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# Dynamic Data-Driven Deterioration Model for Sugarcane Shredder Hammers Oriented to Lifetime Extension

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**Abstract:** Several sugar mills operate as waste-to-energy plants. The shredder is the initial highenergy machine in the production chain and prepares sugarcane. Its hammers, essential spare parts, require continuous replacement. Then, the search for intelligent strategies to extend the lifetime of these hammers is fundamental. This paper presents (a) a dynamic data-driven model for estimating the deterioration and predicting remaining life of the sugarcane shredder hammers during operation, for which the real data of the entering sugarcane flow and the power required to prepare the sugarcane are analyzed, and (b) a management architecture intended for online decision-making assistance to extend the hammers' life by making a trade-off between the desired lifetime, along with a nominal shredder work satisfaction criterion. The deterioration model is validated with real data achieving an accuracy of 84.41%. The remaining life prognostic is within a confidence zone calculated from the historical sugarcane flow, with a probability close to 99%, fitting a lognormal probability distribution. A numerical example is also provided to illustrate a closed loop control, where the proposed architecture is used to extend the useful life of the hammers during operation, adjusting the incoming sugarcane flow while maintaining the nominal work satisfaction of the shredder.

**Keywords:** data-driven method; deterioration model; extension of lifetime; maintenance; management of lifetime; prognostics and health management (PHM); reliability; reliability adaptive system (RAS); RUL prognostics; sugarcane shredder

MSC: 60K10; 62N05; 90B25; 93-10; 93C95; 70Q05

# 1. Introduction

Sugarcane exposed to physical-chemical and biological processes can be transformed into white sugar, alcohol, energy, and other byproducts such as organic fertilizers. In addition, several sugar mills generate electricity from the combustion of sugarcane waste. Therefore, the transformation process involves technical issues related to product quality management, energy consumption and energy production. One of the most significant issues is how to perform the maintenance of the machinery involved in production during its life. For example, extending the lifetime of its spare parts is useful for reducing energy, maintenance, and consumables costs. In this context, intelligent maintenance aims to extend the lifetime of spare parts, while maintaining a trade-off between sustainability, quality and process efficiency. Beyond the corrective, scheduled and predictive maintenance approaches, which are based on reaction to failure, precaution, and a prediction of faults (or failure), respectively, the proactive maintenance focus seeks to generate changes, during the lifetime of assets (or system), by making intelligent decisions that may involve one or several optimal objectives.

The sugar cane shredding machine, or shredder, is in charge of preparing the harvested sugarcane stalks and consists of an electric motor coupled to a longitudinal shaft on which



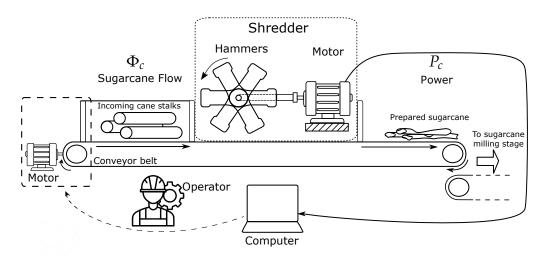
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**Copyright:** © 2024 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https:// creativecommons.org/licenses/by/ 4.0/). a set of tilting hammers are located. These tilting hammers impact the cane stalks at high speed to break the cane bark and facilitate the extraction of juice in the mills. Figure 1 depicts the process performed by a shredder. Hammers are the most critical spare parts of a shredder, as they are the ones that perform the main work and, at the same time, deteriorate the most. In this paper, the deterioration of hammers of a shredder is studied as a process characterized by the progressive loss of the ability of the hammers to perform their task properly within a given range.



**Figure 1.** Schematic diagram of sugarcane preparation stage and the shredder. The sugarcane flow is handled by a conveyor belt. The shredder uses an electric motor and hammers to prepare the incoming sugarcane flow. The power required by the motor is measured. The prepared cane is conveyed to the milling stage.

The lifetime of the hammers of the studied shredder is typically between 8 and 25 days (192 h and 600 h) of continuous duty. The more the hammers deteriorate, the higher the electrical power, and the lower the quality of cane stalk preparation. Over time, this could eventually lead to operating faults or a failure. The reduction in the the quality of the prepared cane is detrimental to the subsequent juice extraction process in the mills. Furthermore, the energy consumption of the preparation stage is significantly high compared to other machines involved in the sugarcane factory. This is why the shredder must be constantly monitored. Moreover, there is a limited availability of hammers, mainly because their cost is high. Once a defective set is removed, it must be repaired. In the meantime, a new set of hammers is installed.

Therefore, one of the goals of shredder management is to extend the lifetime of the hammers, thereby extending the service time of the shredder while respecting the machine's nominal performance. To achieve this goal, it is convenient to continuously estimate the Remaining Useful Life (RUL) of the hammers. The RUL is defined as the time remaining from a current moment until spare parts or devices, i.e., the hammers, no longer perform their intended function. Therefore, to predict the RUL of hammers, it is important to perform accurate, precise, online (in-operation), and non-invasive diagnostics of their current state of deterioration. In turn, this type of RUL prognosis can facilitate decision-making regarding the health state of the machine, aimed at extending the mean time to failure, improving maintenance towards proactive focus, reducing energy costs, reducing downtime, managing assets more effectively, and increasing profitability and the efficiency of production. These tasks are particularly valuable within the framework of intelligent and sustainable resource management.

The prognostics of RUL of assets for industrial machinery and its relation with the reliability of systems is a widely studied topic in the Prognostics and Health Management (PHM) area. The general goal of PHM is getting systems which can autonomously manage their state of health (variable of reliable behavior) according to its current condition and

by considering the influence of the system input over them. This could also be called health-aware or RUL-aware management.

Here, so-called self-optimizing mechatronic systems offer the possibility of adapting the system behavior in terms of deterioration, lifetime or reliability, to the current situation. In [1], the concept of remaining useful life (RUL) control is discussed and analogously in [2], the focus is on system reliability control. In both cases, a closed-loop control is performed that takes estimates of the deterioration, and generates pre-calculated working points to modify the system behavior and act on the RUL or reliability, respectively. In past years [3], these systems have been called Reliability Adaptive Systems (RASs). To obtain a more effective RAS, it is necessary to look for better applications that develop an online estimation of the internal and external operating conditions of the system.

RUL estimation is generally studied as a mainly stochastic problem. Even if a given mechanical system model is well known, there are several sources of exogenous and endogenous uncertainties that affect the precision of the RUL prediction. Moreover, this estimation is subject to the influence of several random variables. Conventional modelbased methods are hampered by limitations or the inability to handle the nonlinear nature, measurement uncertainty, fault coupling, and other application problems. For example, successful models can be highly complex and others are too dependent on historical calculations rather than the current system [4]. However, new similar approaches seek to analyze how few degradation parameters can absorb all possible sources of degradation; see, for instance, [5]. Other methods seek to utilize the data-driven technologies of recent years, aimed at reducing the complexity in the prognosis of the RUL as in [6]. In this work, we intend to address the uncertainties in the framework of a sufficiently agile model focused on online prediction, considering the current condition of the deterioration and its dynamic behavior, as well as the current and future operating conditions.

Some contributions like [7,8] show the importance of focusing on the RUL analysis of the actuator as a critical component, and manage a more dynamic relationship with the current state of deterioration and the environment, focusing on the advantages for the maintenance of the system. In [9], a solution is proposed for the online control of the remaining useful life of a mechanical system, based on a model conceived for the forecasting task, which allows online estimation of the deterioration, taking the uncertainty of the calculation to generate the respective forecast and, by means of a closed-loop action, control the RUL of the system. Hence, there is a need for agile diagnostic and prognostic models that enable intelligent decisions to be made during the work more efficiently. A compilation of new intelligent fault detection methods can be found in [10]. For example, in [11], the concept of forecasting based on learning on the relationship between the input and output of a deteriorating system is used to calculate in real-time the distribution of its RUL.

To the authors' knowledge, there is no similar solution that is focused on the spare parts of the sugarcane shredder under the framework of the described scenario. There, the consumption per ton of cane is understood as an indicator of machine deterioration; however, the corrective maintenance decision is made according to its increase and not according to the preventive satisfaction of the nominal work. Moreover, for the characteristics of the system input as described, there are no studies on the type of probabilistic distribution that the RUL would have.

In this context, this paper describes an agile data-driven model of the hammer deterioration process using the power required to prepare the cane and the sugarcane flow entering the shredder. This model is aimed to perform agile prognostics of hammers' RUL for subsequent decision-making, such as feedback and manipulation of sugarcane flow to eventually extend the hammers' lifetime. The model is used within an architecture that estimates deterioration and prognosticates the RUL during the daily work of the shredder, based on the input signal (sugarcane flow) and the output signal (required power). Additionally, the architecture allows for estimating the input-dependent operating conditions useful for RUL prognosis. The RUL estimation is used to weigh a trade-off between a desired RUL and nominal work satisfaction using a controller. Finally, the parameter provided by the controller can then be the feedback to the system input to generate the final effect, e.g., extension of the hammers' lifetime.

In this work, the sugarcane flow is considered the input of the system. The motion control actions and the characteristics of the input are seen as a source of stress that deteriorates the actuator; see for instance [2,3,8]. Complementarily, in [12,13], the authors assume a relationship between the degradation and the control input of the system. Therefore, we assume that managing the RUL of a component could be achieved by modifying in a suitable way the input, including also a feedback action. Note that this approach is based on the component level because in this case, hammer deterioration is clearly the cause of decreased service time. For systems where it is necessary to be able to first identify the critical component, and regarding how to identify it in a complex dynamic system, recent developments can be found, for instance, in [11].

This paper is organized as follows: Section 2, System Description, includes the preparation stage, the shredder general characteristics for the case of study, the analysis of the phenomena of deterioration of hammers, and considerations and definitions for the general problem. Section 3, Problem Statement, describes in a general and systematic way the architecture proposed as a solution. Section 4, Data-Driven for Deterioration Modeling, describes the data sources for the case study (sugarcane flow and power) and they are analyzed. A model to estimate the deterioration and subsequently the RUL of hammers is performed, tested, and validated from real data. Finally, in Section 5, Numerical Example, the integration of prognostics on the extension of the RUL of hammers during work is simulated and evaluated using the validated model.

#### 2. System Description

The function of a sugar factory is to produce sugar crystals from the sugar cane. Sugar cane is a combination of juice and fiber. The mixture of soluble solids (sucrose and others) and water constitutes the juice of the sugar cane [14]. Sugar cane is harvested in the field mechanically and taken to the factory for global processing. After the cane arrives, it is transported on belt conveyors to the cane preparation stage, where a shredding machine or shredder opens the cane cells to facilitate the extraction process in the subsequent cane milling stage. The goal of the milling stage is to extract the greatest amount of sugar cane juice, and this process depends on how well prepared the sugar cane is in the cane preparation stage; in other words, good cane preparation is a prerequisite for good cane juice extraction.

## 2.1. Cane Preparation Stage

The cane preparation stage is composed of a sugar cane belt conveyor and a shredding machine or shredder. Figure 1 depicts the incoming cane, the preparation process and the outgoing cane. The incoming sugarcane stalks are conveyed through a chute that eliminates the excess and ensures a certain regularity in the cane flow just before entering the shredder. The sugarcane flow entering the shredder is adjusted by changing the speed of the motor that drives the belt conveyor.

From the mechanical point of view, the shredder essentially consists of an electric motor that provides mechanical power through a shaft to a set of rotational hammers that impact the cane. In shredding, sugarcane stalks are subjected to successive and strong impacts to open the cane cells and thus facilitate the extraction of sugarcane juice. To do this, the shredder transfers its kinetic energy from the hammers to the cane stems through impact forces. As a result, the sucrose cells of the sugarcane are exposed and the outgoing product is sent to the milling stage.

The studied sugar shredder consists of a set of 8 axes on which 164 tilting hammers are disposed. The weight of each new hammer is approximately 26 kg (the total weight of new hammers is 4264 kg approximately). The shredder is driven by two electric motors with a power of 1305 kW each and rotates at a constant speed of 900 RPM. In the shredding

process, the hammers lose material due to wear caused by the sliding of the sugarcane stalks on the surface of the hammers, which changes their shape. When hammers deteriorate to a tolerable maximum, the total hammer set is repaired. After the deterioration process, the final weight of each hammer is approximately 25 kg. The lifetime of a set of hammers is 8 to 25 days of continuous work. The lifetime depends on the type of cane fiber that enters, the type of soil in which the cane was harvested, the environmental conditions, and the material of the hammers, among others.

There is only one set of spare hammers, which means limited availability. The process of changing the hammer set takes approximately 4–8 h, and during this time, the preparation process stops completely. The hammers are repaired by applying special welding. This welding is expensive and requires additional work by operators. The cost of repairing the hammers is considerably high, and since their lifetime is relatively short, the accumulated cost is a motivating criterion for extending the lifetime. If the lifetime of the hammers is increased, the total time for changing hammers per year is reduced, generating a reduction in maintenance costs.

Changing the surface of the hammers modifies the nominal rotational behavior of the machine, which is reflected in a progressive increase in the power required to prepare the same amount of cane. Therefore, as the sugar cane preparation process progresses, the hammers deteriorate, and the power required to prepare the sugar cane increases consistently during the lifetime of hammers, i.e., a power demand that tends to grow. The energy consumption of a shredder is around 12% of the global industrial process, which is considered relatively high.

The energy consumption of a shredder is significantly high in absolute value, i.e., between 24 MWh and 40 MWh daily, which is equivalent to approximately 640 MWh on average for the useful life of the hammers. Hence, any improvement in hammer life management could significantly improve the energy consumption profile.

The shredder machine is equipped with electronic instrumentation to measure the power, and an angular speed sensor for the rotation axis. It also has a vibration monitoring system on three axes, and a speed sensor for the sugarcane conveyor belt. The measured signals are sent to a computer that processes the information and stores the data. Operators use this information for monitoring, support maintenance, and production decisions.

#### 2.2. Initial Considerations

In the sugarcane preparation process, the deterioration D of the hammers can be considered a measure of the loss of their ability to perform shredding adequately within a given range. The deterioration D can then be also considered an image of the heat and the worn material at the contact surface of the hammers during their lifetime. In this process, the deterioration of the materials is assumed to increase monotonically, i.e., always increasing.

As proposed in [15], the deterioration D can be modeled as a function of the energy dissipated by the shredder machine and transferred through the hammers to the sugarcane stalks. This assumption is consistent with the Archard equation that is commonly used in the railway industry to predict wear (see, for example, [16,17]).

Abbreviations at the end of the paper shows the nomenclature (acronyms, symbols for variables, parameters, units, and their meaning) used in the paper.

Consider  $P_c(t)$  to be the power required by the sugarcane preparation process when the hammers of the shredder machine perform work, in other words, when they impact the cane. Consider D(t) to be proportional to the energy consumption of the sugarcane preparation process for a given interval from time 0 to time *t*, namely,

$$D(t) = c \int_0^t P_c(t) dt, \qquad (1)$$

where *c* is a constant.

The sugarcane flow is the input of the system according to Definition 1.

**Definition 1.** The sugarcane flow  $\Phi_c$  is defined as the mass of sugarcane per unit of time, a random and bounded exogenous input, which is assumed to be continuously measured and manipulable at each time step through the change in the speed of the conveyor belt.

In this paper, the unit for  $\Phi_c$  is tonnes (metric tons) per hour, i.e., [t/h] for convenience within the field of application. Moreover, according to Section 2.1, the initial considerations, and the definition of  $\Phi_c$ , it can be assumed that  $P_c(t)$  is a function of the sugarcane flow  $\Phi_c$  as

$$P_c(t) = f(\Phi_c(t)).$$
<sup>(2)</sup>

Therefore, from Equation (1), it is possible to find a function f that links the derivative  $\dot{D}(t)$  with  $\Phi_c$  as follows:

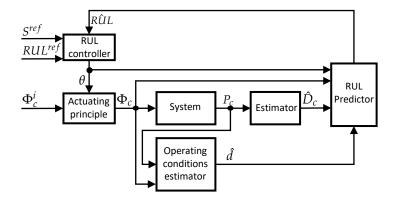
$$\dot{D}(t) = f(\Phi_c(t)). \tag{3}$$

Equation (3) indicates that the rate of deterioration of the hammers is a function of the sugarcane flow, i.e., by manipulating the sugarcane flow, the rate of deterioration of the hammers can be increased or decreased.

Section 3 describes the general problem and sets out the definitions for the study of the deterioration of hammers of the shredder.

### 3. Problem Statement

This work describes a method to estimate the deterioration of hammers, their RUL during their lifetime, and how to extend their useful life. For this purpose, the source of information is the sugarcane flow as the input signal and the power required in the sugarcane preparation process as the output signal. The hammers' lifetime extension should satisfy as much as possible the normal operation of the machine (short-term requirements) while seeking to obtain the desired lifetime objective (long-term requirements). Figure 2 shows the global architecture in which this research is framed. Note that this architecture is similar to the work presented in [18], this time modified and adapted to the particular case of the shredder.



**Figure 2.** Architecture for management of the hammers *RUL*. The architecture is used to estimate during work the current deterioration of the hammers and their lifetime. It also allows to establish a compromise between a desired  $RUL^{ref}$  and the nominal satisfaction of the process  $S^{ref}$ .

The RUL control architecture includes an actuation principle, a RUL predictor, and a RUL controller. The objective of the actuation principle is to modify the input (the sugarcane flow) to generate a change in the system behavior (increase or decrease the hammer wear) to modify its RUL. The RUL modification is based on the online prediction of the RUL, a desired reference *RUL*<sup>ref</sup>, and a desired satisfaction criteria *S*<sup>ref</sup>. The RUL predictor uses information from the deterioration estimator and an operating condition estimator to make a prognostic of the current RUL. The RUL controller determines an actuation modifier

scheduling parameter by solving an online optimization problem according to the dual criteria on  $RUL^{ref}$  and  $S^{ref}$ .

In the following, each block of the architecture proposed in this paper is described under the framework of the global problem.

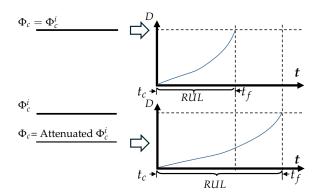
### 3.1. The Actuating Principle

The actuating principle  $H(\theta)$  is intended to modify the incoming sugarcane flow  $\Phi_c^i$  for generating a change in the behavior of the system. Here,  $\theta$  is a time-varying parameter, generated by the RUL controller, that in turn generates  $\Phi_c$  from  $\Phi_c^i$ . The modifier can be represented as  $\Phi_c = H(\theta) \Phi_c^i$ .

Figure 3 depicts two possible scenarios of deterioration with respect to the action of  $H(\theta)$  for a constant input  $\Phi_c^i$ . The first case concerns an input  $\Phi_c = \Phi_c^i$ , and the second case when  $\Phi_c$  is a modified  $\Phi_c^i$ . Remark that higher values of  $\Phi_c$  produce more deterioration, and then the RUL is shortened. On the other hand, lower values of  $\Phi_c$  result in less deterioration and, therefore, the RUL is extended. This example shows that properly attenuating the incoming flow  $\Phi_c^i$  by  $H(\theta)$  decreases the progress of deterioration, and in turn, the lifetime of the hammers is extended.

Note that in practice, one way to implement  $H(\theta)$  can be performed, for instance, by modifying the conveyor belt speed either automatically or with a human in the loop (see Figure 1).

Section 3.2 describes the considerations on the *RUL* predictor. Note that the value of  $\theta$  is calculated and finally given by the *RUL* controller, which is described in Section 3.3.



**Figure 3.** Illustration of the obtained RUL for two different levels of applied input. The obtained RUL increases in cases where the input is a filtered (or attenuated) signal of  $\Phi_c^i$ .

## 3.2. The RUL Predictor

Generally, the RUL is a random variable, which can be characterized, for example, by a probability distribution or confidence metrics. Variables such as the type of cane fiber, the climate, the type of cane crop terrain, and the humidity of the cane, among others, could influence wear. Consequently, there are several sources of uncertainty in the estimation of deterioration and RUL.

However, in this paper, RUL is studied from the point of view of the control systems theory, and therefore, a deterministic analysis of RUL is carried out first. In this sense, the knowledge and manipulation of the RUL are assumed, avoiding the analysis of another stochastic process.

**Definition 2.** At a given current time  $t_c$ , the predicted RUL, denoted by RÛL, is the predicted remaining period of time from the current time instant  $t_c$  until a threshold time  $t_f$ , before the system can no longer perform its intended function.

It is assumed that  $R\hat{U}L$  can be estimated at any time  $t_c$  from the estimation of the current  $\hat{D}_c$  and its simulated trajectory within a horizon until an acceptable threshold before failure. For this simulation,  $\hat{D}_c$  (current deterioration) and a prediction model are needed.

From this point, the model can be run with a regular input such as a step input. This regular input must be a representation of a feature extracted from the data. For example, the amplitude of the step can be the average of the past data, or the minimum value, or the maximum value, or all three values. It is also assumed that in the prediction of the trajectory of D(t), operation conditions remain unvarying along the given horizon from  $t_c$  until  $\hat{t}_f$  (the estimated threshold time).

The time  $\hat{t}_f$  is estimated, and finally,  $R\hat{U}L$  is calculated as

$$R\hat{U}L = \hat{t}_f - t_c. \tag{4}$$

The RUL predictor also uses an operating condition estimator. The operating conditions represent all those circumstances that are not taken into account explicitly in the model but that can be parameterized and introduced systematically in the prediction. For instance, additional information (vibration, temperature, etc.) about the current operating conditions *d* could be defined in a parametric form through a vector of quantized operating states as  $d = [d_1, d_2, ..., d_i]$ . Note that the operating condition estimator could be useful to support the current machine safety system.

The current value of  $\theta$  (the output of the RUL controller) is necessary for the estimation. This is known because is the only tunable parameter which allows the modification of the predicted  $R\hat{U}L$ . It is seen also as a decision-making parameter.

Note also that it is assumed that the power required for the cane preparation process and the cane flow are measured. This allows to predict  $D_c$  online from measured data.

#### 3.3. The RUL Controller

The *RUL* controller determines the scheduling parameter  $\theta$  of the actuating principle described in Section 3.1 by solving an online optimization problem according to twofold criteria on the desired RUL *RUL*<sup>*ref*</sup> and the demanded satisfaction *S*<sup>*ref*</sup>. They are defined as follows: at a given time *t*<sub>c</sub>, the desired *RUL*, denoted *RUL*<sup>*ref*</sup>, is the desired remaining period of time before the system can no longer perform its intended function, i.e., transmitting mechanical power from the motor to the hammers and thus to the preparation process.

The demanded satisfaction denoted  $S^{ref}$  is a value that quantifies the closeness of the generated  $\Phi_c$  from a given reference  $\Phi_c^i$ . Let us consider  $S^{ref} \in [0, 1]$ , where  $S^{ref} = 1$  means that it is desired to obtain  $\Phi_c = \Phi_c^i$ . A value of  $S^{ref}$  close to zero means that the applied input  $\Phi_c$  is moving away from  $\Phi_c^i$ .

Work satisfaction can be linked, for instance, to the quality of cane preparation by the shredder, which can be verified by periodic measurements on the outgoing cane. It has been proven that as the hammers deteriorate, cane preparation becomes worse.

The solution adopted here can be seen as a particular realization of a Model Predictive Controller (MPC). In the global architecture, other possible short-time state and/or control constraints could be admitted.

Now, the problem of controlling the RUL can be formulated as follows.

**Problem 1.** Given the system described by Equations (1)–(3), at every time instant, the sugarcane flow  $\Phi_c$  that guarantees that the predicted RÛL follows a desired one RUL<sup>ref</sup>, respecting, in turn, S<sup>ref</sup>.

This controller considers a trade-off between the desired  $RUL^{ref}$  and the demanded motion satisfaction  $S^{ref}$  (a double-objective criteria) from the predicted  $R\hat{U}L$ . The controller continuously decides the values of the vector  $\theta$  minimizing a given cost function J by solving, at every time-instant, the following optimization problem:

$$\begin{array}{ll} \underset{\theta}{\text{minimize}} & J\left(RUL^{ref}, R\hat{U}L(\theta), S^{ref}, S(\theta)\right), \\ \text{subject to} & f_i(\epsilon_c, u) \le 0, \ i = 1, \dots, n, \end{array}$$
(5)

where  $R\hat{U}L(\theta)$  and  $S(\theta)$  represent the predicted RUL and the obtained motion satisfaction as a function of  $\theta$ , respectively. In this paper, it is assumed that the cost function J includes also scalar values. The functions  $f_i(x, u)$  allow the inclusion of other constraints on the system control input u. Remark that the optimization problem could be solved in real-time or by using an *a priori* calculated look-up table.

Since the shredder machine has to follow possible short-time work demands, Problem 1 has to be reformulated in order to include these work constraints. Thus, Problem 1 is reformulated as Problem 2 as follows.

**Problem 2.** Given a system described by Equations (1)–(3), find at any time instant the parameters of the modifier  $H(\theta)$  such that the obtained sugarcane flow  $\Phi_c$  guarantees that system follows  $RUL^{ref}$  and respects as much as possible the demanded  $\Phi_c^i$  and the demanded motion satisfaction  $S^{ref}$  from the estimation of  $R\hat{U}L$ , which in turn is calculated from the D of the system and the operation conditions quantified as d.

Once the problem has been defined, let us note that it is firstly necessary to model the deterioration of the system in order to estimate its RUL. Section 4 describes the data-driven modeling for the system. Both the RUL controller and the actuating principle are defined and tuned for the particular case. The operation condition estimator can be assumed to be known.

## 4. Data-Driven for Deterioration Modeling

This section describes the data sources for the case study and the resulting deterioration model. In this context, the real production data of sugarcane flow  $\Phi_c$  and the power required in the cane preparation process  $P_c$  are analyzed as input and output signals, respectively. Figure 4 shows the real pre-processed data  $\Phi_c$  and  $P_c$  (gray lines) for a lifetime of hammers. A moving average of 6 h is used for processing all raw datasets (blue lines). According to the assumptions to define Equation (1), power data corresponding to moments where there is no cane flow are removed. The lifetime is defined as the period between the change of shredder hammers. Note that in this case, the life cycle of the hammers is approximately 580 h, which is equivalent to approximately 24 duty days.

As seen in Figure 4, the sugarcane flow  $\Phi_c$  is a random variable bounded between 200 t/h and 350 t/h, approximately.

 $P_c$  is measured directly using the instrumentation of the shredder motor. The energy  $\epsilon_c$  consumed by the process is computed every 1 h. Therefore, 1 h is taken as the step time for data analysis. The cumulative energy consumed by the preparation process stage for the lifetime is approximately 805.51 MWh. This value is useful to provide an energy deterioration threshold reference (see Section 4.2). Remark that, according to Section 2.1, the total weight of the 164 hammers will decrease from approximately 4264 kg (26 kg each) to 4100 kg (25 kg each).

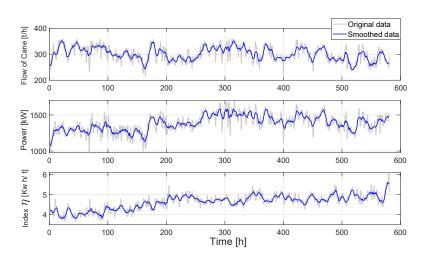
Let us consider the index  $\eta$  as the ratio of power  $P_c$  per sugarcane flow  $\Phi_c$ , which is equivalent to the energy consumed  $\epsilon_c$  to prepare the total mass of sugar cane  $Q_c$  that passes through the shredder per hour. Then, the index  $\eta$  can be defined as

$$\eta = P_c / \Phi_c \equiv \epsilon_c / Q_c. \tag{6}$$

where the units of  $\eta$  are [kWh/t], the units of  $\epsilon_c$  are [kWh].

Here, it is assumed that  $Q_c$  is always available, and there are no delays in the weighing process. In this process, it is assumed that the cane is weighted before entering the conveyor as in real life. For this life cycle, the cumulative sum of the mass of cane processed is 175,210 t (tonnes or metric tonne).

Figure 4 shows also a clearer upward trend of  $\eta$  within the interval between the change of hammers. Values start close to and below 4 kWh/t and end up around 5.5 kWh/t.



**Figure 4.** Pre-processed data. Moving average of sugarcane flow, and power of motor is used to model the process.

# 4.1. Identification

In terms of identification, the objective of this work is to propose a model that is fitted, fast, and simple enough to be used within the control architecture. An acceptable physical representation of the process is sought, and low-order models are preferred.

The ARX model structure (autoregressive model with exogenous inputs) is chosen because it is one of the simplest and most effective structures that allows an agile and dynamic inclusion of the current and past states of the output information (power), which is the variable of interest, with the current and past states of the exogenous input (sugarcane flow) to perform the identification for the system, i.e., for finding the future values of the output. This is particularly useful since the system is progressively changing as it deteriorates.

Remark that for the case study, we are more interested in the dynamic analysis of the data. In the initial analysis of the data, it is concluded that there is a direct relationship between the output and the input; however, it is known from expert judgment in several previous experiments that the relationship changes over time as the system deteriorates. This means that it is possible to model the system like an LPV system (linear parameter variant model). This approach is widely known in the control systems field. This type of model states that there are linear relationships that have slowly varying parameters (much slower than other important quick dynamics within the system). In this case, we are looking for a time-varying parameter linked to the deterioration that affects the variation in the output. Hence, we can use the ARX model, assuming that a deterioration parameter that changes slowly over time will give acceptable enough results.

In this section, the parameters of an ARX model, using the relationship expressed by Equation (2), are estimated. The ARX model is performed, tested, and validated from real data.

The ARX model structure is given by

$$A(q) P_{c}(k) = B(q) \Phi_{c}(k) + e(k),$$
(7)

where *k* represents the discrete current time instant; *q* is the delay operator; *A* is the autoregressive polynomial, the part of the model that captures the dependence of the current output  $P_c(k)$  on its past values; *B* is the input polynomial, the part of the model that captures the dependence of the current output  $P_c(k)$  on past values of the input  $\Phi_c(k)$ ; and term e(k) represents the part of the current output that cannot be explained by the autoregressive and input relationship of the model, and represents a white-noise disturbance value. Dead time in the system, the number of time steps by which the input affects the output, is here assumed to be equal to 0.

Then, polynomials *A* and *B* are defined as

$$A(q) = 1 + a_1 q^{-1} + \ldots + a_{na} q^{-na}$$

$$B(q) = b_1 + b_2 q^{-1} + \ldots + b_{nb} q^{-nb+1}$$
(8)

where *na* and *nb* are the orders of the autoregressive polynomial and the order of the input polynomial, respectively; *na* stands for the number of past output terms used to predict the current output; *nb* represents the number of past input terms used to predict the current output; and  $[a_1 \dots a_{na}]$  and  $[b_1 \dots b_{nb}]$  represent the coefficients of the polynomials.

The identification process is performed using real data, namely, the first half of the pre-processed data for  $\Phi_c$  and  $P_c$  of the studied period. This is performed to validate the model with the second half of the data. The identification process is carried out using the software MATLAB 2020, with the toolbox System Identification. This toolbox is useful for comparing various values for the orders *na* and *nb*.

Table 1 shows the best pairs of values na and nb along with their corresponding quantitative metrics NRMSE fitness value (Normalized Root Mean Squared Error), FPE (Final Prediction Error) and Mean Squared Error (MSE). NRMSE is a measure of how well the predicted model output fits the estimation data. Several na and nb pairs are tested starting with the least complex orders. The table shows five pairs, for which the best values are obtained. An ARX model with na = 3, nb = 4 is chosen. We remark that this pair sets a high NRMSE. Also, the FPE metric is analyzed because the lower its value, the better the quantitative measure of the predictive capability of each model. Moreover, its FPE has a low value, which reflects a good compromise between good predictive capability and low complexity. Finally, the lower MSE indicates a better fit of the model to the data; here, the MSE is a low value among the contiguous pairs, which confirms the choice.

na	nb	NRMSE (%)	FPE	MSE
2	2	83.35	166.0	159.3
2	3	84.20	156.3	148.9
3	2	83.40	167.2	158.2
3	3	84.16	155.2	145.8
3	4	84.41	151.2	141.1 *
4	3	84.19	156.8	145.4
4	4	84.43	153.0	140.9
4	5	84.42	154.0	140.8

**Table 1.** Best pairs of values *na*, *nb* for the ARX model with the corresponding NRMSE, FPE and MSE values.

\* Best pair na, nb.

The resultant model equations for A(q) and B(q) are, respectively,

$$A(q) = 1 - 1.157q^{-1} - 0.05919q^{-2} + 0.2258q^{-3}$$
(9)

$$B(q) = 3.015 - 3.83q^{-1} + 0.1712q^{-2} + 0.6851q^{-3}.$$
 (10)

then, the ARX model structure is given as

$$P_{c}(k) = 1.157 P_{c}(k-1) + 0.05919 P_{c}(k-2) \dots$$

$$\dots - 0.2258 P_{c}(k-3) + 3.015 \Phi_{c}(k) - 3.83 \Phi_{c}(k-1) \dots$$

$$\dots + 0.1712 \Phi_{c}(k-2) + 0.6851 \Phi_{c}(k-3) + e(k),$$
(11)

The *p*-values found for the coefficients of A(z) are 0, 0.5151, and 0.0001, and the *p*-values for coefficients of B(z) are 0, 0, 0, 0.6448, and 0.0022. Coefficients that are significant (p < 0.05) and are crucial to the model and have a statistically relevant impact include coefficients 1 and 3 of A(z) and coefficients 1, 2, and 4 of B(z). The non-significant coefficients

(p > 0.05), which have a minor impact and could be reconsidered in the model, include coefficient 2 of A(z) and coefficient 3 of B(z). One could consider eliminating or revising the non-significant coefficients but since the current model reproduces the phenomenon well, it is chosen to leave them. It is emphasized here that the model fits the needs within the framework of control systems theory. Furthermore, according to the suggested methodology, each new dataset could be modeled with the same structure and order, which would help to make a comparison between models.

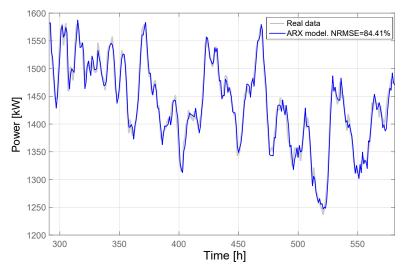
# 4.2. Validation

Validation is performed with the second half of the pre-processed data for  $\Phi_c$  and  $P_c$  of the studied interval. Note that in practice, this kind of validation can mimic developing a model within the first few days of use and applying the model to predict future behavior from the current date until a failure threshold is reached. This process is also close to a prognosis in a real scenario.

This section includes includes a probabilistic certification of the model based on Monte Carlo simulations. Monte Carlo simulations are mathematical modeling techniques that allow understanding the impact of uncertainty and risk in predictions and forecasts. They are based on running a large number of simulations to model the distribution of possible outcomes. The steps in this section are the fitting validation of the ARX model (shown in Section 4.1), analysis of the residual errors, test scenario generation, multiple trajectories simulation, and inference about the results. These simulations help to better understand the robustness of the model and how it might perform under different conditions.

#### 4.2.1. Fitting Validation

Figure 5 shows the real output data (gray line) in comparison to the output of the ARX model (blue line). Here, NRMSE = 84.41%, indicating that the model fits the data appropriately. Note that there is a discrepancy in the capturing of the peaks, which suggests areas where model refinement could improve the overall accuracy. However, according to the methodological criteria, the model has a satisfactory fit, good predictive capability according to the metrics mentioned in the paper, and is low order. Hence, it is considered that the model fits sufficiently well to the needs of inclusion in the architecture in the framework of control systems.



**Figure 5.** Comparison between the model ARX response (1-step predicted) and the original data. The model follows the real data with a NRMSE of 84.41%.

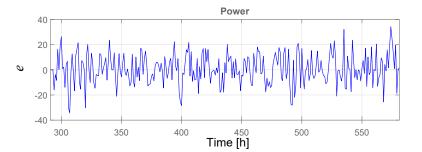
The model was also validated with several parts of the dataset (initial, middle, and final parts) and minimal differences in the NRMSE were found. Moreover, note also that each change in the hammers is a new experiment of the system. We validate the model with

some of these extra datasets, where the differences are slightly larger but not significant, and the fits are still good enough. Moreover, since the latter tests refer to other experiments, the results are not included in the paper.

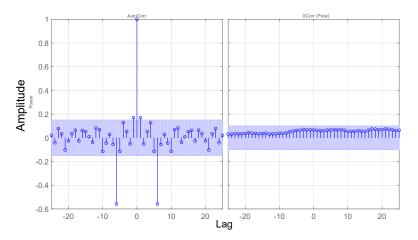
# 4.2.2. Analysis of Residual Errors

Figure 6 shows the model residuals for output  $P_c$ . Note that the output of the model has mainly a residual zero-mean and Gaussian noise. A slight deviation and amplitude increase is observed at the end of the frame, which is expected for a deteriorating system. This deviation is not considered significant.

Figure 7 shows the autocorrelation of the residuals, and the cross-correlation of the residuals with the input signal. The correlations are generated for lags from -25 to 25. As it is shown in the left sub-figure, the residuals are mostly white noise, i.e., independent and almost identically distributed with a mean that is close to zero. The autocorrelation of the residuals is mostly within the confidence bands (the shaded areas with 99% confidence region, marking statistically insignificant correlations) for most lags, except for lag zero, where it is one. The amplitude peaks outside the confidence bands indicate that there is some information in the residuals that the model has not captured; however, this is minimal. The right sub-figure shows the cross-correlation (the correlation between residuals and past inputs). The cross-correlation is within the confidence bands (the shaded areas) for all lags. These results show that the model fits the data and that its performance is sufficiently appropriate with respect to autocorrelation.



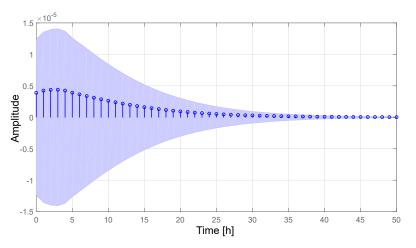
**Figure 6.** Model residuals of the output  $P_c$  for validation. Note that the output has mainly a residual zero-mean and Gaussian noise.



**Figure 7.** Residual correlation. The residuals are mostly white noise. The autocorrelation and crosscorrelation of the residuals are mostly within the confidence bands (shaded areas), which means appropriate performance.

Figure 8 shows the impulse response of the model. The impulse response shows that the model is stable, as it converges to zero without leaving the confidence region. The slight peak at 4 s indicates a mild transient response, suggesting that the system has a minor

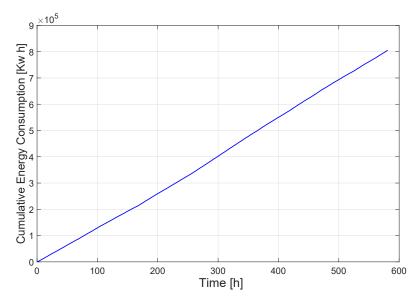
feedback component that slightly amplifies the initial response before damping out. The progressive convergence to zero in 30 s with slow and stable damping, without negative oscillations, suggests that there are no significant delays in the system, as the response is smooth and controlled.



**Figure 8.** Impulse response. The response merges to zero progressively without significant feedback effects and delays.

# 4.2.3. Test Scenario Generation

Figure 9 shows the cumulative energy which is calculated as the sum of all  $\epsilon_c$  of each sample time (1 h). According to Equation (1), it can be considered an image of D which is monotonically increasing. The value reached at the end of this dataset (at the time for hammers change) is 805 MW. This value is considered the maximum value for D normalization purposes.

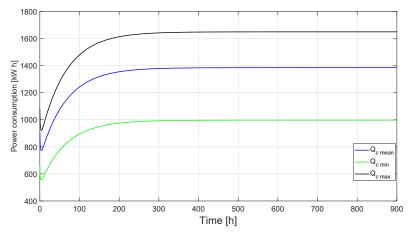


**Figure 9.** Cumulative energy for the interval of duration. The value reached at the time of the hammers' change is 805 MW.

Figure 10 shows the response of the model to three-step inputs with values of amplitude of  $\Phi_c$ , here called  $Q_{c-min}$  (minimum sugarcane flow data),  $Q_{c-max}$  (maximum sugarcane flow data), and  $Q_{c-mean}$  (mean sugarcane flow data). As can be seen, the response is stable in a steady state. The response is considered slow considering that the settling time is approximately 190 h. To calculate  $Q_{c-min}$ ,  $Q_{c-max}$ , and  $Q_{c-mean}$ , we propose

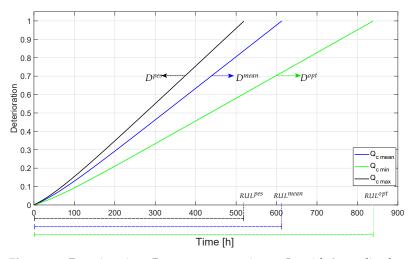
to use either historical sugarcane flow data or a moving average for a given past time or both. Here, historical data are used for this first prognosis.

As can be seen, at the beginning the curves are increasing, which means that a system with a constant input tends to dissipate more power as it deteriorates. Similarly, the higher the value of the step amplitude, the higher the settling value of the response. Finally, the higher the value of the step amplitude, the greater the difference between the initial and final values of the response, indicating a greater growth in energy consumption and therefore a greater deterioration.



**Figure 10.** Power of shredder. Response to step input  $\Phi_c$  with 3 amplitude values  $Q_{c-mean}$ ,  $Q_{c-min}$ , and  $Q_{c-max}$ .

Figure 11 shows the resulting deterioration D corresponding to the step inputs in Figure 10. Note that in this case, the deterioration is calculated in normalized form, taking the maximum value of the optimistic path  $D_{max}$  as  $\overline{D} = D/D_{max}$ . As the amplitude  $Q_c$ becomes higher, the system deteriorates with a higher slope, which means that the *RUL* from the current time  $t_c$  will be shorter. Note that with  $Q_{c-mean}$ ,  $Q_{c-min}$ , and  $Q_{c-max}$ , amplitudes of the flow  $\Phi_c$ , the resulting deterioration trajectories will be, respectively, the mean trajectory  $D^{mean}$  (blue line), the optimistic trajectory  $D^{opt}$  (green line), and the pessimistic trajectory  $D^{pes}$  (black line). Note that  $D^{mean}$  will be within the area between  $D^{opt}$  and  $D^{pes}$  if the operating conditions are the same. Then, the resulting  $R\hat{U}L^{mean} = 613$  h,  $R\hat{U}L^{opt} = 835$  h, and  $R\hat{U}L^{pes} = 519$  h are reached.



**Figure 11.** Deterioration. Response to step input  $\Phi_c$  with 3 amplitude values  $Q_{c-mean}$ ,  $Q_{c-min}$ , and  $Q_{c-max}$ .

Note that in Figure 11,  $t_c = 0$ , which means in that particular case, D = 0, and consequently, the entire useful life of the parts is being predicted. If  $t_c > 0$ , the current deterioration could be calculated taking a metric of the precision of the estimation giving three main values:  $[\hat{D}_c^{mean}, \hat{D}_c^{min}, \hat{D}_c^{max}]$ .

Remark that the prognosis of the trajectory is agile for a basic computer system, so it can be updated quickly. In practice, the updating of the prognosis would be performed when the operating conditions change, or at the discretion of the operator.

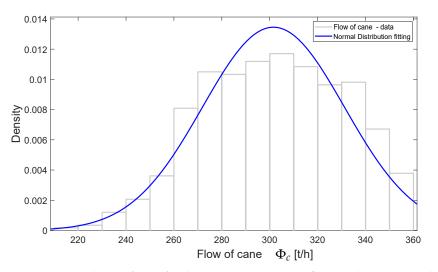
# 4.2.4. Multiple Trajectories Simulations and Inferences

Figure 12 shows the data distribution of the incoming sugarcane flow  $\Phi_c$ . A mean value  $\mu = 301.56$  and a standard deviation  $\sigma = 29.6501$  are found. A fitting with a normal distribution seems to correctly represent the incoming  $\Phi_c$  data. A kurtosis value of 2.3046 is obtained. This means that the data have lighter tails than a normal distribution. This is known as a platykurtic distribution. This means that there are fewer extreme values than would be expected in a normal distribution.

As suggested in [19,20], the number of simulations N which guarantee a confidence parameter  $\delta$  = 0.001 (i.e., 99.9% of the reliability of the estimation procedure) can be computed as

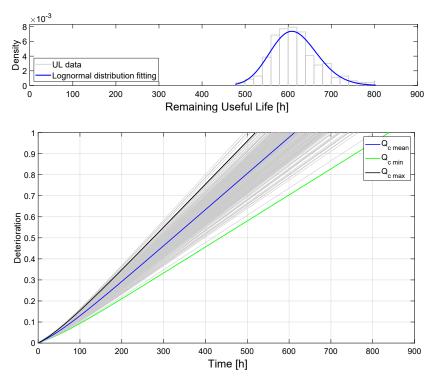
$$\mathcal{N} \ge (1/\gamma) \cdot (1 + n(1/\delta) + (2ln(1/\delta))^{1/2}),\tag{12}$$

where  $\gamma$ , in this case chosen equal to 95 %, represents the percentage of population that fails to meet the condition (e.g., 5% of the population will deteriorate outside the predicted range).



**Figure 12.** Distribution fitting for the incoming sugarcane flow  $\Phi_c$  data. A normal distribution fitting is used.

Figure 13 shows the resulting deterioration corresponding to 233 randomly generated amplitude step inputs, with a normal distribution function, with mean value  $\mu$  = 301.56 and a standard deviation  $\sigma$  = 29.6501. From this simulation, it can be observed that the simulated deterioration trajectories at  $t_c$  = 0, assuming invariant conditions over the prediction horizon, are within the limits defined in Figure 11. In addition, the distribution of the RUL values (equivalent in this case to the total useful life) obtained is shown at the top.



**Figure 13.** Deterioration. Response to step input  $\Phi_c$  of 233 normally distributed amplitude amplitude  $Q_c$  random values. Most of values are inside the zone given by bounds. A lognormal distribution fits the RUL results.

A lognormal distribution fit is used given the non-symmetrical characteristic of the resulting values and their heavier tails. The estimated parameter values are *mean* = 615.73 h, *variance* = 2979.44, *loglocation* = 6.42 and *logscale* = 0.09. This means that it is possible to make a probabilistic prediction of the deterioration trajectory of the hammers, for instance, with lognormal function, for a normally distributed amplitude of sugarcane flow  $\Phi_c$ . According to Figure 11, this is equivalent to 613 h, showing a difference of 5.5% (32.2 h) with respect to 580.8 h (the real value) with a procedure reliability of 99.9%.

This also means that it is expected that the real trajectory is more likely to be closer to the predicted one with the main value of  $D^{mean}$ . As shown in Figure 13, the real trajectory can be expected with a high probability to be within the zone delimited by the simulated pessimistic and optimistic trajectories. Similarly, it is possible to deterministically take  $\hat{R}UL^{mean}$  as the estimate corresponding to the mean value of the  $\Phi_c$  amplitude.

# 5. Numerical Example

In this numerical example, the initial purpose is to evaluate and describe how the system system described by Equation (2) would work with respect to the modification of the *RUL*<sup>*ref*</sup> and *S*. Then, the following descriptions, assumptions and conditions are defined for the scenario.

5.1. Chosen Scenario

- The cane flow  $\Phi^{ref}$  is a step input, whose amplitude  $Q_c$  can vary and is measured during the process.
- The operation conditions estimator provides the exact value of *Q<sub>c</sub>* as a moving average of the last 6 h.
- $\hat{D}$  is calculated from  $P_c$ .
- The reference *RUL*<sup>*ref*</sup> of the hammers can be set in this example as a failure time measured in hours [h]. Changes in *RUL*<sup>*ref*</sup> will be aimed at extending the lifetime of the hammers.
- Satisfaction *S* is set once at the beginning of the hammers' lifetime.

# 5.2. Chosen Actuating Principle

In this example, we use  $\theta \in [0.5, 1]$ , where  $\theta = 1$  means that  $\Phi_c = \Phi_c^i$ . A value of  $\theta$  that tends to zero attenuates  $\Phi_c^i$  for applying  $\Phi_c$  to the system. This is oriented to attenuate the deterioration *D* and, in turn, the extension of the lifetime of hammers.

According to the definition of *S*, a value of the demanded motion satisfaction  $S^{ref}$  tending to one favors the nominal operation of the system rather than the  $RUL^{ref}(t)$ . In the opposite sense, a  $S^{ref}$  value tending to zero is more permissive to changes in performance. Here, we use  $S^{ref} \in [0.5, 1]$ .

Note that we can define the current value of *S* as a function of the current value of  $\theta$  as  $S(\theta) = \theta$ .

#### 5.3. Used RUL Predictor

According to Section 4.2, and for a given a current time  $t_c$ , the *RUL* prediction can be predicted by simulating the model with Equation (11) from a set of initial conditions  $[\hat{D}_c^{mean}, \hat{D}_c^{min}, \hat{D}_c^{max}]$ . However, for this example, only the value of  $\hat{D}_c^{mean}$  is taken as the initial condition at  $t = t_c$ , according to the probabilistic observations. This is equivalent to simulating in  $t_c$  the trajectories of deterioration  $D^{mean}$ ,  $D^{opt}$ , and  $D^{pes}$ , from  $\hat{D}_c^{mean}$  until an acceptable threshold. We run the simulation with the  $\Phi_c$  amplitude values  $Q_{c-mean}, Q_{c-min}$ , and  $Q_{c-max}$ . In this case, it is possible to obtain these values from historical information or by letting the system work to obtain the initial moving averages. For this numerical example, we choose the historical information used to test the model. Of course, the more real datasets we have, the better the model can be fitted. Historical sugarcane flow values from the same experiment (same set of hammers) can be used to calculate the moving average of the above values, as well as the maximum and minimum values. Similarly, with more datasets from other experiments, inferences, comparisons and predictions on the current experiment could be also improved.

The prediction is stopped once the maximal deterioration is achieved, i.e.,  $D(t_f) = \sup\{D\}$ . To define  $\sup\{D\}$ , it is recommended to take a historical parameter on the total energy consumption of another total cycle of the hammers' life. For this paper, the value of  $\sup\{D\}=805$  MW is used according to real data analyzed for modeling. Once the simulation is stopped, the corresponding time  $t_f$  for which the threshold would be reached is registered. Thus, the predicted *RUL* is computed as  $R\hat{U}L = t_f - t_c$  (Equation (4)).

According to Section 3.2, in this paper, it is assumed that the states of operation are  $d = [d_1, d_2, ..., d_n]$  for the health states, from "good" to "alarm". The operating conditions estimator calculates  $d_i = \eta_i \in [\eta_m, \eta_M]$ , where  $\eta_m$  and  $\eta_M$  are the minimum and the maximum nominal values of  $\eta$ , respectively. Here, for instance  $\eta_m = 3.5$ ,  $\eta_M = 6$ , and n = 25 are chosen by convenience. Remember that the  $\eta$  value is increasing over the lifetime of the hammers. A moving average of  $\eta$  is calculated. Then, the *RUL* predictor updates its prognosis each time that  $\eta$  exceeds a value  $d_i$ . Once its maximum value  $\eta_M$  is reached, the forecast stops updating because the end of life is reached. In this sense, the  $\eta$  index is a trigger for a new update of the RUL prognostic. This procedure also makes it possible to monitor the consistency of the model and incipient faults and improving the current safety system of shredder.

The described algorithm can be summarized as

$$R\hat{U}L = SIM(\hat{D}, \Phi_c, t)|_{\theta}|_{d_i}.$$
(13)

# 5.4. Implemented RUL Controller

We propose to use an optimal controller to solve Problem 2, which minimizes a cost function including the double objective (i.e.,  $S^{ref}$ , and  $RUL^{ref}$ ). The problem can be reformulated as follows.

**Problem 3.** Given  $\hat{D}$  and  $\hat{d}$  at a time  $t_c$ , find the value of  $\theta$  which minimizes the cost function:

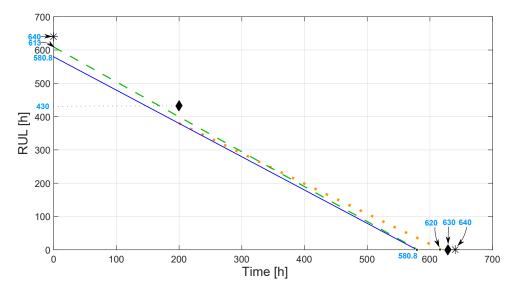
$$J(\theta) = \left(\frac{RUL^{ref} - R\hat{U}L(\theta)}{RUL^{ref}}\right) + \rho\left(S^{ref} - S(\theta)\right),\tag{14}$$

subject to

$$0 \le \theta \le \bar{\theta},\tag{15}$$

where  $\rho > 0$  is a real value which allows considering a trade-off between satisfying desired RUL<sup>ref</sup> and/or the satisfaction S. In this example, it is chosen as  $\rho = 0.5$ . The chosen weighting scalar  $\rho$  suggests that we put more focus on the desired RUL<sup>ref</sup> rather than on the desired satisfaction  $S^{ref}$ . Here, we assume also that the desired RUL<sup>ref</sup> will be higher than the estimated  $R\hat{U}L(\theta)$  to maintain the positivity of this cost function.

Figure 14 shows the evolution of the hammers RUL for the example. The solid line shows the real value at each time instant from the beginning of the life to the end, corresponding to the machine shutdown due to reaching the threshold. This curve is taken with the value reached in the real lifetime of the hammers used to obtain the model, corresponding to 580.8 h.



**Figure 14.** *RUL* along the time: (1) the real trajectory of *RUL* (solid line), (2) the estimated  $R\hat{U}L$  (dashed line), and (3) the real trajectory of *RUL* after the change in  $RUL^{ref}$  (dotted line). The symbol \* represents the desired *RUL* at the beginning, and the diamond symbol represents the *RUL*<sup>ref</sup> after the change.

The scenario shows that in the first instance, a  $RUL^{ref} = 640$  h is defined as the *a priori* expected target. This is marked with an asterisk at the corresponding point. The dashed green line represents the predicted  $R\hat{U}L$  value at each time instant. It shows that this prediction starts at  $R\hat{U}L = 613$  h  $\pm 5.5$  %. The uncertainty is described in Section 4.2; it is used for all estimates from now on and is not explicitly written, for practicality. This shows an optimistic prediction at the beginning, but it is adjusted with respect to the real values as they become known. From the beginning, a difference between the a priori set value  $RUL^{ref}$  and the real value (580.8 h) is shown. This means that the initial target of the example is very optimistic. Of course, during the work, it is not possible to know the real value but only the estimated one.

Figure 14 shows also the scenario where  $RUL^{ref}$  changes during the lifetime of the hammers with respect to the time. At t = 200 h of use, a change in the setting of  $RUL^{ref} = 430$  h is simulated, which is equivalent to 630 h of lifetime. A diamond mark is placed at the corresponding point. This is simulated as a more realistic target since measurements show

that the lifetime would be around 613 h and not 640 h as originally intended. At this point, it is still intended to achieve a higher total lifetime than the one predicted initially equal to 613 h. Once the change in  $RUL^{ref}$  is detected, the decision problem is solved to find the value of  $\theta$  that minimizes *J* according to Equation (14). The dotted red line represents the real trajectory after the change in  $RUL^{ref}$ . It can be seen that the achieved lifetime is finally 620 h.

Note that although the initial target (set a priori as 640 h) is not reached, the value achieved is 39 h (1.6 days) higher than the value that would have been achieved without modification (580.8 h). This is equivalent to an increase of 6.8% in the useful life with respect to the trajectory without management.

Hence, this example simulates a successful case in which the *RUL*<sup>ref</sup> of the hammers is extended during its lifetime. An increase in the total lifetime of the hammers is obtained by making a trade-off between the RUL requirements and satisfaction of the nominal operation.

As can be seen, the implementation of the life extension architecture is feasible for the case study. Summarizing, this is feasible in practice because there is a high-quality instrumentation system currently implemented, there is technical data collection, and the flow can be controlled by means of the conveyor belt. In addition, there is an electrical safety system in cases such as overloading. On the other hand, the current main limitation is the longitudinal validation of the architecture for several hammer changes since long times and high-cost power systems are involved and in continuous production. Other main limitations to implement the architecture in other similar industrial systems would be given by the shortcomings or absence of the mentioned features. Also, other limitations would include low control and communication capabilities (e.g., delays) of the information management systems. For the latter, it would be necessary to quantify them and take them into account in data management. The implementation may also be limited by the need to include a human in the loop (due to safety regulations or administration).

#### 6. Conclusions and Future Work

A data-driven model for the deterioration of hammers was developed, tested, and validated from real data. Real data on the sugarcane flow shredded per hour and electrical power were analyzed for a period between maintenance for the replacement of the hammers. An architecture aimed at extending the residual lifetime of hammers and using the deterioration model was presented. The lifetime extension was realized through an effective trade-off between the desired lifetime and nominal work satisfaction. It was observed that there is an increasing trend in the index of electrical consumption per ton of cane as the hammers deteriorate. The work shows that this index can be integrated within the architecture as an indicator of the machine's health state and as a prognosis updating parameter. The development of the model was oriented to be used within a global architecture intended to extend the *RUL* of the hammers of the shredder. The used specific RUL estimation method works online, offering agile updating estimations. Nevertheless, the complexity of this method is dependent on the variation in the operating conditions. The prognostic of the RUL is probabilistically certified. It was demonstrated that it is possible to make a probabilistic prediction of the deterioration trajectory of the hammers. The uncertainty in the prediction of *RUL* was around 5.5%, assuming the knowledge of the current state, with around 99% procedure reliability, according to a lognormal distribution fitting, for a normally distributed amplitude of sugarcane flow. A numerical example was used to show that it is possible to use the architecture to increase the lifetime of the hammers by around 6.8%, which is equivalent to extending the service time of the shredder. This value is slightly larger than the estimation error, but tends to be smaller the closer we are to the end of the hammer's life. Future work concerns the evaluation of the model within an architecture of extension of useful life with automatic control over the conveyor or with a human in the loop. This is also useful to prove the reduction or increase in lifetime with real tests and from a statistical point of view. It is also intended to work with the evaluation of other model structures in an optimal way.

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**Data Availability Statement:** Data supporting the conclusions of this article may be made available by the authors upon request to them and with permission from *"Grupo Agroindustrial Riopaila Castilla"* (GARC). Other aspects concerning the collection of other data and more specific characteristics, as well as complementary data could be requested to GARC, being at its discretion the provision of the same, as it is a private company.

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### Abbreviations

The following abbreviations are used in this manuscript:

Symbol	Units	Physical Meaning	
Â	_	Autoregressive polynomial	
В	-	Input polynomial	
d	-	Parameter of operating conditions	
D	-	Deterioration	
$P_c$	kW *	Power of motor, Output of system	
Qc	t **	Mass of cane	
t	h ***	Time	
$\Phi_c$	t/h	sugarcane flow, Input of system	
$\epsilon_{c}$	kWh	Energy	
η	kWh/t	Energy consumption per tonne of cane	
$\eta S$	-	Satisfaction index	
e(t)	-	Error of model	
θ	-	Parameter of actuating principle	
^	_	Superscript for estimations	
•	-	Superscript for derivative	
ref	-	Superscript for reference	
FPE	_	Final Prediction Error	
MSE	_	Mean Squared Error	
NRMSE	-	Root Mean Squared Error	
PHM	_	Prognostics and Health Management	
RAS	_	Reliability Adaptive System	
RUL	h	Remaining Useful Life	
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Note: \* kilowatt, \*\* tonne (metric ton), \*\*\* hour.

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