

Details of Experiments with Real-World Datasets

Clustering details include the parameters and the following attributes of the clustering process:

- k is the number of clusters;
- f^* is the best known objective function value multiplied by the number provided after the name of the dataset in the caption of each table;
- s is the sample size;
- n_{exec} is the number of executions for each choice of k ;
- n_s is the number of used samples;
- T is the maximal CPU time allowed for the execution of an algorithm;
- T_1 and T_2 are the maximal CPU times allowed for the first and second phases of the HPClust-hybrid algorithm, respectively;
- n_d is the number of distance function evaluations.

0.1. CORD-19 Embeddings

Dimensions: $m = 599616$, $n = 768$.

Description: COVID-19 Open Research Dataset (CORD-19) is a resource of more than half a million scholarly articles about COVID-19, SARS-CoV-2, and related coronaviruses represented as embeddings in vectorized form.

Table 1: Summary of the results with CORD-19 Embeddings ($\times 10^9$)

k	f^*	\bar{f}	HPClust-inner						HPClust-competitive						HPClust-cooperative					
			ε		\bar{t}		t		ε		\bar{t}		t		ε		\bar{t}		t	
			med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std
2	2.03893*	0.1082	0.006	0.001	15.834	0.0	15.834	6.423	0.006	0.001	15.225	3.105	20.295	9.046	0.006	0.0	15.556	15.461	27.333	14.943
3	1.9093*	0.10141	0.019	0.007	23.69	13.913	33.566	13.481	0.011	0.006	10.312	4.304	16.699	11.92	0.019	0.004	13.461	7.223	15.598	10.088
5	1.77676*	0.09433	0.145	0.066	22.952	10.562	27.398	11.366	0.015	0.003	11.529	2.162	12.459	7.228	0.015	0.002	21.706	4.474	27.337	5.387
10	1.62555*	0.08679	0.463	0.253	3.419	2.783	29.185	8.184	0.067	0.057	7.935	1.012	34.535	10.962	0.063	0.045	7.959	1.433	21.939	10.646
15	1.55295*	0.08276	0.282	0.104	12.552	7.758	31.399	8.814	0.073	0.124	19.717	5.888	34.974	7.468	0.128	0.061	14.53	1.741	20.63	8.791
20	1.49987*	0.07991	0.414	0.075	17.89	8.654	38.208	13.756	0.196	0.146	25.292	4.827	28.762	2.341	0.143	0.101	21.548	1.319	32.616	4.514
25	1.46394*	0.07789	0.166	0.195	16.347	7.16	31.45	9.114	0.092	0.07	32.139	5.446	36.157	4.77	0.153	0.097	32.654	4.919	33.721	6.338
Mean:			0.213		16.098		29.577		0.066		17.45		26.269		0.075		18.202		25.596	

k	f^*	\bar{f}	HPClust-hybrid						Forgy K-means						PBK-BDC					
			ε		\bar{t}		t		ε		\bar{t}		t		ε		\bar{t}		t	
			med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std
2	2.03893*	0.1082	0.007	0.001	6.118	10.005	22.541	6.125	0.0	0.0	–	–	14.984	1.23	0.0	0.06	–	–	2.533	1.639
3	1.9093*	0.10141	0.011	0.005	7.245	4.692	18.988	8.346	0.011	1.406	–	–	45.337	10.538	0.058	0.028	–	–	9.368	4.529
5	1.77676*	0.09433	0.015	0.045	24.03	5.652	23.789	9.182	-0.002	0.234	–	–	104.197	13.797	2.161	1.416	–	–	15.807	1.399
10	1.62555*	0.08679	0.057	0.021	8.259	0.877	26.434	6.826	0.576	0.844	–	–	487.922	226.84	1.937	1.23	–	–	61.921	10.023
15	1.55295*	0.08276	0.111	0.153	15.05	2.097	27.231	7.537	0.342	0.229	–	–	887.432	1046.333	2.333	1.291	–	–	110.804	27.208
20	1.49987*	0.07991	0.161	0.184	25.027	3.976	38.075	7.841	0.233	0.319	–	–	1405.199	794.487	3.496	1.356	–	–	143.392	28.935
25	1.46394*	0.07789	0.18	0.079	26.508	3.56	32.961	7.091	0.056	0.217	–	–	1987.234	837.859	2.21	0.842	–	–	189.484	31.495
Mean:			0.078		16.034		27.146		0.174		–		704.615		1.742		–		76.187	

Table 2: Clustering details with CORD-19 Embeddings

k	n_{exec}	HPClust-inner				HPClust-competitive				HPClust-cooperative				HPClust-hybrid					Forgy K-means	PBK-BDC
		s	n_s	T	n_d	s	n_s	T	n_d	s	n_s	T	n_d	s	n_s	T_1	T_2	n_d	n_d	n_d
2	7	32000	82	40.0	3.7E+07	32000	434	40.0	1.6E+08	32000	569	40.0	1.5E+08	32000	511	37.333	2.667	1.6E+08	1.4E+07	1.3E+07
3	7	32000	147	40.0	5.4E+07	32000	257	40.0	2.0E+08	32000	233	40.0	2.0E+08	32000	297	32.0	8.0	2.0E+08	5.6E+07	4.9E+07
5	7	32000	104	40.0	8.0E+07	32000	129	40.0	2.5E+08	32000	321	40.0	2.6E+08	32000	267	21.333	18.667	2.6E+08	1.3E+08	1.0E+08
10	7	32000	75	40.0	1.2E+08	32000	215	40.0	3.5E+08	32000	123	40.0	3.4E+08	32000	147	24.0	16.0	3.3E+08	6.9E+08	4.2E+08
15	7	32000	41	40.0	1.4E+08	32000	110	40.0	3.7E+08	32000	35	40.0	3.6E+08	32000	73	26.667	13.333	3.5E+08	1.3E+09	7.9E+08
20	7	32000	46	40.0	1.7E+08	32000	37	40.0	3.8E+08	32000	54	40.0	3.8E+08	32000	45	8.0	32.0	3.3E+08	2.1E+09	1.0E+09
25	7	32000	32	40.0	1.9E+08	32000	30	40.0	3.7E+08	32000	23	40.0	3.7E+08	32000	25	32.0	8.0	3.4E+08	2.9E+09	1.5E+09

0.2. HEPMASS

Dimensions: $m = 10500000$, $n = 27$.

Description: The data set contains the 28 normalized features of physical particles that can be used for discovering the exotic ones in the field of high-energy physics.

Table 3: Summary of the results with HEPMASS ($\times 10^8$)

k	f^*	\bar{f}	HPClust-inner						HPClust-competitive						HPClust-cooperative					
			ε		\bar{t}		t		ε		\bar{t}		t		ε		\bar{t}		t	
			med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std
2	2.48889*	0.01512	0.004	0.001	8.694	6.958	19.003	8.514	0.004	0.0	8.529	5.832	26.287	3.089	0.003	0.001	11.257	5.634	18.327	7.876
3	2.36789*	0.01439	0.009	0.62	16.367	2.089	19.541	6.92	0.005	0.003	7.582	3.877	13.209	6.614	0.008	0.436	4.764	3.203	16.956	8.51
5	2.21106*	0.01349	0.341	0.437	5.451	2.961	20.678	10.902	0.012	0.161	1.352	4.625	17.709	7.415	0.333	0.378	2.665	0.943	14.077	7.155
10	2.00353*	0.01223	0.289	0.078	5.034	6.332	18.004	8.518	0.086	0.069	2.335	0.651	16.864	10.671	0.122	0.066	1.619	0.369	13.306	5.406
15	1.89922*	0.01157	0.397	0.191	5.965	0.0	12.862	7.789	0.094	0.068	6.256	6.402	23.113	5.153	0.155	0.12	4.364	1.089	16.703	6.439
20	1.82904*	0.01114	0.322	0.051	15.461	4.451	17.015	8.134	0.156	0.087	6.688	5.447	20.223	7.415	0.209	0.089	3.768	5.851	20.596	4.425
25	1.77524*	0.01082	0.189	0.179	3.61	4.257	22.461	6.812	0.111	0.032	3.925	1.402	24.055	4.933	0.279	0.151	5.554	4.163	19.474	7.38
Mean:			0.222		8.655		18.509		0.067		5.238		20.209		0.159		4.856		17.063	

k	f^*	\bar{f}	HPClust-hybrid						Forgy K-means						PBK-BDC					
			ε		\bar{t}		t		ε		\bar{t}		t		ε		\bar{t}		t	
			med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std
2	2.48889*	0.01512	0.003	0.001	19.63	6.539	21.339	7.536	0.0	0.0	—	—	14.412	1.882	0.0	0.0	—	—	3.128	0.266
3	2.36789*	0.01439	0.005	0.001	9.107	4.481	18.658	7.2	0.0	0.436	—	—	30.808	15.829	0.347	1.087	—	—	5.228	0.278
5	2.21106*	0.01349	0.008	0.161	2.253	4.243	15.988	8.928	0.323	0.114	—	—	79.275	8.253	0.985	0.993	—	—	8.819	0.499
10	2.00353*	0.01223	0.112	0.072	1.658	0.439	23.171	6.864	0.217	0.257	—	—	398.39	138.675	2.767	1.451	—	—	28.289	1.781
15	1.89922*	0.01157	0.115	0.163	3.747	5.177	18.969	7.195	0.289	0.188	—	—	706.048	332.25	1.634	1.369	—	—	50.802	3.377
20	1.82904*	0.01114	0.12	0.101	4.547	9.065	16.949	9.017	0.121	0.166	—	—	1103.684	294.319	2.351	0.863	—	—	67.597	2.791
25	1.77524*	0.01082	0.186	0.139	5.863	4.647	24.828	7.007	0.344	0.269	—	—	1229.349	342.28	2.092	0.584	—	—	85.084	5.815
Mean:			0.078		6.686		19.986		0.185		—		508.852		1.454		—		35.564	

Table 4: Clustering details with HEPMASS

k	n_{exec}	HPClust-inner				HPClust-competitive				HPClust-cooperative				HPClust-hybrid				Forgy K-means	PBK-BDC
		s	n_s	T	n_d	s	n_s	T	n_d	s	n_s	T	n_d	s	n_s	T_1	T_2	n_d	n_d
2	7	64000	25	30.0	3.2E+07	64000	256	30.0	1.1E+08	64000	172	30.0	1.0E+08	64000	188	17.0	13.0	1.0E+08	5.7E+08
3	7	64000	27	30.0	5.6E+07	64000	116	30.0	2.0E+08	64000	155	30.0	1.9E+08	64000	164	7.0	23.0	2.0E+08	1.4E+09
5	7	64000	26	30.0	9.6E+07	64000	166	30.0	3.8E+08	64000	121	30.0	3.6E+08	64000	135	7.0	23.0	3.7E+08	4.3E+09
10	7	64000	23	30.0	2.0E+08	64000	135	30.0	8.0E+08	64000	112	30.0	7.4E+08	64000	189	16.0	14.0	7.7E+08	2.4E+10
15	7	64000	16	30.0	3.1E+08	64000	189	30.0	1.3E+09	64000	142	30.0	1.2E+09	64000	148	9.0	21.0	1.3E+09	4.4E+10
20	7	64000	21	30.0	4.4E+08	64000	154	30.0	1.9E+09	64000	162	30.0	1.8E+09	64000	129	28.0	2.0	1.8E+09	7.0E+10
25	7	64000	26	30.0	5.7E+08	64000	175	30.0	2.4E+09	64000	140	30.0	2.2E+09	64000	170	22.0	8.0	2.3E+09	7.8E+10

0.3. US Census Data 1990

Dimensions: $m = 2458285$, $n = 68$.

Description: The data set was obtained from the (U.S. Department of Commerce) Census Bureau website and contains a one percent sample of the Public Use Microdata Samples (PUMS) person records drawn from the entire 1990 U.S. census sample.

Table 5: Summary of the results with US Census Data 1990 ($\times 10^8$)

k	f^*	\bar{f}	HPClust-inner						HPClust-competitive						HPClust-cooperative					
			ε		\bar{t}		t		ε		\bar{t}		t		ε		\bar{t}		t	
			med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std
2	18.39812*	0.04235	0.107	0.107	1.569	0.665	1.547	0.737	0.278	0.191	1.058	0.722	2.102	0.673	0.233	0.174	0.983	0.685	1.825	0.68
3	6.1591*	0.01444	0.069	66.222	0.813	1.121	1.191	0.959	0.078	0.03	1.292	0.691	1.852	0.859	0.074	0.023	0.856	0.741	1.376	0.951
5	3.35214*	0.00827	2.179	9.495	0.142	0.271	1.937	0.929	0.105	0.044	0.112	0.035	1.966	0.753	0.13	1.488	0.108	0.319	1.343	0.708
10	2.36352*	0.00599	4.682	2.985	0.166	0.588	1.926	0.867	2.413	1.546	0.179	0.137	2.063	0.961	3.296	2.01	0.172	0.039	2.068	0.86
15	2.04097*	0.00508	4.538	4.329	0.368	0.71	1.774	0.718	2.141	1.11	0.258	0.164	1.769	0.75	1.829	1.228	0.246	0.074	1.957	0.719
20	1.81278*	0.00446	5.921	3.048	0.89	0.499	1.407	0.672	2.17	0.794	0.358	0.347	2.132	0.935	2.858	1.073	0.455	0.351	1.755	0.609
25	1.64602*	0.00408	4.439	1.401	0.487	0.586	1.613	0.952	3.178	0.842	0.457	0.381	1.806	0.875	3.034	0.946	0.434	0.188	1.928	0.73
Mean:			3.134		0.634		1.628		1.48		0.531		1.956		1.636		0.465		1.75	

4

k	f^*	\bar{f}	HPClust-hybrid						Forgy K-means						PBK-BDC					
			ε		\bar{t}		t		ε		\bar{t}		t		ε		\bar{t}		t	
			med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std
2	18.39812*	0.04235	0.193	0.101	0.529	0.722	2.371	0.787	0.0	0.0	–	–	0.824	0.235	0.0	0.0	–	–	0.334	0.019
3	6.1591*	0.01444	0.083	0.027	1.114	0.72	1.452	0.882	162.972	61.576	–	–	1.837	1.254	170.038	55.912	–	–	0.652	0.036
5	3.35214*	0.00827	0.117	0.037	0.106	0.037	1.563	0.7	356.299	149.498	–	–	16.932	7.178	216.673	185.79	–	–	1.675	0.106
10	2.36352*	0.00599	3.426	1.645	0.168	0.042	2.665	1.017	12.78	250.618	–	–	41.385	15.171	21.403	168.656	–	–	4.014	0.287
15	2.04097*	0.00508	2.063	1.189	0.279	0.159	2.463	0.757	9.039	185.544	–	–	74.563	18.755	17.079	191.017	–	–	6.282	0.398
20	1.81278*	0.00446	3.548	1.232	0.572	0.434	1.873	0.766	12.895	6.605	–	–	117.544	73.095	16.362	10.313	–	–	9.041	0.82
25	1.64602*	0.00408	2.641	1.094	0.41	0.262	2.147	0.78	11.136	5.757	–	–	179.535	60.011	15.337	6.458	–	–	11.032	0.508
Mean:			1.724		0.454		2.077		80.732		–		61.803		65.27		–		4.719	

Table 6: Clustering details with US Census Data 1990

k	n_{exec}	HPClust-inner				HPClust-competitive				HPClust-cooperative				HPClust-hybrid					Forgy K-means	PBK-BDC
		s	n_s	T	n_d	s	n_s	T	n_d	s	n_s	T	n_d	s	n_s	T_1	T_2	n_d	n_d	n_d
2	20	6000	19	3.0	5.9E+06	6000	162	3.0	1.1E+07	6000	120	3.0	1.1E+07	6000	170	0.2	2.8	1.1E+07	1.5E+07	1.9E+07
3	20	6000	14	3.0	8.7E+06	6000	136	3.0	1.7E+07	6000	106	3.0	1.7E+07	6000	100	2.1	0.9	1.7E+07	3.7E+07	4.4E+07
5	20	6000	18	3.0	1.5E+07	6000	143	3.0	3.2E+07	6000	97	3.0	2.9E+07	6000	113	0.6	2.4	3.0E+07	3.6E+08	1.5E+08
10	20	6000	20	3.0	3.5E+07	6000	120	3.0	7.5E+07	6000	132	3.0	7.0E+07	6000	176	2.4	0.6	7.9E+07	9.5E+08	5.1E+08
15	20	6000	23	3.0	5.5E+07	6000	88	3.0	1.2E+08	6000	104	3.0	1.2E+08	6000	128	1.9	1.1	1.2E+08	1.7E+09	9.0E+08
20	20	6000	16	3.0	7.8E+07	6000	92	3.0	1.7E+08	6000	66	3.0	1.5E+08	6000	78	0.1	2.9	1.5E+08	2.7E+09	1.3E+09
25	20	6000	12	3.0	9.4E+07	6000	60	3.0	2.0E+08	6000	64	3.0	2.0E+08	6000	72	2.5	0.5	2.0E+08	4.1E+09	1.7E+09

0.4. Gisette

Dimensions: $m = 13500, n = 5000$.

Description: patterns for handwritten digit recognition problem.

Table 7: Summary of the results with Gisette ($\times 10^{12}$)

k	f^*	\bar{f}	HPClust-inner						HPClust-competitive						HPClust-cooperative					
			ε		\bar{t}		t		ε		\bar{t}		t		ε		\bar{t}		t	
			med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std
2	4.19944	3.1048	0.009	0.005	2.673	1.476	3.135	1.445	0.007	0.006	3.941	0.683	4.994	0.75	0.009	0.005	4.327	0.746	4.623	0.759
3	4.11596	3.04579	0.036	0.148	1.315	1.4	2.646	1.244	0.029	0.013	5.224	0.563	5.991	0.964	0.023	0.017	5.001	0.426	5.271	0.749
5	4.02303	2.97834	0.077	0.03	3.049	0.836	3.353	0.894	0.064	0.037	8.468	0.901	8.529	0.875	0.081	0.037	8.397	1.456	8.484	1.568
10	3.87672	2.87532	0.165	0.099	4.147	0.874	4.501	0.82	0.156	0.045	16.714	0.814	18.757	2.217	0.132	0.061	16.53	1.032	18.299	1.802
15	3.81766	2.81586	-0.282	0.051	5.514	0.535	5.47	0.646	-0.297	0.048	25.545	3.208	26.532	5.26	-0.315	0.048	25.196	2.045	28.983	2.846
20	3.81436	2.77677	-1.6	0.048	6.593	0.908	6.593	0.713	-1.628	0.045	32.543	3.114	38.244	4.044	-1.627	0.066	32.486	1.482	34.75	2.311
25	3.74937	2.74501	-1.002	0.072	7.495	0.988	6.831	1.347	-1.022	0.053	43.202	3.331	46.72	3.559	-1.055	0.055	40.976	4.052	45.528	3.191
Mean:			-0.371		4.398		4.647		-0.384		19.377		21.395		-0.393		18.988		20.848	

k	f^*	\bar{f}	HPClust-hybrid						Forgy K-means						PBK-BDC					
			ε		\bar{t}		t		ε		\bar{t}		t		ε		\bar{t}		t	
			med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std
2	4.19944	3.1048	0.008	0.004	4.459	0.432	4.499	0.702	0.0	0.0	–	–	3.691	1.633	0.006	0.003	–	–	3.726	1.139
3	4.11596	3.04579	0.023	0.016	5.623	1.684	5.777	1.243	0.0	0.0	–	–	7.901	2.493	0.01	0.002	–	–	8.518	2.308
5	4.02303	2.97834	0.071	0.032	8.021	0.722	8.617	1.411	0.011	0.027	–	–	32.88	20.091	0.037	0.041	–	–	16.183	8.116
10	3.87672	2.87532	0.127	0.046	16.899	1.893	19.183	2.955	0.038	0.049	–	–	45.665	21.177	0.116	0.042	–	–	31.801	17.762
15	3.81766	2.81586	-0.301	0.026	25.974	2.107	27.735	2.35	-0.442	0.046	–	–	59.415	23.158	-0.332	0.069	–	–	43.361	10.205
20	3.81436	2.77677	-1.658	0.044	32.582	2.488	36.983	4.158	-1.782	0.045	–	–	106.076	27.759	-1.69	0.058	–	–	60.231	25.017
25	3.74937	2.74501	-1.049	0.042	41.765	2.526	45.036	3.52	-1.215	0.082	–	–	114.912	27.943	-1.11	0.079	–	–	68.469	23.838
Mean:			-0.397		19.332		21.119		-0.484		–		52.934		-0.423		–		33.184	

Table 8: Clustering details with Gisette

k	n_{exec}	HPClust-inner				HPClust-competitive				HPClust-cooperative				HPClust-hybrid					Forgy K-means	PBK-BDC
		s	n_s	T	n_d	s	n_s	T	n_d	s	n_s	T	n_d	s	n_s	T_1	T_2	n_d	n_d	n_d
2	15	10000	8	5.0	1.1E+06	10000	20	5.0	3.4E+06	10000	18	5.0	3.4E+06	10000	15	4.5	0.5	3.3E+06	7.0E+05	6.7E+05
3	15	10000	5	5.0	1.5E+06	10000	7	5.0	4.0E+06	10000	4	5.0	4.1E+06	10000	5	1.833	3.167	3.9E+06	1.6E+06	1.6E+06
5	15	10000	5	5.0	2.0E+06	10000	4	5.0	6.3E+06	10000	6	5.0	6.4E+06	10000	5	3.167	1.833	6.4E+06	6.8E+06	3.2E+06
10	15	10000	1	5.0	2.9E+06	10000	6	5.0	1.6E+07	10000	6	5.0	1.6E+07	10000	5	3.833	1.167	1.7E+07	9.7E+06	6.8E+06
15	15	10000	1	5.0	3.9E+06	10000	5	5.0	2.5E+07	10000	7	5.0	2.5E+07	10000	6	3.0	2.0	2.4E+07	1.3E+07	9.5E+06
20	15	10000	1	5.0	4.4E+06	10000	6	5.0	3.5E+07	10000	3	5.0	3.3E+07	10000	6	2.333	2.667	3.5E+07	2.3E+07	1.3E+07
25	15	10000	1	5.0	6.1E+06	10000	6	5.0	4.4E+07	10000	6	5.0	4.4E+07	10000	5	4.667	0.333	4.4E+07	2.5E+07	1.5E+07

0.5. Music Analysis

Dimensions: $m = 106574, n = 518$.

Description: a dataset for music analysis which contains different spectral and statistical attributes for each music track.

Table 9: Summary of the results with Music Analysis ($\times 10^{11}$)

k	f^*	\bar{f}	HPClust-inner						HPClust-competitive						HPClust-cooperative					
			ε		\bar{t}		t		ε		\bar{t}		t		ε		\bar{t}		t	
			med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std
2	5.00474*	0.26351	0.065	7.815	4.602	2.312	4.495	2.16	0.097	0.054	1.824	1.797	4.255	2.473	0.073	0.025	1.266	1.203	5.064	2.03
3	3.83748*	0.20356	0.076	0.037	2.503	2.658	4.018	2.625	0.163	0.077	4.177	1.92	4.841	1.995	0.132	0.047	2.619	1.984	4.483	2.256
5	2.74249*	0.14584	0.274	1.459	1.709	2.075	4.109	2.177	0.195	0.137	2.379	2.027	4.536	2.182	0.21	1.192	1.539	0.946	4.358	1.809
10	1.87296*	0.10086	1.911	0.823	2.045	2.229	4.75	2.346	0.51	0.645	1.938	1.989	4.025	2.093	0.658	0.766	2.446	1.825	4.716	1.959
15	1.54422*	0.08235	1.181	0.352	5.033	2.527	5.469	1.896	1.002	0.506	5.801	2.243	6.335	2.091	1.104	0.615	5.006	1.346	5.922	1.963
20	1.35315*	0.07212	1.416	0.683	2.412	2.113	4.326	2.584	1.287	0.5	6.013	1.922	5.91	2.243	1.398	0.84	4.482	1.778	5.824	2.016
25	1.22622*	0.06535	1.466	0.814	4.223	1.683	4.973	1.925	1.912	0.483	5.984	1.433	6.462	1.676	2.224	0.697	5.937	1.84	6.796	1.793
Mean:			0.913		3.218		4.591		0.738		4.017		5.195		0.829		3.328		5.309	

k	f^*	\bar{f}	HPClust-hybrid						Forgy K-means						PBK-BDC					
			ε		\bar{t}		t		ε		\bar{t}		t		ε		\bar{t}		t	
			med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std
2	5.00474*	0.26351	0.081	0.042	2.83	1.904	3.613	2.307	-0.0	9.283	–	–	2.863	0.593	1.266	6.592	–	–	0.446	0.06
3	3.83748*	0.20356	0.142	0.048	3.071	1.871	4.941	1.695	-0.0	4.505	–	–	4.797	2.883	2.19	13.482	–	–	1.174	0.314
5	2.74249*	0.14584	0.208	0.494	2.046	1.981	5.308	2.497	-0.001	1.834	–	–	11.907	2.354	1.223	29.104	–	–	2.204	0.38
10	1.87296*	0.10086	0.604	0.786	2.652	1.685	5.809	2.564	1.448	1.175	–	–	56.994	26.9	9.839	8.146	–	–	6.313	0.994
15	1.54422*	0.08235	1.565	0.612	4.07	2.336	6.531	2.011	0.649	0.425	–	–	138.407	34.68	5.648	4.717	–	–	10.277	1.354
20	1.35315*	0.07212	1.405	0.658	5.708	2.343	6.508	2.543	0.597	0.594	–	–	151.525	44.902	7.064	3.501	–	–	13.586	2.985
25	1.22622*	0.06535	1.945	0.795	6.795	2.159	6.584	2.231	0.611	0.548	–	–	239.709	65.327	6.692	4.415	–	–	17.212	2.663
Mean:			0.85		3.882		5.613		0.472		–		86.6		4.846		–		7.316	

Table 10: Clustering details with Music Analysis

k	n_{exec}	HPClust-inner				HPClust-competitive				HPClust-cooperative				HPClust-hybrid				Forgy K-means	PBK-BDC
		s	n_s	T	n_d	s	n_s	T	n_d	s	n_s	T	n_d	s	n_s	T_1	T_2	n_d	n_d
2	20	6000	278	8.0	2.0E+07	6000	964	8.0	8.7E+07	6000	1249	8.0	8.6E+07	6000	834	1.333	6.667	8.5E+07	3.3E+06
3	20	6000	157	8.0	2.7E+07	6000	644	8.0	9.7E+07	6000	629	8.0	9.7E+07	6000	680	1.6	6.4	9.6E+07	8.0E+06
5	20	6000	116	8.0	3.8E+07	6000	318	8.0	1.1E+08	6000	282	8.0	1.1E+08	6000	345	1.333	6.667	1.1E+08	1.7E+07
10	20	6000	50	8.0	5.5E+07	6000	59	8.0	1.2E+08	6000	84	8.0	1.2E+08	6000	99	6.133	1.867	1.1E+08	6.3E+07
15	20	6000	34	8.0	6.0E+07	6000	59	8.0	1.2E+08	6000	46	8.0	1.2E+08	6000	46	4.533	3.467	1.1E+08	1.0E+08
20	20	6000	14	8.0	6.4E+07	6000	29	8.0	1.2E+08	6000	30	8.0	1.2E+08	6000	21	0.533	7.467	1.0E+08	1.5E+08
25	20	6000	16	8.0	6.6E+07	6000	21	8.0	1.2E+08	6000	23	8.0	1.2E+08	6000	10	0.267	7.733	8.8E+07	1.9E+08

0.6. Protein Homology

Dimensions: $m = 145751, n = 74$.

Description: a data set for protein homology prediction which contains a features describing the match (e.g. the score of a sequence alignment) between the native protein sequence and the sequence that is tested for homology.

Table 11: Summary of the results with Protein Homology ($\times 10^{11}$)

k	f^*	\bar{f}	HPClust-inner						HPClust-competitive						HPClust-cooperative					
			ε		\bar{t}		t		ε		\bar{t}		t		ε		\bar{t}		t	
			med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std
2	15.20433*	4.88318	1.848	0.686	2.345	0.497	2.272	0.746	1.878	0.029	0.873	0.732	1.514	0.949	1.866	0.663	1.5	0.721	2.351	0.583
3	8.07129*	2.89651	0.521	0.553	2.183	0.98	1.949	0.908	0.851	0.609	1.203	0.666	2.23	1.017	0.621	0.457	1.155	0.866	2.187	0.952
5	5.30537*	1.86379	0.804	0.622	1.397	0.999	1.554	0.888	0.784	0.838	1.432	0.799	1.569	0.916	0.651	0.424	1.105	0.391	1.401	0.649
10	3.3767*	1.26637	0.198	0.787	1.784	0.887	2.311	0.915	0.244	0.21	2.264	0.808	2.69	0.521	0.235	0.21	2.417	0.506	2.718	0.502
15	2.86473*	1.08655	1.166	0.849	1.665	0.798	2.274	0.927	0.905	0.429	3.293	0.636	3.547	0.727	0.733	0.468	3.288	0.775	3.379	0.777
20	2.5732*	0.98195	0.782	0.531	1.535	0.899	2.782	0.896	0.761	0.414	3.874	0.656	4.003	0.823	1.012	0.495	3.692	0.6	4.596	1.048
25	2.38539*	0.90731	1.035	0.745	1.045	0.398	2.556	0.886	0.719	0.736	4.189	0.826	4.557	1.009	1.22	0.664	4.481	1.436	4.775	1.223
Mean:			0.908		1.708		2.242		0.878		2.447		2.873		0.906		2.52		3.058	

k	f^*	\bar{f}	HPClust-hybrid						Forgy K-means						PBK-BDC					
			ε		\bar{t}		t		ε		\bar{t}		t		ε		\bar{t}		t	
			med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std
2	15.20433*	4.88318	1.874	0.025	0.995	1.107	2.419	0.768	1.824	0.0	–	–	0.44	0.048	1.825	0.004	–	–	0.307	0.037
3	8.07129*	2.89651	0.811	0.617	2.182	1.037	2.114	1.027	0.0	0.0	–	–	1.241	0.216	0.017	53.129	–	–	0.899	0.154
5	5.30537*	1.86379	1.689	0.508	1.242	0.849	2.808	0.666	0.001	0.0	–	–	3.087	0.22	15.471	10.927	–	–	1.797	0.297
10	3.3767*	1.26637	0.387	0.211	2.967	0.52	3.022	0.539	18.12	0.0	–	–	12.253	2.843	25.368	10.08	–	–	5.637	2.004
15	2.86473*	1.08655	0.949	0.462	3.528	1.02	3.837	1.13	23.941	0.064	–	–	31.292	7.263	31.217	10.86	–	–	11.102	1.696
20	2.5732*	0.98195	0.623	0.542	4.412	0.955	4.681	1.218	28.605	0.245	–	–	35.658	11.432	33.088	11.515	–	–	15.362	3.452
25	2.38539*	0.90731	0.982	0.808	5.021	0.448	4.72	1.107	31.848	0.149	–	–	51.196	12.361	37.4	7.777	–	–	20.165	4.787
Mean:			1.045		2.907		3.371		14.906		–		19.31		20.627		–		7.896	

Table 12: Clustering details with Protein Homology

k	n_{exec}	HPClust-inner				HPClust-competitive				HPClust-cooperative				HPClust-hybrid					Forgy K-means	PBK-BDC
		s	n_s	T	n_d	s	n_s	T	n_d	s	n_s	T	n_d	s	n_s	T_1	T_2	n_d	n_d	n_d
2	15	56000	79	3.5	4.1E+07	56000	259	3.5	2.1E+08	56000	341	3.5	2.2E+08	56000	428	3.267	0.233	2.2E+08	6.7E+06	5.6E+06
3	15	56000	70	3.5	6.8E+07	56000	244	3.5	2.6E+08	56000	272	3.5	2.7E+08	56000	259	2.567	0.933	2.6E+08	2.0E+07	1.6E+07
5	15	56000	45	3.5	9.1E+07	56000	103	3.5	3.0E+08	56000	57	3.5	3.0E+08	56000	186	0.933	2.567	2.8E+08	5.8E+07	3.9E+07
10	15	56000	27	3.5	1.4E+08	56000	38	3.5	3.3E+08	56000	28	3.5	3.1E+08	56000	13	0.233	3.267	2.0E+08	2.5E+08	1.7E+08
15	15	56000	11	3.5	1.8E+08	56000	14	3.5	3.6E+08	56000	10	3.5	3.4E+08	56000	6	0.233	3.267	3.1E+08	6.5E+08	3.6E+08
20	15	56000	15	3.5	1.9E+08	56000	5	3.5	3.8E+08	56000	6	3.5	4.0E+08	56000	6	1.167	2.333	3.7E+08	7.5E+08	4.9E+08
25	15	56000	10	3.5	2.0E+08	56000	3	3.5	4.5E+08	56000	6	3.5	4.5E+08	56000	4	3.15	0.35	4.3E+08	1.0E+09	7.0E+08

0.7. MiniBooNE Particle Identification

Dimensions: $m = 130064$, $n = 50$.

Description: a data set for distinguishing electron neutrinos (signal) from muon neutrinos (background) which contains different particle variables for each event.

Table 13: Summary of the results with MiniBooNE Particle Identification ($\times 10^{10}$)

k	f^*	\bar{f}	HPClust-inner						HPClust-competitive						HPClust-cooperative					
			ε		\bar{t}		t		ε		\bar{t}		t		ε		\bar{t}		t	
			med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std
2	8.92236	8.90824	0.0	0.0	0.961	0.493	1.719	0.593	0.0	0.0	0.546	0.757	1.931	0.836	0.0	0.0	0.624	0.462	1.507	0.735
3	5.22601	5.2178	0.0	5.409	0.881	0.86	1.598	0.83	0.0	0.0	0.497	0.317	1.844	0.773	0.0	0.001	0.559	0.448	2.014	0.656
5	1.82252	1.82055	0.005	29.133	0.542	1.107	1.646	1.114	0.005	0.006	1.349	0.526	1.784	0.63	0.003	0.003	1.362	0.464	2.068	0.43
10	0.9092	0.90911	0.094	702427.909	1.583	0.613	2.219	0.65	0.033	0.043	2.63	0.329	3.05	0.945	0.051	0.031	2.705	0.412	3.065	0.461
15	0.63506	0.64964	2.395	1.575	1.162	0.483	2.284	0.567	0.173	0.667	3.973	0.442	4.099	0.67	0.141	0.391	3.87	0.793	4.345	1.781
20	0.50863	0.54514	1.034	3.267	1.003	0.285	2.073	0.444	0.085	1255623.008	4.681	0.536	6.863	1.949	0.214	0.291	4.608	0.827	6.948	1.716
25	0.44425	0.44476	0.026	2.267	1.208	0.698	2.961	0.565	-0.303	0.133	7.802	1.262	8.719	1.595	-0.391	0.113	6.073	0.774	7.816	2.224
Mean:			0.508		1.048		2.071		-0.001		3.068		4.041		0.003		2.829		3.966	

k	f^*	\bar{f}	HPClust-hybrid						Forgy K-means						PBK-BDC					
			ε		\bar{t}		t		ε		\bar{t}		t		ε		\bar{t}		t	
			med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std
2	8.92236	8.90824	0.0	0.0	0.453	0.468	1.686	0.803	0.0	140555.682	–	–	0.135	0.109	286908.084	140555.682	–	–	0.197	0.069
3	5.22601	5.2178	0.0	0.0	0.817	0.383	1.932	0.66	0.0	166530.775	–	–	0.416	0.236	0.0	122199.794	–	–	0.343	0.125
5	1.82252	1.82055	0.008	0.005	2.417	0.64	2.724	0.639	116.777	55.608	–	–	1.563	0.473	116.777	57.841	–	–	1.636	0.439
10	0.9092	0.90911	0.03	0.053	2.787	1.204	4.616	0.816	0.002	0.0	–	–	13.383	3.122	0.002	0.001	–	–	12.268	2.158
15	0.63506	0.64964	0.123	0.048	3.514	0.685	5.845	1.681	3.883	0.764	–	–	17.098	4.58	3.883	0.777	–	–	15.296	4.864
20	0.50863	0.54514	0.091	0.169	5.003	0.552	5.87	1.558	7.051	0.439	–	–	24.047	4.968	7.052	0.603	–	–	24.811	6.02
25	0.44425	0.44476	-0.267	0.182	6.377	0.932	7.623	1.661	8.936	0.202	–	–	29.885	4.702	8.936	0.38	–	–	28.882	7.385
Mean:			-0.002		3.053		4.328		19.521		–		12.361		41006.39		–		11.919	

Table 14: Clustering details with MiniBooNE Particle Identification

k	n_{exec}	HPClust-inner				HPClust-competitive				HPClust-cooperative				HPClust-hybrid					Forgy K-means	PBK-BDC
		s	n_s	T	n_d	s	n_s	T	n_d	s	n_s	T	n_d	s	n_s	T_1	T_2	n_d	n_d	n_d
2	15	130000	59	3.0	5.5E+07	130000	201	3.0	2.1E+08	130000	182	3.0	2.1E+08	130000	203	2.5	0.5	1.9E+08	2.3E+06	4.7E+06
3	15	130000	42	3.0	7.2E+07	130000	161	3.0	2.5E+08	130000	173	3.0	2.6E+08	130000	159	2.5	0.5	2.5E+08	8.6E+06	8.2E+06
5	15	130000	39	3.0	1.1E+08	130000	74	3.0	3.1E+08	130000	98	3.0	3.1E+08	130000	74	0.2	2.8	2.2E+08	4.5E+07	4.8E+07
10	15	130000	29	3.0	1.7E+08	130000	8	3.0	3.8E+08	130000	12	3.0	3.7E+08	130000	11	2.0	1.0	3.4E+08	4.1E+08	3.8E+08
15	15	130000	21	3.0	2.1E+08	130000	4	3.0	5.8E+08	130000	4	3.0	5.9E+08	130000	6	0.5	2.5	6.2E+08	5.4E+08	4.9E+08
20	15	130000	14	3.0	2.3E+08	130000	5	3.0	8.2E+08	130000	5	3.0	8.5E+08	130000	4	2.9	0.1	8.5E+08	7.6E+08	7.8E+08
25	15	130000	16	3.0	2.5E+08	130000	5	3.0	1.1E+09	130000	5	3.0	9.9E+08	130000	3	1.7	1.3	1.1E+09	9.5E+08	9.3E+08

0.8. MiniBooNE Particle Identification (normalized)

Dimensions: $m = 130064$, $n = 50$.

Description: a data set for distinguishing electron neutrinos (signal) from muon neutrinos (background) which contains different particle variables for each event. Min-max scaling was used for normalization of data set values for better clusterization.

Table 15: Summary of the results with MiniBooNE Particle Identification (normalized) ($\times 10^2$)

k	f^*	\bar{f}	HPClust-inner						HPClust-competitive						HPClust-cooperative					
			ε		\bar{t}		t		ε		\bar{t}		t		ε		\bar{t}		t	
			med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std
2	28.01938*	2.49407	0.014	150.663	0.37	0.279	0.417	0.303	0.019	0.01	0.142	0.209	0.546	0.302	0.027	0.009	0.219	0.156	0.625	0.321
3	19.85673*	1.75033	0.031	3.034	0.352	0.262	0.492	0.231	0.026	0.014	0.15	0.168	0.534	0.293	0.031	0.014	0.152	0.135	0.52	0.303
5	12.10267*	1.11597	0.12	1.745	0.023	0.013	0.647	0.301	0.089	0.028	0.066	0.016	0.604	0.299	0.087	0.043	0.064	0.027	0.459	0.249
10	8.57382*	0.76679	0.668	0.528	0.612	0.378	0.479	0.322	0.471	0.33	0.444	0.196	0.627	0.263	0.647	0.564	0.692	0.202	0.837	0.209
15	7.24131*	0.64941	0.619	0.26	0.27	0.294	0.467	0.26	0.75	0.287	0.55	0.184	0.76	0.222	0.772	0.445	0.66	0.197	0.763	0.221
20	6.30493*	0.56979	1.164	0.703	0.463	0.253	0.586	0.298	1.282	0.747	1.0	0.273	1.045	0.208	0.963	0.577	0.848	0.209	0.951	0.222
25	5.71335*	0.51724	1.147	0.47	0.693	0.269	0.738	0.253	1.209	0.447	1.108	0.231	1.171	0.232	1.363	0.605	0.915	0.219	1.157	0.303
Mean:			0.538		0.398		0.546		0.549		0.494		0.755		0.556		0.507		0.759	

k	f^*	\bar{f}	HPClust-hybrid						Forgy K-means						PBK-BDC					
			ε		\bar{t}		t		ε		\bar{t}		t		ε		\bar{t}		t	
			med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std
2	28.01938*	2.49407	0.02	0.009	0.239	0.194	0.582	0.231	0.0	336.097	–	–	0.245	0.099	2.67	1.876	–	–	0.064	0.019
3	19.85673*	1.75033	0.03	0.014	0.222	0.185	0.473	0.3	6.987	389.115	–	–	0.391	0.08	9.686	10.446	–	–	0.084	0.033
5	12.10267*	1.11597	0.084	0.039	0.064	0.028	0.727	0.236	-0.002	1.531	–	–	0.824	0.251	12.653	27.352	–	–	0.234	0.062
10	8.57382*	0.76679	0.648	0.403	0.588	0.237	0.689	0.208	1.487	1.117	–	–	4.124	1.152	7.647	5.029	–	–	1.027	0.229
15	7.24131*	0.64941	0.499	0.313	0.594	0.301	0.85	0.229	0.33	1.282	–	–	10.031	4.537	7.806	7.356	–	–	1.735	0.332
20	6.30493*	0.56979	1.232	0.793	0.994	0.298	1.075	0.354	0.803	0.492	–	–	14.275	4.417	7.111	3.61	–	–	2.324	0.485
25	5.71335*	0.51724	1.068	0.299	0.989	0.331	1.144	0.532	0.118	0.256	–	–	20.063	10.132	6.67	3.059	–	–	2.98	0.44
Mean:			0.512		0.527		0.791		1.389		–		7.136		7.749		–		1.207	

Table 16: Clustering details with MiniBooNE Particle Identification (normalized)

k	n_{exec}	HPClust-inner				HPClust-competitive				HPClust-cooperative				HPClust-hybrid					Forgy K-means	PBK-BDC
		s	n_s	T	n_d	s	n_s	T	n_d	s	n_s	T	n_d	s	n_s	T_1	T_2	n_d	n_d	n_d
2	20	12000	54	1.0	7.5E+06	12000	437	1.0	4.3E+07	12000	594	1.0	4.5E+07	12000	488	0.033	0.967	4.3E+07	4.4E+06	4.1E+06
3	20	12000	56	1.0	1.6E+07	12000	282	1.0	6.7E+07	12000	291	1.0	7.0E+07	12000	260	0.033	0.967	6.9E+07	9.9E+06	7.2E+06
5	20	12000	56	1.0	2.3E+07	12000	194	1.0	9.0E+07	12000	164	1.0	9.1E+07	12000	264	0.167	0.833	8.9E+07	2.3E+07	1.8E+07
10	20	12000	26	1.0	5.5E+07	12000	50	1.0	1.2E+08	12000	72	1.0	1.3E+08	12000	50	0.667	0.333	1.1E+08	1.2E+08	8.6E+07
15	20	12000	14	1.0	7.5E+07	12000	20	1.0	1.3E+08	12000	24	1.0	1.3E+08	12000	23	0.867	0.133	1.2E+08	3.1E+08	1.5E+08
20	20	12000	12	1.0	8.3E+07	12000	14	1.0	1.3E+08	12000	14	1.0	1.3E+08	12000	10	0.233	0.767	1.0E+08	4.2E+08	2.3E+08
25	20	12000	11	1.0	8.8E+07	12000	10	1.0	1.4E+08	12000	12	1.0	1.4E+08	12000	10	0.733	0.267	1.4E+08	6.1E+08	3.0E+08

0.9. MFCCs for Speech Emotion Recognition

Dimensions: $m = 85134$, $n = 58$.

Description: a data set for predicting females and males speech emotions based on Mel Frequency Cepstral Coefficients (MFCCs) values.

Table 17: Summary of the results with MFCCs for Speech Emotion Recognition ($\times 10^9$)

k	f^*	\bar{f}	HPClust-inner						HPClust-competitive						HPClust-cooperative					
			ε		\bar{t}		t		ε		\bar{t}		t		ε		\bar{t}		t	
			med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std
2	0.74513*	0.10188	0.029	0.015	0.55	0.262	0.623	0.289	0.04	0.016	0.444	0.279	0.609	0.273	0.04	0.023	0.302	0.222	0.561	0.214
3	0.50215*	0.06923	0.037	0.022	0.352	0.308	0.366	0.297	0.043	0.027	0.186	0.133	0.458	0.3	0.051	0.029	0.25	0.162	0.555	0.184
5	0.3456*	0.04777	0.059	0.03	0.499	0.253	0.499	0.291	0.063	0.043	0.32	0.213	0.579	0.256	0.057	0.022	0.281	0.176	0.596	0.255
10	0.21763*	0.03009	1.209	1.243	0.366	0.133	0.363	0.252	0.11	0.033	0.57	0.22	0.662	0.23	0.129	0.046	0.51	0.165	0.644	0.179
15	0.17608*	0.02458	1.2	0.733	0.301	0.19	0.564	0.205	0.237	0.37	0.49	0.158	0.746	0.21	0.519	0.534	0.464	0.207	0.812	0.218
20	0.15383*	0.0214	0.8	1.017	0.315	0.185	0.706	0.29	0.789	0.397	0.863	0.151	0.903	0.247	0.644	0.364	0.755	0.186	0.982	0.217
25	0.14109*	0.01968	1.142	0.742	0.351	0.218	0.526	0.247	1.09	0.41	1.104	0.165	0.974	0.198	0.97	0.443	0.893	0.368	1.079	0.303
Mean:			0.639		0.39		0.521		0.339		0.568		0.704		0.344		0.494		0.747	

k	f^*	\bar{f}	HPClust-hybrid						Forgy K-means						PBK-BDC					
			ε		\bar{t}		t		ε		\bar{t}		t		ε		\bar{t}		t	
			med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std
2	0.74513*	0.10188	0.039	0.02	0.275	0.278	0.345	0.294	0.001	0.0	–	–	0.174	0.065	0.001	0.0	–	–	0.049	0.007
3	0.50215*	0.06923	0.044	0.032	0.242	0.25	0.643	0.261	0.001	0.0	–	–	0.236	0.058	0.001	34.911	–	–	0.069	0.008
5	0.3456*	0.04777	0.057	0.028	0.331	0.22	0.538	0.242	-0.002	0.0	–	–	0.774	0.129	25.789	19.059	–	–	0.2	0.03
10	0.21763*	0.03009	0.102	0.024	0.724	0.222	0.78	0.209	3.278	1.286	–	–	2.347	0.54	11.788	8.119	–	–	0.693	0.145
15	0.17608*	0.02458	0.251	0.263	0.644	0.235	0.855	0.228	1.7	1.843	–	–	5.663	2.193	12.054	8.998	–	–	1.202	0.215
20	0.15383*	0.0214	0.791	0.603	0.832	0.25	0.91	0.344	2.096	1.593	–	–	10.052	2.014	11.16	4.233	–	–	2.035	0.301
25	0.14109*	0.01968	1.017	0.327	0.801	0.166	0.989	0.366	3.385	1.68	–	–	15.119	5.08	9.933	5.571	–	–	2.359	0.407
Mean:			0.329		0.55		0.723		1.494		–		4.909		10.104		–		0.944	

Table 18: Clustering details with MFCCs for Speech Emotion Recognition

k	n_{exec}	HPClust-inner				HPClust-competitive				HPClust-cooperative				HPClust-hybrid					Forgy K-means	PBK-BDC
		s	n_s	T	n_d	s	n_s	T	n_d	s	n_s	T	n_d	s	n_s	T_1	T_2	n_d	n_d	n_d
2	20	12000	109	1.0	1.4E+07	12000	469	1.0	6.1E+07	12000	386	1.0	5.7E+07	12000	230	0.767	0.233	5.6E+07	3.5E+06	3.1E+06
3	20	12000	60	1.0	1.9E+07	12000	252	1.0	7.2E+07	12000	312	1.0	7.6E+07	12000	350	0.2	0.8	7.3E+07	5.4E+06	4.8E+06
5	20	12000	54	1.0	2.7E+07	12000	186	1.0	9.1E+07	12000	172	1.0	8.8E+07	12000	150	0.833	0.167	8.9E+07	1.9E+07	1.5E+07
10	20	12000	26	1.0	5.0E+07	12000	67	1.0	1.0E+08	12000	60	1.0	1.1E+08	12000	78	0.967	0.033	1.1E+08	5.7E+07	4.8E+07
15	20	12000	21	1.0	6.0E+07	12000	24	1.0	1.1E+08	12000	30	1.0	1.1E+08	12000	19	0.033	0.967	7.9E+07	1.5E+08	9.9E+07
20	20	12000	20	1.0	6.6E+07	12000	17	1.0	1.2E+08	12000	18	1.0	1.1E+08	12000	13	0.9	0.1	1.1E+08	2.6E+08	1.6E+08
25	20	12000	8	1.0	7.1E+07	12000	10	1.0	1.2E+08	12000	13	1.0	1.2E+08	12000	9	0.833	0.167	1.2E+08	4.2E+08	2.1E+08

0.10. ISOLET

Dimensions: $m = 7797$, $n = 617$.

Description: data set of patterns for spoken letter recognition which contains the spectral coefficients and other additional features.

Table 19: Summary of the results with ISOLET ($\times 10^5$)

k	f^*	\bar{f}	HPClust-inner						HPClust-competitive						HPClust-cooperative					
			ε		\bar{t}		t		ε		\bar{t}		t		ε		\bar{t}		t	
			med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std
2	7.2194	3.66767	0.033	0.008	1.796	1.551	2.954	1.115	0.033	0.007	1.503	1.185	2.82	1.431	0.032	0.007	0.459	0.692	1.448	1.457
3	6.78782	3.4509	0.054	0.279	0.969	0.694	2.918	1.369	0.044	0.008	2.079	1.157	3.222	0.959	0.043	0.006	0.793	0.573	2.038	1.324
5	6.13651	3.11969	0.456	0.41	0.624	0.341	2.13	1.492	0.066	0.135	1.094	1.663	3.945	1.463	0.071	0.098	0.889	0.608	2.951	1.373
10	5.28577	2.70109	0.622	0.502	0.82	1.018	2.565	1.323	0.189	0.087	1.256	0.335	2.976	1.041	0.343	0.236	0.805	0.342	3.197	1.258
15	4.87391	2.49236	1.4	0.56	0.313	1.402	3.013	1.381	0.647	0.373	1.625	0.748	3.45	1.206	0.552	0.321	1.563	0.437	2.674	0.99
20	4.60857	2.35574	1.162	0.868	1.516	1.047	2.941	1.138	0.357	0.365	2.369	0.887	4.128	0.785	0.391	0.376	2.135	0.599	3.588	0.914
25	4.44323	2.25735	1.0	0.372	1.028	0.139	2.428	1.433	0.28	0.329	3.2	0.591	4.233	1.024	0.332	0.224	3.263	0.714	4.174	0.931
Mean:			0.675		1.01		2.707		0.231		1.875		3.539		0.252		1.415		2.867	

k	f^*	\bar{f}	HPClust-hybrid						Forgy K-means						PBK-BDC					
			ε		\bar{t}		t		ε		\bar{t}		t		ε		\bar{t}		t	
			med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std
2	7.2194	3.66767	0.032	0.006	1.592	1.149	2.841	1.579	-0.0	0.0	–	–	0.119	0.022	0.026	0.01	–	–	0.062	0.008
3	6.78782	3.4509	0.045	0.007	1.657	1.094	4.006	1.218	0.552	0.27	–	–	0.311	0.283	0.047	0.245	–	–	0.151	0.049
5	6.13651	3.11969	0.07	0.13	1.762	1.292	2.122	1.538	0.392	0.797	–	–	0.63	0.345	0.444	0.691	–	–	0.31	0.087
10	5.28577	2.70109	0.166	0.122	0.949	0.483	3.472	1.262	0.936	1.01	–	–	1.475	1.051	1.281	0.647	–	–	0.609	0.367
15	4.87391	2.49236	0.731	0.354	2.276	1.118	3.789	0.858	1.403	1.382	–	–	1.611	1.074	2.444	1.378	–	–	0.964	0.392
20	4.60857	2.35574	0.34	0.346	2.067	0.664	4.11	0.879	1.079	0.845	–	–	3.058	1.241	1.816	1.161	–	–	1.208	0.389
25	4.44323	2.25735	0.259	0.261	3.407	0.958	4.48	1.001	1.252	0.854	–	–	5.116	0.925	1.127	0.67	–	–	1.989	0.743
Mean:			0.235		1.959		3.546		0.802		–		1.76		1.026		–		0.756	

Table 20: Clustering details with ISOLET

k	n_{exec}	HPClust-inner				HPClust-competitive				HPClust-cooperative				HPClust-hybrid					Forgy K-means	PBK-BDC
		s	n_s	T	n_d	s	n_s	T	n_d	s	n_s	T	n_d	s	n_s	T_1	T_2	n_d	n_d	n_d
2	15	4000	283	5.0	1.3E+07	4000	977	5.0	4.9E+07	4000	518	5.0	5.1E+07	4000	1120	1.167	3.833	5.0E+07	1.7E+05	1.1E+05
3	15	4000	240	5.0	1.4E+07	4000	879	5.0	5.4E+07	4000	523	5.0	5.4E+07	4000	1096	1.667	3.333	5.3E+07	4.9E+05	2.9E+05
5	15	4000	128	5.0	1.8E+07	4000	606	5.0	5.6E+07	4000	431	5.0	5.6E+07	4000	300	1.833	3.167	5.5E+07	1.1E+06	4.8E+05
10	15	4000	79	5.0	2.5E+07	4000	147	5.0	5.9E+07	4000	209	5.0	5.9E+07	4000	186	4.667	0.333	5.7E+07	2.5E+06	1.1E+06
15	15	4000	66	5.0	2.8E+07	4000	112	5.0	5.9E+07	4000	58	5.0	5.9E+07	4000	87	1.167	3.833	4.8E+07	2.9E+06	1.9E+06
20	15	4000	40	5.0	3.1E+07	4000	77	5.0	5.8E+07	4000	75	5.0	5.9E+07	4000	80	3.333	1.667	5.7E+07	5.6E+06	2.4E+06
25	15	4000	27	5.0	3.3E+07	4000	43	5.0	5.6E+07	4000	40	5.0	5.8E+07	4000	37	3.167	1.833	4.5E+07	7.8E+06	3.2E+06

0.11. Sensorless Drive Diagnosis

Dimensions: $m = 58509$, $n = 48$.

Description: a data set for sensorless drive diagnosis with features extracted from motor current.

Table 21: Summary of the results with Sensorless Drive Diagnosis ($\times 10^7$)

k	f^*	\bar{f}	HPClust-inner						HPClust-competitive						HPClust-cooperative					
			ε		\bar{t}		t		ε		\bar{t}		t		ε		\bar{t}		t	
			med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std
2	3.88116	3.87915	-0.0	15.678	0.305	0.296	0.392	0.313	-0.0	2.101	0.226	0.151	0.544	0.234	-0.0	1.438	0.26	0.169	0.716	0.257
3	2.91313	3.22719	-0.0	5.899	0.022	0.161	0.516	0.227	-0.0	0.55	0.077	0.008	0.578	0.247	-0.0	0.869	0.082	0.013	0.659	0.267
5	1.93651	1.93613	0.022	8.618	0.307	0.219	0.653	0.285	0.015	7.434	0.559	0.187	0.764	0.219	0.011	1.235	0.48	0.147	0.805	0.184
10	0.98472	1.0394	5.588	8.042	0.177	0.279	0.58	0.257	-2.401	1.407	0.74	0.203	1.017	0.15	-2.394	1.676	0.717	0.179	1.018	0.183
15	0.62816	0.63072	0.481	4.002	0.291	0.251	0.681	0.196	0.034	0.858	1.28	0.448	1.616	0.661	0.028	7.247	1.475	0.412	1.731	0.566
20	0.49884	0.50187	0.486	1.649	0.413	0.135	0.734	0.203	-0.557	1.962	1.78	0.45	2.104	0.596	-0.053	1.871	1.966	0.425	2.326	0.584
25	0.42225	0.43197	2.193	1.768	0.508	0.139	0.811	0.191	1.049	0.546	2.509	0.452	2.867	0.808	0.94	0.502	2.384	0.74	2.826	0.794
Mean:			1.253		0.289		0.624		-0.266		1.024		1.356		-0.21		1.052		1.44	

k	f^*	\bar{f}	HPClust-hybrid						Forgy K-means						PBK-BDC					
			ε		\bar{t}		t		ε		\bar{t}		t		ε		\bar{t}		t	
			med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std
2	3.88116	3.87915	-0.0	0.636	0.213	0.139	0.628	0.276	100.19	0.0	–	–	0.136	0.038	100.19	0.0	–	–	0.14	0.022
3	2.91313	3.22719	-0.0	0.968	0.081	0.014	0.751	0.258	10.865	71.87	–	–	0.433	0.158	10.865	67.653	–	–	0.483	0.152
5	1.93651	1.93613	0.016	6.804	0.704	0.201	0.844	0.207	37.859	0.003	–	–	0.496	0.068	37.853	35.156	–	–	0.524	0.132
10	0.98472	1.0394	-2.404	0.932	0.69	0.194	1.581	0.459	127.202	0.358	–	–	1.696	0.338	127.256	0.036	–	–	1.77	0.387
15	0.62816	0.63072	0.029	0.082	1.339	0.335	1.989	0.587	235.577	0.573	–	–	1.93	0.551	235.435	5.741	–	–	1.789	0.399
20	0.49884	0.50187	-0.058	0.448	1.895	0.392	2.297	0.56	309.27	23.686	–	–	3.769	1.075	309.269	23.374	–	–	3.477	1.446
25	0.42225	0.43197	0.92	0.55	2.407	0.607	2.924	0.944	315.617	35.414	–	–	6.618	2.05	315.673	0.549	–	–	6.369	1.881
Mean:			-0.214		1.047		1.573		162.369		–		2.154		162.363		–		2.079	

Table 22: Clustering details with Sensorless Drive Diagnosis

k	n_{exec}	HPClust-inner					HPClust-competitive				HPClust-cooperative				HPClust-hybrid					Forgy K-means	PBK-BDC
		s	n_s	T	n_d		s	n_s	T	n_d	s	n_s	T	n_d	s	n_s	T_1	T_2	n_d	n_d	n_d
2	40	58500	32	1.0	1.9E+07		58500	177	1.0	8.5E+07	58500	228	1.0	8.5E+07	58500	199	0.267	0.733	8.4E+07	3.9E+06	3.9E+06
3	40	58500	38	1.0	2.6E+07		58500	134	1.0	9.8E+07	58500	140	1.0	1.0E+08	58500	170	0.2	0.8	9.5E+07	1.4E+07	1.5E+07
5	40	58500	36	1.0	4.0E+07		58500	59	1.0	1.1E+08	58500	84	1.0	1.1E+08	58500	80	0.833	0.167	1.0E+08	1.6E+07	1.7E+07
10	40	58500	18	1.0	5.7E+07		58500	12	1.0	1.3E+08	58500	13	1.0	1.3E+08	58500	10	0.633	0.367	1.2E+08	5.7E+07	5.9E+07
15	40	58500	14	1.0	7.4E+07		58500	5	1.0	2.2E+08	58500	3	1.0	2.3E+08	58500	5	0.4	0.6	2.2E+08	6.6E+07	5.9E+07
20	40	58500	10	1.0	7.7E+07		58500	4	1.0	2.8E+08	58500	5	1.0	2.9E+08	58500	4	0.533	0.467	2.7E+08	1.3E+08	1.2E+08
25	40	58500	7	1.0	8.3E+07		58500	5	1.0	3.7E+08	58500	4	1.0	3.5E+08	58500	5	0.767	0.233	3.6E+08	2.2E+08	2.1E+08

0.12. Sensorless Drive Diagnosis (normalized)

Dimensions: $m = 58509$, $n = 48$.

Description: a data set for sensorless drive diagnosis with features extracted from motor current. Min-max scaling was used for normalization of data set values for better clusterization.

Table 23: Summary of the results with Sensorless Drive Diagnosis (normalized) ($\times 10^3$)

k	f^*	\bar{f}	HPClust-inner						HPClust-competitive						HPClust-cooperative					
			ε		\bar{t}		t		ε		\bar{t}		t		ε		\bar{t}		t	
			med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std
2	15.64798*	0.89303	0.067	0.035	0.1	0.082	0.122	0.08	0.091	0.046	0.073	0.052	0.122	0.082	0.088	0.051	0.076	0.069	0.15	0.087
3	12.19375*	0.70587	3.467	2.324	0.058	0.097	0.139	0.098	0.187	1.016	0.054	0.053	0.159	0.076	0.187	1.145	0.044	0.044	0.163	0.091
5	7.85054*	0.45202	0.363	1.748	0.099	0.078	0.166	0.076	0.343	0.255	0.066	0.058	0.172	0.08	0.293	0.21	0.056	0.062	0.181	0.087
10	4.71275*	0.28067	3.764	2.034	0.089	0.08	0.165	0.08	0.609	1.073	0.067	0.058	0.212	0.074	1.936	1.295	0.064	0.038	0.201	0.075
15	3.62541*	0.21493	3.765	2.962	0.106	0.07	0.229	0.091	1.445	0.992	0.111	0.048	0.203	0.072	1.85	1.395	0.091	0.045	0.223	0.071
20	2.971*	0.17797	4.762	2.238	0.059	0.068	0.169	0.087	2.142	0.786	0.101	0.051	0.23	0.065	2.391	1.266	0.099	0.034	0.233	0.066
25	2.60929*	0.15364	5.017	2.274	0.111	0.065	0.2	0.086	2.629	1.204	0.155	0.078	0.25	0.076	2.993	1.446	0.185	0.07	0.246	0.061
Mean:			3.029		0.089		0.17		1.064		0.09		0.193		1.391		0.088		0.2	

k	f^*	\bar{f}	HPClust-hybrid						Forgy K-means						PBK-BDC					
			ε		\bar{t}		t		ε		\bar{t}		t		ε		\bar{t}		t	
			med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std
2	15.64798*	0.89303	0.089	0.059	0.067	0.081	0.175	0.086	0.0	11.811	–	–	0.051	0.019	0.002	17.132	–	–	0.012	0.001
3	12.19375*	0.70587	0.15	0.596	0.078	0.065	0.191	0.074	0.979	3.288	–	–	0.09	0.066	1.574	11.101	–	–	0.033	0.008
5	7.85054*	0.45202	0.297	0.238	0.06	0.044	0.157	0.087	0.535	2.44	–	–	0.21	0.15	11.592	16.743	–	–	0.055	0.012
10	4.71275*	0.28067	1.148	1.18	0.062	0.051	0.253	0.08	6.68	3.559	–	–	0.563	0.402	13.32	7.63	–	–	0.131	0.025
15	3.62541*	0.21493	1.781	0.887	0.094	0.056	0.244	0.079	8.774	3.827	–	–	0.978	0.407	14.032	8.194	–	–	0.201	0.032
20	2.971*	0.17797	3.125	1.012	0.101	0.063	0.251	0.073	12.594	5.298	–	–	1.644	0.589	18.915	6.425	–	–	0.265	0.051
25	2.60929*	0.15364	2.768	1.106	0.161	0.072	0.275	0.065	13.879	6.179	–	–	1.806	0.741	19.277	4.994	–	–	0.318	0.05
Mean:			1.337		0.089		0.221		6.206		–		0.763		11.245		–		0.145	

Table 24: Clustering details with Sensorless Drive Diagnosis (normalized)

k	n_{exec}	HPClust-inner				HPClust-competitive				HPClust-cooperative				HPClust-hybrid					Forgy K-means	PBK-BDC
		s	n_s	T	n_d	s	n_s	T	n_d	s	n_s	T	n_d	s	n_s	T_1	T_2	n_d	n_d	n_d
2	40	3500	38	0.3	2.2E+06	3500	252	0.3	1.4E+07	3500	298	0.3	1.4E+07	3500	362	0.16	0.14	1.4E+07	1.2E+06	1.1E+06
3	40	3500	42	0.3	3.4E+06	3500	248	0.3	1.9E+07	3500	293	0.3	2.0E+07	3500	321	0.13	0.17	1.9E+07	2.7E+06	2.6E+06
5	40	3500	36	0.3	5.4E+06	3500	174	0.3	2.6E+07	3500	194	0.3	2.6E+07	3500	158	0.01	0.29	2.5E+07	5.9E+06	5.4E+06
10	40	3500	27	0.3	9.7E+06	3500	110	0.3	3.4E+07	3500	114	0.3	3.2E+07	3500	134	0.23	0.07	3.3E+07	1.9E+07	1.4E+07
15	40	3500	31	0.3	1.5E+07	3500	52	0.3	3.7E+07	3500	60	0.3	3.7E+07	3500	70	0.26	0.04	3.6E+07	3.2E+07	2.3E+07
20	40	3500	20	0.3	1.8E+07	3500	39	0.3	3.9E+07	3500	40	0.3	3.8E+07	3500	34	0.1	0.2	3.4E+07	5.6E+07	2.9E+07
25	40	3500	20	0.3	2.1E+07	3500	30	0.3	4.0E+07	3500	28	0.3	3.9E+07	3500	28	0.1	0.2	3.6E+07	6.2E+07	3.7E+07

0.13. Online News Popularity

Dimensions: $m = 39644$, $n = 58$.

Description: this dataset summarizes a heterogeneous set of features about articles published by Mashable in a period of two years for predicting the number of shares in social networks (popularity).

Table 25: Summary of the results with Online News Popularity ($\times 10^{14}$)

k	f^*	\bar{f}	HPClust-inner						HPClust-competitive						HPClust-cooperative					
			ε		\bar{t}		t		ε		\bar{t}		t		ε		\bar{t}		t	
			med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std
2	9.53913	2.23789	0.012	0.011	0.328	0.146	0.345	0.177	0.023	0.015	0.134	0.131	0.254	0.164	0.019	0.009	0.064	0.098	0.187	0.132
3	5.91077	1.35797	0.05	7.212	0.403	0.156	0.357	0.174	0.061	0.035	0.211	0.175	0.392	0.219	0.072	0.031	0.256	0.146	0.415	0.164
5	3.09885	0.70224	0.08	6.232	0.473	0.147	0.483	0.162	0.068	0.018	0.193	0.101	0.34	0.186	0.08	0.034	0.146	0.093	0.374	0.181
10	1.17247	0.27667	3.005	5.598	0.198	0.172	0.359	0.188	1.531	0.834	0.265	0.131	0.571	0.178	1.001	1.526	0.267	0.128	0.424	0.126
15	0.77637	0.1913	2.99	5.129	0.149	0.162	0.468	0.167	2.225	1.195	0.265	0.156	0.473	0.171	1.863	1.21	0.259	0.123	0.564	0.161
20	0.59809	0.14447	4.752	2.196	0.156	0.147	0.441	0.226	2.587	1.268	0.406	0.122	0.568	0.152	3.388	1.168	0.418	0.15	0.552	0.167
25	0.49616	0.1202	5.599	1.786	0.205	0.161	0.262	0.24	5.083	2.276	0.551	0.147	0.62	0.149	4.767	7.225	0.529	0.149	0.594	0.16
Mean:			2.355		0.273		0.388		1.654		0.289		0.46		1.598		0.277		0.444	

k	f^*	\bar{f}	HPClust-hybrid						Forge K-means						PBK-BDC					
			ε		\bar{t}		t		ε		\bar{t}		t		ε		\bar{t}		t	
			med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std
2	9.53913	2.23789	0.015	0.008	0.131	0.128	0.316	0.158	-0.0	0.0	—	—	0.036	0.015	0.001	0.0	—	—	0.017	0.004
3	5.91077	1.35797	0.089	0.033	0.21	0.182	0.495	0.193	0.0	0.0	—	—	0.206	0.062	1.679	29.518	—	—	0.057	0.015
5	3.09885	0.70224	0.076	0.026	0.187	0.136	0.378	0.147	12.069	7.238	—	—	0.253	0.167	80.754	51.871	—	—	0.117	0.046
10	1.17247	0.27667	0.928	0.717	0.276	0.177	0.581	0.186	12.363	17.442	—	—	0.877	0.705	38.376	20.585	—	—	0.511	0.163
15	0.77637	0.1913	1.91	3.185	0.355	0.2	0.611	0.24	16.822	9.168	—	—	1.898	0.58	43.769	12.082	—	—	0.819	0.229
20	0.59809	0.14447	3.687	1.27	0.395	0.204	0.675	0.297	25.504	6.434	—	—	3.253	1.371	46.453	19.54	—	—	1.822	0.357
25	0.49616	0.1202	5.135	5.994	0.495	0.14	0.643	0.248	37.787	10.213	—	—	7.378	2.996	53.319	12.025	—	—	2.015	0.471
Mean:			1.692		0.293		0.529		14.935		—		1.986		37.764		—		0.765	

Table 26: Clustering details with Online News Popularity

k	n_{exec}	HPClust-inner				HPClust-competitive				HPClust-cooperative				HPClust-hybrid					Forge K-means	PBK-BDC
		s	n_s	T	n_d	s	n_s	T	n_d	s	n_s	T	n_d	s	n_s	T_1	T_2	n_d	n_d	n_d
2	20	10000	98	0.7	8.7E+06	10000	382	0.7	4.5E+07	10000	281	0.7	4.4E+07	10000	455	0.63	0.07	4.5E+07	5.6E+05	4.4E+05
3	20	10000	54	0.7	1.3E+07	10000	266	0.7	5.9E+07	10000	258	0.7	6.2E+07	10000	312	0.14	0.56	6.0E+07	4.9E+06	2.0E+06
5	20	10000	74	0.7	2.0E+07	10000	130	0.7	6.6E+07	10000	191	0.7	6.8E+07	10000	168	0.467	0.233	6.6E+07	6.0E+06	4.1E+06
10	20	10000	32	0.7	3.0E+07	10000	97	0.7	7.3E+07	10000	60	0.7	7.5E+07	10000	84	0.49	0.21	6.6E+07	2.4E+07	1.9E+07
15	20	10000	26	0.7	4.2E+07	10000	26	0.7	7.7E+07	10000	33	0.7	7.9E+07	10000	14	0.047	0.653	5.7E+07	4.8E+07	3.4E+07
20	20	10000	14	0.7	4.7E+07	10000	16	0.7	8.2E+07	10000	16	0.7	8.0E+07	10000	20	0.63	0.07	8.1E+07	9.1E+07	6.6E+07
25	20	10000	5	0.7	5.1E+07	10000	10	0.7	8.4E+07	10000	11	0.7	8.5E+07	10000	13	0.653	0.047	8.2E+07	2.1E+08	8.2E+07

0.14. Gas Sensor Array Drift

Dimensions: $m = 13910, n = 128$.

Description: this data set contains measurements from chemical sensors utilized in simulations for drift compensation in a discrimination task of different gases at various levels of concentrations.

Table 27: Summary of the results with Gas Sensor Array Drift ($\times 10^{13}$)

k	f^*	\bar{f}	HPClust-inner						HPClust-competitive						HPClust-cooperative					
			ε		\bar{t}		t		ε		\bar{t}		t		ε		\bar{t}		t	
			med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std
2	7.91186	4.78811	0.088	0.064	0.46	0.394	0.844	0.552	0.127	0.091	0.438	0.614	1.283	0.614	0.203	0.084	0.462	0.576	1.118	0.573
3	5.02412	3.01636	0.133	9.866	0.627	0.548	0.896	0.572	0.194	0.11	0.44	0.398	0.924	0.582	0.255	0.109	0.631	0.445	1.119	0.582
5	3.22394	2.03175	6.935	3.508	0.727	0.638	0.797	0.572	7.107	3.625	0.718	0.48	1.151	0.544	0.178	1.763	0.589	0.446	1.145	0.525
10	1.65524	1.06767	3.155	2.805	0.169	0.367	0.797	0.527	0.434	1.262	0.325	0.167	1.193	0.46	0.274	1.403	0.285	0.214	1.128	0.562
15	1.13801	0.74507	4.665	3.27	0.192	0.362	1.024	0.552	0.202	1.306	0.453	0.199	1.269	0.492	0.418	1.634	0.43	0.095	1.307	0.445
20	0.87916	0.56988	3.129	2.638	0.717	0.544	1.319	0.604	1.84	0.792	1.023	0.415	1.488	0.422	2.375	1.02	0.853	0.342	1.488	0.396
25	0.72274	0.47044	4.598	1.838	0.396	0.568	1.008	0.548	2.526	0.797	1.42	0.45	1.685	0.492	2.684	0.897	1.165	0.357	1.528	0.406
Mean:			3.243		0.47		0.955		1.776		0.688		1.285		0.912		0.631		1.262	

k	f^*	\bar{f}	HPClust-hybrid						Forgy K-means						PBK-BDC					
			ε		\bar{t}		t		ε		\bar{t}		t		ε		\bar{t}		t	
			med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std
2	7.91186	4.78811	0.192	0.1	0.394	0.516	0.952	0.654	-0.0	0.0	-	-	0.053	0.007	0.029	0.048	-	-	0.028	0.009
3	5.02412	3.01636	0.245	0.11	0.451	0.523	1.044	0.566	-0.001	0.0	-	-	0.104	0.016	0.03	0.034	-	-	0.088	0.024
5	3.22394	2.03175	0.366	3.35	0.817	0.525	1.245	0.557	8.108	0.387	-	-	0.247	0.064	8.156	0.394	-	-	0.118	0.03
10	1.65524	1.06767	0.232	0.696	0.287	0.21	1.592	0.468	37.905	17.254	-	-	0.595	0.415	41.32	13.482	-	-	0.371	0.23
15	1.13801	0.74507	-0.175	1.603	0.46	0.184	1.887	0.59	27.472	10.362	-	-	1.653	0.679	30.525	24.91	-	-	0.704	0.319
20	0.87916	0.56988	2.03	1.033	1.132	0.391	1.673	0.48	45.732	7.967	-	-	1.684	0.509	45.904	6.855	-	-	0.982	0.472
25	0.72274	0.47044	2.639	0.954	1.54	0.425	1.951	0.446	50.936	12.149	-	-	2.544	1.197	52.691	14.414	-	-	1.796	0.664
Mean:			0.79		0.726		1.478		24.307		-		0.983		25.522		-		0.584	

Table 28: Clustering details with Gas Sensor Array Drift

k	n_{exec}	HPClust-inner				HPClust-competitive				HPClust-cooperative				HPClust-hybrid				Forgy K-means	PBK-BDC
		s	n_s	T	n_d	s	n_s	T	n_d	s	n_s	T	n_d	s	n_s	T_1	T_2	n_d	n_d
2	30	9000	140	2.0	2.3E+07	9000	810	2.0	9.0E+07	9000	606	2.0	9.2E+07	9000	642	1.0	1.0	9.2E+07	5.0E+05
3	30	9000	128	2.0	2.9E+07	9000	424	2.0	9.6E+07	9000	468	2.0	9.9E+07	9000	544	0.867	1.133	1.0E+08	6.2E+05
5	30	9000	84	2.0	3.8E+07	9000	300	2.0	1.0E+08	9000	292	2.0	1.0E+08	9000	292	0.867	1.133	9.8E+07	1.3E+06
10	30	9000	46	2.0	5.5E+07	9000	102	2.0	1.1E+08	9000	106	2.0	1.1E+08	9000	144	1.4	0.6	1.0E+08	3.9E+06
15	30	9000	42	2.0	5.8E+07	9000	65	2.0	1.1E+08	9000	65	2.0	1.1E+08	9000	96	1.867	0.133	1.0E+08	7.6E+06
20	30	9000	40	2.0	6.2E+07	9000	44	2.0	1.1E+08	9000	38	2.0	1.1E+08	9000	30	0.2	1.8	7.9E+07	1.1E+07
25	30	9000	20	2.0	6.5E+07	9000	29	2.0	1.1E+08	9000	30	2.0	1.1E+08	9000	16	0.8	1.2	7.3E+07	1.9E+07

0.15. 3D Road Network

Dimensions: $m = 434874$, $n = 3$.

Description: 3D road network from Denmark with highly accurate elevation information which contains longitude, latitude and altitude for each road segment or edge in the graph. Usually this data set used in eco-routing and fuel/Co2-estimation routing algorithms.

Table 29: Summary of the results with 3D Road Network ($\times 10^6$)

k	f^*	\bar{f}	HPClust-inner						HPClust-competitive						HPClust-cooperative					
			ε		\bar{t}		t		ε		\bar{t}		t		ε		\bar{t}		t	
			med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std
2	49.13298	11.15303	0.004	0.006	0.257	0.123	0.265	0.125	0.005	0.015	0.195	0.151	0.304	0.149	0.008	0.005	0.177	0.107	0.228	0.131
3	22.77818	5.1707	0.005	0.007	0.19	0.133	0.318	0.141	0.011	0.011	0.15	0.102	0.285	0.145	0.015	0.015	0.13	0.093	0.263	0.145
5	8.82574	1.99891	0.02	0.014	0.182	0.146	0.294	0.136	0.021	0.021	0.249	0.115	0.341	0.121	0.018	0.024	0.182	0.099	0.346	0.139
10	2.56661	0.58256	0.167	0.116	0.167	0.113	0.234	0.122	0.159	0.103	0.301	0.137	0.418	0.142	0.164	0.185	0.315	0.099	0.419	0.121
15	1.27069	0.28889	0.334	0.38	0.276	0.104	0.382	0.144	0.223	0.377	0.502	0.346	0.504	0.332	0.343	0.331	0.442	0.198	0.503	0.193
20	0.80865	0.18573	1.243	0.823	0.287	0.109	0.343	0.098	0.542	0.652	0.46	0.168	0.591	0.184	0.382	0.644	0.463	0.208	0.541	0.191
25	0.59259	0.13625	1.038	0.84	0.242	0.13	0.405	0.118	0.557	0.489	0.603	0.264	0.757	0.351	0.588	0.487	0.603	0.255	0.755	0.276
Mean:			0.402		0.229		0.32		0.217		0.351		0.457		0.217		0.33		0.436	

k	f^*	\bar{f}	HPClust-hybrid						Forgy K-means						PBK-BDC					
			ε		\bar{t}		t		ε		\bar{t}		t		ε		\bar{t}		t	
			med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std
2	49.13298	11.15303	0.01	0.011	0.157	0.115	0.195	0.154	0.0	0.0	–	–	0.113	0.033	0.0	0.0	–	–	0.062	0.007
3	22.77818	5.1707	0.012	0.012	0.123	0.082	0.233	0.121	0.0	0.0	–	–	0.187	0.065	0.0	77.393	–	–	0.106	0.009
5	8.82574	1.99891	0.031	0.027	0.215	0.106	0.309	0.129	0.0	0.0	–	–	0.517	0.087	77.246	43.749	–	–	0.268	0.034
10	2.56661	0.58256	0.227	0.176	0.546	0.188	0.503	0.196	0.008	0.0	–	–	5.994	0.261	62.553	44.418	–	–	1.802	0.34
15	1.27069	0.28889	0.224	0.25	0.455	0.443	0.501	0.48	0.002	0.0	–	–	7.558	0.788	57.087	42.217	–	–	2.768	0.357
20	0.80865	0.18573	0.468	0.501	0.43	0.315	0.797	0.418	0.005	0.0	–	–	25.175	1.893	42.58	23.013	–	–	4.995	0.876
25	0.59259	0.13625	0.523	0.481	0.685	0.483	0.874	0.6	1.615	0.25	–	–	24.839	1.744	45.092	33.97	–	–	6.186	0.732
Mean:			0.214		0.373		0.487		0.233		–		9.198		40.651		–		2.313	

Table 30: Clustering details with 3D Road Network

k	n_{exec}	HPClust-inner				HPClust-competitive				HPClust-cooperative				HPClust-hybrid					Forgy K-means	PBK-BDC
		s	n_s	T	n_d	s	n_s	T	n_d	s	n_s	T	n_d	s	n_s	T_1	T_2	n_d	n_d	n_d
2	40	100000	17	0.5	1.8E+07	100000	100	0.5	9.6E+07	100000	72	0.5	9.6E+07	100000	57	0.033	0.467	1.0E+08	2.1E+07	1.8E+07
3	40	100000	15	0.5	2.6E+07	100000	84	0.5	1.6E+08	100000	78	0.5	1.6E+08	100000	68	0.467	0.033	1.6E+08	4.2E+07	4.2E+07
5	40	100000	14	0.5	5.7E+07	100000	74	0.5	2.8E+08	100000	71	0.5	2.7E+08	100000	52	0.117	0.383	2.7E+08	1.5E+08	1.3E+08
10	40	100000	8	0.5	1.7E+08	100000	26	0.5	6.0E+08	100000	24	0.5	5.6E+08	100000	10	0.15	0.35	4.3E+08	2.2E+09	1.2E+09
15	40	100000	6	0.5	2.6E+08	100000	8	0.5	8.5E+08	100000	10	0.5	8.1E+08	100000	11	0.45	0.05	7.9E+08	3.0E+09	2.4E+09
20	40	100000	6	0.5	3.6E+08	100000	6	0.5	9.5E+08	100000	6	0.5	9.9E+08	100000	6	0.333	0.167	9.7E+08	1.1E+10	5.0E+09
25	40	100000	5	0.5	3.6E+08	100000	5	0.5	1.2E+09	100000	5	0.5	1.3E+09	100000	5	0.317	0.183	1.2E+09	1.1E+10	7.8E+09

0.16. Skin Segmentation

Dimensions: $m = 245057, n = 3$.

Description: Skin and Nonskin dataset is generated using skin textures from face images of diversity of age, gender, and race people and constructed over B, G, R color space.

Table 31: Summary of the results with Skin Segmentation ($\times 10^9$)

k	f^*	\bar{f}	HPClust-inner						HPClust-competitive						HPClust-cooperative					
			ε		\bar{t}		t		ε		\bar{t}		t		ε		\bar{t}		t	
			med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std
2	1.32236	0.04216	0.031	0.019	0.105	0.045	0.097	0.054	0.035	0.013	0.04	0.026	0.105	0.055	0.034	0.022	0.028	0.041	0.106	0.05
3	0.89362	0.02822	0.054	0.032	0.058	0.052	0.084	0.06	0.043	0.024	0.066	0.035	0.099	0.049	0.038	0.03	0.038	0.046	0.107	0.065
5	0.50203	0.0161	0.124	2.491	0.048	0.046	0.134	0.053	0.073	0.586	0.018	0.013	0.143	0.056	0.078	0.815	0.018	0.025	0.104	0.051
10	0.25121	0.00817	6.804	5.439	0.039	0.075	0.113	0.061	0.212	1.399	0.026	0.016	0.142	0.053	0.247	2.335	0.023	0.037	0.136	0.064
15	0.16964	0.00544	3.665	2.287	0.064	0.044	0.128	0.059	1.201	1.962	0.046	0.029	0.142	0.042	0.734	2.771	0.038	0.032	0.12	0.053
20	0.12615	0.004	4.366	2.928	0.11	0.055	0.126	0.054	2.311	1.567	0.092	0.051	0.138	0.051	2.202	2.534	0.07	0.034	0.121	0.047
25	0.10228	0.00335	5.333	2.735	0.067	0.035	0.104	0.051	3.485	1.754	0.052	0.016	0.15	0.051	4.461	1.755	0.056	0.034	0.155	0.046
Mean:			2.911		0.07		0.112		1.052		0.049		0.131		1.113		0.039		0.121	

k	f^*	\bar{f}	HPClust-hybrid						Forgy K-means						PBK-BDC					
			ε		\bar{t}		t		ε		\bar{t}		t		ε		\bar{t}		t	
			med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std
2	1.32236	0.04216	0.036	0.022	0.034	0.04	0.144	0.057	-0.0	0.0	-	-	0.042	0.008	-0.0	0.007	-	-	0.014	0.001
3	0.89362	0.02822	0.069	0.031	0.041	0.042	0.106	0.054	-0.001	0.0	-	-	0.081	0.032	0.003	59.847	-	-	0.025	0.003
5	0.50203	0.0161	0.092	0.298	0.021	0.022	0.124	0.062	1.65	6.344	-	-	0.117	0.036	18.075	22.33	-	-	0.036	0.003
10	0.25121	0.00817	0.202	2.214	0.029	0.03	0.176	0.059	9.122	7.003	-	-	0.219	0.062	26.085	8.432	-	-	0.062	0.007
15	0.16964	0.00544	0.888	2.013	0.04	0.039	0.152	0.051	13.463	7.936	-	-	0.389	0.187	29.36	13.524	-	-	0.104	0.011
20	0.12615	0.004	2.121	1.667	0.089	0.04	0.151	0.033	16.816	7.379	-	-	0.548	0.224	34.997	18.226	-	-	0.145	0.018
25	0.10228	0.00335	3.727	1.445	0.061	0.023	0.164	0.049	22.066	6.921	-	-	0.698	0.237	35.345	16.71	-	-	0.182	0.023
Mean:			1.019		0.045		0.145		9.016		-		0.299		20.552		-		0.081	

Table 32: Clustering details with Skin Segmentation

k	n_{exec}	HPClust-inner					HPClust-competitive					HPClust-cooperative					HPClust-hybrid					Forgy K-means	PBK-BDC
		s	n_s	T	n_d		s	n_s	T	n_d		s	n_s	T	n_d		s	n_s	T_1	T_2	n_d	n_d	n_d
2	30	8000	10	0.2	1.6E+06		8000	89	0.2	9.8E+06		8000	80	0.2	8.8E+06		8000	114	0.047	0.153	9.4E+06	6.9E+06	5.4E+06
3	30	8000	10	0.2	3.5E+06		8000	75	0.2	1.8E+07		8000	82	0.2	1.7E+07		8000	76	0.033	0.167	1.8E+07	1.8E+07	1.7E+07
5	30	8000	16	0.2	5.3E+06		8000	111	0.2	2.8E+07		8000	78	0.2	2.5E+07		8000	88	0.153	0.047	2.7E+07	2.7E+07	2.4E+07
10	30	8000	8	0.2	1.1E+07		8000	81	0.2	6.3E+07		8000	90	0.2	5.9E+07		8000	96	0.127	0.073	6.0E+07	7.6E+07	6.5E+07
15	30	8000	14	0.2	2.4E+07		8000	64	0.2	1.1E+08		8000	53	0.2	1.0E+08		8000	72	0.153	0.047	1.1E+08	1.5E+08	1.3E+08
20	30	8000	11	0.2	3.0E+07		8000	48	0.2	1.5E+08		8000	44	0.2	1.3E+08		8000	56	0.053	0.147	1.3E+08	2.3E+08	1.8E+08
25	30	8000	10	0.2	4.6E+07		8000	44	0.2	1.5E+08		8000	48	0.2	1.5E+08		8000	48	0.16	0.04	1.5E+08	2.8E+08	2.4E+08

0.17. KEGG Metabolic Relation Network (Directed)

Dimensions: $m = 53413$, $n = 20$.

Description:

Table 33: Summary of the results with KEGG Metabolic Relation Network (Directed) ($\times 10^8$)

k	f^*	\bar{f}	HPClust-inner						HPClust-competitive						HPClust-cooperative					
			ε		\bar{t}		t		ε		\bar{t}		t		ε		\bar{t}		t	
			med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std
2	11.3853	11.29955	0.0	8.626	0.246	0.336	0.434	0.282	0.0	0.115	0.277	0.242	0.655	0.273	0.24	0.115	0.112	0.116	0.384	0.289
3	4.9006	4.84007	0.001	27.183	0.296	0.183	0.486	0.226	0.559	0.242	0.201	0.179	0.4	0.252	0.559	0.277	0.226	0.235	0.69	0.272
5	1.88367	1.86304	0.005	0.315	0.521	0.276	0.585	0.292	0.016	0.708	0.321	0.196	0.499	0.238	0.014	0.707	0.381	0.198	0.557	0.232
10	0.60513	0.61753	0.07	7.977	0.077	0.226	0.681	0.307	0.022	1.556	0.269	0.083	0.683	0.174	0.041	0.024	0.254	0.17	0.643	0.244
15	0.35393	0.35466	4.554	6.115	0.591	0.25	0.538	0.182	-0.418	0.998	0.451	0.196	0.87	0.223	-0.491	2.633	0.387	0.164	0.853	0.198
20	0.25027	0.25131	2.103	6.812	0.152	0.267	0.76	0.3	0.149	0.63	0.799	0.198	1.006	0.195	0.433	0.795	0.792	0.213	0.966	0.272
25	0.19289	0.19795	4.091	2.5	0.217	0.155	0.545	0.217	1.372	1.097	0.914	0.313	1.143	0.284	1.64	5.875	0.818	0.233	1.064	0.236
Mean:			1.546		0.3		0.575		0.243		0.462		0.751		0.348		0.424		0.737	

k	f^*	\bar{f}	HPClust-hybrid						Forgy K-means						PBK-BDC					
			ε		\bar{t}		t		ε		\bar{t}		t		ε		\bar{t}		t	
			med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std
2	11.3853	11.29955	0.0	0.11	0.249	0.126	0.477	0.233	18.854	0.0	–	–	0.041	0.004	18.854	0.003	–	–	0.033	0.006
3	4.9006	4.84007	0.559	0.256	0.226	0.218	0.373	0.297	124.789	0.0	–	–	0.08	0.023	124.789	9.606	–	–	0.075	0.008
5	1.88367	1.86304	0.072	0.715	0.376	0.249	0.731	0.205	0.0	9.787	–	–	0.201	0.042	0.0	0.001	–	–	0.189	0.07
10	0.60513	0.61753	0.041	8.926	0.276	0.056	0.766	0.194	36.81	3.376	–	–	0.607	0.158	36.81	3.067	–	–	0.582	0.039
15	0.35393	0.35466	-0.359	1.22	0.504	0.186	0.935	0.211	96.641	4.224	–	–	1.873	0.168	97.957	4.103	–	–	1.69	0.245
20	0.25027	0.25131	0.15	25.29	0.631	0.364	1.26	0.303	162.301	4.756	–	–	3.433	0.856	162.039	3.883	–	–	3.784	0.849
25	0.19289	0.19795	1.312	0.908	0.814	0.224	1.421	0.32	230.281	6.699	–	–	5.045	0.638	223.96	5.88	–	–	5.15	0.602
Mean:			0.254		0.44		0.852		95.668		–		1.611		94.916		–		1.643	

Table 34: Clustering details with KEGG Metabolic Relation Network (Directed)

k	n_{exec}	HPClust-inner					HPClust-competitive				HPClust-cooperative				HPClust-hybrid					Forgy K-means	PBK-BDC
		s	n_s	T	n_d		s	n_s	T	n_d	s	n_s	T	n_d	s	n_s	T_1	T_2	n_d	n_d	n_d
2	20	53350	54	1.0	3.1E+07		53350	586	1.0	2.0E+08	53350	348	1.0	2.0E+08	53350	377	0.767	0.233	1.9E+08	1.9E+06	2.0E+06
3	20	53350	62	1.0	4.4E+07		53350	222	1.0	2.3E+08	53350	412	1.0	2.2E+08	53350	206	0.967	0.033	2.3E+08	5.4E+06	5.4E+06
5	20	53350	64	1.0	7.4E+07		53350	144	1.0	2.6E+08	53350	190	1.0	2.8E+08	53350	260	0.333	0.667	2.7E+08	1.5E+07	1.5E+07
10	20	53350	52	1.0	1.1E+08		53350	77	1.0	2.8E+08	53350	73	1.0	3.0E+08	53350	108	0.867	0.133	2.7E+08	5.4E+07	5.3E+07
15	20	53350	34	1.0	1.3E+08		53350	62	1.0	2.9E+08	53350	56	1.0	3.0E+08	53350	35	0.6	0.4	2.3E+08	1.7E+08	1.5E+08
20	20	53350	34	1.0	1.6E+08		53350	31	1.0	3.0E+08	53350	27	1.0	2.9E+08	53350	13	0.1	0.9	2.3E+08	3.2E+08	3.5E+08
25	20	53350	18	1.0	1.7E+08		53350	14	1.0	3.1E+08	53350	12	1.0	3.1E+08	53350	10	0.033	0.967	2.8E+08	4.6E+08	4.8E+08

0.18. Shuttle Control

Dimensions: $m = 58000$, $n = 9$.

Description: each entity in the dataset contains several shuttle control attributes.

Table 35: Summary of the results with Shuttle Control ($\times 10^8$)

k	f^*	\bar{f}	HPClust-inner						HPClust-competitive						HPClust-cooperative					
			ε		\bar{t}		t		ε		\bar{t}		t		ε		\bar{t}		t	
			med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std
2	21.34329	19.86153	5.043	12.056	0.724	0.333	0.716	0.364	5.043	1.082	0.22	0.357	1.0	0.34	0.0	0.744	0.093	0.067	0.821	0.37
3	10.85415	10.49161	0.28	29.163	0.532	0.425	0.852	0.398	3.658	1.293	0.418	0.338	1.02	0.48	3.658	1.613	0.418	0.308	0.854	0.311
4	8.8691	8.62423	0.32	4.741	0.816	0.342	0.695	0.397	0.343	7.623	0.793	0.417	0.879	0.447	0.0	0.075	0.307	0.361	0.687	0.446
5	7.24479	7.28912	1.484	7.359	0.017	0.037	0.73	0.456	0.178	7.394	0.034	0.077	0.757	0.403	0.392	0.21	0.033	0.005	0.714	0.429
10	2.83216	2.99551	8.736	21.835	0.148	0.337	0.859	0.422	1.623	2.889	0.082	0.011	0.944	0.438	0.671	0.475	0.081	0.012	0.412	0.399
15	1.53154	1.69671	16.164	8.425	0.053	0.289	0.883	0.411	5.617	2.582	0.149	0.022	0.738	0.363	5.814	2.605	0.146	0.016	1.054	0.427
20	1.06012	1.07621	3.493	7.041	0.181	0.409	0.952	0.41	-0.758	3.626	0.225	0.045	1.102	0.419	-1.494	2.123	0.21	0.079	1.04	0.437
25	0.77978	0.79776	9.944	4.377	0.083	0.0	0.688	0.394	2.84	3.246	0.378	0.361	1.212	0.35	3.339	3.844	0.387	0.311	1.173	0.268
Mean:			5.683		0.319		0.797		2.318		0.287		0.957		1.548		0.209		0.844	

k	f^*	\bar{f}	HPClust-hybrid						Forgy K-means						PBK-BDC					
			ε		\bar{t}		t		ε		\bar{t}		t		ε		\bar{t}		t	
			med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std
2	21.34329	19.86153	1.86	2.186	0.115	0.269	0.916	0.412	51.112	11.489	–	–	0.022	0.007	51.112	0.025	–	–	0.026	0.006
3	10.85415	10.49161	3.658	1.368	0.245	0.183	0.553	0.373	100.557	38.781	–	–	0.07	0.053	100.558	44.75	–	–	0.041	0.039
4	8.8691	8.62423	0.0	7.064	0.204	0.374	0.36	0.463	143.415	59.951	–	–	0.057	0.032	143.415	50.661	–	–	0.043	0.019
5	7.24479	7.28912	0.178	5.719	0.032	0.003	0.767	0.352	38.691	61.282	–	–	0.058	0.018	38.774	47.006	–	–	0.074	0.022
10	2.83216	2.99551	0.692	0.98	0.084	0.008	0.884	0.3	135.103	37.214	–	–	0.137	0.079	135.73	31.725	–	–	0.177	0.07
15	1.53154	1.69671	3.768	2.945	0.145	0.016	1.193	0.401	225.668	38.69	–	–	0.228	0.102	243.615	42.423	–	–	0.213	0.051
20	1.06012	1.07621	0.017	2.21	0.226	0.256	1.054	0.4	324.175	40.507	–	–	0.308	0.078	284.79	24.232	–	–	0.275	0.09
25	0.77978	0.79776	4.719	2.264	0.654	0.328	1.21	0.254	391.295	22.132	–	–	0.674	0.23	396.372	19.782	–	–	0.555	0.232
Mean:			1.862		0.213		0.867		176.252		–		0.194		174.296		–		0.176	

Table 36: Clustering details with Shuttle Control

k	n_{exec}	HPClust-inner				HPClust-competitive				HPClust-cooperative				HPClust-hybrid					Forgy K-means	PBK-BDC
		s	n_s	T	n_d	s	n_s	T	n_d	s	n_s	T	n_d	s	n_s	T_1	T_2	n_d	n_d	n_d
2	15	57950	200	1.5	9.1E+07	57950	1378	1.5	5.2E+08	57950	1169	1.5	5.3E+08	57950	1370	0.75	0.75	5.3E+08	2.8E+06	3.2E+06
3	15	57950	175	1.5	1.1E+08	57950	953	1.5	6.0E+08	57950	1008	1.5	6.3E+08	57950	618	0.65	0.85	6.2E+08	6.1E+06	5.9E+06
4	15	57950	139	1.5	1.4E+08	57950	835	1.5	7.2E+08	57950	681	1.5	6.8E+08	57950	338	0.6	0.9	7.1E+08	7.9E+06	7.2E+06
5	15	57950	145	1.5	1.8E+08	57950	600	1.5	7.6E+08	57950	568	1.5	7.8E+08	57950	643	0.6	0.9	8.0E+08	9.3E+06	8.1E+06
10	15	57950	106	1.5	2.8E+08	57950	432	1.5	9.6E+08	57950	172	1.5	9.2E+08	57950	416	0.1	1.4	9.0E+08	2.8E+07	3.6E+07
15	15	57950	105	1.5	3.8E+08	57950	204	1.5	9.4E+08	57950	310	1.5	1.0E+09	57950	336	1.1	0.4	9.6E+08	4.9E+07	4.5E+07
20	15	57950	80	1.5	4.5E+08	57950	186	1.5	9.8E+08	57950	157	1.5	9.7E+08	57950	146	0.6	0.9	9.7E+08	6.4E+07	5.9E+07
25	15	57950	56	1.5	5.0E+08	57950	157	1.5	9.8E+08	57950	135	1.5	1.0E+09	57950	118	0.35	1.15	9.1E+08	1.0E+08	1.2E+08

0.19. Shuttle Control (normalized)

Dimensions: $m = 58000$, $n = 9$.

Description: each entity in the dataset contains several shuttle control attributes. Min-max scaling was used for normalization of data set values for better clusterization.

Table 37: Summary of the results with Shuttle Control (normalized) ($\times 10^1$)

k	f^*	\bar{f}	HPClust-inner						HPClust-competitive						HPClust-cooperative					
			ε		\bar{t}		t		ε		\bar{t}		t		ε		\bar{t}		t	
			med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std
2	104.41601	3.33677	0.106	0.211	0.244	0.103	0.218	0.101	0.184	0.167	0.068	0.065	0.164	0.103	0.245	0.102	0.112	0.078	0.182	0.122
3	73.28769	2.33445	0.697	0.868	0.132	0.065	0.135	0.129	0.675	0.297	0.029	0.049	0.18	0.111	0.514	0.519	0.039	0.036	0.153	0.1
4	50.076	1.5748	0.781	12.197	0.212	0.116	0.242	0.116	0.675	0.508	0.019	0.03	0.204	0.111	0.46	0.347	0.023	0.026	0.132	0.109
5	39.78043	1.24889	1.301	1.451	0.057	0.068	0.189	0.134	1.224	0.875	0.023	0.02	0.248	0.121	1.679	0.823	0.019	0.013	0.149	0.116
10	15.04997	0.44476	2.315	11.582	0.215	0.147	0.245	0.123	0.824	0.969	0.143	0.1	0.297	0.088	2.23	0.96	0.057	0.114	0.267	0.095
15	9.81804	0.28928	5.001	3.919	0.066	0.094	0.25	0.111	3.02	1.72	0.042	0.018	0.217	0.096	2.906	1.421	0.027	0.025	0.26	0.116
20	7.233	0.19874	6.611	3.444	0.114	0.099	0.243	0.102	2.84	1.499	0.062	0.049	0.244	0.102	4.49	2.5	0.051	0.043	0.245	0.118
25	5.86461	0.15054	5.645	3.749	0.14	0.13	0.207	0.126	4.909	1.212	0.14	0.076	0.255	0.108	5.227	1.257	0.094	0.088	0.246	0.101
Mean:			2.807		0.147		0.216		1.794		0.066		0.226		2.219		0.053		0.204	

k	f^*	\bar{f}	HPClust-hybrid						Forgy K-means						PBK-BDC					
			ε		\bar{t}		t		ε		\bar{t}		t		ε		\bar{t}		t	
			med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std
2	104.41601	3.33677	0.231	0.153	0.148	0.084	0.221	0.124	14.732	10.172	–	–	0.018	0.005	7.998	2.641	–	–	0.006	0.001
3	73.28769	2.33445	0.505	0.279	0.07	0.091	0.305	0.107	1.765	12.847	–	–	0.027	0.007	14.401	16.099	–	–	0.008	0.001
4	50.076	1.5748	0.514	0.415	0.025	0.058	0.208	0.124	0.0	6.86	–	–	0.032	0.006	36.908	23.315	–	–	0.008	0.001
5	39.78043	1.24889	1.166	0.975	0.027	0.04	0.261	0.102	0.826	4.343	–	–	0.061	0.024	18.537	18.539	–	–	0.011	0.002
10	15.04997	0.44476	0.621	1.146	0.091	0.062	0.239	0.088	47.02	19.014	–	–	0.077	0.028	51.611	26.149	–	–	0.018	0.002
15	9.81804	0.28928	2.955	1.34	0.029	0.012	0.284	0.089	21.544	37.916	–	–	0.092	0.061	33.001	42.832	–	–	0.028	0.003
20	7.233	0.19874	2.245	1.949	0.07	0.062	0.332	0.094	22.889	57.432	–	–	0.162	0.079	41.776	47.828	–	–	0.035	0.005
25	5.86461	0.15054	4.658	1.881	0.123	0.078	0.223	0.111	23.942	59.568	–	–	0.237	0.087	51.317	75.254	–	–	0.044	0.005
Mean:			1.612		0.073		0.259		16.59		–		0.088		31.943		–		0.02	

Table 38: Clustering details with Shuttle Control (normalized)

k	n_{exec}	HPClust-inner				HPClust-competitive				HPClust-cooperative				HPClust-hybrid				Forgy K-means	PBK-BDC
		s	n_s	T	n_d	s	n_s	T	n_d	s	n_s	T	n_d	s	n_s	T_1	T_2	n_d	n_d
2	20	2000	98	0.4	1.6E+06	2000	624	0.4	1.3E+07	2000	664	0.4	1.3E+07	2000	806	0.04	0.36	1.3E+07	1.6E+06
3	20	2000	74	0.4	5.0E+06	2000	598	0.4	3.2E+07	2000	541	0.4	3.6E+07	2000	1066	0.28	0.12	3.4E+07	2.2E+06
4	20	2000	92	0.4	5.0E+06	2000	718	0.4	4.3E+07	2000	456	0.4	4.1E+07	2000	686	0.24	0.16	4.1E+07	3.5E+06
5	20	2000	74	0.4	8.4E+06	2000	716	0.4	5.8E+07	2000	468	0.4	5.4E+07	2000	786	0.187	0.213	5.8E+07	5.9E+06
10	20	2000	74	0.4	1.3E+07	2000	718	0.4	1.0E+08	2000	676	0.4	1.0E+08	2000	595	0.16	0.24	1.0E+08	1.1E+07
15	20	2000	70	0.4	2.6E+07	2000	358	0.4	1.5E+08	2000	436	0.4	1.4E+08	2000	493	0.32	0.08	1.5E+08	1.8E+07
20	20	2000	62	0.4	3.7E+07	2000	295	0.4	1.8E+08	2000	310	0.4	1.7E+08	2000	414	0.253	0.147	1.7E+08	2.6E+07
25	20	2000	50	0.4	4.3E+07	2000	255	0.4	1.9E+08	2000	249	0.4	1.9E+08	2000	237	0.08	0.32	1.9E+08	5.2E+07

0.20. EEG Eye State

Dimensions: $m = 14980, n = 14$.

Description: the data set consists of 14 electroencephalogram (EEG) values for predicting the corresponding eye state.

Table 39: Summary of the results with EEG Eye State ($\times 10^8$)

k	f^*	\bar{f}	HPClust-inner						HPClust-competitive						HPClust-cooperative					
			ε		\bar{t}		t		ε		\bar{t}		t		ε		\bar{t}		t	
			med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std
2	7845.09934	8178.13658	4.245	4.728	0.661	0.358	0.719	0.359	4.247	0.002	0.411	0.21	0.706	0.374	4.246	0.002	0.183	0.199	0.995	0.37
3	1833.88058	1833.87892	0.0	0.003	0.486	0.355	0.638	0.347	0.0	0.003	0.246	0.254	0.855	0.379	0.0	0.003	0.42	0.313	0.755	0.375
4	2.23605	2.23431	0.0	0.001	0.629	0.383	0.678	0.428	0.002	0.001	0.352	0.307	0.563	0.474	0.0	0.001	0.206	0.276	0.615	0.363
5	1.33858	1.33703	-0.0	14.651	0.669	0.35	0.508	0.339	-0.0	120196.81	0.276	0.354	0.583	0.433	-0.0	0.0	0.1	0.221	0.668	0.481
10	0.4531	0.4527	0.001	0.554	0.679	0.363	0.865	0.366	-0.004	88058.848	0.612	0.347	1.088	0.469	-0.005	0.005	0.397	0.307	0.95	0.406
15	0.34653	0.34837	0.622	0.502	0.032	0.015	0.498	0.425	0.055	0.143	0.113	0.037	1.079	0.324	0.135	0.126	0.111	0.19	0.857	0.351
20	0.28986	0.29175	0.785	0.717	0.064	0.367	0.98	0.345	0.02	0.133	0.216	0.055	1.089	0.321	0.06	0.205	0.193	0.063	0.887	0.39
25	0.25989	0.26088	0.636	0.604	0.204	0.274	1.099	0.377	0.156	0.095	0.222	0.049	0.87	0.353	0.137	0.082	0.225	0.042	0.874	0.303
Mean:			0.786		0.428		0.748		0.559		0.306		0.854		0.571		0.229		0.825	

k	f^*	\bar{f}	HPClust-hybrid						Forgy K-means						PBK-BDC					
			ε		\bar{t}		t		ε		\bar{t}		t		ε		\bar{t}		t	
			med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std
2	7845.09934	8178.13658	4.247	3.973	0.179	0.249	0.842	0.435	-0.0	1.274	–	–	0.003	0.0	-0.0	16.348	–	–	0.004	0.001
3	1833.88058	1833.87892	0.0	0.003	0.245	0.27	0.773	0.42	227.909	98.687	–	–	0.004	0.001	227.909	49.672	–	–	0.005	0.001
4	2.23605	2.23431	0.0	0.001	0.151	0.218	0.798	0.466	268809.803	133731.189	–	–	0.021	0.009	268809.803	128214.104	–	–	0.019	0.007
5	1.33858	1.33703	-0.0	6.519	0.161	0.213	1.077	0.422	449091.754	223405.448	–	–	0.029	0.006	449091.754	205786.243	–	–	0.028	0.005
10	0.4531	0.4527	-0.002	0.006	0.561	0.31	0.96	0.32	1326681.022	632723.737	–	–	0.074	0.035	1326681.023	607938.794	–	–	0.079	0.019
15	0.34653	0.34837	0.058	0.097	0.103	0.051	1.077	0.347	1734685.672	751140.757	–	–	0.197	0.06	1.077	849818.798	–	–	0.145	0.054
20	0.28986	0.29175	0.025	0.031	0.192	0.055	1.105	0.372	2073832.95	989155.199	–	–	0.375	0.117	2073833.29	989155.218	–	–	0.448	0.143
25	0.25989	0.26088	0.109	0.13	0.202	0.07	1.211	0.312	1156493.228	1156492.007	–	–	0.371	0.117	2312984.49	1059942.517	–	–	0.358	0.088
Mean:			0.555		0.224		0.98		876227.792		–		0.134		803953.668		–		0.136	

Table 40: Clustering details with EEG Eye State

k	n_{exec}	HPClust-inner				HPClust-competitive				HPClust-cooperative				HPClust-hybrid				Forgy K-means	PBK-BDC
		s	n_s	T	n_d	s	n_s	T	n_d	s	n_s	T	n_d	s	n_s	T_1	T_2	n_d	n_d
2	20	14979	414	1.5	5.8E+07	14979	2898	1.5	4.0E+08	14979	4582	1.5	4.0E+08	14979	4036	0.7	0.8	4.2E+08	1.5E+05
3	20	14979	368	1.5	8.7E+07	14979	3046	1.5	4.9E+08	14979	2518	1.5	4.8E+08	14979	2674	0.05	1.45	5.0E+08	3.6E+05
4	20	14979	342	1.5	9.9E+07	14979	1678	1.5	5.7E+08	14979	1809	1.5	5.6E+08	14979	2296	0.3	1.2	5.5E+08	1.5E+06
5	20	14979	266	1.5	1.2E+08	14979	1426	1.5	5.7E+08	14979	1598	1.5	5.8E+08	14979	2723	0.85	0.65	5.8E+08	3.3E+06
10	20	14979	308	1.5	1.5E+08	14979	1563	1.5	6.9E+08	14979	1400	1.5	7.0E+08	14979	1357	0.85	0.65	6.9E+08	9.2E+06
15	20	14979	174	1.5	2.6E+08	14979	935	1.5	7.2E+08	14979	742	1.5	7.0E+08	14979	996	0.35	1.15	7.3E+08	2.4E+07
20	20	14979	298	1.5	3.3E+08	14979	678	1.5	7.5E+08	14979	558	1.5	7.2E+08	14979	753	1.4	0.1	7.4E+08	3.5E+07
25	20	14979	286	1.5	3.7E+08	14979	370	1.5	7.3E+08	14979	408	1.5	7.4E+08	14979	490	0.2	1.3	6.7E+08	5.0E+07

0.21. EEG Eye State (normalized)

Dimensions: $m = 14980, n = 14$.

Description: the data set consists of 14 electroencephalogram (EEG) values for predicting the corresponding eye state. Min-max scaling was used for normalization of data set values for better clusterization.

Table 41: Summary of the results with EEG Eye State (normalized) ($\times 10^1$)

k	f^*	\bar{f}	HPClust-inner						HPClust-competitive						HPClust-cooperative					
			ε		\bar{t}		t		ε		\bar{t}		t		ε		\bar{t}		t	
			med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std
2	1.15267	1.15216	6.104	9.638	0.258	0.307	0.574	0.311	0.002	8.681	0.193	0.199	0.581	0.287	0.002	0.001	0.139	0.152	0.599	0.274
3	0.82423	0.87097	5.716	13.325	0.005	0.278	0.551	0.267	0.001	9.655	0.009	0.005	0.331	0.29	0.001	1.026	0.009	0.008	0.482	0.278
4	0.5429	0.57038	5.15	14.019	0.005	0.192	0.507	0.297	0.001	10.29	0.012	0.001	0.438	0.259	0.001	0.001	0.011	0.001	0.549	0.301
5	0.28952	0.28903	0.002	33.997	0.413	0.331	0.504	0.315	0.002	15.033	0.161	0.163	0.339	0.255	0.002	0.0	0.195	0.186	0.472	0.281
10	0.10269	0.10335	0.707	0.479	0.029	0.19	0.601	0.303	-0.003	67.68	0.064	0.015	0.449	0.294	-0.004	0.126	0.059	0.014	0.671	0.29
15	0.07469	0.07479	0.2	0.789	0.05	0.24	0.606	0.254	0.036	0.053	0.134	0.049	0.712	0.209	0.052	0.066	0.139	0.045	0.495	0.276
20	0.06125	0.06154	0.457	0.629	0.059	0.077	0.566	0.313	0.177	0.146	0.166	0.06	0.654	0.222	0.205	0.167	0.166	0.054	0.623	0.211
25	0.05385	0.0543	0.873	0.774	0.065	0.186	0.575	0.267	-0.154	0.152	0.224	0.048	0.777	0.192	-0.151	80.458	0.201	0.044	0.637	0.196
Mean:			2.401		0.11		0.56		0.008		0.12		0.535		0.014		0.115		0.566	

k	f^*	\bar{f}	HPClust-hybrid						Forgy K-means						PBK-BDC					
			ε		\bar{t}		t		ε		\bar{t}		t		ε		\bar{t}		t	
			med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std
2	1.15267	1.15216	0.001	2.823	0.258	0.256	0.578	0.275	25.398	0.011	–	–	0.016	0.003	25.398	0.0	–	–	0.018	0.003
3	0.82423	0.87097	0.001	7.024	0.009	0.001	0.435	0.25	69.038	0.035	–	–	0.017	0.004	69.038	4.986	–	–	0.015	0.003
4	0.5429	0.57038	0.001	6.519	0.011	0.001	0.56	0.248	152.474	0.048	–	–	0.022	0.007	152.479	0.049	–	–	0.02	0.007
5	0.28952	0.28903	0.002	13.447	0.295	0.235	0.561	0.315	367.097	32.271	–	–	0.033	0.011	367.097	24.212	–	–	0.036	0.009
10	0.10269	0.10335	-0.004	0.131	0.063	0.051	0.789	0.265	633.846	193.064	–	–	0.116	0.037	879.525	132.438	–	–	0.098	0.038
15	0.07469	0.07479	0.037	0.068	0.138	0.146	0.688	0.194	853.035	256.91	–	–	0.179	0.105	853.015	297.335	–	–	0.154	0.062
20	0.06125	0.06154	0.226	0.146	0.176	0.034	0.781	0.239	1044.241	312.789	–	–	0.301	0.151	1044.477	285.929	–	–	0.242	0.092
25	0.05385	0.0543	-0.122	0.197	0.21	0.047	0.925	0.236	1190.906	385.599	–	–	0.44	0.156	1190.787	365.14	–	–	0.303	0.119
Mean:			0.018		0.145		0.665		542.004		–		0.141		572.727		–		0.111	

Table 42: Clustering details with EEG Eye State (normalized)

k	n_{exec}	HPClust-inner				HPClust-competitive				HPClust-cooperative				HPClust-hybrid					Forgy K-means	PBK-BDC
		s	n_s	T	n_d	s	n_s	T	n_d	s	n_s	T	n_d	s	n_s	T_1	T_2	n_d	n_d	n_d
2	30	14979	337	1.0	3.6E+07	14979	2470	1.0	2.6E+08	14979	2653	1.0	2.6E+08	14979	2379	0.233	0.767	2.6E+08	9.4E+05	1.0E+06
3	30	14979	314	1.0	5.2E+07	14979	1104	1.0	3.1E+08	14979	1702	1.0	3.1E+08	14979	1378	0.533	0.467	3.0E+08	1.3E+06	1.5E+06
4	30	14979	252	1.0	7.2E+07	14979	1234	1.0	3.5E+08	14979	1438	1.0	3.5E+08	14979	1590	0.933	0.067	3.4E+08	2.3E+06	2.0E+06
5	30	14979	224	1.0	8.3E+07	14979	770	1.0	3.7E+08	14979	1136	1.0	3.8E+08	14979	1370	0.733	0.267	3.7E+08	3.7E+06	3.9E+06
10	30	14979	192	1.0	1.2E+08	14979	560	1.0	4.3E+08	14979	862	1.0	4.3E+08	14979	1040	0.5	0.5	4.3E+08	1.4E+07	1.3E+07
15	30	14979	166	1.0	1.7E+08	14979	602	1.0	4.6E+08	14979	358	1.0	4.5E+08	14979	531	0.333	0.667	4.7E+08	2.0E+07	2.0E+07
20	30	14979	152	1.0	2.0E+08	14979	361	1.0	4.6E+08	14979	318	1.0	4.6E+08	14979	442	0.733	0.267	4.7E+08	3.1E+07	3.3E+07
25	30	14979	174	1.0	2.5E+08	14979	282	1.0	4.6E+08	14979	228	1.0	4.7E+08	14979	415	0.933	0.067	4.7E+08	4.3E+07	4.1E+07

0.22. Pla85900

Dimensions: $m = 85900$, $n = 2$.

Description: a data set contains cities coordinates for traveling salesman problem.

Table 43: Summary of the results with Pla85900 ($\times 10^{15}$)

k	f^*	\bar{f}	HPClust-inner						HPClust-competitive						HPClust-cooperative					
			ε		\bar{t}		t		ε		\bar{t}		t		ε		\bar{t}		t	
			med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std
2	3.74908	0.60031	0.054	0.718	0.59	0.405	0.776	0.44	0.013	0.021	0.19	0.276	0.603	0.429	0.011	0.224	0.108	0.17	0.73	0.44
3	2.28057	0.36407	0.026	0.036	0.809	0.508	0.941	0.444	0.024	0.021	0.259	0.307	0.552	0.428	0.025	0.032	0.274	0.233	0.735	0.391
5	1.33972	0.21512	0.09	0.751	0.292	0.34	0.821	0.438	0.046	0.029	0.099	0.083	0.683	0.42	0.051	0.305	0.051	0.331	0.718	0.427
10	0.68294	0.10944	0.587	0.371	0.85	0.459	0.802	0.468	0.111	0.148	0.221	0.297	0.923	0.48	0.151	0.317	0.146	0.274	0.844	0.433
15	0.46029	0.07355	0.291	0.476	0.557	0.504	0.919	0.433	0.251	0.153	0.539	0.349	0.769	0.36	0.268	0.183	0.335	0.398	0.979	0.438
20	0.34988	0.05595	0.656	0.422	0.545	0.379	0.833	0.421	0.316	0.254	0.507	0.36	0.971	0.411	0.304	0.338	0.3	0.191	0.655	0.38
25	0.28259	0.04518	0.89	0.318	0.853	0.383	0.884	0.373	0.617	0.281	0.618	0.392	0.806	0.408	0.763	0.455	0.432	0.304	0.711	0.402
Mean:			0.371		0.642		0.854		0.197		0.348		0.758		0.225		0.235		0.767	

k	f^*	\bar{f}	HPClust-hybrid						Forgy K-means						PBK-BDC					
			ε		\bar{t}		t		ε		\bar{t}		t		ε		\bar{t}		t	
			med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std
2	3.74908	0.60031	0.016	0.014	0.139	0.224	0.988	0.428	0.0	0.686	–	–	0.024	0.007	6.458	2.618	–	–	0.01	0.002
3	2.28057	0.36407	0.024	0.029	0.287	0.353	0.671	0.41	0.0	0.0	–	–	0.078	0.025	0.005	20.855	–	–	0.022	0.006
5	1.33972	0.21512	0.036	0.027	0.091	0.089	1.106	0.456	0.407	1.133	–	–	0.082	0.06	6.719	5.906	–	–	0.027	0.012
10	0.68294	0.10944	0.136	0.186	0.273	0.318	1.022	0.348	0.42	0.774	–	–	0.201	0.087	14.084	10.514	–	–	0.067	0.017
15	0.46029	0.07355	0.226	0.143	0.555	0.281	1.004	0.337	0.495	0.806	–	–	0.313	0.156	17.409	9.89	–	–	0.098	0.022
20	0.34988	0.05595	0.331	0.126	0.469	0.305	0.728	0.394	0.45	0.601	–	–	0.453	0.213	15.152	8.883	–	–	0.125	0.043
25	0.28259	0.04518	0.618	0.299	0.645	0.359	0.996	0.369	0.932	0.495	–	–	0.697	0.229	13.672	7.007	–	–	0.163	0.038
Mean:			0.198		0.351		0.931		0.386		–		0.264		10.5		–		0.073	

Table 44: Clustering details with Pla85900

k	n_{exec}	HPClust-inner				HPClust-competitive				HPClust-cooperative				HPClust-hybrid					Forgy K-means	PBK-BDC
		s	n_s	T	n_d	s	n_s	T	n_d	s	n_s	T	n_d	s	n_s	T_1	T_2	n_d	n_d	n_d
2	40	14000	237	1.5	4.4E+07	14000	1470	1.5	3.4E+08	14000	1718	1.5	3.2E+08	14000	2442	1.0	0.5	3.4E+08	5.1E+06	4.1E+06
3	40	14000	240	1.5	8.1E+07	14000	1190	1.5	5.9E+08	14000	1476	1.5	6.0E+08	14000	1413	1.4	0.1	6.0E+08	1.6E+07	1.1E+07
5	40	14000	217	1.5	1.4E+08	14000	1238	1.5	9.3E+08	14000	1272	1.5	9.1E+08	14000	1943	1.05	0.45	9.1E+08	1.9E+07	1.6E+07
10	40	14000	186	1.5	3.7E+08	14000	1000	1.5	1.7E+09	14000	907	1.5	1.7E+09	14000	1098	1.2	0.3	1.7E+09	7.6E+07	5.8E+07
15	40	14000	159	1.5	5.6E+08	14000	498	1.5	2.4E+09	14000	748	1.5	2.3E+09	14000	728	0.6	0.9	2.3E+09	1.4E+08	1.0E+08
20	40	14000	117	1.5	7.6E+08	14000	482	1.5	2.7E+09	14000	336	1.5	2.7E+09	14000	359	1.4	0.1	2.7E+09	2.2E+08	1.4E+08
25	40	14000	110	1.5	9.3E+08	14000	270	1.5	2.8E+09	14000	264	1.5	2.9E+09	14000	324	1.15	0.35	2.8E+09	3.5E+08	1.9E+08

0.23. D15112

Dimensions: $m = 15112, n = 2$.

Description: a data set with German cities coordinates for travelling salesman problem.

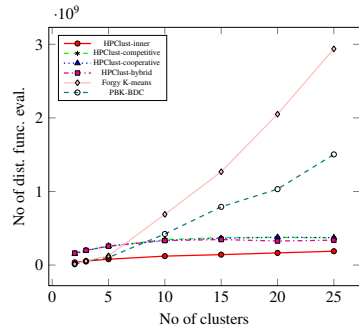
Table 45: Summary of the results with D15112 ($\times 10^{11}$)

k	f^*	\bar{f}	HPClust-inner						HPClust-competitive						HPClust-cooperative					
			ε		\bar{t}		t		ε		\bar{t}		t		ε		\bar{t}		t	
			med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std
2	3.68403	1.91227	0.011	0.012	0.84	0.495	0.84	0.44	0.014	0.014	0.218	0.268	0.602	0.222	0.02	0.023	0.258	0.231	0.745	0.256
3	2.5324	1.30699	0.021	0.023	0.289	0.54	1.009	0.425	0.023	0.019	0.277	0.238	0.45	0.299	0.036	0.027	0.205	0.166	0.619	0.379
5	1.32707	0.68683	0.041	0.023	0.507	0.402	0.907	0.427	0.034	0.022	0.07	0.123	0.801	0.399	0.045	0.02	0.178	0.159	0.777	0.329
10	0.64491	0.33574	0.734	1.319	0.495	0.411	0.649	0.458	0.118	0.145	0.104	0.205	0.973	0.434	0.098	0.278	0.15	0.356	1.158	0.315
15	0.43136	0.22393	0.776	0.79	0.205	0.127	0.546	0.389	0.235	0.091	0.247	0.194	0.365	0.369	0.309	0.2	0.163	0.323	0.596	0.444
20	0.32177	0.16878	0.888	0.619	0.214	0.171	0.558	0.449	0.28	0.144	0.098	0.081	1.023	0.398	0.626	0.497	0.063	0.041	0.623	0.4
25	0.25308	0.13159	0.868	0.851	0.396	0.516	0.623	0.409	0.306	0.206	0.487	0.257	0.626	0.338	0.867	0.432	0.675	0.342	0.945	0.361
Mean:			0.477		0.421		0.733		0.144		0.214		0.691		0.286		0.242		0.78	

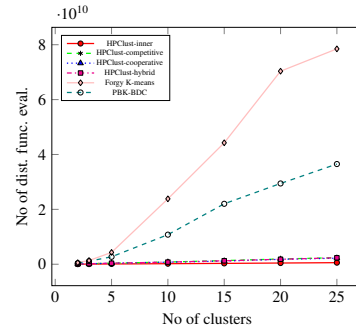
k	f^*	\bar{f}	HPClust-hybrid						Forgy K-means						PBK-BDC					
			ε		\bar{t}		t		ε		\bar{t}		t		ε		\bar{t}		t	
			med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std	med	std
2	3.68403	1.91227	0.021	0.012	0.293	0.276	0.909	0.428	0.0	0.0	–	–	0.003	0.0	0.013	0.008	–	–	0.003	0.0
3	2.5324	1.30699	0.03	0.04	0.342	0.24	0.799	0.341	0.001	0.0	–	–	0.007	0.002	0.038	0.084	–	–	0.004	0.001
5	1.32707	0.68683	0.052	0.029	0.254	0.22	1.129	0.346	-0.0	7.357	–	–	0.005	0.002	0.048	4.148	–	–	0.004	0.001
10	0.64491	0.33574	0.1	0.033	0.126	0.214	1.1	0.296	1.411	1.559	–	–	0.032	0.02	0.955	1.46	–	–	0.018	0.006
15	0.43136	0.22393	0.283	0.147	0.381	0.372	0.663	0.467	2.788	1.452	–	–	0.045	0.013	2.639	1.792	–	–	0.015	0.005
20	0.32177	0.16878	0.3	0.155	0.071	0.049	0.818	0.377	1.635	2.513	–	–	0.05	0.014	3.321	2.902	–	–	0.019	0.006
25	0.25308	0.13159	0.297	0.339	0.226	0.293	0.91	0.432	2.208	1.762	–	–	0.093	0.037	2.838	1.386	–	–	0.04	0.012
Mean:			0.155		0.242		0.904		1.149		–		0.033		1.407		–		0.015	

Table 46: Clustering details with D15112

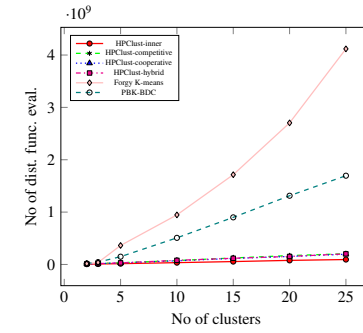
k	n_{exec}	HPClust-inner				HPClust-competitive				HPClust-cooperative				HPClust-hybrid					Forgy K-means	PBK-BDC
		s	n_s	T	n_d	s	n_s	T	n_d	s	n_s	T	n_d	s	n_s	T_1	T_2	n_d	n_d	n_d
2	15	8000	1083	1.5	8.7E+07	8000	6286	1.5	4.6E+08	8000	7270	1.5	4.7E+08	8000	9539	0.95	0.55	7.3E+08	4.8E+05	2.4E+05
3	15	8000	1184	1.5	1.5E+08	8000	3347	1.5	8.5E+08	8000	4976	1.5	8.3E+08	8000	6275	0.7	0.8	1.1E+09	1.9E+06	9.3E+05
5	15	8000	759	1.5	2.0E+08	8000	5774	1.5	1.3E+09	8000	5444	1.5	1.4E+09	8000	7672	0.7	0.8	1.4E+09	1.5E+06	9.2E+05
10	15	8000	392	1.5	4.5E+08	8000	3398	1.5	2.3E+09	8000	3959	1.5	2.3E+09	8000	3865	0.9	0.6	2.3E+09	8.9E+06	3.8E+06
15	15	8000	304	1.5	6.2E+08	8000	622	1.5	2.7E+09	8000	1352	1.5	2.9E+09	8000	1290	1.35	0.15	2.6E+09	1.5E+07	6.9E+06
20	15	8000	231	1.5	8.9E+08	8000	1400	1.5	2.9E+09	8000	1015	1.5	3.1E+09	8000	1087	1.35	0.15	3.0E+09	2.5E+07	9.6E+06
25	15	8000	207	1.5	9.7E+08	8000	615	1.5	3.0E+09	8000	1099	1.5	3.1E+09	8000	964	0.85	0.65	3.1E+09	2.6E+07	1.3E+07



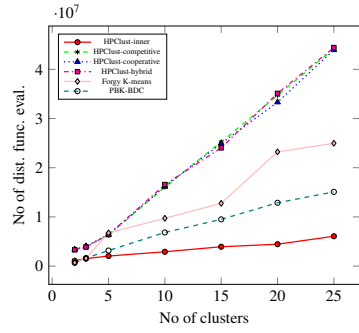
(a) CORD-19 Embeddings



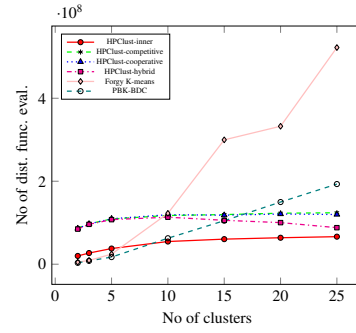
(b) HEPMASS



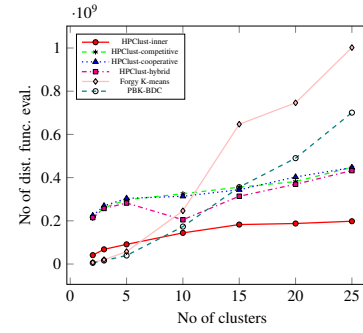
(c) US Census Data 1990



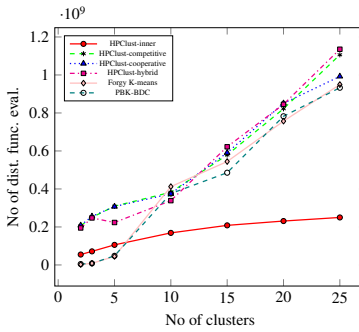
(d) Gisette



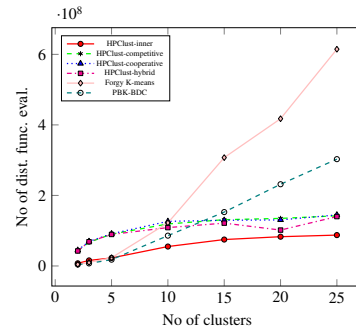
(e) Music Analysis



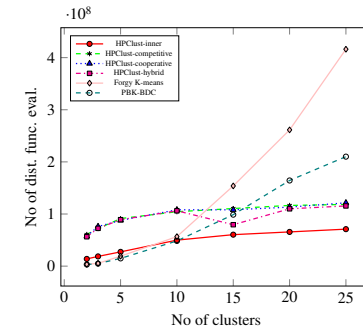
(f) Protein Homology



(g) MiniBooNE Particle Identification

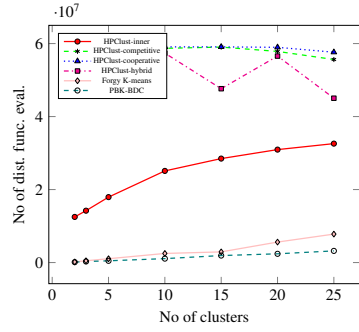


(h) MiniBooNE Particle Identification (normalized)

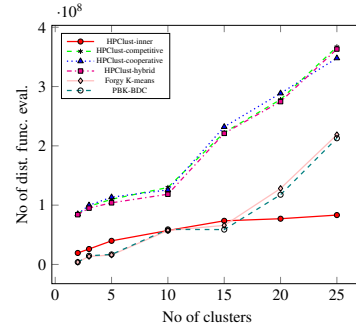


(i) MFCCs for Speech Emotion Recognition

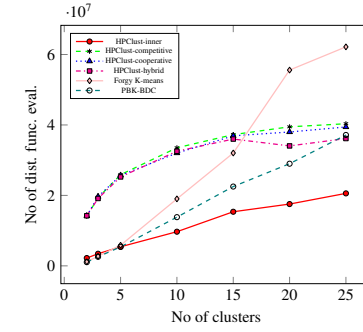
Figure 1: Number of distance evaluations, 1



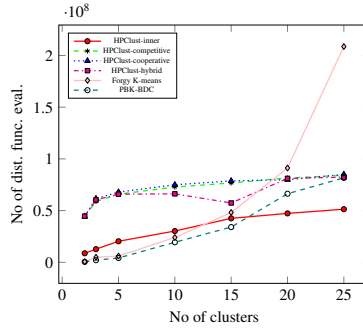
(a) ISOLET



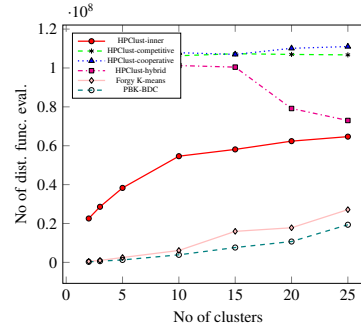
(b) Sensorless Drive Diagnosis



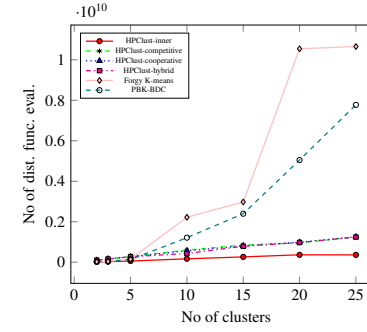
(c) Sensorless Drive Diagnosis (normalized)



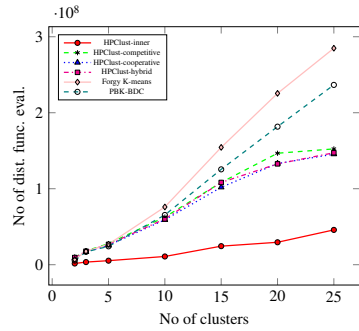
(d) Online News Popularity



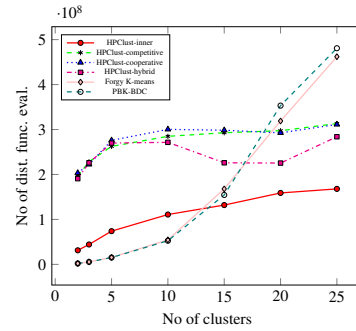
(e) Gas Sensor Array Drift



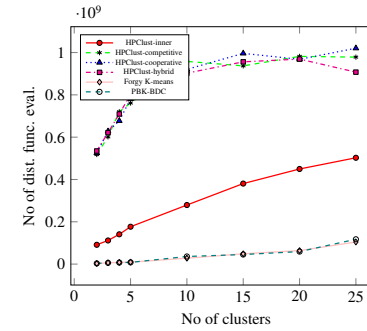
(f) 3D Road Network



(g) Skin Segmentation

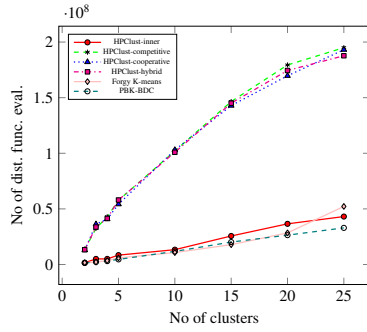


(h) KEGG Metabolic Relation Network (Directed)

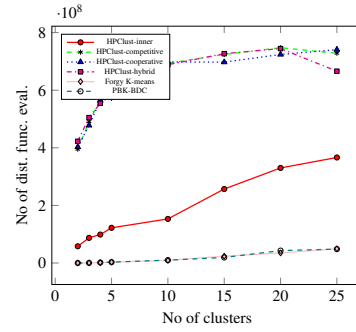


(i) Shuttle Control

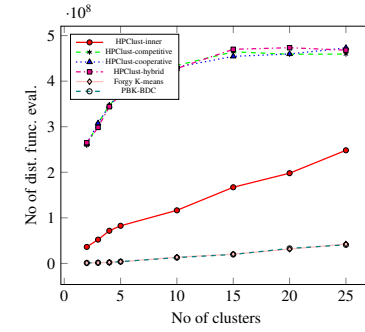
Figure 2: Number of distance evaluations, 2



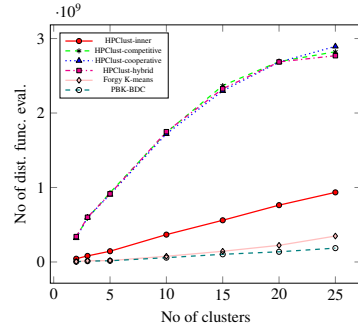
(a) Shuttle Control (normalized)



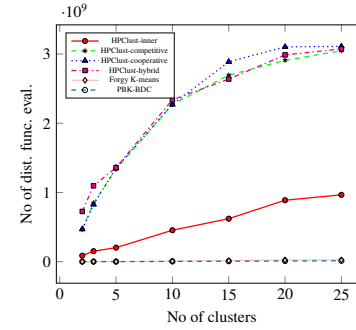
(b) EEG Eye State



(c) EEG Eye State (normalized)



(d) Pla85900



(e) D15112

Figure 3: Number of distance evaluations, 3