mathematics

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Optimal Carbon Pricing and Carbon Footprint in a Two-Stage Production System Under Cap-and-Trade Regulation

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Abstract: Integrating low-carbon design into products is crucial for reducing carbon emissions throughout their life cycle and promoting sustainable development. Addressing the uncertainty in the carbon footprint resulting from the unknown choice of product material solutions. This paper considers ABC (activity-based costing) along with the components' carbon footprint and scrap return issues to illustrate the above challenge in a two-stage production-inventory system with imperfect processes. We determine the optimal production and sales strategies that maximize total profit per unit time. An algorithm is developed to identify these optimal solutions. To illustrate the effectiveness of the proposed model and algorithm, two numerical examples from the Taiwan die casting industry are presented. Additionally, a sensitivity analysis is conducted to provide valuable managerial insights.

Keywords: sustainable supply chain; carbon footprint; activity-based costing; carbon price; EPQ

MSC: 90B30; 90B90; 90B05

Citation: Tseng, H.-Y.; Huang, Y.-F.; Fu, C.-J.; Weng, M.-W. Optimal Carbon Pricing and Carbon Footprint in a Two-Stage Production System Under Cap-and-Trade Regulation. *Mathematics* **2024**, *12*, 3567. https://doi.org/10.3390/math12223567

Academic Editors: Meichuan Cheng and Alexis P.I. Goh

Received: 30 September 2024
Revised: 8 November 2024
Accepted: 12 November 2024
Published: 15 November 2024

1. Introduction

As the emission of greenhouse gases (GHGs) reshapes climate patterns, manufacturers are increasingly considering the product carbon footprint (PCF) within sustainable supply chains (SSC). The product carbon footprint (PCF) became one of the most widely used environmental indicators, emerging as a method for assessing GHG emissions from goods and services. Traditional accounting approaches often view environmental costs as normal overhead, with little focus on environmental issues like carbon emissions (De and Friend [1]). Various industries have utilized activity-based costing methods to more accurately estimate their production and environmental costs. These industries include construction, DRAM manufacturing, tourism, automotive, aviation, and metal manufacturing (Tsai et al. [2]). Prior research has primarily considered carbon taxes, but not the potential for companies to purchase carbon rights to increase their carbon emission allowances. This study examines the impact of carbon emissions on profits under different scenarios, incorporating the cap-and-trade concept, where companies have a limited number of emission permits allocated by the government and must acquire additional permits from the market if their emissions exceed the allotted amount. Conventional management accounting often categorizes environmental costs as general overhead, with manufacturers demonstrating limited concern for environmental issues, including the financial implications of carbon emissions. Nevertheless, numerous industries have leveraged activity-based costing to more precisely quantify production costs in conjunction with environmental expenditures, encompassing sectors such as construction, DRAM technology, tourism, automotive, aviation, and metal manufacturing (Tsai et al. [2,3]). Moreover, previous researchers did not consider the possibility of purchasing carbon rights to receive a larger CE allowance for production, but only considered carbon tax (CT) in their models when using the ABC approach to estimate CEC (Tsai et al. [2–4]). For this reason, the models in this study discussed the effect of CE on profit through three kinds of situations. We incorporated the concept of cap-and-trade (CAT), in which companies have a limited number of permits for carbon emissions allocated from the government. The companies are required to hold permits in an amount equal to their emissions; otherwise, they need to buy carbon rights (CR) from the market. The economic production quantity model has been extensively adopted in practical applications due to its conceptual simplicity. However, the underlying assumptions of the original EPQ model exhibit certain limitations, prompting numerous researchers to explore approaches for enhancing and refining this inventory management framework from diverse perspectives. The standard EPQ model presumes that the setup cost is a constant. However, the setup cost is typically a function of setup time. Consequently, the setup cost varies due to differing setup times. Several research studies have addressed this issue. Darwish [5] investigated the EPQ model with fluctuating setup costs. Kreng et al. [6] evaluated the EPQ model under various demand scenarios, with the ratio of setup time reduction as a decision variable. Various strategies were recommended to minimize the PCF across SSC, which also depend on the procurement frequency of raw materials from suppliers. Afshar-Nadjafi [7] proposed a mathematical representation of the economic production quantity model in which the unit production cost and set-up cost were expressed as continuous functions of the production rate. The design variable has a significant impact on total production costs. The activity-based costing (ABC) can provide more accurate information on decision-making, so it is extensively noted and also applied to different fields. Some scholars explored the application of ABC in logistics. Kang [8] combined it with the current logistics management, from the application of an activity-based costing method in the process of enterprise logistics management of realistic urgency, and already have the conditions; we construct the enterprise logistics cost management mode based on ABC. The economic production quantity model examines a single product within a single-stage production system with an optimal process, assuming constant demand. However, extensive evidence suggests that demand can be influenced by various market factors and consumer behaviors, including selling price, quality, time, and visible inventory levels. Chang et al. [9] investigates a two-stage production model proposed by Pearn et al. [10] The initial stage is an automated manufacturing process where the necessary components are fabricated. The component production processes commence concurrently and operate independently. The subsequent stage is a manual assembly process that combines the components into the final product. Su et al. [11] examines an imperfect multi-stage production system that manufactures complementary products from blended materials containing scrap, which is generated from defective items. The scrap return feed rates differ between the two products, and the product with the higher rate is sequenced second in the production process to prevent the unlimited accumulation of scrap. While some scholars have explored the aforementioned direct factors, real-world demand patterns are also impacted by indirect influences, such as economic conditions, corporate activities, and international politics. Certain scholars have also incorporated pricing considerations in order to align demand-related assumptions more closely with real-world conditions. Saha and Goyal [12] investigated supply chain coordination contracts influenced by both stock and price factors affecting demand. Some researchers deal with an economic production quantity (EPQ) model with reworkable defective items and pricing policy (Taleizadeh et al. [13]; Shah et al. [14]; Jazinaninejad et al. [15]; Ruidas et al. [16]; Su et al. [17]; Duan and Cao [18]). Numerous studies (Seyedhosseini et al. [19]; Ruidas et al. [16]; Su et al. [11]) stress that when we are selling something, we evaluate market prices by comparing them to some reference price that we have in mind. Since we often make decisions by comparing possible outcomes to the reference price, our decisions behavior can change according to our perception of gains and losses. Some carbon pricing (Daryanto and Wee [20]; Sinha and Modak [21]; Sepehri and Gholamian [22]; Shah et al. [14]; Ebrahimi et al. [23]; Mesrzade et al. [24]; Bhavani et al. [25]; Li et al. [26]) usually takes the form of a tax imposed on emissions or an emissions trading scheme in which companies can buy and sell the right to generate emissions. All forms of carbon pricing provide incentives to reduce emissions. Du et al. [27] proposed a carbon-related price–discount sharing-like scheme to achieve the channel coordination and discuss the possibility of Pareto improvement. Wang et al. [28] investigated the production and carbon emission reduction strategies in a two-echelon supply chain model under cap-and-trade regulation. They showed that the carbon emission abatement level can be increased with an increased carbon trading price. Lou et al. [29] and Yang et al. [30] defined price- and carbon emission-dependent demand and proposed a model considering emissions due to manufacturing and allowable reduction investment. Tao et al. [31] consider carbon transfer cost and carbon holding cost in a supply chain with carbon footprint constraints. Lu et al. [32] explored potential competitive and co-operative dynamics in sustainable product-inventory models, focusing on collaborative investments in carbon emission reduction technology within the frameworks of carbon cap-and-trade and carbon offset policies. Priyan [33] developed a solution procedure of the problem associated with the amount of CO_2 where all the CO_2 factors might increase or decrease fuzziness in an imperfect production model. He et al. [34] devoted to presenting a master–slave design pattern integrated with a carbon footprint for low-carbon design. Producing new materials results in carbon emissions; circular economies minimize the need for producing new materials by maximizing the reuse of resources, thus eliminating the carbon costs of producing new materials. Huang et al. [35] presents a two-stage inventory model for designing a green supply chain within a carbon trading environment. Xiang et al. [36] addressed the challenges of uncertainty, dynamism, and interdependencies in carbon data during product carbon footprint accounting; a method is proposed that uses multi-level dynamic allocation of carbon data between organizations and products. Some issues that need to be discussed in EPQ model are (a) imperfect process; (b) scrap returns; (c) carbon issues; and (d) circular economy. Biswas and Schultz [37] studied an imperfect two-stage manufacturing process that orders raw materials from a supplier and produces finished goods with defective items that can be reworked. Martí et al. [38] presented a mathematical model to assist companies facing these joint environmental and operational trade-offs, and help them define carbon abatement strategies in a cost-effective manner. Indeed, internal carbon pricing (ICP) can be the price determined by the amount of money an organization spends for reducing greenhouse gas emissions, such as the budget spent on CSR activities, the funds used to invest in GHG reduction projects such as renewable energy projects, or a forest planting project, etc. To ensure the ICP remains aligned with market dynamics, regulatory frameworks, and organizational goals, this annual revenue targets should be regularly reassessed. Ruidas et al. [39] aimed at developing an imperfect production inventory model under the various carbon emission regulatory policies where the various carbon emission parameters are interval numbers. Corona et al. [40] provided an excellent review of methods, findings, and instructional issues related to CE concept. To protect our environment, current firms have committed to the circular economy for scrap recycling and low waste during the manufacturing process. Soleymanfar et al. [41] developed a new model to determine the optimal economic order quantity and production quantity for multiple products in a two-echelon supply chain with a retailer and supplier, considering product returns. Table 1 indicates the main classification of inventory models from relevant studies.

Table 1. The main classification of inventory models from relevant studies.

References	Pricing Policy	Single/ Multiple Stage	Imperfect Process	Scrap Returns	Carbon Footprint	ABC
	PD/CP					
Saha and Goyal [12]	PD	Single				
Lou et al. [29]	CP	Single				
Taleizadeh et al. [13]	PD	Single	√	√		
Yang et al. [30]	CP	Single			√	
Du et al. [27]	CP	Single				
Seyedhosseini et al. [19]	PD	Single				
Ruidas et al. [39]	PD	Multiple	√			
Wang et al. [28]	CP	Single				
Lu et al. [32]	CP	Single			√	
Shah et al. [42]	PD/CP	Single	√			
Ebrahimi et al. [23]	CP	Single				
Mesrzade et al. [24]	CP	Single			√	
Priyan [33]	CP	Single			√	
He et al. [34]	CP	Single			√	
Huang et al. [35]	CP	Multiple				
Present model	EOQ	Multiple	√	√	√	√

Note: price discount (PD); carbon price (CP); activity-based costing (ABC).

2. Problem Description

The manufacturing process of die cast aluminum alloys poses significant challenges to sustainability, including energy consumption, emissions, and waste management. The high energy consumption contributes to greenhouse gas emissions and increases the carbon footprint of the manufacturing process. Manufacturers can make more informed and sustainable decisions by optimizing the material choice and understanding its impact on both carbon emissions and costs. However, existing efforts on low carbon design usually do not consider the interaction between material selection and structural design and it may lead to increased carbon emissions. In order to identify the high-emission components, an activity-based method is used to allocate the carbon footprint. This gap in the research motivated us to derive the inventory management with carbon footprint constraints problem in a two-stage (multiple-stage) assembly production system. More importantly, a contract with the ABC approach and fixed design cost is analyzed and examined to coordinate the carbon footprint supply chain. Through the lifecycle analysis, the major parts and components characterized with high carbon emission are picked out according to the allocation result. Three issues that need to be resolved in this paper are as follows:

- What is the optimal production run time?
- What is the optimal carbon price?
- How can we reduce the production cost and optimize the recovery rate?

In this paper, we address the ABC problem, incorporating carbon footprint and scrap return issues, within a two-stage production-inventory system with imperfect processes. The remainder of the paper is organized as follows. Section 3 outlines the notations and assumptions, which form the basis for the mathematical formulations and theoretical results presented in Section 4. Section 5 introduces a real-world PCF scenario in the die-casting industry to demonstrate the practical relevance of our model. Numerical examples from this die-casting case are provided to illustrate the solution procedure, followed by a sensitivity analysis to offer managerial insights and decision-making guidance in Section 6. Finally, Section 7 presents the conclusions and suggestions for future research.

3. Notation and Assumptions

Before developing the mathematical model, this section first lists the notation used and the assumptions required to the proposed model. It is hereby stated as follows.

3.1. Notation

s_p	selling price per unit item.
c_r	production cost for returning a defective product to scrap returns.
n	requisite elements for a final manufactured good.
p_i	rate at which the component h is produced, measured in units per unit time, where i $= 1, 2, \cdots, $ n. i $= 1, 2, \cdots, $ n and $p_1 > p_2 > \cdots > p_e$.
p_e	assembly rate of the finished product in units per unit time.
p_f	feed rate of raw material in production in units.
h_i	component i holding cost per unit, where i $= 1, 2, \cdots, $ n.
h_r	holding costs of scrap materials per unit time.
h_e	holding cost of end product per unit time.
θ	defect rate of the finished product.
t_1	time period prior to depletion of inventory of each component, $t_1 \geq 0$.
t_2	time period prior to store WIP inventory in a warehouse, $t_2 \geq 0$.
t_3	time period prior to depletion of inventory of each component, $t_3 \geq 0$.
t_{ed}	time period prior to depletion of inventory of finished product, $t_{ed} \geq 0$.
t_e	production run time of finished product, $t_e \geq 0$.
T	length of cycle, $T \geq 0$.
Q_i	maximum inventory level of component i, where i $= 1, 2, \cdots, $ n.
Q_{sr}	maximum inventory level of scrap returns.
Q_e	maximum inventory level of finished product.
Decision Variables	
t_i	the production run time of component i, $t_i \geq 0$ where i $= 1, 2, \cdots, $ n.
s	unit internal carbon price, s ≥ 0.
r	the recovery rate of scrap returns.

This paper is based on the following assumptions.

3.2. Assumptions

- To prevent a stage from starving due to insufficient input from the previous stage, the minimum production rate in Stage 1, the assembly rate in Stage 2, and the demand rate must satisfy the following condition: $\mathrm{p_n} > \mathrm{p_e} > \mathrm{D}$.
- Automated inspections can improve manufacturing efficiency and productivity. By automating inspections, companies can inspect more products faster, reducing rework time and enhancing production flow. Process quality is assumed to be independent in the two stages, and the inspection time is so short that it can be disregarded. The rework time for defective items is also disregarded.
- Transforming a firm into a green industrial entity requires reducing emissions to a specified standard, with the aim of enhancing consumer purchase intentions by improving the corporate image. This transition inevitably necessitates investment in fixed equipment costs F_g (e.g., apportion charges per cycle for air purifiers, washing towers, or filtration) and variable operational costs V_g (e.g., material, water, and energy). Generally, such investments are allocated based on their contribution to the overall benefits. In this paper, we examine whether investment in green industrial development is advantageous by comparing the total profit per unit time with v $= 1$ and without v $= 0$ such investment.
- Building upon the demand model proposed by Modak et al. [43], the literature suggests that demand is influenced by the selling price, fluctuations in carbon prices, and investment in green industrial development. The model framework assumes a simple linear function for demand, $\mathrm{D} = \mathrm{a} - \mathrm{b}s_p + \rho s + wv$, where a > 0 represents the market potential, b denotes the elasticity of selling price, ρ captures the elasticity of carbon leakage, and δ reflects the elasticity of investment in green industrial development.

The research reveals that utilizing carbon price as an investment in sustainable product activities can be beneficial, with the condition $\rho > b$ being a prerequisite. Furthermore, it is common for ODM companies to determine the optimal recovery rate rather than adjusting the selling price in response to changes in carbon prices to maximize profit, leading to the selling price being treated as a fixed parameter in this model.

- When estimating the carbon footprint of a mechanical product at the conceptual design stage, a carbon footprint calculation model is first required. Based on the definition of carbon footprint (PAS-2050, 2011 [44]) and the product life cycle, the carbon footprint contribution can be classified into five stages across the product's entire life cycle, starting with the raw materials acquisition stage (design stage δ_1), manufacturing stage (end product δ_{21}; components δ_{22}; scrap returns δ_{23}), transportation stage, usage stage, and recycle and disposal stage (He et al. [34]).
- Manufacturers use the activity-based costing (ABC) approach to obtain more accurate and detailed cost assignments for activities (Kamal et al. [45]). This approach leverages activity drivers to help manufacturers effectively collect and control relevant cost data. The Green Production Decision Model with Carbon Footprint (GPDMCF) proposed in this paper could be used to explore the impact of carbon emissions in the die casting industry. The overhead activities were divided into the unit, batch, and product levels using the ABC approach, and their resources and activity drivers were also investigated and selected using the ABC approach. Manufacturers have a fixed capacity of machine hours in the short term. Depreciation and machine costs are fixed, and there are limited machine hours for manufacturing. This paper examines the effects of carbon footprint and cap-and-trade on environmental performance. The environmental permits cannot be sold to other companies after purchase. GPDMCF provides data on the direct materials, direct labor, and product machining that manufacturers can use to produce final goods. These data can effectively reflect the manufacturing situation and help researchers simulate the production process.

4. Model Formulation

The inventory levels of components, finished products, and scrap returns in a two-stage assembly production system are depicted in Figure 1. This figure reveals the relationships between new materials, finished goods, and scrap returns. To mitigate global warming, incorporating low-carbon materials into product design is crucial for sustainable innovation. This process spans all stages, from raw material extraction to production, transportation, use, and eventual disposal or recycling. Measuring and reducing the carbon footprint of finished products is central to sustainable design, offering substantial environmental and economic benefits. Effectively managing scrap and returns is crucial for minimizing the carbon footprint of finished goods.

- On the above assumptions the required components, product yield, and total demand remain constant within a given cycle; i.e.,

  $p_1t_1 = p_2t_2 = p_it_i = \cdots = p_nt_n = p_et_e = \mathrm{DT}$, then we have

$$t_i = \frac{p_nt_n}{p_i}, \tag{1}$$

and

$$\mathrm{T} = \frac{p_nt_n}{\mathrm{D}}, \tag{2}$$

where $\mathrm{i} = 1, 2, \cdots, \mathrm{n}$..

- The maximum quantity of components i that can be held in inventory can be expressed as follows:

$$Q_i = (p_i - p_e)t_i = p_et_{id}. \tag{3}$$

After rearranging Equation (3), we obtain the following:

$$t_{id} = \frac{(p_i - p_e)t_i}{p_e},$$
$$= \frac{(p_i - p_e)p_n t_n}{p_e p_i}, \quad (4)$$

where $\text{i} = 1, 2, \cdots, \text{n}$.

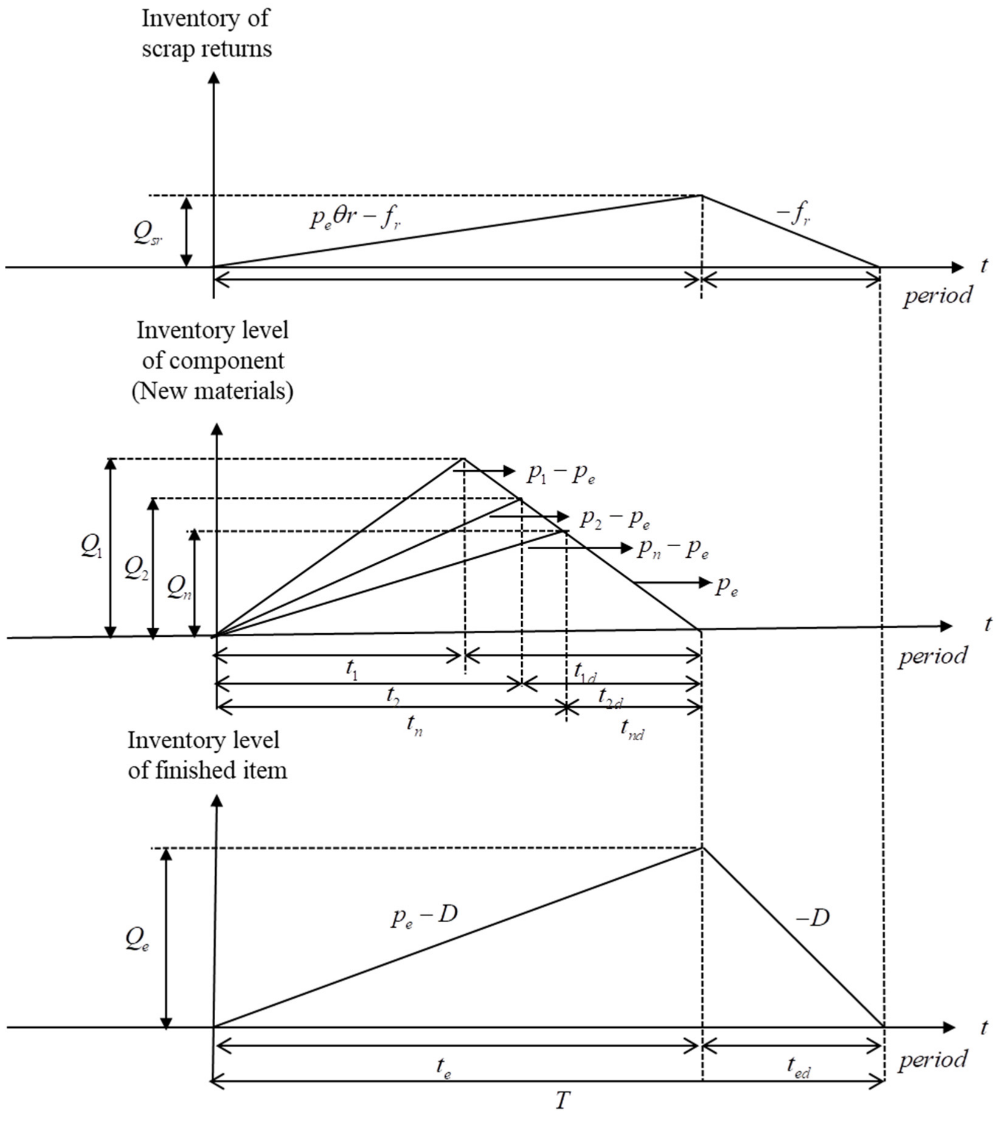


Figure 1. Graph of inventory levels for components and finished product.

- The finished product's maximum inventory level can be described as follows:

$$Q_e = (p_e - \text{D})t_e = (p_e - \text{D})(t_n + t_{nd}),$$
$$= (p_e - D)\frac{p_n t_n}{p_e}.(\text{from Equation } (4)). \quad (5)$$

- The maximum inventory level of the scrap returns can be described as follows:

$$Q_{sr} = (p_e \theta \text{r} - f_r)t_e. \quad (6)$$

The total profit per cycle is determined by the above findings.

1. Sales revenue (denoted by SR) is equal to the actual operating revenue $s_r - \mathrm{s}$ multiplied by the total demand DT, as follows:

$$\begin{aligned} \mathrm{SR} &= (s_r - \mathrm{s})\mathrm{DT} \\ &= (s_r - \mathrm{s})p_n t_n. \end{aligned} \tag{7}$$

2. Design cost (denoted by DC): Traditional approaches have considered set-up costs as an important factor in later development stages, independent from design decisions. This can lead to estimated set-up costs that are significantly higher than target costs. In some cases, an additional design stage may be required, which would become an additional investment. The Design to Cost strategy aims to incorporate cost considerations throughout the project development cycle so that cost targets guide the decision-making process. In the design stage, the total carbon footprint, δ_1, includes the costs associated with changing tools or molds, moving materials or components, and checking the initial output.

$$\mathrm{DC} = \delta_1 \sum_{i=1}^{n} Q_i = \delta_1 (p_n - p_e) t_n, \tag{8}$$

3. Holding cost of end product (denoted by HC_e): Figure 1 presents the per cycle holding cost of the end product, which is calculated as follows:

$$\begin{aligned} HC_e &= \frac{\delta_{21} h_e H_e T}{2}, \\ &= \frac{\delta_{21} h_e (p_e - D)}{2 p_e D} p_n^2 t_n^2. (\text{from Equations (2) and (5)}). \end{aligned} \tag{9}$$

4. The total holding cost (denoted by HC_s) for n components per cycle is calculated as follows:

$$\begin{aligned} HC_s &= \sum_{i=1}^{n} \frac{\delta_{22} Q_i h_i (t_i + t_{id})}{2}, \\ &= \delta_{22} \frac{p_n^2 t_n^2}{2} \left[\sum_{i=1}^{n} h_i \left(\frac{1}{p_e} - \frac{1}{p_i} \right) \right]. \end{aligned} \tag{10}$$

5. The holding cost of scrap returns (denoted by HC_r) can be calculated as follows:

$$HC_r = \delta_{23} h_r \frac{(p_e \theta \mathrm{r} - f_r) t_e^2 + f_r t_{ed}}{2}. \tag{11}$$

6. Recovery cost (denoted by RC): The total quantity of the defective products in a cycle is expressed as follows:

$$\mathrm{RC} = \delta_3 c_r p_e \theta t_e. \tag{12}$$

7. Investment to reduce the emission of pollutants (denoted by IC): The investment cost is the sum of the fixed cost, F_g (annual investment amount for carbon emission reduction per cycle, such as apportion charges for air purifiers, washing towers, or filtration equipment) and variable cost, $V_g p_e t_e$ (reduce the emission of pollutants per unit item, such as power or material for equipment), with V_g representing the investment to reduce the emission of pollutants and IC calculated as follows:

$$\begin{aligned} \mathrm{IC} &= \mathrm{v}\delta_4 \left(F_g + V_g p_e t_e \right) \\ &= \mathrm{v}\delta_4 \left(F_g + V_g p_n t_n \right). \end{aligned} \tag{13}$$

where v $= \begin{cases} v = 0, \text{ no invest} \\ v = 1, \text{ invest} \end{cases}$

8. Logistic and inventory management cost (denoted by LC)

The costs associated with spare parts logistics and inventory management throughout their life cycle should be estimated using an activity-based costing (ABC) model. According to Afonso and Paisana [46], the ABC matrix can facilitate this cost computation. The first step involves identifying the set of n resources and mapping the mmm activities required for managing the logistics of K spare parts. Finally, it is essential to establish the relationship between these activities and the corresponding spare parts. The resources vector column contains data on the total monetary value of each resource across multiple n rows (n = number of resources). For each resource r_j, a specific resource driver j is used to allocate costs to the various activities. The resources–activities matrix shows the proportion of each resource allocated to each activity i, calculated by dividing the quantity of the resource driver j associated with that activity i$\left(r'_{ij}\right)$ by the total quantity of the resource driver j$\left(r'_j\right)$.

$$r_{ij} = \frac{r'_{ij}}{r'_j},$$

then, the cost allocated to each activity a_i will be obtained as follows:

$$a_i = \sum_{j=1}^{n} r'_{ij} \cdot r'_j.$$

On the other hand, in the activities vector column, the element a_i represents the cost allocated to activity i. In the activities–products matrix (or activities–cost objects matrix), each element a_{ki} is the proportion of the activity-driver related to the cost object. It is calculated by dividing the quantity of the activity driver i related to the component n$\left(a'_{ni}\right)$ by the total quantity of the activity driver i$\left(a'_i\right)$. The cost per cost object n (in this case, component n) can then be obtained by multiplying the activities–products matrix by the activities vector column.

$$a_{ni} = \frac{a'_{ni}}{a'_i}.$$

Then, the cost allocated to each component will be expressed as follows:

$$sp_n = \sum_{i=1}^{m} a_{ni} \cdot a_i.$$

Then, replacing a_i in the previous equation LC gives the following equation:

$$\text{LC} = \delta_5 \text{s} \sum_{i=1}^{m} \sum_{j=1}^{n} a_{ni} \cdot r_{ij} \cdot r_j \tag{14}$$

where sp_n this reflects the logistics costs associated with a component. The cost per cost object is determined by multiplying the activities–products matrix by the activities vector column. This establishes the relationships among resources, activities, and cost objects:

$$\begin{matrix} Activity\,1 \\ Activity\,2 \\ \vdots \\ \vdots \\ Activity\,m \end{matrix} \begin{bmatrix} \text{Resource}\,1 & \text{Resource}\,2 & \text{Resource}\,j & \cdots & \cdots & \text{Resource}\,k \\ \surd & \surd & & & & \\ \surd & & \surd & & & \\ & & & & & \\ & & \surd & & & \surd \end{bmatrix}$$

Considering the concept of life cycle, there will be a logistic cost value for each period (e.g., year). Overall, the total profit per unit time can be obtained as follows:

$$\mathrm{AP}(\mathrm{s},t_n,\mathrm{r}) = \frac{1}{T}\times\{SR-DC-HC_e-HC_s-HC_r-RC-IC-LC\}$$
$$= \mathrm{D}p_nt_n\left\{\frac{(s_r-\mathrm{s})}{p_nt_n}-\frac{c_f(p_n-p_e)\delta_1}{p_n{}^2t_n}-\frac{h_e(p_e-D)\delta_{21}}{2p_eD}-\frac{\delta_{22}}{2}\left[\sum_{i=1}^{n}h_i\left(\frac{1}{p_e}-\frac{1}{p_i}\right)\right]\right.$$
$$-\delta_{23}h_r\frac{(p_e\theta\mathrm{r}-f_r)t_e^2+f_rt_{ed}}{2p_n{}^2t_n{}^2}-\delta_3\frac{c_rp_f\mathrm{r}\theta t_e}{p_n{}^2t_n{}^2}-\frac{\mathrm{v}\delta_4}{p_n{}^2t_n{}^2}\left(F_g+V_gp_nt_n\right)-\mathrm{s}p_nt_n\delta_5\sum_{i=1}^{m}\sum_{j=1}^{n}a_{ni}\cdot r_{ij}\cdot r_j \tag{15}$$

This study aimed to determine the optimal values s, t_n, and r that maximize total profit per unit time $\mathrm{AP}(\mathrm{s},t_n,\mathrm{r})$. Differentiating the values given in (15) with respect to s, $\mathrm{t_n}$, and r, respectively, we have the following equation:

$$\frac{\partial\mathrm{AP}(\mathrm{s},t_n,\mathrm{r})}{\partial\mathrm{s}} = -\left\{\delta_{23}h_r\frac{(p_e\theta\mathrm{r}-f_r)t_e^2+f_rt_{ed}}{2p_nt_n}+\delta_3\frac{c_rp_e\mathrm{r}\theta t_e}{p_nt_n}+\mathrm{v}\delta_4\frac{F_g}{p_nt_n}+D\right.$$
$$-(s_r-\mathrm{s})\rho-\delta_1\frac{\rho^2(p_n-p_e)}{p_n}-\delta_{21}\frac{h_e\rho}{2p_e}p_nt_n-\delta_{22}\frac{\rho p_nt_n}{2}\left[\sum_{i=1}^{n}h_i\left(\frac{1}{p_e}-\frac{1}{p_i}\right)\right]$$
$$\left.+\mathrm{v}\delta_4\rho V_g+\mathrm{D}\delta_5\sum_{i=1}^{m}\sum_{j=1}^{n}a_{ni}\cdot r_{ij}\cdot r_j\right\}, \tag{16}$$

$$\frac{\partial\mathrm{AP}(\mathrm{s},t_n,\mathrm{r})}{\partial t_n} = -\frac{D}{t_n\rho}\left\{\delta_{23}\rho h_r\frac{(p_e\theta\mathrm{r}-f_r)t_e^2+f_rt_{ed}}{2p_nt_n}+\delta_3\frac{c_rp_e\mathrm{r}\theta t_e\rho}{p_nt_n}+\mathrm{v}\rho\delta_4\frac{F_g}{p_nt_n}\right.$$
$$\left.-\rho\delta_{21}\frac{h_e(p_e-D)}{2p_eD}p_nt_n-\rho\delta_{22}\frac{p_nt_n}{2}\left[\sum_{i=1}^{n}h_i\left(\frac{1}{p_e}-\frac{1}{p_i}\right)\right]\right\}, \tag{17}$$

and

$$\frac{\partial\mathrm{AP}(\mathrm{s},t_n,\mathrm{r})}{\partial\mathrm{r}} = \mathrm{D}\left[r^2-\delta_{23}h_r\frac{p_e\theta t_e^2}{2p_nt_n}-\delta_3\frac{c_rp_e\theta t_e}{p_nt_n}\right]. \tag{18}$$

First, for any given $\mathrm{r}\geq 0$, it is well known that the necessary condition for (s,t_n) to be optimal must satisfy the equation $\partial\mathrm{AP}(\mathrm{s},t_n|\mathrm{r})/\partial\mathrm{s}$ and $\partial\mathrm{AP}(\mathrm{s},t_n|\mathrm{r})/\partial t_n$, simultaneously, which implies the following equation:

$$\delta_{23}\rho h_r\frac{(p_e\theta\mathrm{r}-f_r)t_e^2+f_rt_{ed}}{2p_nt_n}+\delta_3\rho\frac{c_rp_e\mathrm{r}\theta t_e}{p_nt_n}+\mathrm{v}\rho\delta_4\frac{F_g}{p_nt_n}=$$
$$-\delta_{22}\rho\frac{p_nt_n}{2}\left[\sum_{i=1}^{n}h_i\left(\frac{1}{p_e}-\frac{1}{p_i}\right)\right]-\mathrm{v}\rho\delta_4V_g-\mathrm{D}\rho\delta_5\sum_{i=1}^{m}\sum_{j=1}^{n}a_{ni}\cdot r_{ij}\cdot r_j$$
$$+\rho\delta_{21}\frac{h_ep_nt_n}{2p_e}-\rho\frac{\delta_1{}^2(p_n-p_e)}{p_n}+(s_r-\mathrm{s})\rho-D, \tag{19}$$

and

$$\delta_{23}h_r\left[(p_e\theta\mathrm{r}-f_r)t_e^2+f_rt_{ed}\right]+2\left(\delta_3c_rp_e\mathrm{r}\theta t_e+\mathrm{v}\delta_4F_g\right)=$$
$$p_n{}^2t_n{}^2\left\{\frac{h_e(p_e-D)\delta_{21}}{p_eD}+\delta_{22}\left[\sum_{i=1}^{n}h_i\left(\frac{1}{p_e}-\frac{1}{p_i}\right)\right]\right\}, \tag{20}$$

respectively. Since the left-hand sides of Equations (19) and (20) are the same, the right-hand sides are also equal:

$$\left\{\delta_{21}\frac{h_e(p_e-2D)}{2p_eD}+\left[\sum_{i=1}^{n}h_i\left(\frac{1}{p_e}-\frac{1}{p_i}\right)\right]\right\}\rho\delta_{22}p_nt_n$$

$$= (s_r - \mathrm{s})\rho - \mathrm{D}\delta_5\sum_{i=1}^{m}\sum_{j=1}^{n} a_{ni}{\cdot}r_{ij}{\cdot}r_j - \mathrm{v}\rho\delta_4 V_g - \rho^2\delta_1\frac{(p_n - p_e)}{p_n} - D.$$

From (12), we obtain the following equation:

$$\mathrm{r} = \sqrt{\frac{\theta p_e t_e}{2p_n t_n}(h_r - 2c_r)}.$$

It is easy to see that r is the difference in the holding cost of scrap returns per unit per unit time and returning a defective product to scrap returns. The below two cases show all of the possible situations of r.

Case 1: $h_r \geq 2c_r$

In this scenario, the value r is greater than or equal to zero. The key finding is that the reduction in holding costs of scrap returns per unit over time compensates for the production costs associated with returning defective products to scrap. Consequently, manufacturers should focus on preventing metal waste generation, encouraging more convenient reuse, and shifting away from the disposable culture that relies on single-use metals. Investing in alternative systems for reuse and refill services is essential.

Case 2: $h_r < 2c_r$

In this case, r is negative and against $\mathrm{r} \geq 0$. The most important finding of this case is the manufacturer cannot find reusable materials that can reduce waste yet, causing environmental pollution and ecological destruction, and threatening the sustainable development of human generations. The government should provide incentives for manufacturers to seek reusable options to encourage manufacturers to invest in recyclable material options. Finding the closed-form solution for $\mathrm{t_n}$ from Equation (20) is challenging. However, we can demonstrate that a unique value of $\mathrm{t_n}$ satisfying (20) exists, as shown in the following lemma.

Lemma 1. *The solution of t_n in (20) not only exists but also is unique.*

Proof. Let

$$\begin{aligned}\mathrm{F}(t_n) = \Big\{&\delta_{23}\rho h_r\tfrac{(p_e\theta\mathrm{r}-f_r)t_e^2+f_r t_{ed}}{2p_n t_n} + \delta_3\rho\tfrac{c_r p_e \mathrm{r}\theta t_e}{p_n t_n} + \mathrm{v}\rho\delta_4\tfrac{F_g}{p_n t_n}\\ &-\rho\delta_{21}p_n t_n\tfrac{h_e(p_e-D)}{2p_e D} + \rho\delta_{22}\tfrac{p_n t_n}{2}\Big[\sum_{i=1}^{n} h_i\Big(\tfrac{1}{p_e}-\tfrac{1}{p_i}\Big)\Big]\Big\}.\end{aligned}$$

for $t_n \in (0,\infty)$. Taking the first derivative of $\mathrm{F}(t_n)$ with respect to $t_n \in (0,\infty)$, gives

$$\begin{aligned}\tfrac{\mathrm{dF}(t_n)}{dt_n} = -\Big\{&\delta_{23}\rho h_r t_n^{-2}\tfrac{(p_e\theta\mathrm{r}-f_r)t_e^2+f_r t_{ed}}{2p_n} + \delta_3\rho t_n^{-2}\tfrac{c_r p_e \mathrm{r}\theta t_e}{p_n} + \mathrm{v}\rho\delta_4 t_n^{-2}\tfrac{F_g}{p_n}\\ &+\rho\delta_{21}p_n\tfrac{h_e(p_e-D)}{2p_e D} + \rho\delta_{22}\tfrac{p_n}{2}\Big[\sum_{i=1}^{n} h_i\Big(\tfrac{1}{p_e}-\tfrac{1}{p_i}\Big)\Big]\Big\} < 0.\end{aligned}$$

That is, $\mathrm{F}(t_n)$ is a strictly decreasing function of $t_n \in (0,\infty)$. Further, we have

$$\lim_{t_n\to 0}\mathrm{F}(t_n) = \left[\rho\delta_{23}h_r(p_e\theta\mathrm{r} - f_r) + \frac{\rho\delta_3 c_r p_e \mathrm{r}\theta}{p_n}\right] > 0$$

and $\lim_{t_n\to-\infty}\mathrm{F}(t_n) = -\infty < 0$. By applying the intermediate value theorem, there exists a unique $t_n \in (0,\infty)$ such that $\mathrm{F}(t_n) = 0$. This completes the proof. (See Appendix A). □

The algorithm is also presented graphically in Figure 2 (Algorithm 1).

Algorithm 1. A fast algorithm to obtain optimal solutions

Step 1. Start with $\tau = 0$ and $s_{j,\tau} = 0$.

Step 2. Put $s = s_{j,\tau}$ into Equation (20) to obtain the corresponding value of t_n, i.e., $\tilde{t}_{n,j}$, and then use Equation (20) to calculate $F(\tilde{t}_{n,j})$.

Step 3. If $F(\tilde{t}_{n,j}) \geq 0$, put $\tilde{t}_{n,j}$ into Equation (19) to obtain the corresponding value of s, i.e., $s_{j,\tau+1}$. Otherwise, let $\tilde{s}_j = 0$.

Step 4. If the difference between $s_{j,\tau}$ and $s_{j,\tau+1}$ is sufficiently small, set $\tilde{s}_j = s_{j,\tau+1}$. Otherwise, set $s_{j,\tau+1} = s_{j,\tau} + \varepsilon$, where ε is any small positive number, and set $\tau = \tau + 1$; then, go back to Step 3.

Step 5. Substitute $t_n = \tilde{t}_{n,j}$ and $s = \tilde{s}_j$ into Equation (15) to calculate the value of $AP(\tilde{s}_j, \tilde{t}_{n,j}, \tilde{r}_j)$.

Step 6. If $AP(\tilde{s}_0, \tilde{t}_{n0}, \tilde{r}_0) < AP(\tilde{s}_1, \tilde{t}_{n1}, \tilde{r}_1)$, then $(s^*, t_n{}^*, r^*) = (\tilde{s}_1, \tilde{t}_{n1}, \tilde{r}_1)$ is the optimal solution. Otherwise, $(s^*, t_n{}^*, r^*) = (\tilde{s}_0, \tilde{t}_{n0}, \tilde{r}_0)$. Substitute s^*, $t_n{}^*$, and r^* into Equations (1), (2) and (15) to calculate the values of $t_1{}^*, t_2{}^*, \ldots, t_{n-1}{}^*, T^*$, and AP $(s^*, t_n{}^*, r^*)$.

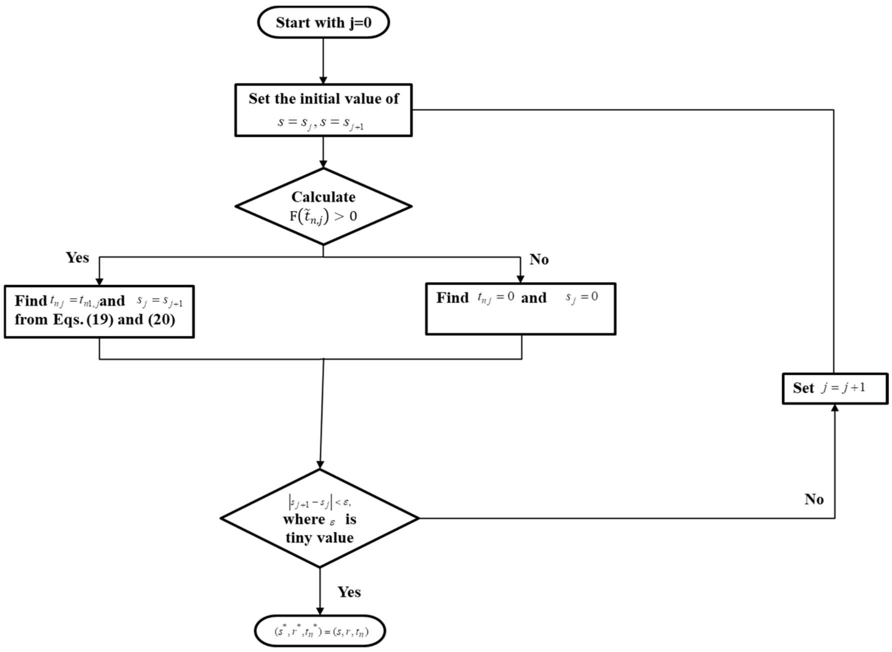


Figure 2. Algorithm 1 for generating the optimal solution.

5. Application Example

The model was tested through a case study of a Taiwanese machinery manufacturer. A numerical example validated our findings, and sensitivity analysis examined optimal policy trends, providing insights for the company.

5.1. Carbon Footprint Calculation in the Context of a Machinery OBM

Mengshen Machinery, established in 1973, specializes in the research, development, and manufacturing of dustless automatic painting equipment. All design, production, assembly, and testing processes are carried out within the factory. The design and manufacturing departments directly engage with the customer base, allowing them to accumulate experience and introduce new and improved products to meet industry needs. In recent years, the company has focused its technical capabilities on panel automation equipment and related process tools. It has a strong development team and specializes in OEM/ODM manufacturing of various automation equipment. Mengshen has established production bases in both Taiwan and mainland China, allowing it to accumulate extensive experience in equipment development. Toray's strengths provide advantages for carbon fiber composite blocks, including global operations and production facilities in Japan, the United

States, and Europe, as well as vertical integration of prepares and intermediate materials. Carbon fiber composite recycling technology enables the efficient separation and recovery of composite and resin materials, with recycled materials retaining similar characteristics to their original state. Recycling methods include mechanical processing, pyrolysis, and solvent dissolution. The mechanical method crushes and grinds waste composites into powder, but the resulting products are resin-rich and have limited commercial value, serving primarily as fillers or energy sources. Pyrolysis techniques, such as conventional, fluidized bed, and microwave-assisted pyrolysis, offer alternative recycling approaches. In the semiconductor industry, the primary source of the carbon footprint is electricity usage during manufacturing, with power consumption for cleanrooms and production processes being equal. This highlights the importance of energy-efficient facility and equipment design. The carbon emission contributions from various production processes are, in decreasing order, as follows: diffusion, etching, thin film, and photolithography. While thin film processes emit high-global-warming-potential PFC gases, other processes primarily contribute emissions from electricity consumption, i.e., using a chip inspection and rework machine to inspect reworked chips, including those with front-side or back-side breakage. The proper disposal of damaged chips is essential to minimize the environmental impact, which should be carried out following relevant hazardous waste regulations to ensure responsible management. On the other hand, recycled fibers and recovered resin, as well as easily recyclable composites made from degradable thermoset materials, are considered promising solutions to the problem of composite material recycling. Degradable thermoset plastics have cleavable bonds within their cross-linked networks. To reach net zero targets requires actions to reduce emissions; companies need to evaluate the impact of climate risks. It is advisable to implement internal carbon pricing and determine an optimal production rate as tools for assessing capital expenditures and promoting emissions reductions. This approach will strengthen their carbon risk management capabilities and enhance their resilience to climate challenges.

5.2. Numerical Examples

5.2.1. Example 1

Let us consider an inventory system with the following data. A two-stage assembly system is observed. It consists of three component processes (n = 3) in stage 1, and three components are required to assemble an end product in stage 2. Authors should discuss the results and how they can be interpreted from the perspective of previous studies and the working hypotheses. The findings and their implications should be discussed in the broadest context possible. Future research directions may also be highlighted.

Demand function: a = 10, b = 0.2, ρ = 0.3, w = 0.6, s_p = \$400.

Component 1 process: $p_1 = 400$, $h_1 = \$0.01$, $\theta_1 = 0.5$, $r_1 = \$0.1$.

Component 2 process: $p_2 = 900$, $h_2 = \$0.2$, $\theta_2 = 0.03$, $r_2 = \$0.2$.

Component 3 process: $p_3 = 800$, $h_3 = \$0.3$, $\theta_3 = 0.02$, $r_3 = \$0.3$.

Assembly process: $p_e = 700$, $h_e = \$0.2$, $\theta_e = 0.01$, $r_e = \$0.4$.

Other costs: k = \$300, $\beta_0 = \$10$, $\beta_1 = \$500$, $F_g = \$500$, $V_g = \$2$.

Carbon footprint coefficient: $\delta_1 = 0.1$, $\delta_{21} = 0.2$, $\delta_{22} = 0.3$, $\delta_{23} = 0.2$, $\delta_3 = 0.1$, $\delta_4 = 0.05$, $\delta_5 = 0.05$.

5.2.2. Example 2

- To clarify the relative contributions within the resources–activities matrix, the above Example 1 and an activity-based costing (ABC) method were applied. The cost computation mechanism using the AB-LCC model will be described in detail for the first year of the life cycle, with the understanding that this calculation should be similarly applied for the remaining years. Analysis of example data in Tables 2 and 3.

Table 2. Data for computing the resources–activities matrix.

	r_{i1_1}	r_{i2_1}	r_{i3_1}	r_{i4_1}	r_{i5_1}	r_{i6_1}	r_{i7_1}	r_{i8_1}
a_{1_1}	40	0.24	10	30	1	25	5	15
a_{2_1}	20	0.12	5	20	6	25	5	5
r_{3_1}	10	0.06	2	40	5	25	5	35
r_{4_1}	10	0.06	5	45	3	25	5	45
r_{5_1}	34	0.2	25	40	4	25	5	5
r_{6_1}	23	0.14	30	45	25	25	5	15
r_{7_1}	25	0.15	5	40	5	25	5	5
r_{8_1}	20	0.1	7	45	25	25	5	5

Table 3. Activity costs per spare part in the process.

Activity	a_{i_1}	Tappet	Slide
Rolling mill process	USD 70,111	USD 66,123	USD 3988
Heat treatment	USD 45,124	USD 40,134	USD 4990
Shock	USD 32,120	USD 30,146	USD 1974
Mill	USD 23,123	USD 19,134	USD 3989
Leak test	USD 62,111	USD 57,321	USD 4790
Picking	USD 43,345	USD 41,321	USD 2024
Marking and packing	USD 24,451	USD 21,321	USD 3130
Shipping	USD 56,114	USD 53,124	USD 2990
	USD 356,499	USD 328,624	USD 27,875

5.3. Sensitivity Analysis

The numerical Examples 1 and 2 presented in Section 5.2 was used to assess the effects of changes to system parameters (h_r, s_p, θ_i, ρ, w, F_g, V_g, c_r, and p_f) on the values of s^*, $t_n{}^*$, r^*, and $AP(s^*, t_n{}^*, r^*)$. Each parameter was adjusted separately (i.e., the other parameters were left unchanged) by +50%, +25%, −25%, or −50%. Our analytical results in Tables 4 and 5 permit the following interesting observations and managerial insights that could be used to guide decision making. The analytical results for Examples 1 and 2 are shown in Table 6.

Table 4. Computation results of Example 1 for traditional cost.

		v = 1				v = 0			
Parameter		s	t_3	r	AP	s	t_3	r	AP
h_r	+50%	33.3327	0.5525	0.2567	104.316	63.3326	0.4467	0.3811	80.883
	+25%	33.3327	0.5698	0.2504	102.558	63.3326	0.4068	0.4246	73.675
	−25%	33.3327	0.5951	0.2418	102.111	63.3328	0.2370	0.5619	42.657
	−50%	33.3327	0.6544	0.2238	101.801	63.3327	0.1968	0.6958	35.438
s_p	+50%	33.3324	0.4683	0.2141	84.321	63.3323	0.3381	0.3911	54.453
	+25%	33.3326	0.5494	0.2230	98.904	63.3325	0.3471	0.4110	64.323
	−25%	33.3330	0.6257	0.5605	131.232	63.3328	0.3610	0.5121	74.133
	−50%	33.3331	0.7006	0.6354	148.228	63.3331	0.3701	0.5911	84.231
θ_i	+50%	33.3329	0.1641	0.2432	35.658	63.3736	0.0174	0.2385	85.300
	+25%	33.3329	0.2371	0.2465	95.746	63.3416	0.0264	0.3668	68.877
	−25%	33.3327	0.3522	0.6563	107.137	63.3106	0.0360	0.6386	63.303
	−50%	33.3326	0.4733	0.9572	108.919	63.3026	0.0533	0.8773	61.621
ρ	+50%	22.2221	0.2923	0.2437	526.727	42.2217	0.5514	0.2179	273.153
	+25%	26.6663	0.2331	0.2455	421.147	50.6661	0.5307	0.3244	96.311
	−25%	44.4436	0.2292	0.2554	36.2223	84.4435	0.3131	0.5458	56.678
	−50%	66.6655	0.2151	0.2587	32.4278	126.665	0.3048	0.6207	54.198

Table 4. *Cont.*

		v = 1				v = 0			
Parameter		**s**	$\mathbf{t_3}$	**r**	**AP**	**s**	$\mathbf{t_3}$	**r**	**AP**
w	+50%	31.1367	0.5995	0.2671	106.211	78.2136	0.4499	0.3600	81.461
	+25%	32.3125	0.5895	0.2572	105.213	70.3136	0.4031	0.4202	72.994
	−25%	34.3025	0.5695	0.2172	103.117	55.1436	0.3529	0.5431	62.125
	−50%	36.3327	0.5195	0.2071	102.013	48.2316	0.2778	0.5678	56.775
F_g	+50%	38.3327	0.6101	0.2670	94.317	65.2216	0.6091	0.5237	110.522
	+25%	36.3327	0.6001	0.2570	99.317	64.1126	0.5165	0.5162	93.636
	−25%	32.3327	0.5491	0.2270	106.317	62.2326	0.3305	0.3872	59.843
	−50%	31.3327	0.5391	0.2170	108.317	61.3326	0.2275	0.3732	41.179
V_g	+50%	30.1327	0.5292	0.2910	99.104	61.3128	0.4816	0.3542	87.2872
	+25%	31.0327	0.5592	0.2859	100.104	62.3326	0.3602	0.4906	65.2563
	−25%	32.8327	0.6092	0.2460	108.317	65.4347	0.3571	0.4988	64.7677
	−50%	33.3327	0.6791	0.2338	110.317	68.1353	0.3461	0.5579	64.1929
c_r	+50%	30.1327	0.6462	0.2210	94.452	61.2328	0.4592	0.2447	73.3645
	+25%	33.3324	0.6378	0.2371	99.381	62.5367	0.4529	0.2992	72.1321
	−25%	33.3328	0.5613	0.2514	138.356	63.3316	0.3445	0.5113	60.4589
	−50%	33.3329	0.5024	0.2636	166.605	66.1326	0.3182	0.6708	50.1495
p_f	+50%	33.3326	0.6461	0.2307	104.445	63.1426	0.4591	0.2447	83.3471
	+25%	33.3327	0.6178	0.2461	104.338	63.2347	0.4529	0.2992	82.1249
	−25%	33.3328	0.5615	0.2513	103.235	63.5126	0.3445	0.5112	60.4649
	−50%	33.3329	0.5102	0.2635	103.160	63.6338	0.3383	0.6706	58.1555

Table 5. Computation results of Example 2 for ABC policy.

		v = 1				v = 0			
Parameter		**s**	$\mathbf{t_3}$	**r**	**AP**	**s**	$\mathbf{t_3}$	**r**	**AP**
h_r	+50%	17.2198	0.1333	0.0491	101.888	63.3332	0.3596	2.6607	646.848
	+25%	17.2632	0.1659	0.0601	106.269	63.3331	0.3254	2.2373	585.478
	−25%	17.4421	0.2667	0.0105	123.754	63.3328	0.2370	0.5620	426.571
	−50%	17.6694	0.4001	0.0162	145.602	63.3327	0.1969	0.4958	354.379
s_p	+50%	17.3292	0.1983	0.0767	112.831	63.3327	0.3028	0.2410	544.735
	+25%	17.3294	0.2094	0.0769	112.831	63.3328	0.2856	0.6034	513.819
	−25%	17.3298	0.3257	0.0770	112.829	63.3330	0.2585	2.1783	465.273
	−50%	17.3321	0.6006	0.0773	112.829	63.3330	0.2541	8.4230	463.758
θ_i	+50%	17.3896	0.2041	0.0511	117.532	63.3329	0.2562	0.6816	460.996
	+25%	17.3786	0.2171	0.0613	116.631	63.3329	0.2689	1.0303	483.889
	−25%	17.3286	0.2522	0.0102	111.234	63.3330	0.2933	2.3324	527.721
	−50%	17.3166	0.2733	0.0153	110.732	63.3330	0.3607	3.3465	1092.55
ρ	+50%	11.8242	0.2923	0.0790	84.6499	42.2217	0.2514	0.1643	449.910
	+25%	14.0213	0.2331	0.0778	95.9593	50.6661	0.2724	1.3470	490.223
	−25%	22.8601	0.2292	0.0756	140.816	84.4435	0.2767	3.8662	590.028
	−50%	33.9461	0.2151	0.0747	196.584	126.665	0.4884	4.7066	879.080
w	+50%	20.3296	0.4995	0.0867	126.241	78.3329	0.2690	3.0576	860.521
	+25%	19.3125	0.3895	0.0827	115.233	70.8329	0.2721	2.4188	507.633
	−25%	14.3025	0.2241	0.0717	93.018	55.8329	0.2770	0.5660	492.547
	−50%	12.3327	0.2195	0.0707	82.013	48.3329	0.2879	0.1175	418.030
F_g	+50%	19.2318	0.6101	0.0867	94.317	63.3327	0.7114	0.2951	128.051
	+25%	18.5326	0.6001	0.0825	99.317	63.3327	0.6438	0.3237	158.920
	−25%	12.3327	0.5491	0.0727	116.426	63.3330	0.2599	4.3029	557.712
	−50%	11.3327	0.5391	0.0717	118.337	63.3330	0.2281	4.4941	636.371

Table 5. *Cont.*

Parameter		v = 1				v = 0			
		s	t_3	r	AP	s	t_3	r	AP
V_g	+50%	10.1327	0.2092	0.0791	99.104	63.3327	0.6496	0.3164	116.932
	+25%	11.0327	0.2292	0.0785	100.104	63.3327	0.6197	0.3311	131.554
	−25%	22.8327	0.3092	0.0746	128.317	63.3330	0.2373	1.4508	552.999
	−50%	23.3327	0.3791	0.0733	130.317	63.3330	0.2128	1.4664	626.847
c_r	+50%	16.1327	0.2462	0.0721	112.832	63.3328	0.2294	0.2290	413.058
	+25%	16.3324	0.2378	0.0737	112.831	63.3328	0.2338	0.3856	420.931
	−25%	17.3328	0.1613	0.0851	112.829	63.3332	0.3497	3.3868	629.232
	−50%	17.3329	0.1024	0.0863	112.829	63.3337	0.4784	7.3682	860.159
p_f	+50%	17.7626	0.2461	0.2307	126.681	63.3328	0.2295	0.2289	413.111
	+25%	17.5327	0.2178	0.2461	119.862	63.3328	0.2338	0.3856	420.958
	−25%	17.3328	0.1615	0.2513	105.568	63.3332	0.3497	3.3871	629.206
	−50%	17.3329	0.1102	0.2635	98.069	63.3337	0.4786	7.3718	860.521

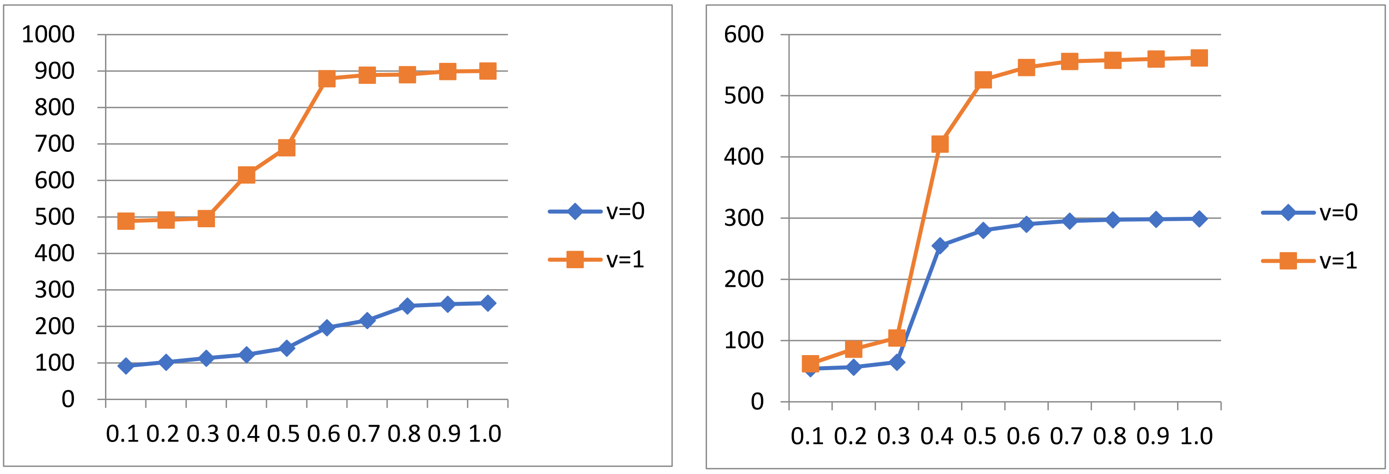
Table 6. Some managerial insights of sensitivity analysis.

Increasing Parameter(s)	Example 1 (Non-ABC)	Example 2 (ABC)
h_r	After recycling discarded automobile parts, the recycling company dismantles major large metal components, tires, lead-acid batteries, lubricants, refrigerants, and other directly recyclable items. The remaining items, along with the waste automobile parts, are then handed over to processing companies. Through multiple treatment processes such as shredding and sorting, profits have significantly increased, leading companies to be more willing to invest in the recycling of discarded parts.	It is required that each operational cost be linked to value-generating activities. Adopting this system allows for visibility into cost, quality, and time-related information. Through the ABC costing system, companies can effectively manage metal components and reduce unnecessary operational activities to lower operating costs.
s_p	Effective carbon pricing and price differentials not only incentivize companies to save energy and reduce carbon emissions but also to promote innovation in carbon reduction technologies and the carbon footprint of products.	The ABC-integrated carbon cost accounting system is highly operational. It not only provides comprehensive and detailed carbon cost information but also allows for tracing the sources of carbon costs. This helps to uncover potential for reducing carbon costs and achieving the goal of controlling carbon emissions.
θ_i	Reducing defective products not only saves on the cost of replacing materials and avoids additional expenses for inspection and correction of defects, but also increases profits.	By applying the principles of activity-based costing for cost allocation, the accuracy of cost information for automotive parts can be improved. Additionally, it can extend management applications, such as customer profitability analysis and process non-value-added activity improvements.
ρ, w	To avoid the high costs of carbon taxes, companies may shift their production and investment to low-cost countries, leading to carbon leakage. Investing in carbon reduction technologies can focus on energy saving, emission reduction, circular utilization, and environmentally friendly materials, thereby reducing environmental impact and increasing corporate profits.	The implementation of CBAM aims to curb ‘carbon leakage’, which occurs when companies, to avoid costs associated with carbon emissions, relocate production to countries and regions with lower carbon requirements. This shift leads to an increase in the consumption of imported products with higher carbon footprints and incentivizes companies to adopt the ABC costing method.

Table 6. *Cont.*

Increasing Parameter(s)	Example 1 (Non-ABC)	Example 2 (ABC)
F_g, V_g	Investing in energy-saving and carbon reduction technologies does indeed increase costs, but innovative energy-saving technologies can help companies reduce the costs of carbon reduction and increase operational profits.	Corporate investments in energy-saving and carbon reduction primarily focus on energy efficiency and low-carbon process analysis. Using scrap automobile parts for steelmaking saves more than 60% of the energy used in conventional processes. The ABC costing method is employed to correct the cost distortion associated with traditional manufacturing cost allocation.
c_r, p_f	Defective and surplus materials hold no economic value for the company. However, if waste can be effectively reused, it can reduce environmental impact, increase company profits, and promote a circular economy.	By using the ABC (activity-based costing) method to allocate manufacturing expenses based on different production activities, managers can clearly and accurately see the sources of manufacturing costs. For example, the costs associated with reworking defective products can accurately reflect where the expenses are incurred.

We then examine how investing versus not investing in green industrial development impacts the optimal strategies for both ABC and non-ABC systems. Industries with a higher risk of carbon leakage receive a greater share of free allowances, shifting the financial burden from large polluters to smaller downstream enterprises. To simplify the explanation, we use the numerical values in Example 1, adopt a value for $\rho \in \{0.1, 0.2, \cdots, 1\}$, and draw a line chart according to the analysis results, as shown in Figure 3. This shift increases the overall impact on industries and society; companies that have implemented energy-saving and carbon-reduction technologies achieve higher profitability compared to those that have not. To promote environmentally responsible practices, governments worldwide have introduced carbon footprint labels and carbon tax systems. As a result, businesses must not only manage production costs efficiently but also factor in environmental protection and carbon tax expenses. As the government reduces the number of emissions permits each year, the total cap is lowered, causing the permits to become more expensive. Over time, this incentivizes companies to invest in cleaner technologies and reduce their emissions more efficiently, as it becomes cheaper than purchasing permits. Companies are taxed if their emissions exceed their allotted permits, and they may face penalties for violations. Conversely, companies that lower their emissions can sell or trade their excess permits to other firms with higher emissions. They can also save these permits for future use. The cap-and-trade system establishes a market for emissions. Companies that have emissions credits can sell them for profit, creating a new economic resource for industries. Opponents of cap and trade argue that it may lead to excessive pollution up to the maximum levels set by the government, as the allowable levels could be too lenient, hindering the transition to cleaner energy. Additionally, emissions credits are often less expensive than converting to cleaner technologies and resources, particularly for industries that rely on fossil fuels. This suggests that cap and trade may not provide a meaningful incentive for those industries to change their practices.



(**a**) (**b**)

Figure 3. (**a**) Effect of ρ on total profit per unit time (ABC). (**b**) Effect of ρ on total profit per unit time (ABC).

6. Management Implications

The paper provides guidance for manufacturing companies on how to focus their management efforts and use an ABC approach. It suggests ways for companies to set optimal internal carbon prices and develop future projects to improve performance. The paper also identifies the following important areas for future research:

- Changing internal behaviors to accelerate greenhouse gas emissions reduction
- Mitigating risks after implementing carbon pricing
- Identifying cost-saving and investment opportunities in the value chain
- Incorporating climate risks into financial and operational decision-making

By internalizing the external costs of carbon, companies can encourage low-carbon development and green investments. This can enhance their social image, competitiveness, and ability to meet government emissions regulations and market demands, supporting sustainable development. The research finds that well-designed and equitable carbon pricing approaches can reconcile environmental sustainability with economic development objectives. Policymakers are advised to pursue comprehensive strategies incorporating carbon pricing into broader economic and environmental frameworks, emphasizing the significance of global collaboration, and supporting ongoing studies to refine carbon pricing models and approaches.

7. Conclusions

In this paper, we developed a two-stage production-inventory system addressing carbon footprint and scrap return issues using the activity-based costing (ABC) method. First, we established the necessary and sufficient conditions for the existence and uniqueness of the optimal solution. A key finding is that the reduced holding cost of scrap returns per unit over time compensates for the production costs of returning defective products to scrap. Next, we presented a straightforward algorithm to identify the optimal solution $(s^*, {t_n}^*, r^*)$ that maximizes the total profit per unit of time. The effects of the model parameters on the optimal solution and maximum total profit per unit of time were analyzed through two numerical examples in the casting industry, providing valuable managerial insights for Mengshen firms. The present study enhances the previous studies' findings by providing a much more detailed examination of internal carbon pricing. The limitation concerns the internal carbon pricing used in the current study. However, governmental carbon pricing is a necessary part of the climate policy toolkit required to achieve net-zero emissions and reach the Paris Agreement goals. Based on the analysis results, the findings of this study are as follows:

1. This shift leads to an increase in the consumption of imported products with higher carbon footprints and incentivizes companies to adopt the ABC costing method. The company considers that the selection of materials based on product carbon footprint should prioritize consumer-end products, extend footprint calculations to the supply chain, and allocate more resources, both in terms of manpower and material, which will incur higher costs.
2. The company needs to re-evaluate the various possibilities of manufacturing from the very beginning, considering, during the design and manufacturing stages, how the product can be reused, repaired, remanufactured, and recycled. This approach ensures that the product does not become waste after use but is instead as fully circulated and reused as possible.
3. It is recommended that the company implement a voluntary reduction plan to be eligible for adjustments in the carbon-leakage risk factor for chargeable emissions. Even if the entities subject to the charges qualify for the aforementioned transitional adjustment mechanism, they are still required to pay a certain proportion of the carbon fees and cannot be exempted from this obligation. Additionally, they must carry out an approved voluntary reduction plan to achieve actual reductions.

The findings of this study suggest several avenues for future research. First, the effects of variable deterioration rates and the stochastic nature of demand warrant further exploration. Researchers could also examine how production and sales strategies should be adapted when the time to market for new finished products is unpredictable. Lastly, investigating a multistage production system for multiple products represents a valuable area for further study.

Author Contributions: Conceptualization, H.-Y.T., C.-J.F. and Y.-F.H.; methodology, M.-W.W.; software, M.-W.W.; validation, H.-Y.T., C.-J.F., Y.-F.H. and M.-W.W.; formal analysis, M.-W.W.; investigation, M.-W.W.; resources, M.-W.W.; data duration, H.-Y.T.; writing—original draft preparation, H.-Y.T., M.-W.W. and C.-J.F.; writing—review and editing, H.-Y.T., M.-W.W. and C.-J.F.; visualization, Y.-F.H.; supervision, Y.-F.H.; project administration, Y.-F.H. All authors have read and agreed to the published version of the manuscript.

Funding: This work was supported by the Ministry of Science and Technology of the Republic of China under Grant NSTC 113-2637-H-324-003-.

Data Availability Statement: The original contributions presented in the study are included in the article, further inquiries can be directed to the corresponding author.

Acknowledgments: The authors would like to thank the editor and anonymous reviewers for their valuable and constructive comments, which have led to a significant improvement in the manuscript.

Conflicts of Interest: The authors declare that there are no conflicts of interest.

Appendix A

Proof of Lemma 1. For a given s, taking the second-order partial derivatives of $\mathrm{AP}(\mathrm{s}, t_n, \mathrm{r})$ with respect to t_n, and r, we have

$$\frac{\partial^2 \mathrm{AP}(\mathrm{s}, t_n, \mathrm{r})}{\partial t_n^2} = \frac{D}{\rho}\left\{-\delta_{23}\rho h_r \frac{(p_e\theta \mathrm{r} - f_r)t_e^2 + f_r t_{ed}}{2p_n} + 2\rho\delta_{21}\frac{h_e(p_e - D)}{p_e D}p_n t_n + \rho\delta_{22} p_n t_n\left[\sum_{i=1}^{n} h_i\left(\frac{1}{p_e} - \frac{1}{p_i}\right)\right]\right\},$$

and

$$\frac{\partial^2 \mathrm{AP}(\mathrm{s}, t_n, \mathrm{r})}{\partial r^2} = 2r^2\mathrm{D}.$$

Furthermore, for a given s, the following equation can be obtained:

$$\frac{\partial^2 \mathrm{AP}(\mathrm{s}, t_n, \mathrm{r})}{\partial \mathrm{r} \partial t_n} = \frac{\partial^2 \mathrm{AP}(\mathrm{s}, t_n, \mathrm{r})}{\partial t_n \partial \mathrm{r}} = 0.$$

Therefore, for a given s, the determinant of the Hessian matrix at the point (t_n, r) is

$$\begin{vmatrix} \frac{\partial^2 \mathrm{AP}(\mathrm{s},t_n,\mathrm{r})}{\partial \mathrm{r}^2} & \frac{\partial^2 \mathrm{AP}(\mathrm{s},t_n,\mathrm{r})}{\partial \mathrm{r} \partial t_n} \\ \frac{\partial^2 \mathrm{AP}(\mathrm{s},t_n,\mathrm{r})}{\partial t_n \partial \mathrm{r}} & \frac{\partial^2 \mathrm{AP}(\mathrm{s},t_n,\mathrm{r})}{\partial {t_n}^2} \end{vmatrix} = \frac{r^2 D^2}{\rho} \left\{ -\delta_{23} \rho h_r \frac{(p_e \theta \mathrm{r} - f_r) t_e^2 + f_r t_{ed}}{p_n} + 4\rho\delta_{21} \frac{h_e (p_e - D)}{p_e D} p_n t_n \right.$$
$$\left. +2\rho\delta_{22} p_n t_n \left[\sum_{i=1}^{n} h_i \left(\frac{1}{p_e} - \frac{1}{p_i} \right) \right] \right\} > 0.$$

Thus, we can prove that the value of (t_n, r) that satisfies Equations (17) and (18) not only exists but also is unique as Lemma 1. □

References

1. De Beer, P.; Friend, F. Environmental accounting: A management tool for enhancing corporate environmental and economic performance. *Ecol. Econ.* **2006**, *58*, 548–560. [CrossRef]
2. Tsai, W.-H.; Chang, Y.-C.; Lin, S.-J.; Chen, H.-C.; Chu, P.-Y. A green approach to the weight reduction of aircraft cabins. *J. Air Transp. Manag.* **2014**, *40*, 65–77. [CrossRef]
3. Tsai, W.-H.; Lin, T.W.; Chou, W.-C. Integrating activity-based costing and environmental cost accounting systems: A case study. *Int. J. Bus. Syst. Res.* **2010**, *4*, 186–208. [CrossRef]
4. Tsai, W.-H.; Yang, C.-H.; Chang, J.-C.; Lee, H.-L. An Activity-Based Costing decision model for life cycle assessment in green building projects. *Eur. J. Oper. Res.* **2014**, *238*, 607–619. [CrossRef]
5. Darwish, M.A. EPQ models with varying setup cost. *Int. J. Prod. Econ.* **2008**, *113*, 297–306. [CrossRef]
6. Kreng, B.W.; Wu, S.Y. Implementing the optimal policy of set-up time reduction in an economic production quantity model. *Int. J. Syst. Sci.* **2000**, *31*, 605–612. [CrossRef]
7. Afshar-Nadjafi, B. An EPQ model with unit production cost and set-up cost as functions of production rate. *Model. Simul. Eng.* **2013**, *8*, 1–7. [CrossRef]
8. Kang, M. Activity-based costing research on enterprise logistics cost management. *Bus. Manag. Res.* **2015**, *4*, 18–21. [CrossRef]
9. Chang, H.J.; Su, R.H.; Yang, C.T.; Weng, M.W. An economic manufacturing quantity model for a two-stage assembly system with imperfect processes and variable production rate. *Comput. Ind. Eng.* **2012**, *63*, 285–293. [CrossRef]
10. Pearn, W.; Su, R.H.; Weng, M.W.; Hsu, C.H. Optimal production run time for two-stage production system with imperfect processes and allowable shortages. *Central Eur. J. Oper. Res.* **2011**, *19*, 533–545. [CrossRef]
11. Su, R.H.; Weng, M.W.; Yang, C.T.; Li, H.T. An imperfect production–inventory model with mixed materials containing scrap returns based on a circular economy. *Processes* **2021**, *9*, 1275. [CrossRef]
12. Saha, S.; Goyal, S.K. Supply chain coordination contracts with inventory level and retail price dependent demand. *Int. J. Prod. Econ.* **2015**, *161*, 140–152. [CrossRef]
13. Taleizadeh, A.A.; Kalantari, S.S.; Cárdenas-Barrón, L.E. Pricing and lot sizing for an EPQ inventory model with rework and multiple shipments. *TOP* **2016**, *24*, 143–155. [CrossRef]
14. Shah, N.H.; Patel, D.G.; Shah, D.B. EPQ model for returned/reworked inventories during imperfect production process under price-sensitive stock-dependent demand. *Oper. Res.* **2018**, *18*, 343–359. [CrossRef]
15. Jazinaninejad, M.; Seyedhosseini, S.M.; Hosseini-Motlagh, S.M.; Nematollahi, M. Coordinated decision-making on manufacturer's EPQ-based and buyer's period review inventory policies with stochastic price-sensitive demand: A credit option approach. *RAIRO-Oper. Res.* **2019**, *53*, 1129–1154. [CrossRef]
16. Ruidas, S.; Seikh, M.R.; Nayak, P.K. An EPQ model with stock and selling price dependent demand and variable production rate in interval environment. *Int. J. Syst. Assur. Eng. Manag.* **2020**, *11*, 385–399. [CrossRef]
17. Su, R.H.; Weng, M.W.; Huang, Y.F. Innovative maintenance problem in a two-stage production-inventory system with imperfect processes. *Ann. Oper. Res.* **2020**, *287*, 379–401. [CrossRef]
18. Duan, Y.; Cao, Y. Optimal pricing and deteriorating inventory control when inventory stimulates stochastic demand with reference price effect. *RAIRO-Oper. Res.* **2021**, *55*, S1803–S1821. [CrossRef]
19. Seyedhosseini, S.M.; Hosseini-Motlagh, S.M.; Johari, M.; Jazinaninejad, M. Social price-sensitivity of demand for competitive supply chain coordination. *Comput. Ind. Eng.* **2019**, *135*, 1103–1126. [CrossRef]
20. Daryanto, Y.; Wee, H.M. Low carbon economic production quantity model for imperfect quality deteriorating items. *Int. J. Ind. Eng. Eng. Manag.* **2019**, *1*, 1–8. [CrossRef]
21. Sinha, S.; Modak, M.M. An EPQ model in the perspective of carbon emission reduction. *Int. J. Math. Oper. Res.* **2019**, *14*, 338–358. [CrossRef]

22. Sepehri, A.; Gholamian, M.R. Joint pricing and lot-sizing for a production model under controllable deterioration and carbon emissions. *Int. J. Syst. Sci. Oper. Logist.* **2022**, *9*, 324–338. [CrossRef]
23. Ebrahimi, E.; Shirazi, B.; Mahdavi, I.; Khamseh, A.A. A green economic production quantity model: A case study. *Int. J. Glob. Warm.* **2022**, *28*, 275–296. [CrossRef]
24. Mesrzade, P.; Dehghanian, F.; Ghiami, Y. A Bilevel model for carbon pricing in a green supply chain considering price and carbon-sensitive demand. *Sustainability* **2023**, *15*, 16563. [CrossRef]
25. Bhavani, G.D.; Mahaoatra, G.S.; Kumar, A. A sustainable two-echelon green supply chain coordination model under fuzziness incorporating carbon pricing policies. *Environ. Sci. Pollut. Res.* **2023**, *30*, 89197–89237. [CrossRef]
26. Li, J.; Lai, K.K.; Li, Y. Remanufacturing and low-carbon investment strategies in a closed-loop supply chain under multiple carbon policies. *Int. J. Logist. Res. Appl.* **2024**, *27*, 170–192. [CrossRef]
27. Du, S.; Hu, L.; Wang, L. Low-carbon supply policies and supply chain performance with carbon concerned demand. *Ann. Oper. Res.* **2017**, *255*, 569–590. [CrossRef]
28. Wang, Z.; Brownlee, A.E.I.; Wu, Q. Production and joint emission reduction decisions based on two-way cost-sharing contract under cap-and-trade regulation. *Comput. Ind. Eng.* **2020**, *146*, 106549. [CrossRef]
29. Lou, G.X.; Xia, H.Y.; Zhang, J.Q.; Fan, T.J. Investment strategy of emission-reduction technology in a supply chain. *Sustainability* **2015**, *7*, 10684–10708. [CrossRef]
30. Yang, L.; Zhang, Q.; Ji, J. Pricing and carbon emission reduction decisions in supply chains with vertical and horizontal cooperation. *Int. J. Prod. Econ.* **2017**, *191*, 286–297. [CrossRef]
31. Tao, F.; Fan, T.; Lai, K.K. Optimal inventory control policy and supply chain coronation problem with carbon footprint constraints. *Int. Trans. Oper. Res.* **2018**, *25*, 1831–1853. [CrossRef]
32. Lu, C.J.; Yang, C.T.; Yen, H.F. Stackelberg game approach for sustainable production-inventory model with collaborative investment in technology for reducing carbon emissions. *J. Clean. Prod.* **2020**, *270*, 121963. [CrossRef]
33. Priyan, S. Effect of green investment to reduce carbon emissions in an imperfect production system. *J. Clim. Financ.* **2023**, *2*, 100007. [CrossRef]
34. He, B.; Mao, H.; Li, T.; Xiao, J. A closed-loop digital twin modeling method integrated with carbon footprint analysis. *Comput. Ind. Eng.* **2023**, *182*, 109389. [CrossRef]
35. Huang, Y.F.; Weng, M.W.; Fu, C.J. A two-stage sustainable production-inventory model with carbon credit demand. *Int. J. Ind. Eng. Manag.* **2024**, *15*, 96–108. [CrossRef]
36. Xiang, D.; Xu, F.; Sah, R.K.; Wei, C.; Fang, K. Carbon footprint accounting method based on dynamic allocation of multi-level carbon data from manufacturing organizations to products. *J. Clean. Prod.* **2024**, *467*, 142865. [CrossRef]
37. Biswas, P.; Schultz, S.R. An EPQ model for an imperfect product system with rework, scrap and backorder. *Int. J. Ind. Syst. Eng.* **2021**, *37*, 411–431. [CrossRef]
38. Martí, J.M.C.; Tancrez, J.S.; Seifert, R.W. Carbon footprint and responsiveness trade-offs in supply chain network design. *Int. J. Prod. Econ.* **2015**, *166*, 129–142. [CrossRef]
39. Ruidas, S.; Seikh, M.R.; Nayak, P.K. A production inventory model with interval-valued carbon emission parameters under price-sensitive demand. *Comput. Ind. Eng.* **2021**, *154*, 107154. [CrossRef]
40. Corona, B.; Shen, L.; Reike, D.; Carreón, J.R.; Worrell, E. Towards sustainable development through the circular economy—A review and critical assessment on current circularity metrics. *Resour. Conserv. Recycl.* **2019**, *151*, 104498. [CrossRef]
41. Soleymanfar, V.R.; Makui, A.; Taleizadeh, A.A.; Tavakkoli-Moghaddam, R. Sustainable EOQ and EPQ models for a two-echelon multi-product supply chain with return policy. *Environ. Dev. Sustain.* **2022**, *24*, 5317–5343. [CrossRef]
42. Shah, N.; Patel, E.; Rabari, K. EPQ model to price-sensitive stock dependent demand with carbon emission under green and preservation technology investment. *Econ. Comput. Econ. Cybern. Stud. Res.* **2022**, *56*, 209–222. [CrossRef]
43. Modak, N.M.; Kelle, P. Using social work donation as a tool of corporate social responsibility in a closed-loop supply chain considering carbon emissions tax and demand uncertainty. *J. Oper. Res. Soc.* **2019**, *72*, 61–77. [CrossRef]
44. *PAS-2050:2011*; Specification for the Assessment of the Life Cycle Greenhouse Gas Emissions of Goods and Services. BSI-British Standards Institution: London, UK, 2011. [CrossRef]
45. Kamal Abd Rahman, I.; Omar, N.; Zainal Abidin, Z. The applications of management accounting techniques in Malaysian companies: An industrial survey. *J. Financ. Report. Account.* **2003**, *1*, 1–12. [CrossRef]
46. Afonso, P.S.; Paisana, A.M. An algorithm for activity based costing based on matrix multiplication. In Proceedings of the IEEE International Conference on Industrial Engineering and Engineering Management, Hong Kong, China, 8–11 December 2009; pp. 1–17. [CrossRef]