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A Bilevel DEA Model for Efficiency Evaluation and Target Setting with Stochastic Conditions

Andreas C. Georgiou *, Konstantinos Kaparis, Eleni-Maria Vretta, Kyriakos Bitsis and George Paltayian

Quantitative Methods and Decision Analysis Lab, Department of Business Administration, University of Macedonia, GR-54636 Thessaloniki, Greece; k.kaparis@uom.edu.gr (K.K.); emvretta@uom.edu.gr (E.-M.V.); kmpitsis@uom.edu.gr (K.B.); gpaltag@uom.edu.gr (G.P.)
* Correspondence: acg@uom.edu.gr

Abstract: The effective allocation of limited resources and the establishment of targeted goals play a pivotal role in enhancing the overall efficiency of large enterprises and organizations. To achieve optimal organizational efficiency, managers seek dynamic strategies that adapt to the constraints of limited and uncertain historical data. This paper introduces an evaluation of organizational efficiency through a stochastic framework, employing a bilevel data envelopment analysis (DEA) approach. This decision-making process is centralized within a decision-making unit (DMU) overseeing subordinate decision-making units (subDMUs). Discrete scenarios, each associated with a realization probability, define the uncertain parameters in the bilevel DEA-based model. This stochastic approach allows for recourse actions upon scenario realization leading to an enhanced overall organizational strategy. Decision-makers acting within uncertain and dynamic environments can benefit from this research since it allows the investigation of efficiency assessment under alternative scenarios in the presence of volatility and risk. The potential impact of applying this methodology varies depending on the specific domain. Although, the context of this paper focuses on banking, in general, enhancing resource allocation and target setting under stochasticity, contributes to advancing sustainability across all its three dimensions (economic, environmental, social). As mentioned earlier, the practical application of our approach is demonstrated via a case study in the banking sector.

Keywords: DEA; bilevel optimization; stochastic conditions; resource allocation

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1. Introduction

Large enterprises and organizations are the backbone of local and national economies; they generate substantial profits, contribute to economic development, and foster innovation. Their investments in human capital, productivity, and facilities contribute to technological advancements. Additionally, these entities actively promote green growth and a circular economy, aligning with sustainable development goals and the protection of natural assets. Beyond economic contributions, their impact on societal welfare is significant, providing job opportunities, healthcare coverage, and social insurance that underpin overall well-being. The ongoing changes in economic policies, prices, and market fluctuations necessitate these entities to optimally allocate resources and set targets. Such strategic decisions are paramount, influencing productivity, efficiency, future planning, and profitability. The perpetual limitation of resources underscores the critical importance of optimal resource allocation for large enterprises and organizations, facilitating their ability to achieve objectives and remain competitive in a dynamic market.

Major organizations usually comprise a central decision-making unit (DMU) and several subordinate decision-making units (subDMUs). The primary DMU, which is responsible for overseeing and managing the subDMUs, plays a pivotal role in allocating finite resources and defining appropriate output targets. Concurrently, it may also set minimum thresholds for the *efficiency* of each subDMU. The latter term (i.e., efficiency) is quantified as the weighted ratio of the outputs generated to inputs utilized by the DMU. On the other hand, *effectiveness* in the context of a DMU is articulated in terms of profitability however in a broader spectrum, effectiveness encapsulates the extent to which an organization achieves its objectives.

The problem of resource allocation and target setting, calls for redesigning policies of organizations with multi-stage or multi-level structures, in a way that optimizes organizational efficiency and effectiveness.

Data envelopment analysis (DEA) introduced by Charnes, Cooper, and Rhodes [1] is a widely used non-parametric method for assessing the efficiency of homogeneous DMUs with multiple inputs and outputs. Traditional DEA models, treat DMUs as black boxes, disregarding internal structures, interconnections, and interactions among operational and organizational stages. On the other hand, in a *Network* DEA (NDEA) environment, each DMU comprises various stages or levels, so optimizing the performance of a DMU could theoretically result from evaluating the performance of each individual stage or level (sub-DMUs). In an NDEA scheme, intermediate outputs play a vital role in the DMU evaluation as they are generated from a sub-DMU, and act as inputs to another sub-DMU of the system, as defined by Färe and Grosskopf [2]. Kao and Hwang in [3] showed that ignoring the sub-processes of a DMU may lead to an overall efficient system, even though a DMU might be inefficient in an individual stage. Kao and Hwang in [4] showed that it is important to consider the internal structure of a DMU to identify any inefficiencies, since a DMU may have better overall efficiency compared to another DMU, although the sub-processes of the first DMU may have worse individual efficiencies. The internal structure of a DMU can be decomposed into two stages in a simple case, while in a more complex case, it may consist of multiple stages. Halkos et al. [5] provide a comprehensive classification of two-stage DEA models.

Despotis et al. [6] presented a novel definition for overall system efficiency in network DEA literature, inspired by the concept of the "weak link" in supply chains and the maximum-flow/minimum-cut problem in networks. Employing a two-phase max-min optimization technique within a multi-objective programming framework, they estimate individual stage efficiencies and overall system efficiency in two-stage processes of varying complexity. Additional research on composition and decomposition techniques in both two-stage and multi-stage environments can be found in [7–9]. In the context of assessing a parallel network structure integrated with a hierarchical one, Kremantzis et al. [10] propose a linear additive decomposition DEA model as well as a non-linear multiplicative aggregation DEA model. Both constitute alternative approaches to evaluating the performance in parallel network DEA problems.

Fukuyama and Matousek [11] studied the strengths between network and traditional DEA. Based on their research, the precision and accuracy of DEA results are better when network models are used compared to traditional DEA models. In the same manner, Kao [12] showed that it is possible for a DMU to be considered as efficient using traditional DEA and not efficient using the network DEA approach. Hence, the efficiency can be overestimated by the classical models, and this problem is perpetuated as the stages increase. More comprehensive research about NDEA models can be found in [13–18]. Typically, each DMU independently optimizes its input and output levels to maximize its efficiency. However, our study concerns the cases of major enterprises, where a central DMU governs a group of subDMUs to maximize overall organizational efficiency and profitability. DEA serves as a mathematical programming technique extensively applied to address centralized resource allocation and target-setting challenges. In most resource allocation DEA models, precise input and output data are assumed, whereas real-world data are often unavailable or inaccurate. Relying on calculated optimal solutions based on such data may lead to profit loss, planning inconsistencies, and reduced production. Therefore, acknowledging the uncertainty in achieving output targets becomes imperative. Large organizations must be capable of redesigning consumption and production processes, and taking remedial actions to maximize overall efficiency. Uncertainty is a fundamental factor in addressing challenges related to resource allocation, production design, and output targeting. One of the most challenging issues faced by traditional optimization problems is the tendency of optimal solutions to perturbations in the values of the problem's parameters, often exhibiting a high degree of sensitivity. This characteristic underscores the crucial importance of identifying "robust solutions" in the realm of optimization theory. To mitigate such uncertainties, the optimization community employs various mathematical frameworks, including stochastic programming, chance-constrained programming, and robust optimization. Stochastic programming optimizes the expected outcome of an objective function. On the other hand, chance constraint programming ensures that the derived solution satisfies certain constraints within a given probability level. Finally, robust optimization is a risk-averse strategy, focusing on optimizing the "worst- case" scenario within a predefined uncertainty set. In the stochastic programming approach, the uncertain parameter vector is modeled using discrete probabilistic scenarios, while in the robust optimization approach, its values are defined by a continuous set [19].

The latest approaches tackle data uncertainty by incorporating methods that account for and mitigate the impact of fluctuations, or imprecisions in the input data. This is achieved by considering a range or set of possible values for the input parameters rather than relying on precise, fixed values. The robust DEA approach aims to provide reliable and stable efficiency assessments even when faced with uncertainties in the data, thus enhancing the model's resilience to variations that consider the dynamic and uncertain nature of the banking environment. Therefore, a robust solution remains optimal regardless of the stochasticity governing the problem's parameters, although this optimal performance is restricted to a specific parameter range. The latter represents a significant advantage over traditional DEA methods that do not handle data uncertainty.

In their recent work, Zhang et al. [20] addressed the challenge of allocating limited medical reserves in the context of a public health emergency. It takes into account uncertainties in both demand and donated supplies, as well as the priorities of healthcare centers. The formulation of the problem involves a two-stage stochastic program, treating donated supplies as an effective recourse action with the ultimate goal of minimizing overall losses. According to Shakouri et al. [21], in situations where uncertainties exist in the data of a problem, traditional DEA models may yield inaccurate results. For this reason, they proposed two stochastic p-robust two-stage network DEA (NDEA) models to estimate the efficiency of DMU in an uncertain environment. These models are developed within the context of a bilevel framework. Their approach facilitates more effective mitigation of the adverse impact on the objective function, addressing uncertainties often neglected in traditional NDEA models. The practical application of these models is demonstrated through an analysis of the performance of bank branches. Finally, robust and stochastic optimization techniques have been successfully applied in various DEA models, such as [22–25].

This paper introduces a stochastic bilevel DEA model aimed at optimizing overall organizational efficiency. The efficiency metric, defined as profitability (total revenues minus total input costs), is evaluated within a stochastic framework in a bilevel structure (DMU and sub DMUs and under uncertainty). Building upon Hakim et al.'s deterministic model [26], the proposed DEA model accommodates stochastic conditions for uncertain parameters by incorporating alternative scenarios with associated occurrence probabilities. Specifically, the model assumes imprecise and unknown data for output targets, requiring the decision-maker of the central unit to formulate a strategy without perfect information. The motivation is an application in the banking sector where DEA methods have been extensively applied ([11,27–33]).

Our research is driven by the recent performance evaluations conducted by Greek banking institutions, which are a response to the ongoing transformative phase within the Greek banking system. This restructuring is mandated by regulatory directives issued by European supervisory authorities and is deemed crucial due to the economic crisis of the past fifteen years and the prolonged debt crisis. At its essence, this restructuring is guided by two principles: the reduction of operational costs and the strategic deployment of technology. Consequently, Greek banks have embarked on a new era characterized by a comprehensive overhaul of their network infrastructure. The primary aim is to enhance organizational effectiveness and branch efficiency, thereby boosting revenue generation from retail banking products while optimizing resource utilization. Additionally, in accordance with European guiding principles, banks are revamping their branch networks by introducing innovative outlets that seamlessly integrate state-of-the-art technologies to serve customers, along with augmenting their specialized staff. This transformative process is geared toward achieving key objectives, including heightened net profitability and the equitable distribution of dividends to shareholders.

The rest of the paper is organized as follows: Section 2 discusses pertinent DEA-based models, exploring various approaches to resource allocation, targeting, and uncertainty capture. We review fundamental concepts and mathematical formulations of bilevel programming and optimization under uncertainty. Section 3 provides the problem description and notation. Section 4 details the bilevel DEA-based model with stochastic conditions and outlines the proposed solution methodology. Our computational study and results are presented in Section 5, while Section 6 encapsulates concluding remarks based on the paper's findings and contributions.

2. Literature

2.1. Resource Allocation

Numerous approaches have been proposed to tackle resource allocation and target-setting challenges. Golany et al. [34] introduced a DEA-based model optimizing overall organizational profitability and technical efficiency. Athanassopoulos [35] integrated goal programming and DEA for multi-level resource allocation, applied to central fund allocation in Greek local authorities. Yu et al. [36] employed a centralized DEA model with a Russell measure for human resource reallocation in Taiwanese airports. Amirteimoori and Tabar [37] addressed fixed resource allocation in organizations with multiple DMUs, while Beasley [38] maximized average efficiency for DEA-based models, incorporating fixed-cost resources and output targets in centralized decision-making. Lozano and Villa [39] presented DEA models for centralized resource allocation, aiming to minimize input consumption, maximize output production, and enhance individual DMU efficiency. Varmaz et al. [40] incentivized subDMUs in large organizations, adapting Lozano and Villa's model [39] to compute super-efficiency. Afsharian et al. [41] proposed a DEA-based model for incentivizing DMUs under central management, addressing shortcomings in Varmaz et al. [40]. Similarly, Afsharian et al. [42] extended this approach to hierarchically structured organizations, illustrating it with data from a German retail bank. Asmild et al. [43] expanded Lozano and Villa's [39] models, suggesting modifications for inefficient DMUs and providing a procedure for alternative optimal solutions in an input-oriented BCC framework. Wu et al. [44] incorporated economic and environmental factors in DEA models for resource allocation, considering three scenarios for resource availability. Fang [45] proposed a generalized centralized resource allocation model, decomposing technical efficiency into components and illustrating the approach with a supermarket example.

Two-stage network DEA approaches addressing the resource allocation problem have been introduced by various researchers. Chen et al. [46] proposed a DEA model evaluating the efficiency of two-stage network processes with shared inputs across both stages, encompassing inputs utilized collectively and those specific to each stage. Zha and Liang [47] outlined a cooperative model allocating freely shared inputs in a series production process. This product-form model calculates the overall efficiency for the assessed DMU, illustrating collaboration between the two stages.

Wu et al. [48] presented an approach to managing undesirable intermediate outputs in a two-stage production process with shared resources. They employed additive and non-cooperative models to gauge the efficiency of each DMU and subDMU, applying these models to industrial production in thirty provincial regions in China. Yu et al. [49]

addressed the allocation of fixed costs among subDMUs, considering efficiency. They introduced a two-stage network DEA model grounded in cross-efficiency concepts.

Recent studies, particularly [50,51], have introduced notable advancements in tackling the issue of resource allocation prompted by the Internet of Things.

2.2. Bilevel Network DEA

In his seminal work, Dempe [52] describes a bilevel programming problem (BLP) as a setting where an optimization problem includes within its constraint set a second, partial optimization problem. The outer optimization task is commonly denoted as the upper level, while the inner optimization task is referred to as the lower level. The idea can be traced back to the early work of Freiherr von Stackelberg [53] in economic game theory. According to Stackelberg's conceptualization, the hierarchical structure encompasses two distinct decision-makers: the leader and the follower, corresponding to the upper and lower-level problems, respectively. The standard mathematical formulation of a bilevel problem is as follows:

$$\min_{x,y} \quad F(x,y) \tag{1}$$

$$\text{s.t.} \quad G(x,y) \leq 0 \tag{2}$$

$$H(x,y) = 0 \tag{3}$$

$$\min_{y} \quad f(x,y) \tag{4}$$

$$\text{s.t.} \quad g(x,y) \leq 0 \tag{5}$$

$$h(x,y) = 0 \tag{6}$$

where $x \in R^n$ and $y \in R^m$ are the set of upper- and lower-level variables, respectively. Moreover, the upper-level problem (leader's problem) is specified via (1)–(3) and its domain is partially specified by the optimal solutions of the lower-level problem (follower's problem) outlined by (4)–(6).

The motivation behind the employment of this optimization schema is its ability to capture the hierarchical relations between the centralized decision-maker and the multiple sub-DMUs with great accuracy.

Shafiee et al. [54] introduced a bilevel DEA model for evaluating bank branch performance, employing a mixed-integer linear programming (MILP) approach for its solution. The study incorporates internal structures and Stackelberg relationships, providing insightful information about each component of the banking chain. Zhou et al. [55] devised a bilevel DEA model tailored for systems with bilevel structures, exemplified by manufacturing supply chains with multiple distribution centers. Their approach, rooted in the Stackelberg competition game theory, features multiple followers. The case study involves a supply chain with a plant and two distribution centers. Sinha et al. [56] developed an oligopolistic market model with multiple leaders and followers over multiple time periods under Stackelberg relations. Their model, applicable to industries like aircraft manufacturing, accounts for leaders acting in a Stackelberg manner toward followers while engaging in the Cournot competition among themselves. Experimental results illustrate the impact of player entrance or exit on profits and costs, with nonlinear handling of demand and cost functions for accurate problem simulation. Hajiagha et al. [57] proposed an efficiency-based planning method, considering current DMU performance and projecting future efficiency while also considering profit performance. This bilevel approach maximizes efficiency at the upper level and optimizes inputs and outputs based on costs and profits at the lower level. Addressing the limitations of classic DEA models, the authors emphasize the simultaneous consideration of profit and technical efficiency. In recent developments, bilevel DEA-based models for resource allocation and target setting have emerged. Hakim et al. [26] proposed a deterministic bilevel DEA model for centralized resource allocation and target setting, optimizing organizational effectiveness by maximizing total profit while ensuring each

DMU operates efficiently within predefined bounds. Ang et al. [58] extended this work for organizational systems with higher-level entities and subordinate two-stage DMUs. Their bilevel model aims to maximize both organizational and two-stage DMU efficiency.

2.3. Stochastic Optimization for DEA

According to Olesen and Petersen [59], stochastic DEA extends the original idea in three different directions. First, is the deviations from the production frontier, while in the second case, DEA can handle random noise coming from measurement or specification errors. In the latter case, the production possibility set (PPS) is adjusted according to the random data.

Chance constraints were introduced by Charnes and Cooper [60] and are routinely used ever since in the context of stochastic DEA. By using this method, we can formulate a problem with stochastic constraints assuming that we may have constraints' violation within a certain probability level. Beraldi and Bruni [61] proposed a stochastic DEA method using chance constraints formulation that transformed into a deterministic equivalent under the discrete distribution assumption.

Zhou et al. [62] suggest a stochastic network DEA model to facilitate a two-stage system under data uncertainty. The model is based on a centralized control mechanism and a transformation to a deterministic equivalent linear programming model. The transformation relies on the assumption that some problem parameters, e.g., inputs/outputs are related to stochastic factors.

The conventional DEA formulations exhibit determinism and static characteristics, rendering them highly sensitive to minor parameter fluctuations. Acknowledging this susceptibility to small changes, the incorporation of robustness in DEA models becomes imperative. The objective is to maintain solution stability in the face of uncertain conditions. Marbini et al. [63] pioneered the development of novel robust non-radial DEA models, specifically designed to gauge the performance of decision-making units under conditions of data uncertainty. Their approach involves the utilization of Interval DEA, enabling the assessment of interval efficiencies based on both optimistic and pessimistic viewpoints. Ultimately, the authors introduce the concept of the "price of robustness" to comprehensively evaluate the effectiveness and robustness of the proposed models.

In their study, Tseng et al. [64] investigated the dynamics of economic efficiency and revenue sharing in the electricity market, employing a sophisticated bilevel scheme. Their primary objective was to pinpoint the Nash equilibrium while contending with capacity constraints estimated through DEA. To tackle the challenges arising from price uncertainties, the researchers introduced a cutting-edge approach by developing stochastic mixed complementarity models. These models seamlessly integrate stochastic programming and robust optimization techniques, offering a robust solution to address the intricate issue of price uncertainty in the electricity market.

As highlighted by Omrani et al. [65], the sole computation of a single efficiency metric proves insufficient in certain contexts for assessing the overall efficiency of decision-making units (DMUs). Consequently, a multi-objective DEA model has been devised to concurrently evaluate profit, operational, and transactional efficiencies within the realm of bank branches, particularly under conditions of data uncertainty. To address the challenges posed by data uncertainty, a robust approach has been employed in the formulation of the model, enhancing its capacity to provide a more comprehensive and nuanced assessment of efficiency in banking operations.

2.4. The Proposed Stochastic Framework

In traditional DEA models, such as [1], each DMU decides on its own input and output levels to maximize its own efficiency. In single-stage resource allocation DEA models, DMUs are considered as black boxes, namely the internal structures, and the interconnections and interactions among the stages of the operational and organizational structures are ignored [34–45]. However, in large enterprises and organizations, a group of subDMUs is under the control of a central DMU, aiming to maximize overall organizational efficiency and profitability. Nonetheless, in the two-stage DEA models [46,47] where interactions and interconnections are incorporated, hierarchical relations among the departments or the organizational levels are not considered. In the majority of resource allocation DEA models, the input and output data are known and precise, while in real-world problems, these data are often unavailable or erroneous. The stochastic DEA models on resource allocation [59–65] deal with uncertainty, however, to the best of our knowledge, the DMUs under examination have no bilevel structure.

Within our proposed stochastic framework, we incorporate a bilevel approach to DEA methodology alongside stochastic conditions, aimed at capturing the uncertainty surrounding the achievement of output targets and thereby optimizing organizational efficiency. The uncertainty within our framework is delineated by discrete realizations of uncertain parameters across various scenarios. Each uncertain parameter within the model is assigned a value corresponding to a scenario, with each scenario linked to a realization probability reflecting managerial estimations. Furthermore, the decision-maker retains the ability to select a strategy either prior to or independently of knowing the exact values assumed by uncertain parameters when a scenario materializes.

3. Problem Description

Within this section, we offer a comprehensive description of the problem at hand. Firstly, in Section 3.1, we provide a detailed description of our case, pointing out all the major elements. We explain how an input, which is associated with a cost, is converted into a valuable output. Moreover, in Section 3.2, we present the notation and model assumptions to establish a clear understanding of the problem.

3.1. An Application in Banking

In general, the banking sector plays a pivotal role in conducting diverse financial transactions, aggregating funds, and financing both short- and long-term public and private investments. In particular, the Greek banking sector is further characterized by fierce competition, driven by the pursuit of increased profitability for stakeholders. In recent years, this competition has intensified as management endeavors to optimize returns. Given the paramount significance of this sector, the proposed stochastic bilevel DEA model is motivated by an application specific to banking.

In the application scenario, the central administration aims to maximize overall efficiency by optimizing profit. This necessitates strategic resource allocation among subDMUs (which actually are specific branches) and the establishment of output targets aligned with their capabilities. Concretely, the bank management defines future performance targets for each individual DMU, considering the resources available to them. The institution selected for the implementation of the proposed model is among the Greek systemic banks. It maintains an active network of more than 250 branches and significant metrics in terms of human capital, deposits, and loans, capturing an estimated 25% of the total market share. For the specific case under consideration, the implementation focuses on a network situated in one of the largest urban centers in Greece. The planning process includes ten branches (DMUs) of diverse sizes, (b)ig, (m)edium, or (s)mall, determined by factors, such as staffing levels, customer base, and the volume of deposits and loans across various categories (including mortgage loans, consumer loans, and small business loans). Additionally, the branches are geographically classified as eastern, central, or western, reflecting their specific locations in the region. The spatial planning of these branches was designed to allow coverage of geographical districts within the urban landscape under study. The data mirror typical real-world scenarios; however, they have been simulated for disclosure purposes.

In our model (Figure 1), we established five key inputs that induce expenses under typical operation circumstances for each bank branch (DMU):

X_1: Specialized personnel (relationship managers);
X_2: Supporting personnel (base officers);

X_3: ATMs;
X_4: Administrative costs (thousands of euros);
X_5: Interests for deposits (millions of euros).

In a similar manner, six outputs were selected and are outlined as follows:

Y_1: Mortgage loans (ML);
Y_2: Small business loans (SB);
Y_3: Consumer loans (CL);
Y_4: Mutual funds (MF);
Y_5: Net fee income (NFI);
Y_6: Surplus deposits (SD).

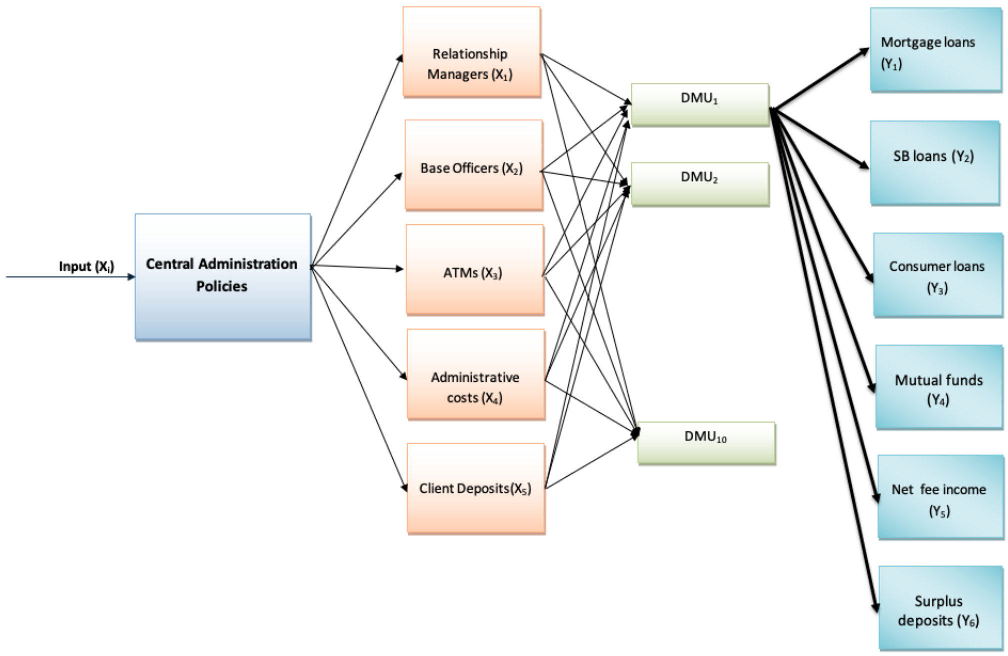

Figure 1. Input and output structures.

During the conversion of inputs into expenses, we adhered to industry-standard practices prevalent in the banking sector. For the two staff categories, namely relationship managers and base officers, we considered the average yearly salary expenses. Regarding ATMs and administrative expenses, we factored in the costs associated with installation, operation, and distribution per staff member. Regarding deposits, we accounted for the average weighted interest rate of bank deposits, set at 0.35%. All expenses are presented on an annual basis.

The bank manager's objective encompasses two primary goals: to enhance both efficiency and profitability, thus safeguarding sustainability and ensuring resilience in the face of dynamic economic and political conditions. To aid the decision-maker, we propose an optimal resource allocation strategy and establish output targets. It is important to note that the pursuit of profit maximization is tempered by an additional constraint, mandating the fulfillment of a minimum efficiency rate for each DMU. In the process, each DMU utilizes inputs X_1 to X_5 (see Table 1). Table 1 includes typical values for the above inputs for 10 DMUs.

Table 1. Input data for each DMU.

Label	DMU	X_1	X_2	X_3	X_4	X_5
East B1	1	4	12	3	16	0.788
East B2	2	3	10	2	15.6	0.525
East M	3	2	8	1	8	0.420
East S	4	1	5	1	2.4	0.280
Central B	5	4	10	4	21	0.875
Central M	6	5	13	4	36	1.225
Central S	7	1	4	2	1.4	0.350
West B	8	3	9	2	12	0.455
West M	9	2	7	1	7.2	0.385
West S	10	0	6	2	7.2	0.420
Total	10	25	84	22	126.8	5.723

One could argue that outputs ML, SB, and CL, correspond to revenues coming from loans while MF and NFI refer to commissions from banking transactions and mutual funds management. Finally, SD pertains to the revenue that stems from the surplus of deposits that a branch holds and are placed as deposits in the European Central Bank, through the Bank of Greece.

Table 2 contains the balances for each loan type (in millions of euros), the balances of mutual funds under management (in millions of euros), the surplus of deposits (in millions of euros), and the net commissions from banking activities (in thousands of euros).

Table 2. Output balances.

DMU	ML	SB	CL	MF	NFI	SD
1	30	19	9.8	40	9.1	67.5
2	40	15	4.5	35	7.56	45
3	18	6	2.3	20	4.54	36
4	10	3	1.5	8	2.25	24
5	30	12	11	50	8.81	50
6	24	10	9	35	7.58	35
7	12	8	5	15	1.85	15
8	50	10	12	20	5.4	20
9	34	5.8	8	9	4.5	9
10	15	3	5	5	1.3	5
Total	263	91.8	68.1	237	52.889	302.45

The income of each DMU is calculated as a percentage of the output balances. Moreover, these percentages are summarized in Table 3.

Table 3. Net margin rate profit for each output.

Output Income	Percentage
Mortgages income	2.00%
SB Loans income	4.00%
Consumer income	7.00%
Mutual income	0.50%
Deposit income	3.75%

Additionally, we added the net fee income for each DMU to the above balances in order to calculate the total income. Using the output balances and income rates, we can deduce the final output data recorded in Table 4.

Table 4. Output data for each DMU.

		DMU									
Scenario	**Output**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	Y_1	0.42	0.56	0.25	0.14	0.42	0.34	0.17	0.70	0.48	0.21
	Y_2	0.53	0.42	0.17	0.08	0.34	0.28	0.22	0.28	0.16	0.08
	Y_3	0.48	0.22	0.11	0.07	0.54	0.44	0.25	0.59	0.39	0.25
	Y_4	0.14	0.12	0.07	0.03	0.18	0.12	0.05	0.07	0.03	0.02
	Y_5	6.37	5.29	3.18	1.58	6.17	5.31	1.3	3.78	3.15	0.91
	Y_6	1.77	1.18	0.95	0.63	1.97	2.76	0.79	1.02	0.87	0.95
2	Y_1	0.6	0.8	0.36	0.2	0.6	0.48	0.24	1	0.68	0.3
	Y_2	0.76	0.6	0.24	0.12	0.48	0.4	0.32	0.4	0.23	0.12
	Y_3	0.69	0.32	0.16	0.11	0.77	0.63	0.35	0.84	0.56	0.35
	Y_4	0.2	0.18	0.10	0.04	0.25	0.18	0.08	0.1	0.05	0.03
	Y_5	9.1	7.56	4.54	2.25	8.81	7.58	1.85	5.4	4.5	1.3
	Y_6	2.53	1.69	1.35	0.9	2.81	3.94	1.13	1.46	1.24	1.35
3	Y_1	0.69	0.92	0.41	0.23	0.69	0.55	0.28	1.15	0.78	0.35
	Y_2	0.87	0.69	0.28	0.14	0.55	0.46	0.37	0.46	0.27	0.14
	Y_3	0.79	0.36	0.19	0.12	0.89	0.72	0.4	0.97	0.64	0.4
	Y_4	0.23	0.2	0.12	0.05	0.29	0.2	0.09	0.12	0.05	0.03
	Y_5	10.47	8.69	5.22	2.59	10.13	8.72	2.13	6.21	5.18	1.5
	Y_6	2.91	1.94	1.55	1.04	3.23	4.53	1.29	1.68	1.42	1.55

3.2. Notations and Assumptions

Table 5 presents a comprehensive compilation of the notation, accompanied by brief descriptions.

Table 5. Notation summary.

Notation	**Description**
Indices	
n	number of DMUs
m	number of input resources
s	number of output targets
ω	scenario index
Sets	
$\mathcal{J}$	set of DMUs
$\mathcal{I}$	set of inputs
$\mathcal{O}$	set of outputs
Ω	set of scenarios
Parameters	
p_r	unit price for output r
c_i	unit cost for input i
X_{ik}	observed input i for DMU k
Y_{rk}^{ω}	observed output r for DMU k of scenario ω
Le_k	lower bound for efficiency of DMU k
Lx_{ik}	lower bound for input resource i of DMU k
Ux_{ik}	upper bound for input resource i of DMU k
Ly_{rk}	lower bound for output target r of DMU k
Uy_{rk}	upper bound for output target r of DMU k
b_i	availability for input resource i
ϵ	infinitesimal number
q^{ω}	realization probability of scenario ω

Table 5. *Cont.*

Notation	Description
Variables	
x_{ik}	input resource i for DMU k
y_{rk}^{ω}	output target r for DMU k of scenario ω
v_{ik}	weight attached to input resource i for DMU k
u_{rk}^{ω}	weight attached to output target r for DMU k of scenario ω
l_k^t	unrestricted variable
λ_{jk}^{ω}	is used for defining the possibility set of input resources or output targets of DMU k of scenario ω
e_k^*	optimal efficiency for DMU k
$e_k^{\omega*}$	optimal efficiency for DMU k of scenario ω
e_{kj}	cross-efficiency of DMU j with respect to DMU k

4. Methodology

4.1. Stochastic Bilevel DEA Model

In the context described in Section 3.1, it is apparent that the decisions for the allocation of inputs within the branches (subDMUs) are made at the beginning of the period by the central unit and then are followed by the resolution of the inherent uncertainty. It makes sense that target setting should take into account the observed outputs under the realized scenario. Thus, target setting constitutes the second-stage (aka recourse) variables in our formulation. Therefore, we extend, analogously, the deterministic bilevel model of Hakim et al. [26] to a two-stage stochastic bilevel DEA model with recourse actions.

4.1.1. The Upper-Level Model

Within a defined set of scenarios denoted as Ω, the upper-level structure follows the framework given in (7)–(15). The upper-level model includes decision variables for the input resources (x_{ik}) and the output targets (y_{rk}^{ω}). The optimization criterion involves profit maximization through optimal resource allocation, output targeting, and efficiency lower bounds of each subDMU.

$$\max_{x_{ik}, y_{rk}^{\omega}, \lambda_{jk}^{\omega}} \sum_{\omega=1}^{|\Omega|} q^{\omega} \left[\sum_{r=1}^{s} p_r \sum_{k=1}^{n} y_{rk}^{\omega} \right] - \sum_{i=1}^{m} c_i \sum_{k=1}^{n} x_{ik} \tag{7}$$

$s.t.$

$$Le_k \leq e_k^{\omega*} \qquad \forall k \in \mathcal{J} \tag{8}$$

$$x_{ik} \geq \sum_{j=1}^{n} \lambda_{jk}^{\omega} X_{ij} \qquad \forall i \in \mathcal{I}, k \in \mathcal{J} \tag{9}$$

$$y_{rk}^{\omega} \leq \sum_{j=1}^{n} \lambda_{jk}^{\omega} Y_{rj}^{\omega} \qquad \forall r \in \mathcal{O}, k \in \mathcal{J}, \omega \in \Omega \tag{10}$$

$$\sum_{j=1}^{n} \lambda_{jk}^{\omega} = 1 \qquad \forall k \in \mathcal{J}, \omega \in \Omega \tag{11}$$

$$\lambda_{jk}^{\omega} \geq 0 \qquad \forall k \in \mathcal{J}, \omega \in \Omega \tag{12}$$

$$\sum_{k=1}^{n} x_{ik} \leq b_i \qquad \forall i \in \mathcal{I} \tag{13}$$

$$Lx_{ik} \leq x_{ik} \leq Ux_{ik} \qquad \forall i \in \mathcal{I}, k \in \mathcal{J} \tag{14}$$

$$Ly_{rk} \leq y_{rk}^{\omega} \leq Uy_{rk} \qquad \forall r \in \mathcal{O}, k \in \mathcal{J}, \omega \in \Omega \tag{15}$$

The objective function (7) maximizes the expected overall organizational profits, where c_i and p_r denote the unit input costs and the unit output prices, respectively. Constraint (8) sets an efficiency lower bound for each subDMU decided by the central DMU. Constraints (9) and (10) ensure that the optimal allocation of resources and targeting is feasible with respect to the production possibility set constructed by the observed input and output values of the subDMUs. Constraint (11) poses the variable returns to scale (VRS) assumption for the model; nevertheless, the constant returns to scale (CRS) assumption can also be considered ignoring the latter constraint. Constraint (13) sets an upper bound for the availability of resources. Constraints (14) and (15) set upper and lower bounds for input resources and output targets decided by the central DMU.

4.1.2. The Lower-Level Model

The lower-level model is the multiplier DEA-based model under VRS assumption as presented in Beasley [38]. In the lower level, the optimal weights associated with inputs and outputs are determined. The main objective here is that each DMU tries to maximize its efficiency, given the input resources and target setting based on the upper-level decisions. For every DMU $k(k = 1, \dots, n)$ and scenario $\omega \in \Omega$, the lower-level problem is described by (16)–(19).

$$e_k^{\omega*} = \max_{v_{ik}, u_{rk}^{\omega}, t_k^{\omega}} \frac{\sum_{r=1}^{s} u_{rk}^{\omega} y_{rk}^{\omega} - l_k^{\omega}}{\sum_{i=1}^{m} v_{ik} x_{ik}} \tag{16}$$

$$s.t.$$

$$0 \le e_{kj}^{\omega} = \frac{\sum_{r=1}^{s} u_{rk}^{\omega} y_{rj}^{\omega} - l_k^{\omega}}{\sum_{i=1}^{m} v_{ik} x_{ij}} \le 1 \qquad \forall k, j \in \mathcal{J}, \omega \in \Omega \tag{17}$$

$$v_{ik} \ge \epsilon \qquad \forall i \in \mathcal{I}, k \in \mathcal{J} \tag{18}$$

$$u_{rk}^{\omega} \ge \epsilon \qquad \forall r \in \mathcal{O}, k \in \mathcal{J}, \omega \in \Omega \tag{19}$$

The objective function (16) of this model calculates the optimal efficiency score $e_k^{\omega*}$ for each subDMU k and each scenario ω. The model, which runs for each subDMU k, computes the optimal input (v_{ik}) and output weights (u_{rk}^{ω}) for each scenario ω that maximizes the efficiency for each subDMU k. Constraint (17) restricts the values of subDMU efficiency between zero and one. Constraints (18) and (19) ensure that the weights take values larger than a nonnegative infinitesimal number for input and output respectively. The existence of the free variable l_k^{ω} imposes the variable returns to scale assumption for the efficiency of the subDMU k.

4.2. Solution Approach

In this section, we generalize Theorem 1 of Hakim et al. [26] in our stochastic framework.

Lemma 1. *The solution $(x_{ik}^*, y_{rk}^{\omega*}, \lambda_{jk}^{\omega*}; \forall i, k, j, r)$ of the upper-level model (7)–(15) is optimal, assuming that $(u_{rk}^{\omega*}, v_{ik}^*, l_k^{\omega*}; \forall i, k, r)$ is an optimal solution of the lower level model (16)–(19) if and only if $(x_{ik}^*, y_{rk}^{\omega*}, \lambda_{jk}^{\omega*}, u_{rk}^{\omega*}, v_{ik}^*, l_k^{\omega*}; \forall i, k, j, r)$ is an optimal solution of the single-level model (20)–(31).*

Proof. Let us assume that $(x_{ik}^*, y_{rk}^{\omega*}, \lambda_{jk}^{\omega*}; \forall i, k, j, r)$ is an optimal solution of the upper-level model (7)–(15) and U^* is the corresponding objective value. Moreover, let $(u_{rk}^{\omega*}, v_{ik}^*, l_k^{\omega*}; \forall i, k, r)$ be the optimal solution of the lower-level model (16)–(19), given that $(x_{ik}^*, y_{rk}^*, \lambda_{jk}^*; \forall i, k, j, r)$ is a feasible solution of the upper-level model (7)–(15). Then, $(x_{ik}^*, y_{rk}^{\omega*}, \lambda_{jk}^{\omega*}, u_{rk}^{\omega*}, v_{ik}^*, l_k^{\omega*}; \forall i, k, j, r)$ satisfies all the constraints of the single-level model (20)–(31) since it satisfies constraints (9)–(15) and (17)–(19), which are the same with (22)–(31). Furthermore, the optimal solution of the lower-level model e_k^* equals e_{kk} when the weights are replaced with their optimal values $(u_{rk}^{\omega*}, v_{ik}^*, l_k^{\omega*}; \forall i, k, r)$; therefore, constraint (21) is also satisfied. Hence, $(x_{ik}^*, y_{rk}^{\omega*}, \lambda_{jk}^{\omega*}, u_{rk}^{\omega*}, v_{ik}^*, l_k^{\omega*}; \forall i, k, j, r)$ is a feasible solution for the single-level model and the corresponding objective value is equal to the optimum value U^* of the bilevel model. If A^* is the optimum value of the single-level model then it holds that $A^* \ge U^*$.

Conversely, we have to show that the optimal solution $(x^*_{ik}, y^{\omega*}_{rk}, \lambda^{\omega*}_{jk}, u^{\omega*}_{rk}, v^*_{ik}, l^{\omega*}_k;$ $\forall i,k,j,r)$ of the single-level model (20)–(31) induces an optimal solution $(x^*_{ik}, y^{\omega*}_{rk}, \lambda^{\omega*}_{jk};$ $\forall i,k,j,r)$ of the upper-level model (7)–(15) and an optimal solution $(u^{\omega*}_{rk}, v^*_{ik}, l^{\omega*}_k; \forall i,k,r)$ of the lower-level model (16)–(19). The optimal solution $(x^*_{ik}, y^{\omega*}_{rk}, \lambda^{\omega*}_{jk}, u^{\omega*}_{rk}, v^*_{ik}, l^{\omega*}_k; \forall i,k,j,r)$ of the single-level model (20)–(31) satisfies constraints (9)–(15) and (17)–(19), which are the same with (22)–(31). Furthermore, e^*_k equals e_{kk} for the optimal weights, thereby constraint (8) is also satisfied. Thus, the optimal solution $(x^*_{ik}, y^{\omega*}_{rk}, \lambda^{\omega*}_{jk}, u^{\omega*}_{rk}, v^*_{ik}, l^{\omega*}_k; \forall i,k,j,r)$ of the single-level model (20)–(31) induces a feasible solution $(x^*_{ik}, y^{\omega*}_{rk}, \lambda^{\omega*}_{jk}; \forall i,k,j,r)$ of the upper-level model (7)–(15), where $(u^{\omega*}_{rk}, v^*_{ik}, l^{\omega*}_k; \forall i,k,r)$ is an optimal solution of the lower-level model (16)–(19). Assuming that U^* is the optimum value of the upper-level model and the objective value of the feasible solution $(x^*_{ik}, y^{\omega*}_{rk}, \lambda^{\omega*}_{jk}; \forall i,k,j,r)$ of the upper-level model (7)–(15) is A^*, then it holds that $A^* \leq U^*$. $\square$

By Lemma 1, the stochastic bilevel DEA programming problem is converted to a single-level as follows:

$$\max_{x_{ik}, y^{\omega}_{rk}, \lambda^{\omega}_{jk}} \sum_{\omega=1}^{|\Omega|} q^{\omega} \left[\sum_{r=1}^{s} p_r \sum_{k=1}^{n} y^{\omega}_{rk} \right] - \sum_{i=1}^{m} c_i \sum_{k=1}^{n} x_{ik} \tag{20}$$

$s.t.$

$$Le_k \leq e^{\omega}_{kk} = \frac{\sum_{r=1}^{s} u^{\omega}_{rk} y^{\omega}_{rk} - l^{\omega}_k}{\sum_{i=1}^{m} v_{ik} x_{ik}} \qquad \forall k \in \mathcal{J}, \omega \in \Omega \tag{21}$$

$$x_{ik} \geq \sum_{j=1}^{n} \lambda^{\omega}_{jk} X_{ij} \qquad \forall i \in \mathcal{I}, k \in \mathcal{J} \tag{22}$$

$$y^{\omega}_{rk} \leq \sum_{j=1}^{n} \lambda^{\omega}_{jk} Y^{\omega}_{rj} \qquad \forall r \in \mathcal{O}, k \in \mathcal{J}, \omega \in \Omega \tag{23}$$

$$\sum_{j=1}^{n} \lambda^{\omega}_{jk} = 1 \qquad \forall k \in \mathcal{J}, \omega \in \Omega \tag{24}$$

$$\lambda^{\omega}_{jk} \geq 0 \qquad \forall k \in \mathcal{J}, \omega \in \Omega \tag{25}$$

$$\sum_{k=1}^{n} x_{ik} \leq b_i \qquad \forall i \in \mathcal{I} \tag{26}$$

$$Lx_{ik} \leq x_{ik} \leq Ux_{ik} \qquad \forall i \in \mathcal{I}, k \in \mathcal{J} \tag{27}$$

$$Ly_{rk} \leq y^{\omega}_{rk} \leq Uy_{rk} \qquad \forall r \in \mathcal{O}, k \in \mathcal{J}, \omega \in \Omega \tag{28}$$

$$0 \leq e^{\omega}_{kj} = \frac{\sum_{r=1}^{s} u^{\omega}_{rk} y^{\omega}_{rj} - l^{\omega}_k}{\sum_{i=1}^{m} v_{ik} x_{ij}} \leq 1 \qquad \forall j \in \mathcal{J}, \omega \in \Omega \tag{29}$$

$$v_{ik} \geq 0 \qquad \forall i \in \mathcal{I}, k \in \mathcal{J} \tag{30}$$

$$u^{\omega}_{rk} \geq 0 \qquad \forall r \in \mathcal{O}, k \in \mathcal{J}, \omega \in \Omega \tag{31}$$

The above optimization problem is non-linear and can be shown to be non-convex as well by considering the Hessian matrix of constraints (21) and (29). In either case, the Hessian matrix is not positive semidefinite and, thus, the constrained set is not convex.

5. Computational Study

The proposed work was encoded in Python 3.7.0, and for the stochastic bilevel model, Pyomo 5.7.3 was used, combined with the optimization engine of Gurobi 10.0.1. This solver version can deal with quadratic non-convex constraint problems by using global optimization techniques. All the experiments were conducted on an Intel Core i5-8350 CPU @ 1.70 GHz with 16 GB of RAM, running on 64-bit Ubuntu 22.04.1 (Intel, Santa Clara, CA, USA).

Our test instance comprises three different distinct scenarios denoted by ω, where $\omega = 1, 2, 3$ and the realization probabilities are $q^1 = 0.2, q^2 = 0.5, q^3 = 0.3$, respectively. We assume that the input availabilities are independent of scenarios and are given as $b_i = (25, 84, 22, 126.8, 5.723) \quad \forall i \in \mathcal{I}$, respectively. The lower bound on each input i and DMU k is given by $Lx_{ik} = 0.8X_{ik}$, while the upper bound is calculated as $Ux_{ik} = 1.2X_{ik}$. In a similar manner, the lower and upper bound for output r and DMU k are given by $Ly_{rk} = 0.8Y^1_{rk}$, and $Uy_{rk} = 1.2Y^3_{rk}$, respectively. The output bounds are independent of the scenarios. The unit costs for each of the five inputs are c_i = (40,000, 28,000, 27,500, 1000, and 1,000,000) $\forall i \in \mathcal{I}$. In addition, the corresponding unit output prices are set to EUR 100,000, except the last output price, which is $p_6 = 1,000,000$. We are aligned with the VRS assumption and the efficiency lower bound is $Le_k = 0.95$ for all DMU and scenarios. Finally, we should point out that our test instance has 5 inputs, 6 outputs, and 10 DMUs.

Additionally, a sensitivity analysis is undertaken to evaluate the model's performance, with a specific emphasis on both profitability and efficiency, while maintaining all other parameters at a constant level. In the initial scenario, we systematically vary the efficiency lower bound. Subsequently, a series of test instances is executed, each characterized by distinct input resources.

Utilizing the data outlined in Tables 1 and 4, we have derived an optimal solution addressing the challenge of resource allocation and target setting, accounting for an efficiency lower bound ($LBe_k = 0.95$) across all decision-making units. Referencing Table 6, the allocation of input resources among bank branches by the central administration is depicted. Notably, all resources are fully utilized, except for AC, which exhibits a slack of 7.63 in relation to the upper availability bound of 126.8.

Table 6. Resource allocation for each DMU.

DMU	X_1	X_2	X_3	X_4	X_5
1	4.00	12.00	3.00	16.00	0.788
2	3.30	10.37	2.30	13.93	0.630
3	2.20	7.82	1.20	8.38	0.437
4	1.10	5.20	1.00	2.88	0.291
5	4.01	10.72	3.65	19.42	0.848
6	4.35	11.24	3.90	28.80	0.987
7	1.05	4.14	1.95	1.68	0.352
8	2.80	9.00	1.80	12.79	0.546
9	2.20	7.50	1.20	8.08	0.425
10	0.00	6.00	2.00	7.20	0.420
Total	25.00	84.00	22.00	119.17	5.723

In addition to managing resource allocation among DMUs, our model emphasizes a crucial aspect: the establishment of output targets designed to enhance organizational effectiveness and profit maximization. Table 7 showcases the optimal output plan for each output and DMU, considering various scenarios. The final column displays the summation of outputs for each distinct output. The probability of occurrence for each scenario reflects the economic uncertainty anticipated during the future implementation of the strategic plan. Specifically, we consider scenarios representing a pessimistic outlook ($q^1 = 0.2$), a normal economic environment ($q^2 = 0.5$), and an optimistic scenario ($q^3 = 0.3$). This probability distribution accounts for the potential economic conditions that may influence the execution of the strategic plan.

Table 7. Output targets for every DMU and scenario.

		DMU										
Scenario	**Output**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**Total**
1	Y_1	0.420	0.469	0.392	0.174	0.419	0.391	0.183	0.560	0.470	0.210	3.687
	Y_2	0.532	0.421	0.199	0.092	0.403	0.337	0.221	0.239	0.199	0.084	2.726
	Y_3	0.480	0.394	0.222	0.105	0.518	0.499	0.252	0.470	0.401	0.245	3.587
	Y_4	0.140	0.111	0.056	0.028	0.162	0.153	0.051	0.063	0.042	0.018	0.825
	Y_5	6.370	5.373	3.471	1.733	6.228	5.889	1.385	3.766	3.472	0.910	38.597
	Y_6	1.772	1.418	0.982	0.654	1.909	2.221	0.791	1.154	0.957	0.945	12.802
2	Y_1	0.600	0.669	0.497	0.248	0.599	0.558	0.261	0.648	0.672	0.300	5.053
	Y_2	0.760	0.601	0.287	0.131	0.576	0.481	0.316	0.443	0.285	0.120	4.000
	Y_3	0.686	0.435	0.222	0.145	0.740	0.713	0.360	0.610	0.573	0.350	4.833
	Y_4	0.200	0.159	0.078	0.041	0.232	0.219	0.074	0.107	0.061	0.025	1.195
	Y_5	9.100	7.676	4.973	2.475	8.897	8.413	1.978	6.340	4.960	1.300	56.112
	Y_6	2.531	2.025	1.403	0.934	2.727	3.173	1.130	1.755	1.367	1.350	18.395
3	Y_1	0.690	0.770	0.497	0.276	0.689	0.642	0.300	0.745	0.773	0.345	5.727
	Y_2	0.874	0.691	0.331	0.151	0.662	0.552	0.363	0.510	0.320	0.138	4.592
	Y_3	0.789	0.435	0.222	0.145	0.850	0.820	0.414	0.702	0.658	0.403	5.438
	Y_4	0.230	0.183	0.090	0.047	0.267	0.242	0.085	0.123	0.062	0.029	1.356
	Y_5	10.465	8.827	5.719	2.846	10.232	9.675	2.275	7.291	5.704	1.495	64.529
	Y_6	2.911	2.329	1.614	1.074	3.136	3.649	1.300	2.018	1.572	1.553	21.155

The efficiency analysis in Table 8 provides a comprehensive overview of DMU performance under varying operating scenarios. It is essential to note that $\tilde{e_k}$ represents the weighted average efficiency across all scenarios. Notably, DMU 10 consistently demonstrates high efficiency across all scenarios, whereas DMU 7 exhibits lower efficiency in the first scenario but achieves efficiency in subsequent scenarios. DMUs 3 and 6 showcase increased efficiency in scenarios 2 and 3, respectively. The primary objective of the proposed model is to maximize overall profit, and Table 9 elucidates the total revenues, costs, profits, and profitability. Notably, the input allocation remains constant across scenarios, resulting in a fixed total input cost of 9,799,166. As anticipated, revenues and, consequently, profits, vary with scenarios, with lower profitability in the worst economic scenario, moderate profitability in the moderate scenario, and high profitability in the most optimistic scenario. It is crucial to highlight that the 'Expected' row represents the *weighted* sum of revenues and profits, respectively. In reference to our benchmark instance, with an efficient lower bound, $LBe_k = 0.95$, the total expected profit amounts to EUR 15,302,644, reflecting a 60.69% profitability. This underscores the model's effectiveness in achieving optimal outcomes even in diverse operating conditions.

Table 8. Efficiencies for every DMU in each scenario.

DMU	e_k^{1*}	e_k^{2*}	e_k^{3*}	$\tilde{e_k}$
1	0.95	0.95	0.95	0.95
2	0.95	0.95	0.95	0.95
3	0.95	0.98	0.95	0.97
4	0.95	0.95	0.95	0.95
5	0.95	0.95	0.95	0.95
6	0.95	0.95	0.96	0.95
7	0.95	1	1	0.99
8	0.95	0.95	0.95	0.95
9	0.95	0.95	0.95	0.95
10	1	1	1	1

$\tilde{e_k}$ is the weighted average efficiency.

Table 9. Revenues, profits, and profitability for the three scenarios.

Scenario	Revenues	Profit	Profitability (%)
1	17,744,100	7,944,934	44.78
2	25,514,665	15,715,499	61.59
3	29,318,860	19,519,693	66.58
Expected	25,101,811	15,302,644	60.96

5.1. Sensitivity Analysis

We performed a sensitivity analysis by changing the efficiency lower bound to be achieved by the bank branches from 0.95 to 0.7 and 0, respectively. In Table 10, we can see all optimal DMU efficiencies taking into consideration the output scenarios and the enforced efficiency lower bound. We can observe that in the first case, e.g., $LBe_k = 0.7$, we do not have significant variations except for DMUs 2 and 4, which have average efficiencies of 0.78 and 0.89, respectively. More precisely, DMU 4 performs very well in the moderate and optimistic scenario. In Table 11, it appears that the organization has higher profits for $LBe_k = 0$ than for $LBe_k = 0.7$. This means that a strict policy about branch efficiency does not necessarily yield greater profitability. Another argument of the latter statement is that the relaxed problem for $LBe_k = 0.7$ seems to have an inferior solution to the one we obtain when $LBe_k = 0.95$ and a better one in the case where $LBe_k = 0$.

It is noteworthy that—during the experiments—we identified that the system can work in a completely efficient manner, having all $e_k^{\omega *} = 1 \quad \forall k \in J$ and $\omega \in \Omega$. In order to achieve this ambitious feat, we need to increase the input availability b_5 from 5.723 to 6, yielding a profit of 15,452,279.

Table 10. Efficiencies for each DMU for $LBe_k = 0.7$ and $LBe_k = 0$.

	$LBe_k = 0.7$				$LBe_k = 0$			
DMU	e_k^{1*}	e_k^{2*}	e_k^{3*}	$\tilde{e_k}$	e_k^{1*}	e_k^{2*}	e_k^{3*}	$\tilde{e_k}$
1	0.7	0.72	0.72	0.72	0.27	0.3	0.34	0.31
2	0.78	0.76	0.81	0.78	0.54	0.18	0.37	0.31
3	0.7	0.7	0.78	0.72	0.57	0.61	0.63	0.61
4	0.7	0.94	0.94	0.89	0.7	0.84	0.84	0.81
5	0.72	0.7	0.72	0.71	0.28	0.32	0.34	0.32
6	0.7	0.7	0.75	0.72	0.32	0.37	0.4	0.37
7	0.71	0.7	0.7	0.70	0.93	0.62	0.08	0.52
8	0.72	0.7	0.7	0.70	0.47	0.55	0.58	0.54
9	0.7	0.7	0.7	0.70	0.86	0.94	0.66	0.84
10	0.79	0.73	0.7	0.73	0.57	0.49	0.46	0.50

Table 11. Revenues, costs, and profits for the three scenarios for $LBe_k = 0.7$ and $LBe_k = 0$.

	$LBe_k = 0.7$			$LBe_k = 0$		
Scenario	Revenues	Profit	Yield (%)	Revenues	Profit	Yield (%)
1	17,744,100	7,945,593	44.78	17,744,100	7,945,084	44.78
2	25,514,665	15,716,158	61.60	25,514,665	15,715,649	61.59
3	29,314,435	19,515,928	66.57	29,318,946	19,519,931	66.58
Expected	25,100,483	15,301,976	60.96	25,101,836	15,302,821	60.96

Input cost = 9,798,507.

We also implemented a sensitivity analysis for the case study presented in Section 3.1, considering seven different strategies to maximize the overall organizational efficiency. In the first case, two base officers were replaced with an ATM and, therefore, the ATMs are increased by one in each bank branch. Then the overall profits of the bank are increased in contrast to the profits in the base case study (see Table 12). In this scenario, the overall profits are even higher than the base case study. Thereby this strategy creates the highest profit for the bank. In the second case, one base officer is replaced with a relationship manager in each bank branch. Following this strategy, the overall profitability of the bank decreases in comparison to the base case study. This reduction is expected since relationship managers have a higher cost for the bank. In the third scenario, we differentiate the cost of deposits among the bank branches with respect to their location. More precisely, the cost of deposits increases to 1% from 0.35% for the West bank branches, to 1.5% for the City Center bank branches, and to 2% for the East bank branches. In the fourth scenario, we combine the replacement of a base officer with a relationship manager in each bank branch with the increase in the cost of deposits performed in the third scenario. In the third and fourth scenarios, the overall profit becomes negative. Since the cost of deposits is augmented, the costs exceed the revenues of the bank eliminating the profit. In the fifth scenario, a combination of the first and third scenarios is implemented and a loss is observed, however lower than that of scenario four. In the sixth scenario, two base officers were replaced with an ATM, and one base officer was replaced with a relationship manager in each bank branch. Thus, the tendency of the banks to replace employees with ATMs and digital services is also due to profit gain. In the seventh scenario, a combination of scenarios one, two, and three is performed, leading to a loss higher than those observed in scenarios three, four, and five. Furthermore, in conjunction with Table 12, Figure 2 illustrates the fluctuations in revenues, costs, and profits. In this sensitivity analysis, we modify input resources, noting that expected revenues remain consistent compared to input allocation costs. However, the variability in costs has a substantial impact on expected profits, resulting in losses in some scenarios.

Table 12. Sensitivity analysis for the seven scenarios assumed.

Scenario	Exp. Revenues	Total Cost	Exp. Profit
1	25,098,657	9,495,636	15,603,021
2	25,106,434	9,917,651	15,188,784
3	23,951,562	25,579,251	−1,627,688
4	23,982,217	25,693,324	−1,711,107
5	24,074,086	25,261,859	−1,187,773
6	25,103,557	9,615,635	15,487,922
7	24,117,803	25,382,135	−1,264,331

'Exp.' = 'Expected'.

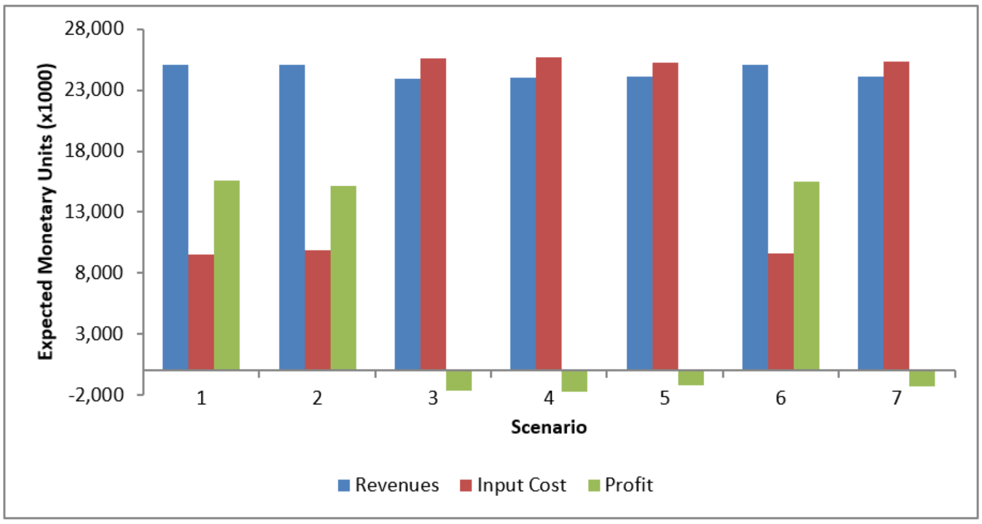

Figure 2. Revenues, costs, and profits for every scenario.

5.2. Theoretical and Managerial Implications

In a theoretical context within the exploration of bilevel modeling, decisions that need to be made at the outset of the examined time period and before any of the scenarios unfold are termed first-stage decisions. Moreover, decisions executed after a scenario materializes, known as recourse decisions, allow corrective actions and are referred to as second-stage decisions. The initial-stage solutions, implemented at the beginning, remain consistent across all scenarios. In contrast, second-stage decisions vary and are contingent on the specific scenario. The bilevel DEA model, incorporating stochastic conditions, enables recourse decisions and actions based on information obtained following the realization of a particular scenario.

As a result, managers can adjust their strategic planning in response to information revealed over time. On the other hand, the deterministic counterpart of the proposed DEA model lacks the flexibility for dynamic changes in strategy and adjustments to emerging economic conditions. For DMUs with multi-stage or multi-level structures, managers aim for optimal organizational efficiency by pursuing dynamic strategies that can adapt to the constraints of limited and uncertain historical data. To this end, the total input consumption is reduced and/or the total production is augmented and simultaneously the overall profits are maximized. Decision-makers acting within uncertain and dynamic environments can benefit from the suggested approach since it allows the investigation of efficiency assessment under alternative scenarios in the presence of volatility and risk. The potential impact of applying this methodology varies depending on the specific domain. Although, the context of this paper focuses on banking, in general, enhancing resource allocation and target setting under stochasticity, contributes to advancing sustainability across all its three dimensions (economic, environmental, social).

The bilevel DEA model, incorporating stochastic conditions, calculates the anticipated organizational profit by considering total expected income from outputs and subtracting total input costs. These computations account for various scenarios determined by the manager. In contrast, the deterministic model optimizes future organizational profits based on historical data with fixed input and output values. In both models, the central DMU must formulate a strategy before uncertain parameters are revealed, ensuring consideration of potential future outcomes for more informed predictions of organizational profits. This approach enhances the accuracy of predicting expected organizational efficiency and allows for more precise resource allocation and target setting through adjustments when new information is revealed.

6. Conclusions

The presented bilevel DEA model with stochastic conditions simultaneously optimizes resource allocation and output targeting, taking into consideration the efficiency lower bound posed by the central manager of large DMUs comprising multiple subDMUs. It considers the hierarchical relations that appear in such large organizations and enterprises that can be captured uniquely through the bilevel framework. Within this framework, objectives are optimized while simultaneously ensuring that DMU's operational efficiency aligns with the managerial strategy. The interconnections and conflicting interests inherent in this complex organizational structure involving the central administration and subordinate DMUs cannot be adequately captured by the network DEA optimization schema. In our approach, the uncertainty and unavailability of data are considered when evaluating the efficiency of large DMUs with a hierarchical structure. The proposed stochastic approach allows for the realization of uncertain parameters through discrete scenarios associated with an occurrence probability. One of the main advantages of this model is that it enables decision-makers of large DMUs to obtain an optimal economic strategy that permits readjustment to the new data upon the realization of one of the scenarios. Based on the scenario to be realized, recourse actions can be taken to adjust input consumption and output targets accordingly. To examine the performance of the stochastic approach, we apply the proposed model to evaluate the efficiency of a bank based on data that mirror typical real-world scenarios.

One limitation of this study is its exclusive focus on for-profit organizations, particularly large enterprises, which directly influence the objective function. Alternative applications of the model could explore diverse efficiency realizations, incorporating variations of the objective function. Additionally, when addressing banking issues, our concentration was on stochastic fluctuations in output targets while assuming deterministic input values. However, in many real-world scenarios, inputs may also exhibit stochastic tendencies. Furthermore, although our analysis was based on three scenarios, there are typically numerous potential scenarios to consider. For our case study, we chose to examine only a small number of decision-making units (DMUs) and scenarios. However, increasing the dimension of the problem will substantially increase the computational resources needed. This may necessitate the use of additional methodologies rooted in machine learning, offering an intriguing avenue for further research. For instance, Hao and An [66] suggested a pre-scoring method for DMUs, referred to as the angle-index synthesis method. They performed several numerical experiments, highlighting that their algorithm demonstrates excellent performance in computational time, exhibiting a linear increase in computational time, even for a staggering case involving 1 billion DMUs.

Taking into account the above, other avenues for future research could involve introducing stochastic elements at the input level, providing a representation of a potentially more realistic economic environment. Furthermore, we are investigating the possible integration of chance constraints concerning the targeted efficiency levels for each DMU. This approach allows the central administration to specify a range of desired efficiency levels for each distinguished DMU rather than a precise value. Finally, another path for future exploration might involve substituting the lower level with a two-stage problem while preserving the bilevel hierarchy. This approach would entail addressing a stochastic bilevel network DEA problem, thus finding applications in diverse sectors beyond banking. Ultimately, an alternative approach could involve demonstrating a tighter formulation to effectively address instances with larger dimensions.

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