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The Rise of the Superstars: Uncovering the Composition Effect of International Trade That Cements the Dominant Position of Big Businesses

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Abstract: International markets are extremely polarised, with a few big *superstar* businesses operating alongside numerous small competitors, and globalisation has been highlighted as a powerful force behind the superstars' increasingly dominant presence. The empirical literature has established that superstars are more efficient compared to their smaller counterparts, and, unlike them, they exhibit strategic behaviour. Building on this evidence, we develop a model to examine how an initial productivity advantage allows a select few firms to expand, via innovation, to the extent that it becomes optimal to adopt strategic behaviour, and show how polarised markets emerge endogenously as the unique subgame perfect equilibrium in pure strategies. We then introduce international trade and show that, in polarised markets, trade liberalisation puts into motion a novel composition effect, reallocating market share from smaller to larger rivals and raising large firms' profits. This effect suppresses the pro-competitive welfare gains from trade and cements the dominant position of big businesses, who come out as the big winners of globalisation. We find that, although trade increases welfare, by reducing average markup and markup heterogeneity, in the presence of a handful of large powerful firms, welfare gains are severely diminished, and subsidising smaller enterprises may turn out to be welfare-enhancing.

Keywords: monopolistic competition; oligopoly; subgame perfect Nash equilibrium; superstar firms; international trade; trade and welfare

MSC: 91A80; 91B54; 91B60

JEL Classification: C7; D43; F1; F6; L10

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1. Introduction

Global markets are characterised by extreme and growing concentration. Almost twenty years ago, Bernard et al. [1] documented that the largest 1 percent of trading firms, formally defined in the literature as *superstar* firms [2], was responsible for over 80 percent of the value of total US trade. Since their seminal publication, a similar pattern has consistently come up in most developed and developing nations [3–5]. Recently, an emerging empirical literature has suggested a rising trend in the market power of superstar firms [5–9] and has highlighted globalisation as, arguably, the most powerful force behind their increasingly dominant presence [10,11]. The theoretical literature, on the other hand, by emphasising the intensification of competition as a robust effect of international trade [12–14], has not gone far enough in reconciling this theoretical pro-competitive effect with the documented progressing global influence of big businesses.

In this paper, we build a simple theoretical mechanism to show how a select few firms grow into superstars while operating alongside a fringe of small competitors. We use this to uncover a novel *composition effect* that enables those lucky few to come out as the big

winners of trade liberalisation at the price of lowering the profitability of small firms and suppressing aggregate welfare gains from trade.

To construct a model of how superstars emerge, our natural starting point is to examine how they differ from small and medium enterprises. Superstars are characterised by a superior productive efficiency [15] that generates increased market shares and also, unlike small firms, by the ability to internalise the impact of their behaviour on the market, as manifested in their unique ability to charge higher markups [4,16,17] selected strategically [18] to vary with their market shares. Building on this empirical evidence, we develop a stylised two-period model, where a priori identical (non-strategic) monopolistic competitors draw from a common productivity distribution (period one), as in [19], and show how exogenous initial differences in productivity draws are endogenously magnified, resulting in different abilities to invest in cost-reducing innovations. Thus, our study contributes to the existing literature by rationalising the appearance of polarised markets as a subgame perfect general equilibrium result: successful innovators, and only them, expand to the extent that they find it optimal to start behaving in a strategic (oligopolistic) fashion. We end up (period two) with an industry governed by mixed market competition [20,21] with an oligopolistic sub-sector of superstars co-existing with a monopolistically competitive fringe of small firms.

Moving on, in this setup of two sub-sectors, we introduce international trade. Our paper is, to our knowledge, the first to show that, when explicitly modelling the increased productivity and oligopolistic nature of superstars, trade triggers two different types of market share reallocation away from smaller and towards larger firms: one which resembles Melitz-type [19] selection and is due to the efficiency advantage of superstar firms, and one that is solely attributed to the differential abilities of the two types of firms to charge variable markups. Hence, a newly documented composition effect is generated, materialised through a market share reallocation from smaller towards larger rivals, which dampens the standard pro-competitive effect of trade liberalisation and cements the dominant position of big businesses by boosting their profitability. We find that, although trade always increases welfare by reducing the average markup and markup heterogeneity, gains from trade are significantly suppressed in the shadow of a few powerful large firms. We conclude that, in polarised international markets, size-dependent policies, like, for example, subsidies for smaller exporters, may have a welfare-enhancing role to play.

The structure of our paper is as follows. In Section 2, based on the assumption that firms grow because of luck and successful innovation, we construct a two-period general equilibrium model with free entry where the most efficient firms grow into superstars and choose to take action in order to exploit their market power. At the same time, the majority of their competitors end up supplying only a small portion of the market and refrain from internalising their impact on the aggregates. We end up with a polarised market where a handful of big businesses compete alongside a myriad of small enterprises, very much like what the empirical literature reports. In Section 3, conditions are derived for this polarised market to emerge as the subgame perfect equilibrium in pure strategies. We find that productivity heterogeneity, which is commonly assumed in trade models, emerges as a necessary condition for the endogenous emergence of mixed markets with an oligopolistic and a monopolistically competitive sub-sector. We go on to conduct a comparative statics analysis of our closed economy equilibrium: we illustrate the potentially devastating effect of large firms on aggregate welfare and indicate that our model could serve as an alternative interpretation of the inverted-U relationship between innovation and competition [22].

Moving on, in Section 4, we introduce international trade between two symmetric countries and, in Section 5, we discuss how bringing together the two established market structures of oligopoly and monopolistic competition in an open economy setup does not result in a simple combination of the variety gains predicted by the monopolistically competitive models [23], with the pro-competitive gains expected to materialise under pure oligopoly [24]. The co-existence of firms that differ in their strategic behaviour, as well as their efficiency level, gives rise to a composition effect of these two traditional

sources of gains, resulting in a market share reallocation from smaller to larger players that dampens the pro-competitive effect and reduces welfare gains from trade. We show that trade liberalisation disproportionately benefits large firms, since small firms largely absorb globalisation-related competitive pressures, shielding large firms from increased import penetration. Our results replicate the widely documented and discussed global rise of big firm profitability and market concentration [6,25,26]. Finally, in Section 6, we show how subsidising the exports of small enterprises may induce a raise in welfare gains from trade. Section 7 concludes the paper.

2. The Model Economy

Given the indisputable evidence pointing towards an extremely and increasingly skewed firm distribution, we argue that there is policy and descriptive need for a simple theoretical mechanism that, in line with the data, allows a small number of firms to endogenously grow into superstars. To this end, we refer to the empirical literature regarding the defining characteristics of big businesses: they possess an increased productivity and the ability to charge a higher markup of price over marginal cost [4,16]. More crucially, they are capable of exploiting their strategic market power [18] and internalise the effect of their decisions on market aggregates. As a result, unlike their smaller competitors charging a markup consistent with monopolistic competition, currently the workhorse of international economics, the largest, most efficient suppliers (and exclusively them) exhibit oligopolistic behaviour and choose their markup in a strategic fashion. We start off with a two-period model of a closed economy with free entry that highlights the specific conditions required to obtain the market power heterogeneity that we observe in the data.

2.1. Preferences and Demand

We analyse a two-period game. At each point in time ($t = 1, 2$), the economy involves one horizontally differentiated and one homogeneous good. The homogeneous good (O) sums up the rest of the economy, is produced under constant returns, and is supplied by perfectly competitive firms, and its production only requires labour, which is perfectly mobile. The horizontally differentiated good (H) is produced under increasing returns. At time t, H_t can be supplied by a Cournot oligopolist or by a monopolistic competitor. When each type of firm is active and why will become clear in Section 2.2. For now, it suffices to assume that H_t is formed by two constant elasticity of substitution (CES) composites, X and Y, which at time t are:

$$X_t = \left(\sum_{i=1}^{n_{xt}} x_{it}^a \right)^{\frac{1}{a}}$$

$$Y_t = \left(\sum_{i=1}^{n_{yt}} y_{it}^a \right)^{\frac{1}{a}}$$

where x_{it} is the output level of a Cournot oligopolistic (CO) firm $i = 1, 2, \ldots, n_{xt}$ at time t and y_{it} is the output level of a monopolistically competitive (MC) firm $i = 1, 2, \ldots, n_{yt}$ at time t. $a \in (0, 1)$, implying that the goods are imperfect substitutes and the elasticity of substitution between any two goods is given by $\sigma \equiv \frac{1}{(1-a)} > 1$. Combining X and Y gives the output index H of the differentiated sector at time t:

$$H_t = (X_t^a + Y_t^a)^{\frac{1}{a}}$$

The representative consumer is endowed with L units of labour, and, at each period $t = 1, 2$, maximises the per-period utility function:

$$U_t = \ln H_t + \beta \ln O_t$$

subject to the budget constraint:

$$O_t + E_t = I_t$$

where E represents the expenditure on the differentiated good ($E_t = P_t H_t$), I is the (endogenous) income level in terms of the numeraire O and we define P to be the aggregate price of the differentiated sector. In the utility function, the parameter $\beta > 0$ expresses the weight of the homogeneous good. A higher β shifts consumption away from the differentiated and towards the homogeneous industry.

Note that the assumption of constant elasticity of substitution (CES) preferences is critical because it implies the lack of demand-driven market power and, hence, of demand-driven markup heterogeneity. Therefore, any markup heterogeneity can be attributed to supply side determinants, namely productivity and strategic market power. Assuming that there are no means available to the consumer to transfer wealth from one period to the other, we can focus on the per-period optimisation problem, which yields the inverse demand for each differentiated variety, produced either by a CO firm:

$$p_{xit} = x_{it}^{a-1} H_t^{1-a} P_t$$

or by an MC firm:

$$p_{yit} = y_{it}^{a-1} H_t^{1-a} P_t$$

Looking at the inverse demand functions, we obtain, firstly, that the quantity demanded from each firm decreases with the firm's own price but increases with the aggregate statistics H_t and P_t, which encapsulate the aggregate behaviour of the differentiated sector. Finally, we obtain the key property of the CES demand, namely that the elasticity of demand that each firm faces is constant, not changing as the firm moves down its demand function, and it also equals the elasticity of substitution.

2.2. Production

We start off with a closed economy two-period model, which is represented in Figure 1. The first period can be understood as a newly created industry and the second as a mature one. At the beginning of the first period, N firms enter the market. Entry only occurs at the beginning of the game and there are no exogenous shocks that could force a firm to exit before the end of the game. To enter, firms must make an initial investment in the form of a fixed sunk entry cost $f_e > 0$ (measured in units of the homogeneous good), which, in the literature, usually corresponds to the number of days needed to create a new firm [27]. The number of entrants N is subject to free entry: N is such that the total expected profits from both periods of the game are equal to the entry cost. Prior to entry, firms are identical and they are considered symmetrically non-strategic. Once they enter the market, firms draw an initial productivity parameter $z > 0$ from a common distribution, where z is defined as the inverse of a firm's marginal cost. With costless differentiation, each firm chooses to produce a different variety. To keep the model tractable, we assume that z can only take two discrete values, low z_l and high z_h, each with a known probability:

$$z \in \{z_l, z_h\} \text{ where } z_l < z_h \text{ and } P(z = z_h) = \gamma \in (0,1)$$

One can think of a high versus a low random productivity draw as a *good*, as opposed to a *bad*, business idea (relaxing this assumption by incorporating a continuous productivity distribution à la Melitz [19] would severely affect the model's tractability without adding to its key insights).

Production only requires labour which, at each period, is inelastically supplied at the aggregate level L. The technology of each firm is represented by its productivity z. Production also requires a fixed cost $f > 0$ which is the same across all firms. Thus, the labour used for the production of output q equals:

$$l(q;z) = \frac{1}{z} q + f$$

Setting the common (across sectors) wage w equal to 1, the above equation is also the cost function of a productivity z firm.

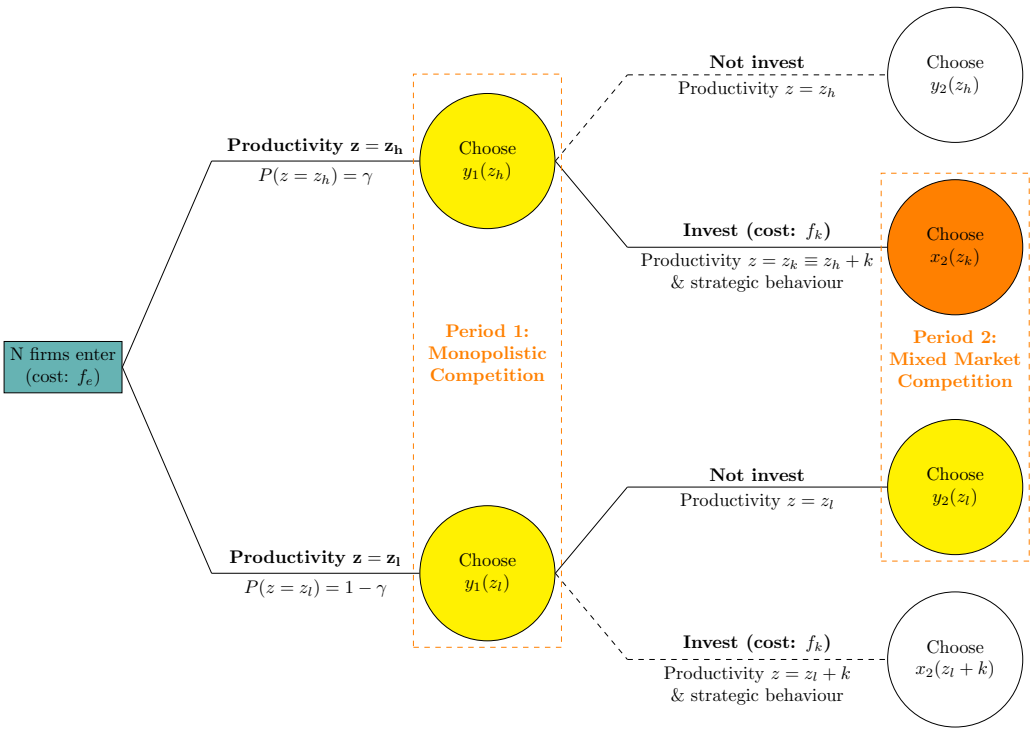

Figure 1. Entry occurs at the beginning of a two-period game: N firms enter, paying f_e, and draw productivity z, either high (z_h) or low (z_l). N is subject to free entry. In period one, firms select their quantity ($y_1(z_h)$ or $y_1(z_l)$) and engage in monopolistic competition. At the beginning of period two, firms choose whether to invest in innovation, at a cost f_k, which raises productivity by k and enables a firm to act oligopolistically. In equilibrium, in period two, firms select their quantity ($x_2(z_k)$ or $y_2(z_l)$) and engage in mixed market competition: all low-productivity firms choose not to invest and remain small monopolistic competitors with productivity z_l and all high-productivity firms choose to invest, grow larger with an increased productivity $z_k \equiv z_h + k$ and behave as Cournot oligopolists.

2.2.1. Period One

Given our assumption that firms are a priori non-strategic, we obtain that, in the first period, all firms operate under monopolistic competition (Figure 1). To use our notation from Section 2.1, only the MC sub-sector Y is active ($n_{x1} = 0$ and $n_{y1} = N$).

Moving on to the firm optimisation problem for period one, as shown in Figure 1, a firm with productivity z will choose its first period output y_1 so as to maximise its first period profit π_{y1} subject to the inverse demand function. The y index represents the fact that the firm acts as a monopolistic competitor. First-period profits for high-productivity firms are presented below:

$$\pi_{y1}(z_h) = \left(\frac{1}{az_h}\right)^{\frac{a}{a-1}}(1-a)E_1P_1^{\frac{a}{1-a}} - f$$

where E_1 is the expenditure on the differentiated good and P_1 is the aggregate price of the differentiated sector in period one. Similarly, for low-productivity firms:

$$\pi_{y1}(z_l) = \left(\frac{1}{az_l}\right)^{\frac{a}{a-1}}(1-a)E_1P_1^{\frac{a}{1-a}} - f$$

From period one profit maximisation, we obtain the optimal pricing rule $p_{y1}(z) = \frac{1}{az}$, where productivity $z \in \{z_l, z_h\}$. Thus, in the first period, the more productive firms

(with productivity $z_h > z_l$) produce a higher output $y_1(z_h) > y_1(z_l)$, charge a lower price $p_{y1}(z_h) < p_{y1}(z_l)$, and earn higher period one profits $\pi_{y1}(z_h) > \pi_{y1}(z_l)$ compared to the less productive firms. However, all firms, regardless of their productivity, will choose the same profit maximising markup of price over marginal cost, equal to $\theta_{y1}(z_l) = \theta_{y1}(z_h) = \frac{1}{a}$.

2.2.2. Period Two

The beginning of the second period finds the lucky high-productivity firms with an increased market share due to their higher period one production compared to the low-productivity firms, and an accumulated higher profit. Now, let us assume that when period two starts, each firm has the option to invest in a discrete efficiency-enhancing innovation working as follows. Incurring a fixed sunk cost $f_k \geq 0$ raises a firm's productivity by $k \geq 0$ and, more importantly, it enables the firm to act as a Cournot oligopolist, in other words, to internalise its impact on market aggregates and behave strategically (Figure 1). This innovation is successful with probability 1 (a non-deterministic innovation would result in a heterogeneous monopolistically competitive fringe with two levels of productivity and unnecessarily complicate the analysis). We make the following assumption, which we prove in Section 3.1.

Assumption 1. *In period two, all high-productivity firms choose to incur f_k and compete as Cournot oligopolists, with a post-innovation productivity equal to $z_k \equiv z_h + k$, and all low-productivity firms remain monopolistic competitors with productivity z_l.*

Under Assumption 1, we obtain that, in terms of our Section 2.1 notation, both the CO sub-sector X and the MC sub-sector Y are now active:

$$n_{x2} = \gamma N \text{ and } n_{y2} = (1 - \gamma)N$$

where N is the number of active firms pinned down by free entry. In Section 3.1, we go on to derive the conditions under which Assumption 1 becomes a subgame perfect equilibrium result in pure strategies: the decision to become an oligopolist is optimal for a firm if and only if this firm has an initial productivity equal to z_h.

The obvious intuition behind this innovation is that of a technological adoption cost, in line with the endogenous growth literature [28]: a high productivity draw in period one enables firms to accumulate profit, which they may choose to invest to further increase their productivity gap from their competitors. Consequently, successful innovators become responsible for a disproportionately large market share and, as a result, find themselves with a significant influence on market aggregates. Under general assumptions to be presented in Section 3.1, it is optimal for these firms to internalise this effect. This is equivalent to adopting an oligopolistic behaviour.

Alternatively, to process innovation, a discrete investment that offers the ability to decrease production costs and exploit market power could manifest itself in a variety of empirically relevant ways. For example, [29] explains it as the cost of offshoring, since this is an option affordable only to the most profitable firms and triggers a radical increase in market concentration. The lobbying literature [30] incorporates fixed costs of making political contributions that generate protection equivalent to a cost reduction. Ref. [14] adds that, not only lobbying, but also complicated regulation works in favour of large firms since they require a high fixed cost of compliance and create market power. Finally, Ref. [31] uses insight from the managerial literature in order to argue that the mere realisation of one's impact on the market implies costly information acquisition and processing. Indeed, we show in Appendix C that it may still be optimal to pay f_k, even if $k = 0$, as long as the investment generates the ability to charge an oligopolistic markup.

We go on to set up the period two profit maximisation problem for each type of firm. Starting with the low-productivity firms, they compete monopolistically (MC firms) and choose their period two production $y_2(z_l) \equiv y_2$, treating market aggregates as given, exactly as they did in period one, aiming at maximising their period two profit $\pi_{y2}(z_l) \equiv \pi_{y2}$.

Therefore, the optimisation problem of an MC firm yields the equilibrium pricing rule $p_{y2}(z_l) = \frac{1}{az_l}$. Since, under Assumption 1, only low-productivity firms operate as MC firms, we obtain that all MC firms produce the same output, which is sold at the same price, and, hence, make the same period two profits.

Moving on to the high-productivity firms, under Assumption 1, in period two, they behave as Cournot oligopolists (CO firms), selecting their period two quantity $x_2(z_k) \equiv x_2$ in order to maximise their period two profits $\pi_{x2}(z_k) \equiv \pi_{x2}$ subject to the inverse demand function and taking as a given that the MC firms maximise their profits subject to the inverse demand and that every other CO firm produces according to their reaction function. The x index represents the fact that the firm acts as a Cournot oligopolist. (Every CO firm understands, first, that their input choice affects the industry price index and is therefore involved in a game-theoretic environment, and, second, that the industry price index is influenced by the aggregate behaviour of MC firms [20]. Third, a CO firm understands that, since the income share spent on the differentiated product is constant due to CES preferences, we obtain that the income level influences firms' demands and hence their profits. As a result, all firms must correctly anticipate what the total income will be. Because of their market power, CO firms could in theory manipulate the income level and hence their demands through their choices. To avoid the non-existence of an equilibrium that is a usual result of accounting for these feedback effects, we take the approach followed by [32] and empirically tested by [18] that each firm that is large within the sector is small in the economy as a whole. This income-taking assumption means that no large firm seeks to manipulate its demand through the income level. In our model, this assumption is equivalent to selecting a sufficiently high β.)

2.3. Closed Economy Equilibrium

We consider a non-cooperative game in which all firms choose their output simultaneously. We close the model with the free entry condition (oligopoly with free entry raises an integer problem; for a discussion, see, for example, [33]), which states that the total number of firms N to enter the differentiated sector is adjusted up to the point where the total expected profits (prior to entry) become equal to zero.

We consider a symmetric equilibrium where, in the second period, every CO firm i, where $i = 1, 2, \ldots, n_{x2}$, chooses the same output x_2 sold at the same price p_{x2} and, hence, earns the same profit π_{x2}. Under the symmetry assumption, the second period profits for CO firms are given by:

$$\pi_{x2} \equiv \pi_{x2}(z_k) = x_2^a E_2^{1-a} P_2^a - \frac{1}{z_k} x_2 - f - f_k$$

whereas the second period profits of MC firms are equal to:

$$\pi_{y2} \equiv \pi_{y2}(z_l) = \left(\frac{1}{az_l}\right)^{\frac{a}{a-1}} (1-a) E_2 P_2^{\frac{a}{1-a}} - f$$

For equilibrium characterisation purposes, we list the equilibrium conditions for each one of the two periods and express them only in terms of N. For a given N, we form a system of nine equations in nine unknowns: four equations for the per-period profits (one for each of the two periods for each type of firm), the aggregate price indices for each period, the levels of expenditure on the differentiated good for each period, and the level of each oligopolist's production in period two x_2. Once we solve this system of equations for a given N, we close the system by adding the free entry condition which pins down N.

Note that, before we add the free entry condition, we can present an interesting result that holds in the short term, or, in other words, for a fixed N. The proof is in Appendix A.

Proposition 1. *Assume that the market size for the differentiated good is exogenous (i.e., the total number of firms N is given). Then, the industry price index, as well as the price and markup charged by a CO firm, decreases when the total number of firms increases.*

The combination of CES and monopolistic competition implies that, for an MC firm, the second period profit-maximising markup $\theta_{y2} = \frac{1}{a}$, exactly as in period one, and, hence, its second period price will be $p_{y2} = \frac{1}{az_l}$. However, in period two, CO firms charge a markup higher than their smaller low-productivity competitors:

$$\theta_{x2} = \frac{1}{a}\frac{z_k}{z_l}\left(\frac{\gamma M}{(1-\gamma)(1-M)}\right)^{\frac{1-a}{a}} > \frac{1}{a} = \theta_{y2} \tag{1}$$

where M is a decreasing function of the number of firms N (see Appendix A). The above expression also implies that, when the productivity gap between the two types of firms is sufficiently high, the larger CO firms will charge a lower price:

$$p_{x2} = \frac{1}{az_l}\left(\frac{\gamma M}{(1-\gamma)(1-M)}\right)^{\frac{1-a}{a}} < \frac{1}{az_l} = p_{y2} \tag{2}$$

Note that if all firms have the same marginal cost, CO firms will choose a higher markup but also a higher price. However, in the presence of a productivity advantage, oligopolistic firms will select a lower price compared to their non-strategic competitors (we find that the residual markup inequality that cannot be attributed to productivity, as documented by [16], can be expressed as the difference between the markups for each type of firm $\theta_{x2} - \theta_{y2}$, in the counterfactual exercise where all firms are equally productive: $z_k = z_l$).

As mentioned above, the final step for computing the closed economy equilibrium is, having expressed the system of nine equations only in terms of the number of firms N, we close the system by adding a tenth equation that will determine N. We show that there exists a value of N that solves our system of equations. This N is the solution to the following free entry condition:

$$\gamma\left[\pi_{y1}(z_h) + \pi_{x2}(z_k)\right] + (1-\gamma)\left[\pi_{y1}(z_l) + \pi_{y2}(z_l)\right] - f_e = 0 \tag{3}$$

We show in Appendix B that there exists at least one $N > 0$ such that the free entry condition holds. This N is unique for sufficiently low levels of a and γ.

We can, finally, calculate the indirect utility of the representative consumer and, given that preferences are homothetic, the indirect utility function will describe aggregate welfare. Not surprisingly, aggregate welfare decreases with the aggregate price, decreases with the average markup and increases with the average productivity.

3. Equilibrium Analysis

3.1. Subgame Perfection

We have computed the mixed market equilibrium, where all firms compete monopolistically in period one and, in period two, all high-productivity firms grow large and adopt an oligopolistic behaviour, whereas all low-productivity firms remain small monopolistic competitors throughout their lives. We now specify the conditions under which this mixed market equilibrium is subgame perfect and Assumption 1 holds as a subgame perfect Nash equilibrium (SPNE) in pure strategies.

Proposition 2. *Investing in acquiring market power is subgame perfect if and only if:*

1. *z_h is sufficiently higher than z_l so that $f_k \in (\pi_{y1}(z_l), \pi_{y1}(z_h))$ is unaffordable for firms with initial productivity draw z_l.*
2. *Given f_k, z_k could be higher or even equal to z_h, as long as f_k, γ and a are sufficiently low.*

We present the formal proof of Proposition 2 in Appendix C, but the intuition is straightforward. If there is only one possible initial productivity draw and all firms are equally productive, then at the end of period one, they will all have accumulated the same profits. As a result, there is no reason why there will be different abilities to innovate in period two. Thus, in a perfectly homogeneous setup, mixed markets can only arise as an equilibrium in mixed strategies. The only possible equilibria in pure strategies are pure monopolistic competition and pure oligopoly. Which of the two will emerge depends on the demand elasticity and the equilibrium number of firms. (Note that, for our results, the assumption of a simultaneous choice of output is crucial. In sequential models, if large firms act first, they choose to mimic the behaviour of the competitive fringe [34].) With (at least) two possible productivity draws, firms accumulate different levels of period one profits. Therefore, productivity heterogeneity, which is commonly assumed in trade models, emerges as a necessary condition for the existence of mixed markets. Furthermore, in our model, a high initial productivity advantage can be endogenously magnified via the decision of a firm to engage in innovation. We end up with the empirically confirmed pattern where most firms are small, characterised by an inferior efficiency and non-strategic behaviour embodied in constant markups, and a few superstars that are not mere productive clones of smaller enterprises, as they exhibit different (oligopolistic) behaviour that translates to higher and variable markups.

3.2. Polarised Market Analysis

Our model is too stylised to be fully calibrated, but we are able to use simulations to draw interesting results which are extremely robust to changes in the parameter values. We normalise $L = 1$ and calibrate our key parameters, as presented in Table 1. The selection of productivity levels z_l, z_h and z_k is arbitrary. However, as long as the ranking is preserved, a change in the parameter values only rescales the numerical analysis. The fixed cost of production f and the cost of entry f_e are also hypothetical but, again, the model is insensitive to changes in the cost values (for the graphs presented here, the parameter values are the following: $z_k = 7$ $z_h = 3$, $z_l = 0.1$, $f = 0.001$, $f_k = 0.015$ and $f_e = 0.03$).

Table 1. Calibrated parameter values

Parameter	Interpretation	Value
a	Variety Substitutability	0.5 [35]
β	Share of Homogenous Good	0.5 [36]
γ	Percentage of CO Firms	0.01 [3]

To analyse competition among big and small businesses, we look into the effect of the probability of an initial high draw γ, which is essentially the percentage of CO firms. In our baseline model, superstars correspond to the top 1 percent of the firm distribution and therefore $\gamma = 0.01$. We can trace the whole spectrum of $\gamma \in (0, 1)$ in Figure 2. As $\gamma \to 1$, our model tends to pure oligopoly, and, as $\gamma \to 0$, it tends to pure monopolistic competition. We find that, as γ increases and the firm distribution becomes less concentrated, competitive pressures increase within the oligopolistic sub-sector and decrease within the competitive fringe. However, the non-strategic nature of the MC firms implies that market competitiveness is driven by the oligopolistic sub-sector. Hence, as γ increases, the markup difference between the big and the small (markup gap in the top left plot of Figure 2) decreases at a decreasing rate. This implies that, from a level of γ onwards, the two sub-sectors behave very similarly. As $\gamma \to 1$, markups tend to equalise and the markup gap tends to 1, although they are never exactly equal, since, by construction, the two types of firms behave differently. The decrease in markups suppresses prices (aggregate price in the top right plot of Figure 2), which has a positive effect on welfare (total welfare in the bottom left plot of Figure 2).

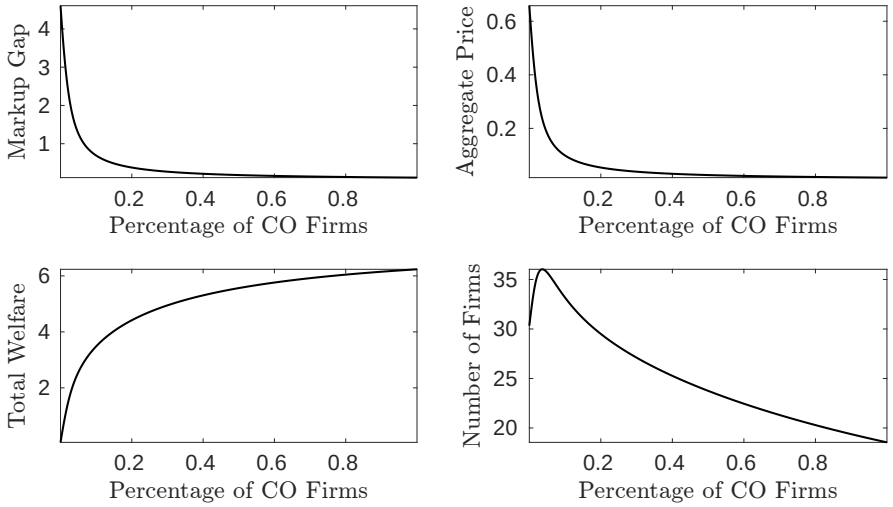

Figure 2. The effect of changing the percentage of large (CO) firms γ from 0 to 1 on the markup gap between large and small firms, the aggregate price, total welfare and the equilibrium number of firms.

Finally, we present the relationship between the level of γ and the equilibrium number of firms, adjusting to the free entry condition (number of firms in the bottom right plot of Figure 2). In our model, entry is driven by the prospect of oligopolistic profit. Therefore, more firms will enter as (1) the probability of obtaining a high productivity draw, which allows a firm to grow, increases and (2) large-firm profitability rises. As we increase γ, these two effects move in the opposite direction. We show that for low levels of γ, where competition among large firms is weak, an increase in the probability of a high draw dominates and N increases. However, beyond a certain level of γ, the result is overturned and any further increase in γ drives the firm number down. The result is an inverted-U curve which expresses the relationship between the level of competition (which increases with γ) and the level of Romer-type [37] innovation, as reflected in the number of differentiated varieties produced in the economy (N). This inverted-U pattern, peaking at relatively low levels of competition, is in line with the endogenous growth literature [22].

4. The Open Economy

We use the two-period framework presented in Section 2 in order to analyse a global economy consisting of two symmetric countries with identical preference and production structure. We assume that the countries open up to trade only in the second period of the game, when both large CO and small MC firms are operating (allowing for trade in the first period would involve two monopolistically competitive industries, as in [19], and we have nothing new to say here about the gains from this type of trade). In an open economy, at the beginning of period two, once the innovation decisions have been made, firms have the option to export to a foreign, perfectly symmetric economy. Trade costs are of the iceberg type, $\tau > 1$, and units of goods must be shipped abroad in order for one unit to be consumed. We abstract from the presence of a fixed entry cost in the export market and, therefore, all firms choose to export.

Starting with large CO firms, they select the quantity sold in the domestic and the foreign market, denoted by x^d and x^f, respectively, to maximise profits:

$$\pi_{x\tau} = \left(p_x^d - \frac{1}{z_k}\right)x^d + \left(p_x^f - \frac{\tau}{z_k}\right)x^f - f - f_k$$

subject to the inverse demand function of each country and taking as a given that both domestic and foreign MC firms maximise their profits subject to the inverse demand at home and abroad and that every other oligopolist (foreign and domestic) produces

according to their reaction functions. We denote the price charged by a large CO firm in the domestic market by p_x^d, and in the foreign market by p_x^f.

Small MC firms choose the level of output supplied in the domestic market y^d and the export market y^f to maximise the following profit function:

$$\pi_{y\tau} = \left(p_y^d - \frac{1}{z_l}\right)y^d + \left(p_y^f - \frac{\tau}{z_l}\right)y^f - f$$

subject to the inverse demand function of each country. We denote the price charged by a small MC firm in the domestic market by p_y^d, and in the foreign market by p_y^f. We find that each domestic small firm sets a domestic price equal to the one in a closed economy setup, but a higher price is set in the foreign market, reflecting the increased marginal cost of exporting.

We consider a non-cooperative game in which big and small firms choose their output simultaneously. We characterise a mixed market equilibrium using the profit maximisation conditions of the small and large firms, as well as the free entry condition. We find that, as in the closed economy case, oligopolists charge a markup that is higher than monopolistic competitors. More importantly, large firms are able to charge a markup that varies with τ, unlike the fringe firms who charge a constant markup of price over marginal cost. These results replicate common empirical patterns [18].

Finally, we use the indirect utility of the representative consumer to obtain the aggregate welfare function V. For the evaluation of welfare gains from trade, we use the compensating variation ω caused by going from the trade equilibrium Ω_1 to the autarky equilibrium Ω_2:

$$\ln \omega = V(\Omega_1) - V(\Omega_2)$$

Welfare and welfare gains from trade decrease with prices and the average markup and increase with average productivity. For the evaluation of a trade policy regime, in Section 6, we use the exact same approach, where Ω_1 is the equilibrium under the policy regime and Ω_2 is the equilibrium without the policy.

5. The Composition Effect of Trade

In this section, we look into the effect of international trade liberalisation. We consider a setup where, in order to export, there is an iceberg-type cost τ but no fixed cost of exporting. (In the presence of fixed costs, we would find one of two possible cases: (1) The fixed cost of exporting is too high for small firms; in which case, international trade only occurs among oligopolists. (2) The fixed cost is low enough for all firms to export; in which case, the no fixed cost analysis is exactly applicable, only scaled down. The case where only oligopolists trade is an even more extreme version of the results under no fixed costs, and small firms are even less flexible to react to trade liberalisation and, hence, even more severely harmed by it.) In Figure 3, which should be read from right to left, we illustrate the effects of decreasing the iceberg cost τ to the point where $\tau = 1$ and trade becomes free (as in Section 3.2, our results are robust to changes in the parameters provided subgame perfection is guaranteed; we set our parameters as follows: $L = 1$, $a = 0.5$, $\beta = 0.5$, $\gamma = 0.01$, $z_k = 7$ $z_h = 3$, $z_l = 0.1$, $f = 0.001$, $f_k = 0.015$ and $f_e = 0.03$).

As τ decreases, tending to 1, two things happen. First, export penetration from both small and large foreign firms increases, suppressing the market shares of domestic firms and raising the market shares of foreign firms in the home market. Second, with CES preferences, small firms cannot adjust their markups, as opposed to large firms who can.

Consequently, large firms engage in reciprocal dumping by charging a lower markup abroad than at home. More importantly, as trade liberalisation proceeds, large firms lower their markups in the domestic market to cope with increased competition at home and raise them in the foreign market as a result of decreased export costs. Differential abilities to adjust markups create a market share reallocation from small towards large firms that boosts large firms' profits as trade becomes cheaper. This result is shown in Figure 3, where,

as the trade cost decreases, the total profit (from the whole two-period game) of a small MC firm decreases (total MC profit in the top left plot of Figure 3) and the total profit of a large CO firm increases (total CO profit in the top middle plot of Figure 3).

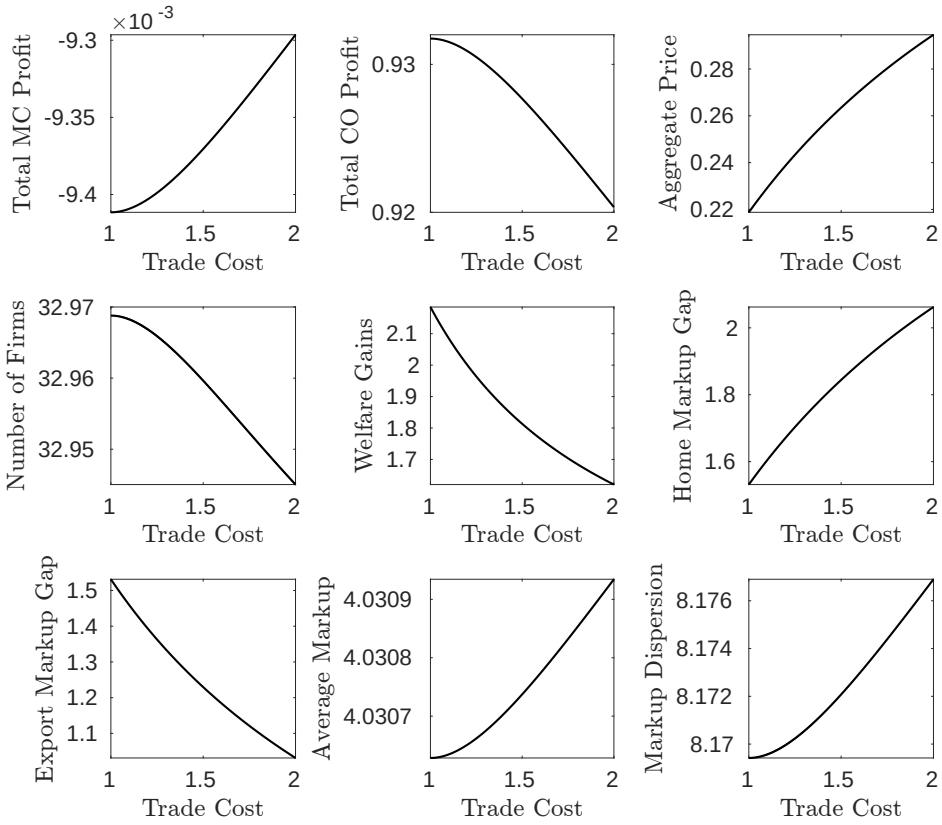

Figure 3. Reading the plots from right to left, we show the effect of suppressing the trade cost τ, tending to $\tau = 1$ (free trade), on the total small (MC) firms' profits, total large (CO) firms' profits, aggregate price, the equilibrium number of firms, welfare gains from trade, the home and export markup gap between large and small firms, average markup and markup dispersion.

To link our findings to the trade literature, when big businesses co-exist with small non-atomic firms, trade liberalisation gives rise to a composition effect of the two traditional sources of gains, namely the variety gains predicted by monopolistically competitive models and the pro-competitive gains expected to materialise under oligopoly. Hence, our predicted effect of trade liberalisation differs from both monopolistically competitive Krugman-type [23] and olipolistic Brander-and-Krugman-type [24] models. We find that, in the presence of market power heterogeneity, there is a new channel through which trade liberalisation affects welfare. This channel, that we refer to as the composition effect, works through the differential abilities of firms to price-to-market and materialises via a market share reallocation from small towards big businesses. (In our baseline calibration, MC firms have positive total operational profits, but, when the entry cost is subtracted, their total profits are negative. The reason they do not exit is that they have to repay the sunk fixed entry cost. On the contrary, even after the deduction of the entry cost, oligopolists make positive profits. Our model is insensitive to changes in the entry cost value.)

To shed more light on the interaction between the big and the small, in the remainder of Figure 3, we focus on the markup adjustment taking place with the increase in trade openness. We obtain that large firms' variable markups imply that the change in trade costs is only partly being passed on to the prices. Thus, the aggregate price decreases but less than the trade cost (aggregate price in the top right plot of Figure 3). Regarding markups, we find that the average markup decreases but only marginally (average markup price in the top right plot of Figure 3); the increase in large firms' foreign markups is slightly

lower than the decrease in their domestic markups (home markup gap and export markup gap in the middle right plot and bottom left plot of Figure 3, respectively). Given that the average markup (average markup in the bottom middle plot of Figure 3) and also markup dispersion (markup dispersion in the bottom right plot of Figure 3) both decrease as τ decreases, we find that some pro-competitive gains from trade survive (welfare gains, middle plot of Figure 3). These gains, however, are crucially diminished due to market power asymmetries.

The intuition behind this result is that, in the presence of a competitive fringe, the fringe largely absorbs competitive pressures. Thus, as τ decreases, large-firm profitability rises, and this happens at the expense of small firms, whose profits decline as trade becomes cheaper, causing a dampening in the pro-competitive gains from trade. Note that the increase in oligopolistic profits dominates the decrease in monopolistically competitive profits, causing expected profits, prior to entry, to increase and inducing more entry (number of firms in the middle left plot of Figure 3). This result is driven by the presence of small firms, because in strictly oligopolistic models, like [38], trade liberalisation creates less entry. However, even though the number of operating firms increases, intensifying competition among large firms, this effect is not sufficient to overturn the increase in oligopolistic profits due to trade liberalisation, much like the evidence documented in [6]. (In terms of the mixed market literature, our model shares an intuition similar to [21]. However, this model operates under the assumptions of a homogeneous productivity and no entry, and, therefore, lowering trade costs drives consumer surplus down. Being a more general framework, our paper performs better in replicating the vast majority of empirical works in the field of international trade, according to which trade is, on aggregate, welfare-enhancing).

A final interesting point is that, comparing the number of available varieties (N versus $2N_\tau$), variety gains from trade survive, although the adjustment in N_τ as τ changes is linked to the existence of market power, unlike in [23].

We conclude that leaving out strategic asymmetries leads to a systematic error in the estimation of trade-induced competition. Hence, our model contributes to the literature on incomplete pass-through (for a survey, see [39]) by being the first, to our knowledge, to study the co-existence of fixed and variable markup firms in a heterogeneous productivity framework.

The importance of modelling market power becomes obvious when we compare welfare gains for different levels of γ (Figure 4). We compare the scenario where large firms are more frequent and correspond to the top 10 percent of the firm distribution ($\gamma = 0.1$) to the one where the firm distribution is extremely skewed and large firms only correspond to the top 0.1 percent ($\gamma = 0.001$). Our baseline calibration is one where superstars are the top 1 percent ($\gamma = 0.01$), as implied by the data. We use a logarithmic representation of our y-axis for illustration purposes. We conclude that, as concentration increases (γ decreases) and large firms become more sheltered from competition, welfare gains from trade decrease, although trade liberalisation is always welfare-enhancing. These results are in line with [38,40,41], who find that markups decrease due to pro-competitive gains from trade. However, contrary to the findings in [41], in our model, pro-competitive gains are lower in the presence of extensive misallocation, which is due to large inefficiencies associated with markups. The characteristic of our model driving these results is that we abstract from pure oligopoly and incorporate the documented inability of most firms to change their pricing behaviour in response to changes in trade costs. (However, as in [42], gains from trade are not monotonically linked to the level of γ. Although, given the existence of large firms ($\gamma > 0$), more inefficiency (lower γ) implies lower gains from trade, in the binary comparison of zero versus positive percentage of large firms, trade creates more gains in an economy with oligopolists than in a purely monopolistically competitive framework.)

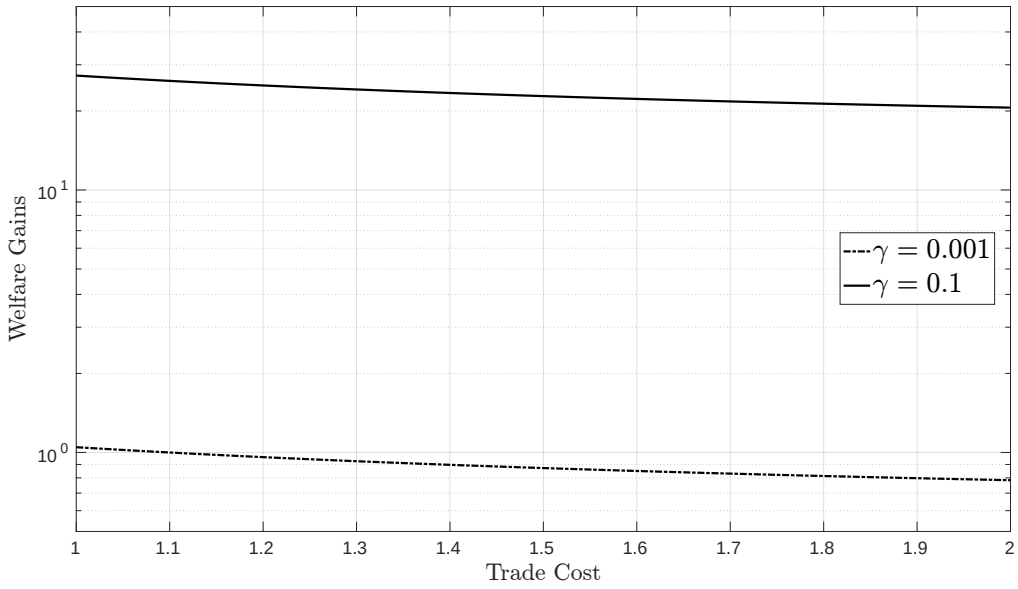

Figure 4. As γ decreases (from 0.1 to 0.001), welfare gains from trade drop significantly.

Having established that, at least in polarised markets, the presence of large firms limits welfare gains from trade, it may be implied that competition policy measures, in the shape of antitrust actions and regulatory reforms, which increase the percentage of large firms and boost competition among them, could be nothing but beneficial. However, bear in mind that large firms are the innovators in our market. Thus, even in our simple setup, it is clear that any attempt at a competition policy should aim to decrease markup dispersion among firms, but without altering the incentives to innovate. Any policy maker should progress with caution, carefully evaluating the trade off between efficiency gains from big firm innovation and market-power-related welfare losses, as the two tend to co-exist [38].

6. Policy Implications

We can now use our model to show how differential market power could serve to justify the use of size-dependent policies. In Section 5, large firms are more efficient and, therefore, they expand as trade becomes less costly. This selection-type reallocation is welfare-enhancing and any policy intervention raising obstacles to this mechanism is arguably harmful. However, selection is not the whole story. In the counterfactual exercise where all firms in the market are equally productive ($z_l = z_h = z_k$), market share reallocation still occurs. This is an inefficiency triggered by trade liberalisation: even without a cost advantage, strategic firms will always set a higher markup and will benefit from pricing-to-market. Consequently, our model emphasises the need to know why firms excel before drawing welfare conclusions regarding any resource reallocation among competitors.

In this final part of our analysis, we use our market structure to show that there exist crucial complementarities between trade and competition policies. We go on to investigate whether size-dependent trade policies could magnify the welfare gains from trade. In reality, such policies, that either restrict big establishments and/or promote small ones, are widespread across countries. They take different forms, including trade restrictions and subsidies, and an extensive literature has taken aim at evaluating their costs and benefits, as well as their impact on the size distribution of firms [43,44]. There is a consensus that such size-dependent policies tend to suppress both aggregate output and average productivity. We show that, in the presence of strategic market power heterogeneity, size-dependent policies might actually increase social welfare.

Our policy is simple and abstract, taking the form of an export subsidy. The level of the subsidy is revealed once each firm has drawn an initial productivity. Government budget balances through a lump-sum tax T to the consumers, whose lifetime income drops by T.

We compare the results from a subsidy to low-productivity firms to the situation where all firms face the same trade cost, for example, $\tau = 2$. In Figure 5, we present the effect of an export subsidy on the monopolistically competitive firms, where the subsidy takes the form of a decreased trade cost faced by these firms. In other words, oligopolistic firms face the real trade cost $\tau = 2$, whereas the MC firms face a trade cost $\tilde{\tau} \in [1, 2]$. The difference between the two costs $\tau - \tilde{\tau}$ is equal to the subsidy per unit of exported output.

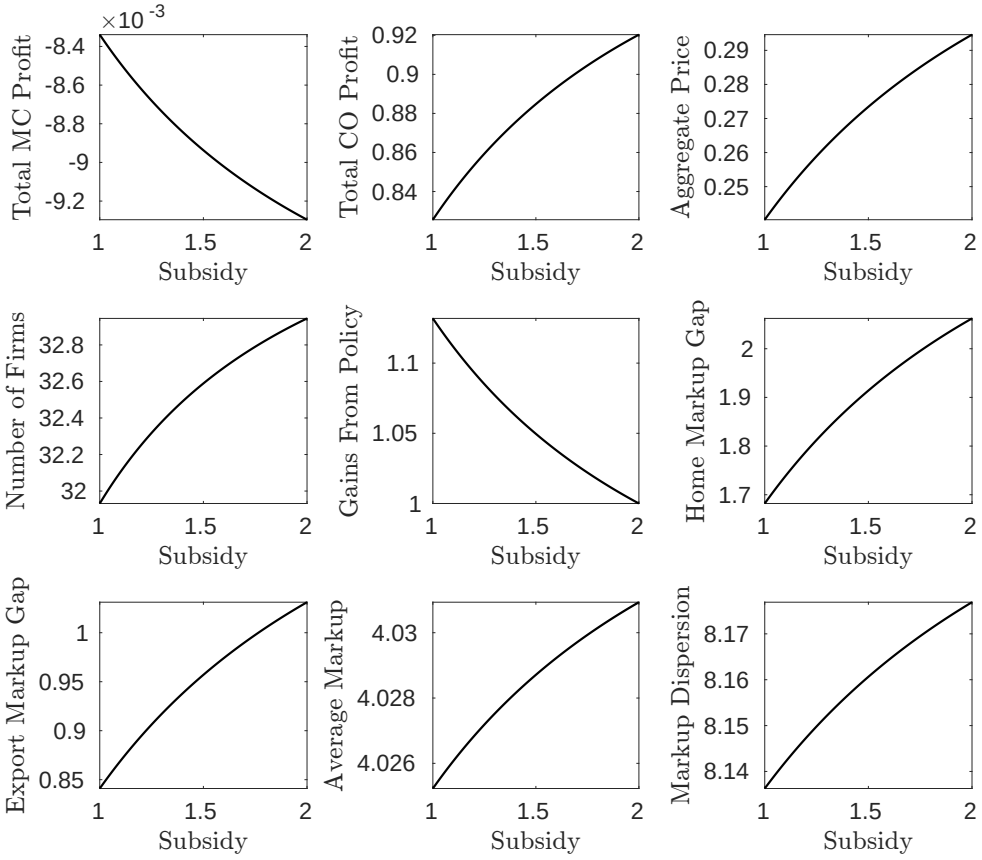

Figure 5. Set the trade cost $\tau = 2$. Reading the plots from right to left, as small firms face a higher export subsidy, which is as if they faced a lower trade cost $\tilde{\tau} \in [1, 2]$, observe the effect on the total small (MC) firms' profits, total large (CO) firms' profits, aggregate price, the equilibrium number of firms, welfare gains from the policy, the home and export markup gap between large and small firms, average markup and markup dispersion.

We find that, contrary to the homogeneous market power case assumed in existing literature, a size-dependent policy leads to an increase in output, which drives aggregate prices down. Reading Figure 5 from right to left reveals that substituting small firms lowers the average markup and markup dispersion by suppressing both the home and the foreign markup gap between small and big businesses. As a result, a market share reallocation in the opposite direction to the one that takes place during trade liberalisation increases the monopolistically competitive profit at the expense of the oligopolistic sub-sector. As a result, aggregate welfare increases and the welfare gains from the policy increase with the magnitude of the subsidy. Finally, subsidising small firms leads to a decrease in the expected profits prior to entry, driven by the decrease in oligopolistic profits. Hence, although they are found to be socially beneficial, size-dependent policies, in our setup, discourage entry. These findings are aligned with [45], who argue that global market integration should be accompanied by competition policies. (When discussing policy implications, there is also the crucial matter of economic stability. Small firms are less resilient to crises than larger ones, partly due to their lower productivity and credit constraints, a matter that has been thoroughly investigated, especially since the COVID-19 pandemic [46–50]. At the

same time, large firms are discrete and granular, and their idiosyncratic shocks reshape the evolution of the aggregate economy, often in an unpredictable way [51,52]. Consequently, the matter of economic stability is impossible to discuss in a setup without random shocks affecting each type of firm, and through them, the aggregate income.

7. Conclusions

In this paper, we develop a model in order to examine how productivity differences could endogenously lead to differences in strategic behaviour, resulting in polarised markets. In our paper, some firms grow large due to a lucky draw from a common productivity distribution and successful cost-reducing innovations, whereas most of their competitors remain small, constrained by their low productivity draw and their limited access to credit. As a result, a small number of extremely productive firms grow into superstars while they compete with a fringe of small and relatively inefficient counterparts, and polarised markets emerge endogenously as the subgame perfect equilibrium in pure strategies.

We go on to introduce international trade to this model economy and calculate the welfare gains from trade liberalisation. We find that superstars emerge as the big winners of globalisation, first, because of their increased productivity, as demonstrated by their lower prices generating increased market shares, and, second, due to their oligopolistic behaviour providing them with the unique ability to vary their markups with their market shares. We show that incorporating productivity heterogeneity jointly with differences in strategic behaviour generates a composition effect that dampens the pro-competitive effect of trade liberalisation. As trade becomes less costly, superstar firms benefit more from exporting to the foreign market than they are harmed by the increase in domestic competition, since competitive pressures are largely buffered by the fringe of smaller enterprises. Therefore, when trade occurs between countries with similar concentration, big businesses are not threatened enough by the competitive tensions induced by trade liberalisation. We find that using purely monopolistically competitive or oligopolistic models to predict welfare gains from trade leads to a significant systematic error in the estimation of trade-induced competition. As [53] observes, the net effect of trade on welfare crucially depends on which firms see the largest increase in perceived competition due to increased import penetration.

Finally, we show that, in our simple setup, it is beneficial to promote size-dependent policies supporting small and medium enterprises (SMEs). However, in a richer setup, one should be very careful in the evaluation of such practices. On the one hand, a lower concentration decreases welfare, but, on the other, in a dynamic setup, size-dependent and competition policies could affect innovation incentives. There is an important, although nuanced, trade-off between efficiency gains from large firm innovation and welfare losses due to decreased competition and markup-related misallocation. This concern emphasises the need for future research to design and pursue trade reforms jointly with competition policy and innovation incentives.

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Abbreviations

The following abbreviations are used in this manuscript:

MDPI	Multidisciplinary Digital Publishing Institute
DOAJ	Directory of open access journals
TLA	Three letter acronym
LD	Linear dichroism

Appendix A. Proof of Proposition 1

In this part, we look into the short-term properties of our model before applying the free entry condition, or, in other words, how the model behaves for a given number of firms N. We define:

$$M \equiv (az_l P_2)^{\frac{a}{1-a}}(1-\gamma)N$$

Therefore, we can write x_2 as follows:

$$x_2 = E_2 P_2^{-1}\left[\frac{1-M}{\gamma N}\right]^{\frac{1}{a}}$$

Substituting the above into E_2 and then substituting P_2 into M yields:

$$\frac{1-\frac{z_l}{z_k}\left(\frac{1-\gamma}{\gamma}\frac{1-M}{M}\right)^{\frac{1-a}{a}}}{1-M} = \frac{1}{\gamma N}$$

Define:

$$G(M) \equiv \frac{1-\frac{z_l}{z_k}\left(\frac{1-\gamma}{\gamma}\frac{1-M}{M}\right)^{\frac{1-a}{a}}}{1-M}$$

The function G increases with M and is such that $\lim_{M\to 0^+} G(M) = -\infty$ and $\lim_{M\to 1^-} G(M) = \infty$. Therefore, for any given N, the equation $G(M) = \frac{1}{\gamma N}$ has a unique solution $M(N) \in (0,1)$ which decreases with N. It then follows from the definition of M that the period two aggregate price index $P_2(N)$ decreases with N. Using the inverse demand function faced by an oligopolist and the definition of M, we find that the equilibrium price $p_{x2}(N)$ set by a large firm from Equation (2), as well as its markup θ_{x2} from Equation (1), are also decreasing functions of N.

Appendix B. Proof of Existence and Uniqueness of Equilibrium

To prove the existence of equilibrium, we go on to add the free entry condition, given by Equation (3), stating that the expected profit prior to entry is zero. We define:

$$R(N) \equiv a\frac{z_l}{z_k}\left(\frac{\gamma}{1-\gamma}\right)^{\frac{a-1}{a}}(1-M(N))^{\frac{1}{a}}M(N)^{\frac{a-1}{a}} + aM(N)$$

Using $R(N)$, the expected profit, prior to entry, can be written in terms of N only as follows:

$$E\pi(N) = \frac{1}{N}[-2L - Nf_e] + \frac{(\beta+1)}{N}$$
$$\left[\frac{L-Nf}{\beta+1-\left[\gamma\left(\frac{1}{az_h}\right)^{\frac{-a}{1-a}}+(1-\gamma)\left(\frac{1}{az_l}\right)^{\frac{-a}{1-a}}\right]^{-1}\left[(1-\gamma)\left(\frac{1-a}{(az_l)^{\frac{a}{a-1}}}\right)+\gamma\left(\frac{1-a}{(az_h)^{\frac{a}{a-1}}}\right)\right]} + \frac{L-\gamma N f_k - Nf}{\beta+R(N)}\right]$$

Now, we have to show that there exists a value of N, denoted by N^*, that solves $E\pi(N^*) = 0$. Given that for any given N, we have that $M(N) \in (0,1)$, we obtain that $\lim_{N\to\infty} E\pi(N) = -\infty$, and since $E\pi(0) > 0$, we have that there exists at least one $N^* > 0$ such that the free entry condition holds.

To prove uniqueness, we need to show that N^* is unique. It suffices to show that $R(N)$ is strictly increasing in N or, equivalently, strictly decreasing in M. If this holds, the expected profits prior to entry will be strictly decreasing in N and therefore N^* will be the unique general equilibrium number of firms. We have that for a sufficiently high productivity gap between large and small firms, large firms charge a lower price, which implies that:

$$\left(\frac{\gamma M}{(1-\gamma)(1-M)}\right)^{\frac{1-a}{a}} < 1$$

We define:

$$F(M) \equiv \left(\frac{\gamma M}{(1-\gamma)(1-M)}\right)^{-\frac{1-a}{a}} > 1$$

$F(M)$ will be decreasing in M (increasing in N). Using $F(M)$, we rewrite $R(N)$ as a function of M and show that it is in fact decreasing in M. To show that $R(M)$ decreases with M, we need to show that the term:

$$\frac{z_l}{z_k}\left(\frac{\gamma}{1-\gamma}\right)^{\frac{a-1}{a}}(1-M)^{\frac{1}{a}}M^{\frac{a-1}{a}}$$

is decreasing in M and at a rate which is higher than 1. We show that the first term is indeed decreasing, since $\frac{\partial R(M)}{\partial M} < 0$. From the comparison of the CO and MC firm markups, we have that $\frac{z_l}{z_k}F(M) < 1$. Therefore, we can guarantee uniqueness for the parameter values for which the term inside the brackets is above 1, so that:

$$\frac{z_l}{z_k}F(M)\left(\frac{1-a}{a}M^{-1}+1\right) > 1$$

This is true for sufficiently low levels of a and γ.

Appendix C. Proof of Proposition 2

We select the cost of innovation so that $f_k \in (\pi_{y1}(z_l), \pi_{y1}(z_h))$. Since the innovation decision must be made before the second period production and given that there is no borrowing in the model, low-productivity firms will be credit-constrained and therefore unable to invest. As a result, all z_l-productivity firms will compete monopolistically in the second period and their efficiency will be pinned down by their initial productivity z_l. Having constrained the behaviour of low-productivity firms, we go on to show under which conditions all z_h-productivity firms will choose to innovate and behave as Cournot oligopolists, resulting in the endogenous emergence of mixed market competition. The game we are considering is the following: in the first stage, high-productivity firms choose whether to innovate and become Cournot oligopolists or not to innovate and compete monopolistically. Given their decision, in the second stage, firms choose their output simultaneously. Solving backwards, we start with the second stage and consider a generic partition of the high-productivity firms in which a percentage $g \in [0,1]$ of these firms act as Cournot oligopolists considering the effect of their choice on the market aggregates, whereas a percentage $(1-g)$ of these firms choose to neglect their impact. We define ε, ϱ and η to be the expenditure, aggregate price index and total production of the differentiated good under this partition. We show that there is no subgame perfect Nash equilibrium (SPNE) where $g \in (0,1)$.

Under the above partition, we have that the total production of the differentiated industry is equal to:

$$\eta = (Ng\gamma x^{\alpha} + N(1-g)\gamma y_h^a + N(1-\gamma)y_l^a)^{\frac{1}{a}}$$

where x is the production of z_k-productivity Cournot oligopolists, y_h is the production of z_h-productivity monopolistic competitors and y_l is the production of z_l-productivity

monopolistic competitors. As a result, the inverse demand function faced by each firm can be expressed as $p = q^{a-1}\eta^{1-a}\varrho$. We assume symmetry across the $g\gamma N$ high-productivity firms that act as Cournot oligopolists and across the $(1-g)\gamma N$ high-productivity firms that act as monopolistic competitors and a fixed N. The maximised profit for each type of firms is presented below:

$$\pi_{yh}(g\gamma N, N) = \left(\frac{1}{az_h}\right)^{\frac{a}{a-1}}(1-a)\varepsilon\varrho^{\frac{a}{1-a}} - f$$

$$\pi_{yl}(g\gamma N, N) = \left(\frac{1}{az_l}\right)^{\frac{a}{a-1}}(1-a)\varepsilon\varrho^{\frac{a}{1-a}} - f$$

and defining:

$$S \equiv (az_h\varrho)^{\frac{a}{1-a}}N\gamma(1-g) - (az_l\varrho)^{\frac{a}{1-a}}N(1-\gamma)$$

we obtain:

$$\pi_x(g\gamma N, N) = \left[\frac{1-S}{g\gamma N} - \frac{1}{z_k}\varrho^{-1}\left(\frac{1-S}{g\gamma N}\right)^{\frac{1}{a}}\right]\varepsilon - f - f_k$$

Having computed the second-stage profit, we move to the first stage, where high-productivity firms simultaneously decide to innovate or not. Given a total number N of firms and that $(1-\gamma)N$ of these firms have a low productivity and behave as monopolistic competitors, there exists an SPNE partition $\{g\gamma N, (1-g)\gamma N\}$ in which $g\gamma N$ high-productivity Cournot oligopolists co-exist with $(1-g)\gamma N$ high-productivity monopolistic competitors if:

1. No high-productivity monopolistic competitor has a unilateral incentive to deviate and become a Cournot oligopolist because:

$$\pi_x(g\gamma N + 1, N) - \pi_{yh}(g\gamma N, N) < 0. \tag{A1}$$

2. No Cournot oligopolist has a unilateral incentive to deviate and become a high-productivity monopolistic competitor because

$$\pi_{yh}(g\gamma N - 1, N) - \pi_x(g\gamma N, N) < 0. \tag{A2}$$

There is no SPNE in pure strategies in which some high-productivity firms incorporate the aggregate impact of their behaviour while others do not ($g \in (0,1)$). The proof follows [31] under the generalisation of non-linear functions as long as $\partial p_i(q_i, Q)/\partial q_i < 0$ and $\partial p_i(q_i, Q)/\partial Q < 0$, where Q is the total production, under [54]. Consider any given partition $\{g\gamma N, (1-g)\gamma N\}$ with $g \in (0,1)$. Given the expressions for maximised profits, assuming that Conditions (A1) and (A2) are simultaneously satisfied leads to a contradiction and, hence, either a high-productivity monopolistic competitor or a Cournot oligopolist has a unilateral incentive to deviate from $\{g\gamma N, (1-g)\gamma N\}$. Now, consider the partition $\{g\gamma N, (1-g)\gamma N\} = \{\gamma N, 0\}$ in which all high-productivity firms compete as Cournot oligopolists. We refer to this as the mixed market competition outcome, as opposed to $\{g\gamma N, (1-g)\gamma N\} = \{0, \gamma N\}$, which is the pure monopolistically competitive outcome. Unilateral deviation from mixed market competition is not profitable if and only if the profit of a single z_h-productivity monopolistic competitor is lower than any z_k-productivity firm's profit when all high-productivity firms have chosen to act as oligopolists with a productivity z_k. Formally, $\pi_{yh} - \pi_x < 0$, which is equivalent to:

$$E\left[\frac{1-M(N)}{\gamma N} - \frac{1}{z_k}P^{-1}\left(\frac{1-M(N)}{\gamma N}\right)^{\frac{1}{a}} - (az_hP)^{\frac{a}{1-a}}(1-a)\right] > f_k \tag{A3}$$

where P is the aggregate price index and E is the expenditure on the differentiated good. This is more likely to happen when the cost of innovation f_k is higher and the produc-

tivity increase due to the innovation (from z_h to z_k) being larger. Note, however, that Equation (A3) can hold even when $z_h = z_k$ as long as f_k, γ and a are sufficiently low.

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