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Dynamic Event-Triggered Consensus Control for Markovian Switched Multi-Agent Systems: A Hybrid Neuroadaptive Method

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Abstract: This paper presents a solution to the consensus problem for a particular category of uncertain switched multi-agent systems (MASs). In these systems, the communication topologies between agents and the system dynamics are governed by a time-homogeneous Markovian chain in a stochastic manner. To address this issue, we propose a novel neuroadaptive distributed dynamic event-triggered control (DETC) strategy. By leveraging stochastic Lyapunov theory and matrix inequality methodology, we establish sufficient conditions for practical ultimate mean square consensus (UMSBC) of MASs using a combination of neural networks (NNs) adaptive control strategy and DETC method. Our approach employs a distributed adaptive NNs DETC mechanism in MASs with unknown nonlinear dynamics and upgrades it at the moment of event sampling in an aperiodic manner, resulting in significant savings in computation and resources. We also exclude the Zeno phenomenon. Finally, we provide numerical examples to demonstrate the feasibility of our proposed approach, which outperforms existing approaches.

Keywords: neuroadaptive control; dynamic event-triggered; consensus; Markovian switched; multi-agent systems

MSC: 93A14; 90B18; 68T42

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1. Introduction

Cooperative behaviors in MASs have been extensively studied by scholars, particularly in computer science and engineering, to address large-scale issues [1–6]. Synchronization or consensus is a crucial problem for MASs and has gained increasing scholarly attention. Although research on MASs has produced significant results [7–10], most existing consensus results pertain to linear systems with identical dynamics and fixed topologies [11–13]. However, in reality, internal or external factors such as environmental changes may introduce unknown, nonlinear, heterogeneous, or switched components into MASs, which must be addressed. Therefore, research on MASs has made significant strides in making systems more realistic [14–19]. For instance, ref. [14] thoroughly explored the fixed-time consensus issue for a family of nonhomogeneous nonlinear MASs, while ref. [16] considered a group of nonlinear MASs vulnerable to external shocks for the leader-following bipartite consensus under directed fixed and switching topologies. In cooperative-competitive networks, ref. [17] investigated couple-group consensus for heterogeneous MASs with switching topology, and ref. [18] mainly focused on a type of nonlinear switching MASs about event-triggered adaptable tracking control. Moreover, ref. [15] researched the consensus control issue of second-order MASs with switched dynamics. Additionally, ref. [19] proved the existence of a unique global positive solution for a new stochastic epidemic model explaining the dynamics of hepatitis B with the inclusion of white noise, Markovian switching, and vaccination control. This paper concentrates on Markovian switching MASs and demonstrates interacting systems with unknown, heterogeneous, nonlinear dynamics, as well as arbitrarily switched systems and network architectures.

Achieving consensus control in MASs requires each agent to constantly receive information about its nearby neighbors, resulting in additional computing costs, processing power requirements, and communication delays. To address these challenges, event-triggered control methods [20–26] have been developed as an approach to avoid continuous communication. Two prominent event-triggered control methods are static event-triggered control (SETC) [21] and distributed event-triggered control (DETC) [27–31]. Compared to SETC, DETC can drastically reduce the occurrence of events without significantly affecting the controller's functionality. In this paper, we propose a dynamic event-triggered (DET) consensus control method that combines the benefits of event-triggered control with the ability to handle unknown, nonlinear, heterogeneous, or switched systems. The main challenge in the DET synchronization issue is to establish a distributed dynamic event-based protocol for MASs that combines an event-based control rule and triggering function. Prior research on DETC includes observer-based DET semiglobal bipartite consensus (SGBC) for linear MASs with input saturation in a competitive network [27], DETC's application to universal linear MASs [28], and event-triggered consensus of generalized linear MASs in leaderless and leader-following networks within the setting of adaptive control [29]. However, to the best of our knowledge, there is still a need for a breakthrough in developing a distributed DET synchronization control procedure for generic nonlinear Markovian switching systems.

NNs have become a popular technique for dealing with nonlinearity and uncertainties in systems and networks due to their high approximation capacity [32–37]. For example, in [32], an adaptive NN-based control strategy was proposed to achieve asymptotic consensus for a MASs with uncertain nonlinear dynamics and a high-dimensional leader under a directed switching topology. Control concerns for MASs subjected to unknown nonlinearity and external disturbances in a directed communication topology were addressed in [33]. An analytical tool was developed in [34] for the neuroadaptive tracking control of hybrid Markovian switching networks with heterogeneous nonlinear dynamics and randomly switched topologies. In [35], the observer-based event-triggered tracking control problem for nonlinear MASs subject to denial-of-service (DoS) attacks was examined.

Based on the analysis presented earlier, the main focus of this study is on the adaptive NN-based distributed DET consensus problem for switching MASs. We propose a novel distributed DET consensus methodology that updates only when the DET criteria are violated. Information from neighboring agents is used to distribute the control rule and event-triggering function, thereby avoiding Zeno behavior. The main contributions of this study, compared to relevant works, are as follows:

1. The consensus control problem of switched MASs prompted by a discrete Markovian process in the network and system is addressed. Notably, existing consensus control approaches have primarily focused on MASs communicated via a fixed connected graph [10–14,16,20–23]. However, few consensus control techniques have been investigated for unknown nonlinear switching MASs. As a result, it is increasingly crucial to investigate the synchronous control of unknown nonlinear MASs with switching topologies and systems.
2. In this paper, the consistency problem of a class of MASs with unknown nonlinearities is handled by employing a neural adaptive DET strategy. Compared with continuous-time neuroadaptive consensus control algorithms [32–34,36], the proposed consensus control rule not only accomplishes consensus control but also successfully reduces communication.
3. A new adaptive distributed DET conditions to steer the proposed nonlinear systems to consensus is presented. The key to achieving event-triggered consensus control is designing suitable triggering circumstances. Notably, while there have been a few event-triggered consensus control methods for nonlinear MASs researched [20–22], current event-triggered synchronization control approaches mostly focus on linear MASs [27–30] on undirected graphs. Therefore, it is more relevant to research DET

consensus control of nonlinear MASs on switched undirected networks, as opposed to the findings in [27–30].

The rest of this paper is organized as follows. Section 2 presents the study topic, preliminary information, and a discussion of a hybrid adaptive DETC. Theoretical study of the consensus of switched MASs with unknown nonlinear dynamics is provided in Section 3. Examples of simulations are provided in Section 4, and the conclusions are drawn in Section 5.

Notations: The following standard notations are used in this text. q-dimensional real vectors make up the collection $\mathfrak{R}^q$, while nonnegative real numbers make up the collection $\mathfrak{R}_+$. Euclidean norm, Frobenuis norm and Kronecker product are described by $\|\cdot\|$, $\|\cdot\|_F$ and $\otimes$, respectively. $x = \mathrm{col}(x_1, x_2, \cdots, x_N), x_i \in \mathfrak{R}^q$, is a column vector of dimension qN. $\mathrm{diag}\{\cdots\}$ represents the diagonal matrix. $I_N \in \mathfrak{R}^{N\times N}$ is the identity matrix and $\underline{1}_N = [1, 1, \cdots, 1]^\top \in \mathfrak{R}^N$. For symmetric matrix A, $A \succ 0 (A \succeq 0)$ means that A is positive definite (positive semidefinite) matrix, and $\lambda_{\max}(A)$ ($\lambda_{\min}(A)$) is the maximum (minimum) eigenvalue of matrix A. $\mathbb{P}$ stands for probability, and $\mathbb{E}$ stands for expectation, both of which are specified in a complete probability space.

2. Problem Formulation and Preliminaries

2.1. Markovian Switched MASs

This paper considers Markovian switched MASs with unknown nonlinear dynamics, consisting of N agents. The system dynamics are expressed as follows:

$$\dot{x}_i(\tau) = A_{\delta(\tau)} x_i(\tau) + B_{\delta(\tau)}(h_i(x_i(\tau), \tau) + u_i(\tau)), \quad \tau \geqslant \tau_0, \tag{1}$$

where $x_i \in \mathfrak{R}^q$ and $u_i \in \mathfrak{R}^m$ represent the state vector and the control input vector of the i-th agent, $x_i(\tau_0)$ is the initial state, $A_{\delta(\tau)} \in \mathfrak{R}^{q\times q}$ and $B_{\delta(\tau)} \in \mathfrak{R}^{q\times m}$ are the system state matrix and the system control input matrix, and $h_i : \mathfrak{R}^q \times \mathfrak{R}_+ \to \mathfrak{R}^m$ denotes a smooth unknown nonlinear function, $i = 1, 2, \cdots, N$. The time-depend switched signal is represented as $\delta(\tau) : [0, \infty) \to \aleph = \{1, 2, \cdots, \vartheta\}$ ($\vartheta \in \mathbb{N}$), where $\vartheta > 1$ is the number of modes. For the time sequence $\{\tau_v, v \in \mathbb{Z}_+\}$ satisfying $0 = \tau_0 < \tau_1 < \cdots < \tau_v < \cdots$ and $\lim_{v\to+\infty} \tau_v = +\infty$. Suppose that $\{\tau_v\}_{v\geqslant 1}$ is the Markovian switched time sequence and the form $\{\delta(\tau_v)\}_{v\geqslant 1}$ is a homogeneous Markovian chain, with a state set $\aleph$, initial distribution probability $\pi^{(0)} = \left[\pi_1^{(0)}, \pi_2^{(0)}, \cdots, \pi_\vartheta^{(0)}\right]^\top$ and transition probability matrix $P = [p_{ij}] \in \mathfrak{R}^{\vartheta\times\vartheta}$, where $p_{ij} = \mathbb{P}\{\delta(\tau_{v+1}) = j \mid \delta(\tau_v) = i\}$ and $\pi_i^{(0)} = \mathbb{P}\{\delta(\tau_0) = i\}$. In this case, for $\delta(\tau) = l \in \aleph$, $A_{\delta(\tau)}$ and $B_{\delta(\tau)}$ are denoted as A_l and B_l.

The network topology of MASs (1) is denoted by an undirected switched graph $\mathcal{G}_{\delta(\tau)} = \{\mathcal{V}, \mathcal{E}_{\delta(\tau)}, \mathcal{A}_{\delta(\tau)}\}$. The edge set of $\mathcal{G}_{\delta(\tau)}$ denotes as $\mathcal{E}_{\delta(\tau)} = \{(i, j) \in \mathcal{V} \times \mathcal{V}\}$, where $\mathcal{V} = \{1, 2, \cdots, N\}$ represents the agent set. $(j, i) \in \mathcal{E}_{\delta(\tau)}$ represents that the information of agent j is accessible to agent i at time τ and define $\mathcal{N}_i^{\delta(\tau)} = \{j \in \mathcal{V} : (j, i) \in \mathcal{E}_{\delta(\tau)}, j \neq i\}$, for $i \in \mathcal{V}$. $\mathcal{A}_{\delta(\tau)} = \left[a_{ij}^{\delta(\tau)}\right] \in \mathfrak{R}^{N\times N}$ is the adjacency matrix of the graph $\mathcal{G}_{\delta(\tau)}$, where $a_{ii}^{\delta(\tau)} = 0$ and if $(j, i) \in \mathcal{E}_{\delta(\tau)}$, $a_{ij}^{\delta(\tau)} = a_{ji}^{\delta(\tau)} = 1, \forall i, j \in \mathcal{V}$. The Laplacian matrix of graph $\mathcal{G}_{\delta(\tau)}$ can be denoted as $L_{\delta(\tau)} = \left[l_{ij}^{\delta(\tau)}\right] \in \mathfrak{R}^{N\times N}$, where $l_{ij}^{\delta(\tau)} = -a_{ij}^{\delta(\tau)}$ if $i \neq j$ and $l_{ii}^{\delta(\tau)} = \sum_{j\neq i}^{N} a_{ij}^{\delta(\tau)}$. The graph $\mathcal{G}_{\delta(\tau)}$ randomly switches among ϑ graphs, resulting in a Markovian switched topology for the MASs (1).

Remark 1. *Each agent in a networked system can only perceive the behavior of its neighboring agents, which is why the system is often referred to as distributed or cooperative. However, the control issues in network (1) arise from factors such as heterogeneity, unidentified nonlinearity, and Markovian switching. While existing works mostly focus on fixed systems of MASs [1,11,27–30], it is more meaningful to consider switched systems of MASs since systems can change in practical situations. Furthermore, it is important to investigate how Markovian switching topology affects* *consensus effectiveness. Therefore, the main objective of this study is to construct a distributed control technique for the MASs in (1) that maintains consensus despite these challenges.*

Assumption 1 ([34,38])**.** *The average dwell time of the Markovian switched time sequence* $\{\delta(\tau_v)\}_{v\geqslant 1}$ *is more than* $\chi_d > 0$ *if there exist a positive integer* N_0 *and a positive number* $\chi_d > 0$*, such that*

$$\frac{\tau - \tau_0}{\chi_d} \leqslant N(\tau, \tau_0) \leqslant N_0 + \frac{\tau - \tau_0}{\chi_d}, \quad \tau > \tau_0 \geq 0, \tag{2}$$

where $N(\tau, \tau_0)$ *denotes the number of switched times of the switched sequence* $\{\tau_v\}_{v\geqslant 1}$ *in the time interval* $(\tau_0, \tau]$*.*

Assumption 2 ([34,39])**.**

(1) the pair (A_l, B_l) *is controllable for all* $l \in \aleph$, *;*
(2) the graph $\mathcal{G}_l$ *is connected for all* $l \in \aleph$*;*
(3) the discrete Markovian process $\{\tau_v\}_{v\geqslant 1}$ *is ergodic.*

Remark 2. *According to Assumption 1, there is no strict requirement for the the Markovian switched time sequence* $\{\tau_v\}_{v\geqslant 1}$*, which has an average dwell time smaller than* $\chi_d > 0$ *, but it must be larger than* $\frac{\tau - \tau_0}{N(\tau,\tau_0) - N_0}$*.*

Remark 3. *These assumptions are commonly used in the analysis of Markovian switched systems. They ensure that the system is controllable, the switching topologies are well-connected, and the Markovian chain is well-behaved. Furthermore, Assumption 2 (3) ensures that the Markovian chain* $\{\delta(\tau_v)\}_{v\geqslant 1}$ *has a unique, positive, invariant probability distribution* $\pi = (\pi_1, \pi_2, \ldots, \pi_\vartheta)$ *satisfying* $\pi P = \pi$*. Each mode may be accessed from any other, in other words.*

Lemma 1 ([39,40])**.** $\forall l \in \aleph$, *the Laplacian matrix* L_l*, linked to the undirected graph* $\mathcal{G}_l$*, has an eigenvalue 0 with* $\underline{1}_N$ *as a corresponding right eigenvector, while all the other eigenvalues are positive. Plus, when* $\mathcal{G}_l$ *is connected, 0 is a simple eigenvalue of* L_l*.*

2.2. Preliminaries

Let the error be explained by $z_i = x_i - \frac{1}{N}\sum_{j=1}^{N} x_j$, $i \in \mathcal{V}$ and $z = \mathrm{col}(z_1, \cdots, z_N)$ can be used to denote the compact form of error vector. The error can be given by $z = (M \otimes I_q)x$, where $M = I_N - \frac{1}{N}\underline{1}_N\underline{1}_N^\top \geqslant 0$ and $x = \mathrm{col}(x_1, x_2, \cdots, x_N)$. Noticeably, with Assumption 2 (2), for $\forall l \in \aleph$, $L_l M = L_l = M L_l$, $M^2 = M$. The subsequent consensus definition is thereafter provided to clarify the control goal of this study.

Definition 1. *The Markovian switched MASs (1) is said to achieve practical ultimate mean square bounded consensus (UMSBC) at an error level* $\epsilon_0 > 0$ *if there exists a compact set* $\mathcal{O}$ *such that for any initial condition* $z(\tau_0)$*, the error signal* z *converges to*

$$\mathcal{O} = \left\{ z(\tau) \in \mathfrak{R}^{qN} : \mathbb{E}[\|z(\tau)\|^2] \leqslant \epsilon_0 \right\} \quad \text{as } \tau \to \infty.$$

Remark 4. *The above definition introduces the concept of practical UMSBC involving a key constant* ϵ_0*, where* $\epsilon_0 \leqslant 0$ *is a predetermined threshold. In particular, if* $\epsilon_0 = 0$*, one may have the common UMSBC. By using the practical UMSBC as a foundation, the practical consensus is described.*

2.3. Hybrid Adaptive Control Using Neural Networks

In light of the fact that the uncertainties $h_i(x_i, \tau)$ are permitted to differ, it should be noted that the agents are basically heterogeneous. In addition, $h_i(x_i, \tau)$ might be unknown towards the control architect. Within this study, NNs are used as a direct means of offsetting the unidentified nonlinear functions of system. The following lemma is required to do this.

Lemma 2 ([39]). *The unidentified becoming function $h_i(x_i,\tau)$ in (1) can be linearly parameterized by a NNs approximation given*

$$h_i(x_i,\tau) = W_i^\top \phi_i(x_i) + \varepsilon_i(\tau), \quad \forall x_i \in \Omega_i,$$

where the optimum weight matrix $W_i \in \Re^{r\times m}$ is unknown, which is constrained by $\|W_i\|_F \leq W_M^i$ and the basis function vector $\phi_i(x_i) : \Re^q \mapsto \Re^r$ with $\phi_i(x_i) = (\phi_{i1}(x_i), \cdots, \phi_{ir}(x_i))^\top$ and satisfies $\|\phi_i(x_i)\| \leq \phi_M^i$. $\varepsilon_i(\tau)$ is the approximation error vector meeting $\|\varepsilon_i(\tau)\| \leq \varepsilon_M^i$ and Ω_i is a big enough compact set in $\Re^q$, $i \in \mathcal{V}$.

Remark 5. *The universal approximation theorem, which is often used when handling approximation problems with NNs, may ensure Lemma 2 [32–36]. NNs approximation of unknown smooth function $h_i(x_i,\tau)$ is depicted in Figure 1. Here, for the controller design, the constants W_M^i, ϕ_M^i, and ε_M^i could be unknown, which is solely used for theoretical study.*

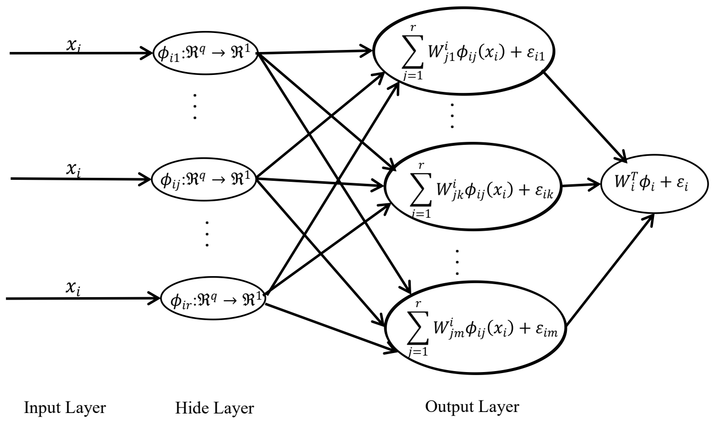


Figure 1. Neural networks estimation.

Assume that $x_i(\tau_0) \in \Omega_i$ and x_i stays inside Ω_i during the trajectory of system (1). The NNs estimator is constructed as $\widehat{W}_i^\top(\tau)\phi_i(x_i)$ to obtain real data $h_i(x_i,\tau)$, while the approximating error is identified by

$$h_i(x_i,\tau) - \widehat{W}_i^\top(\tau)\phi_i(x_i) = \overline{W}_i^\top(\tau)\phi_i(x_i) + \varepsilon_i(\tau),$$

where $\overline{W}_i(\tau) = W_i - \widehat{W}_i(\tau)$. Denote $\varepsilon = \mathrm{col}(\varepsilon_1, \varepsilon_2, \cdots, \varepsilon_N)$, then $\|\varepsilon\|^2 \leqslant \sum_{i=1}^{N} {\varepsilon_M^i}^2 = \bar{\varepsilon}, i \in \mathcal{V}$.

In order to implement practical tracking, a hybrid neuroadaptive control scheme is developed, which includes a distributed control protocol and a hybrid NNs adaption law:

$$u_i(\tau) = \alpha K_{\delta(\tau)}\widetilde{e}_i(\tau) - \widehat{W}_i^\top(\tau_k^i)\phi_i(x_i(\tau_k^i)), \quad \tau \in [\tau_k^i, \tau_{k+1}^i), \tag{3}$$

$$\dot{\widehat{W}}_i = -\beta\widehat{W}_i + \phi_i x_i^\top P_{\delta(\tau)}B_{\delta(\tau)}, \tag{4}$$

where $\widetilde{e}_i(\tau) = \sum_{j\in\mathcal{N}_i^{\delta(\tau)}} a_{ij}^{\delta(\tau)}(x_j(\tau_k^i) - x_i(\tau_k^i))$, $\tau \in [\tau_k^i, \tau_{k+1}^i)$, $K_{\delta(\tau)} = B_{\delta(\tau)}^\top P_{\delta(\tau)}$ denotes the control gain matrix, $P_{\delta(\tau)} \succ 0$ will be designed latter, $\alpha, \beta > 0$ are the control and tuning gains, and $\{\tau_k^i\}_{k\geqslant 1}$ is the event-triggering instants series of agent i, which will be discussed in more detail in the event-triggering condition, $i \in \mathcal{V}$.

In these circumstances, for $\tau \in [\tau_k^i, \tau_{k+1}^i)$, the closed-loop system is enabled by the control input (3)

$$\begin{aligned}\dot{x}_i(\tau) =& A_{\delta(\tau)}x_i(\tau) + \alpha B_{\delta(\tau)}K_{\delta(\tau)}\widetilde{e}_i(\tau) \\ &+ B_{\delta(\tau)}\Big(W_i^\top\phi_i(x_i(\tau)) - \widehat{W}_i^\top(\tau_k^i)\phi_i(x_i(\tau_k^i)) + \varepsilon_i(\tau)\Big).\end{aligned} \tag{5}$$

Then, from (5), with some simple calculations, the compact form of the error system can be rewritten as

$$\begin{aligned}\dot{z}(\tau) =& \Big[\Big(I_N \otimes A_{\delta(\tau)}\Big) - \alpha\Big(L_{\delta(\tau)} \otimes B_{\delta(\tau)}K_{\delta(\tau)}\Big)\Big]z(\tau) + \alpha\Big(M \otimes B_{\delta(\tau)}K_{\delta(\tau_v)}\Big)e^*(\tau) \\ &+ \Big(M \otimes B_{\delta(\tau)}\Big)\varepsilon + \Big(M \otimes B_{\delta(\tau)}\Big)\overline{W}(\tau)^\top \Phi(x(\tau)) \\ &+ \Big(M \otimes B_{\delta(\tau)}\Big)\widehat{W}^\top(\tau)\Phi^*(x(\tau)) \qquad (6) \\ &+ \Big(M \otimes B_{\delta(\tau)}\Big)\widehat{W}^{*\top}(\tau)\overline{\Phi}(x(\tau)), \quad \tau \in [\tau_k^i, \tau_{k+1}^i), \qquad (7)\end{aligned}$$

where $e^*(\tau) = \mathrm{col}(e_1^*(\tau), e_2^*(\tau), \cdots, e_N^*(\tau))$ with $e_i^*(\tau) = \sum_{j\in\mathcal{N}_i^{\delta(\tau)}} a_{ij}^{\delta(\tau)}(x_j^i(\tau) - x_i^i(\tau))$ and $x_j^i(\tau) = x_j(\tau_k^i) - x_j(\tau)$, which is the measurement error of x_j, $\Phi(x(\tau)) = \mathrm{col}(\phi_1(x_1(\tau)), \phi_2(x_2(\tau)), \cdots, \phi_N(x_N(\tau)))$; $\overline{\Phi}(x(\tau)) = \mathrm{col}(\phi_1(x_1(\tau_k^1)), \phi_2(x_2(\tau_k^2)), \cdots, \phi_N(x_N(\tau_k^N)))$; $\Phi^*(x(\tau)) = \mathrm{col}(\phi_1^*(x_1(\tau)), \phi_2^*(x_2(\tau)), \cdots, \phi_N^*(x_N(\tau)))$ with $\phi_i^*(x_i(\tau)) = \phi_i(x_i(\tau)) - \phi_i(x_i(\tau_k^i))$, which is the measurement error of ϕ_i; $W(\tau) = \mathrm{diag}(W_1(\tau), W_2(\tau), \cdots, W_N(\tau))$; $\overline{W}(\tau) = \mathrm{diag}(\overline{W}_1(\tau), \overline{W}_2(\tau), \cdots, \overline{W}_N(\tau))$; $\widehat{W}(\tau) = \mathrm{diag}(\widehat{W}_1(\tau), \widehat{W}_2(\tau), \cdots, \widehat{W}_N(\tau))$ and $\widehat{W}^*(\tau) = \mathrm{diag}(\widehat{W}_1^*(\tau), \widehat{W}_2^*(\tau), \cdots, \widehat{W}_N^*(\tau))$ with $\widehat{W}_i^*(\tau) = \widehat{W}_i(\tau) - \widehat{W}_i(\tau_k^i)$, which is the measurement error of $\widehat{W}_i$, $i, j \in \mathcal{V}$.

2.4. *Dynamic Event-Triggered Control Protocols*

To lessen the volume of communications for saving energy, the distributed DETC law to be designed as follows:

$$\begin{aligned}\tau_{k+1}^i \triangleq \inf\Big\{\tau > \tau_k^i :\ & \xi\|e_i^*(\tau)\|^2 > d_1\|e_i(\tau)\|^2 + b_1^{-1}\psi_i^{(1)}(\tau) \quad \text{or} \\ & \xi\Big(\|\widehat{W}_i^\top(\tau)\phi_i^*(x_i(\tau))\|^2 + \|\widehat{W}_i^{*\top}(\tau)\phi_i(x_i(\tau_k^i))\|^2\Big) \\ & > d_2\|e_i(\tau)\|^2 + b_2^{-1}\psi_i^{(2)}(\tau)\Big\}, \end{aligned} \qquad (8)$$

with

$$\dot{\psi}_i^{(1)}(\tau) = -a_1\psi_i^{(1)}(\tau) + a_2\Big(d_1\|e_i(\tau)\|^2 - \xi\|e_i^*(\tau)\|^2\Big), \qquad (9)$$

$$\begin{aligned}\dot{\psi}_i^{(2)}(\tau) =& -a_3\psi_i^{(2)}(\tau) + a_4\big[d_2\|e_i(\tau)\|^2 - \xi\big(\|\widehat{W}_i^\top(\tau)\phi_i^*(x_i(\tau))\|^2 \\ &+ \|\widehat{W}_i^{*\top}(\tau)\phi_i(x_i(\tau_k^i))\|^2\big)\big],\end{aligned} \qquad (10)$$

where $\xi = \max_{l\in\aleph}\{\rho_1\alpha^2\|P_lB_lK_l\|^2, \rho_2\|P_lB_l\|^2, \rho_3\|P_lB_l\|^2\}$, e_i is the consensus error of agent i, i.e., $e_i = \sum_{j\in\mathcal{N}_i^{\delta(\tau)}} a_{ij}^{\delta(\tau)}(x_j(\tau) - x_i(\tau))$, $i \in \mathcal{V}$ and a_i $(i = 1, 2, 3, 4)$, b_j $(j = 1, 2)$ are positive constants satisfying $a_1b_1 + a_2 > 1$, $a_3b_2 + a_4 > 1$ and ρ_1, ρ_2, ρ_3, d_1, d_2 are given positive parameters.

Remark 6. *As a result, the time-triggered mechanism may lead to unnecessary waste of resources due to the lack of explicit regulations for choosing appropriate triggered intervals. The event-triggered mechanism, on the other hand, adopts the zero-order hold (ZOH) method, which means that the control input remains constant until the next triggered instant, reducing the communication load. Figure 2 illustrates the structure of the MASs with DET mechanism. Moreover, it can be observed from (9) and (10) that* $\psi_i^{(1)}(\tau) > 0$ *and* $\psi_i^{(2)}(\tau) > 0$ *act as dynamic thresholds for the DETC scheme, serving as positive scalar functions. A similar proof can be made as in [41]. The dynamic threshold helps to decrease the number of triggers compared to the static case [20–26], resulting in less frequent triggers overall.*

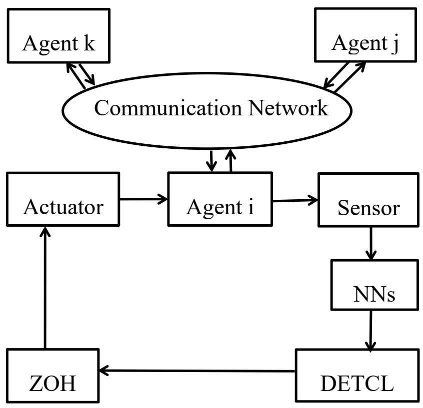


Figure 2. Structure of the MASs (1) with DET mechanism.

3. Main Results

The purpose of this study is to develop a neuroadaptive DET feedback control rule to achieve consensus of Markovian switched networked systems. The major theorem of this section is now prepared for presentation.

Theorem 1. *If Assumptions 1 and 2 are valid, for* $\forall l \in \aleph$*, if there exist matrices* $P_l \succ 0$*,* $Q_l \succ 0$ *and parameters* $\rho_1, \rho_2, \rho_3, \rho_4, d_1, d_2 > 0$ *satisfy*

$$P_l A_l + A_l^\top P_l - 2\alpha\lambda_2(L_l) P_l B_l (B_l)^\top P_l + \kappa I_q = -Q_l, \tag{11}$$

$$\chi_d > \frac{\ln \mu}{\theta}, \tag{12}$$

where $\lambda_2(L_l)$ *is the minimal positive eigenvalue of* L_l*,* $l \in \aleph$*,* $\kappa = (d_1 + d_2)\max_{l\in\aleph}\|L_l\|^2 + \kappa_1$*,* $\kappa_1 = \sum_{i=1}^4 \rho_i^{-1}$*,* $\mu = \frac{\pi_M}{\pi_m}$*,* $\pi_M = \max\{\pi_1, \pi_2, \ldots, \pi_\vartheta\}$*,* $\pi_m = \min\{\pi_1, \pi_2, \ldots, \pi_\vartheta\}$*,* $\theta = \min\left\{\frac{\min_{l\in\aleph}\lambda_{\min}(Q_l)}{\max_{l\in\aleph}\lambda_{\max}(P_l)}, \theta_1\right\}$ *and* $\theta_1 = \min\left\{\beta, \frac{a_1 b_1 + a_2 - 1}{b_1}, \frac{a_3 b_2 + a_4 - 1}{b_2}\right\}$*, then the practical UMSBC of MASs (1) can be accomplished with the distributed control protocol (3), the hybrid NNs adaptation law (4) and the distributed DETC law (8).*

Proof. The Lyapunov function candidate is constructed as

$$\begin{aligned} V(\tau) &= V_1(\tau) + V_2(\tau) + V_3(\tau), \\ V_1(\tau) &= z^\top(\tau)(I_N \otimes P_{\delta(\tau)})z(\tau), \\ V_2(\tau) &= \|\overline{W}(\tau)\|_F^2, \\ V_3(\tau) &= \sum_{i=1}^N \psi_i^{(1)}(\tau) + \sum_{i=1}^N \psi_i^{(2)}(\tau). \end{aligned} \tag{13}$$

At time τ, exists $v \in \mathbb{N}_+$, such that when $\tau \in [\tau_v, \tau_{v+1})$, $\delta(\tau) = l \in \aleph$. Therefore, the right-Dini derivative of $V_1(\tau)$ along (6) and Young's inequality [42] can be obtained as

$$\begin{aligned} D^+V_1 =& 2z^\top[(I_N \otimes P_l A_l) - \alpha(L_l \otimes P_l B_l K_l)]z + 2\alpha z^\top (M \otimes P_l B_l K_l) e^* + 2z^\top (M \otimes P_l B_l)\overline{W}^\top \Phi \\ &+ 2z^\top (M \otimes P_l B_l)\widehat{W}^\top \Phi^* + 2z^\top (M \otimes P_l B_l)\widehat{W}^{*\top}\overline{\Phi} + 2z^\top (M \otimes P_l B_l)\varepsilon \\ \leqslant& 2z^\top[(I_N \otimes P_l A_l) - \alpha(L_l \otimes P_l B_l K_l)]z + 2x^\top (I_N \otimes P_l B_l)\overline{W}^\top \Phi + \rho_4^{-1}\|P_l B_l\|^2 \bar{\varepsilon} + \kappa_1 \|z\|^2 \\ &+ \xi\Big(\|e^*\|^2 + \|\widehat{W}^\top \Phi^*\|^2 + \|\widehat{W}^{*\top}\overline{\Phi}\|^2\Big), \end{aligned} \tag{14}$$

where ξ and $\bar{\varepsilon}$ have been defined above.

Additionally, in the view of (4), it is known that

$$\begin{aligned} D^+V_2 &= -2\beta\|\overline{W}\|_F^2 + 2\beta\,\mathrm{tr}\Big(\overline{W}^\top W\Big) - 2\sum_{i=1}^{N}\mathrm{tr}\Big(\overline{W}_i^\top \phi_i x_i^\top P_l B_l\Big) \\ &\leqslant -\beta\|\overline{W}\|_F^2 + \beta\|W\|_F^2 - 2\sum_{i=1}^{N}\mathrm{tr}\Big(\overline{W}_i^\top \phi_i x_i^\top P_l B_l\Big) \\ &= -\beta\|\overline{W}\|_F^2 + \beta\|W\|_F^2 - 2x^\top (I_N \otimes P_l B_l)\overline{W}^\top \Phi. \end{aligned} \tag{15}$$

One can immediately obtain by (9) and (10) that

$$\begin{aligned} D^+V_3 =& (a_2 d_1 + a_4 d_2)\|e\|^2 - a_2\xi\|e^*\|^2 - a_4\xi\Big(\|\widehat{W}^\top \Phi^*\|^2 + \|\widehat{W}^{*\top}\overline{\Phi}\|^2\Big) \\ &- a_1\sum_{i=1}^{N}\psi_i^{(1)} - a_3\sum_{i=1}^{N}\psi_i^{(2)}. \end{aligned} \tag{16}$$

Combining with (14)–(16) and the distributed DETC law (8), it immediately holds that

$$\begin{aligned} D^+V \leqslant& z^\top\Big\{\Big[I_N \otimes (P_l A_l + A_l^\top P_l)\Big] - 2\alpha(L_l \otimes P_l B_l K_l) + \kappa_1 I_{qN}\Big\}z + (d_1 + d_2)\|e\|^2 - \beta\|\overline{W}\|_F^2 \\ &- \frac{a_1 b_1 + a_2 - 1}{b_1}\sum_{i=1}^{N}\psi_i^{(1)} - \frac{a_3 b_2 + a_4 - 1}{b_2}\sum_{i=1}^{N}\psi_i^{(2)} + \zeta \\ \leqslant& z^\top\Big\{\Big[I_N \otimes (P_l A_l + A_l^\top P_l)\Big] - 2\alpha(L_l \otimes P_l B_l K_l) + \kappa I_{qN}\Big\}z - \theta_1(V_2 + V_3) + \zeta, \end{aligned} \tag{17}$$

where $\zeta = \rho_4^{-1}\| P_l B_l\|^2 \bar{\varepsilon} + \beta\|W\|_F^2$. Furthermore, based on lemma 1, there is an orthogonal matrix $T_l = [T_l^{(1)}, T_l^{(2)}] \in \mathfrak{R}^{N\times N}$ with the property that $T_l^\top L_l T_l = \Lambda_l$, where $T_l^{(1)} = \frac{1}{\sqrt{N}}1_N \in \mathfrak{R}^N$, $T_l^{(2)} \in \mathfrak{R}^{N\times(N-1)}$, $\Lambda_l = \mathrm{diag}(\lambda_1(L_l), \lambda_2(L_l), \cdots, \lambda_N(L_l))$ and $0 = \lambda_1(L_l) < \lambda_2(L_l) \leqslant \cdots \leqslant \lambda_N(L_l)$ are the eigenvalues of L_l. Then, let $y(\tau) = (T_l^\top \otimes I_q)z(\tau) = \mathrm{col}(y^{(1)}(\tau), y^{(2)}(\tau)) \in \mathfrak{R}^{qN}$, where noticing $y^{(1)}(\tau) = (T_l^{(1)\top} \otimes I_q)z(\tau) = \mathbf{0} \in \mathfrak{R}^q$ and $y^{(2)}(\tau) = (T_l^{(2)\top} \otimes I_q)z(\tau) \in \mathfrak{R}^{q\times(N-1)}$, one can derive that

$$\begin{aligned} D^+V \leqslant& y^\top\Big\{\Big[I_N \otimes \Big(P_l A_l + A_l^\top P_l\Big)\Big] - 2\alpha(\Lambda_l \otimes P_l B_l K_l) + \kappa I_{qN}\Big\}y - \theta_1(V_2 + V_3) + \zeta \\ \leqslant& y^{(2)\top}\Big[I_{N-1} \otimes (P_l A_l + A_l^\top P_l - 2\alpha\lambda_2(L_l)P_l B_l K_l + \kappa I_q)\Big]y^{(2)} - \theta_1(V_2 + V_3) + \zeta. \end{aligned}$$

Due to (11) and $T_l^{(2)} T_l^{(2)\top} = M$, it holds that

$$\begin{aligned} D^+V \leqslant& - z^\top (T_l^{(2)} T_l^{(2)\top} \otimes Q_l)z - \theta_1(V_2 + V_3) + \zeta \\ \leqslant& - z^\top (I_N \otimes Q_l)z - \theta_1(V_2 + V_3) + \zeta \end{aligned}$$

$$\begin{aligned}&\leqslant -\frac{\underset{l\in S}{min}\lambda_{\min}(Q_l)}{\underset{l\in S}{max}\lambda_{\max}(P_l)}z^\top(I_N\otimes P_l)z-\theta_1(V_2+V_3)+\zeta\\&\leqslant -\theta V+\zeta.\end{aligned}$$

By integrating on both sides and using Fubini's Theorem [43], it can be obtained that

$$\mathbb{E}[V(\tau)]\leqslant\int_{\tau_v}^{\tau}(-\theta\mathbb{E}[V(s)]+\zeta)ds+\mathbb{E}[V(\tau_v)].$$

Furthermore, we can get

$$D^+\mathbb{E}[V(\tau)]\leqslant-\theta\mathbb{E}[V(\tau)]+\zeta. \tag{18}$$

So, one also has

$$\begin{aligned}\mathbb{E}\left[V(\tau_v^+)\right]=&\mathbb{E}\left[z^\top(\tau_v^+)\left(I_N\otimes P_{\delta(\tau_v^+)}\right)z(\tau_v^+)\right]+\mathbb{E}\left[V_2(\tau_v^+)+V_3(\tau_v^+)\right]\\=&\sum_{l=1}^{S}\pi_l z^\top(\tau_v^+)(I_N\otimes P_l)z(\tau_v^+)+\mathbb{E}\left[V_2(\tau_v^+)+V_3(\tau_v^+)\right]\\\leqslant&\pi_M\sum_{l=1}^{S}z^\top(\tau_v^+)(I_N\otimes P_l)z(\tau_v^+)+\mathbb{E}\left[V_2(\tau_v^+)+V_3(\tau_v^+)\right]\\=&\pi_M\sum_{l=1}^{S}z^\top(\tau_v^-)(I_N\otimes P_l)z(\tau_v^-)+\mathbb{E}\left[V_2(\tau_v^+)+V_3(\tau_v^+)\right]\\\leqslant&\frac{\pi_M}{\pi_m}\mathbb{E}\left[V_1(\tau_v^-)\right]+\mathbb{E}\left[V_2(\tau_v^+)+V_3(\tau_v^+)\right]\\\leq&\mu\mathbb{E}\left[V(\tau_v^-)\right].\end{aligned} \tag{19}$$

Since (12), (18), (19) and $\mu<1$, we have

$$\begin{aligned}\mathbb{E}[V]\leqslant&\mathrm{e}^{-\theta(\tau-\tau_v)}\mathbb{E}\left[V(\tau_v^+)\right]+\zeta\int_{\tau_v}^{\tau}\mathrm{e}^{-\theta(\tau-s)}ds \qquad (20)\\\leqslant&\mathrm{e}^{-\theta(\tau-\tau_v)}\mu\mathbb{E}\left[V(\tau_v^-)\right]+\zeta\int_{\tau_v}^{\tau}\mathrm{e}^{-\theta(\tau-s)}ds\\\leqslant&\mathrm{e}^{-\theta(\tau-\tau_{v-1})}\mu^2\mathbb{E}\left[V(\tau_{v-1}^-)\right]+\zeta\mu\int_{\tau_{v-1}}^{\tau_v}\mathrm{e}^{-\theta(\tau-s)}ds+\zeta\int_{\tau_v}^{\tau}\mathrm{e}^{-\theta(\tau-s)}ds\\\leqslant&\mathrm{e}^{-\theta(\tau-\tau_0)}\mu^{N(\tau,\tau_0)}\mathbb{E}[V(\tau_0)]+\zeta\mu^{N(\tau,\tau_0)}\int_{\tau_0}^{\tau_1}\mathrm{e}^{-\theta(\tau-s)}ds+\zeta\mu^{N(\tau,\tau_1)}\int_{\tau_1}^{\tau_2}\mathrm{e}^{-\theta(\tau-s)}ds+\dots\\&+\zeta\int_{\tau_v}^{\tau}\mathrm{e}^{-\theta(\tau-s)}ds\\\leqslant&\mathrm{e}^{-\theta(\tau-\tau_0)}\mu^{N(\tau,\tau_0)}\mathbb{E}[V(\tau_0)]+\zeta\int_{\tau_0}^{\tau}\mu^{N(\tau,s)}\mathrm{e}^{-\theta(\tau-s)}ds\\\leqslant&\mathrm{e}^{-\left(\theta-\frac{\ln\mu}{\chi_d}\right)(\tau-\tau_0)}\mu^{N_0}\mathbb{E}[V(\tau_0)]+\zeta\mu^{N_0}\int_{\tau_0}^{\tau}\mathrm{e}^{-\left(\theta-\frac{\ln\mu}{\chi_d}\right)(\tau-s)}ds\\\leqslant&\mathrm{e}^{-\left(\theta-\frac{\ln\mu}{\chi_d}\right)(\tau-\tau_0)}\mu^{N_0}\mathbb{E}[V(\tau_0)]+\frac{\zeta\mu^{N_0}}{\theta-\frac{\ln\mu}{\chi_d}}, \qquad (21)\end{aligned}$$

which, along with (6) and (13), concludes the proof. □

Remark 7. *Theorem 1 presents a comprehensive approach for analyzing mean-square bounded synchronization in switched MASs. By making certain assumptions, we obtain adequate conditions for reducing the impact of switching. Fubini's theorem is employed to solve the exchangeability* *of expectation and integral, which is facilitated by the continuity of system dynamics within each switching interval. This criterion is straightforward to apply.*

Remark 8. *Note that in Equation (19), the impulse gain μ represents the effect of the Markovian switched system $A_{\delta(\tau)}$, $B_{\delta(\tau)}$, and topology $\mathcal{G}_{\delta(\tau)}$ on the overall system. To demonstrate how the Markovian switch affects the error and consensus performance, relationships between the systems, network structures, and dwell time χ_d are derived in Equation (12).*

Theorem 2. *Under Assumptions 1 and 2, the Zeno performance is eliminated for nonlinear MASs (1) by utilizing the triggering protocol (8).*

Proof. We prove the statement by contradiction. Suppose that $\lim_{k\to\infty}\tau_k^i = \tau_\infty^i < \infty$, and then, $\lim_{k\to\infty}\tau_k^i = \lim_{k\to\infty}\left(\tau_{k+1}^i - \tau_k^i\right) = 0$. It is easy to check that $\|e_i^*(\tau)\| \leqslant \sum_{j\in\mathcal{N}_i^{\delta(\tau)}} a_{ij}^{\delta(\tau)}\left(\|(x_j(\tau_k^i) - x_j(\tau))\| + \|(x_i(\tau_k^i) - x_i(\tau))\|\right)$ and $x_i(\tau)$ is continuous for $\tau \in \left[\tau_k^i, \tau_{k+1}^i\right]$, so are $\widehat{W}_i^*(\tau)$ and $\phi_i^*(x_i(\tau))$ for $\tau \in \left[\tau_k^i, \tau_{k+1}^i\right]$, $i \in \mathcal{V}$. Therefore, as $\tau \to \infty$, it can be seen that

$$\left\|e_i^*(\tau_\infty^i)\right\| \leqslant \lim_{k\to\infty} \sum_{j\in\mathcal{N}_i^{\delta(\tau)}} a_{ij}^{\delta(\tau)}\left(\left\|(x_j(\tau_{k+1}^i) - x_j(\tau_k^i))\right\| + \left\|(x_i(\tau_{k+1}^i) - x_i(\tau_k^i))\right\|\right) \to 0, \quad (22)$$

or

$$\begin{aligned}
&\left\|\widehat{W}_i^\top(\tau_\infty^i)\phi_i^*(x_i(\tau_\infty^i))\right\|^2 + \left\|\widehat{W}_i^{*\top}(\tau_\infty^i)\phi_i(x_i(\tau_\infty^i))\right\|^2 \\
&= \lim_{k\to\infty}\left(\left\|\widehat{W}_i^\top(\tau_{k+1}^i)\phi_i^*(x_i(\tau_{k+1}^i))\right\|^2 + \left\|\widehat{W}_i^{*\top}(\tau_{k+1}^i)\phi_i(x_i(\tau_k^i))\right\|^2\right) \\
&= \lim_{k\to\infty}\left(\left\|\widehat{W}_i^\top(\tau_{k+1}^i)(\phi_i(x_i(\tau_{k+1}^i)) - \phi_i(x_i(\tau_k^i)))\right\|^2\right. \\
&\quad \left. + \left\|(\widehat{W}_i^\top(\tau_{k+1}^i) - \widehat{W}_i^\top(\tau_k^i))\phi_i(x_i(\tau_k^i)))\right\|^2\right) \to 0. \qquad (23)
\end{aligned}$$

On the one hand, from (8), we have

$$\begin{aligned}
\lim_{k\to\infty}\left\|e_i^*(\tau_{k+1}^i)\right\|^2 &\triangleq \lim_{k\to\infty}\left(\frac{d_1}{\xi}\left\|e_i(\tau_{k+1}^i)\right\|^2 + \frac{1}{b_1\xi}\psi_i^{(1)}(\tau_{k+1}^i)\right) \\
&\geqslant \frac{1}{b_1\xi}\psi_i^{(1)}(\tau_\infty^i) \\
&> 0, \qquad (24)
\end{aligned}$$

or

$$\begin{aligned}
&\lim_{k\to\infty}\left(\left\|\widehat{W}_i^\top(\tau_{k+1}^i)\phi_i^*(x_i(\tau_{k+1}^i)))\right\|^2 + \left\|\widehat{W}_i^{*\top}(\tau_{k+1}^i)\phi_i(x_i(\tau_{k+1}^i)))\right\|^2\right) \\
&\triangleq \lim_{k\to\infty}\left(\frac{d_2}{\xi}\left\|e_i(\tau_{k+1}^i)\right\|^2 + \frac{1}{b_2\xi}\psi_i^{(1)}(\tau_{k+1}^i)\right) \\
&\geqslant \frac{1}{b_2\xi}\psi_i^{(2)}(\tau_\infty^i) \\
&> 0. \qquad (25)
\end{aligned}$$

Derived by (22) and (23), $\lim_{k\to\infty}\left\|e_i^*(\tau_{k+1}^i)\right\|^2 = 0$ and $\lim_{k\to\infty}\left(\left\|\widehat{W}_i^\top(\tau_{k+1}^i)\phi_i^*(x_i(\tau_{k+1}^i))\right\|^2 + \left\|\widehat{W}_i^{*\top}(\tau_{k+1}^i)\phi_i(x_i(\tau_{k+1}^i))\right\|^2\right) = 0$, which contradicts the outcome in (24) and (25).

From the talks above, it can be deduced from the contradictions that $\lim_{k\to\infty}\tau_k^i = \infty$. As a result, there will not be any Zeno behavior within any short period and each agent will only produce a certain amount of occurrences. □

If the MASs (1) under the fixed topology $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, \mathcal{A}\}$, one can get a common system which can achieve the practical UMSBC under the distributed control protocol (3), the hybrid NNs adaptation law (4) and the distributed DETC law (8) as well. Meanwhile, we get the outcome shown below.

Corollary 1. *If Assumptions 1 and 2 are valid, for* $\forall l \in \aleph$*, there exist matrices* $P_l \succ 0$ *and* $Q_l \succ 0$ *which satisfy*

$$P_l A_l + A_l^\top P_l - 2\alpha\lambda_2(L) P_l B_l B_l^\top P_l + \kappa I_q = -Q_l,$$

$$\chi_d > \frac{\ln\mu}{\theta},$$

where L is the Laplacian matrix of graph $\mathcal{G}$, $\kappa = (d_1 + d_2)\|L\|^2 + \kappa_1$, $\kappa_1 = \sum_{i=1}^4 \rho_i^{-1}$, $\mu = \frac{\pi_M}{\pi_m}$, $\theta = \min\left\{\frac{\min_{l\in\aleph}\lambda_{\min}(Q_l)}{\max_{l\in\aleph}\lambda_{\max}(P_l)}, \theta_1\right\}$*, and* $\theta_1 = \min\left\{\beta, \frac{a_1 b_1 + a_2 - 1}{b_1}, \frac{a_3 b_2 + a_4 - 1}{b_2}\right\}$ *, then the practical UMSBC of MASs (1) can be accomplished with respect to the distributed control protocol (3), the hybrid NNs adaptation law (4) and the distributed DETC law (8).*

Remark 9. *It will become a well-researched mean-square synchronization of MASs with nonidentical agents, if there is no impact of switching topology. Great studies have been conducted on this issue by several scholars, see [44].*

DETC is thought about in the argument above. Besides, the system (1) can realize the practical UMSBC by the following SETC mechanism:

$$\tau_{k+1}^i \triangleq \inf\left\{\tau > \tau_k^i \mid \gamma_i^{(1)}(\tau) > 0 \quad \text{or} \quad \gamma_i^{(2)}(\tau) > 0\right\}, \tag{26}$$

with

$$\begin{aligned}
\gamma_i^{(1)}(\tau) =& \xi\|e_i^*(\tau)\|^2 - c_1\|e_i(\tau)\|^2 - c_2 \mathrm{e}^{-c_3\tau},\\
\gamma_i^{(2)}(\tau) =& \xi\|\widehat{W}_i^\top(\tau)\phi_i^*(x_i(\tau))\|^2 + \xi\|\widehat{W}_i^{*\top}(\tau)\phi_i(x_i(\tau_k^i))\|^2 - c_4\mathrm{e}^{-c_5\tau},
\end{aligned}$$

where $c_1, c_2, c_3, c_4, c_5 > 0$ are the designed parameter. Next, we get the outcome shown below.

Corollary 2. *If Assumptions 1 and 2 are valid, for* $\forall l \in \aleph$*, there exist matrices* $P_l \succ 0$ *and* $Q_l \succ 0$ *which satisfy*

$$P_l A_l + A_l^\top P_l - 2\alpha\lambda_2(L_l) P_l B_l K_l + \kappa I_q = -Q_l,$$

$$\chi_d > \frac{\ln\mu}{\theta'},$$

where $\kappa = (d_1 + d_2)\max_{l\in\aleph}\|L_l\|^2 + \kappa_1$, $\kappa_1 = \sum_{i=1}^4 \rho_i^{-1}$, $\mu = \frac{\pi_M}{\pi_m}$*, and* $\theta' = \min\left\{\frac{\min_{l\in\aleph}\lambda_{\min}(Q_l)}{\max_{l\in\aleph}\lambda_{\max}(P_l)}, \beta\right\}$, *then the practical UMSBC of MASs (1) can be accomplished with respect to the distributed control protocol (3), the hybrid NNs adaptation law (4) and the distributed* SETC *law (26).*

Remark 10. *It should be noted that the authors of [23] looked at synchronizing MASs with SETC using the mean-square method. Different from the prior study, this research expands on the findings to DETC. The number of DETCs that trigger an event is much lower than the number of SETCs,* *which will be represented in the numerical simulation later, even if the system (1) can attain consensus via the SETC law (26). As a result, using the DETC technique (8) to create consensus requires less effort and money. Our work has been a substantial expansion of [23] in this way.*

4. Illustrative Examples

Two simulation experiments are provided in this part to demonstrate the usefulness of the created hybrid neuroadaptive DETC and the practical convergence outcome. Taking the ten nonlinear Markovian switching systems into account

$$\begin{aligned}
\dot{x}_1 &= A_{\delta(\tau)}x_1 + B_{\delta(\tau)}\big(u_1 + 3sin(4x_{11}^2(\tau)) + 0.6\big),\\
\dot{x}_2 &= A_{\delta(\tau)}x_2 + B_{\delta(\tau)}(u_2 - 4sin(4x_{21}(\tau)) + 0.61),\\
\dot{x}_3 &= A_{\delta(\tau)}x_3 + B_{\delta(\tau)}(u_3 + 2sin(4x_{32}(\tau)) + 0.51),\\
\dot{x}_4 &= A_{\delta(\tau)}x_4 + B_{\delta(\tau)}(u_4 - 5sin(4x_{41}(\tau)) + 0.45),\\
\dot{x}_5 &= A_{\delta(\tau)}x_5 + B_{\delta(\tau)}(u_5 + 3.5sin(2x_{51}(\tau)) + 0.6),\\
\dot{x}_6 &= A_{\delta(\tau)}x_6 + B_{\delta(\tau)}(u_6 + 2.5sin(2x_{61}(\tau)) + 0.6),\\
\dot{x}_7 &= A_{\delta(\tau)}x_7 + B_{\delta(\tau)}\big(u_7 + 2sin(4x_{71}^3(\tau)) + 0.5\big),\\
\dot{x}_8 &= A_{\delta(\tau)}x_8 + B_{\delta(\tau)}(u_8 - 5sin(4x_{81}(\tau)) + 0.4),\\
\dot{x}_9 &= A_{\delta(\tau)}x_9 + B_{\delta(\tau)}(u_9 + 3.5sin(3x_{91}(\tau)) + 0.3),\\
\dot{x}_{10} &= A_{\delta(\tau)}x_{10} + B_{\delta(\tau)}(u_{10} + 4sin(2x_{101}(\tau)) + 0.6),
\end{aligned}$$

with

$$A_1 = \begin{bmatrix} -0.7 & -0.3 \\ -0.1 & -0.5 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -0.5 & -0.4 \\ -0.2 & -0.6 \end{bmatrix}, \quad A_3 = \begin{bmatrix} -0.5 & -0.2 \\ -0.15 & -0.4 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0.02 \\ 0.05 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.05 \\ 0.04 \end{bmatrix}, \quad B_3 = \begin{bmatrix} 0.01 \\ 0.03 \end{bmatrix},$$

where $x_i(\tau) = [x_{i1}(\tau), x_{i2}(\tau)]^\top \in \Re^2, i \in \{1, 2, \ldots, 10\}$. Random selection is used to choose the starting states from the range $[-10, 10] \times [-10, 10]$. Additionally, the nonlinearity is provided just for illustrative purposes and may not apply to the actual system. Twenty covert neurons with activation functions are used in each NNs approximator to estimate the uncertain nonlinearity. The Markovian process δ jumps among modes $\{1, 2, 3\}$. Three undirected graphs with the following Laplacian matrices are displayed in Figure 3, and the highlighting network architecture $\mathcal{G}_{\delta(\tau)}$ alternates between them at random.

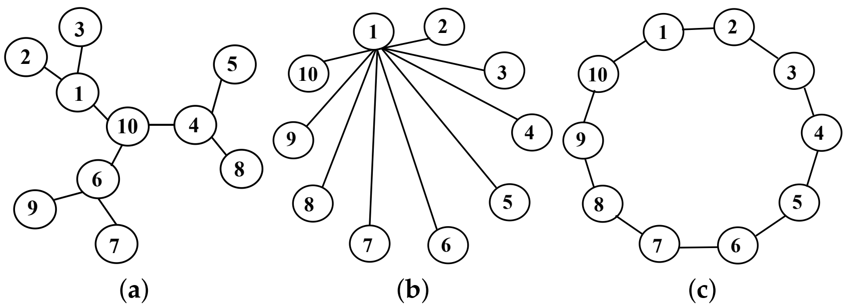
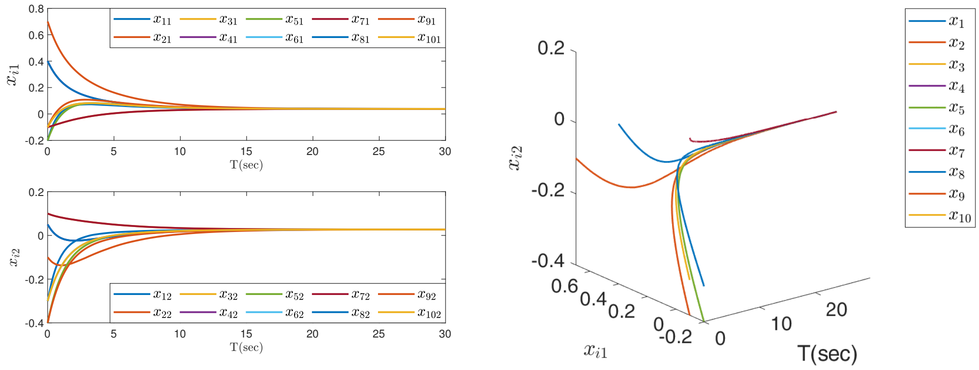


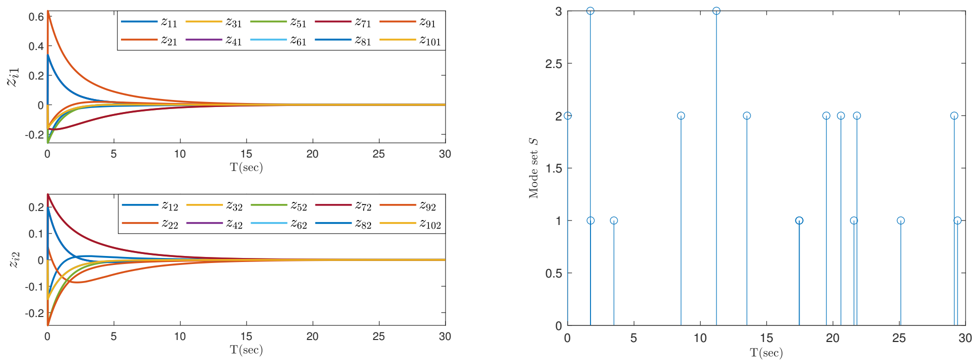
Figure 3. Possible interaction topologies between the ten agents. (**a**) $\mathcal{G}_1$. (**b**) $\mathcal{G}_2$. (**c**) $\mathcal{G}_3$.

Example 1. *We create the distributed DETC (8) to achieve system (1) consensus. Let* $a_1 = 0.1$*;* $a_2 = 0.9$*;* $a_3 = 0.1$*;* $a_4 = 0.9$*;* $b_1 = 2$*;* $b_2 = 2$*;* $d_1 = 0.2$*;* $d_2 = 0.1$*;* $\rho_1 = 0.55$*;* $\rho_2 = 0.5$*;* $\rho_3 = 0.5$*;* $\rho_4 = 0.5$*;* $\alpha = 3$*;* $\beta = 50$*;* $\chi_d = 2$ *and use the LMI toolbox to solve the inequality (11). We obtain* $\theta = 1.1827$*. Moreover,* $\kappa = 18.8$ *and the sampling time of simulation is set at 0.001 s. The simulation findings are shown in Figures 4–7. The uniformly bounded consensus control is demonstrated by the state trajectories in Figure 4 and the error trajectories in Figure 5*a*, which provide evidence that our control strategy is effective. Nevertheless, Figures 5*b*, 6*a*,b and 7*a *exhibit the mode of Markovian switching, the control inputs* u_i*, the NNs estimator, and the triggering* *instants of each agent under the DETC law (8), showing that the interval between two consecutive triggering instants increase and the number of triggerings decrease, indicating the efficacy of our DET method.*



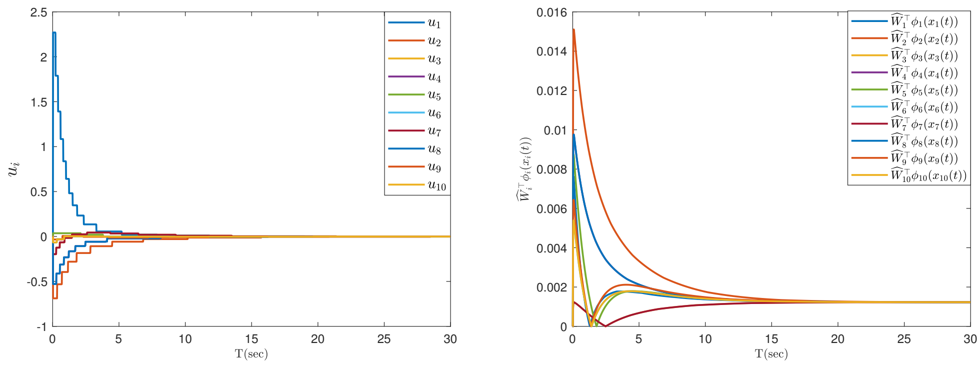
(**a**) (**b**)

Figure 4. (**a**) Trajectories of x_{i1} and x_{i2} by using (8) for Example 1; (**b**) Trajectories of x_i by using (8) for Example 1.



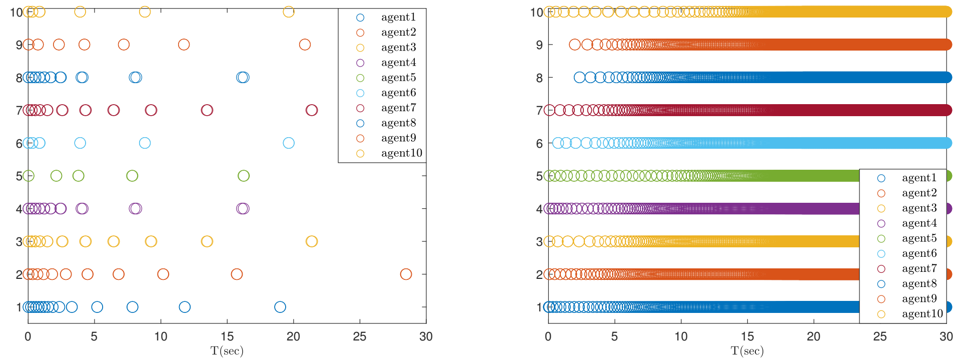
(**a**) (**b**)

Figure 5. (**a**) Trajectories of z_{i1} and z_{i2} by using (8) for Example 1; (**b**) Mode of Markovian switching by using (8) for Example 1.



(**a**) (**b**)

Figure 6. (**a**) Trajectories of u_i by using (3) for Example 1; (**b**) Trajectories of the NNs estimator by using (4) for Example 1.



(**a**) (**b**)

Figure 7. (**a**) Triggering instants by using (8) for Example 1; (**b**) Triggering instants by using (26) for Example 2.

Example 2. *Figure 7b illustrates the triggering instants for each agent under the SETC methodology (26), with the devised parameters selected as* $c_1 = 0.01$; $c_2 = 100$; $c_3 = -0.02$; $c_4 = 1000$; $c_5 = -0.02$.

In Table 1, the triggering number of each agents are separately listed under the DETC law (8) and the SETC law (26). It can be seen that the dynamic excitation times are far less than the static excitation times. Thus, it can be established that the control approach in this study needs fewer control updates to assure system performance than in [23], resulting in a significant reduction in energy costs.

Table 1. Triggering number for Examples 1 and 2.

Agent	1	2	3	4	5	6	7	8	9	10
the DETC law (8)	30	22	22	20	10	12	22	20	14	12
the SETC law (26)	1458	1459	1368	1449	1333	1411	1399	1381	1412	1281

5. Conclusions

This study presents an adaptive neural network-based distributed DET consensus control technique for Markovian switching systems with unidentified nonlinearity. To address the issue of nonlinearity, we propose unique adaptive DETC methods that reduce triggering instants and ensure consistent performance. Unlike previous time-triggered consensus control approaches, the proposed technique minimizes computation and transmission while achieving consensus control. Additionally, we demonstrate the existence of a compact set, confirming the effectiveness of neural network approximation for hybrid dynamical systems. However, there are still some issues remaining, such as further minimizing the number of triggers and achieving finite-time control, which require further research.

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