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An Analysis of Power Friction Losses in Gear Engagement with Intermediate Rolling Elements and a Free Cage

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Abstract: Currently, mechanical gears with cycloid engagement are increasingly used in mechanisms along with involute ones. In modern drive mechanisms, using pin gears and gears with intermediate rolling elements (IRE) is widespread, which simultaneously use cycloid gears. To a greater extent, pin gears are now being investigated, but IRE gears have their undeniable advantages. Many works are devoted to the study of cycloid toothing for certain gears, but the efficiency, especially that of IRE gears, has practically not been investigated. Therefore, the analysis of power losses in the engagement of a gear with IRE and a free cage (IREFC) is relevant. In this analysis, the authors of the work have used laws of mechanics, methods of energy flows and a secant normal. Mathematical expressions have been obtained to estimate slip speeds and power friction losses in the engagement of a gear with IREFC, and a formula has been derived to determine the efficiency of a mentioned mechanical transmission. The calculation of slip speeds and power losses at the points of contact of a rolling element with cycloid profiles of wheels for selected initial parameters of a gear with IREFC has been presented. The friction power and the overall efficiency of the entire gear engagement have also been calculated. This work shows that power friction losses at the points of contact of a rolling element with cycloid profiles of tooth wheels of a gear with IREFC are not the same. The friction power in the contact of a rolling element with a cycloid profile of a cam is an order of magnitude higher than the friction power in the contact of a rolling element with a cycloid profile of a crown.

Keywords: friction power; efficiency; cycloid gears; rolling elements; free cage; cycloid engagement; sliding speed; cycloid profile; cycloid wheel; rolling friction; sliding friction

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1. Introduction

Mechanical transmissions with cycloid toothing are widely used when designing modern devices and mechanisms. This is explained by the increased resistance of such mechanisms to high loads due to the perception of only contact loads by links in the gearing [1]. Cycloid gears with intermediate rolling elements (IRE) have a set of high technical characteristics; however, the issue of power loss in a cycloid gear with intermediate rolling elements remains insufficiently studied.

Many works are devoted to the study of the engagement of cycloid gears [2–8]. But pin gears, in which there are no intermediate rolling elements, are mainly studied [9–18]. In [9], the efficiency coefficient in the engagement of a pin gear with pins without bushings and with bushings was investigated. It showed that using pins with bushings makes the transmission work smoother, and friction losses decrease. In another work [10], authors developed a nonlinear dynamic model of a needle bearing, taking into account the friction among a cycloid wheel, a needle roller and a bearing cage in a pin gear. The study [11] presented a method for diagnosing damage to a cycloid tooth wheel of pin toothing on a laboratory bench based on a frequency analysis of vibration signals. In [12], the distribution of forces in pin toothing was considered, taking into account the finger deflections of a pin wheel; an improved model of the load distribution of an uncoordinated kinematic cycloid–finger pair was proposed. In works [13,14], the authors considered the operation of a pin cycloid reducer and the influence of various factors on it. The authors of [15] investigated the impact of a gap in the engagement of a cycloid gearing on the distribution of forces, taking into account errors in the assembly, the manufacturing and the modification of a cycloid profile. In another paper [16], the authors proposed a semi-analytical model of the load distribution based on a three-dimensional elastic solution that considers the longitudinal configuration of a cycloid tooth. The model takes into account the contact deformation of a cycloid tooth, and loads are calculated in compliance with the conditions of compatibility and equilibrium. The authors in [17] proposed a new method for modifying the profile of a tooth of a pin gear, based on considering the complex effect of the distribution of the pressure angle, the gap in the engagement and the gap between a tip and a leg of a cycloid tooth. In [18], a new model of cycloid contact in an engagement with several rows of teeth was proposed. This model allows for the studying of the influence of the load on various methods of modifying cycloid teeth, including the accuracy of the pin gear, mechanical characteristics and its efficiency.

When studying cycloid gears with IRE, attention was mainly paid to research of engagement forces and stresses arising in the contact of a cycloid tooth profile with rolling elements [19,20]. The most promising gears with IREB are those with intermediate rolling elements and a free cage (IREFC), and the engagement of this gear was studied mainly to change the accuracy of the profile and the distribution of forces and contact stresses [3,4,21]. Gears with IRE, in general, and a gear with IREFC, in particular, can be used in different mechanisms of shut-off and adjusting fittings, in driving units of transport systems, in rotary drives and in drives of various installations, where it is necessary to use compact mechanisms capable of withstanding significant loads. Such mechanisms can work in an intermittent mode and in an average continuous operating mode. Mechanisms that are based on gears with IRE can operate both indoors and outdoors in various weather conditions. However, despite extensive studies of gears with IRE, the investigation of the friction in an engagement has not been given sufficient attention. Therefore, the analysis of power friction losses in the engagement of gears with IRE is relevant. Power losses in engagement and efficiency can be considered taking into account various factors: materials of contacting links, lubricants, etc. But in this paper, we will consider only the efficiency of the engagement of a gear with IREFC. Therefore, the purpose of this work is to determine the friction losses of engaging the links made of steel, without taking into account losses in lubrication and rolling bearings, on which the links of a gear with IREFC are installed.

Let us consider a gear with IREFC (Figure 1), which consists of an input shaft 1 with an eccentric disc 2 located on it, a tooth wheel 3 with a cycloid profile, which is mounted on the eccentric disc 2 through a rolling bearing 4, a chain of intermediate rolling elements 5 located in a cage 6 and a wheel 7 with internal cycloid teeth fixed motionlessly in a gear housing.

The motion in a gear with IREFC (Figure 1) is transmitted from the input shaft 1 through the eccentric 2 to the chain of rolling elements 5, which, rolling along the cycloid profile of the tooth wheel 7, transmit motion to the tooth wheel 3. Hence, rolling elements are engaged with cycloid profiles of two tooth wheels. Therefore, a gear with IREFC has two contact points of the engagement of rolling elements with tooth wheels, and this is a peculiarity of this gear, which should be taken into account when determining gear efficiency.

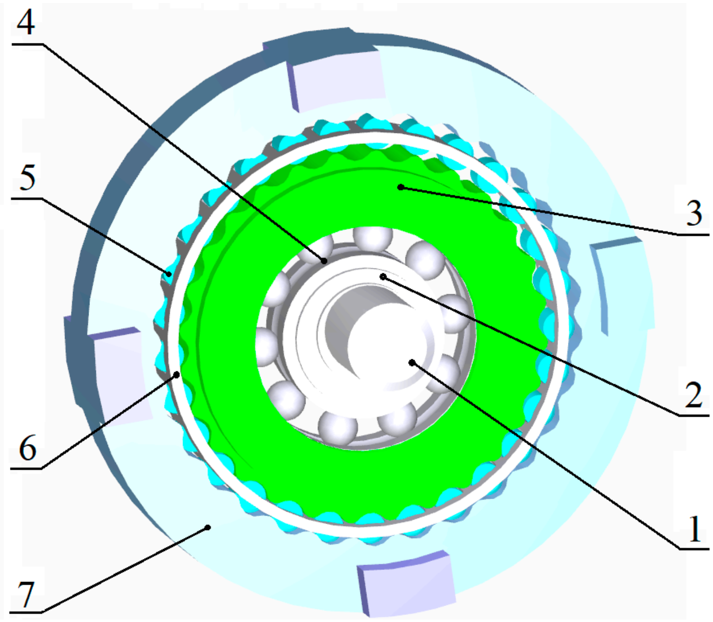


Figure 1. Scheme of a gear with intermediate rolling elements and a free cage: 1—input shaft; 2—eccentric disc; 3—cycloidal wheel-cam; 4—rolling bearing; 5—intermediate rolling elements; 6—cage; 7—cycloidal wheel-crown.

2. Determination of Efficiency and Friction Power at Points of Contact of Rolling Element with Cycloid Profiles of Wheels

Let us determine the efficiency of a gear with IREFC by analogy with rolling bearings. This is justified because, as in the case of rolling bearings, intermediate elements are located between two profiles. The only difference is that bearing rings have straight roller paths (if we consider a reamer) and cycloid wheels have a wavy roller path (in a reamer). But when considering a single rolling element in an enlarged way, the comparison is fair (Figure 2). In general, equations of cycloid profiles for a gear with IREFC are derived in [3].

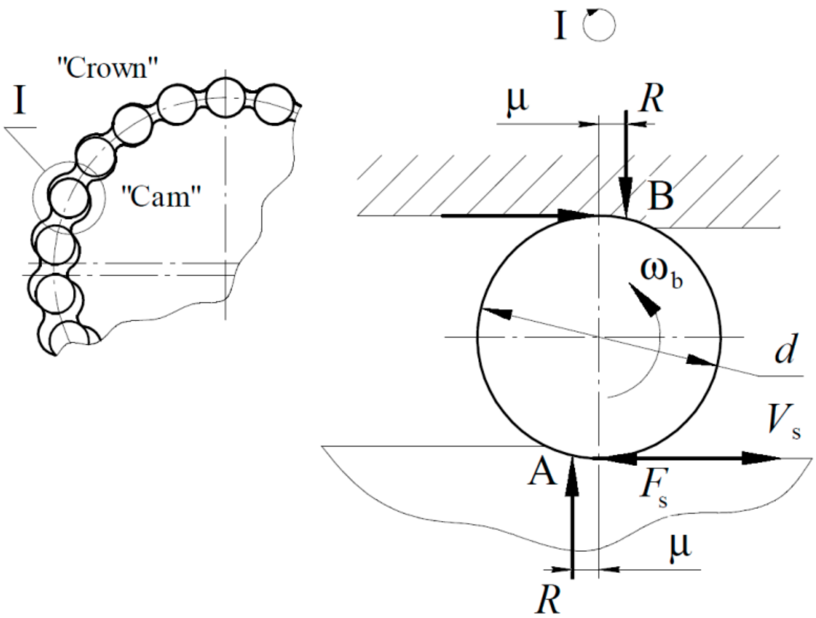


Figure 2. Forces and speeds at the points of contact of a rolling element with a profile of wheels.

Using a method of "energy flows" [22,23], let us consider the engagement of rolling elements with cycloid profiles in a gear with IREFC. The essence of the method is reduced to an analytical representation of efficiency in the form of a power ratio: the power of useful resistances N_{out} to the power of driving forces N_{in}:

$$\eta = \frac{N_{out}}{N_{in}} \tag{1}$$

In the case of a gear with IREFC, the power of useful resistances is N_{out}, which is the power on the tooth wheel 3 (Figure 1), since it is connected to the output shaft of the mechanism. And the power of driving forces N_{in} is the power at the input shaft of the gear supplied to the eccentric disc 2 (Figure 1). The power of useful resistances, i.e., the power spent on performing useful work, is the product of the torque at the output link of the gear (or the output shaft of the mechanism) and the angular velocity of this link. However, it can be presented in a different way. So, the power of useful resistances can be represented as the difference in powers of driving forces N_{in} and friction forces N_{s} arising in the engagement of rolling elements (Figure 2) with two cycloid profiles of gears 3 and 7 (Figure 1).

Then, taking into account the above, Expression (1) will take the following form:

$$\eta = \frac{N_{in} - N_s}{N_{in}} = 1 - \frac{N_s}{N_{in}} \tag{2}$$

The points of contact in the engagement of a gear with IREFC can be schematically represented (Figure 2) similarly to ball guides [23] of a translational or rotational motion. Taking advantage of the similarity of the engagement with a rolling support, the friction power can be represented as the sum of friction powers at the points of contact of rolling elements with cycloid profiles of wheels (Figure 2):

$$N_{\mathrm{s}} = N_{\mathrm{SA}} + N_{\mathrm{SB}} \tag{3}$$

Figure 2 shows that the rolling element is in contact with the cycloid profile of the cam at point A and with the cycloid profile of the crown at point B. According to the principle of operation of a gear with IREFC, a rolling element is rolled along a cycloid profile of a crown without sliding, i.e., only rolling friction is present at point B. When interacting with a cycloid cam profile, a rolling element rolls and slips along this profile; therefore, rolling friction and sliding friction are present at point A. When composing Formula (3), it is necessary to consider the friction power N_{SB} at point B as a single-component power that depends only on rolling friction. At the same time, the friction power N_{SA} should be considered as complex, depending on the rolling friction N_{rf} and the sliding friction N_{sf} of a rolling element along a cycloid cam profile. Therefore, the friction power at point A can be defined as follows:

$$N_{\mathrm{SA}} = N_{rf} + N_{sf}$$

The rolling friction at point A is determined from an expression:

$$N_{rf} = R\mu \cdot \omega_b$$

where

R—the engagement force of a rolling element and a cam, which is also a normal force to contacting surfaces;
μ—the coefficient of the rolling friction;
ω_{b}—the rolling element rotation rate.

The engagement force R is determined by Hooke's law, based on the proportional dependence of force and deformation [3,4].

And the sliding friction at point A is defined as:

$$N_{sf} = F_s \cdot V_s = R f_s \cdot V_s$$

where

F_s—the friction force in contact with a rolling element and a cycloid cam profile;
F_s—the coefficient of the sliding friction;
V_s—the sliding speed of a rolling element along a cam profile.

Then, the friction power at point A is calculated using the following formula:

$$N_{SA} = R(\mu \cdot \omega_b + f_s \cdot V_s) \quad (4)$$

The friction power at point B, as already mentioned, depends only on rolling friction forces and is determined by a formula:

$$N_{SB} = R\mu\omega_b \quad (5)$$

Engagement forces are determined based on the Hertz theory, assuming a linear relationship between a force and a deformation. A detailed definition of these efforts was described earlier in [3]. Let us choose friction coefficients based on the fact that steel links of high hardness are involved in contact. Supposing this, the coefficient of rolling friction can be assumed to be μ = 0.001 m, and the coefficient of sliding friction is f_s = 0.1 [23]. Speed parameters of a gear with IREFC will be determined based on the geometry of its engagement.

3. The Determination of the Slip Speed in an Engagement

To determine the slip speed in the engagement of a gear with IREFC, let us consider contact points of an arbitrary rolling element with cycloid profiles of wheels (Figure 3). As is known [3], these contact points are located on a general normal to profiles passing through a pitch point of a gear with IREFC. Figure 3 presents an arbitrary rolling element on a circumference of centers r_c. Points of contact with cam A and crown B are indicated on a rolling element. Also in Figure 3, P indicates the pitch point of a gear with IREFC; V^{12} is the speed at a point of contact of a rolling element with a cam profile; V^{32} is the speed of a rolling element with a crown profile.

The slip speed is determined using the secant normal method [24]. Since velocities V^{12} and V^{32} are applied to a rolling element (Figure 3), the slip speed is determined by adding a vector of these speeds:

$$\overline{V_S} = \overline{V}^{12} + \overline{V}^{32} \quad (6)$$

The speeds at points of contact of a rolling element with profiles are analytically determined relative to pitch point P, using a quadratic dependence of the distance from the pitch to contact points on the angle of a rolling element position φ_2. The distance from the pitch point P to the center of the rolling element (Figure 3), expressed through initial parameters of a gear with IREFC, has the form [3]:

$$O'P = L = r_2\sqrt{\chi + 1 - 2\chi\cos\varphi_2} \quad (7)$$

The initial parameters of a gear with IREFC are as follows [3]: the radius of the generating circle r_2, the displacement coefficient χ, the number of rolling elements Z_2 and the radius of the rolling element r_b.

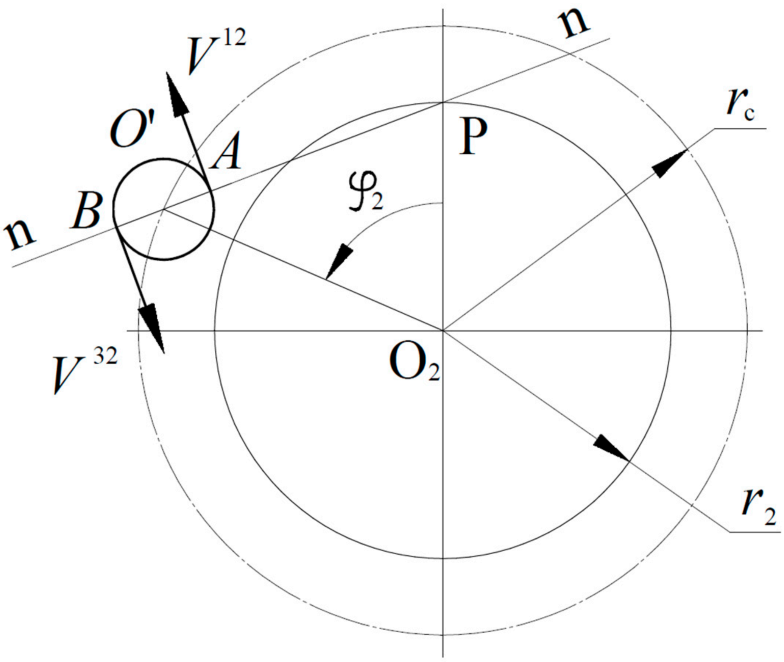


Figure 3. Calculation scheme for determining the slip speed in the engagement of a gear.

Then, the distance from the pitch point P to points of contact A and B (Figure 3), respectively, is equal to:

$$L_A = L - r_b; L_B = L + r_b \tag{8}$$

Cam–cage and cage–crown gear ratios are determined by the following ratios:

$$i_{12} = \frac{\omega_c}{\omega_2}; i_{23} = \frac{\omega_2}{\omega_{cr}} \tag{9}$$

where

ω_c—the angular velocity of the cam;
ω_2—the angular velocity of the cage;
ω_{cr}—the angular velocity of the crown.

Then, the linear velocities at the points of contact of the rolling element with profiles of cycloid wheels through the angular velocity of the cage are expressed, taking into account Formulas (8) and (9), respectively:

$$V^{12} = \omega_2(\mathrm{L} - r_b)(i_{12} - 1)V^{23} = \omega_2(\mathrm{L} + r_b)(i_{32} - 1) \tag{10}$$

Considering the differential dependence of the velocity on a path and the difference in a traveled path by the point of a rolling element in contact with cycloid profiles of a cam and a crown, we can determine an analytical expression for a slip speed. To do this, let us substitute Expression (10) for linear velocities at contact points into Formula (6) and obtain an analytical expression of the slip speed through gear ratios of contacting links and the angular velocity of a cage:

$$V_s = \omega_c\left[L_A\left(1 - \frac{1}{i_{12}}\right) + L_B\left(\frac{i_{32} - 1}{i_{12}}\right)\right] \tag{11}$$

Replacing angular velocities in Expression (9) with radii of corresponding circles, we will rewrite formulas of gear ratios between links as follows:

$$i_{12} = \frac{r_2}{r_c}$$

$$i_{32} = \frac{r_2}{r_{cr}}$$

In turn, the radii of the generating circles of gears have the following dependence on the initial parameters of a gear with IREFC:

$$r_c = r_2\left(1 - \frac{1}{Z_2}\right)$$

$$r_{cr} = r_2\left(1 + \frac{1}{Z_2}\right)$$

Taking into account the proportional dependence of the radii of the generating circles of links, let us substitute expressions for gear ratios in the following dependence (11):

$$V_{\mathrm{s}} = \omega_{\mathrm{c}}\left[L_A \cdot \frac{1}{Z_2} + L_B\left(\frac{1}{\left(1 + \frac{1}{Z_2}\right)} - 1\right) \cdot \left(1 - \frac{1}{Z_2}\right)\right]$$

and after transformations, we will obtain an analytical expression of the slip speed through the number of intermediate rolling elements and the angular velocity of a cam:

$$V_{\mathrm{s}} = \omega_{\mathrm{c}}\left[\frac{L_A}{Z_2} + \frac{L_B(1 - Z_2)}{Z_2^2 + Z_2}\right] \quad (12)$$

It follows from Formula (12) that the sliding velocity V_{S} of a rolling element along a cam profile is obtained with a negative sign. This indicates that the angular velocity of a cam rotation is directed in the opposite direction of the sliding velocity of a rolling element.

In Expression (12), the angular velocity of a cam is expressed through the angular velocity of an input shaft (Figure 1):

$$\omega_{\mathrm{c}} = \omega_1 \frac{2}{Z_2 - 1} \quad (13)$$

where ω_1—the angular velocity of the input link (input shaft).

Let us substitute (13) into Equation (12) and obtain an expression of the slip speed in the engagement of a gear with IREFC through the number of teeth and the angular velocity of an input link:

$$V_{\mathrm{s}} = L_A\omega_1 \frac{2}{Z_2(Z_2 - 1)} - L_B\omega_1 \frac{2}{Z_2(Z_2 + 1)}$$

After the conversion, we will obtain:

$$V_{\mathrm{s}} = \frac{2\omega_1}{Z_2}\left[\frac{L_A}{Z_2 - 1} - \frac{L_B}{Z_2 + 1}\right] \quad (14)$$

The angular velocity of the input link ω_1, as a rule, is equal to the angular velocity of a motor shaft, to which a gear is attached.

4. The Determination of the Total Efficiency in the Engagement of a Gear

After determining the slip speed (14) of a rolling element by a cam profile, it is possible to proceed to derive an analytical expression for friction powers at contact points and to obtain the full efficiency of engaging a gear with IREFC.

Determining the friction power at the points of contact of a rolling element with cycloid profiles of wheels requires determining only the angular velocity of the rolling element ω_b. The angular velocity of the rolling element can be found through the proportional dependence of the radius (r_b) and the linear velocity (V^{23}) at a point of contact with a crown profile:

$$\omega_b = \frac{V^{23}}{r_b} \tag{15}$$

The linear velocity of the rolling element (10) at a point of contact with a cycloid profile of a crown, expressed through the angular velocity (13) of an input link, is analytically determined as:

$$V^{23} = L_B\omega_1 \cdot \frac{2}{Z_2(Z_2+1)} \tag{16}$$

where

Z_2—the number of rolling elements of a gear;
L_B—the distance from the pitch point to the point of contact of a rolling element with a crown profile.

The distance from the pitch point L_B, expressed taking into account (7) and (8), is the following:

$$L_B = r_b + r_2\sqrt{\chi + 1 - 2\chi\cos\varphi_2}$$

Let the angular velocity of a rolling element be expressed with initial parameters and the angular velocity of the input link of a gear with IREFC. For this purpose, Expression (15) of the angular velocity of the rolling element, taking into account (16) and without the consideration of the sign of the relative movement of the links, will be written as follows:

$$\omega_b = L_B \cdot \omega_1 \cdot \frac{2}{r_b Z_2(Z_2+1)} \tag{17}$$

Then, analytical Expression (4) of the friction power at point A, considering (14) and (17), will take the form of:

$$N_{SA} = R\left(\mu L_B\omega_1 \frac{2}{r_b Z_2(Z_2+1)} + f_s \cdot \frac{2\omega_1}{Z_2}\left[\frac{L_A}{Z_2-1} - \frac{L_B}{Z_2+1}\right]\right)$$

and after the conversion, we will finally write an equation for calculating the friction power at point A:

$$N_{SA} = \frac{2R\omega_1}{Z_2}\left(\frac{L_B}{Z_2+1}\left(\frac{\mu}{r_b} - f_s\right) + \frac{f_s L_A}{Z_2-1}\right) \tag{18}$$

Taking into account (13), Expression (5) can be rewritten as:

$$N_{SB} = R\mu \cdot \frac{2L_B \cdot \omega_1}{r_b Z_2(Z_2+1)} \tag{19}$$

Therefore, considering (18) and (19), after the conversion, Expression (3) will have the following view:

$$N_m = \frac{2R\omega_1}{Z_2}\left[\frac{L_B}{Z_2+1}\left(\frac{2\mu}{r_b} - f_s\right) + \frac{f_s L_A}{Z_2-1}\right] \tag{20}$$

In Equation (20), the friction power is expressed through the angular velocity of the input link and represents the quadratic dependence on the number of rolling elements. Then, to determine the efficiency of a gear with IREFC, it is necessary to specify the power of its driving forces. The power of the driving forces is defined as the product of the driving torque (M_{in}), which is the torque on an electric motor shaft, by the angular velocity of the input shaft of an electric motor (the input link of a gear with IREFC), while the angular velocity of the input link will decrease. Then, Expression (2), which determines the efficiency of the gear in question, can be written as an analytical expression:

$$\eta = 1 - \frac{2R}{M_{in}Z_2}\left[\frac{L_B}{Z_2+1}\left(\frac{2\mu}{r_b} - f_s\right) + \frac{f_s L_A}{Z_2-1}\right] \tag{21}$$

The obtained dependence (21) allows for estimating the efficiency of a gear with IREFC in contact with a rolling element with profiles of cycloid wheels. But one must understand that half of the rolling elements are involved in the engagement and transmission of power. In this regard, in order to completely determine the friction losses in the engagement of a gear with IREFC, it is necessary to calculate these losses on each rolling element transmitting power in the engagement:

$$N_{m_i} = \frac{2R_i\omega_1}{Z_2}\left[\frac{L_{B_i}}{Z_2+1}\left(\frac{2\mu}{r_b} - f_s\right) + \frac{f_s L_{A_i}}{Z_2 - 1}\right] \tag{22}$$

In this case, i covers the values from 1 to $Z_2/2$ in increments of 1, which corresponds to the number of the rolling element, when numbering from a rolling element, located in a depression, to a rolling element, located on a protrusion of a cycloid profile. In view of this, an algebraic sum of expressions will yield the total power of friction losses in the engagement of a gear with IREFC.

And to determine the efficiency of the complete engagement of a gear with IREFC, it is necessary to obtain an algebraic product of the efficiency on each rolling element transmitting power in the engagement of a gear:

$$\eta = \prod_{1}^{Z_2/2}\left(1 - \frac{2R_i}{M_{in}Z_2}\left[\frac{L_{B_i}}{Z_2+1}\left(\frac{2\mu}{r_b} - f_s\right) + \frac{f_s L_{A_i}}{Z_2 - 1}\right]\right) \tag{23}$$

The resulting Expression (23) allows for the determining of the efficiency of a gear with IREFC depending on its initial parameters.

5. An Analysis of the Change in the Friction Power on Loaded Rolling Elements of a Gear

Let us consider a gear with IREFC, which has the following initial parameters:

- r_2 = 20 mm;
- Z_2 = 26;
- χ = 1.4;
- r_b = 2 mm.

To determine the engagement losses of a gear, having specified initial parameters, we will also set the power and the speed of an electric motor. Let the power of the electric motor be 1500 watts, and the number of revolutions of the electric motor shaft will be n = 1500 rpm. Then, the angular velocity of the rotation of the input link of a gear with IREFC will be ω_1 = 157.1 s^{-1}, and the torque on the electric motor shaft and, consequently, on the input transmission link will be M_{in} = 9.55 Nm.

According to Formulas (8) and (14), we will calculate the geometric parameters of the L_A and L_B engagement, as well as the value of the slip speed for each rolling element transmitting a load in the engagement of a gear with IREFC. The obtained values for each rolling element are presented in the form of Table 1. Further, after calculating the friction power in the contact of each rolling element with cycloid wheel profiles according to Formula (22), Table 1 can be supplemented. For more information on the distribution of the friction power in the engagement, based on analytical Expressions (18) and (19), we will also provide power values at points A and B.

As already noted, a sign before the value of the slip speed indicates the multidirectionality of the vectors of the linear velocity of the cam relative to the linear velocity of the rolling element at the point of contact with a cycloid profile of the cam. As Table 1 shows, the highest slip speed occurs at the first rolling element, i.e., the one located in a depression. And as the rolling element moves closer to the top of a tooth, the slip speed decreases.

Table 1. Values of geometric, kinematic and energy parameters of the engagement of a gear with IREFC.

No. of the Rolling Element	L_A, m	L_B, m	V_S, 10^{-3}, m/s	N_S, *W*	N_{SA}, *W*	N_{SB}, *W*	*R*, *N*
1	0.006	0.01	−1.58	0	0	0	0
2	0.007826	0.011826	−1.51	3.3487	0.1455	0.0013	651.33
3	0.011867	0.015867	−1.37	6.2414	0.1867	0.0024	896.22
4	0.016592	0.020592	−1.2	8.6756	0.1818	0.0033	953.81
5	0.021404	0.025404	−1.02	10.5946	0.1624	0.0039	940.35
6	0.026051	0.030051	−0.86	11.9113	0.1384	0.0044	891.39
7	0.030388	0.034388	−0.7	12.5565	0.1140	0.0046	819.65
8	0.034318	0.038318	−0.56	12.4937	0.0909	0.0046	730.96
9	0.037764	0.041764	−0.44	11.7252	0.0701	0.0043	628.82
10	0.040664	0.044664	−0.33	10.2936	0.0519	0.0037	515.85
11	0.04297	0.04697	−0.25	8.2790	0.0363	0.0030	394.33
12	0.044645	0.048645	−0.19	5.7950	0.0228	0.0021	266.43
13	0.04566	0.04966	−0.16	2.9821	0.0110	0.0011	134.28
14	0.046	0.05	−0.14	0	0	0	0

A change in the friction power in the engagement of rolling elements with cycloid profiles has a nature that differs from a change in the slip speed. On the first and last rolling elements, the friction power is practically absent, which is explained by the lack of forces in the engagement in these places. That is, in these positions, rolling elements do not transmit load or power. The maximum power loss can be attributed to rolling elements, located approximately in the middle of the chain of elements, transmitting the power in the engagement of a gear with IREFC.

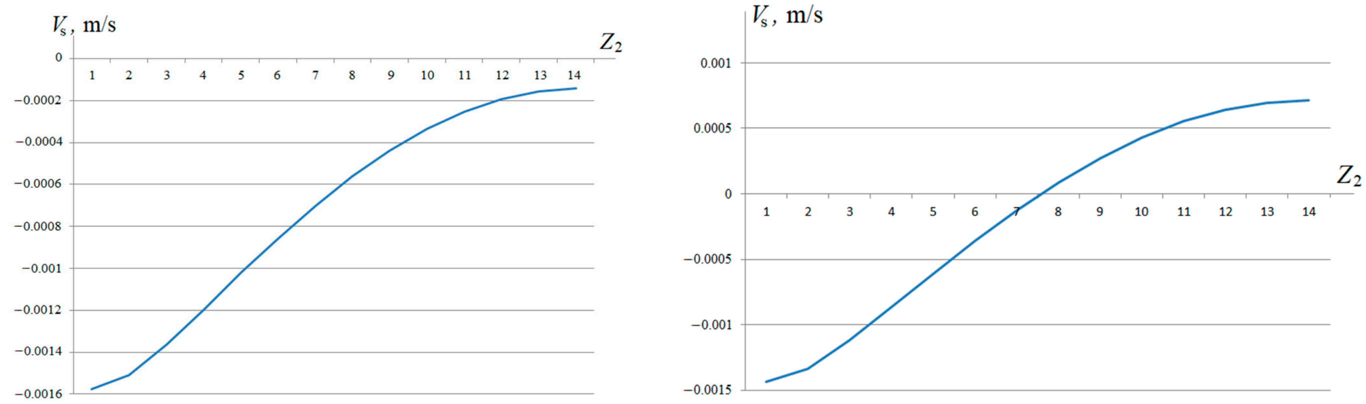
Table 1 demonstrates that the friction power at the point of contact of a rolling element with a cam profile is much greater than the friction power at the point of contact with a crown profile.

Having performed the calculation according to Formula (21) for each rolling element and, finally, according to Formula (23), we will obtain a total efficiency of a gear with IREFC with specified initial parameters equal to $\eta = 0.93$.

6. Results and Discussions

By linearly changing the values of the initial parameters of a gear with IREFC, one can see nonlinear changes in the values of the slip speed and the friction power. For example, by changing the radius of the generating circle r_2 from 20 to 30 mm, we can observe a significant change in the slip speed (Figure 4). When r_2 = 20 mm, the entire graph of the slip speed lies in a negative range of values (Figure 4a). And when the radius increases to 30 m, the graph moves from an area of negative values to an area of positive values (Figure 4b). However, the friction power in the engagement practically does not change (Figure 5). When r_2 = 20 mm, the maximum friction power in the engagement of a gear is N_S = 12.56 W, and N_S = 12.37 W when r_2 = 30 mm. The presented data were determined at the rated power of an electric motor of 1.5 kW, as indicated earlier.

On the other hand, if the radius of the rolling element is changed, when other initial parameters remain unchanged, both the slip speed and the friction power undergo significant changes. For example, if we change r_b from 1.2 to 2.8 mm in increments of 0.4 mm, it is possible to observe a sequential change in the slip speed *V*s. in the engagement of a rolling element with a cycloid cam profile (Figure 6) and the friction power N_S on each rolling element in the engagement of a gear with IREFC (Figure 7).



(**a**) (**b**)

Figure 4. The change in *V*s depending on the position of the rolling element: (**a**) slip speed when r_2 = 20 mm; (**b**) slip speed when r_2 = 30 mm.

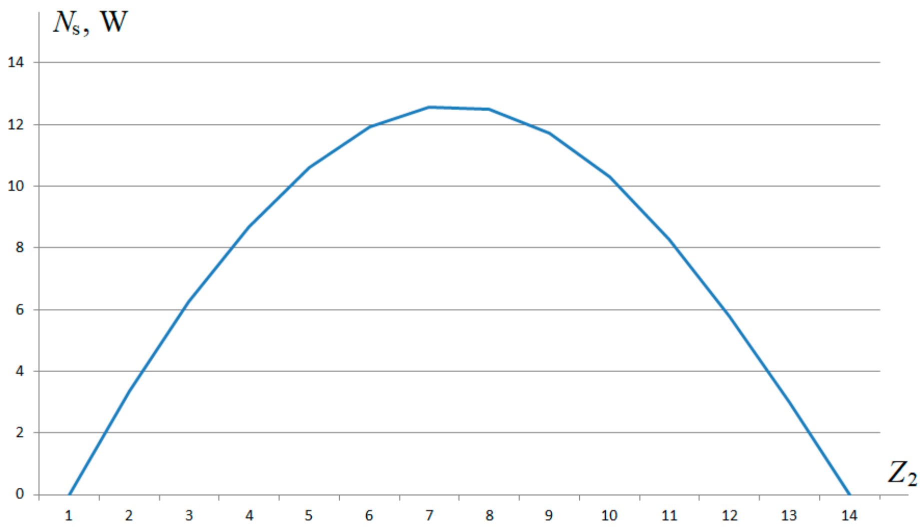


Figure 5. The change in N_S depending on the position of the rolling element.

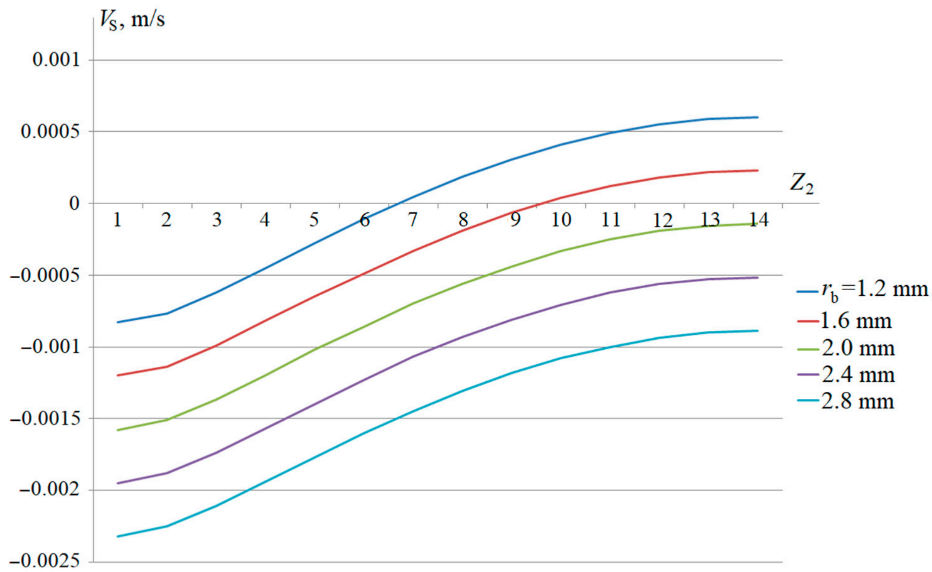


Figure 6. The change in *V*s depending on the position of the rolling element when r_b values are different.

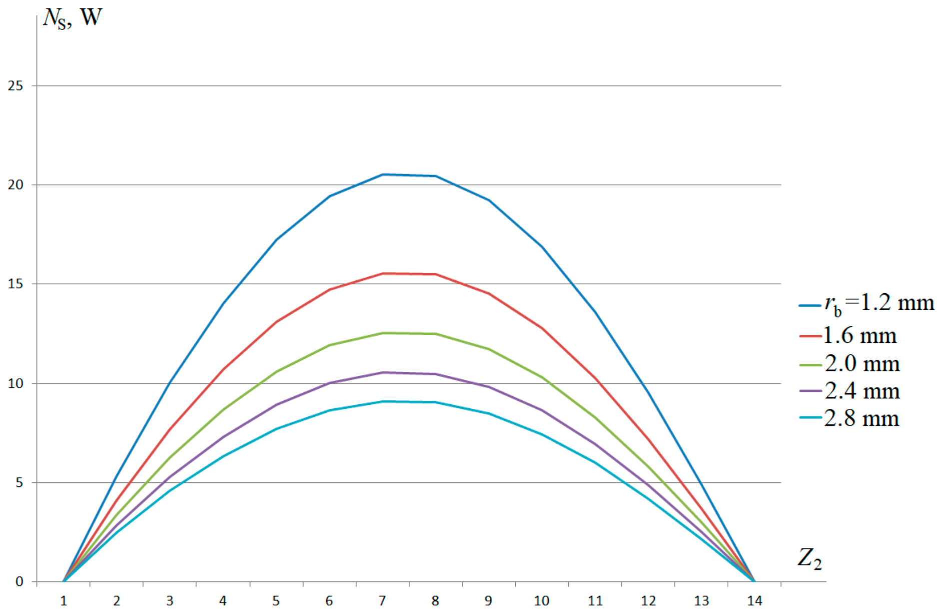


Figure 7. The change in N_S depending on the position of the rolling element at different r_b values.

The graphs of a cosinusoid (Figure 6) show that when the radius of the rolling element increases, the values of the slip speed in the engagement of a rolling element with a cycloid cam profile shift to the negative zone. As already noted, the sign of the slip speed shows the relative direction of its vector. However, this sign has no effect on the determination of the friction power in the engagement, since for this purpose, the slip speed is taken modulo. It should be noted that when changing only the radius r_b, the range of changing the slip speed between the first rolling element and the last one in the power transmission section does not change and is ΔV_S = 0.00143 m/s. Also, the law of changing the slip speed along the cycloid profile of a tooth does not change.

Parabolic graphs (Figure 7) show that when the radius of a rolling element increases, the value of the maximum slip power decreases. When r_b = 1.2 mm, the maximum friction power in the gear engagement is N_S = 20.54 W, and N_S = 9.11 W when r_b = 2.8 mm. This is explained by the direct proportional dependence of the power on the force, the speed and the inverse dependence on the geometry of a rolling element. That is, as the radius of a rolling element increases, the circumferential velocity on its surface increases and, accordingly, the slip speed in contact decreases, as can be seen in the graphs of Figure 6. At the same time, the force in contact with a cycloid profile with a rolling element remains unchanged, since the radius of a generating circle is constant (r_2 = const). Therefore, the friction power decreases when the radius of a rolling element increases by 1.6 mm and the maximum friction power decreases by 11.4 W (Figure 8).

For example, when designing mechanisms based on a gear with IREFC, in order to reduce power losses, it is necessary to choose the largest possible diameter of a rolling element. This will allow for reducing the friction power in the gear engagement and therefore increasing the efficiency of the entire mechanism.

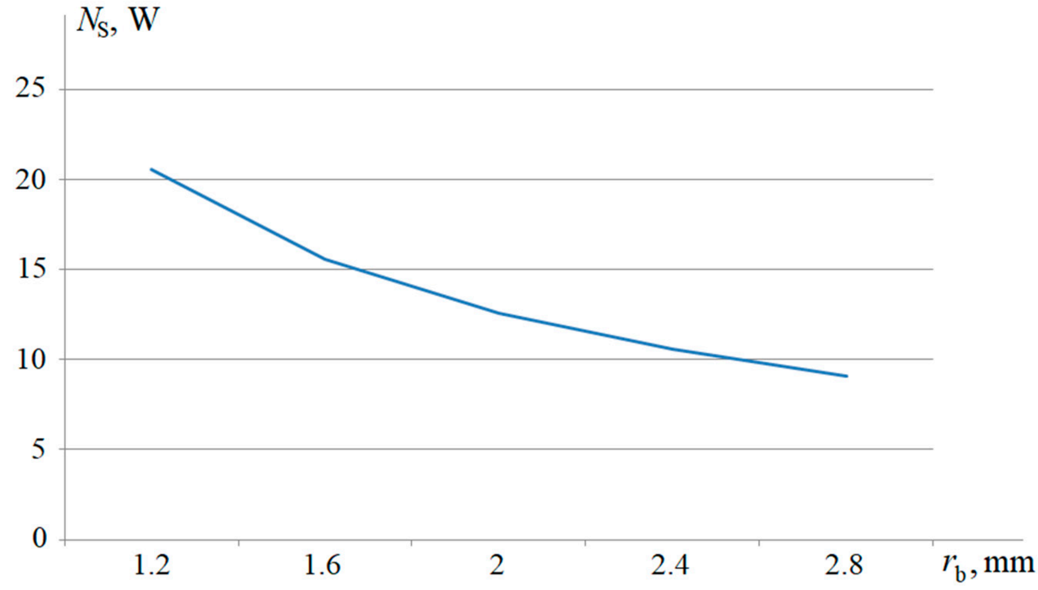


Figure 8. The change in the maximum power N_S depending on r_b in the gear engagement.

7. Conclusions

In conclusion, this work provides analytical expressions for estimating power losses in the engagement of a gear with IREFC, and a formula for determining the efficiency of a specified mechanical transmission has been derived. The calculation of the efficiency, the friction power in the engagement and the power losses at the points of contact of a rolling element with cycloid profiles of wheels for selected initial parameters of a gear with IREFC has been presented.

The authors have shown that the power friction losses at the points of contact of a rolling element with cycloid profiles of tooth wheels of a gear with IREFC are not the same and vary according to the parabolic law. The friction power in the contact of a rolling element with a cycloid profile of a cam is an order of magnitude higher than the friction power in the contact of a rolling element with a cycloid profile of a crown. For example, in contact with the cam, it is 0.091 W, and in contact with the crown, it amounts to 0.0046 W. The authors have also demonstrated that a change in the radius of the generating circle r_2, other things being equal, does not influence the friction power in the engagement of a gear in any way. By changing the radius r_2 1.5 times, we obtain a change in the friction power in the engagement by 0.19 W. On the other hand, a change in the radius of the rolling element r_b influences simultaneously the values of the slip speed in the contact of a cycloid cam profile with a rolling element and the friction power in the gear engagement. This work demonstrates that the slip speed in the engagement varies along a cosinusoid with a possible transition from the fourth quarter to the first quarter of the coordinate plane. Hence, a method for determining the efficiency of engaging a gear with intermediate rolling elements and a free cage was proposed.

The obtained results allow for increasing the accuracy of detection by 1.5% determining the efficiency of a projected gear with IREFC and, therefore, more precisely selecting the power of an electric motor for electromechanical drives based on the mentioned mechanical transmission. The obtained results should be used for designing electromechanical drives, as well as for studying the influence of other initial parameters on the change in power distribution and the efficiency in the engagement of a cycloid gear with intermediate rolling elements and a free cage.

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