Online Supplementary S2

Proof for Proposition 2

The definition of R_p tells us that Eq (7) holds only if

$$\lim_{n\to\infty} Pr\left(\bigcup_{n>m}\left\{\sum_{l\in\frac{M_n}{N_n}} I\left(R(y_l,\omega_l)<R_p\right)<F\left(R(y,\omega)<R_p\right)\cdot\left|\frac{M_n}{N_n}\right|\right\}\right)=0$$

A sufficient large m makes $\frac{|BM_n|}{\left|\frac{M_n}{N_n}\right|}\sim O\left(\frac{1}{n}\right)$, where BM_n represents the total number of new savings customers in the nth period ($n>m$). For each n, of all the $\left|\frac{M_n}{N_{n-1}}\right|$ customers who have not received a loan, only those with the top $|BM_n|$ expected returns may receive a loan. The event $\sum_{l\in\frac{M_n}{N_n}} I\left(R(y_l,\omega_l)<R_p\right)<F\left(R(y,\omega)<R_p\right)\cdot\left|\frac{M_n}{N_n}\right|$ occurring at this point requires that the risk of at least one lending customer whose retained interest rate satisfies the condition of $R(y,\omega)<R_p$ is misjudged to the extent that its expected return can be ranked in top $|BM_n|$, which occurs with a probability of occurrence less than $\alpha n\tau^n$ where $0<\alpha,\tau<1$. Therefore,

$$Pr\left(\bigcup_{n>m}\left\{\sum_{l\in\frac{M_n}{N_n}} I\left(R(y_l,\omega_l)<R_p\right)<F\left(R(y,\omega)<R_p\right)\cdot\left|\frac{M_n}{N_n}\right|\right\}\right)\leq\sum_{n>m}(n-m)n\cdot\alpha\cdot\tau^n$$

holds and converges to 0 as $m\to\infty$. □