision-making method constructed in this paper is reasonable. The ranking results of PHFWBM operator, PHFWGBM operator, and PHFWMGSM operator are $A_4 \succ A_3 \succ A_1 \succ A_2$, and the ranking results of PHFOWA operator and PHFOWG operator are $A_4 \succ A_2 \succ A_3 \succ A_1$, which are different from the ranking results proposed by this paper ($A_4 \succ A_3 \succ A_2 \succ A_1$).

The main reasons for this difference are:

(1) Different comparison methods: The score function and deviation function defined in Ref. [21] are used in this paper to compare the size of the PHFE, while Ref. [22] used the distance of the PHFE proposed in it to compare the size of the PHFE, and the calculation is more complicated. Refs. [26,27] used score function and deviation function to compare the size of PHFE, but the function formulas used are different from those in this paper and are more outdated than those in this paper.

(2) Different aggregation methods: The above methods only focus on the relation between attributes when aggregating information, ignoring the influence of extreme value in group decision-making on the decision results. However, this paper not only considers the relation between attributes, but also pays attention to the extreme value in evaluation information, which is more consistent with reality.

(3) The irrationality of decision-makers can have an impact on decision results. The method proposed in this paper is a generalized TODIM multi-attribute group decision-making model, while the TODIM method is an effective tool for dealing with the multi-attribute decision-making problem considering the preference of the DMs. However, Refs. [22,26,27] did not consider the impact of decision-maker irrationality on decision results.

In fact, according to the assessment information in the original evaluation matrix of the experts, the values of pharmaceutical e-commerce platform A_3 are higher than pharmaceutical e-commerce platform A_2, and the values of pharmaceutical e-commerce platform A_2 are higher than pharmaceutical e-commerce platform A_1, namely $A_4 \succ A_3 \succ A_2 \succ A_1$, which agree with the decision results proposed in this paper. However, the results obtained in Refs. [22,26,27] are inconsistent with the original data, so the decision method proposed in this paper is more reliable.

7. Conclusions

In this paper, a probabilistic hesitant fuzzy pharmaceutical e-commerce platform selection method based on prospect theory is proposed. In the probabilistic hesitant fuzzy environment, firstly, the credibility of the decision-makers is proposed. Based on the credibility, the weights of the decision-makers are adjusted to eliminate the influence of insufficient information acquisition or personal bias on the decision results. Secondly, the PHFPA operator and PHFPWA operator are defined for information aggregation to eliminate the influence of extreme values on the decision results. Thirdly, considering that the decision-makers are not completely rational, a generalized TODIM method developed from prospect theory is introduced to construct a probabilistic hesitant fuzzy generalized TODIM multi-attribute group decision-making model. Finally, the method is applied to the selection of a pharmaceutical e-commerce platform.

In the future, we will further extend the model proposed in this paper from the two following aspects: (1) This paper used a single PA operator to assemble information. In the future, the PA operator will be considered to integrate with other operators with different characteristics, such as Bonferroni mean operator and Heronian mean operator, to solve the multi-attribute group decision-making problem more comprehensively. (2) This paper proved the effectiveness of the method with a numerical example. In order to more fully prove the feasibility and innovation of the method, actual data will be obtained for empirical research in the future.

Author Contributions: Conceptualization, S.L. and Z.G.; methodology, S.L. and Z.G.; writing—original draft preparation, S.L.; writing—review and editing, S.L. and Z.G. All authors have read and agreed to the published version of the manuscript.

Funding: This research was supported by the National Social Science Foundation of China [grant number: 20BTJ012]; Post-graduate's Innovation Fund Project of Hebei University [grant number: HBU2021bs003].

Data Availability Statement: Data is contained within the article.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. China Internet Network Information Center. The 49th Statistical Report on China's Internet Development[R/OL]. 25 February 2022. Available online: https://www.cnnic.net.cn/NMediaFile/old_attach/P020220721404263787858.pdf (accessed on 3 April 2023).
2. Zadeh, L.A. Fuzzy sets. *Inf. Control.* **1965**, *8*, 338–353. [CrossRef]
3. Webb, L. Green Purchasing: Forging a New Link in the Supply Chain. *Resource* **1994**, *6*, 14–18.
4. Handfield, R.B. Green Supply Chain: Best Practices from the Furniture Industry. *Proc.-Annu. Meet. Decis. Sci. Inst.* **1996**, *3*, 1295–1297.

5. Wu, J. A new approach for priorities trapezoidal fuzzy number reciprocal judgement matrix. *Chin. J. Manag. Sci.* **2010**, *3*, 95–100. [CrossRef]
6. Gao, X.L.; Wang, H.L.; Gan, W.H. Group decision model for green supplier selection based on trapezoidal fuzzy soft sets. *Comput. Eng. Appl.* **2017**, *1*, 265–270.
7. Li, L.; Zhang, S.Y. Aggregation method of intuitionistic fuzzy numbers with minimized maximal compromise under consensus condition. *Chin. J. Manag. Sci.* **2021**, *29*, 168–176.
8. Liu, J.B.; Peng, L.S.; Li, H.X.; Huang, B.; Zhou, X.Z. Interval-valued intuitionistic fuzzy three-way group decisions considering the unknown weight information. *Oper. Res. Manag. Sci.* **2022**, *31*, 50–57.
9. Peng, Y.; Liu, X.H.; Sun, J.B. Interval-valued intuitionistic fuzzy multi-attribute group decision making approach based on the hesitancy degrees and correlation coefficient. *Chin. J. Manag. Sci.* **2021**, *29*, 229–240.
10. Pan, F.P.; Gong, R.Z.; Tan, K.X. Multi-attribute group decision-making method for tourism projects under interval intuitionistic fuzzy information. *Stat. Decis.* **2021**, *37*, 173–176.
11. Shi, M.H.; Xiao, Q.X. Pythagorean group decision making method and its application in green supplier selection. *Oper. Res. Manag. Sci.* **2019**, *28*, 47–56.
12. Torra, V. Hesitant fuzzy sets. *Int. J. Intell. Syst.* **2010**, *25*, 529–539. [CrossRef]
13. Zhu, B. *Research and Application of Decision-Making Method Based on Preference Relation*; Southeast University: Nanjing, China, 2014.
14. Yager, R.R. The power average operator. *IEEE Trans. Syst. Man Cybern. Part A Syst. Hum.* **2001**, *31*, 724–731. [CrossRef]
15. Xu, Z.S.; Yager, R.R. Power-geometric operators and their use in group decision making. *IEEE Trans. Fuzzy Syst.* **2010**, *18*, 94–105.
16. Wan, B.T.; Lu, R.Y.; Lai, Z.Q. Interval-valued Pythagorean fuzzy power-weighted averaging operator and group decision method. *Comput. Appl. Res.* **2020**, *37*, 183–187.
17. Liu, P.D.; Liu, X. The neutrosophic number generalized weighted power averaging operator and its application in multiple attribute group decision making. *Int. J. Mach. Learn. Cybern.* **2018**, *9*, 347–358. [CrossRef]
18. Liang, D.C.; Darko, A.P.; Zeng, J. Interval-valued pythagorean fuzzy power average-based MULTIMOORA method for multi-criteria decision-making. *J. Exp. Theor. Artif. Intell.* **2020**, *32*, 845–874. [CrossRef]
19. Wang, J.; Shang, X.P.; Bai, K.Y.; Xu, Y. A new approach to cubic q-rung orthopair fuzzy multiple attribute group decision-making based on power Muirhead. *Neural Comput. Appl.* **2020**, *32*, 14087–14112. [CrossRef]
20. Li, L.; Ji, C.J.; Wang, J. A Novel Multi-attribute Group Decision-Making Method Based on q-Rung Dual Hesitant Fuzzy Information and Extended Power Average Operators. *Cogn. Comput.* **2021**, *13*, 1345–1362. [CrossRef]
21. Xu, Z.S.; Zhou, W. Consensus building with a group of decision makers under the hesitant probabilistic fuzzy environment. *Fuzzy Optim. Decis. Mak.* **2017**, *16*, 481–503. [CrossRef]
22. Wu, J.; Liu, X.D.; Zhang, S.T.; Wang, Z.Z. Probabilistic hesitant fuzzy Bonferroni mean operators and their application in decision making. *Fuzzy Syst. Math.* **2019**, *33*, 116–126.
23. Liu, S.J.; Guo, Z.X. Probabilistic hesitant fuzzy multi-attribute decision-making method based on improved distance measurement. *J. Intell. Fuzzy Syst.* **2022**, *43*, 5953–5964. [CrossRef]
24. Llamazares, B. An analysis of the generalized TODIM method. *Eur. J. Oper. Res.* **2018**, *269*, 1041–1049. [CrossRef]
25. Ministry of Commerce of the People's Republic of China. 2021 Statistical Report on Pharmaceutical Circulation Industry[R/OL]. 6 September 2022. Available online: http://images.mofcom.gov.cn/scyxs/202209/20220906181153114.pdf (accessed on 3 April 2023).
26. Wu, W.Y.; Li, Y.; Ni, C.W.; Zhu, X.H.; Wu, Z.J. Probabilistic hesitant fuzzy Maclaurin geometric symmetric averaging operator and its group decision model. *Syst. Sci. Math.* **2020**, *40*, 1074–1089.
27. Wu, W.Y.; Li, Y.; Jin, F.F.; Ni, Z.W.; Zhu, X.H. Group Decision Making Model Based on Probabilistic Hesitant Fuzzy Information Aggregation Operations. *Pattern Recognit. Artif. Intell.* **2017**, *30*, 894–906.