mathematics

Article

Hybrid Fuzzy Method for Performance Evaluation of City Construction

Chun-Ming Yang [1], Chang-Hsien Hsu [2,*], Tian Chen [3] and Shiyao Li [1,4]

[1] School of Economics and Management, Dongguan University of Technology, Dongguan 523808, China; 2020812@dgut.edu.cn or vicyang0706@gmail.com (C.-M.Y.); 15007214200@163.com (S.L.)
[2] Department of Business Administration, Asia University, Taichung 413305, Taiwan
[3] School of Marxism, Dongguan University of Technology, Dongguan 523808, China; chentian@dgut.edu.cn
[4] School of Management and Economics, Kunming University of Science and Technology, Kunming 650093, China
* Correspondence: pci@asia.edu.tw

Abstract: Evaluating the performance of city construction not only helps optimize city functions and improve city quality, but it also contributes to the development of sustainable cities. However, most of the scoring rules for evaluating the performance of city construction are overly cumbersome and demand very high data integrity. Moreover, the properties, change scale, and scope of different evaluation indicators of city construction often lead to uncertain and ambiguous results. In this study, a hybrid fuzzy method is proposed to conduct the performance evaluation of city construction in two phases. Firstly, a city performance index (CPI) was developed by combining the means and standard deviations of indicators of city construction to address the volatility of historical statistical data as well as different types of data. Considering the sampling errors in data analysis, the parameter estimation method was used to derive the $100\% \times (1-\alpha)$ confidence interval of the CPI. Buckley's fuzzy approach was then adopted to extend the statistical estimators from the CPI into fuzzy estimators, after which a fuzzy CPI was proposed. To identify the specific improvement directions for city construction, the fuzzy axiom design (fuzzy AD) method was applied to explore the relationship between the targets set by city managers and actual performance. Finally, an example of six cities in China is provided to illustrate the effectiveness and practicality of the proposed method. The results show that the performance of Chongqing on several evaluation indicators is lower than that of other cities. The proposed method takes into account the issues of uniformity and diversity in the performance evaluation of city construction. It can enable a quantitative assessment of the city construction level in all cities and provide theoretical support and a decision-making basis for relevant government departments to optimize city construction planning and scientifically formulate city construction policies.

Keywords: fuzzy method; parameter estimation; confidence interval; fuzzy axiom design; city; performance evaluation

MSC: 60A86; 62P25; 62C86

check for updates

Citation: Yang, C.-M.; Hsu, C.-H.; Chen, T.; Li, S. Hybrid Fuzzy Method for Performance Evaluation of City Construction. *Mathematics* **2024**, *12*, 2792. https://doi.org/10.3390/math12172792

Academic Editor: Aleksandar Aleksić

Received: 16 July 2024
Revised: 5 September 2024
Accepted: 6 September 2024
Published: 9 September 2024

1. Introduction

With the rapid development and progress of technology and the increasing demand for high-quality living environments, planning and constructing various modern and distinctive cities based on natural environments and socio-economic conditions have become important directions for governments worldwide by which to promote sustainable city development. However, the exponential growth in urbanization has exacerbated issues such as environmental pollution, resource shortages, and traffic congestion, placing enormous pressure on city governance [1,2]. To address this issue, many countries have implemented various enhancement and improvement measures. For example, the UK has established an Ultra-Low Emission Zone in London [3]. South Korea designated the Gyeongpo area in Gangwon Province as a pilot project site for low-carbon green city construction [4]. Denmark implemented a set of policies and actions to promote low-carbon city construction in Copenhagen [5]. China launched a low-carbon pilot city construction program in 2010, with 6 provinces and 81 cities currently participating in the initiative [6].

Improving the level of city construction and governance can help to integrate city land resources and remove abandoned facilities, houses, factories, and stations that are detrimental to city development [7]. Inefficient city construction disrupts normal on-road vehicle traffic flow and increases travel costs, as well as affects air pollution levels in adjacent areas [8,9]. High-quality city construction plays an important role in sustainable development [10]. Effective city planning and management are crucial for promoting the economic growth of the city as well as mitigating the energy demand for cooling or heating and improving carbon efficiency [11,12]. Several studies have employed various methods to explore the performance of city construction. Li et al. [13] investigated the impact of low-carbon city construction on public transportation using a progressive difference-in-difference method based on data from 265 Chinese cities from 2004 to 2016. Yang et al. [14] analyzed the effect of low-carbon city construction on air quality using difference-in-difference and panel threshold models. Through the networked and intelligent upgrading and transformation of city infrastructure, the transformation of city management can be accelerated, thereby enhancing the level of city construction and operational efficiency. Indeed, smart city construction promotes city informatization and the construction of intelligent infrastructure in addition to facilitating city green development. Based on their study of 15 smart cities in the UK, Yigitcanlar and Kamruzzaman [15] found that the construction of smart cities can effectively reduce carbon emissions. Mao et al. [16] constructed an evaluation index system for the level of smart city construction using multivariate data and evaluated the efficiency of the construction of 37 Chinese smart cities from 2016 to 2021 using the super-efficiency slack-based model considering undesired output and the Malmquist index. Wang [17] used the heterogeneity-robust estimator in a differences-in-differences model to investigate whether smart city construction could expand green space. Zhang et al. [18] used a difference-in-differences model to evaluate the impact of information consumption policies in demonstration cities on CO_2 and SO_2 emissions based on panel data from 204 Chinese cities from 2010 to 2019.

In addition to focusing on low-carbon green and smart city construction, López-Ruiz et al. [19] proposed a new methodology based on intellectual capital to measure the growth capabilities of knowledge city construction by ranking 158 European cities using 73 different indicators. AlKheder et al. [20] evaluated 17 incidents of social disputes in city construction projects and 12 factors of social risk in Kuwait using social risk analysis theory. Zou et al. [21] studied the impact of China's high-speed rail construction on the construction of resource-based cities from both temporal and spatial dimensions using geographic information system technology and spatial Durbin models. Chen et al. [22] explored the effectiveness of China's new energy demonstration city construction using a variety of models, such as the synthetic control method, propensity score matching difference-in-difference model, and spatial Durbin difference-in-difference model. Wu et al. [23] proposed a novel evaluation model with 36 indicators following their study on city-scale construction and the effectiveness of managing demolition waste in the Guangdong–Hong Kong–Macao Greater Bay Area of China using an analytic hierarchy process.

Performance refers to the input–output relationship of an organization or system in achieving its goals [24]. Performance evaluation aims to assess this relationship and provide a basis for improving performance levels, in which good performance indicates achieving higher output with lower inputs [25]. Performance evaluation has been widely applied to assess city construction projects [26]. Evaluating and grading the performance of city construction through quantitative indicators provides managers with crucial information to understand the effectiveness of city construction projects. Therefore, the performance evaluation of city construction refers to evaluating the "input–output" relationship during the city construction process and determining whether the construction outcomes achieve the expected construction goals.

Although evaluating the performance of city construction is a very novel topic, it is affected by multiple indications with diversity, multiplicity, and correlation [16]. First, the properties, change scale, and scope of different indicators often lead to uncertain and erroneous results [27]. Second, the performance evaluation of city construction is highly comprehensive because of close interactions with environmental factors, thus limiting the availability of reliable information [19]. Third, despite the fact that the field of city construction has provided a range of, albeit limited, methods attempting to evaluate, monitor, and measure the performance of city construction while effectively controlling the decision-making process of city construction, most of the scoring rules are overly cumbersome and demand very high data integrity [23]; this poses significant challenges for cities with incomplete data [28]. Fourth, data for the indicators of city construction evaluation typically come from cross-sectional or panel databases and exhibit high levels of uncertainty and fuzziness, making them challenging to address with traditional mathematical models [18,29]. Fifth, evaluating city construction performance is crucial for sustainable city development. Scientifically and accurately assessing the level of construction performance as well as understanding the gap between city construction performance and target performance can assist in not only identifying shortcomings in city management but also planning future improvement directions [26]. Sixth, no studies have combined fuzzy AD for the performance evaluation of city construction.

Researchers have attempted to use traditional crisp measures to effectively convey performance evaluations, but these approaches can be challenging [30]; therefore, the application of fuzzy techniques is preferred to express preferences and/or evaluations in fuzzy environments [31]. The contribution of this study lies in the two-phase pre-assessment, which uses a hybrid fuzzy method for the performance evaluation of city construction. First, we consider the fuzziness and uncertainty of data, extend crisp values into fuzzy numbers, and propose a fuzzy CPI. By combining the fuzzy AD method, the fuzzy CPI measures the performance of cities in various evaluation indicators against their targets using probability ratios. This approach not only promotes a more scientific evaluation of city construction performance but also simplifies and facilitates the decision-making activities of city managers and policymakers. The main contributions are summarized as follows:

- This study proposes a fuzzy CPI for conducting structured performance evaluations of city construction based on limited data and fuzzy data, which helps government departments take precise measures to address the shortcomings of city construction.
- This study considers various types of indicators involved in the performance evaluation of city construction and discusses how changes in these indicators impact city construction.
- Considering the limited resources and optimization, this study uses fuzzy AD to conduct a computational analysis of city construction performance. This not only provides a new perspective for evaluating city construction performance but also extends and deepens the findings of previous research.
- Finally, based on empirical analysis, this study provides powerful recommendations for city construction across different countries and regions.

The remainder of this paper is organized as follows: Section 2 elaborates on the fuzzy CPI. Section 3 presents the evaluation level of city construction performance based on the fuzzy AD and fuzzy CPI. Section 4 proposes a procedure for the performance evaluation of city construction. Moreover, an example is provided for examining the applicability of our approach. Conclusions and recommendations are drawn in Section 5.

2. Fuzzy City Performance Index

Most data on the evaluation indicators of city construction typically come from official or industry institution statistical databases. To avoid the impact of the variability from the mean or average in the dataset for the performance evaluation of city construction, this study employed the standard deviation as well as the mean, which took into account the volatility and dispersion of the data, to reflect the dynamic changes in the city. Because uncertainty and sample errors are inevitable when sampling from a population, fuzzy numbers provide a more flexible and objective measurement method than crisp numbers and can better reflect actual conditions. In this study, the proposed CPI was first classified into cost data and benefit data based on the different types of data. A $100\% \times (1 - \alpha)$ confidence interval for CPI was then proposed using parameter estimations. Following the combination of fuzzy parameter estimations, a fuzzy CPI was introduced.

2.1. City Performance Index and Its $100\% \times (1 - \alpha)$ Confidence Interval

First, the CPI is defined as follows:

$$T_{cpi} = \left\langle \begin{array}{ll} \frac{UCL-\mu}{\sigma},\ i = c & \text{for cost data} \\ \frac{\mu-LCL}{\sigma},\ i = b & \text{for benefit data} \end{array} \right. \tag{1}$$

where UCL and LCL, respectively, denote the upper and lower control limits of each evaluation indicator of city construction performance, and μ and σ are, respectively, the mean and standard deviation of historical statistical data of each evaluation indicator for a city.

If $\mu + w\sigma = USL$, the CPI for cost data is denoted as follows:

$$T_{cpc} = \frac{UCL - \mu}{\sigma} = \frac{(\mu + w\sigma) - \mu}{\sigma} = \frac{w\sigma}{\sigma} = w \tag{2}$$

Similarly, if $\mu - w\sigma = LSL$, the CPI for benefit data is denoted as follows:

$$T_{cpb} = \frac{\mu - LCL}{\sigma} = \frac{\mu - (\mu - w\sigma)}{\sigma} = \frac{w\sigma}{\sigma} = w \tag{3}$$

Therefore, there is a one-to-one correspondence between CPI and standard deviation. This means that when the CPI value is higher, data volatility is lower, indicating better performance of the city with respect to the indicator.

Although CPI is powerful, it does not consider the biases and sampling errors that may arise from estimations. As a result, we use a parameter estimation method to derive the $100\% \times (1 - \alpha)$ confidence interval of CPI. Suppose that $X_1, \ldots, X_j, \ldots, X_n$ is a random sample from a normal of random variable X with mean μ and variance σ^2, i.e., $X \sim N(\mu, \sigma^2)$, then the maximum likelihood estimator (MLE) of mean μ and standard deviation σ are the sample mean $\overline{X}$ and the sample standard deviation s, respectively. Hence, the MLE of CPI with μ and σ can be represented as follows:

$$\hat{T}_{cpi} = \left\langle \begin{array}{ll} \frac{UCL-\overline{X}}{s},\ i = c & \text{for cost data} \\ \frac{\overline{X}-LCL}{s},\ i = b & \text{for benefit data} \end{array} \right. \tag{4}$$

Considering that it is difficult to obtain complete data for various city construction indicators, under the assumption of normality, let t follow a t distribution with $n - 1$ degrees of freedom (i.e., t_{n-1}). Therefore, we obtain the following:

$$t = \frac{\overline{X} - \mu}{s/\sqrt{n}} \sim t_{n-1} \tag{5}$$

By combining Equations (4) and (5), we have the following:

$$\hat{T}_{cpi} = \left\langle \begin{array}{ll} \frac{\sqrt{n}\left[(UCL-\mu)-(UCL-\overline{X})\right]}{s},\ i = c & \text{for cost data} \\ \frac{\sqrt{n}\left[(\overline{X}-LCL)-(\mu-LCL)\right]}{s},\ i = b & \text{for benefit data} \end{array} \right. \tag{6}$$

The t-distribution is symmetric; therefore, we have the following [32]:

$$\Pr(-t_{\alpha'/2;n-1} \leq t \leq t_{\alpha'/2;n-1}) = \sqrt{1-\alpha} \tag{7}$$

where $t_{\alpha'/2;n-1}$ is the upper $\alpha'/2$ quantile of t_{n-1}, $\alpha' = 1 - \sqrt{1-\alpha}$, and α is the significance level.

Substitute Equation (6) into (7), we obtain the following:

$$\Pr\left\{ (UCL - \overline{X}) - t_{\alpha'/2;n-1} \times \left(\frac{s}{\sqrt{n}}\right) \leq (UCL - \mu) \leq (UCL - \overline{X}) + t_{\alpha'/2;n-1} \times \left(\frac{s}{\sqrt{n}}\right) \right\} = \sqrt{1-\alpha} \tag{8}$$

On the assumption of normality, let K follow a chi-square distribution with $n-1$ degrees of freedom (i.e., $\chi^2_{(n-1)}$). Therefore, we obtain the following:

$$K = \frac{(n-1)s^2}{\sigma^2} \sim \chi^2_{(n-1)} \tag{9}$$

and

$$\Pr(\chi^2_{\alpha'/2;n-1} \leq K \leq \chi^2_{1-\alpha'/2;n-1}) = \sqrt{1-\alpha} \tag{10}$$

where $\chi^2_{\alpha'/2;n-1}$ is the lower $\alpha'/2$ quantile of $\chi^2_{(n-1)}$.

Substitute Equation (9) into (10), we obtain the following:

$$\Pr\left\{ \sqrt{\frac{\chi^2_{\alpha'/2;n-1}}{(n-1)s^2}} \leq \frac{1}{\sigma} \leq \sqrt{\frac{\chi^2_{1-\alpha'/2;n-1}}{(n-1)s^2}} \right\} = \sqrt{1-\alpha} \tag{11}$$

Let Equations (8) and (11) be as follows:

$$\Omega = \left\{ (UCL - \overline{X}) - t_{\alpha'/2;n-1} \times \left(\frac{s}{\sqrt{n}}\right) \leq (UCL - \mu) \leq (UCL - \overline{X}) + t_{\alpha'/2;n-1} \times \left(\frac{s}{\sqrt{n}}\right) \right\} \tag{12}$$

$$\mho = \left\{ \sqrt{\frac{\chi^2_{\alpha'/2;n-1}}{(n-1)s^2}} \leq \frac{1}{\sigma} \leq \sqrt{\frac{\chi^2_{1-\alpha'/2;n-1}}{(n-1)s^2}} \right\} \tag{13}$$

and Ω' and $\mho'$ denote the complement of sets Ω and $\mho$, respectively.

Because $\Pr(\Omega' \cup \mho') \leq \Pr(\Omega') + \Pr(\mho') = \alpha$ and $\Pr(\Omega \cap \mho) \geq 1 - \Pr(\Omega') - \Pr(\mho') = 1 - \alpha$, Equations (12) and (13) can be rewritten as follows:

$$\Pr\left\{ \sqrt{\frac{\chi^2_{\alpha'/2;n-1}}{n-1}} \times \left(\hat{T}_{cpi} - \frac{t_{\alpha'/2;n-1}}{\sqrt{n}}\right) \leq T_{cpi} \leq \sqrt{\frac{\chi^2_{1-\alpha'/2;n-1}}{n-1}} \times \left(\hat{T}_{cpi} + \frac{t_{\alpha'/2;n-1}}{\sqrt{n}}\right) \right\} \geq 1-\alpha \tag{14}$$

Therefore, the $100\% \times (1-\alpha)$ confidence interval of CPI can be derived as follows:

$$T^*_{cpi} = \left[T^{LCL}_{cpi}, T^{UCL}_{cpi}\right] = \left[\sqrt{\frac{\chi^2_{\alpha'/2;n-1}}{n-1}} \times \left(\hat{T}_{cpi} - \frac{t_{\alpha'/2;n-1}}{\sqrt{n}}\right), \sqrt{\frac{\chi^2_{1-\alpha'/2;n-1}}{n-1}} \times \left(\hat{T}_{cpi} + \frac{t_{\alpha'/2;n-1}}{\sqrt{n}}\right) \right] \tag{15}$$

2.2. Fuzzy Estimator for City Performance Index

Previous studies on city construction have typically assumed that the data used for evaluation indicators are precise. However, it is difficult to clearly quantify statistical data [33,34]. Another possibility is that cross-sectional or panel databases contain some ambiguities, which can lead to uncertainties and misjudgments in traditional city construction methods [34]. To address the fuzziness and uncertainty of statistical data, Buckley [35] presented the use of the triangular fuzzy numbers (TFNs) generated by sets of confidence intervals of parameters to construct the fuzzy parameter estimates [36]. Therefore, we combine the proposed $100\% \times (1-\alpha)$ confidence interval of CPI in Section 2.1 with Buckley [35]' fuzzy parameter estimations; we further propose a fuzzy CPI $\widetilde{T}_{cpi}$ and its $\alpha - cut$ as follows:

$$\widetilde{T}_{cpi}[\alpha] = \left[\hat{T}^{L}_{cpi}(\alpha), \hat{T}^{R}_{cpi}(\alpha)\right],\ 0.01 \leq \alpha \leq 1 \tag{16}$$

$$\hat{T}^{L}_{cpi}(\alpha) = \left(\hat{T}_{cpi} - \frac{t_{\alpha/2;n-1}}{\sqrt{n}}\right) \times \sqrt{\frac{\chi^2_{\alpha/2;n-1}}{n-1}} \tag{17}$$

$$\hat{T}^{R}_{cpi}(\alpha) = \left(\hat{T}_{cpi} + \frac{t_{\alpha/2;n-1}}{\sqrt{n}}\right) \times \sqrt{\frac{\chi^2_{1-\alpha/2;n-1}}{n-1}} \tag{18}$$

We can clearly see that $\hat{T}^{L}_{cpi}(1) = \hat{T}^{R}_{cpi}(1) = \hat{T}_{cpi} \times \sqrt{\frac{\chi^2_{0.5;n-1}}{n-1}}$ in the case when $\alpha = 1$ [36]. Therefore, TFN of $\hat{T}_{cpi}$ can be expressed as $\widetilde{T}_{cpi} = \Delta(\widetilde{T}^{L}_{cpi}, \widetilde{T}^{M}_{cpi}, \widetilde{T}^{R}_{cpi})$, where

$$\widetilde{T}^{L}_{cpi}(\alpha) = \hat{T}^{L}_{cpi}(0.01) = \left(\hat{T}_{cpi} - \frac{t_{0.005;n-1}}{\sqrt{n}}\right) \times \sqrt{\frac{\chi^2_{0.005;n-1}}{n-1}} \tag{19}$$

$$\widetilde{T}^{M}_{cpi}(\alpha) = \hat{T}_{cpi} \times \sqrt{\frac{\chi^2_{0.5;n-1}}{n-1}} \tag{20}$$

$$\widetilde{T}^{R}_{cpi}(\alpha) = \hat{T}^{R}_{cpi}(0.01) = \left(\hat{T}_{cpi} + \frac{t_{0.005;n-1}}{\sqrt{n}}\right) \times \sqrt{\frac{\chi^2_{0.995;n-1}}{n-1}} \tag{21}$$

The membership function of fuzzy number $\widetilde{T}_{cpi}$ is as follows:

$$\eta_{\widetilde{T}_{cpi}}(x) = \begin{cases} 0, & if\ x < \widetilde{T}^{L}_{cpi} \\ \alpha_L, & if\ \widetilde{T}^{L}_{cpi} \leq x < \widetilde{T}^{M}_{cpi} \\ 1, & if\ x = \widetilde{T}^{M}_{cpi} \\ \alpha_R, & if\ \widetilde{T}^{M}_{cpi} \leq x \leq \widetilde{T}^{R}_{cpi} \\ 0, & if\ \widetilde{T}^{R}_{cpi} < x \end{cases} \tag{22}$$

where α_L and α_R are determined by $\widetilde{T}^{L}_{cpi}(\alpha_L) = x$ and $\widetilde{T}^{R}_{cpi}(\alpha_R) = x$, respectively.

3. Fuzzy AD Based on the Performance Evaluation Level of City Construction

The performance evaluation of city construction is a process of comprehensively assessing and ranking cities based on a series of indicators. The purpose of evaluating the performance of city construction is not simply to rank the results of city construction but instead, more importantly, to understand the gap between the actual performance and the target performance of the city with respect to each indicator. Unlike traditional multiple attribute decision-making methods that select maximum, minimum, or average values as reference points (such as a technique for order of preference by similarity to ideal solution (TOPSIS), *vlsekriterijumska pptimizacija i kompromisno resenje* (VIKOR), and *tomada de decisão iterativa multicritério* (TODIM)), fuzzy AD uses probability ratios, which can better measure how well the output performance of the city meets the target performance [37]. In recent years, fuzzy AD has been widely applied in various fields, for example, maintenance strategy for public buildings [38], financial decision-making in supply chains with credit risk assessments [39], sites of electric vehicle charging stations [40], blockchain deployment projects [41], development of an instructional design model for maritime education and training [42], and human–machine interface design [43].

AD mainly includes two axioms: the independence axiom and the information axiom [44]. The independence axiom indicates that the mutual influence between various evaluation indicators of the city is minimized, while the information axiom suggests that the city with the least information content IC_i is the best one under various evaluation indicators. In this study, the system range (SR) in the AD represents the actual performance of the CPI, the target range (TR) represents the target performance of the CPI, and the coverage area (CA) indicates the extent to which the actual performance meets the target performance of the CPI. Let g_i represent the success probability of the city meeting the CPI; thus, the IC_i is defined as follows:

$$IC_i = \log_2\left(\frac{1}{g_i}\right) \tag{23}$$

$$g_i = \frac{CA}{SR} \tag{24}$$

The success probability g_i can be obtained by combining the SR and TR. Hence, the IC_i can be rewritten as follows:

$$IC_i = \log_2\left(\frac{SR}{CA}\right) \tag{25}$$

In the process of evaluating the performance of city construction, the evaluation indicators are composite indicators (i.e., qualitative and quantitative indicators). To effectively quantify the evaluation results and address uncertainty, the crisp values of the evaluation indicators can be transformed into fuzzy numbers through fuzzy AD. First, the values of the evaluation indicators are mapped to the corresponding TFNs to obtain the fuzzy target range (FTR) and the fuzzy system range (FSR). The intersection of the FTR and FSR is then defined as the fuzzy coverage area (FCA). Let a TFN be $a = \left(a^L, a^M, a^R\right)$, $a^L \le a^M \le a^R$, and its membership function is represented as follows:

$$\mu_a(x) = \begin{cases} \left(x - a^L\right)/\left(a^M - a^L\right), & a^L \le x \le a^M \\ \left(a^R - x\right)/\left(a^R - a^M\right), & a^M \le x \le a^R \\ 0 \quad , & others \end{cases} \tag{26}$$

The fuzzy AD is shown in Figure 1. The fuzzy information content FIC_i is expressed as follows:

$$FIC_i = \log_2\left(\frac{FSR}{FCA}\right) = \log_2\left(\frac{\Delta a^L c a^R}{\Delta A^L m a^R}\right) \tag{27}$$

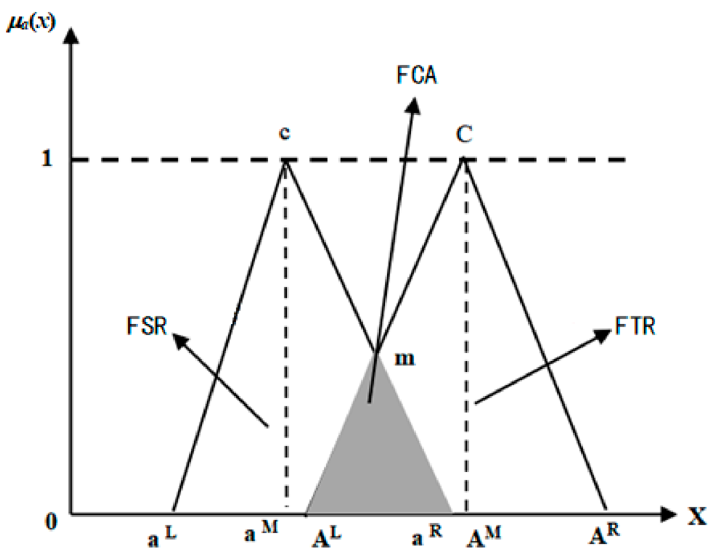

Figure 1. *FCA* of *FSR* and *FTR*.

In the performance evaluation of city construction, the *FTR* (i.e., target performance) of cost-type or benefit-type indicators is determined based on the performance level of city construction, as shown in Table 1.

Table 1. Performance level of city construction.

Level	T_{cpi}	TFN	Achievement Rate (AR)
Excellent	$6 > T_{cpi}$	(0, 6, 6)	AR ≥ 99.9999998%
Very good	$5 \leq T_{cpi} < 6$	(0, 5, 5)	AR ≥ 99.9999426%
Good	$4 \leq T_{cpi} < 5$	(0, 4, 4)	AR ≥ 99.9936657%
Acceptable	$3 \leq T_{cpi} < 4$	(0, 3, 3)	AR ≥ 99.7300203%
Poor	$T_{cpi} < 3$	<(0, 3, 3)	AR < 99.7300203%

Figure 2 is the fuzzy AD based on the performance levels of city construction. As shown in Figure 2, the FIC_i can be rewritten as follows:

$$FIC_i = \log_2\left(\frac{FSR}{FCA}\right) = \log_2\left(\frac{S_{\Delta a^L c a^R}}{S_{\square a^L m n a^R}}\right) \tag{28}$$

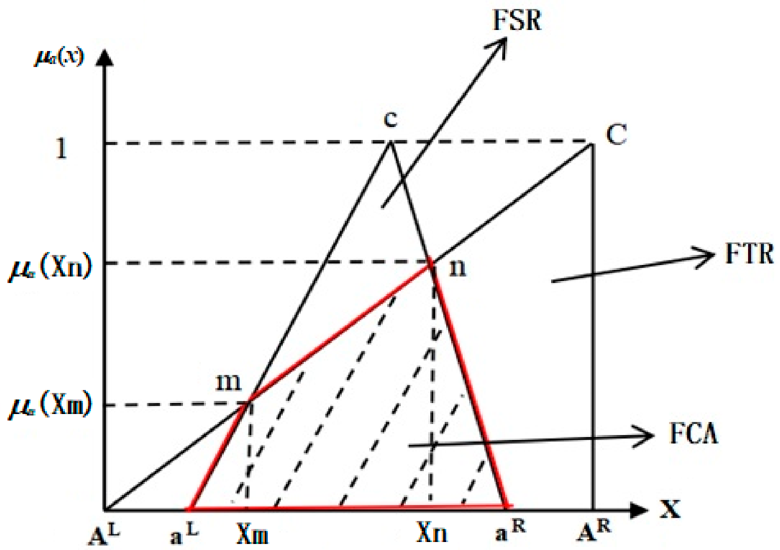

Figure 2. Fuzzy AD based on performance level of city construction.

4. Results

4.1. Operational Procedure for Performance Evaluation of City Construction

The performance evaluation of city construction involves management, renovation, and improvement, with each indicator corresponding to different evaluation standards. To facilitate implementation, we standardized the operational procedure for the proposed method as follows:

4.1.1. Phase 1: Pre–Assessment

Step 1: Determine the evaluation indicators and collect relevant data for the performance evaluation of city construction.

Step 2: Calculate the value of CPI T_{cpi}. First, identify the evaluation indicator type and then compute the T_{cpi} values for each city with respect to each evaluation indicator using Equation (1). A negative T_{cpi} value indicates that the performance of the evaluation indicator is inadequate and should be prioritized for improvement. In other words, a larger negative value equates to a higher priority for improvement.

4.1.2. Phase 2: Rank

Step 3: Compute the value of fuzzy CPI $\widetilde{T}_{cpi}$. By using Equations (19)–(22), the remaining T_{cpi} of each city with respect to each evaluation indicator can be transformed into TFNs. The $\widetilde{T}_{cpi}$ (i.e., FSR) for each city with respect to each indicator can then be obtained.

Step 4: Calculate the value of FIC_i. First, determine the performance level of city construction (i.e., FTR) for each evaluation indicator based on Table 1 and convert them into TFNs. Then, calculate the value of FIC_i for each city with respect to each evaluation indicator using Equation (28).

Step 5: Rank all cities with respect to each evaluation indicator according to the ascending order of FIC_i. Determine the improvement priority for all cities with respect to each evaluation indicator based on the value of FIC_i. The smaller the value of FIC_i, the higher priority the indicator for each city has for improvement.

4.2. An Application Example

To demonstrate the effectiveness of the proposed method in this study, the performance evaluation of low-carbon city construction in China is used in the following as an example. In recent years, rapid population growth, urbanization, and high-intensity human activities have caused many severe environmental problems in China. As a result, China has been implementing the first batch of low-carbon city pilot projects since 2010. The first batch of low-carbon pilots in China includes five provinces and eight cities. As the implementation of this policy has been in place for over a decade, the data are relatively comprehensive and complete. In addition, based on comparability and data completeness, we selected six cities for comparison: Tianjin, Chongqing, Shenzhen, Xiamen, Hangzhou, and Guiyang. In the following, the method proposed in Section 4 was used to perform performance evaluations of these cities.

Step 1: Six indicators were selected to evaluate the performance of low-carbon city construction (see Table 2). The data for these evaluation indicators were sourced from the statistical yearbooks of each city, the National Economic and Social Development Statistical Bulletin (2015–2019), the Bureau of Landscape Architecture and Forestry Bulletin (2015–2019), and the China Urban Rail Transit Yearbook (2015–2019).

Table 2. Performance evaluation indicator system of low-carbon city construction.

Indicator	Calculation Formula	Unit	Type	Benchmark Value
CO_2 emissions per capita (X1)	CO_2 emissions/total population	Ton/person	Cost	<7.28
Proportion of tertiary industry (X2)	Tertiary sector output/total gross domestic product (GDP)	%	Benefit	>52.6%
Electricity consumption per capita (X3)	Total electricity consumption/total population	KWh/person	Cost	<7500 KWh
Proportion of coal consumption in total primary energy consumption (X4)	Coal consumption/primary energy consumption	%	Cost	<27%
Public buses per capita (X5)	Buses/total population	Buses/10,000 people	Benefit	>12 buses
Rail length per capita (X6)	Rail length/total population	mm	Benefit	>17 mm

Step 2: Table 3 shows the values of T_{cpi} for each city with respect to each indicator. Negative values indicate inadequate performance for a city in that indicator, which should be prioritized for improvement in the first phase. For example, Tianjin, Shenzhen, and Xiamen exhibited poor performance in CO_2 emissions per capita (X1), with Shenzhen being the worst performer among them.

Step 3: Table 4 shows the values of $\widetilde{T}_{cpi}$.

Step 4: Considering that low-carbon pilot cities in China have been in operation for more than a decade and that these pilot cities should perform to an excellent level on all indicators, we used "Excellent" as the standard for each indicator. According to Table 1, the

FTR was determined as (0, 6, 6). Table 5 shows the values of IC_i for each city with respect to each indicator.

Step 5: Table 6 shows the ranking results of each city with respect to each indicator using the proposed two-phase method. Note that a larger T_{cpi} value only suggests that the city performs relatively well on that specific evaluation indicator. In fact, city construction should be considered from an overall perspective, with a rational allocation of resources. Furthermore, emphasizing overly high-performing indicators should be reduced, and improving underperforming indicators should be a priority. For example, the performance of Chongqing in electricity consumption per capita (X3) was the best among all cities; however, after applying Fuzzy AD, its improvement priority dropped to fifth place.

Table 3. Values of T_{cpi}.

Indicator	City					
	Tianjin	Chongqing	Shenzhen	Xiamen	Hangzhou	Guiyang
X1	−0.872	11.571	−4.566	−1.132	0.480	7.710
X2	1.391	−1.125	7.331	3.433	3.645	5.638
X3	−2.734	21.702	−15.365	−23.021	−11.276	3.772
X4	−13.291	−42.391	5.436	6.892	0.738	−2.192
X5	−4.029	−32.056	7.138	3.398	2.261	−22.807
X6	1.265	−10.378	12.158	0.118	−1.790	−4.517

Table 4. Values of $\widetilde{T}_{cpi}$.

Indicator	City					
	Tianjin	Chongqing	Shenzhen	Xiamen	Hangzhou	Guiyang
X1	–	(2.63, 10.6, 22.31)	–	–	(0.11, 0.47, 0.93)	(1.75, 7.06, 14.87)
X2	(0.32, 1.27, 2.69)	–	(1.67, 6.72, 14.14)	(0.78, 3.14, 6.62)	(0.83, 3.34, 7.03)	(1.28, 5.16, 10.87)
X3	–	(1.54, 18.07, 49.96)	–	–	–	(0.27, 3.14, 8.69)
X4	–	–	(0.38, 4.53, 12.52)	(0.49, 5.74, 15.87)	(0.05, 0.61, 1.71)	–
X5	–	–	(0.51, 5.94, 16.44)	(0.24, 2.83, 7.83)	(0.16, 1.88, 5.21)	–
X6	(0.09, 1.05, 2.92)	–	(0.86, 10.12, 27.99)	(0.01, 0.1, 0.28)	–	–

Table 5. Values of IC_i.

Indicator	City					
	Tianjin	Chongqing	Shenzhen	Xiamen	Hangzhou	Guiyang
X1	–	1.79	–	–	2.53	0.63
X2	1.21	–	0.50	0.32	0.09	0.01
X3	–	2.99	–	–	–	0.24
X4	–	–	0.03	0.27	1.79	–
X5	–	–	0.34	0.32	0.64	–
X6	1.19	–	1.52	4.09	–	–

Table 6. Final ranking of the cities.

Indicator	City					
	Tianjin	Chongqing	Shenzhen	Xiamen	Hangzhou	Guiyang
X1	3	5	1	2	4	6
X2	2	1	3	4	5	6
X3	4	5	2	1	3	6
X4	2	1	6	5	4	3
X5	3	1	5	6	4	2
X6	6	1	5	4	3	2

5. Conclusions

This study proposes a two-phase fuzzy hybrid method that integrates fuzzy CPI and fuzzy AD by which to leverage the advantages of each method. Specifically, the second phase considers target performance to make the evaluation process more insightful, which is the most significant aspect of the proposed method. The results of the case application demonstrate the practical feasibility of this method, enabling city managers to accurately identify the indicators that need the most immediate improvement and to coordinate suitable resource allocation among them.

Although this study provides some reference and enlightenment for researchers to study the performance of city construction in the future, it also faces some limitations. First, this study presents the fuzzy CPI to address the volatility of historical statistical data as well as different types of data. Due to the limitation of fuzzy CPI, future research could incorporate qualitative data to gain deeper insights into the underlying motivations and values of the population, providing a valuable supplement to quantitative results and supporting city managers in making scientific decisions. Second, this study only uses TFNs in the analysis. Further investigation into the nature of fuzzy preference degrees between two fuzzy numbers or the inclusion of different fuzzy numbers, such as trapezoidal fuzzy numbers, LR-type fuzzy numbers, and pentagonal fuzzy numbers, can also open up a new research field. The proposed method only uses Chinese as an example; therefore, future research could expand to address issues in the performance evaluation of other city construction in other countries or regions.

Author Contributions: Conceptualization, C.-M.Y.; data curation, T.C. and S.L.; formal analysis, C.-M.Y., C.-H.H. and S.L.; funding acquisition, C.-M.Y. and C.-H.H.; investigation, T.C. and S.L.; methodology, C.-M.Y. and C.-H.H.; resources, T.C.; software, S.L.; supervision, C.-M.Y. and C.-H.H.; validation, C.-H.H.; writing—original draft preparation, C.-M.Y., T.C. and S.L.; writing—review and editing, C.-M.Y., C.-H.H. and T.C. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by the National Natural Science Foundation of China, grant No. 71762008.

Data Availability Statement: Data are contained within the article.

Conflicts of Interest: The authors declare no conflicts of interest.

References

1. Shimada, K.; Tanaka, Y.; Gomi, K.; Matsuoka, Y. Developing a long-term local society design methodology towards a low-carbon economy: An application to Shiga prefecture in Japan. *Energy Policy* **2007**, *35*, 4688–4703. [CrossRef]
2. Yigitcanlar, T.; Kamruzzaman, M. Does smart city policy lead to sustainability of cities? *Land Use Policy* **2018**, *73*, 49–58. [CrossRef]
3. Hajmohammadi, H.; Heydecker, B. Evaluation of air quality effects of the London ultra-low emission zone by state-space modelling. *Atmos. Pollut. Res.* **2022**, *13*, 101514. [CrossRef]
4. Wang, K. Available online: https://www.neaspec.org/sites/default/files/2.%20KRIHS%20Kwangik%20Wang%20-%20Korea_Green_City.pdf (accessed on 6 June 2024).
5. Su, M.; Liang, C.; Chen, B.; Chen, S.; Yang, Z. Low-carbon development patterns: Observations of typical Chinese cities. *Energies* **2012**, *5*, 291–304. [CrossRef]
6. Yin, H.; Qian, Y.; Zhang, B.; Pérez, R. Urban construction and firm green innovation: Evidence from China's low-carbon pilot city initiative. *Pac-Basin Financ. J.* **2023**, *80*, 102070. [CrossRef]
7. Manman, L.; Goswami, P.; Mukherjee, P.; Mukherjee, A.; Yang, L.; Ghosh, U.; Menon, V.G.; Qi, Y.; Nkenyereye, L. Distributed artificial intelligence empowered sustainable cognitive radio sensor networks: A smart city on-demand perspective. *Sustain. Cities Soc.* **2021**, *75*, 103265. [CrossRef]
8. Du, B.; Steven, I.; Chien, J. Feasibility of shoulder use for highway work zone optimization. *J. Traffic Transport. Eng.* **2014**, *1*, 235–246. [CrossRef]
9. Noland, R.B.; Hanson, C.S. Life-cycle greenhouse gas emissions associated with a highway reconstruction: A New Jersey case study. *J. Clean. Prod.* **2015**, *107*, 731–740. [CrossRef]
10. Ramaswami, A.; Russell, A.G.; Culligan, P.J.; Sharma, K.R.; Kumar, E. Meta-principles for developing smart, sustainable, and healthy cities. *Science* **2016**, *352*, 940–943. [CrossRef]

11. Hukkalainen, M.; Virtanen, M.; Paiho, S.; Airaksinen, M. Energy planning of low carbon urban areas-Examples from Finland. *Sustain. Cities Soc.* **2017**, *35*, 715–728. [CrossRef]
12. Kim, K.; Yi, C.; Lee, S. Impact of urban characteristics on cooling energy consumption before and after construction of an urban park: The case of Gyeongui line forest in Seoul. *Energy Build.* **2019**, *191*, 42–51. [CrossRef]
13. Li, J.; Tang, F.; Zhang, S.; Zhang, C. The effects of low-carbon city construction on bus trips. *J. Public Transport.* **2023**, *25*, 100057. [CrossRef]
14. Yang, Z.; Yuan, Y.; Tan, Y. The impact and nonlinear relationship of low-carbon city construction on air quality: Evidence from a quasi-natural experiment in China. *J. Clean. Prod.* **2023**, *422*, 138588. [CrossRef]
15. Yigitcanlar, T.; Foth, M.; Kamruzzaman, M. Towards post-anthropocentric cities: Reconceptualizing smart cities to evade urban ecocide. *J. Urban Technol.* **2018**, *26*, 147–152. [CrossRef]
16. Mao, C.; Wang, Z.; Yue, A.; Liu, H.; Peng, W. Evaluation of smart city construction efficiency based on multivariate data fusion: A perspective from China. *Ecol. Indic.* **2023**, *154*, 110882. [CrossRef]
17. Wang, F. Does the construction of smart cities make cities green? Evidence from a quasi-natural experiment in China. *Cities* **2023**, *140*, 104436. [CrossRef]
18. Zhang, K.; Zhu, P.H.; Qian, X.Y. National information consumption demonstration city construction and urban green development: A quasi-experiment from Chinese cities. *Energy Econ.* **2024**, *130*, 107313. [CrossRef]
19. López-Ruiz, V.R.; Alfaro-Navarro, J.L.; Nevado-Peña, D. Knowledge-city index construction: An intellectual capital perspective. *Expert Syst. Appl.* **2014**, *41*, 5560–5572. [CrossRef]
20. AlKheder, S.; Alzarari, A.; AlSaleh, H. Urban construction-based social risks assessment in hot arid countries with social network analysis. *Habitat Int.* **2023**, *131*, 102730. [CrossRef]
21. Zou, Y.; Song, M.; Zhang, W.; Wang, Z. The impact of high-speed rail construction on the development of resource-based cities: A temporal and spatial perspective. *Socio-Econ. Plan. Sci.* **2023**, *90*, 101742. [CrossRef]
22. Chen, M.; Su, Y.; Piao, Z.; Zhu, J.; Yue, X. The green innovation effect of urban energy saving construction: A quasi-natural experiment from new energy demonstration city policy. *J. Clean. Prod.* **2023**, *428*, 139392. [CrossRef]
23. Wu, H.; Chen, R.; Yuan, H.; Yong, Q.; Weng, X.; Zuo, J.; Zillante, G. An evaluation model for city-scale construction and demolition waste management effectiveness: A case study in China. *Waste Manag.* **2024**, *182*, 284–298. [CrossRef] [PubMed]
24. Yan, L.; Zhang, L.; Liang, W.; Li, W.; Du, M. Key factors identification and dynamic fuzzy assessment of health, safety and environment performance in petroleum enterprises. *Saf. Sci.* **2017**, *94*, 77–84. [CrossRef]
25. Yang, J.; Chen, F.; Wang, Y.; Mao, J.; Wang, D. Performance evaluation of ecological transformation of mineral resource-based cities: From the perspective of stage division. *Ecol. Indic.* **2023**, *154*, 110540. [CrossRef]
26. Yang, C.M.; Li, S.; Huang, D.; Lo, W. Performance evaluation of carbon-neutral cities based on fuzzy AHP and HFS-VIKOR. *Systems* **2024**, *12*, 173. [CrossRef]
27. Li, Y.; Li, J. Method development and empirical research in examining the construction of China's "Zero-waste Cities". *Sci. Total Environ.* **2024**, *906*, 167345. [CrossRef]
28. Wilson, D.C.; Rodic, L.; Cowing, M.J.; Velis, C.A.; Whiteman, A.D.; Scheinberg, A.; Vilches, R.; Masterson, D.; Stretz, J.; Oelz, B. 'Wasteaware' benchmark indicators for integrated sustainable waste management in cities. *Waste Manag.* **2015**, *35*, 329–342. [CrossRef]
29. Seker, S.; Aydin, N.; Tuzkaya, U.R. What is Needed to design sustainable and resilient cities: Neutrosophic fuzzy based DEMATEL for designing cities. *Int. J. Disast. Risk Reduct.* **2024**, *108*, 104569. [CrossRef]
30. Shen, L.; Huang, Z.; Wong, S.W.; Liao, S.; Lou, Y. A holistic evaluation of smart city performance in the context of China. *J. Clean. Prod.* **2018**, *200*, 667–679. [CrossRef]
31. Dou, S.; Shen, Y.; Zhu, H. Fuzzy-based multi-criteria humanistic assessment system for city tunnels: From methodology to application. *Tunn. Undergr. Sp. Technol.* **2023**, *134*, 104993. [CrossRef]
32. Martin, B.R. *Statistics for Physical Science: An Introduction*; Academic Press: Amsterdam, The Netherlands, 2012.
33. Kutty, A.A.; Kucukvar, M.; Onat, N.C.; Ayvaz, B.; Abdella, G.M. Measuring sustainability, resilience and livability performance of European smart cities: A novel fuzzy expert-based multi-criteria decision support model. *Cities* **2023**, *137*, 104293. [CrossRef]
34. Otay, İ.; Onar, S.Ç.; Öztayşi, B.; Kahraman, C. Evaluation of sustainable energy systems in smart cities using a Multi-Expert Pythagorean fuzzy BWM & TOPSIS methodology. *Expert Syst. Appl.* **2024**, *250*, 123874.
35. Buckley, J.J. Fuzzy statistics: Hypothesis testing. *Soft Comput.* **2005**, *9*, 512–518. [CrossRef]
36. Chen, K.S.; Wang, C.H.; Tan, K.H.; Chiu, S.F. Developing one-sided specification six-sigma fuzzy quality index and testing model to measure the process performance of fuzzy information. *Int. J. Prod. Econ.* **2019**, *208*, 560–565. [CrossRef]
37. Büyüközkan, G. An integrated fuzzy multi-criteria group decision-making approach for green supplier evaluation. *Int. J. Prod. Res.* **2012**, *50*, 2892–2909. [CrossRef]
38. Ighravwe, D.E.; Oke, S.A. A multi-criteria decision-making framework for selecting a suitable maintenance strategy for public buildings using sustainability criteria. *J. Build. Eng.* **2019**, *24*, 100753. [CrossRef]
39. Wu, X.; Liao, H. Utility-based hybrid fuzzy axiomatic design and its application in supply chain finance decision making with credit risk assessments. *Comput. Ind.* **2020**, *114*, 103144.
40. Feng, J.; Xu, S.X.; Li, M. A novel multi-criteria decision-making method for selecting the site of an electric-vehicle charging station from a sustainable perspective. *Sustain. Cities Soc.* **2021**, *65*, 102623. [CrossRef]

41. Gölcük, İ. An interval type-2 fuzzy axiomatic design method: A case study for evaluating blockchain deployment projects in supply chain. *Inform. Sci.* **2022**, *602*, 159–183. [CrossRef]
42. Kandemir, I.; Cicek, K. Development an instructional design model selection approach for maritime education and training using fuzzy axiomatic design. *Educ. Inf. Technol.* **2023**, *28*, 11291–11312. [CrossRef]
43. Liu, Q.; Chen, J.; Yang, K.; Liu, D.; He, L.; Qin, Q.; Wang, Y. An integrating spherical fuzzy AHP and axiomatic design approach and its application in human-machine interface design evaluation. *Eng. Appl. Artif. Intel.* **2023**, *125*, 106746. [CrossRef]
44. Kulak, O.; Kahraman, C. Fuzzy multi-attribute selection among transportation companies using axiomatic design and analytic hierarchy process. *Inform. Sci.* **2005**, *170*, 191–210. [CrossRef]