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Dominations in Intutionistic Fuzzy Directed Graphs with Applications towards Influential Graphs

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Abstract: In this manuscript, we introduce a few new types of dominations in intuitionistic fuzzy directed graphs (IFDGs) based on different types of strong arcs (SAs). Our work is not only a direct extension of domination in directed fuzzy graphs (DFGs) but also fills the gap that exists in the literature regarding the dominations in different extended forms of fuzzy graphs (FGs). In the beginning, we introduce several types of strong arcs in IFDGs, like semi-β strong arcs, semi-δ strong arcs, etc. Then, we introduce the concepts of domination in IFDGs based on these strong arcs and discuss its various useful characteristics. Moreover, the dominating set (DS), minimal dominating set (MDS), etc., are described with some fascinating results. We also introduce the concept of an independent set in IFDGs and investigate its relations with the DS, minimal independent set (MIS) and MDS. We also provide numerous important characterizations of domination in IFDGs based on minimal and maximal dominating sets. In this context, we discuss the lower and upper dominations of some IFDGs. In addition, we introduce the terms status and structurally equivalent and examine a few relationships with the dominations in IFDGs. Finally, we investigate the most expert (influential) person in the organization by utilizing the concepts of domination in IFGs.

Keywords: IFDGs; strong arcs; domination in IFDG; independent set; minimal and maximal dominating sets

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1. Introduction

The term fuzzy sets (FSs) was first introduced by Zadeh [1] in 1965. The theory of FSs has become useful in different areas, such as management sciences, medical and life sciences, management sciences, social sciences, statistics, artificial intelligence, multiagent systems, expert systems, etc. In FSs, each element has some membership value allocated from the interval [0, 1]. Due to the flexibility of FSs, numerous generalizations of them has been introduced. The very first generalization of FSs, named interval-valued fuzzy sets (IVFSs), was introduced by Zadeh in [2]. In IVFSs, the membership value is a subinterval of [0, 1] instead of a fixed number. Since the concept of the non-membership value is not considered in FSs, it was also observed that in order to describe the particular type of information, one component (i.e., a membership value) is not sufficient. To explain such circumstances, Atanassov [3] introduced the concept of intuitionistic fuzzy sets (IFs), in which both the membership and non-membership values are considered, with the restriction that their sum is less than 1.

On the other hand, fuzzy logic becomes more beneficial and important in describing real-life problems with uncertainties. Recently, different types of networking have been dealt with through fuzzy logic. Consequently, fuzzy graph (FG) theory has become an important mathematical tool to address real-time issues more accurately. This new concept includes the fuzziness of the vertices and edges in fuzzy graphs (FGs). FGs were first introduced by Rosenfeld [4] and Kauffman [5]. They also introduced various graph theoretic tools, such as paths, cycles, bridges, trees, connectedness, etc., in their articles. As compared to classical graph theory, FGs are more effective because of their flexibility. In the literature, numerous applications of FGs have been investigated because of their flexibility. In the theory of FGs, many new terms were introduced by Bhattacharya [6]. In [7], some new operations were initiated and applied to FGs. The notion of Cayley IVFGs was described in [8]. In [9], the term complement of FGs was discussed. Poulik et al. [10] shifted the term average connectivity from classical graphs to FGs. Overall, FGs have become useful in several fields, like networking, modelling, social sciences, the recognition of different patterns, etc. Among the other types of FGs, fuzzy directed graphs (FDGs) or fuzzy digraphs have their own importance. Mordeson and Nair [11] introduced the notion of FDG. FDGs were further discussed in [12]. Numerous new terms related to FDGs, along with their applications, have been explored. Akram, Muhammad et al. [13] discussed the concept of bipolar FDGs in decision support systems. In continuation, a generalization of FGs, termed intuitionistic fuzzy graphs (IFGs), was introduced in [14]. Similarly, the notion of complex intuitionistic fuzzy graphs, along with their application to networking, was explored in [15]. Akram et al. [16–18] introduced many new terms, which included strong IFGs, IF hypergraphs, IF cycles, and IF trees. Afterwards, Akram et al. [19] introduced the concept of intuitionistic fuzzy digraphs (IFDGs) and their application in decision support systems. The application of IFGs in a water supply system was explored in [20]. Interval-valued intuitionistic fuzzy competition graphs were explored in [21]. IVIF-(s, t) graphs were discussed in [22,23]. The concepts of m-polar IFGs were introduced in [24]. Singh, Suneet et al. [25] discussed an interval-valued intuitionistic fuzzy directed graph with application towards transportation systems. Nithyanandham et al. [26] discussed an energy-based bipolar IFDG and presented its application in decision making theory. Some of the main components of picture fuzzy graphs (PFGs) were explored in [27].

In classical graph theory, the term domination has its own importance. Many researchers have presented several extended forms of domination in graphs, such as double Roman domination [28], triple Roman domination [29], broadcast domination [30], outer-convex domination [31], paired domination [32], etc. Kosari and Asgharsharghi introduced different domination numbers of graphs [33]. The notion of influence graphs has also been described in the literature to solve “influential problems” like the influence maximization problem for unknown social networks [34,35], etc. Alternatively, the term domination in FGs based on effective edges was introduced in [36]. The domination in FGs using strong arcs (SAs) was discussed by Nagoorgani et al. [37]. The notion of global domination in FGs based on SAs was discussed in [38]. Similarly, Shanmugam et al. [39] presented the idea of bridge domination in FGs. In [40], domination in FDGs was examined. Recently, in [41], the notions of broadcasts and dominating broadcasts were introduced, and they also provided applications of these concepts in a transportation model. The concept of domination in rough fuzzy digraphs was described by [42]. Similarly, domination in several types of vague graphs was discussed in [43–46]. Domination in IFGs was discussed by Parvathi [47], while double domination in IFGs was described by Nagoorgani [48]. The concept of domination in bipolar picture fuzzy graphs (BPPFGs) with application in social networks was introduced in [49].

In this study, we introduce various types of domination based on different strong arcs in intuitionistic fuzzy digraphs (IFDGs). Firstly, we describe various types of strong arcs in IFDGs. Then, based on these arcs, we introduce the concepts of domination in IFDGs.

These concepts are direct generalizations of the dominations in FDGs. We also provide some important characteristics of dominations in IFDGs based on minimal and maximal dominating sets. In addition, we introduce the terms status, structurally equivalent, and the lower and upper domination number, etc., in the framework of IFDGs. At the end, we provide the application of domination in IFDGs towards an organization in order to identify the most influential person through domination in IFDGs.

Motivations and Novelty:

In an IFG, the membership and non-membership values extend the domain as compared to the other extensions of FGs and make the circumstances more flexible to express problems with uncertainties. The term domination in FGs, IFGs, and BPFGs has been established in the literature, which motivated us to extend these terms towards IFDGs, along with their application. Our study also fills the gaps existing in the literature. We can summarize the novelty of our work as in the following points.

1. Firstly, we introduce different types of strong arcs in IFDGs, like semi-β strong arcs, semi-δ strong arcs, etc. Then, we introduce the concepts of domination in IFDGs based on these strong arcs. Different characterizations of some special IFDGs are also explored.
2. We also provide numerous important characterizations of domination in IFDGs based on minimal and maximal dominating sets. The lower and upper dominations of some IFDGs are also investigated.
3. We introduce the terms status and structurally equivalent and find few relationships with the dominations in IFDGs.
4. To demonstrate the usefulness of the terms that we have introduced, we offer their application in the context of influence graphs.

This article consists of five sections. In Section 2, we add some useful definitions and explanations related to FSs, FGs, FDGs, IFGs, etc. In Section 3, we introduce the concept of domination in an intuitionistic fuzzy digraph (IFDG) based on different types of SAs, which is a direct generalization of domination in FDGs. In the beginning, we introduce different types of SAs, like semi-β strong arcs, semi-δ strong arcs, etc. Then, we provide some important characterizations of domination in IFDGs based on minimal and maximal dominating sets. We also introduce the terms status, structurally equivalent, and the lower and upper domination number, etc., in the framework of IFDGs. At the end, we provide the application of domination in IFDGs towards an organization in order to identify the most influential person through domination in IFDGs. In Section 5, we provide the conclusions, which also include the future prospects of our work.

2. Preliminaries

In this section, we provide some useful terms related to FSs and FGs and their extensions. For the basics of classical graph theory, one may consult [50].

Definition 1 ([49])**.** *An FS F described on a non-empty set Y is a pair F = {(s,σ(s)): s ∈ Y, σ(s) ∈ [0, 1]}, where σ(s) is the membership function from Y to [0, 1].*

Definition 2 ([51])**.** *An intutionistic fuzzy set (IFS) N on a non-empty set Y is a pair* $N = (\beta_N, \delta_N) : Y \to [0,\ 1]$, *where* $\beta_N : Y \to [0,\ 1]$ *is said to be the degree of membership and* $\delta_N : Y \to [0,\ 1]$ *is the degree of non-membership satisfying the condition* $0 \leq \beta_N(s) + \delta_N(s) \leq 1$, *for all* $s \in Y$.

Definition 3 ([51])**.** *A function* $N = (\beta_N, \delta_N) : Y \times Y \to [0,\ 1] \times [0,\ 1]$ *is said to be an intutionistic fuzzy relation (IFR) on Y if* $\beta_N(s,t) + \delta_N(s,t) \leq 1$, *for all* $(s,t) \in Y \times Y$.

Definition 4 ([51])**.** *Let* $N = (\beta_N, \delta_N)$ *and* $M = (\beta_M, \delta_M)$ *be IFSs on the set Y. If* $N = (\beta_N, \delta_N)$ *is an IFR on a set Y, then* $N = (\beta_N, \delta_N)$ *is called an IFR on* $M = (\beta_M, \beta_M)$, *if* $\beta_N(s,t) \leq min\{\beta_M(s), \delta_M(t)\}$ *and* $\delta_N(s,t) \geq max\{\delta_M(s), \delta_M(t)\}$, *for all* $s, t \in Y$. *An IFR N on Y is said* *to be symmetric if* $\beta_N(s,t) = \beta_N(t,s)$ *and* $\delta_N(s,t) = \delta_N(t,s)$*, for all* $s, t \in Y$*.*

Definition 5 ([49])**.** *A fuzzy graph (FG) on a set* V *is a pair* $G^{\bullet} = (A, B)$*, where* $A = \{\rho_A\}$ *and* $B = \{\rho_B\}$*, where* $\rho_A : V \to [0,1]$ *and* $\rho_B : V \times V \to [0,1]$ *with* $\rho_B(s,t) \le \rho_A(s) \wedge \rho_A(t)$*, for all* $s, t \in V$*.*

Definition 6 ([49])**.** *Let* $G^{\bullet} = (A, B)$*, where* $A = \{\rho_A\}$ *and* $B = \{\rho_B\}$*, is the FG of a crisp graph* $G = (V, E)$*. We say that s dominates t in the* $G^{\bullet}$*, if* $\rho_B(st) = \rho_A(s) \wedge \rho_A(t)$*, for* $s, t \in V$*. A subset* V_1 *of* V *is said to be a dominating set (DS) of the FG* $G^{\bullet}$ *if, for each* $s \in V_1$*, there is* $t \in V - V_1$ *such that s dominates t. A DS* A_1 *in an FG* $G^{\bullet}$ *is a minimal dominating set (MDS) if* A_1 *has no proper dominating subset. A DS in FG* $G^{\bullet}$ *having the minimum (fuzzy) cardinality is known as the domination number (DN) of FG* $G^{\bullet}$*.*

Definition 7 ([49])**.** *Let* $G^{\bullet}$ *be an FG without an isolated vertex. Then, the DS* V_1 *is known as the total dominating set (TDS) if a vertex in* V_1 *dominates all vertices of* V*. The minimum (fuzzy) cardinality of the TDS is known as the total domination number (TDN).*

Definition 8 ([49])**.** *Two vertices s and t are called neighbors (Ns) in an FG* $G^{\bullet}$ *if* $\rho(s,t) > 0$*. The set of all Ns of s is denoted by* $Nbhd(s)$*.*

Definition 9 ([49])**.** *A vertex s is known as a strong neighbor (SN) if the arc* (s,t) *is strong. The collection of all strong neighbors (SNs) of s is said to be a strong neighborhood (SNbhd) of s and is represented by* $Nbhd_S(s)$*.*

Definition 10. *The closed strong neighborhood (CSNbhd) is defined as* $Nbhd_S[s] = Nbhd_S(s) \cup \{s\}$*.*

Definition 11 ([51])**.** *An IFG with underlying set V is described as* $\hat{G} = (N, M)$*, where* $N = \{\beta_N, \delta_N\}$ *and* $M = \{\beta_M, \delta_M\}$*, where*
(i) the function $\beta_N : V \to [0, 1]$ *represents the degree of membership of any element* $s \in V$ *and* $\delta_N : V \to [0, 1]$ *represents the degree of non-membership of any element* $s \in V$ *such that* $\beta_N(s) + \delta_N(s) \le 1$*, for all* $s \in V$*;*
(ii) the function $\beta_M : E \subseteq V \times V \to [0, 1]$ *is the degree of membership of any element* $(s,t) \in E$*, while* $\delta_M : E \subseteq V \times V \to [0, 1]$ *is the degree of non-membership of any element* $(s,t) \in E$ *satisfying* $\beta_M(s,t) \le min\{\beta_N(s), \beta_N(t)\}$ *and* $\delta_M(s,t) \ge max\{\delta_N(s), \delta_N(t)\}$ *such that* $0 \le \delta_M(s,t) + \delta_M(s,t) \le 1$*, for all* $(s,t) \in E$*.*

Definition 12 ([51])**.** *If* s, t *are any two vertices of the IFG* $\hat{G} = (N, M)$*, where* $N = \{\beta_N, \delta_N\}$ *and* $M = \{\beta_M, \delta_M\}$*, then the* β_M*-strength of connectedness between s and t is* $\beta_M^{\infty}(s,t)$*, where*

$$\beta_M^{\infty}(s,t) = sup\{\beta_M^k : k = 0, 1, 2, 3......n\}$$

and the δ_M*-strength of connectedness between s and t is*

$$\delta_M^{\infty}(s,t) = inf\{\delta_M^k : k = 0, 1, 2, 3......n\}.$$

If y and z are connected by means of paths of length k, then

$$\beta_M^k(s,t) = sup\{\beta_M(s,t_1) \wedge \beta_M(t_1,t_2) \wedge ....\beta_M(t_{k-1},t) : s, t_1, t_2, ..., t_{k-1}, t \in V\}$$

and

$$\delta_M^k(s,t) = inf\{\delta_M(s,t_1) \wedge \delta_M(t_1,t_2) \wedge ....\delta_M(t_{k-1},t) : s, t_1, t_2, ..., t_{k-1}, t \in V\}.$$

Definition 13 ([51])**.** *If deleting any vertex s of connected IFG* $\hat{G}$ *decreases the strength of connectedness between several pairs of vertices (nodes), then such a vertex s is called a cut vertex.*

Definition 14 ([51]). *Let $\hat{G}$=(N, M) be an IFG. Then, $|N| = \sum_{s \in N} \frac{1+\beta_N(s)-\delta_N(s)}{2}$ is known as the vertex cardinality of N, $|M| = \sum_{(s,t) \in M} \frac{1+\beta_M(s,t)-\delta_M(s,t)}{2}$ is the edge cardinality of M, and $|T| = |N| + |M|$ is the cardinality of IFG $\hat{G}$.*

Definition 15 ([40]). *A directed simple graph is represented by $G_D = (\tilde{V}, \tilde{E})$, where $\tilde{V}$ is a non-empty finite set of vertices and $\tilde{E} = \{(s,t) : s, t \in \tilde{V}, s \neq t\}$ is a set of directed edges. A pair $G_D^{\bullet} = (A, B)$ is called a fuzzy digraph (FDG), where $A = \{\rho_A\}$ and $B = \{\rho_B\}$ are the mappings $\rho_A : \tilde{V} \to [0,1]$ and $\rho_B : \tilde{E} \to [0,1]$, such that $\rho_B(s,t) \leq \rho_A(s) \wedge \rho_A(t)$, for all $s, t \in \tilde{V}$ and $(s,t) \in \tilde{E}$. We call a digraph $G_D = (\tilde{V}, \tilde{E})$ a hidden directed graph of a fuzzy directed graph $G_D^{\bullet} = (A, B)$.*

Definition 16 ([40]). *The sequence of strong arcs such that the end vertex of every arc is the same as the starting vertex of the next arc in a sequence is called a fuzzy dipath (FDP) P.*

Definition 17 ([40]). *A dipath (DP) that begins and ends with the same vertex is called a fuzzy dicycle (FDC) C.*

Definition 18 ([19]). *An intuitionistic fuzzy digraph (IFDG) of a digraph $G_D = (\tilde{V}, \tilde{E})$ is a pair $G_D^{\circ} = (N, M)$, where $N = (\tilde{V}, \beta_N, \delta_N)$ represents an IFS in $\tilde{V}$ and $M = (\tilde{V} \times \tilde{V}, \beta_M, \delta_M)$ represents an IF relation on $\tilde{V}$ such that*

$$\beta_M(st) \leq min(\beta_N(s), \delta_N(t))$$

$$\beta_M(st) \geq max(\beta_N(s), \delta_N(t))$$

and $0 \leq \beta_M(st) + \delta_M(st) \leq 1$, for all $s, t \in \tilde{V}$. We note that M may not be a symmetric relation.

3. Domination in Intutionistic Fuzzy Digraphs

In this section, firstly, we introduce the concepts of strong arcs and their types in IFDGs. Based on these strong arcs, we present the concepts of domination in IFDGs. Moreover, the dominating set (DS), minimal dominating set (MDS), etc., are also described with some interesting results. Then, we also introduce the concept of an independent set in an IFDG and its relations with the DS, minimal independent set (MIS) and MDS. At the end of this section, we present the terms status and structurally equivalent and explore some relations among these terms and the domination in IFDGs.

We begin our discussion with the definition of the degree of a vertex in an IFDG.

Definition 19. *Let $G_D = (\tilde{V}, \tilde{E})$ be a hidden digraph of an IFDG $G_D^{\circ} = (N, M)$. Then, the order q of G_D° is defined as*

$$q = (\sum_{s \in \tilde{V}} \beta_N(s), \sum_{s \in \tilde{V}} \delta_N(s)).$$

Example 1. *In the IFDG shown in Figure 1, we have $q = (2, 0.6)$.*

Now, we present the definition of the size of an IFDG.

Definition 20. *Let $G_D = (\tilde{V}, \tilde{E})$ be a hidden digraph of $G_D^{\circ} = (N, M)$. The size p of G_D° is defined as*

$$p = (\sum_{s \neq t} \beta_M(s,t), \sum_{s \neq t} \delta_M(s,t))$$

for all $(s,t) \in \tilde{E}$.

Example 2. *Referring to the IFDG shown in Figure 1, we have $p = (1.6, 1.6)$.*

Here, we present the definition of a strong arc in an IFDG, which plays a crucial role in the rest of this paper.

Definition 21. *An arc* (s,t) *of an IFDG* G_D° *is said to be a strong arc if* $\beta_M(s,t) = \beta_M^\infty(s,t)$ *and* $\delta_M(s,t) = \delta_M^\infty(s,t)$*; otherwise, the arc* (s,t) *is non-strong.*

Afterwards, we present different types of strong arcs in IFDGs, such as semi β-strong arcs, semi δ-strong arcs, etc.

Definition 22. *An arc* (s,t) *of an IFDG* G_D° *is a semi* β*-strong arc if* $\beta_M(s,t) = \beta_M^\infty(s,t)$ *and* $\delta_M(s,t) \neq \delta_M^\infty(s,t)$*.*

Definition 23. *An arc* (s,t) *of an IFDG* G_D° *is a semi* δ*-strong arc if* $\beta_M(s,t) \neq \beta_M^\infty(s,t)$ *and* $\delta_M(s,t) = \delta_M^\infty(s,t)$*.*

In Example 3, we analyze the strong arcs among those depicted in the IFDG given in Figure 1.

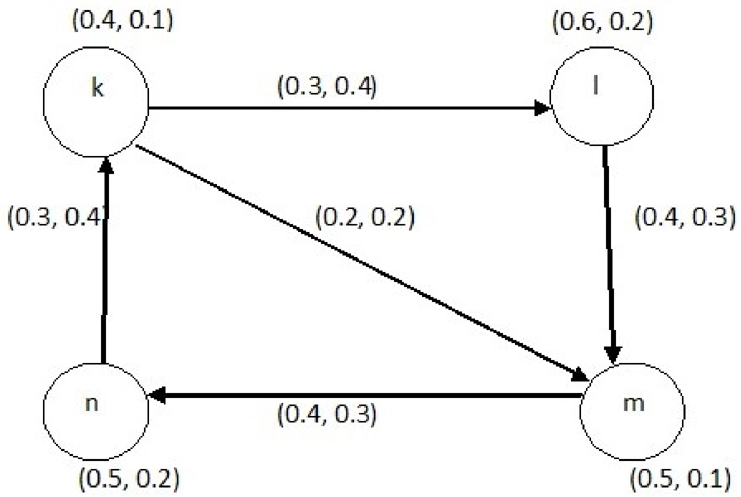

Figure 1. Intuitionistic fuzzy digraph.

Example 3. *We determine which arcs in the IFDG shown in Figure 1 are considered strong arcs and which ones are not.*
Case (i) *Consider the arc* (k,l)*;* $\beta_M(k,l) = 0.3$ *and* $\delta_M(k,l) = 0.4$*. Now,* $\beta_M^\infty(k,l) = sup\{\beta_N(k,l)\} = 0.3$ *and* $\delta_M^\infty(k,l) = sup\{\delta_N(k,l)\} = 0.4$*. Therefore,* $\beta_M(k,l) = \beta_M^\infty(k,l) = 0.3$ *and* $\delta_M(k,l) = \delta_M^\infty(k,l) = 0.3$*. Hence, the arc* (k,l) *is a strong arc.*
Case (ii) *Let us consider an arc* (l,m)*;* $\beta_M(l,m) = 0.4$ *and* $\delta_M(l,m) = 0.3$*. Now,* $\beta_M^\infty(l,m) = sup\{\beta_N(l,m)\} = 0.4$ *and* $\delta_M^\infty(l,m) = inf\{\delta_N(l,m)\} = 0.3$*. Therefore,* $\beta_M(l,m) = \beta_M^\infty(l,m) = 0.4$ *and* $\delta_M(l,m) = \delta_M^\infty(l,m) = 0.3$*. Hence, the arc* (l,m) *is a strong arc.*
Case (iii) *Let us consider the arc* (m,n)*;* $\beta_M(m,n) = 0.4$ *and* $\delta_M(m,n) = 0.3$*. Now,* $\beta_2^\infty(m,n) = sup\{\beta_1(m,n)\} = 0.4$ *and* $\delta_M^\infty(m,n) = inf\{\delta_N(m,n)\} =0.3$*. Therefore,* $\beta_M(m,n) = \beta_M^\infty(m,n) = 0.4$ *and* $\delta_M(m,n) = \delta_M^\infty(m,n) = 0.3$*. Hence, the arc* (m,n) *is a strong arc.*
Case (iv) *Let us consider the arc* (n,k)*;* $\beta_M(n,k) = 0.3$ *and* $\delta_M(n,k) = 0.4$*. Now,* $\beta_M^\infty(n,k) = sup\{\beta_N(n,k)\} = 0.3$ *and* $\delta_M^\infty(n,k) = inf\{\delta_M(n,k)\} = 0.4$*. Therefore,* $\beta_M(n,k) = \beta_M^\infty(n,k) = 0.3$ *and* $\delta_M(n,k) = \delta_M^\infty(n,k) = 0.4$*. Hence, the arc* (n,k) *is a strong arc.*
Case (v) *Consider the arc* (k,m)*;* $\beta_M(k,m) = 0.2$ *and* $\delta_M(k,m) = 0.2$*. Now,* $\beta_M^\infty(m,k) = sup\{\beta_N(k,l) \wedge \beta_N(l,m)\} = sup\{0.3,0.4\} = 0.4$ *and* $\delta_M^\infty(k,m) = inf\{\delta_N(k,l) \vee \delta_N(l,m)\} = inf\{0.4, 0.3\} = 0.3$*. Therefore,* $\beta_M(k,m) \neq \beta_M^\infty(k,m)$ *and* $\delta_M(k,m) = \delta_m^\infty(k,m) = 0.3$*. Hence, the arc* (k,m) *is not a strong arc.*

In Definition 24, we introduce the terms strong neighborhood (SNbhd) and closed neighborhood (CNbhd) along with their types and cardinalities.

Definition 24. *Let $G_D = (\tilde{V}, \tilde{E})$ be a hidden digraph of an IFDG $G_D^\circ = (N, M)$. Then,*
(i) $Nbhd_S(s)$ = $\{t \in \tilde{V}$: $arc(s,t)$ is strong arc$\}$ is the SNbhd of $s \in \tilde{V}$. Similarly, the CNbhd of s is $Nbhd_S[s]$ = $Nbhd_S(s) \cup \{s\}$.
(ii) $Nbhd_{\beta S}(s)$ = $\{t \in \tilde{V}$: arc (s,t) is semi β-strong arc$\}$ is known as the semi β-SNbhd of $s \in \tilde{V}$ and CNbhd of s is $Nbhd_{\beta S}[s] = Nbhd_{\beta S}(s) \cup \{s\}$.
(iii) $Nbhd_{\delta S}(s)$ = $\{t \in \tilde{V}$: $arc(s,t)$ is semi δ-strong arc$\}$ is known as the semi δ-SNbhd of $s \in \tilde{V}$ and CNbhd of s is$Nbhd_{\delta S}[s] = Nbhd_{\delta S}(s) \cup \{s\}$.
(iv) $\eta_S(G_D^\circ)$ = $min\{|Nbhd_S(s)| : s \in \tilde{V}(G_D^\circ)\}$ is the minimum cardinality of the SNbhd.
(v) $\theta_S(G_D^\circ)$ = $max\{|Nbhd_S(s)| : s \in \tilde{V}(G_D^\circ)\}$ is the maximum cardinality of the SNbhd.

Theorem 1. *If two nodes of an IFDG G_D° are linked by one dipath, then every arc of G_D° is a strong arc.*

Proof. Let G_D° be a connected IFDG with n nodes. If we take $n = 2$, then s and t must be adjacent by one arc (because G_D° is a connected IFDG). Clearly, $\beta_M(s,t) = {\beta_M}^\infty(s,t)$ and $\delta_M(s,t) = {\delta_M}^\infty(s,t)$. Hence, an arc (s,t) is a strong arc. Let $n > 2$. In any IF dipath, ${\beta_M}^\infty(s,t) = \beta_M(s,t)$ and ${\delta_M}^\infty(s,t) = \delta_M(s,t)$ for any arc in the dipath (s,t), as they are connected through the same dipath. Thus, it is proven that $\beta_M(s,t) = {\beta_M}^\infty(s,t)$ and $\delta_M(s,t) = {\delta_M}^\infty(s,t)$ for any number of arcs in a given dipath. Hence, all the arcs are strong. □

Corollary 1. *In an IF dipath, each arc is a strong arc.*

Theorem 2. *In a non-trivial connected IFDG G_D° with n nodes such that $n \geq 2$, G_D° has at least one strong arc.*

Proof. Let G_D° be a connected IFDG with vertices $n \geq 2$. Assume that s and t are the two nodes of G_D°.
$Case(i)$: When $n = 2$: Because G_D° is a connected IFDG, s and t are two nodes such that (s,t) is an arc. From Theorem 1, only one strongest dipath between s and t exists such that $\beta_M(s,t) = {\beta_M}^\infty(s,t)$ and $\delta_M(s,t) = {\delta_M}^\infty(s,t)$. Hence, (s,t) is a strong arc.
$Case(ii)$: When $n > 2$: Assume that G_D° has at least one strong arc. Because G_D° is connected with $n > 2$, there exists more than one dipath between s and t such that at least one strong dipath exists. Thus, $\beta_M(s,t) = {\beta_M}^\infty(s,t)$ and $\delta_M(s,t) = {\delta_M}^\infty(s,t)$ (from Theorem 1). If this does not hold, there is no dipath between s and t. Hence, G_D° is a disconnected digraph, which contradicts our hypothesis that G_D° is connected. Therefore, if $n \geq 2$, then non-trivial connected IFDG G_D° has at least one strong arc. □

Theorem 3. *Let (s,t) be the arc of IFDG G_D°. Then, the following conditions are equivalent.*
(i) In G_D°, an arc (s,t) is a strong arc.
(ii) An arc (s,t) must be semi β-strong and semi δ-strong.
(iii) The membership degree and non-membership degree of arc (s,t) must be in between the closed interval $[\beta_{SM}, \delta_{LM}]$, where the smallest value of the membership degree of the IFDG G_D° is β_{SM}, and the largest value of the non-membership degree of the IFDG G_D° is δ_{LM}.

Definition 25. *Let G_D° be an IFDG and s, t be any two vertices of G_D°. Then, s dominates t, if the arc (s,t) is a strong arc.*

Example 4. *Referring to the IFDG given in Figure 1, the arcs (k,l), (l,m), (m,n), (n,k) are strong arcs but the arc (m,k) is a non-strong arc. Thus, l dominates m, m dominates n and n dominates k, but m does not dominate k.*

Definition 26. *A DS of IFDG G_D° is a subset N_1 of $\tilde{V}$ if, for each $t \in \tilde{V} - N_1$, there exists $s \in N_1$ such that s dominates t. A DS N_1 is an MDS if there is no proper subset of N_1 that is a DS. The minimum cardinality from all MDSs is a lower DN of G_D° and it is abbreviated as $L_D(G_D^\circ)$. The maximum cardinality from all MDSs is an upper DN of G_D° and is abbreviated as $U_D(G_D^\circ)$. The minimum fuzzy cardinality from all DSs of an IFDG is known as the strong arc DN and is* *symbolically written as $\omega_S(G^\circ_D)$. The corresponding DS is known as the minimum strong arc DS and the number of elements in the minimum strong arc DS is known as $n[\omega_S(G^\circ_D)]$.*

Example 5. *Consider a set of vertices $N = \{a, b, c, d\}$ in an IFDG, as shown in Figure 2. Let N_1 = $\{a, c\}$ be the DS lying in N. Let $\{b, d\}$ be the set of vertices other than N_1, such that each of its vertices dominates at least one vertex in N, which implies that N_1 is a DS. Again, consider that N_2 = $\{b, d\}$ is the DS lying in N. Let $\{a, c\}$ be the set other than N_2 such that each of its vertices dominates at least one vertex in N, which implies that N_2 is a DS. Thus, the DSs are $\{a, c\}$ and $\{b, d\}$, while $\{b, d\}$ is the MDS of minimum cardinality 1.25 and $\{a, c\}$ is the MDS of maximum cardinality 1.35.*

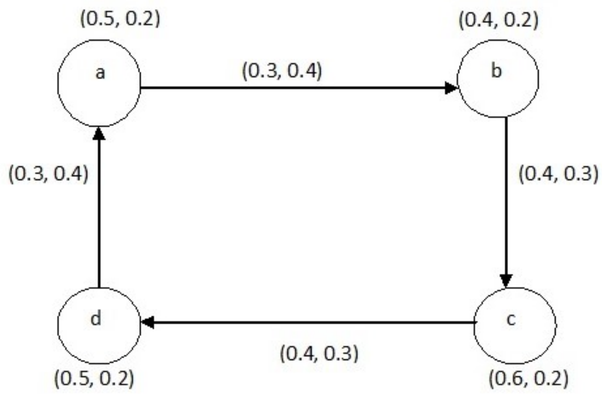

Figure 2. Intuitionistic fuzzy digraph.

Definition 27. *The open Nbhd of s in an IFDG G°_D is represented as $ONbhd(s)$ and is defined as $ONbhd(s) = \{t \in \tilde{V} : \beta(s,t) > 0, \delta(s,t) > 0\}$. The vertex t is known as the SN of s if an arc (s,t) is a strong arc, and the set of all SNs of s is known as the SNbhd of s and is abbreviated as $Nbhd_S(s)$. Similarly, $Nbhd_S[s] = Nbhd_S(s) \cup \{s\}$ is the CSNbhd of s.*

Example 6. *Referring to the IFDG given in Figure 1, one can easily deduce that the arcs $(k,l), (l,m), (m,n), (n,k)$ are strong arcs, while the arc (k,m) is a non-strong arc. The SN of k is l. Thus, $Nbhd_S[k] = \{l\} \cup \{k\} = \{l, k\}$ is the CSNbhd of k.*

Definition 28. *Let G°_D be an IFDG and s, t be any two vertices of G°_D. Then,*
(i) s semi β- dominates t, if the arc (s,t) is a semi β-strong arc;
(ii) s semi δ- dominates t, if an arc (s,t) is a semi δ-strong arc.

Remark 1. *(i) Semi β- strong arc DN is $\omega_{\beta S}(G^\circ_D)$. The number of elements in the minimum semi β- strong arc DS is symbolically written as $n[\omega_{\beta S}(G^\circ_D)]$.*
(ii) Semi δ - strong arc DN is described as $\omega_{\delta S}(G^\circ_D)$. The number of elements in the minimum semi δ - strong arc DS is represented as $n[\omega_{\delta S}(G^\circ_D)]$.

Definition 29. *Two vertices s and t of an IFDG $G^\circ_D = (N, M)$ are called isolated vertices if $\beta_M(s,t) = 0$ and $\delta_M(s,t) = 0$. Secondly, $Nbhd(s) = \emptyset$, which implies that there does not exist any Nbhd of s. Thus, an isolated vertex cannot dominate any other vertex of G°_D.*

Theorem 4. *In an IFDG $G^\circ_D = (N, M)$, a DS N_1 is an MDS if, for each $s \in N_1$, one of the following conditions holds.*
(i) s is not an SN of any vertex in N_1;
(ii) there exists a vertex $t \in N - N_1$ such that $Nbhd(t) \cap N_1 = \{s\}$.

Proof. Assume that N_1 is an MDS of G°_D. For each vertex $s \in N_1$, $N_1 - s$ is not a DS. Then, $t \in (N_1 - s)$ exists and is not dominated by any vertex in $N_1 - s$. If $t = s$, then t is not an SN of any vertex in N_1. If $t \neq s$, t is not dominated by $N_1 - t$, but it is dominated by N_1, and there is a vertex t that is the only SN of s in N_1. Hence, $Nbhd(t) \cap N_1 = s$.

Conversely, consider that N_1 is a DS. For every vertex $s \in N_1$, one of the two given conditions holds true. Assume that N_1 is not an MDS. Thus, there exists a vertex $s \in N_1$, $N_1 - s$ that is a DS. Hence, s is an SN to one of the vertices in $N_1 - s$, so condition (i) does not hold true. If $N_1 - s$ is a DS, then each vertex of $N - N_1$ is an SN to one of the vertices in $N_1 - s$, and condition (ii) also does not hold true. This is a contradiction of our hypothesis that one of the two conditions holds true. Thus, N_1 is an MDS. □

Theorem 5. *Let $G_D^\circ = (N, M)$ be an IFDG with no isolated vertex. Let N_1 be an MDS. Then, $N - N_1$ is a DS of G_D°.*

Proof. Assume that N_1 is an MDS. Consider that t is a vertex of N_1. As G_D° does not have isolated vertices and there exists a vertex $s \in Nbhd(t)$, t is dominated by one of the vertices in $N_1 - t$, i.e., $N_1 - t$ is a DS. From Theorem 4, $s \in N - N_1$. Thus, each vertex in N_1 is dominated by one of the vertices in $N - N_1$ and $N - N_1$ is a DS. □

Corollary 2. *If there is no isolated vertex in an IFDG $G_D^\circ = (N, M)$, then $L_D(G_D^\circ) \leq q(G_D^\circ)/2$.*

Proof. Let G_D° be an IFDG with no isolated vertex. Then, it has two disjoint DSs, i.e., $L_D(G_D^\circ) \leq q(G_D^\circ)/2$. □

Definition 30. *Two vertices s and t of an IFDG $G_D^\circ = (N, M)$ are called independent if there is no strong edge between these two vertices. A subset N_2 of N is known as an independent set (IS) of an IFDG G_D° if the following conditions hold:*

$$\beta_M(s,t) < \beta_M^\infty(s,t) \text{ and } \delta_M(s,t) < \delta_M^\infty(s,t)$$

for all $(s,t) \in N_2$.

Definition 31. *An IS $N_2 \subseteq N$ in an IFDG $G_D^\circ = (N, M)$ is called a maximal independent set (MIS) if the set $N \cup \{s\}$ is not independent for every $s \in N - N_2$. The minimum cardinality between the MISs is called the lower independent number of an IFDG G_D°, represented by $i(G_D^\circ)$. The maximum cardinality between the MISs is called the upper independent number of an IFDG G_D°, represented as $I(G_D^\circ)$.*

Theorem 6. *An IS is an MIS in an IFDG $G_D^\circ = (N, M)$ if and only if it is an IS and DS.*

Proof. Let N_2 be an MIS of IFDG G_D°. Then, for each vertex $s \in (N - N_2)$, the set $N_2 \cup s$ is not independent. Moreover, for each vertex $s \in (N - N_2)$, there exists a vertex $t \in N_2$ such that t is an SN of s. Hence, N_2 is a DS. Thus, N_2 is both a DS and IS.

Conversely, let N_2 be an IS and DS. If N_2 is not an MIS, there is a vertex $s \in N - N_2$ such that the set $N_2 \cup \{s\}$ is independent. If $N_2 \cup \{s\}$ is independent, then there is no vertex in N_2 that is an SN of s. Hence, N_2 is not a DS, which contradicts our assumption. Thus, N_2 is an MIS. □

Theorem 7. *Every MIS in an IFDG $G_D^\circ = (N, M)$ is an MDS.*

Proof. Let N_2 be an MIS of an IFDG. By assumption, N_2 is a DS but not an MDS. Then, there exists at least one vertex $s \in N_2$ such that $N_2 - \{s\}$ is a DS. If $N_2 - \{s\}$ dominates $N - \{N_2 - (s)\}$, then there is at least one vertex in $N_2 - \{s\}$ that is necessarily an SN of t, which contradicts our assumption. Hence, N_2 is an MDS. □

Definition 32. *In IFDG G_D°, a subset of vertex set N is known as status S if each vertex $g, h \in S$ obeys the property that the vertex g dominates the vertices in $N - S$ and is equal to the set of vertices in $N - S$ that is dominated by h.*

Remark 2. *Each vertex in status S dominates the same set of vertices outside the status. It can be seen that the status must contain at least two vertices.*

Theorem 8. *If a status S of a connected non-trivial IFDG G°_{D} is an MDS, then S is an independent DS with cardinality 2.*

Proof. Let S be a status that is an MDS. As G°_{D} is connected with no isolated vertex, then there exists at least one vertex $g \in N - S$. As S is an MDS, g is adjoint at least in the S, and since S is the status, each node of S is adjoint to the g. Additionally, every vertex of S is adjoint to the each vertex in $N - S$. Thus, $|S| \geq 2$, because S is the status. Now, consider $|S| \geq 3$ and assume that h belongs to S and g belongs to $N - S$. Since S is the status, it implies that h is adjoint to each vertex of $C - S$ and g is adjoint to every other vertex of S. Hence, the DS is $\{g, h\}$, which contradicts our assumption that S is the minimal set. Thus, $|S| = 2$. However, if h is adjoint to the g, then the DS of G°_{D} is g, which is again a contradiction that S is the minimal set. Consequently, $|S| = 2$ is an IS. □

Definition 33. *Let g and h be any two vertices of an IFDG. Then, these two vertices are called structurally equivalent if either $Nbhd_S(g) = Nbhd_S(h)$ or $Nbhd_S[g] = Nbhd_S[h]$. A set S is called structurally equivalent if each of the two vertices in S is structurally equivalent.*

Corollary 3. *Let G°_{D} be a connected IFDG. Let S be an MDS that is structurally equivalent. Then, the set S has two independent vertices such that each vertex has a degree $q(G^{\circ}_{D}) - 1$.*

4. Application of Domination in IFDGs towards Social Networks

Graphs have various applications in many areas of science, such as chemistry, physics, biology, mathematics, computer science and others. In the organization model, it has been noted that, in a group, there is a connection between two workers. It is also necessary to conclude that, in a graph, one worker is more dominant or influential. Using the graph,we can draw this scenario. In a specific group, we can draw a graph in which each vertex represents each worker. In the graph, the directed edges show the relationship between two workers from one particular vertex (worker) to another. Multiple edges or loops are not needed in these types of graphs. In classical graph theory, every vertex has equal importance. It is not possible to draw such types of graphs in an organization model accurately. Additionally, in classical graph theory, every organization in a social unit (individual or organization) should have equal importance, but the situation is different in real life. Similarly, in classical graph theory, every directed edge has equal strength. Thus, the influence of the worker has fuzzy directed boundaries. It is useful to represent these situations in fuzzy directed graphs. Every vertex represents a worker and the strength of his influence in the organization model, and it is represented by the membership value in the fuzzy directed influential graph. Since the developed form of FS is the IFS, domination in IFDGs provides better results as compared to fuzzy directed graphs.

4.1. Fuzzy Influence Digraph

Let us consider an organization with workers and their designations. Let S = {BOD, CEO, CTO, DM, DPD, DHR, Stt } be the set of workers for this organization, as shown in Table 1. By conducting research on the organization, we conclude the following.

(i) The CEO has worked with the DM for about 8 years, and, on strategic initiatives, he gives importance to his input.
(ii) The BOD has been chaired for about 8 years and is associated with the DM. Similarly to the CEO, the BOD also values the DM.
(iii) In reorganization, the whole marketing scheme is vital but the DHR is more vital.
(iv) There is a history of disputes between the CTO and DHR.
(v) The CTO has more influence on the DPD.

Table 1. Designations of workers in an organization and abbreviations used for their designations.

Designation	Abbreviation
Board of Directors	BOD
Chief Executive officer	CEO
Chief Technology officer	CTO
Director of Marketing	DM
Director of Product Development	DPD
Director of Human Resources	DHR
Staff	Stt

An influence digraph can be drawn by observing the above-mentioned points, but this type of digraph does not show the power of the workers in an organization and also the degree of influence of workers on one another. It is important to show them in fuzzy sets as their influence and power have no definite limits. The influence of workers on one another can be represented through a fuzzy digraph, but there exists hesitation in evaluating their influence. We consider a fuzzy directed influential graph of this organization, shown in Figure 3. The organization is represented by the nodes and its membership value represents the degree of influence. The degree of membership represents how influential the worker is? in the organization. The BOD has an 80% level of influence. In the digraph, the directed edges show the influence level of one worker on the other workers within the organization. The membership degree of the directed edges is considered as a positive percentage of influence, e.g., the DM has a 50% influence on both the BOD and CEO. Thus, the DM dominates both the BOD and CEO, which is why it is busier and more influential than the others.

While dealing with the above circumstances through FDGs, we have only the degree of acceptance and there is no information about the degree of non-acceptance of the lower staff members. Hence, there is a lack of information that can be properly manipulated through the IFDGs.

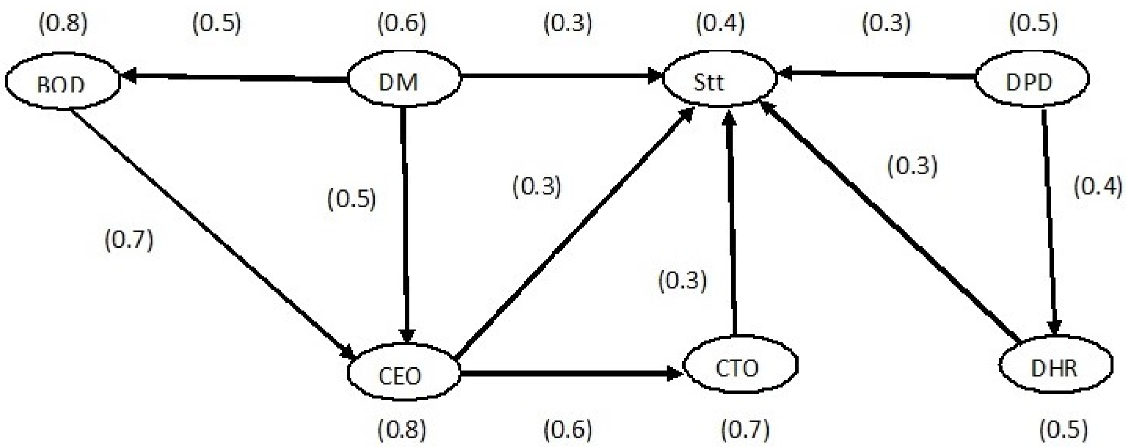

Figure 3. Fuzzy directed influence graph.

4.2. Intuitionistic Fuzzy Influence Digraph

Since the power and influence of workers cannot be properly described in fuzzy digraphs, we use an IFDG, which gives better results as compared to FGs. In an IFDG, the directed edges are used to show the influence. The resulting IFDG is shown in Figure 4, and Table 2 shows the allocated membership and non-membership values. In the IFDG, the vertices also represent the workers along with their power in terms of membership and non-membership degrees, which are described by percentages. For instance, the CEO has 80% power in the organization. Likewise, in the IFDG, the directed edges show the influence of one worker on another. The membership and non-membership degrees can also be referred to as a positive influence and negative influence, respectively. For instance, the BOD has a 50% influence on the DM's opinion but he does not follow his opinion 30% of the time. In Figure 4, it can be seen that the DM has an influence on both the BOD and CEO. As the membership degrees in both cases are 0.5, which is 50%, his influence on both of them is the same. In the case of the CEO, the hesitation degree is 0.2, which is $(\pi = 1 - 0.5 - 0.3)$, but it is 0.1 in the case of the BOD, which is $(\pi = 1 - 0.5 - 0.4)$, which shows that the CEO has more hesitation than the BOD. It is clear that the most influential worker within the organization is the DM. He has a great influence on both the BOD and CEO; each has 80% power. Clearly, all the arcs are strong but the DM dominates the BOD, CEO and Stt. Hence, the most influential worker in the organization is the DM.

Table 2. Power of workers allocated in terms of membership and non-membership degrees.

	BOD	CEO	CTO	DM	DPD	DHR	Stt
β_N	0.8	0.8	0.7	0.6	0.5	0.5	0.4
δ_N	0.1	0.1	0.2	0.2	0.2	0.3	0.2

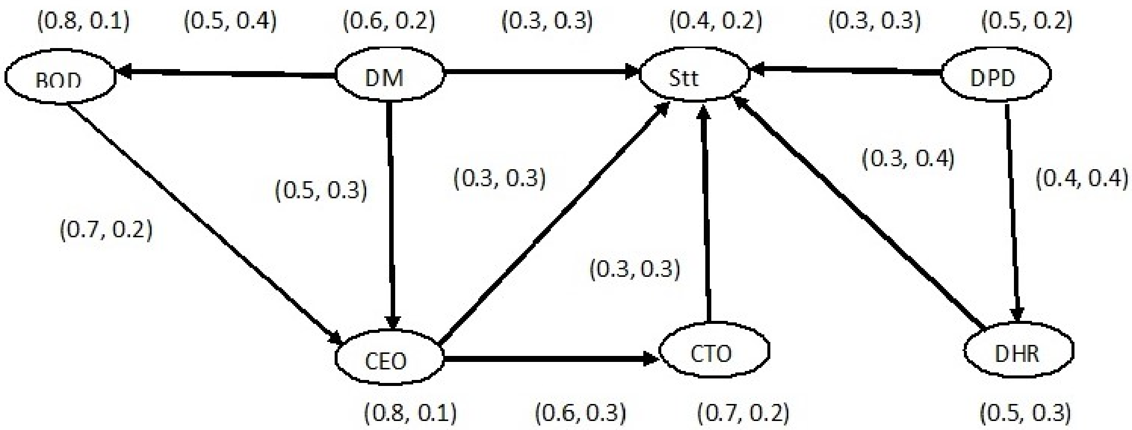

Figure 4. Intuitionistic fuzzy directed influence graph

5. Conclusions

In this article, we have introduced the notion of domination in IFDGs based on SAs, along with several fundamental properties and applications. We have extended the concepts of domination in FDGs. Since dominations in picture fuzzy graphs and bipolar picture fuzzy graphs were introduced in the literature, but the concept of domination in IFDGs was missing, we have also filled this gap the literature related to domination. At the beginning of our study, we introduced several types of strong arcs in IFDGs, like semi-β strong arcs, semi-δ strong arcs, etc. Then, we introduced the concept of domination in IFDGs based on these strong arcs and discussed its various useful characteristics. Moreover, the dominating set (DS), minimal dominating set (MDS), etc., were described with some fascinating results. We have also introduced the concept of an independent set in an IFDG and investigated its relations with the DS, minimal independent set (MIS) and MDS. We have also provided numerous important characterizations of domination in IFDGs based on the minimal and maximal dominating sets. In this context, we have discussed the lower and upper dominations of some IFDGs. In addition, we have introduced the terms status and structurally equivalent and explored a few relationships with the dominations in IFDGs. Finally, we have investigated the most expert (influential) person in the organization by using the concepts of domination in IFGs. One could extend these concepts towards other extended forms of FGs, like spherical picture fuzzy graphs.

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