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Intuitionistic Fuzzy Ordinal Priority Approach with Grey Relational Analysis

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Abstract: Multi-attribute decision-making (MADM) is a methodology for solving decision problems with a finite set of alternatives. The several methods of MADM require weights for the criteria and the alternatives to provide a solution. The Ordinal Priority Approach (OPA) is a recently proposed method for MADM that innovates; it does not require these inputs, just the rankings of criteria and alternatives. This article introduces a new hybrid method for MADM: the Intuitionistic Fuzzy Ordinal Priority Approach with Grey Relational Analysis (OPA-IF-GRA). OPA-IF-GRA combines GRA with OPA-IF, a newer extension of OPA that includes intuitionistic fuzzy sets to incorporate uncertainty into the decision-making process. The article presents an OPA-IF-GRA application for solving an electronics engineering problem, considering four criteria and six alternatives. The solution of OPA-IF-GRA is compared with the solutions obtained with three other MADM methods.

Keywords: ordinal priority approach; intuitionistic fuzzy sets; grey relational analysis; power dividers

MSC: 91B06

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1. Introduction

Multi-criteria decision-making (MCDM) is a methodology for solving decision problems involving multiple, and sometimes conflicting, criteria [1]. Multi-attribute decision analysis (MADM) is part of the MCDM discipline that deals with a finite set of alternatives [2]. To solve a decision problem, MADM methods require weights for the criteria and weights for the alternatives considering each criterion [3]. Identifying these weights is often the hard part of an MADM method application. Sometimes, this is more than uphill. In other words, it is almost impossible, with decision-makers needing to be entirely sure of the different weights they need to identify. To help those situations, fuzzy systems have been inserted in MADM [4,5].

The Ordinal Priority Approach (OPA) is a recently proposed method for MADM that innovates; it does not require weights but just the ranks of the criteria and the alternatives [6]. Identifying rank is more accessible than identifying weight because, for instance, the OPA application does not require pairwise comparisons, as do the Analytic Hierarchy Process (AHP) [7] or the Best–Worst Method (BWM) [8]. The time consumption in decision-making decreases since OPA only requires ordinal information for both sets, criteria, and alternatives. However, decision-makers also cannot identify the ranks. Then, the second author of the OPA proposed the OPA for Fuzzy Linguistic Information (OPA-F) for situations where uncertainty prevails [9]. Therefore, instead of providing crisp ranks (such as first, second, third...) for OPA, the decision-maker shall identify which is the importance level of criteria and alternatives (such as excellent, fairly good, very good...) for OPA-F. The levels of importance are associated with type-1 fuzzy sets [10,11]. So, OPA-F requires weights instead of ranks, even if the weights are in linguistic form, not numerical.

This work proposes the Intuitionistic Fuzzy OPA (OPA-IF), an extension for OPA and OPA-F with triangular intuitionistic fuzzy sets (TIFS) instead of type-1 fuzzy sets. One of the remaining drawbacks of fuzzy sets is that, in some instances, it can be challenging to find an exact membership mapping for a fuzzy set [12]. The TIFS solves this issue by specifying the element's membership and non-membership degrees in a fuzzy set [13,14]. More than that, OPA-IF requires ranks and not weights of criteria as the previous OPA-F.

The weights of the alternatives compose the decision matrix, a central tool in MADM. The decision matrix is unnecessary for OPA since this MADM method only requires ranks. In other words, OPA requires an ordinal decision matrix. However, sometimes a decision matrix may be available, for example, when the attributes are all numeric, such as costs, distance, and time. In this case, obtaining an ordinal matrix from a cardinal matrix can be difficult. For example, if the difference between Weight 1 and Weight 2 is much more significant than that between Weight 2 and Weight 3, a tie for Rank 2 is fair. To avoid this situation and deal with this uncertainty, the grey relational analysis is combined with OPA-IF in a hybrid proposed method for MCDM.

This work has five more sections. Section 2 presents concepts pertaining to the decision problem of power divider design. Section 3 introduces the methodology, presenting concepts on fuzzy sets, OPA, and GRA. Section 4 presents the data collected for the OPA-IF-GRA application to select a BUPD design. Section 5 presents the results of OPA-IF-GRA and compares them with the application of two popular MADM methods: AHP and BWM. Section 6 concludes the work, highlighting its original contributions and proposing themes for new research.

2. Background

A power divider is a three-port device that divides power equally and without phase variation among the output ports. Power dividers are indispensable in microwave communication circuits with butler matrices or balanced amplifiers. Power dividers have incorporated many technologies, including dual-band, harmonic suppression, size miniaturization, and filtering. Balanced circuits have become prominent among the many technologies used in the communication sector due to their increased immunity to environmental and electronic noise. Previously, baluns were combined with power dividers to create a balanced circuit due to their ability to attenuate noise. A balun is a device that transforms single-ended ports into balanced ports and vice versa. However, this demands a substantial size. The characteristics of a power divider and a balun were combined to create a balanced to unbalanced power divider. One benefit of using a balanced to unbalanced power divider over a balanced one is that the former may be used in any circuit. In contrast, the latter can only be used in a balanced circuit. Then, the BUPD is favored over the balanced power divider. Figure 1 presents the design, with power inputs 1 and 1′, and outputs 2 and 3.

A BUPD has one balanced input and two single-ended outputs. It operates in differential mode and common mode. In common mode, the device suppresses the signal and functions as a band-stop filter, while in differential mode, it evenly splits the power without any phase difference. Any circuit's practical implementation must be considered when designing it. Size, cost, and performance are three elements that are important for realistic implementation. A circuit design must be cost-effective, compact, and perform efficiently.

Huang and Zhu [15] presented a novel in-phase BUPD based on two-dimension patch resonators with a bandpass filtering response. The desired out-of-phase input and in-phase output signals were generated and applied for BUPD design using the electric field distribution of the fundamental mode in a square patch cavity. Two coupled patch resonators with the appropriate coupling strength in a stacked arrangement were used to obtain an effective bandpass filtering response. Two grounded isolation resistors helped provide good high isolation between the output ports. A prototype filtering in-phase BUPD operating at 4.0 GHz with a fractional bandwidth (FBW) of 13.7% was used for verification. The measured findings experimentally confirmed the design idea, which agreed well with the simulated ones.

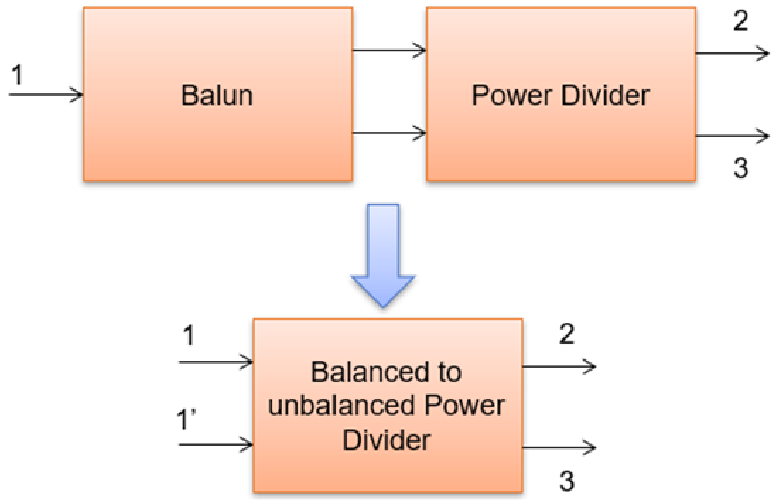

Figure 1. Design of power divider.

It has been suggested that a Bagley BUPD with input-reflectionless filtering properties be used. Three single-ended output ports and a balanced input port are features that are challenging to accomplish with traditional BUPD. Parallel linked lines provide the desired filtering properties. Two transmission zeros are introduced close to the passband using stepped impedance resonators to enhance the selectivity of differential-mode filtering even more. Loading absorptive branches yields the input-reflectionless characteristic in the bandstop region. A microstrip Bagley BUPD prototype working at 1.0 GHz with a 3-dB filtering bandwidth of 72% has been designed and constructed to validate the suggested power divider topology. Moreover, the entire measurement frequency range of 0–2.5 GHz has been covered by a 10-dB input-reflectionless bandwidth. The excellent agreement between the measurement and simulation validated the proposal [16].

An analogous stub-loaded transmission line is used in place of the quarter-wavelength transformer to achieve successful miniaturization. Utilizing a double-sided parallel-strip line 180-degree phase inverting arrangement yields excellent wideband CMS. The manufactured Bagley BUPD has a miniaturized circuit footprint and operates at 1.0 GHz. Its 6-dB fractional bandwidth spans 70% from 0.5 to 1.2 GHz. Additionally, from 0.5 to 1.5 GHz, CMS greater than 20 dB is accomplished [17].

The dual-band filtering power divider with full-frequency isolation features four coupled line sections, two ring resonators, two shunted stepped impedance resonators, two transmission lines, and an isolation network. The design achieves in-band equal-ripple and six out-of-band transmission zeros. Full-frequency isolation is realized using closed-form design equations and circuit simulation results for four types of INW, which are summarized and discussed in detail. A general design flowchart is provided to validate the proposed synthesis theory, and several design examples are selected for detailed comparison. Finally, a DB FPD prototype is designed and fabricated for experimental validation, with measured results closely aligning with the theoretical predictions and showing an isolation level below −20.4 dB across the full-frequency range [18].

Kumaran et al. [19] present a single-supply balun-first three-way parallel Doherty power amplifier (PA) designed for millimeter wave (mm-wave) fifth-generation applications. A bandwidth enhancement technique is incorporated into the design to reduce impedance mismatches between differential PAs, improve broadband power back-off efficiency, and widen the operational frequency range. With a core area of 0.77 mm^2 and a realization in 40 nm complementary metal oxide semiconductor bulk technology, the prototype exhibits a saturated power/peak gain that exceeds 20 dBm/16 dB. It also shows a drain efficiency that surpasses 15%/22%/33% at 9.5 dB/6 dB/0 dB power back-off in the 24–30 GHz band. For a 1-GHz 64-quadrature amplitude modulation orthogonal frequency division multiplexing signal, the suggested mm-wave PA operates at an average output power of 9.4 dBm with an average drain efficiency of 15%, achieving error vector magnitude/adjacent channel leakage ratio values of −24.3 dB/−30.1 dBc. With error vector magnitude/adjacent channel leakage ratio values of −30 dB/−36.3 dBc, an average Pout/DE of 8.6 dBm/12% is obtained for a 50-MHz 1024-quadrature amplitude modulation orthogonal frequency division multiplexing signal.

Using three coupled lines, Shi et al. [20] implemented a wideband BUPD in RO4003C (tanδ = 0.0027) substrate. The circuit was analyzed using even- and odd-mode techniques to attain wide bandwidth in differential mode and suppression in common mode. Further, the common mode suppression bandwidth was enhanced by introducing an open stub, but the operating bandwidth was limited to 30%. The works mentioned in Gao et al. [21] also used coupled lines to design BUPD on a dielectric substrate with tanδ = 0.003. Additionally, an open stub was introduced to create transmission zero. The circuit occupied 0.52 × 0.26 λ_g^2 with an FBW of 80%. However, enhanced performance with affordable substrate is preferred. The design proposed by Feng et al. [22] utilizes a coupled line and two symmetrical transmission lines to enhance the bandwidth. The FBW of the circuit is 89.1%. The planar circuit was fabricated on a dielectric substrate with tanδ = 0.003 and occupies 0.5 × 0.2 λ_g^2 area. Two-port coupled lines with short- and open-circuit stubs enhanced the passband performance and stopband response of BUPD in Zhuang et al. [23]. The circuit was analyzed based on the even- and odd-mode ABCD matrix. The FBW is about 28.5% from 1.67 to 2.24 GHz. The circuit was implemented in a substrate with a loss tangent 0.0037 occupying 0.58 × 0.58 λ_g^2. In Bhowmik et al. [24], coupled lines and an additional shunt TL in the middle created the UWB BUPD. It provided an FBW of 95.6%. The circuit was implemented in a cost-effective substrate of loss tangent 0.02 occupying 0.52 × 0.36 λ_g^2 area. In Xu et al. [25], SIW technology was used to implement BUPD. The microstrip–slotline–SIW transition is a critical part of designing. Here, a multilayered structured configuration was used to excite the SIW. It provided an FBW of 60% with a center frequency of 8.5 GHz. The circuit was implemented in the substrate with loss tanδ 0.0027 and occupies 3.15 × 2.4 λ_g^2 area.

3. Methodology

3.1. Research Methodology and Proposed Model

This study aims to identify the most suitable BUPD design based on substrate, size, CMS, and FBW. Thus, the research has five stages. Figure 2 illustrates the research stages.

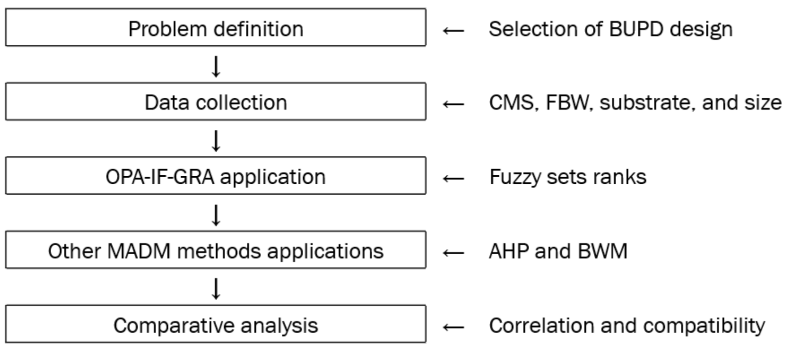

Figure 2. Research methodology.

Independently of the method, the three main steps for the MADM are weighting the criteria (Step 1), weighting the alternatives (Step 2), and the aggregation of the weights (Step 3). Table 1 presents the differences among the steps of AHP, BWM, GRA, OPA, OPA-F, and OPA-IF, first proposed and applied in this paper.

Table 1. Differences among the steps in MADM methods.

Method	Input for Step 1	Input for Step 2	Process and Formulas
AHP [7]	Pairwise comparisons	Numerical data or pairwise comparisons	Normalization of eigenvector and weighted arithmetical average
BWM [8]	Pairwise comparisons and ranks of the criteria	Pairwise comparisons and ranks of the alternatives	Linear programming
GRA [26,27]	Numerical data	Numerical data	max/min relations (Equations (10)–(12))
OPA [6]	Ranks of the criteria	Ranks of the alternatives	Linear programming
OPA-F [9]	Fuzzy weights of the criteria	Fuzzy weights of the alternatives	Linear programming
OPA-IF	Fuzzy ranks of the criteria	Fuzzy ranks of the alternatives	Linear programming

Therefore, Section 3.2 introduces the OPA method; Section 3.3, concepts on TIFS; Section 3.4, OPA-F and OPA-IF; and Section 3.5, GRA. Section 5 presents the results of the OPA-IF-GRA application and the results of the AHP and BWM applications. It also presents a comparative analysis of correlation coefficients and compatibility indices, as performed by martino et al. [28]: Pearson correlation coefficient ρ for overall weights w_k^M and w_k^N of the alternatives k via methods M and N, Spearman correlation coefficient ρ_S for the ranks r_k^M and r_k^N, and Garuti compatibility index G, as in Equations (1)–(3).

$$\rho = \frac{\text{cov}(w_k^M, w_k^N)}{\sqrt{\text{var}(w_k^M)\text{var}(w_k^N)}} \tag{1}$$

$$\rho_S = \frac{\text{cov}(r_k^M, r_k^N)}{\sqrt{\text{var}(r_k^M)\text{var}(r_k^N)}} \tag{2}$$

$$G = \sum_{k=1}^{K} \left[\frac{\min(w_k^M, w_k^N)}{\max(w_k^M, w_k^N)} \frac{w_k^M + w_k^N}{2} \right] \tag{3}$$

3.2. Ordinal Priority Approach

OPA was proposed for multi-criteria group decision-making with I experts ranking K alternatives regarding J criteria. This work proposes OPA-IF-GRA for a case of individual decision-making, i.e., with $I = 1$. Thus, the index i becomes unnecessary in the work. However, in alignment with OPA theory, the j index will be kept for the criteria and the k index for the alternatives.

Unlike previously proposed MADM methods, OPA does not require weights for the criteria or the alternatives. OPA requires the ranks r_j for the criteria j and the ranks r_{jk} for the alternatives k on each criterion j, $\forall\, j = 1, 2, 3, \ldots\ J$ and $k = 1, 2, 3, \ldots\ K$.

The weights w_{jk} for the alternatives k on criteria j are obtained from the linear programming model presented in Equation (4):

$$\begin{array}{ll} \text{maximize } Z & \\ \text{subject to:} & Z \le jk(w_{jk}^r - w_{jk}^{r+1})\ \forall j,k,r \\ & Z \le jk(w_{jk})\ \forall j,k \\ & \sum_{j=1}^{J}\sum_{k=1}^{K} w_{jk} = 1 \\ & w_{jk} \ge 0\ \forall j,k \end{array} \tag{4}$$

Ataei et al. [6] noted that the decision-makers only provided the ranks for the criteria and the alternatives. Then, w_{jk} is the decision variable obtained by solving the model.

One reason the AHP is a leading MADM method is the availability of software to facilitate its application [29]. In this was, OPA's authors developed solvers for their method in diverse platforms such as MATLAB Online (https://www.mathworks.com/products/matlab.html, accessed on 1 July 2024), Microsoft Excel 2013 or greater, and web-based JavaScript [30].

3.3. Fuzzy Sets

Fuzzy sets [10], intuitionist fuzzy sets [31], or other variations are widely used to handle incomplete and imprecise information. An overview of fuzzy MCDM approaches is provided by Mardani et al. [32].

Assuming that $\mathbf{X} \neq \{\}$ is a given set, a type-1 fuzzy set, or simply a fuzzy set, $\tilde{A}$ is defined as $\tilde{A} = \{\langle x, \mu_{\tilde{A}} \rangle; x \in \mathbf{X}\}$, where $\mu_{\tilde{A}}$ is the membership function of x in $\tilde{A}$. As originally proposed by Zadeh [10], $\mu_{\tilde{A}}$ associates with each element x of $\mathbf{X}$ a real number in the interval [0, 1].

The triangular fuzzy set $\tilde{A} = (l, m, u)$ has the membership function $\mu_{\tilde{A}}$, as described in Equation (5):

$$\mu_{\tilde{A}}(x) = \begin{cases} (x-l)/(m-l) & \text{if } l \leq x \leq m \\ (u-x)/(u-m) & \text{if } m \leq x \leq u \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

An intuitionistic fuzzy set $\tilde{D}$ is defined as $\tilde{D} = \{\langle x, \mu_{\tilde{D}}, \nu_{\tilde{D}} \rangle; x \in \mathbf{X}\}$ where $\mu_{\tilde{D}}$ and $\nu_{\tilde{D}}$ are the membership and the non-membership functions of x in $\tilde{D}$, respectively. A Triangular Intuitionistic Fuzzy Set (TIFS) has the same membership function as a triangular fuzzy set. Also, the TIFS $\tilde{D} = (l, m, u; l', m, u')$ has a non-membership $\nu_{\tilde{D}}$ function, as described in Equation (6):

$$\nu(d) = \begin{cases} (m-x)/(m-l') & \text{if } l' \leq x \leq m \\ (x-m)/(u'-m) & \text{if } m \leq x \leq u' \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

If $\tilde{D}_1 = (l_1, m_1, u_1; l'_1, m_1, u'_1)$ and $\tilde{D}_2 = (l_2, m_2, u_2; l'_2, m_2, u'_2)$ are two TIFS, then:

1. $\tilde{D}_1 \oplus \tilde{D}_2 = (l_1 + l_2, m_1 + m_2, u_1 + u_2; l'_1 + l'_2, m_1 + m_2, u'_1 + u'_2)$
2. $\tilde{D}_1 \otimes \tilde{D}_2 = (l_1 l_2, m_1 m_2, u_1 u_2; l'_1 l'_2, m_1 m_2, u'_1 u'_2)$
3. $\tilde{D}_1 \ominus \tilde{D}_2 = (l_1 - u_2, m_1 - m_2, u_1 - l_2; l'_1 - u'_2, m_1 - m_2, u'_1 - l'_2)$
4. $\tilde{D}_1 \oslash \tilde{D}_2 = (l_1/u_2, m_1/m_2, u_1/l_2; l'_1/u'_2, m_1/m_2, u'_1/l'_2)$
5. $k \times \tilde{D}_1 = (kl_1, km_1, ku_1; kl'_1, km_1, ku'_1), k \in \mathbf{R}^+$

Equation (7) presents a formula for the defuzzification $\hat{D}$ of the TIFS $\tilde{D} = (l, m, u; l', m, u')$:

$$\hat{D} = \frac{l + l' + 4m + u + u'}{8}. \tag{7}$$

3.4. OPA with Fuzzy

The OPA method only requires ordinal information: the ranks for criteria and alternatives. For cases where the decision-makers cannot identify the ranks clearly, Mahmoudi et al. [9] proposed the OPA-F and extension of OPA with type-1 fuzzy sets, as presented in Tables 2 and 3.

Table 2. Importance of the alternatives in OPA-F.

Linguistic Variable	Triangular Fuzzy Set	Rank
Very poor	(0.9, 1, 1)	7
Poor	(0.7, 0.9, 1)	6
Medium poor	(0.5, 0.7, 0.9)	5
Fair	(0.3, 0.5, 0.7)	4
Medium good	(0.1, 0.3, 0.5)	3
Good	(0, 0.1, 0.3)	2
Very good	(0, 0, 0.1)	1

Table 3. Importance of the criteria in OPA-F.

Linguistic Variable	Triangular Fuzzy Set	Rank
Very low	(0.9, 1, 1)	7
Low	(0.7, 0.9, 1)	6
Medium-low	(0.5, 0.7, 0.9)	5
Medium	(0.3, 0.5, 0.7)	4
Medium-high	(0.1, 0.3, 0.5)	3
High	(0, 0.1, 0.3)	2
Very high	(0, 0, 0.1)	1

As OPA-F is an extension of OPA, the algorithm of OPA-F also includes a linear programming model. However, as observed in Tables 2 and 3, the triangle fuzzy sets are for the importance of the criteria and the alternatives. In other words, the triangle fuzzy sets were proposed for weights, not ranks.

The ranking in OPA-IF is not quantitative but qualitative or linguistical. Table 4 presents ITFS for ranks to OPA-IF for the criteria and the alternatives.

Table 4. Fuzzy ranks for criteria or alternatives.

Linguistic Variable	Intuitionistic Triangular Fuzzy Set	Defuzzification
Top rank	$(1,1,1;1,1,1)$	1
Middle-to-top rank	$(1,2,3;1,2,4)$	2.25
Middle rank	$(2,3,4;1,3,5)$	3
Bottom-to-middle rank	$(3,4,5;2,4,6)$	4
Bottom rank	$(4,5,6;3,5,7)$	5

The linear programming model for OPA-IF is a fuzzy linear programming model [4], as presented in Equation (8):

$$\begin{array}{rc} \text{maximize } \tilde{Z} & \\ \text{subject to:} & \tilde{Z} \leq jk(\tilde{w}_{jk}^{r} - \tilde{w}_{jk}^{r+1}) \; \forall j,k,r \\ & \tilde{Z} \leq jk(\tilde{w}_{jk}) \; \forall j,k,r \\ & \sum_{j=1}^{J} \sum_{k=1}^{K} \tilde{w}_{jk} = (1,1,1;1,1,1) \\ & u_{jk}^{\tilde{w}} \geq m_{jk}^{\tilde{w}} \geq l_{jk}^{\tilde{w}} \; \forall j,k \\ & l_{jk}^{\tilde{w}} \geq 0 \; \forall j,k \\ & u_{jk}^{\prime\tilde{w}} \geq m_{jk}^{\tilde{w}} \geq l_{jk}^{\prime\tilde{w}} \; \forall j,k \\ & l_{jk}^{\prime\tilde{w}} \geq 0 \; \forall j,k. \end{array} \tag{8}$$

In the proposed OPA-IF-GRA, OPA-IF determines the criteria weights and GRA the alternatives weights. Therefore, the fuzzy linear programming model can be simplified by focusing only on the weights $\tilde{w}_j$ for the criteria j from their ranks $\tilde{r}_j$ provided by the decision-maker, as in Equation (9).

$$\begin{array}{rc} \text{maximize } \tilde{Z} & \\ \text{subject to:} & \tilde{Z} \leq j(\tilde{w}_{j}^{r} - \tilde{w}_{j}^{r+1}) \; \forall j,r \\ & \tilde{Z} \leq jk(\tilde{w}_{jk}) \; \forall j,k \\ & \sum_{j=1}^{J} \tilde{w}_{j} = (1,1,1;1,1,1) \\ & u_{j}^{\tilde{w}} \geq m_{j}^{\tilde{w}} \geq l_{j}^{\tilde{w}} \; \forall j \\ & l_{jk}^{\tilde{w}} \geq 0 \; \forall j \\ & u_{j}^{\prime\tilde{w}} \geq m_{j}^{\tilde{w}} \geq l_{j}^{\prime\tilde{w}} \; \forall j \\ & l_{j}^{\prime\tilde{w}} \geq 0 \; \forall j \end{array} \tag{9}$$

3.5. Grey Relationship Analysis

GRA requires that the available data y_{jk} for alternatives k regarding criteria j be normalized to z_{jk} [26,27], as in Equation (10).

$$z_{jk} = \begin{cases} \frac{y_{jk} - \min_j(y_{jk})}{\max_j(y_{jk}) - \min_j(y_{jk})} & \text{if the greater the better} \\ \frac{\max_j(y_{jk}) - y_{jk}}{\max_j(y_{jk}) - \min_j(y_{jk})} & \text{if the lower the better} \end{cases} \tag{10}$$

Grey relational coefficients γ_{0k} compare two datasets $\mathbf{X}_0 = \{x_{01}, x_{02}, x_{03}, \ldots x_{0n}\}$ and $\mathbf{X}_k = \{x_{k1}, x_{k2}, x_{k3}, \ldots x_{kn}\}$, as in Equation (11), where the dynamic distinguishing coefficient $\xi \in [0,1]$ is often adopted as $\xi = 0.5$ [33].

$$\gamma_{0k}(j) = \frac{\min_k \min_j |x_0(j) - x_k(j)| + \xi \max_k \max_j |x_0(j) - x_k(j)|}{|x_0(j) - x_k(j)| + \xi \max_k \max_j |x_0(j) - x_k(j)|} \tag{11}$$

The grey relational grade Γ_{0k} is a weighted average as in Equation (12), where w_j is criterion j's weight.

$$\Gamma_{0k} = \int_{j=1}^{n} w_j \times \gamma_{0k}(j) \tag{12}$$

3.6. AHP and BWM

In the analytic hierarchy process [7], weights of criteria and alternatives are originally obtained with pairwise comparison matrices. First, the J criteria must be pairwise compared, forming the comparison matrix $\boldsymbol{A}$.

$$\boldsymbol{A} = \begin{bmatrix} a_{11} & a_{12} & \ldots & a_{1J} \\ a_{21} & a_{22} & \ldots & a_{2J} \\ \ldots & \ldots & & \ldots \\ a_{J1} & a_{J2} & \ldots & a_{JJ} \end{bmatrix}$$

The components of a comparison matrix $\boldsymbol{A} = [a_{ij}]$ are estimated ratios of the criteria's weights w_j. For example, a_{12} is an estimate of w_1/w_2. Table 5 presents the Saaty Scale, a linear 1–9 scale on which comparisons are based:

Table 5. Saaty Scale [3,7,34].

Intensity	Definition	Explanation
1	Equal importance	The two compared objects have the same importance
3	Moderate importance	Experience or judgment slightly favors one object over another
5	Strong importance	Experience or judgment strongly favors one object
7	Demonstrate importance	One object is very strongly favored, and its dominance is observed in practice
9	Absolute importance	The evidence favoring one object is of the highest possible order of affirmation

Note: Rationals from the scale may be used as comparisons when more consistency is required.

The Saaty Scale has three corollaries: $a_{ii} = 1, a_{ij} > 0$, and $a_{ji} = 1/a_{ij}, \forall\, i, j = 1, 2, 3, \ldots J$. Equation (13) presents the estimation of $\boldsymbol{w}$ from $\boldsymbol{A}$, where $\lambda_{\max}$ is $\boldsymbol{A}$'s principal eigenvalue.

$$\boldsymbol{A}\boldsymbol{w} = \lambda_{\max}\boldsymbol{w} \tag{13}$$

Consistency checking is one reason the AHP leads the publications on MADM, as consistency measures the quality of data input [34]. The consistency ratio CR is obtained with Equation (14), where n is the number of compared objects and RI is a random index.

$$CR = \frac{\lambda_{\max} - n}{RI(n-1)} \tag{14}$$

If $a_{ij} = w_i/w_j, \forall i, j = 1, 2, 3, \ldots n$, then $\lambda_{\max} = n$ and $CR = 0$. Otherwise, $\lambda_{\max} > n$ and $CR > 0$. Usually, comparison matrices with $CR \leq 0.1$ are accepted according to AHP Theory [7].

The alternatives weighting according to each criterion can also be conducted with comparison matrices. However, if there are available data on the importance, likelihood, performance, or preference of the alternatives, they can be used in an AHP application. According to Salomon [35], these data may be normalized in two ways: ideal (maximum equals one) or normal (sum equals one).

One significant disadvantage of the AHP application against other MADM methods is the need for pairwise comparisons [36]. A decision problem involving four criteria and six alternatives will require five comparison matrices. After all, 66 comparisons will be needed, 6 among the criteria and 60 among the alternatives. Therefore, to decrease the decision-making effort, Harker [37] proposed the Incomplete Pairwise Comparisons (IPC). This algorithm may reduce from $n(n-1)/2$ to n comparisons needed to fulfill a pairwise comparison matrix. With IPC, a four-criterion–six-alternative problem will require only 28 comparisons, 4 among criteria and 24 among alternatives. The reduction in the number of comparisons can be even greater than 57% in problems involving more alternatives and criteria. However, the algorithm's complexity, which involves matrix derivation, made its implementation in software models unfeasible. Therefore, the IPC became impractical.

Rezai [8] proposed the Best–Worst Method (BWM), an MADM method that works with fewer pairwise comparisons than AHP: only $2n-3$ comparison for n elements. Then, a four-criterion–six-alternative problem will require only 41 comparisons: 5 for the criteria and 36 for the alternatives. The 38% reduction in the number of comparisons may still sound attractive. However, the reduction in comparisons is achieved by previously ranking the objects. So, the total required input must be corrected, including two ranks (best and worst) for the criteria and 12 ranks (6×2) for the alternatives. Then, instead of AHP, which only requires 66 comparisons, BWM will require 41 comparisons and 14 ranks, totaling 55 judgments provided by an expert or the decision-maker. Therefore, the effort reduction from AHP by BWM for this problem is only 17%.

BWM application starts with identifying the best (B) and the worst (W) criterion. Rezai [8] suggested using the Saaty Scale to determine the preference of the best criterion over all other criteria. There will be $n-1$ pairwise comparisons: $a_{B1}, a_{B2}, a_{B3} \ldots a_{BJ}$; then, $n-2$ pairwise comparisons with the worst criterion $a_{1W}, a_{2W}, a_{3W} \ldots a_{JW}$.

The weights of the criteria are obtained by solving the linear programming model in Equation (15):

$$\begin{gathered} \text{minimize} \max_{j} \left[\left| \frac{w_B}{w_j} - a_{Bj} \right|, \left| \frac{w_j}{w_W} - a_{jW} \right| \right], \\ \text{subject to:} \\ \sum_{j=1}^{J} w_j = 1, \\ w_j \geq 0 \ \ \forall j = 1, 2, 3, \ldots J. \end{gathered} \tag{15}$$

In this article, AHP and BWM will require pairwise comparisons only to weigh the criteria. This is because there are available data for the alternatives.

4. Data Collection

With a restricted 30% operating bandwidth, Shi et al. [20] showcased a wideband BUPD on RO4003C substrate, emphasizing differential mode bandwidth and common mode suppression enhancement with an open stub. Gao et al. [21] raised concerns about affordability by introducing an open stub for a transmission zero and using coupled lines on a dielectric substrate to achieve an 80% fractional bandwidth (FBW). Feng and colleagues [22] achieved an 89.1% FBW on a dielectric substrate by employing symmetrical transmission lines to increase bandwidth. Zhuang et al. [23] used two-port coupled lines to improve the passband and stopband responses, and they were able to achieve a 28.5% FBW on a substrate with distinct properties. Practicality was emphasized when Bhowmik et al. [24] creatively developed an ultra-wideband BUPD with a 95.6% FBW on an affordable substrate. With a 60% FBW design at 8.5 GHz, Xu et al. [25] demonstrated sophisticated, multilayered structures using substrate-integrated waveguide technology. Those designs provide engineers and researchers various options based on particular application requirements and design priorities. Those options include a range of trade-offs in performance metrics such as FBW, common mode suppression, substrate characteristics, and implementation complexities. Table 6 presents the data for those different approaches of BUPD: Designs 1 to 6.

Table 6. Data of BUPD designs.

Design	CMS [dB]	FBW	Size [mm^2]	Substrate
1 [20]	−20	33%	0.15 × 0.95	3.38
2 [21]	−10	80%	0.52 × 0.26	2.65
3 [22]	−10	50%	0.50 × 0.20	2.65
4 [23]	−12.3	28.5%	0.58 × 0.58	3.66
5 [24]	−10	95%	0.36 × 0.52	4.40
6 [25]	−10	60%	3.15 × 2.40	3.38

For the criteria in Table 6, the decision-maker ranked substrate at the top, followed by tied CMS and FBW, which ranked middle-to-top. The last criterion ranked was size, at the bottom. These ranks in OPA-IF result in the weights presented in Table 7.

Table 7. Criteria weights with OPA-IF.

Criterion	Rank	Weight
CMS	Middle-to-top	0.208
FBW	Middle-to-top	0.208
Size	Bottom	0.116
Substrate	Top	0.468

Considering CMS and FBW as beneficial criteria, and size and substrate as non-beneficial, Table 8 presents the normalized data with Equation (8) for the designs on every criterion.

Table 8. Normalized data of BUPD designs.

Design	CMS	FBW	Size	Substrate
1	0	0.068	0.994	1
2	1	0.774	0.995	0.983
3	1	0.323	1	0.983
4	0.770	0	0.968	0.942
5	1	1	0.988	0
6	1	0.474	0	1

5. Results

Table 9 presents the grey relational grades for the designs and their rank. Therefore, the OPA-IF-GRA application indicates the selection of Design 2.

Table 9. Grey relational grades and ranks for BUPD Designs.

Design k	Γ_{0k}	Rank
1	0.725	5
2	0.919	1
3	0.865	2
4	0.740	4
5	0.685	6
6	0.816	3

By way of comparison, the decision maker was invited to compare the attributes two by two based on Saaty Scale [7]. Table 10 presents the comparisons and the weight vector of the criteria obtained with the normalization of the direct eigenvector according to the AHP theory.

Table 10. Criteria weights with AHP.

Criterion	CMS	FBW	Size	Substrate	Weight
CMS	1	1/6	8	1/2	0.179
FBW		1	4	1/4	0.251
Size			1	1/6	0.090
Substrate				1	0.480

Table 10 presents a pairwise comparison matrix with an eigenvalue approximated to 4.107. According to AHP theory, as it is close to $J = 4$, the inconsistency of the matrix can be accepted. Moving forward in the AHP application, Table 11 presents the weights of the alternatives regarding each criterion and their overall weights. The weights regarding the criteria were obtained normalizing data from Table 6, with ideal synthesis [35]. The overall weights were obtained with the average sum of the weights of the alternatives weighted by the criteria.

Table 11. Weights of BUPD designs with AHP.

Design	CMS	FBW	Size	Substrate	Overall
1	0.500	0.347	0.702	1	0.720
2	1	0.842	0.740	0.900	0.889
3	1	0.526	1	0.900	0.833
4	0.813	0.300	0.297	0.730	0.598
5	1	1	0.534	0.135	0.543
6	1	0.632	0.013	1	0.819

AHP application indicated the selection of Design 2 as the OPA-IF-GRA application.

A BWM application was simulated by taking some comparisons from Table 10. Table 12 presents the necessary pairwise comparisons involving substrate (best criterion) and size (worst criterion) and the criteria weights resulting from the BWM method.

Table 12. Criteria weights with BWM.

Criterion	Substrate	FBW	CMS	Size	Weight
Substrate	1	4	2	6	0.474
FBW				4	0.276
CMS				8	0.184
Size				1	0.066

BWM does not provide direct weighting for alternatives when data are available. Table 13 presents the overall weights for the BUPD designs combining BWM with AHP (Table 11) and GRA (Table 8).

Table 13. Overall weights of BUPD designs with BWM.

Design	BWM–AHP	BWM–GRA
1	0.697	0.708
2	0.898	0.892
3	0.825	0.822
4	0.705	0.598
5	0.683	0.560
6	0.814	0.833

BWM-AHP and BWM-GRA applications also indicated the selection of Design 2, as did the AHP and OPA-IF-GRA applications.

Table 14 presents the ranks obtained with the applications of the MADM methods. Designs 2 and 5 keep their first and sixth ranks for all methods. The other designs vary their ranks from second to third (Designs 3 and 6) and fourth to fifth (Designs 1 and 4). BWM–GRA and OPA-IF-GRA applications resulted in the same ranks.

Table 14. Ranks of BUPD designs with MADM applications.

Design	AHP	BWM–AHP	BWM–GRA	OPA-IF-GRA
1	4	4	5	5
2	1	1	1	1
3	2	3	2	2
4	4	5	4	4
5	6	6	6	6
6	3	2	3	3

Table 15 presents the values for the Pearson correlation coefficient ρ among BUPD designs' overall weights, resulting in AHP (Table 11), BWM–AHP and BWM–GRA (Table 13), and OPA-IF-GRA (Table 9).

Table 16 presents the statistical significance t_ρ of the values of ρ presented in Table 15.

Table 15. Correlation's statistical significance among MADM applications.

Method	BWM–AHP	BWM–GRA	OPA-IF-GRA
AHP	23.045	4.466	4.590
BWM–AHP		5.055	4.680
BWM–GRA			10.901

Table 16. Correlation of MADM applications.

Method	BWM–AHP	BWM–GRA	OPA-IF-GRA
AHP	0.996	0.913	0.917
BWM–AHP		0.930	0.920
BWM–GRA			0.984

The critical value for four degrees of freedom with 95%-significance is $t_{0.05} \approx 3.182$. As all values in Table 15 are greater than that, the overall weights of BUPD designs resulting from different MADM methods applications are strongly correlated.

Table 17 presents values for the Spearman correlation coefficient ρ_S among BUPD designs' ranks from MADM methods applications.

Table 17. Rank correlation of MADM applications.

Method	BWM–AHP	BWM–GRA	OPA-IF-GRA
AHP	0.943	0.943	0.943
BWM–AHP		0.886	0.886
BWM–GRA			1

Table 18 presents values for the Garuti Compatibility Index among the weights of the BUPD designs with different MADM methods.

Table 18. Garuti compatibility indices of MADM applications.

Method	BWM–AHP	BWM–GRA	OPA-IF-GRA
AHP	0.987	0.922	0.922
BWM–AHP		0.928	0.928
BWM–GRA			0.983

All values presented in Table 18 are greater than 0.9, the threshold proposed by Garuti [38]. Therefore, the overall weights of BUPD, resulting from different MADM methods applications, are compatible.

In summary, the proposed OPA-IF-GRA method selected the same BUPD design as two of the most popular MADM methods: AHP and BWM. The OPA-IF-GRA application resulted in ranks and weights with correlation and compatibility, resulting from AHP and BWM. The advantage of OPA-IF-GRA is the requirement of less information than AHP and BWM for the same decision, taking advantage of available data for alternatives.

6. Conclusions

An MCDM hybrid method, OPA-IF-GRA, was proposed and applied to an electronics engineering decision problem. OPA-IF, an extension of OPA, was proposed for criteria weighting, while GRA weighted the alternative. The hybrid method was fascinating for selecting BUPD designs, with the availability of performance data for the alternatives. As a result, Design 2 has a better overall performance for the particular case. Statistical analyses, with correlation coefficients and compatibility indices, validated this result.

The fuzzy sets presented in Table 4 are central to the proposed model. These sets were empirically proposed, focusing on a four-criteria problem. This proposal was validated for this case. However, verifying how the proposed sets fit different cases involving more criteria may be interesting. Therefore, the replication of the OPA-IF in diverse decision problems is a theme for future research.

The MCDM model was elicited from a literature review with four criteria for selecting BUPD design: CMS, FBW, size, and zubstrate. This is an innovative work on this electronics engineering subject. Another great innovation is the methodological combination of ITFS with GRA in a hybrid MDCM method. The problem was addressed with individual decision-making. Therefore, addressing the same problem with group decision-making is one theme for future research. This theme was already explored in the OPA original proposal, but it remains unexplored with the OPA-IF. Interestingly, data availability decreases the need for expert consultants to weigh the alternatives. However, multiple stakeholders may be consulted to weigh the criteria. Then, the problem advances from engineering to other areas, such as finance or marketing.

Staying in electronics engineering, OPA-IF-GRA has various applications in microwave circuit design, such as selecting components for a microwave circuit design. Engineers can define criteria such as cost, size, power consumption, performance parameters, and reliability. Microwave circuits often require the design of filters with specific characteristics, such as passband ripple, stopband attenuation, and group delay. For instance, engineers may assess different filter topologies, substrate materials, and manufacturing processes to select designs based on performance, cost, and manufacturability criteria. Another example is the design of microwave amplifiers, which considers criteria such as gain, noise figure, stability, power consumption, and linearity. Engineers can evaluate different amplifier topologies, transistor technologies, biasing schemes, and circuit configurations using MCDM methods to identify the optimal amplifier design that satisfies the desired objectives.

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Abbreviations

The following abbreviations and variables are used in this manuscript:

$\boldsymbol{A}$	Comparison matrix
$\tilde{A}$	Fuzzy set
AHP	Analytic hierarchy process
BUPD	Balanced to unbalanced power divider
BWM	Best–worst method
CR	Consistency ratio
CMS	Common mode suppression
$\tilde{D}$	TIFS
$\hat{D}$	Defuzzified $\tilde{D}$
FBW	Fractional bandwidth
G	Garuti Compatibility Index
GRA	Grey relational analysis
IPC	Incomplete pairwise comparisons
MADM	Multi-attribute decision-making
MCDM	Multi-criteria decision-making
OPA	Ordinal priority approach
OPA-F	OPA for fuzzy linguistic information
OPA-IF-GRA	Intuitionistic fuzzy OPA with GRA
RI	Random index
r_j	Rank of the criterion j
r_{jk}	Rank of the alternative i regarding the criterion j
t_α	Student's distribution at α
t_ρ	Statistical significance for ρ
TIFS	Triangular intuitionistic fuzzy set
$\boldsymbol{w}$	Vector of weights of criteria
w_j	Weight of the criterion j
w_{jk}	Weight of the alternative k regarding the criterion j
$\mathbf{X}$	Non-empty set
x	Element of $\tilde{A}$, $\tilde{D}$, or $\mathbf{X}$
y_{jk}	Available datum of the alternative k regarding the criterion j
z_{jk}	Normalized datum of the alternative k regarding the criterion j
Γ_{0k}	Grey relational grade for the alternative k
$\gamma_{0k}(j)$	Grey relational coefficient for the alternative k regarding the criterion j
$\lambda_{\max}$	$\boldsymbol{A}$'s principal eigenvalue
$\mu_{\tilde{D}}$	Membership function to $\tilde{D}$
$\nu_{\tilde{D}}$	Non-membership function to $\tilde{D}$
ξ	Dynamic distinguishing coefficient
ρ	Pearson correlation coefficient
ρ_S	Spearman rank correlation coefficient

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