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Optimizing Supply Chain Design under Demand Uncertainty with Quantity Discount Policy

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Abstract: In typical business situations, sellers usually offer discount schemes to buyers to increase overall profitability. This study aims to design a supply chain network under uncertainty of demand by integrating an all-unit quantity discount policy. The objective is to maximize the profit of the entire supply chain. The proposed model is formulated as a mixed integer nonlinear programming model, which is subsequently linearized into a mixed integer linear programming model and hence able to obtain a global solution. Numerical examples in the manufacturing supply chain where customer demand follows normal distributions are used to assess the effect of quantity discount policies. Key findings demonstrate that the integration of quantity discount policies significantly reduces total supply chain costs and improves inventory management under demand uncertainty, and decision makers need to decide a balance level between service levels and profits.

Keywords: supply chain design; optimization; demand uncertainty; quantity discount

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1. Introduction

Supply chain management (SCM) is essential for modern business success and competitiveness. Effective SCM leads to significant cost savings through optimized procurement, production, inventory management, and logistics. Optimal sourcing strategies and coordination with all levels of the supply chain can reduce material and product costs. Additionally, contemporary markets are characterized by ubiquitous volatilities and perpetual shifts in supply and demand, resulting in uncertainty in supply chain (SC) operations. Failure to integrate significant demand fluctuations into SC design can result in unmet customer demand, leading to loss of market share and increase in stock-out cost or incurring excessive inventory holding costs [1]. This negatively affects the operation of the SC business, profits, and customer service level [2,3].

Due to the important role of uncertainty, uncertainty has become a hot topic in the supply chain that many studies focus on. Peidro et al. [3] reviewed quantitative models for supply chain planning under uncertainty. The study classified sources of uncertainty into demand group, manufacturing group, and supply group. Simangunsong et al. [4] reviewed and further revised sources of uncertainty. The study identified 14 sources of uncertainty in the supply chain. In recent years, many studies [5–10] continued to cover new articles and trends related to uncertainty in the supply chain. Without considering uncertainty, it can harm and disrupt the supply chain.

A complex SC that includes many stakeholders can also lead to uncertainty [4]. To mitigate the uncertainty from SC networks, many studies investigate on the design of the SC network. Ates and Luzzini [11] provided a comprehensive framework of the complexity of the supply network, including its definition, effects, antecedents, and categorization. Govindan et al. [12] reviewed studies on supply chain networks under uncertainty and categorized uncertain environments and sources of uncertainty. Chen et al. [10] focused on a comprehensive literature on uncertainty analysis and optimization modeling in supply chain management. To mitigate uncertainty, this study investigates the problem of SC network design.

Among the uncertainty parameters in the SC network, the most popular uncertainty parameter in the supply chain network is the demand parameter [12]. Additionally, customer demand plays a central role in planning and using company resources. If demand uncertainty is not investigated and handled properly, it can cause SC disruption and unfulfillment of customers. Due to the inherent variability in the demand of customers, the demand can be in three environments in which decision makers know or do not know the probability distributions of the demand or have ambiguity and vagueness about the demand. Many studies [13–17] considered that demand will follow some probability distributions.

Stochastic programming is used with scenario modeling to expand on the multi-period, multi-product, and multi-level model, integrating delivery dates, supplier prices, and demand uncertainties [18]. A mixed integer linear programming model (MILP) is constructed, considering multiple sites, multiple products, and multiple periods under uncertainty of demand uncertainty addressed by the safety stock for mid-term SC production planning [19]. Agrawal et al. [20] devised a stochastic linear optimization model that determines manufacturing orders to maximize retailer profits using demand forecasts as parameters. By integrating batch production and capacity expansion, a linearized mathematical model is developed to plan product production, inventory, and transportation in an SC network [21].

Two popular stochastic methodologies are often used to represent demand uncertainty: the scenario-based approach and the distribution-based approach [22]. The former approach requires forecasting potential outcomes of the uncertain parameter, potentially leading to complexity when the scenarios are numerous or unanticipated. Robust programming is used to handle worst-case scenarios [23–25]. The latter approach assumes that the uncertain parameter follows some probability distributions. The demand is often modeled by a normal distribution with a given mean and standard deviation [12]. Therefore, this study adopts the normal distribution to deal with uncertainty of demand.

The ubiquity of the application of quantity discounts in practice results in a substantial number of related literature, from exhaustive reviews [26] to an extensive survey of various issues from a marketing research angle [27]. One way to address this challenge is to implement quantity discount policies. Dolan [27] nominated two types of nonlinear schedules, including (i) a two-block tariff or an incremental quantity discount (i.e., a per unit price p_1 is charged for any units up to a specified quantity x; if the purchased amount exceeds x, then the per unit price changes to p_2, which is lower than p_1, applied for surpassed units), and (ii) an all-unit quantity discount (i.e., if the purchased quantity exceeds quantity threshold, the price applies to all units, not just those incremental to the breakpoint).

A more comprehensive incremental and all-unit discount scheme is presented that incorporates multiple breakpoints, generally called quantity thresholds [28]. Heydari and Momeni [29] analyzed the benefits and challenges of a coalition of retailers when an all-unit quantity discount is applied. Wangsa et al. [30] also considered this discount scheme in an integrated inventory model to achieve a minimal integrated total cost. Pricing remains a sensitive issue in supplier–buyer relationships, often involving a trade-off relationship [31]. Tsai [32] expanded the application of various quantity discount schemes in an SC with multiple echelon and multiple periods and adopted a stepwise function to obtain a global optimum. Altintas et al. [33] examined how the all-unit quantity discount with a single break is considered under demand uncertainty, taking both the supplier's and the buyer's perspectives. This study adopts quantity discount policies to investigate the effect of pricing.

Effective SCM is crucial not only for cost reduction, but also for maintaining high service levels, ensuring demand fulfillment, and ensuring appropriate stock levels under demand uncertainty. Some studies [23–25] used robust optimization with quantity discount to find the optimal solution to the worst-case scenario. The method requires the forecasting methods to predict the range of uncertain parameters. However, many events follow normal distributions such as people's height, bread weights, and cow milk production [34] and do not require forecasting methods. Those events can be closely related to customer demand; that is, height is closely related to the size of clothes. Instead of using robust optimization, this study adopts the model of Lin et al. [21] to optimize the supply chain network under assumption that demands follow normal distributions.

The central problem is to optimize the total profit of the supply chain while finding the optimal balance between maintaining suitable service levels, capitalizing on cost advantages by determining material and product quantities, and considering quantity discounts in vendor–manufacturer relationships. This research proposes a comprehensive model when it comes to discount policies between vendors and manufacturers, as well as optimizing safety stocks to mitigate uncertainty. Since mixed integer nonlinear programming (MINLP) is challenging to solve optimally with a commercial optimization solver and the original model is MINLP [35], the model is then reconstructed as a mixed integer linear programming (MILP) model employing a linearization technique. The globally optimal solution of this study on SC design can help businesses maintain the equilibrium between cost savings and fulfillment of customer needs. The primary contributions of this research are considered as follows.

- Proposing a comprehensive SC network model considering discount policies under demand uncertainty;
- Applying the linearization technique to reconstruct the original model to an MILP formulation to solve and provide optimal global solutions.

2. Model and Methods

2.1. Modeling the Problem

This study develops a model that considers discount policies under demand uncertainty. The model uses a multi-echelon multi-product multi-period SC network system. The SC network (illustrated in Figure 1) consists of 4 echelons, which are V vendors, P producers, D distributors, and C clients. This paper inherits the model of Lin et al. [21]. However, we have excluded batch production and capacity expansion to simplify the model and focus on the core aspects. The extension of this model is the integration of an all-unit discount policy.

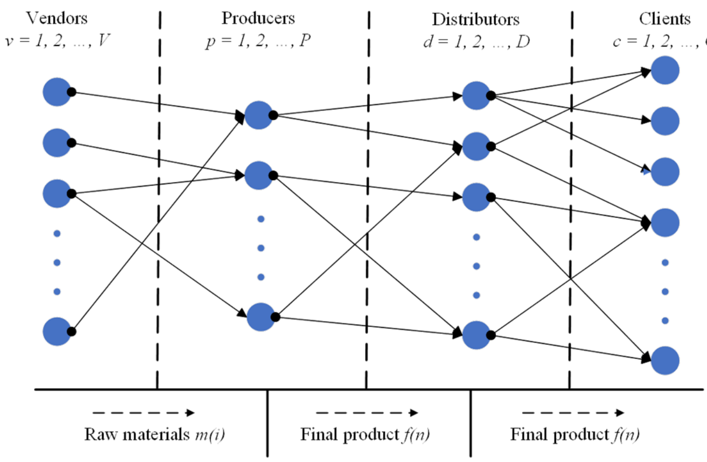

Figure 1. The supply chain network considered in this study.

The model aims to maximize the total profit of the supply chain over the planning timeframe. The objective includes the costs of vendors, producers, and distributors and the income generated from selling products. Producers purchase raw materials $m(i)$ from vendors under their offered discount pricing schemes. The finished product $f(n)$ is then delivered to distributors and sold to customers. This model assumes that inventory is stored only in the warehouses of producers and distributors. The assumptions related to demand are that each product follows a normal distribution as predicted and independent of each other, with the standard deviation and mean of the product $f(n)$ being known.

2.2. Mathematical Model

The nomenclature utilized in this model refers to Lin et al. [21]. This study models a multi-echelon, multi-product, and multi-period supply chain system under demand uncertainty. Demand uncertainty is represented by a normal distribution for each product, with known mean and standard deviation values. The supply chain operates under an all-unit quantity discount policy, wherein the price per unit decreases once an order exceeds a predefined quantity threshold. This integration of demand uncertainty and quantity discount policies aims to optimize both cost efficiency and service levels across the entire supply chain. The following subsections present the original model with batch production excluded and its extension.

First, there is a list of indices that serve to uniquely identify and differentiate between the various parameters and variables of the model.

Indices

v	Vendors that supply raw materials to producers, $v = 1, 2, \ldots, V$;
p	Producers producing finished products delivered to distributors, $p = 1, 2, \ldots, P$;
d	Distributors selling finished products to clients, $d = 1, 2, \ldots, D$;
c	Clients, $c = 1, 2, \ldots, C$;
$m(i)$	Raw materials, indicated by types $i = 1, 2, \ldots, I$;
$f(n)$	Finished products, indicated by types $n = 1, 2, \ldots, N$;
t	Examined phase, $t = 1, 2, \ldots, T$.

Secondly, all parameters are listed as follows.

Parameters

$TC_{tvp}^{m(i)}$	Transportation cost (per unit) for vendor v to ship raw materials $m(i)$ to producer p in phase t;
$TC_{tpd}^{f(n)}$	Transportation cost (per unit) for producer p to ship product $f(n)$ to distributor d in phase t;
$TC_{tdc}^{f(n)}$	Transportation cost (per unit) for distributor d to ship product $f(n)$ to clients c in the phase t;
$MC_{tv}^{m(i)}$	Unit price of raw materials $m(i)$ offered by vendor v in the phase t;
$C_{tp}^{f(n)}$	Production cost (per unit) of product $f(n)$ for producer p in phase t;
$IC_{tp}^{m(i)}$	Inventory storing cost (per unit) of raw materials $m(i)$ for producer p in phase t;
$IC_{tp}^{f(n)}$	Inventory storing cost (per unit) of product $f(n)$ for producer p in phase t;
$IC_{td}^{f(n)}$	Inventory storing cost (per unit) of product $f(n)$ for distributor d in phase t;
$SC_{tp}^{m(i)}$	Stock-out cost (per unit) of raw materials $m(i)$ sent to producer p in phase t;
$SC_{td}^{f(n)}$	Stock-out cost (per unit) of product $f(n)$ sent to distributor d in phase t;
$SC_{tc}^{f(n)}$	Stock-out cost (per unit) of product $f(n)$ sent to client c in phase t;
$CQ_{f(n)}^{m(i)}$	Amount of raw materials $m(i)$ needed to make one unit of product $f(n)$;
$GP_{tc}^{f(n)}$	Charge amount of client c when buying product $f(n)$ in phase t;
T_{vp}	Delivery time from vendor v to producer p;
T_{pd}	Delivery time from producer p to distributor d;
T_{dc}	Delivery time from distributor d to client c;
TP_p	Time taken for producer p to manufacture product $f(n)$;
LT_p	Lead time at producer p, starting from the time raw materials being ordered to the time producer p receiving them;
LT_d	Lead time at distributor d, starting from the time finished products being ordered to the time distributor d receiving them;
LT_c	Lead time at client c, starting from the time finished products being ordered to the time client c receiving them;
$D_{tc}^{f(n)}$	Amount of product $f(n)$ needed by client c in phase t.

Finally, the decision variables that need to be determined are listed below.

Decision variables

$R_{tvp}^{m(i)}$	Quantity of raw materials $m(i)$ sent from vendor v to producer p in phase t;
$R_{tpd}^{f(n)}$	Quantity of product $f(n)$ sent from producer p to distributor d in phase t;
$R_{tdc}^{f(n)}$	Quantity of product $f(n)$ sent from distributor d to client c in phase t;
$O_{tvp}^{m(i)}$	Quantity of raw materials $m(i)$ that producer p places an order from vendor v in phase t;
$O_{tpd}^{f(n)}$	Quantity of product $f(n)$ that distributor d places an order from producer p in phase t;
$O_{tdc}^{f(n)}$	Quantity of product $f(n)$ that client c places and order from distributor d in phase t;
$I_{tp}^{m(i)}$	Raw materials $m(i)$ inventory stored in producer p in phase t;
$I_{tp}^{f(n)}$	Product $f(n)$ inventory stored in producer p in phase t;
$I_{td}^{f(n)}$	Product $f(n)$ inventory stored in distributor d in phase t;
$DQ_{td}^{f(n)}$	Total quantity of product $f(n)$ available for distributor d in phase t;
$SS_{td}^{f(n)}$	Product $f(n)$ that distributor d holds as safety stock in phase t;
$Q_{tp}^{f(n)}$	Quantity of product $f(n)$ that producer p plans to produce in phase t;
$AQ_{tp}^{f(n)}$	Actual quantity of product $f(n)$ that producer p produces in phase t;
$SQ_{tvp}^{m(i)}$	Quantity of raw materials $m(i)$ out of stock from vendor v at producer p in phase t;
$SQ_{tp}^{m(i)}$	Quantity of raw materials $m(i)$ out of stock at producer p in phase t;
$SQ_{td}^{f(n)}$	Quantity of product $f(n)$ out of stock at distributor d in phase t;
$SQ_{tc}^{f(n)}$	Quantity of product $f(n)$ out of stock at client c in phase t;
$Cost$	Total cost of vendors, producers, distributors, and clients.

As the objective is to maximize the total profit of the SC over the planning timeframe, the objective function can be written as follows.

$$\text{Maximize} \sum_{t=1}^{T} \sum_{c=1}^{C} \sum_{n=1}^{N} GP_{tc}^{f(n)} \left(\sum_{d=1}^{D} R_{tdc}^{f(n)} \right) - Cost \tag{1}$$

The objective function contains two parts: (i) the total income of the SC generated from sales of finished products, which is calculated by the price charge of customers $GP_{tc}^{f(n)}$ multiplied by the total amount of sold products $R_{tdc}^{f(n)}$, and (ii) the total cost of vendors, producers, distributors, and clients, which is detailed below:

$$\begin{aligned}
&Cost = TC_V + SC_V + MC_P + PC_P + IC_P + TC_P + SC_P + IC_D + TC_D + SC_D \\
&= \sum_{t=1}^{T} \sum_{p=1}^{P} \left(\sum_{v=1}^{V} \sum_{i=1}^{I} R_{tvp}^{m(i)} TC_{tvp}^{m(i)} \right) + \sum_{t=1}^{T} \sum_{p=1}^{P} \sum_{i=1}^{I} SQ_{tp}^{m(i)} SC_{tp}^{m(i)} \\
&+ \sum_{t=1}^{T} \sum_{p=1}^{P} \left(\sum_{v=1}^{V} \sum_{i=1}^{I} R_{tvp}^{m(i)} MC_{tv}^{m(i)} \right) + \sum_{t=1}^{T} \sum_{p=1}^{P} \sum_{n=1}^{N} Q_{tp}^{f(n)} C_{tp}^{f(n)} \\
&+ \sum_{t=1}^{T} \left(\sum_{p=1}^{P} \sum_{i=1}^{I} I_{tp}^{m(i)} IC_{tp}^{m(i)} + \sum_{p=1}^{P} \sum_{n=1}^{N} I_{tp}^{f(n)} IC_{tp}^{f(n)} \right) \\
&+ \sum_{t=1}^{T} \sum_{p=1}^{P} \sum_{d=1}^{D} \sum_{n=1}^{N} R_{tpd}^{f(n)} TC_{tpd}^{f(n)} + \sum_{t=1}^{T} \sum_{d=1}^{D} \sum_{n=1}^{N} SQ_{td}^{f(n)} SC_{td}^{f(n)} \\
&+ \sum_{t=1}^{T} \sum_{d=1}^{D} \sum_{n=1}^{N} I_{td}^{f(n)} IC_{td}^{f(n)} + \sum_{t=1}^{T} \sum_{d=1}^{D} \sum_{c=1}^{C} \sum_{n=1}^{N} R_{tdc}^{f(n)} TC_{tdc}^{f(n)} \\
&+ \sum_{t=1}^{T} \sum_{c=1}^{C} \sum_{n=1}^{N} SQ_{tc}^{f(n)} SC_{tc}^{f(n)}
\end{aligned}$$

where

Vendors' costs:	TC_V	Total delivery cost
	SC_V	Total stock-out cost
Producers' costs:	MC_P	Total procurement cost
	PC_P	Total production cost as planned
	IC_P	Total inventory cost including raw materials and finished products
	TC_P	Total delivery cost
	SC_P	Total stock-out cost
Distributors' costs:	IC_D	Total inventory cost
	TC_D	Total delivery cost
	SC_D	Total stock-out cost

Model 1 with the objective (1) is subject to constraints (2)–(20) which are discussed next.

(i) Inventory constraints: Inventory in producers and distributors follows the principle that the quantity of inventory in the next period must be equal to the current quantity plus the total purchased amount and then minus the total amount consumed or sold in this period. Therefore, the inventory constraints of raw materials $m(i)$ and finished product $f(n)$ at producer p, and finished product $f(n)$ at distributor d, are shown below, respectively:

$$I_{tp}^{m(i)} + \sum_{v=1}^{V} R_{(t-T_{vp})vp}^{m(i)} - \sum_{n=1}^{N} Q_{tp}^{f(n)} CQ_{f(n)}^{m(i)} = I_{(t+1)p}^{m(i)}, \forall t, p, i \quad (2)$$

$$I_{tp}^{f(n)} + Q_{(t-TP_p)p}^{f(n)} - \sum_{d=1}^{D} R_{tpd}^{f(n)} = I_{(t+1)p}^{f(n)}, \forall t, p, n \quad (3)$$

$$I_{td}^{f(n)} + \sum_{p=1}^{P} R_{(t-T_{pd})pd}^{f(n)} - \sum_{c=1}^{C} R_{tdc}^{f(n)} = I_{(t+1)d}^{f(n)}, \forall t, d, n \quad (4)$$

(ii) Stock-out constraints: The stock-out amount at the current echelon at period $t + LT_m$ is constrained to be equal to the quantity that is ordered at period t minus the quantity delivered from the preceding echelon at period $t + LT_m - T_{sm}$, which means the following:

$$O_{tvp}^{m(i)} - R_{(t+LT_p-T_{vp})vp}^{m(i)} = SQ_{(t+LT_p)vp}^{m(i)}, \ \forall\ t,\ p,\ i \quad (5)$$

$$\sum_{v=1}^{V} (O_{tvp}^{m(i)} - R_{(t+LT_p-T_{vp})vp}^{m(i)}) = SQ_{(t+LT_p)p}^{m(i)}, \ \forall\ t,\ p,\ i \quad (6)$$

$$\sum_{p=1}^{P} (O_{tpd}^{f(n)} - R_{(t+LT_d-T_{pd})pd}^{f(n)}) = SQ_{(t+LT_d)d}^{f(n)}, \ \forall\ t,\ d,\ n \quad (7)$$

$$\sum_{d=1}^{D} (O_{tdc}^{f(n)} - R_{(t+LT_c-T_{dc})dc}^{f(n)}) = SQ_{(t+LT_c)c}^{f(n)}, \ \forall\ t,\ c,\ n \quad (8)$$

Although Equation (5) specifies the quantity of raw materials $m(i)$ that is out of stock from each vendor v at producer p in stage $t + LT_m$, Equation (6) indicates the constraint for the quantity of raw materials $m(i)$ stocking from all vendors at producer p. Similarly, Equations (7) and (8) represent the stock-out quantity of finished products $f(n)$ at distributors and clients in turn. Additionally, the quantities of materials and products received should not exceed those ordered to minimize the cost.

(iii) Mass balance constraints: These constraints ensure an equilibrium and coherence between order and demand quantities in the production and distribution process. Equation (9) says that at any given time t, the total quantity ordered of product $f(n)$ by client c from all distributors should be equal to its demand at the time $t + LT_c$. Equation (10) ensures that the available quantity of product $f(n)$ in phase $t + LT_c - T_{dc}$ (i.e., when distributor d delivers product $f(n)$ to all customers) in distributor d is larger than or equivalent to the amount that all customers order in period t adding the safety stock at $t + LT_c - T_{dc}$. Equation (11) signifies that the total quantity ordered by distributor d is equal to the quantity available for distributor d at a later period (i.e., $t + LT_d$) subtracting the inventory held by distributor d at that time. Equation (12) implies that the total order quantity of materials $m(i)$ from all vendors is equal to the quantity needed for production minus the inventory of raw materials in the producer at that time.

$$\sum_{d=1}^{D} O_{tdc}^{f(n)} = D_{(t+LT_c)c}^{f(n)}, \ \forall\ t,\ c,\ n \quad (9)$$

$$DQ_{(t+LT_c-T_{dc})d}^{f(n)} \geq \left(\sum_{c=1}^{C} O_{tdc}^{f(n)}\right) + SS_{(t+LT_c-T_{dc})d}^{f(n)}, \ \forall\ t, d, n \quad (10)$$

$$\sum_{p=1}^{P} O_{tpd}^{f(n)} = DQ_{(t+LT_d)d}^{f(n)} - I_{(t+LT_d)d}^{f(n)}, \ \forall\ t,\ d,\ n \quad (11)$$

$$\sum_{v=1}^{V} O_{(t-LT_p-TP_p)vp}^{m(i)} = \sum_{n=1}^{N} CQ_{f(n)}^{m(i)} \left(\sum_{d=1}^{D} O_{(t-LT_d+T_{pd})pd}^{p(n)}\right) - I_{(t-TP_p)p}^{m(i)}, \ \forall\ t, p,\ i \quad (12)$$

(iv) Production planning constraints: Equation (13) implies that the quantity of product $f(n)$ planned at time $t - TP_m$ must actually be produced in phase t.

$$Q^{f(n)}_{(t-TP_p)p} = AQ^{f(n)}_{tp},\ \forall\ t,\ p,\ n \tag{13}$$

(v) Lower-bound and upper-bound constraints: The constraints shown below set the limit for the amount of raw materials and finished products moving along the SC, since warehouses and transportation vehicles are limited in space.

$$0 \le R^{m(i)}_{tvp} \le \overline{R^{m(i)}_{tvp}},\ \forall\ t,\ v,\ p,\ i \tag{14}$$

$$0 \le R^{f(n)}_{tpd} \le \overline{R^{f(n)}_{tpd}},\ \forall\ t,\ p,\ d,\ n \tag{15}$$

$$0 \le R^{f(n)}_{tdc} \le \overline{R^{f(n)}_{tdc}},\ \forall\ t,\ d,\ c,\ n \tag{16}$$

$$\underline{I^{m(i)}_{tp}} \le I^{m(i)}_{tp} \le \overline{I^{m(i)}_{tp}},\ \forall\ t, p,\ i \tag{17}$$

$$\underline{I^{f(n)}_{tp}} \le I^{f(n)}_{tp} \le \overline{I^{f(n)}_{tp}},\ \forall\ t, p,\ n \tag{18}$$

$$\underline{I^{f(n)}_{td}} \le I^{f(n)}_{td} \le \overline{I^{f(n)}_{td}},\ \forall\ t, d, n \tag{19}$$

(vi) Distribution-based constraints for demand uncertainty: Customer demands are assumed to follow a normal distribution with a specified mean and standard deviation. Safety stock is used to provide a certain degree of buffer against the deviation of the amount of demand variability. It is calculated by multiplying the standard deviation of the demand $\left(STD^{f(n)}_{td}\right)$, lead time (LT_d), and the number of standard deviations of demand variability—represented by the z-score (z):

$$SS^{f(n)}_{td} = z\left(STD^{f(n)}_{td}\right)\sqrt{LT_d},\ \forall\ t,\ d,\ n \tag{20}$$

This study extends the model of Lin et al. [21] by applying an all-unit discount quantity for the unit price of raw materials $m(i)$, which is denoted as $MC^{m(i)}_{tv}$. However, all-unit discount quantity can add nonlinear terms to the model; hence, the model can be unsolvable by commercial solvers. A technique developed by Tsai [32] is used to transform nonlinear terms to linear terms without changing any assumption of the model. Hence the globally optimal solutions of the model can be found by commercial optimization solvers.

Remark 1 (Tsai [32])**.** *The step function that determines the price according to the quantity $P(Q)$ (Figure 2) can be expressed as follows:*

$$P(Q) = \sum\nolimits_{j=1}^{a-1} P_j u_j,$$

where

$$\sum\nolimits_{j=1}^{a-1} u_j Q_j \le Q \le \sum\nolimits_{j=1}^{a-1} u_j Q_{j+1},$$

$$\sum\nolimits_{j=1}^{a-1} u_j = 1,\ u_j \in \{0,1\}.$$

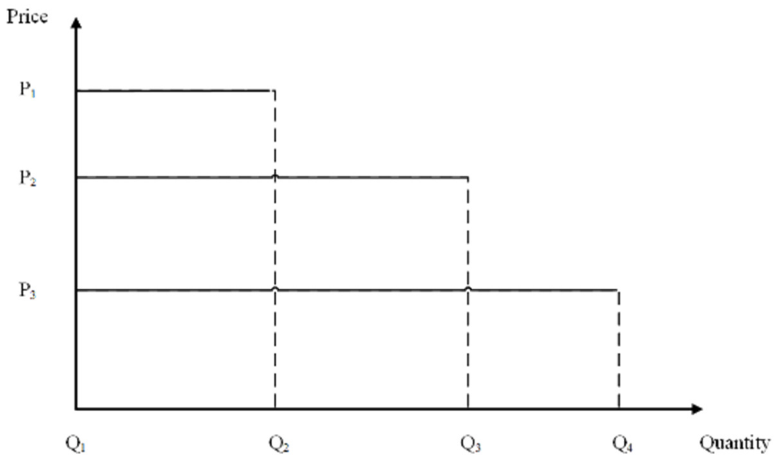

Figure 2. Step quantity discount function.

Proposition 1 (Tsai [32]). *A product term $z = uf(x)$ can be converted into linear inequalities as follows:*

(i) $E(u-1) + f(x) \leq z \leq E(1-u) + f(x)$,

(ii) $-Eu \leq z \leq Eu$,

where $u \in \{0,1\}$, z is an unrestricted in sign variable, and E is a large constant.

Extension 1. *Calculating procurement cost using quantity discount function:*

The unit price of the raw material $m(i)$ offered by the vendor v in phase t is expressed as follows:

$$MC_{tv}^{m(i)} = \begin{cases} MC_{1tv}^{m(i)} \ if \ R_{1tvp}^{m(i)} < R_{tvp}^{m(i)} \leq R_{2tvp}^{m(i)} \\ MC_{2tv}^{m(i)} \ if \ R_{2tvp}^{m(i)} < R_{tvp}^{m(i)} \leq R_{3tvp}^{m(i)} \\ \cdots \\ MC_{(a-1)tv}^{m(i)} \ if \ R_{(a-1)tvp}^{m(i)} < R_{tvp}^{m(i)} \leq R_{atvp}^{m(i)} \end{cases}$$

This is equivalent to

$$MC_{tv}^{m(i)} = \sum_{j=1}^{a-1} MC_{jtv}^{m(i)} u_{jtv}^{m(i)}, \ \forall \ t, v, \ i, u_{jtv}^{m(i)} \in \{0,1\}.$$

The procurement cost can be transformed into a step quantity discount function by deploying the remark and proposition mentioned above. This results in the expression of the procurement cost function as follows.

$$\text{Minimize} \sum_{t=1}^{T} \sum_{p=1}^{P} \sum_{v=1}^{V} \sum_{i=1}^{I} \sum_{j=1}^{a-1} MC_{jtv}^{m(i)} \gamma_j^{m(i)}$$

subject to

$$\sum_{j=1}^{a-1} u_{jtv}^{m(i)} R_{jtvp}^{m(i)} \leq R_{tvp}^{m(i)} \leq \sum_{j=1}^{a-1} u_{jtv}^{m(i)} R_{(j+1)tvp}^{m(i)}, \ \forall \ t, \ v, p, \ i \tag{21}$$

$$\sum_{j=1}^{a-1} u_{jtv}^{m(i)} = 1, \ u_{jtv}^{m(i)} \in \{0,1\} \tag{22}$$

$$E\left(u_{jtv}^{m(i)} - 1\right) + R_{tvp}^{m(i)} \leq \gamma_j^{m(i)} \leq E\left(1 - u_{jtv}^{m(i)}\right) + R_{tvp}^{m(i)}, \ \forall t, \ v, p, \ i, \ j \tag{23}$$

$$-Eu_{jtv}^{m(i)} \leq \gamma_j^{m(i)} \leq Eu_{jtv}^{m(i)}, \ \forall t, \ v, \ i, \ j \tag{24}$$

where $\gamma_j^{m(i)} = R_{tvp}^{m(i)} u_{jtv}^{m(i)}$, and E is a large constant.

While constraints (21) and (22) ensure the principles of the all-unit discount scheme, the latter verify the linearity of the procurement cost function. Model M1 can then be reformulated as the following linear model M2:

$$\text{Maximize} \sum_{t=1}^{T} \sum_{c=1}^{C} \sum_{n=1}^{N} GP_{tc}^{f(n)} \left(\sum_{d=1}^{D} R_{tdc}^{f(n)} \right) - Cost \tag{25}$$

subject to constraints (2) to (24).

Solving Model 2 with the objective (25) provides the global optimality of the SC under demand uncertainty. By linearizing Model 1, Model 2 adds some variables and constraints to decrease the complexity of the nonlinear model. The computational efficiency of the proposed technique is presented in the next section.

3. Numerical Example

The proposed model is applied to a small-scale production planning problem to demonstrate its effectiveness. This study adopts and revises the data used by Lin et al. [21]. The scenario comprises two vendors v_1 and v_2 that provide two types of raw materials $m(1)$ and $m(2)$, one type of producer p_1 that produces one product $f(1)$, and one distributor d_1 that distributes products to three clients c_1, c_2, and c_3. The planning time horizon spans eight time periods. It takes a period of time for the producer to produce a product (i.e., $TP_p = 1$). The lead times at the producer, distributor, and client are one, three, and four, respectively ($LT_p = 1$, $LT_d = 3$, $LT_c = 4$). For the production of each product $f(1)$, the producer consumes two units of materials $m(1)$ and four units of materials $m(2)$, indicated by $CQ_{f(1)}^{m(1)} = 2$ and $CQ_{f(1)}^{m(2)} = 4$, respectively. It costs 0.8 units of currency to manufacture the product $f(1)$, which means $C_{tp}^{m(1)} = 0.8$. The details of the data are described in the following sections.

3.1. Raw Materials Price

The unit prices of the raw materials offered by the two vendors are detailed in Table 1. Vendor v_2 introduces an all-unit quantity discount scheme for material $m(1)$. The pricing scheme has three quantity intervals that correspond to three price levels.

Table 1. Unit price of raw materials offered by vendors.

Vendor	Raw Material	Quantity Discount	$MC_{tv}^{m(i)}$
v_1	$m(1)$	No	20
	$m(2)$	No	12
v_2	$m(1)$	0–100	21
		101–300	20
		Over 300	19
	$m(2)$	No	14

3.2. Inventory

Initially, the producer stores the inventories of raw materials $m(1)$ and $m(2)$, as well as the finished product $f(1)$, ranging from 140, 280, and 20 units, respectively (i.e., $I_{t1p_1}^{m(1)} = 140$, $I_{t1p_1}^{m(2)} = 280$, $I_{t1p_1}^{f(1)} = 20$). The starting product inventory level at distributor d_1 is indicated as $I_{t1d_1}^{f(1)} = 60$. The upper and lower bounds of the inventory in producers and distributors are shown in Table 2.

Table 2. Lower and upper bounds of the inventory at producer and distributor.

	Producer			Distributor
		p_1		d_1
	$m(1)$	$m(2)$	$f(1)$	$f(1)$
Lower bound	20	40	20	20
Upper bound	4000	8000	700	1000

Unit costs associated with inventory and stock-outs are detailed in Table 3. Additionally, Table 3 lists the price of the product sold to each client.

Table 3. Relevant unit costs and unit price in the supply chain.

	Producer	Distributor	Client		
	p_1	d_1	c_1	c_2	c_3
Inventory holding cost (per unit)	$m(1)$ 0.06 $m(2)$ 0.04	$f(1)$ 0.04			
	$f(1)$ 0.02				
Stock-out cost (per unit)	$m(1)$ 44 $m(2)$ 32	$f(1)$ 500	680	640	600
Product sale price (per unit)			120	240	360

Regarding the safety stock considerations at the distributor, the safety stock factor z given in Table 4 reflects different service levels under normally distributed demand. This study primarily explores outcomes aligned with a service level of 98%. However, the next section also presents objective values corresponding to different service levels to reveal the effect of service-level variations on the total profit of the SC.

Table 4. Z-score corresponding to different service levels [22].

	Desired Service Level				
	95	97	98	99	99.9
z-score	1.65	1.88	2.05	2.33	3.09

3.3. Product Demand

Demands are assumed during the initial four periods, as is certainly known. However, from the fifth period onward, demands are anticipated using the simple moving average technique. Average demand over a four-period period is forecasted for the subsequent period. The specific quantities of the three clients are listed in Table 5, where the total demand column represents the aggregate demands of all clients, and the final column indicates the standard deviation of the demands.

Table 5. Customer demands over eight periods of time.

Time Period	Client			Total Demand	Standard Deviation
	c_1	c_2	c_3	$\sum$	$STD_{td_1}^{f(1)}$
t_1	100	60	160	320	
t_2	200	140	120	460	
t_3	240	40	100	380	
t_4	320	100	140	560	
t_5	215	85	130	430	103.92
t_6	244	92	123	459	75.88

Table 5. *Cont.*

Time Period	Client			Total Demand	Standard Deviation
	c_1	c_2	c_3	$\sum$	$STD_{td_1}^{f(1)}$
t_7	255	80	124	459	75.87
t_8	259	90	130	479	57.00

3.4. Transportation

The shipping time from the vendor to the producer, the producer to the distributor, and the distributor to the client is one period, denoted as $T_{vp} = T_{pd} = T_{dc} = 1$. Furthermore, the upper bound and the transportation costs for all levels are, in turn, shown in Tables 6 and 7.

Table 6. Upper bound of the amount of delivery from vendor to producer, from producer to distributor, and from distributor to client.

		Producer	Distributor	Clients		
		p_1	d_1	c_1	c_2	c_3
Vendor	s_1	$m(1)$ 4000 $m(2)$ 8000				
	s_1	$m(1)$ 4000 $m(2)$ 8000				
Producer	p_1		900			
Distributor	d_1			1000	1000	1000

Table 7. Transportation unit cost.

To From		Producer p_1	Distributor d_1	c_1	Clients c_2	c_3
Vendor	v_1	0.02				
	v_2	0.02				
Producer	p_1		0.04			
Distributor	d_1			0.6	0.8	1

3.5. Numerical Results

The problem is solved by a device with an Intel Core i5-8265U CPU at 1.60 GHz and 4 GB memory, using Gurobi Optimizer version 11.0.0.

3.5.1. Numerical Results with an All-Unit Quantity Discount Applied

The solution obtained ensures global optimality by employing a step quantity discount function to linearize the model despite the increase in the number of variables and constraints. The solved model yields an objective value of 207,131.68, achieved within a CPU time of 1.27 s.

Figure 3 illustrates the flow of the SC network spanning from period t_1 to period t_5. To meet the demands of client c_1, c_2, and c_3 in period t_5, anticipated at 215, 85, and 130 units, respectively, 740 units of materials $m(1)$ and 1480 units of materials $m(2)$ are dispatched to producer p_1 from vendor v_2 and vendor v_1 in turn. Despite its higher base price, vendor v_2 secures the selection due to its quantity discount offer. This also reduces the procurement cost to 170,796.

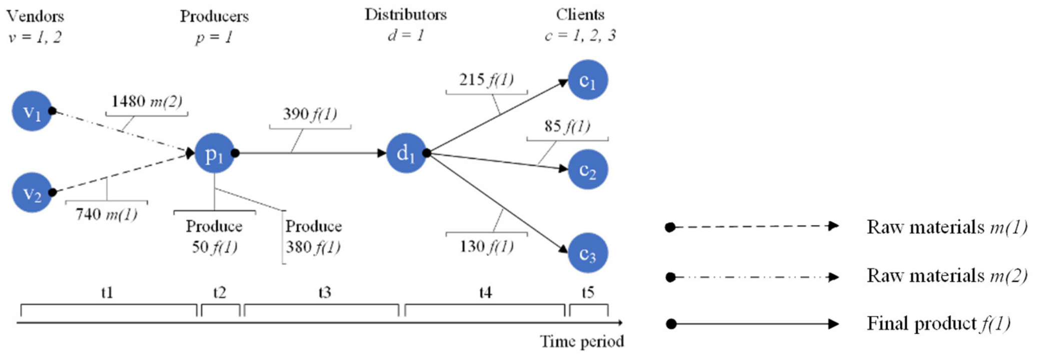

Figure 3. Optimal supply chain network with an all-unit quantity discount, from phase 1 to phase 5 (i.e., when clients receive products for the first time).

After receiving raw materials, p_1 starts producing products $f(1)$ during period t_2 and transports 390 units to distributor d_1 in the next phase. These products are distributed to customers in period t_4, with delivery completed in period t_5. Figure 4 indicates the flow of goods within the SC from period t_2 to period t_8. Table 8 indicates the quantities of orders, deliveries, and production for every entity in the SC network, resulting from the solved model.

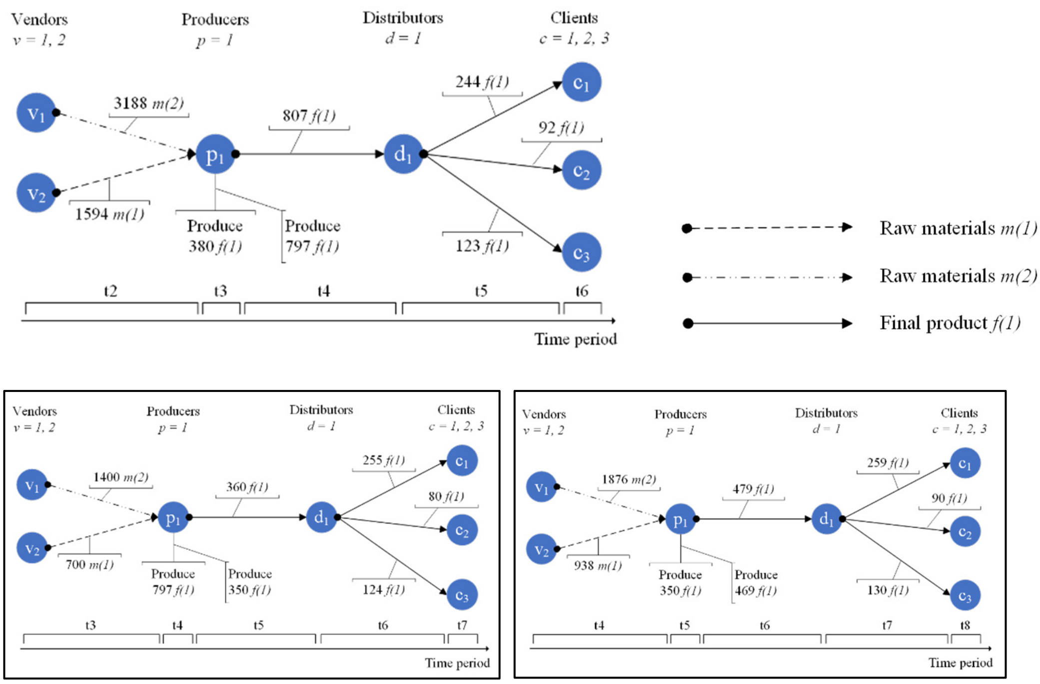

Figure 4. Optimal supply chain network with an all-unit quantity discount, from period t_2 to period t_8.

Figure 5 illustrates the inventory dynamics at the producer and distributor. Throughout the duration examined, the inventory levels of materials at the producer witness a declining trend, as the consumption to produce products decreases and maintaining minimal stock levels for cost optimization is exhibited. The levels of product inventory at the producer peak in period t_4 before gradually dropping to the lowest level in the final interval. In contrast, the distributor's inventory levels inflate in later periods, reflecting the need to fulfill customer demands and safeguard against demand uncertainty by storing safety stock. No stock-out cost occurs during the examined time span.

Table 8. Delivery, order, and production quantity for every echelon of the supply chain with discount in quantity, from period t_1 to t_8.

		From	To		Time Period							
					t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8
Vendor	Delivery											
	$R_{tvp}^{m(i)}$	v_1	p_1	$m(1)$								
				$m(2)$	1480	3188	1400	1876				
		v_2	p_1	$m(1)$	740	1594	700	938				
				$m(2)$								
Producer	Order	v_1	p_1	$m(1)$								
	$O_{tvp}^{m(i)}$			$m(2)$	1480	3188	1400	1876				
		v_2	p_1	$m(1)$	740	1594	700	938				
				$m(2)$								
	Production											
	$Q_{tp}^{f(n)}$	p_1		$f(1)$	50	380	798	349	469			
	$AQ_{tp}^{f(n)}$	p_1		$f(1)$		50	380	798	349	469		
	Delivery											
	$\mathrm{R}_{\mathrm{tpd}}^{\mathrm{m(n)}}$	p_1		$f(1)$			390	807	360	479	10	
Distributor	*Order*											
	$O_{tpd}^{f(n)}$	p_1		$f(1)$	390	808	359	479	10	10		
	Delivery											
	$R_{tpd}^{f(n)}$	d_1	c_1	$f(1)$				215	244	255	259	
			c_2	$f(1)$				85	92	80	90	
			c_3	$f(1)$				130	123	124	130	

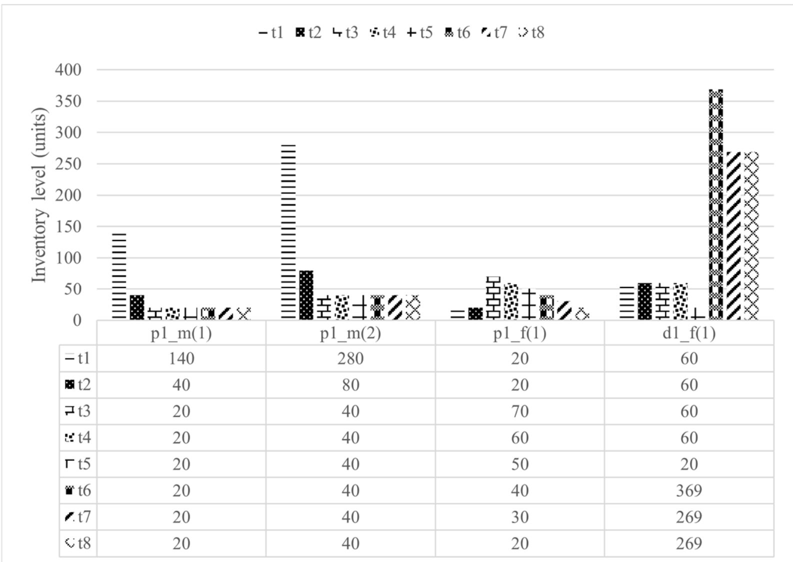

Figure 5. Inventory levels at producer and distributor.

Higher service levels improve the fulfillment of customer demands. However, they lead to an increase in safety stock levels of the distributor. The producer must produce more to fulfill the orders of the distributor. As a result, the costs of material procurement, production, and shipping rise concurrently, leading to a reduction in overall profit. The results listed in Table 9 illustrate the above statement.

Table 9. Total profit of the supply chain corresponding to multiple service levels, with quantity discount.

	Service Level				
	95.00%	**97.00%**	**98.00%**	**99.00%**	**99.90%**
Total profit	211,830.44	209,082.32	207,131.68	203,850.84	194,946.04
Change in total profit	-	−1.30%	−2.22%	−3.77%	−7.97%

3.5.2. Numerical Results without an All-Unit Quantity Discount Policy

There are usual instances where no quantity discount scheme is applied. In this circumstance, the price of materials $m(1)$ offered by vendor v_2 remains at the base rate, which is higher than that of vendor v_1 (as listed in Table 10). This causes the vendor v_2 to be selected, as it has no advantage in price. Therefore, vendor v_1 serves as the solitary source in this model, as elucidated in Figure 6.

Table 10. Raw material price without quantity discount scheme.

Vendor	Raw Material	$MC_{tv}^{m(i)}$	Vendor	Raw Material	$MC_{tv}^{m(i)}$
v_1	$m(1)$ $m(2)$	20 12	v_2	$m(1)$ $m(2)$	21 14

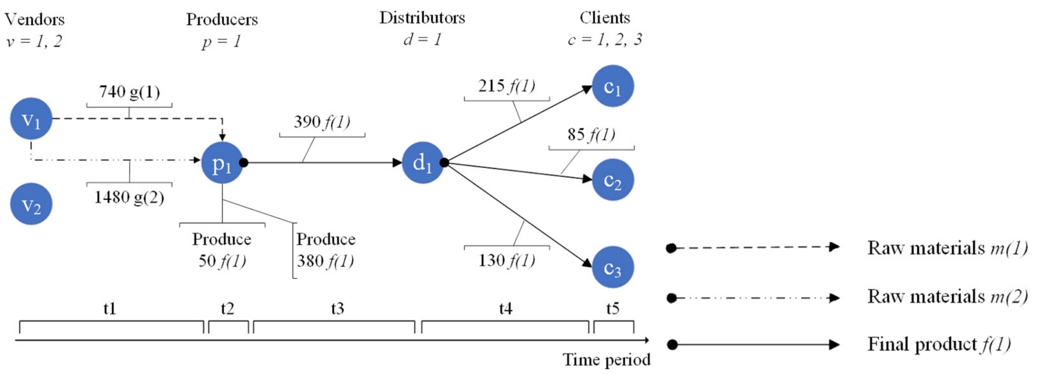

Figure 6. Optimal supply chain network without an all-unit quantity discount, from phase 1 to phase 5 (i.e., when clients receive products for the first time).

Compared to the previous model, these results show the same quantities of materials and finished products in each shipment and inventory levels throughout the periods. This suggests that the behaviors of each level in the SC are not affected by the discount.

In contrast, the materials purchase cost of producer p_1 rises from 170,796 to 174,768 compared to the model with a quantity discount, marking an increase of 2.33% compared to the previous model. Consequently, this results in a reduction in total profit to 203,159.28.

4. Discussions

This study used a linearization technique to transform an MINLP model to an MILP model so the model can find a globally optimal design of the supply chain under demand uncertainty. The comparison of the two models, with and without quantity discounts, indicates that the discount price must encourage the producer more to increase production quantities or purchase materials. However, the results reveal the benefit of offering quantity discount schemes. The vendor becomes significantly more competitive in price, thus significantly enhancing their market position and increasing their chances of being chosen in the sourcing process. Additionally, the total profit of the SC improves thanks to the quantity discount policy. This study provides management solutions to optimize inventory while ensuring customer satisfaction and avoiding stock-out costs, ultimately improving SC companies' resilience and sustainable competitive advantage in today's volatile business landscape.

The managerial implication of this research is the potential for improved cost management within SCs. By incorporating quantity discount policies into their decision-making processes, companies can strategically negotiate with suppliers to secure cost-effective raw materials. It is also crucial for businesses to strike a balance between leveraging discounts and avoiding excess inventory, which increases holding costs. Implementing strategies such as safety stock optimization can mitigate this risk while maintaining service levels. This can lead to significant cost savings and improved profitability over time. Moreover, service levels need to be carefully chosen. If the service levels are increased to a nearly perfect level under demand uncertainty, it can result in a large increase in safety stock levels which leads to a sharp decrease in profit. Hence, companies need to find a balance level between service levels and profits.

5. Conclusions

This study proposes a comprehensive model to design an efficient supply chain amid demand uncertainty, particularly in the context of integrating quantity discount policies. Using a step function technique to linearize the MINLP model, global optimal solutions can be achievable with commercial optimization solvers. The results of numerical examples show that quantity discount policies in vendors increases the total profit of the supply chain. The results also indicate that the decision makers need to find a balance level between service levels and profits since there is a tradeoff between service levels and profits.

Although the proposed model presents a valuable tool for practitioners seeking to navigate the challenges of modern SC operations, limitations still need to be addressed. First, the model optimizes producers' and distributors' planning processes, neglecting vendors' profitability. Second, quantity discount policies can also be applied for producers and distributors. Finally, the numerical examples can be larger and the supply chain network may exhibit complex behaviors. Further research can explore how quantity discount policies affect SC behaviors with respect to changes in demand reflected in price changes in large-scale problems.

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