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Accelerating Consensus Reaching Through Top Persuaders: A Social Persuasion Model in Social Network Group Decision Making

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Abstract: In traditional group decision-making models, it is commonly assumed that all decision makers exert equal influence on one another. However, in real-world social networks, such as Twitter and Facebook, certain individuals—known as top persuaders—hold a disproportionately large influence over others. This study formulates the consensus-reaching problem in social network group decision making by introducing a novel framework for predicting top persuaders. Building on social network theories, we develop a social persuasion model that integrates social influence and social status to quantify individuals' persuasive power more comprehensively. Subsequently, we propose a new CRP that leverages the influence of top persuaders. Our simulations and comparative analyses demonstrate that: (1) increasing the number of top persuaders substantially reduces the iterations required to achieve consensus; (2) establishing trust relationships between top persuaders and other individuals accelerates the consensus process; and (3) top persuaders retain a high and stable level of influence throughout the entire CRP rounds. Our research provides practical insights into identifying and strategically guiding top persuaders to enhance the efficiency in consensus reaching and reduce social management costs within social networked environments.

Keywords: group decision making; trust propagation; trust relationships; opinion dynamics; group consensus

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1. Introduction

In recent years, advancements in communication technology and the proliferation of social media platforms have made information sharing and opinion expression more accessible [1]. These technological developments have increased social interactions among individuals and significantly impacted social network group decision making (SNGDM).

SNGDM refers to a decision-making scenario in which individuals within a social network express their opinions on various alternatives to reach a collective decision [2–6]. Generally, conflicts often arise among individuals, and the consensus reaching process (CRP) is necessary to resolve differences and achieve a unified solution (e.g., [7–10]). Unlike traditional group decision making (GDM), which encourages individuals to modify their preferences to align with group suggestion, the CRP in SNGDM primarily relies on preference adjustment through trust relationships [11–13].

Trust relationships play a crucial role in SNGDM, as individuals are more inclined to agree with those they trust [14–17]. The existing literature on SNGDM has demonstrated that trust relationships can influence decision making through mechanisms such as preference estimation, weight allocation, and trust evolution [18,19]. For example, Dong et al. [2] emphasize the role of leaders and improvements in trust relationships in achieving consensus. Zhang et al. [20] propose a trust evolution model that incorporates trust degrees and opinion similarities, introducing a trust evolution-based exogenous feedback mechanism. Additionally, You et al. [18] develop a reputation-based trust model to establish trust relationships among individuals through direct interactions and word-of-mouth recommendations.

To date, research on trust relationships in SNGDM usually encounters the following issues:

(1) In traditional CRPs of SNGDM, decision makers tend to accept advice from individuals they trust. Based on this idea, several feedback mechanisms have been designed based on trust relationships that make the adjusting opinions more persuasive [2,21,22]. For example, Li et al. [23] proposed a feedback mechanism considering trust relationships and bounded confidence for consensus reaching in SNGDM. However, existing SNGDM research often neglects the critical role of limited attention theory, which suggests that individuals have finite cognitive resources and cannot process all available information uniformly [24]. In complex decision environments, decision makers naturally prioritize opinions they perceive as most relevant or urgent, concentrating their attention on a small subset of their most trusted individuals. This selective attention phenomenon highlights the pivotal role of limited attention in shaping CRPs, especially when combined with trust relationships. Despite its importance, limited attention has rarely been systematically integrated with trust-based feedback mechanisms in SNGDM.

(2) In most studies of SNGDM, trust degrees among individuals are mainly determined by their trust relationships within the social network, such as network centrality metrics [3,12]. In general, individuals with high centrality scores are assumed to gain greater trust from others [25–27]. While centrality-based approaches effectively capture structural influence within social networks [28], they often overlook the role of individual preferences in shaping trust degrees. Recent research indicates that individuals whose opinions align more closely with the group opinion tend to achieve higher social status and earn others' trust more easily [29]. For example, an individual with relatively low centrality but high preference alignment may still exert significant influence in the decision-making process. Therefore, it is crucial to jointly consider both network centrality and individual preference alignment to provide a more accurate and comprehensive representation of trust degrees in SNGDM.

To address these challenges, this study focuses on the phenomenon of social persuasion in social networks. Social persuasion refers to the principles and processes through which an individual's attitudes, beliefs, or behaviors are influenced by other individuals within a social network [30]. According to social network theories, social persuasion arises from various forces, including social influence and social status [31,32]. Notably, social persuasion provides the basis for identifying top persuaders, who exert disproportionately large influence over other individuals [33]. For instance, on platforms such as

Facebook and Twitter, a small number of key opinion leaders (KOLs) dominate discussions and capture the attention of vast audiences on the platform.

In this study, we formulate the consensus-reaching problem with top persuaders and address the following questions: (1) How to predict the top K persuaders? (2) How do these top persuaders affect group consensus? To solve these questions, we first develop a novel trust degree estimation method to quantify individuals' ability to persuade others. Top persuaders are identified based on the estimated trust degrees. Then, we propose a new feedback mechanism based on top persuaders and investigate how they affect consensus reaching and final group consensus by simulations and comparative analyses. The main contributions and highlights of this study are summarized as follows:

(1) A novel method for determining trust degrees among individuals in SNGDM is proposed. The proposed social persuasion model integrates both social influence and social status, where social influence is evaluated using centrality metrics and social status is measured by the consensus degrees of individuals. Compared with existing methods that primarily focus on centrality metrics, our approach provides a more comprehensive framework to capture the critical factors influencing trust degrees. Experimental results demonstrate that the social persuasion model outperforms those considering only social influence or trust relationships in achieving group consensus.

(2) A novel CRP with top persuaders is proposed, which includes two key components: (1) Applying opinion dynamics for individuals based on the social persuasion model; (2) establishing trust relationships between top persuaders and other individuals to help the latter better adjust their preferences and achieve consensus. Our results show that top persuaders maintain their influence throughout the entire CRP rounds.

(3) Our study has practical implications for both business and society. By effectively predicting top persuaders, our method offers substantial value for various applications centered around social networks. For instance, a firm can use our method to identify top persuaders among potential customers, encourage them to adopt a product or service, and leverage their influence to drive wider adoption among other customers.

We organize the remainder of the paper as follows. Section 2 reviews the foundational concepts necessary for our proposal. Section 3 introduces the consensus-reaching problem with social persuasion and outlines our resolution framework. Section 4 details several simulation experiments and comparative analyses. Finally, the main contributions and future studies are drawn in Section 5.

2. Preliminaries

This section briefly introduces preliminaries concerning the traditional GDM problem, social network analysis, and opinion dynamics in a social network, which provide basic knowledge to develop and understand our proposal.

2.1. Traditional GDM Problem

In this article, the GDM problem is defined as a scenario where a group of decision makers $V = \{v_1, v_2, \cdots, v_m\} (m \geq 2)$ express their preferences regarding a set of alternatives $X = \{x_1, x_2, \cdots, x_n\} (n \geq 2)$ to reach a collective decision. Each decision maker $v_k \in V$ provides their preferences over the set of alternatives X. For simplicity, this study assumes that each v_k expresses their preferences using an additive preference relation for pairwise comparisons of alternatives (x_i, x_j).

Definition 1. Let $P^k = (p_{ij}^k)_{n \times n}$ be an additive preference relation, where $p_{ij}^k \in [0,1]$ denotes the preference degree of alternative x_i over alternative x_j provided by decision maker v_k, which has the additive reciprocity property $p_{ij}^k + p_{ji}^k = 1$ and $p_{ii}^k = 0.5$ for

$\forall i,j \in \{1,\dots,n\}$. Specifically, $p_{ij}^k > 0.5$ means alternative x_i is preferred to alternative x_j, and $p_{ij}^k = 0.5$ means there is no difference between alternative x_i and x_j.

Traditionally, GDM problems are divided into two main processes [34,35]: the CRP and the selection process. Furthermore, the CRP comprises two procedures: consensus measure and feedback adjustment. The general framework of GDM is illustrated in Figure 1.

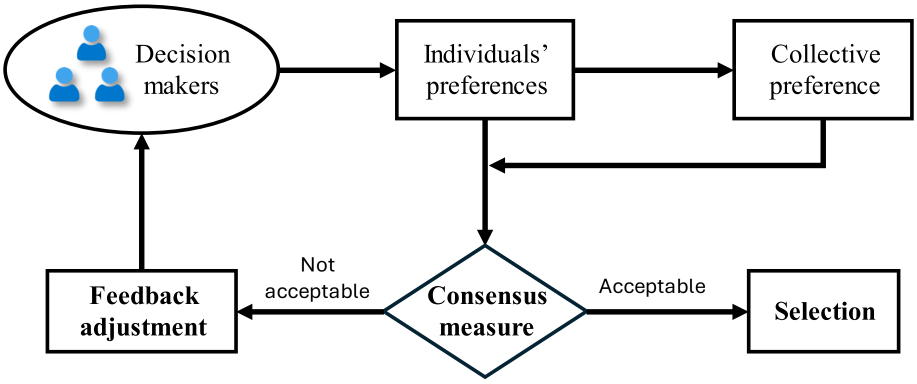

Figure 1. The general framework of GDM.

(1) Consensus measure

The consensus measure is used to assess the consensus degree among decision makers in the group, which is usually calculated by measuring the distances between individual preferences and the collective preference [36,37]. In this study, we employ the weighted average (WA) operator to derive the collective preference $P^c = \left(p_{ij}^c\right)_{n\times n}$, defined as:

$$p_{ij}^c = \sum_{k=1}^m \pi_k p_{ij}^k, \text{ for } i,j = 1,\dots,n \tag{1}$$

Let $\{\pi_1, \pi_2, \dots, \pi_m\}$ be the weights of a set of individuals, where $\pi_k \geq 0$ denotes the weight of individual v_k in the aggregated collective preference and $\sum_{k=1}^m \pi_k = 1$. Next, we measure consensus degree of each individual v_k and calculate the overall consensus degree $CD \in [0,1]$, which are described as follows:

$$CD(v_k) = 1 - \frac{\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\left|p_{ij}^k - p_{ij}^c\right|}{n(n-1)/2} \tag{2}$$

$$CD = \sum_{k=1}^m \pi_k CD(v_k) \tag{3}$$

Here, a larger CD value indicates a higher consensus degree. Achieving complete consensus ($CD = 1$) in real-world GDM problems is both challenging and often unnecessary [38–40]. This study employs a soft consensus approach, where a predefined consensus threshold $\mu \in [0,1]$ is established to determine an acceptable level of consensus, and a maximum consensus time T is set to avoid the failure of the consensus process. Once the overall consensus degree reaches the threshold or the consensus time reaches the maximum round, the current collective preference is considered as the final group solution.

(2) Feedback adjustment

When the group does not reach the consensus threshold μ, individuals will be advised to adjust their preferences to improve the group consensus degree. The implementation of preference adjustment in CRPs consists of two classical rules [41,42]: identification rule (IR) and direction rule (DR). The IR helps to identify individuals who significantly deviate from the collective preference, specifically those who do not reach an acceptable consensus $CD(v_k) < \mu$. The DR, on the other hand, provides the necessary guidance on how individuals identified based on the IR should adjust their preferences. It ensures that individual preferences move closer to the collective preference and then increases the group consensus degree CD.

(3) Selection process

When achieving an acceptable consensus, the selection process will be utilized to derive the final collective preference $P^c = \left(p_{ij}^c\right)_{n\times n}$ and rank the alternatives $X = \{x_1, x_2, \cdots, x_n\}$. The ranking of alternative x_i can be generated based on the dominance degree Q_i over other alternatives, where higher values indicate higher rankings [2,43]. The dominance degree Q_i of alternative x_i is calculated as follows:

$$Q_i = \frac{\sum_{j=1}^{n} p_{ij}^c}{n} \tag{4}$$

2.2. Social Network Analysis

A social network consists of a set of social entities and the relationships among them [44]. Generally, the social network can be depicted by a graph $G(V,E)$, where $V = \{v_1, v_2, \cdots, v_m\}(m \geq 2)$ denotes a set of social entities and $E = \{(v_k, v_l) \mid v_k, v_l \in V; k \neq l\}$ is a set of edges. The directed graph is used in this study, where an edge $(v_k, v_l) \in E$ indicates that individual v_k directly trusts v_l. In our study, the basic definitions and notations regarding social networks are based on the works of Wasserman and Faust [44], Barabási and Márton [45], and Newman [46].

Definition 2. Let $A = (a_{kl})_{m\times m}$ be the adjacency matrix of graph $G(V,E)$. If there is an edge from individual v_k to v_l, a_{kl} is 1; otherwise, it is 0, i.e.,

$$a_{ij} = \begin{cases} 1, & (v_k, v_l) \in E \\ 0, & (v_k, v_l) \notin E. \end{cases} \tag{5}$$

For example, trust relationships among seven social entities are showed in Figure 2, and its adjacent matrix is

$$A = \begin{pmatrix} - & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & - & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & - & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & - & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & - & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & - & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & - \end{pmatrix}.$$

Definition 3. In graph $G(D,E)$, a directed path from individual v_k to v_l is represented by a sequence of edges $\left(v_k, v_{\sigma(1)}\right), \left(v_{\sigma(1)}, v_{\sigma(2)}\right), \cdots, \left(v_{\sigma(q)}, v_l\right)$, and is denoted as $v_k \to v_l$.

Definition 4. The shortest path from individual v_k to v_l is the sequence with the fewest number of edges. The distance from v_k to v_l, denoted by d_{kl}, is the number of edges traversed along the shortest path.

As illustrated by Figure 2, there are two directed paths from v_1 to v_5: $v_1 \to v_2 \to v_3 \to v_4 \to v_2 \to v_5$ and $v_1 \to v_2 \to v_5$. The latter is used as the shortest path and the distance $d_{15} = 2$. It is possible for there to be no shortest path between two individuals, such as v_1 and v_7, who are not connected together by any path through the network. In this case, the distance d_{17} is infinite. In practice, the breadth first search (BFS) is used to calculate the shortest distance between every pair of individuals [47].

Trust relationships play a pivotal role in shaping interactions among individuals and revealing their social influence [44]. In this paper, the edge $(v_k, v_l) \in E$ represents not only the trust relationship from individual v_k to v_l but also the social influence from v_l to v_k. In social network analysis (SNA), centrality-based methods are employed to calculate social influence, considering those individuals with high centrality scores as influential ones [25–27]. For instance, in-degree centrality simply measures the number of incoming edges an entity receives [48]. Closeness centrality of a social entity is calculated as the inverse of the sum of its distances to all other entities in a social network [48], which indicates how close an entity is to all other entities in the network. The closer an entity is, the faster it can potentially spread information. Betweenness centrality of a social entity represents its frequency of falling on the shortest paths that link pairs of other entities in a social network [48,49]. Entities with high betweenness centrality act as bridges and control information flow across the network. Eigenvector centrality measures an entity's influence not just based on the number of connections but also on their centrality scores [50–52]. This means that if an entity is connected to other highly central entities, its own influence is correspondingly enhanced.

In this section, we introduce in-degree centrality, which is one of the simplest and most common centrality measures. The other centrality measures will be used in Section 4.2 for comparison analysis.

Definition 5. The in-degree centrality $C_{in}(v_l)$ measures the number of directed links pointing to v_l, i.e.,

$$C_{in}(v_l) = \sum_{k=1,k\neq l}^{m} a_{kl} / (m-1) \tag{6}$$

Example 1. Consider the seven individuals $V = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$ in Figure 2, the in-degree centrality $C_{in}(v_k)$ of each individual v_k is

$$C_{in} = [0, \tfrac{1}{3}, \tfrac{1}{6}, \tfrac{1}{6}, \tfrac{1}{6}, 0, \tfrac{1}{6}] \tag{7}$$

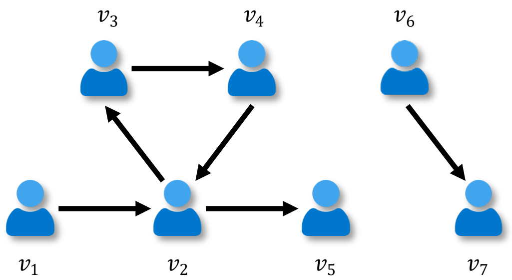

Figure 2. Example of social network.

2.3. Opinion Dynamics in a Social Network

Opinion dynamics, also called opinion evolution, describes how individuals' opinions evolve and update through interactions. This process can be formulated as a discrete-time dynamical process, where consensus, polarization, or fragmentation can occur in the final state.

The DeGroot model is one of the classical models in opinion dynamics [53]. Dong et al. [2,3,54] propose a variant of the DeGroot model called the social network DeGroot model (SNDG). In practical social network scenarios, each decision maker may consider others' opinions to a certain extent and modifies their own opinions accordingly. Let $o_k^t \in R$ denote the opinion of individual v_k at time t. The SNDG model assigns each individual v_k a self-confidence degree $\beta_k \in (0,1)$ for their own opinion and distributes $(1-\beta_k)$ across other individuals. Then, the evolved opinion o_k^{t+1} can be described as

$$o_k^{t+1} = \beta_k o_k^t + \sum_{l=1,l\neq k}^{m} \omega_{kl} o_l^t \tag{8}$$

where $\sum_{l=1,l\neq k}^{m} \omega_{kl} = 1-\beta_k$, and $\omega_{kl} \in (0,1-\beta_k]$ denotes the trust weight individual v_k assigns to v_l. Equation (8) can be equivalently presented as follows:

$$O^{t+1} = WO^t, t = 0,1,2, \ldots \tag{9}$$

where $O^t = (o_1^t, o_2^t, \ldots, o_m^t)$ and $W = \begin{bmatrix} \beta_1 & \omega_{12} & \cdots & \omega_{1m} \\ \omega_{21} & \beta_2 & \cdots & \omega_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \omega_{m1} & \omega_{m2} & \cdots & \beta_m \end{bmatrix}$.

2.4. Theoretical Background

Existing SNGDM research extensively draws upon several foundational theories and models. Social network analysis (SNA) provides indispensable tools for evaluating individuals' social influence within a network. Foundational concepts, such as centrality measures, enable researchers to quantify the structural importance of individuals and their capacity to influence others [27,44,48].

Another critical pillar of SNGDM research is opinion dynamics modeling, which captures how individuals' opinions evolve through iterative interactions with others. Classical models, such as the DeGroot model, have been extended in recent works by Dong et al. [3] and Wu et al. [12] to reflect the complexities of opinion evolution in social networks. These advancements provide insights into how opinions converge, polarize, or fragment under varying network conditions.

By integrating these theoretical frameworks, prior studies have established a robust foundation for analyzing CRPs in SNGDM. Building on this groundwork, our study addresses several existing gaps by introducing a unified social persuasion model that combines social influence and social status to account for trust degrees within networks [30]. Additionally, we extend the application of centrality metrics and opinion dynamics by incorporating the role of top persuaders. These contributions advance the theoretical and practical understanding of SNGDM and provide new perspectives on improving decision efficiency in social networks.

3. The Proposed Framework Based on Social Persuasion

3.1. Problem Description and the Proposed Framework

In traditional GDM, a group of individuals $V = \{v_1, v_2, \cdots, v_m\} (m \geq 2)$ express preferences on a set of alternatives $X = \{x_1, x_2, \cdots, x_n\} (n \geq 2)$ to reach a collective solution. For each decision maker $v_k \in V$, $P^{k,t} = (p_{ij}^{k,t})_{n\times n}$ denotes their preference degree of alternative x_i over alternative x_j and $CD^t(v_k)$ denotes their consensus degree at time t. Conflicts among individuals always exist in GDM, making consensus difficult to achieve. To mitigate these conflicts, the CRP is often employed. In traditional CRPs, individuals are encouraged to adjust their preferences to align more closely with the collective preference, aiming to improve the group consensus degree. However, in real decision-making scenarios, individuals who are in significant conflict are often difficult to coordinate and may be reluctant to make concessions.

Trust relationships play a crucial role in the decision-making process when individuals are required to adjust preferences. Individuals are more inclined to accept the suggestions of individuals they trust. Following this assumption, some CRPs guided by trust relationships have been justified well in the social network group decision making (SNGDM) [2,20,21]. In SNGDM, trust relationships can be depicted by edges in the network $E = \{(v_k, v_l) \mid v_k, v_l \in V; k \neq l\}$, and the strength of trust from v_k to v_l can be depicted by s_{kl} (i.e., the degree of influence v_l has on v_k). Traditional SNGDM models primarily focus on direct interactions among individuals and assume that an individual's social influence is limited to their first-order neighbors [2,20,28]. Specifically, s_{kl} is 0 if there is no trust relationship from v_k to v_l. However, real social networks are far more complex than these models suggest. Social influence exists not only between directly connected individuals but can also spread through indirect social ties. For example, if v_k trusts v_m and v_m trusts v_l, then v_k may be indirectly influenced by v_l even if there is no direct trust relationship between them. This transitivity indicates that social influence can propagate through chains of trust relationships and extend to more distant individuals in the network.

Another important phenomenon of social networks in the context of decision making is social persuasion, which refers to the mechanisms through which an individual's opinions, preferences, or behaviors are influenced by others within the network [30]. Social persuasion effectively illustrates how individuals can influence each other's decision-making process through both direct and indirect social interactions. Moreover, social persuasion allows organizations to identify key social entities, known as top persuaders, whose influence on other entities in the network can significantly facilitate decision making processes and enhance group consensus.

In this study, we formulate the consensus-reaching problem with top persuaders as follows: In a social network, we observe at the current time a set of decision makers who have provided their preferences over a set of alternatives. The objective is to predict the top K persuaders (i.e., K individuals who will lead to the greatest degree of preference adjustment among other decision makers in the network in a future period) and to investigate how top persuaders assist other individuals in SNGDM to reach consensus.

To solve these problems, we need to address two key challenges. First, for each ordered pair of individuals (v_k, v_l), we need to estimate the probability that v_l can persuade v_k to adjust their preferences (i.e., persuasion probability). Then, top persuaders are identified based on the estimated persuasion probabilities. Second, we need to propose a new feedback mechanism based on top persuaders to facilitate consensus reaching. As shown in Figure 3, a resolution framework is proposed to address each of these challenges. In the framework, there are two critical steps:

(1) Establishing the social persuasion model.

According to social network theories, we first construct a model to quantify social persuasion, which can arise from multiple distinct forces including social influence and social status [31,32]. Social influence can be evaluated using centrality metrics (e.g., in-degree centrality) and the influence propagation through trust relationships. Social status, on the other hand, can be measured by the degree of alignment between an individual's preference and the collective preference of the group. Once the social persuasion model is established, the next step is to identify the top persuaders in the social network—those individuals with the strongest social persuasive power.

(2) Implementing the consensus-reaching process with top persuaders.

If the predefined consensus is not achieved, feedback adjustments with top persuaders are adopted to enhance the consensus degree, which includes two key components: (1) Providing suggestions for top persuaders to modify their preferences and applying opinion dynamics for other individuals based on the social persuasion model. (2) Improving trust relationships between top persuaders and other individuals to help the latter better adjust their preferences and achieve consensus.

To further illustrate the resolution framework, Subsection 3.2 will discuss how to establish the social persuasion model, and Subsection 3.3 will show details of the CRP with top persuaders. To improve readability, the main symbols used in this study are listed in Table 1.

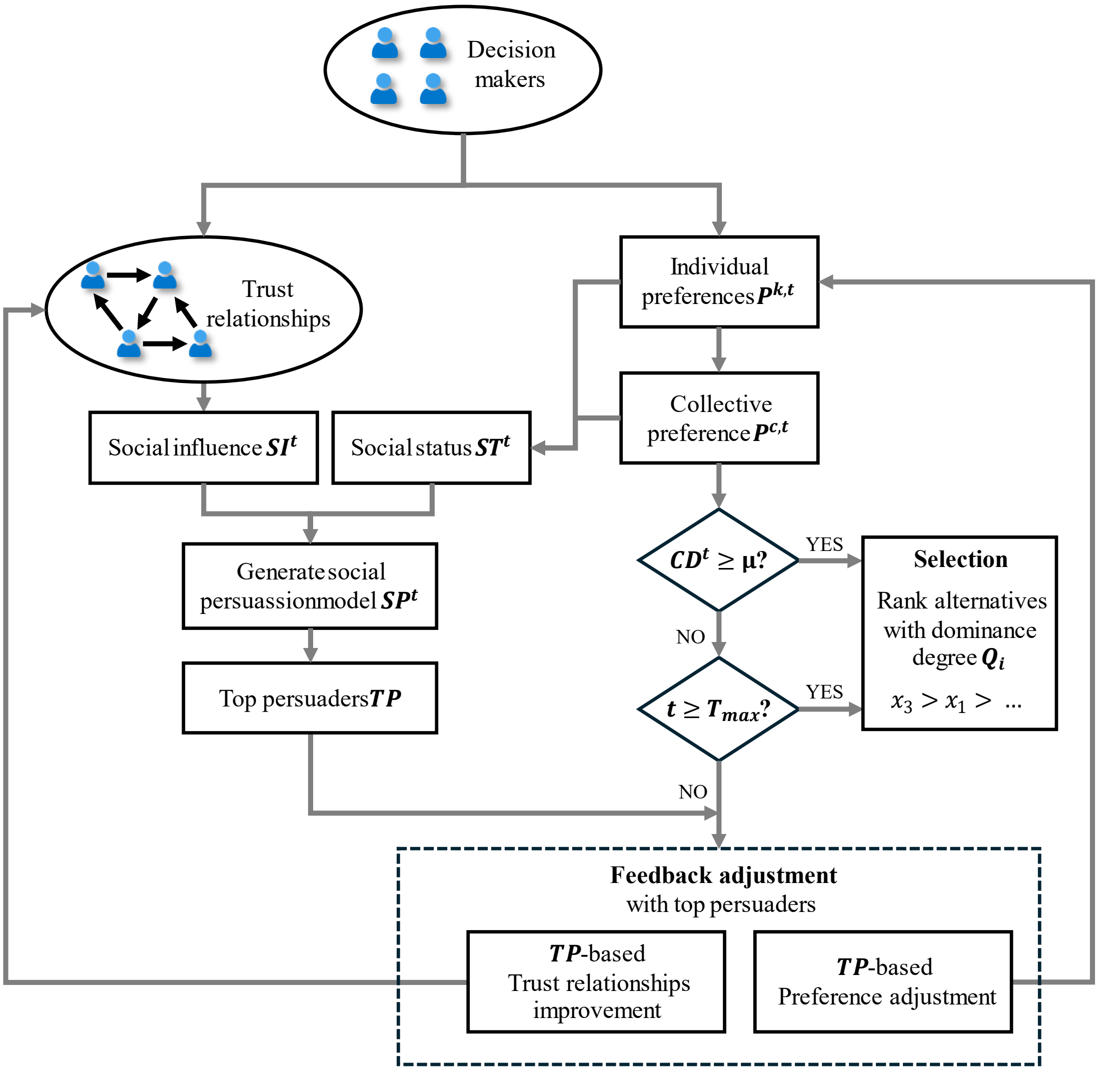

Figure 3. The proposed consensus framework of SNGDM with social persuasion.

Table 1. Summary of the symbols used in this study.

Symbols	Meaning
$V = \{v_1, v_2, \cdots, v_m\}(m \geq 2)$	Set of decision makers in a social network, where v_k denotes the k-th individual.
$X = \{x_1, x_2, \cdots, x_n\}(n \geq 2)$	Set of alternatives, where x_i denotes the i-th alternative.
$P^{k,t} = (p_{ij}^{k,t})_{n\times n}$	The preference of individual v_k, where $p_{ij}^{k,t} \in [0,1]$ denotes v_k's preference for alternative x_i over x_j at time t.
$P^{c,t} = (p_{ij}^{c,t})_{n\times n}$	The collective preference of the group for alternative x_i over x_j at time t.
π_k	The weight of individual v_k in the aggregated collective preference.
CD^t	The consensus degree at time t.
$CD^t(v_k)$	The consensus degree of individual v_k at time t.
μ	The consensus threshold.
T	The maximum consensus time.
Q_i	The dominance degree of each alternative x_i in the collective preference.
$E = \{(v_k, v_l) \mid v_k, v_l \in V; k \neq l\}$	Set of edges, where (v_k, v_l) denotes a trust relationship from v_k to v_l.
$A = (a_{kl})_{m\times m}$	The adjacency matrix, where $a_{kl} = 1$ denotes the edge from individual v_k to v_l.
d_{kl}	The length of the shortest path from individual v_k to v_l.
$C(v_k)$	The centrality score of individual v_k.

β_k	The self-confidence degree of individual v_k.
K	The number of top persuaders and resistant persuadees.
θ	Attenuation factor.
$SI = (s_{kl})_{m\times m}$	The social influence among individuals in the social network, where $s_{kl} \in [0,1]$ measures the strength of social influence that v_l assigns to v_k.
$ST = \{r_1, r_2, \cdots, r_m\}$	The social status of all individuals, where $r_k \in [0,1]$ measures the social status of individual v_k.
U	Uninorm operator.
$SP = (sp_{kl})_{m\times m}$	The social persuasion among individuals in the social network, where $sp_{kl} \in [0,1]$ measures the strength of social persuasion that v_l assigns to v_k.
SP_l	The persuasive power of individual v_l.
TP^A	The set of top persuaders with acceptable consensus degrees.
TP^U	The set of top persuaders with unacceptable consensus degrees.
$SP' = (sp'_{kl})_{m\times m}$	The restricted social persuasion among individuals in the social network.
$W = (\omega_{kl})_{n\times n}$	The influence weight among individuals, where $\omega_{kl} \in (0,1-\beta_k]$ denotes the influence weight individual v_l assigns to v_k.
RP_k	The resistance score of individual v_k.
RP^U	The set of resistant persuadees with unacceptable consensus degrees.
H	The set of recommendations of trust relationships.

3.2. Social Persuasion Model

This section introduces the generation of the social persuasion model and the identification of top persuaders, as depicted in Figure 3.

Step 1: Quantifying social influence using centrality measures

Network centrality measures are essential tools for quantifying social influence, as entities with higher centrality scores generally interact with a greater number of entities and exert more substantial influence within the network. Besides in-degree centrality $C_{in}(v_k)$ defined in Definition 5, there are several other centrality measures that capture the multi-dimensional nature of social influence from different perspectives. Lü et al. [27] compare well-known centrality measures and show that the impact of these measures can differ significantly depending on the network type and research objectives. In this study, we conduct a systematic analysis of the effectiveness of various centrality measures, which is detailed in the comparative analysis presented in Subsection 4.2. To ensure consistency and comparability across different centrality measures, we normalize each centrality score $C(v_k)$ by dividing it by the sum of all centrality scores in the network, as shown in Equation (10). This normalization ensures that the total centrality score across all nodes equals 1, allowing for a comprehensive and unbiased comparison of their influence on the identification of top persuaders and the consensus-reaching process.

$$C_{norm}(v_k) = C(v_k) / \sum_{l=1}^{m} C(v_l) \tag{10}$$

Step 2: Modeling social influence propagation

Prior studies of SNGDM typically focus on the immediate social influence that individuals have on their first-order neighbors. However, in real-world scenarios, social influence extends beyond direct connections and propagates through a chain of trust relationships. For example, as shown in Figure 2, while there is no direct trust relationship between v_1 and v_5, the influence between them (denoted as s_{15}) can be mediated through v_3. This indirect influence cannot be captured by first-order effects alone. Inspired by Fang et al. [33], we introduce an attenuation factor $\theta \in [0,1]$ to model this phenomenon. The factor $\theta^{d_{kl}-1}$ reflects how the influence s_{kl} attenuates as the path distance d_{kl} from v_k to v_l increases. This approach provides a more accurate representation of how social influence propagates through a chain of trust relationships.

Definition 6. Let $SI = (s_{kl})_{m\times m}$ be the social influence matrix of $G(V,E)$, where $s_{kl} \in [0,1]$ measures the strength of social influence that v_l assigns to v_k, which is defined as

$$s_{kl} = C(v_l) \times \theta^{d_{kl}-1} \tag{11}$$

Example 2. In Figure 2, there are two paths from v_1 to v_5 and both can propagate social influence from v_5 to v_1. According to Definition 4, the shortest path is chosen because it propagates social influence more efficiently. Thus, if $\theta = 0.8$ and in-degree centrality is chosen, the value of social influence from v_5 to v_1 is calculated as $s_{15} = C_{in}(v_5) \times \theta^{d_{15}-1} = 0.133$.

Step 3: Quantifying social status

While existing methods focus on social influence, social status also plays a crucial role in social persuasion [31,32]. In reality, an individual's social status affects their ability to shape others' attitudes and behaviors. Higher social status usually reflects greater authority within the social network, enabling these individuals to more easily persuade others to adopt their opinions and decisions. In the SNGDM problem, the social status of individual v_k, denoted as r_k, can be measured by how closely their preferences P^k align with the collective preference P^c. Individuals whose preferences closely match the collective preference P^c typically have a greater influence on the decision-making process because they are assigned more weight. Conversely, individuals who deviate significantly from the collective preference P^c are assigned smaller weights and have a smaller contribution to the consensus.

Definition 7. Let $ST = \{r_1, r_2, \cdots, r_m\}$ be the social status of all individuals, where $r_k \in [0,1]$ measures the similarity between individual preferences P^k and the collective preference P^c, i.e., the consensus degree $CD(v_k)$ of individual v_k. The normalized social status r_k is calculated as follows,

$$r_k = CD(v_k) / \textstyle\sum_{l=1}^{m} CD(v_l) \tag{12}$$

Step 4: Combining social influence and social status

Definition 8. A mapping $U: [0,1] \times [0,1] \rightarrow [0,1]$ satisfying monotonicity, associativity, and commutativity is called the Uninorm operator. There is a neutral element $g \in [0,1]$ that makes $U(x,g) = x$ for $\forall x \in [0,1]$. For $\forall x, y \in [0,1]$, U is defined as follows:

$$U(x,y) = \begin{cases} xy/g & \text{if } 0 \le x, y \le g \\ (x + y - xy - g)/(1-g) & \text{if } g \le x, y \le 1 \\ (x+y)/2 & \text{else} \end{cases} \tag{13}$$

In this study, we propose that social persuasion is determined by two main components: social influence and social status, which can be combined mathematically using the Uninorm operator [55]. The strength of social persuasion that v_l assigns to v_k, denoted as sp_{kl}, is calculated as

$$sp_{kl} = U(s_{kl}, r_l) \tag{14}$$

Depending on the values of s_{kl} and r_l, the Uninorm operator can exhibit three behaviors: reinforcement, weakening, and averaging. As illustrated in Figure 4, if a decision maker has both high social influence and social status, i.e., $s_{kl}, r_l \in [g, 1]$, the Uninorm operator will reinforce their social persuasion. Conversely, if a decision maker lacks both social influence and social status, i.e., $s_{kl}, r_l \in [0, g]$, their social persuasion will be weakened. In other cases, the social persuasion will be an average of their social influence and social status.

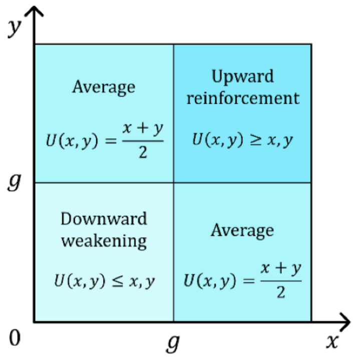

Figure 4. Uninorm operator's behaviors.

Example 3. Seven individuals $V = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$ in Figure 5 are supposed to evaluate three alternatives $X = \{x_1, x_2, x_3\}$, which are given as follows,

$$P^1 = \begin{bmatrix} 0.5 & 0.3 & 0.9 \\ 0.7 & 0.5 & 0.4 \\ 0.1 & 0.6 & 0.5 \end{bmatrix} P^2 = \begin{bmatrix} 0.5 & 0.6 & 0.9 \\ 0.4 & 0.5 & 0.8 \\ 0.1 & 0.2 & 0.5 \end{bmatrix} P^3 = \begin{bmatrix} 0.5 & 0.5 & 0.4 \\ 0.5 & 0.5 & 0.7 \\ 0.6 & 0.3 & 0.5 \end{bmatrix} P^4 = \begin{bmatrix} 0.5 & 0.9 & 0.5 \\ 0.1 & 0.5 & 0.6 \\ 0.5 & 0.4 & 0.5 \end{bmatrix}$$
$$P^5 = \begin{bmatrix} 0.5 & 0.7 & 0.5 \\ 0.3 & 0.5 & 0.7 \\ 0.5 & 0.3 & 0.5 \end{bmatrix} P^6 = \begin{bmatrix} 0.5 & 0.3 & 0.6 \\ 0.7 & 0.5 & 0.4 \\ 0.4 & 0.6 & 0.5 \end{bmatrix} P^7 = \begin{bmatrix} 0.5 & 0.2 & 0.2 \\ 0.8 & 0.5 & 0.8 \\ 0.8 & 0.2 & 0.5 \end{bmatrix}.$$

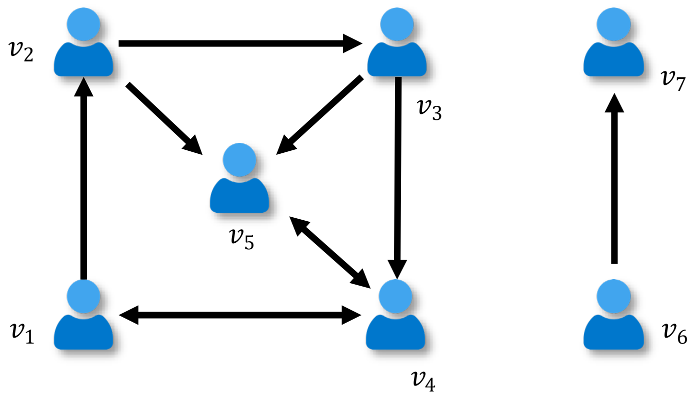

Figure 5. Trust relationships of seven individuals in the social network.

According to Equations (6), (10), and (11), the social influence matrix $SI = (s_{kl})_{m \times m}$ can be obtained. Specifically, based on Equations (6) and (10), the normalized in-degree centrality score of v_5 is $C_{in}(v_5) = 0.3$. Then, we set $\theta = 0.8$, and the social influence propagation from v_5 to v_1 is computed as $s_{15} = C_{in}(v_5) \times \theta^{d_{15}-1} = 0.3 \times 0.8^{2-1} = 0.24$.

According to Equations (1), (2), and (12), the social status of all individuals $ST = \{r_1, r_2, \cdots, r_m\}$ can be obtained. Specifically, based on Equation (1), we assume all individuals have the same weight $\pi_k = 1/m$, and the collective preference is obtained as

$$P^c = \begin{bmatrix} 0.5 & 0.5 & 0.571 \\ 0.5 & 0.5 & 0.629 \\ 0.429 & 0.371 & 0.5 \end{bmatrix}. \quad (15)$$

Then, based on Equations (2) and (12), the social status scores of all individuals are

$$R = [0.13, 0.139, 0.16, 0.145, 0.154, 0.147, 0.125] \quad (16)$$

Step 5: Identifying top persuaders

Each individual's social persuasion is obtained using the Uninorm operator based on social influence and social status. We set $g = 1/m = 0.143$. According to Equations (13) and (14), we combine social influence $s_{15} = 0.24$ and social status $r_5 = 0.154$ to obtain the social persuasion from v_5 to v_1, which is calculated as follows,

$$sp_{15} = U(0.24, 0.154) = 0.25 \tag{17}$$

Similarly, the total social persuasion matrix is obtained as follows,

$$SP = \begin{bmatrix} - & 0.097 & 0.12 & 0.302 & 0.25 & 0.074 & 0 \\ 0.058 & - & 0.13 & 0.242 & 0.309 & 0.074 & 0 \\ 0.073 & 0.062 & - & 0.302 & 0.309 & 0.074 & 0 \\ 0.091 & 0.078 & 0.112 & - & 0.309 & 0.074 & 0 \\ 0.073 & 0.062 & 0.105 & 0.302 & - & 0.074 & 0 \\ 0 & 0 & 0.08 & 0.072 & 0.077 & - & 0.088 \\ 0 & 0 & 0.08 & 0.072 & 0.077 & 0.074 & - \end{bmatrix} \tag{18}$$

Once the social persuasion matrix is obtained, we can further quantify the overall persuasion scores for each individual. In the social persuasion matrix, each element sp_{kl} represents the persuasive power of individual v_l over v_k. To calculate the total persuasive power of v_l, we sum sp_{kl} over all other individuals, i.e.,

$$SP_l = \sum_{k=1,k\neq l}^{m} sp_{kl} \tag{19}$$

Clearly, a higher SP_l indicates a stronger persuasive power. By ranking these scores, we can identify individuals with the highest persuasive power among decision makers, known as the top persuaders. For example, if we set $K = 2$, then individual v_4 ($SP_4 = 1.292$) and v_5 ($SP_5 = 1.331$) will be selected as the top persuaders in the social network.

$$SP_1 = 0.295, SP_2 = 0.3, SP_3 = 0.627, SP_4 = 1.292, SP_5 = 1.331, SP_6 = 0.442, SP_7 = 0.088 \tag{20}$$

3.3. Consensus-Reaching Process with Top Persuaders

In this subsection, we present a novel CRP utilizing top persuaders (TPs) in SNGDM, which involves three main phases: consensus measure, TP-based preference adjustment, and TP-based trust relationships improvement. Among these phases, the consensus measure follows the same approach in traditional GDM, as defined by Equations (1)–(3). The TP-based preference adjustment focuses on guiding TPs to modify their preferences to increase consensus. Additionally, non-TP individuals are encouraged to adjust their preferences based on the opinion dynamics model. In the TP-based trust relationships improvement, we identify resistant persuadees—those unwilling to change their preferences—and aim to manage their trust relationships with TPs to facilitate consensus.

3.3.1. TP-Based Preference Adjustment

If the consensus degree does not meet the predefined threshold, preference adjustment is implemented to help individuals achieve consensus. According to Equation (19), individuals are ranked by their persuasive scores and let TP be the set of top K persuaders in the social network. These top persuaders are then divided into two sets: TP^A, those with acceptable consensus degrees, and TP^U, those with unacceptable consensus degrees, as defined by the following:

$$TP^A = \{v_k \mid v_k \in TP \text{ and } CD(v_k) \geq \mu\} \tag{21}$$

$$TP^U = \{v_k \mid v_k \in TP \text{ and } CD(v_k) < \mu\} \tag{22}$$

Compared with the traditional CRP, this study emphasizes adjusting the preferences of TPs, particularly those in TP^U, to enhance consensus. For $v_k \in TP^U$, it is suggested to guide their preferences $\overline{p_{ij}^k}$ to align more closely with the collective preference p_{ij}^c, i.e.,

$$\overline{p_{ij}^k} \in \left[min\left(p_{ij}^k,\ p_{ij}^c\right), max\left(p_{ij}^k, p_{ij}^c\right)\right] \tag{23}$$

On the other hand, a large number of non-TP individuals play a crucial role in enhancing the overall consensus. Inspired by the opinion dynamics model discussed in Subsection 2.3, we introduce a new social network DeGroot model with top persuaders (i.e., TP-based SNDG) as a feedback mechanism to help non-TP individuals achieve consensus. Specifically, we propose that individuals are more likely to adjust their preferences based on those with higher social persuasion rather than just those they directly trust. For example, in Figure 5, individual v_1 directly trusts v_2 and v_4. However, according to the social persuasion matrix in Equation (18), v_5 has higher persuasive power than v_2. Thus, when adjusting preferences, v_1 will be more inclined to follow preferences of v_4 and v_5 rather than v_2 and v_4. In the TP-based SNDG, individuals will adjust their preferences based on social persuasion, as shown in Figure 6, rather than on direct trust relationships as in Figure 5. For individual v_k, the number of perceived social persuasion adjustments equals their trust relationships (i.e., the out-degree of v_k). Consequently, the restricted social persuasion weights are updated as follows,

$$SP' = \begin{bmatrix} - & - & - & 0.302 & 0.25 & - & - \\ - & - & - & 0.242 & 0.309 & - & - \\ - & - & - & 0.302 & 0.309 & - & - \\ - & - & 0.112 & - & 0.309 & - & - \\ - & - & - & 0.302 & - & - & - \\ - & - & - & - & - & - & 0.088 \\ - & - & - & - & - & - & - \end{bmatrix} \tag{24}$$

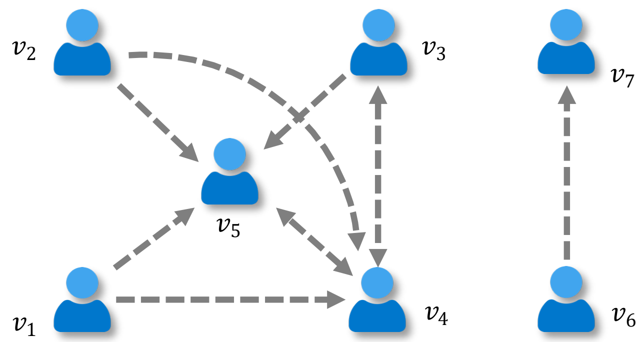

Figure 6. Individuals will be influenced by those with higher social persuasion rather than by direct trust relationships.

In the TP-based SNDG, we assume each individual v_k assigns a self-confidence degree $\beta_k \in [0, 1]$ to their own preference and distributes the remaining weight $(1 - \beta_k)$ across other individuals. Let ω_{kl} $(k, l = 1, \cdots, m; k \neq l)$ be the weight individual v_l assigns to v_k in opinion dynamics, which is calculated as

$$\omega_{kl} = \frac{(1-\beta_k)sp'_{kl}}{\sum_{q=1,q\neq k}^{m} sp'_{kq}} \tag{25}$$

The preference adjustment of individual v_k based on TP-based SNDG is obtained as

$$\overline{p_{ij}^k} = \beta_k p_{ij}^k + \sum_{l=1,l\neq k}^{m} \omega_{kl} p_{ij}^l \tag{26}$$

For Example 3, let the self-confidence degrees $\beta_k = 0.65(k = 1,2,3,4,5)$. Based on Equations (24) and (25), W can be written as

$$W = \begin{bmatrix} 0.65 & - & - & 0.191 & 0.159 & - & - \\ - & 0.65 & - & 0.154 & 0.196 & - & - \\ - & - & 0.65 & 0.173 & 0.177 & - & - \\ - & - & 0.093 & 0.65 & 0.257 & - & - \\ - & - & - & 0.35 & 0.65 & - & - \\ - & - & - & - & - & 0.65 & 0.35 \\ - & - & - & - & - & - & 1 \end{bmatrix} \tag{27}$$

During the preference adjustment, individual v_1 will keep his/her own preference and be influenced by v_4 and v_5, which can be calculated as

$$\overline{p_{12}^1} = \beta_1 p_{12}^1 + \omega_{14} p_{12}^4 + \omega_{15} p_{12}^5 = 0.65 \times 0.3 + 0.191 \times 0.9 + 0.159 \times 0.7 = 0.478 \tag{28}$$

3.3.2. TP-Based Trust Relationships Improvement

In this study, resistant persuadees (RPs) are individuals who stick to their initial preferences and resist to adjustments suggested by top persuaders (TPs). The existence of RPs hinders achieving a higher consensus level, as they are unwilling to align their preferences with the collective decision. Trust relationships play a crucial role in SNGDM, influencing individuals' willingness to accept others' preferences. To address this, the TP-based trust relationship improvement method is proposed to establish new trust relationships from RPs to TPs.

In the social persuasion matrix such as Equation (18), each element sp_{ij} represents the persuasive strength individual v_k receives from v_l. If individual v_k shows low acceptance of v_l's persuasion, it suggests that v_k is more resistant to being persuaded by v_l, resulting in a smaller sp_{wl} value. The resistance degree of individual v_k, denoted as RP_k, is defined as

$$RP_k = 1 / \sum_{l=1, l \neq k}^{m} sp_{kl} \tag{29}$$

According to Equation (29), individuals can be ranked based on their resistance scores, and let RP be the K resistant persuadees with the highest scores in the social network. For example, as shown in Equation (30), if we set $K = 2$, individual v_6 ($RP_6 = 3.157$) and v_7 ($RP_7 = 3.301$) will be identified to be RPs.

$$RP_1 = 1.19, RP_2 = 1.23, RP_3 = 1.22, RP_4 = 1.51, RP_5 = 1.62, RP_6 = 3.16, RP_7 = 3.3. \tag{30}$$

Let RP^U be the set of RPs with unacceptable consensus degrees, i.e.,

$$RP^U = \{v_k \mid v_k \in RP \text{ and } \mathrm{CD}(v_k) < \mu\} \tag{31}$$

Next, TP-based trust relationships improvement is used to identify potential trust relationships between RPs and TPs that can promote the group to reach a consensus. Specifically, the RPs in RP^U are encouraged to trust TPs in TP^A. These potential trust relationships can be defined as follows,

$$H = \{(v_k, v_l) | v_k \in RP^U, v_l \in TP^A, (v_k, v_l) \notin E\} \tag{32}$$

Moreover, social persuasion sp_{kl} is used to recommend the new edges that are easier to establish. Generally, individuals are more likely to accept TPs with higher social persuasive power. For each $v_k \in RP^U$, the TP with highest social persuasion sp_{kl} will be recommended, which can be described as

$$\left\{ (v_k, v_l) \middle| v_k \in RP^U, v_l \in \left\{ v_z | sp_{kz} = \max_y \{ sp_{ky} \mid (v_k, v_y) \in H \} \right\} \right\} \tag{33}$$

4. Simulation and Comparison Analysis

This section explores the role of top persuaders in facilitating consensus reaching in SNGDM through simulation experiments and comparative analysis. We abbreviate our proposed CRP with top persuaders as TPC, the CRP with social information as SIC, and the CRP with trust relationships as TRC [2]. Following the social persuasion model, TP-based preference adjustment, and TP-based trust relationships improvement in Section 3, the TPC model is formally presented in Algorithm 1. The simulation experiments I–IV are developed based on Algorithm 1.

Algorithm 1 General description of TPC model.

Input:	The individual preferences $\{P^1, P^2, \ldots, P^m\}$, the graph of trust relationships $G(V,E)$, the weights of individuals $\{\pi_1, \pi_2, \ldots, \pi_m\}$, the established maximum round T, and the consensus threshold μ.
Output:	The ranking of alternatives $X = \{x_1, x_2, \cdots, x_n\}$.
Step 1:	Let $t = 0$, $P^{k,t} = (p_{ij}^{k,t})_{n\times n}$, and $G^t(V,E^t) = G(V,E)$.
Step 2:	Aggregate the preferences $\{P^{1,t}, P^{2,t}, \ldots, P^{m,t}\}$ to obtain $P^{c,t} = \left(p_{ij}^{c,t}\right)_{n\times n}$ in round t based on Equation (1), i.e., $p_{ij}^{c,t} = \sum_{k=1}^{m} \pi_k p_{ij}^{k,t}$.
Step 3:	Based on Equations (2) and (3), we compute the individual consensus degrees $CD^t(v_k) = 1 - \frac{\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\left\|p_{ij}^{k,t} - p_{ij}^{c,t}\right\|}{n(n-1)/2}$ and group consensus degree $CD^t = \sum_{k=1}^{m} \pi_k CD^t(v_k)$. If $CD^t \geq \mu$ or $t \geq T$, go to Step 7; otherwise go to the next step.
Step 4:	(a) Obtain the social influence matrix $SI^t = (s_{kl}^t)_{m\times m}$ in round t based on Equation (11), i.e., $s_{kl}^t = C^t(v_l) \times \gamma^{d_{kl}^t - 1}$, where $C^t(v_l)$ represents network centrality and d_{kl}^t represents the distance between v_k and v_l. (b) Obtain the social status $ST^t = \{r_1^t, r_2^t, \cdots, r_m^t\}$ in round t based on Equation (12), i.e., $r_k^t = CD^t(v_k) / \sum_{l=1}^{m} CD^t(v_l)$. (c) Obtain the social persuasion matrix $SP^t = (sp_{kl}^t)_{m\times m}$ in round t based on Equation (14), i.e., $sp_{kl}^t = U(s_{kl}^t, r_l^t)$, where $U(x,y)$ represents a Uninorm operator such as Equation (13).
Step 5:	(a) Identify the top persuaders TP^t and resistant persuadees RP^t in round t based on Equation (19) and (29), i.e., $SP_l^t = \sum_{k=1,k\neq l}^{m} sp_{kl}^t$ and $RP_k^t = 1/\sum_{l=1,l\neq k}^{m} sp_{kl}^t$. Further, we classify TP^t and RP^t into different groups: $TP^{A,t} = \{v_k \mid v_k \in TP^t \text{ and } CD^t(d_k) \geq \mu\}$ $TP^{U,t} = \{v_k \mid v_k \in TP^t \text{ and } CD^t(d_k) < \mu\}$ $RP^{U,t} = \{v_k \mid v_k \in RP^t \text{ and } CD^t(d_k) < \mu\}$ (b) TP-based preference adjustment. For $v_k \in TP^{U,t}$, it is suggested to adjust their preferences as $p_{ij}^{k,t+1} \in \left[min\left(p_{ij}^{k,t},\ p_{ij}^{c,t}\right), max\left(p_{ij}^{k,t},\ p_{ij}^{c,t}\right)\right], i \leq j$ and $p_{ij}^{k,t+1} = 1 - p_{ji}^{k,t+1}, j > i$. For non-TP individuals, TP-based SNDG is proposed to adjust their preferences based on Equation (26). (c) TP-based trust relationships improvement. Identify the potential trust relationships$H^t = \{(v_k, v_l) \| v_k \in RP^{U,t}, v_l \in TP^{A,t}, (v_k, v_l) \notin E^t\}$ and recommend each $v_k \in RP^{U,t}$ to trust $v_l \in TP^{A,t}$ from $\left\{(v_k, v_l) \middle\| v_k \in RP^{U,t}, v_l \in \left\{v_z \| sp_{kz}^t = \max_y \{sp_{ky}^t \mid (v_k, v_y) \in H^t\}\right\}\right\}$.
Step 6:	Update trust relationships $G^{t+1}(V, E^{t+1})$ and let $t = t + 1$, then go to Step 2.
Step 7:	Let $Q_i = \sum_{j=1}^{n} p_{ij}^{c,t}/n$. Then, rank alternatives $X = \{x_1, x_2, \cdots, x_n\}$ based on dominance degree Q_i.

4.1. The Design of Simulation Experiments

In simulation experiment I (the TPC model), we randomly generate individual preferences, self-confidence degrees, and trust relationships among them. TP-based preference adjustment and TP-based trust relationships improvement are employed to help the group reach a consensus. Supplementary details for simulation experiment I are as follows.

(1) Generation of trust relationships.

Trust relationships are generated using Erdos–Rényi (ER) random graphs [56], where parameter b represents the probability of an edge generating between two individuals in the graph. This approach ensures a random distribution of trust relationships among individuals, reflecting a variety of potential real-world social network structures.

(2) TP-based preference adjustment.

For $v_k \in TP^{A,t}$, no preference adjustment is necessary. For $v_k \in TP^{U,t}$, they accept the group suggestion with probability ρ in each round. If v_k accepts the group suggestion, their preferences will be influenced both by the group suggestion and by the social persuasion form others. Let ϖ_{kl} $(k, l = 1, 2, \ldots, m; k \neq l)$ be the weight v_k assigns to v_l based on Equation (25), and let $\varpi_{k(m+1)}$ be the weight v_k assigns to the group suggestion $P^{c,t}$, ensuring that $\varpi_{k(m+1)} + \sum_{l=1,l\neq k}^{m} \varpi_{kl} = 1 - \beta_k$. Thus, the preference adjustment for v_k is defined as follows,

$$\begin{cases} p_{ij}^{k,t+1} = \beta_k p_{ij}^{k,t} + \sum_{l=1,l\neq k}^{m} \varpi_{kl} p_{ij}^{l,t} + \varpi_{k(m+1)} p_{ij}^{c,t}, & i \geq j \\ p_{ij}^{k,t+1} = 1 - p_{ji}^{k,t+1}, & i < j. \end{cases} \tag{34}$$

For other individuals ($v_k \in TP^{U,t}$ who do not accept the group suggestion and non-TP individuals), their preferences will be influenced only by the social persuasion from others. Let ω_{kl} be the weight v_k assigns to v_l based on Equation (25), such that $\sum_{l=1,l\neq k}^{m} \omega_{kl} = 1 - \beta_k$ $(k, l = 1, 2, \ldots, m)$. The preference adjustment for these individuals is defined as follows,

$$\begin{cases} p_{ij}^{k,t+1} = \beta_k p_{ij}^{k,t} + \sum_{l=1,l\neq k}^{m} \omega_{kl} p_{ij}^{l,t}, & i \geq j \\ p_{ij}^{k,t+1} = 1 - p_{ji}^{k,t+1}, & i < j. \end{cases} \tag{35}$$

(3) TP-based trust relationships improvement.

For $v_k \in RP^{U,t}$, they accept trust relationship recommendations with probability η in each round. If v_k agrees to trust $v_l \in TP^{A,t}$, we set $a_{kl} = 1$.

4.2. Comparison Analysis

In the proposed consensus framework TPC, we integrate social influence and social status to describe the social persuasion among individuals. Previous studies have primarily focused on trust relationships or social influence alone. To demonstrate the effectiveness of TPC in promoting consensus, we compare it with SIC and TRC through simulation experiment I-III (Appendix A). In simulation experiment II (the SIC model), we consider social influence (without social status), which means that individuals are influenced by those with higher social influence in the social network. In simulation experiment III (the TRC model), we consider the effect of trust relationships and suppose that individuals are solely influenced by their direct neighbors [2].

For the first comparison, we conduct simulation experiment I-III with $n = 4$, $T = 10$, $K = 8$, $\mu = 0.85$, $\theta = 0.9$, and $b = 0.03$. Specifically, in-degree centrality is chosen. Then, we set different values for m, ρ, and η, and run 1000 times to obtain the average values for CD^t in each round t. An efficient CRP should show a rapid improvement in group consensus degree CD^t. The results are presented in Figure 7.

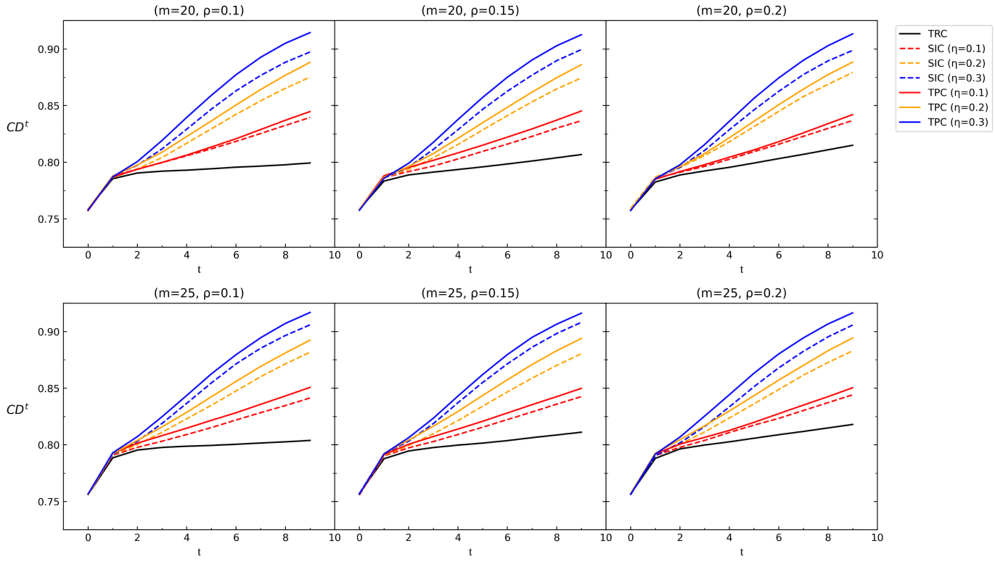

Figure 7. Average CD^t values under different m, ρ, and η values in simulation experiment I–III.

For the second comparison, we further compare the effectiveness of social persuasion (TPC) and social influence (SIC). We conduct simulation experiment I and II with $m = 20$, $n = 4$, $T = 10$, $\mu = 0.85$, $\theta = 0.9$, $b = 0.03$, $\rho = 0.2$, and $\eta = 0.1$. Then, we set different centrality measures and vary the number of top persuaders K from 2 (10% of individuals) to 10 (50% of individuals). Each experiment is run 1000 times to obtain the average iteration number AZ required to achieve consensus. The results are presented in Table 2.

For the third comparison, we conduct simulation experiment I and II with $m = 20$, $n = 4$, $T = 10$, $\mu = 0.85$, $\theta = 0.9$, $b = 0.03$, $\rho = 0.2$, and centrality measure be in-degree. We set different parameters K and η and run 1000 times to obtain values of AZ. The results are described in Figure 8a.

For the fourth comparison, we conduct simulation experiment I and II with $m = 20$, $n = 4$, $T = 10$, $K = 4$, $\mu = 0.85$, $\theta = 0.9$, $\rho = 0.2$, and centrality measure be in-degree. We set different parameters η and b and run 1000 times to obtain values of AZ. The results are described in Figure 8b.

For the fifth comparison analysis, we conduct simulation experiment I and II with $m = 20$, $n = 4$, $T = 10$, $K = 4$, $\mu = 0.85$, $b = 0.03$, $\rho = 0.2$, and centrality measure be in-degree. We set different parameters θ and η and run 1000 times to obtain values of AZ. The results are described in Figure 8c.

From Figures 7 and 8, and Table 2, we can obtain the following observations.

(1) Our proposed consensus framework TPC demonstrates significant efficiency in facilitating consensus under various parameters. Compared to SIC and TRC, TPC achieves consensus more quickly. SIC outperforms TRC by incorporating the effect of social influence. Similarly, TPC outperforms SIC by incorporating the effect of social status. These results highlight the importance of both social influence and social status in the consensus reaching process.

(2) As shown in Table 2, TPC consistently outperforms SIC across all centrality measures. This validates that integrating social persuasion, which integrates both social influence and social status, is more effective than considering social influence alone.

(3) As the number of top persuaders K increases, the average number of iterations to reach consensus AZ significantly decreases. This underscores the crucial role of top persuaders in facilitating group consensus, indicating that a higher number of top persuaders accelerates the consensus process.

(4) As the probability of adding edges η increases, the average CD^t values increase (Figure 7) and AZ decreases (Figure 8). This suggests that enhancing trust relationships with top persuaders can significantly accelerate the consensus-reaching process.

(5) When the attenuation factor θ is greater than 0.9, AZ significantly decreases. However, when θ is below 0.9, AZ does not show significant change. This means that a too-low attenuation factor (severe attenuation) hinders the propagation of social influence through trust relationships, particularly diminishing the influence from higher-order neighbors.

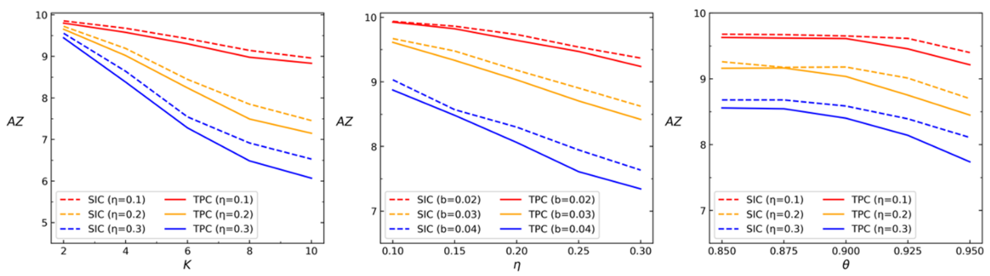

Figure 8. (**a**–**c**) AZ under different parameters K, η, and θ values in simulation experiment I and II.

Table 2. AZ under different centrality measures and parameter K in simulation experiment I and II.

AZ	In-degree		Closeness		Betweenness		Percolation		Eigenvector		Katz		PageRank		Uniform	
K	TPC	SIC	TPC	SIC	TPC	SIC	TPC	SIC	TPC	SIC	TPC	SIC	TPC	SIC	TPC	SIC
2	9.584	9.653	9.512	9.669	8.589	8.793	8.591	8.751	9.353	9.469	9.604	9.706	9.477	9.624	9.226	9.643
4	8.961	9.087	8.882	9.178	7.939	8.115	7.973	8.103	9.179	9.301	9.03	9.113	8.841	9.074	8.579	9.153
6	8.259	8.382	8.19	8.494	7.381	7.61	7.354	7.614	9.044	9.169	8.286	8.422	8.101	8.356	7.897	8.542
8	7.615	7.832	7.6	7.945	6.989	7.236	6.986	7.232	8.954	9.123	7.685	7.808	7.486	7.715	7.354	7.985
10	7.268	7.504	7.229	7.614	6.786	7.077	6.791	7.066	8.951	9.078	7.37	7.42	7.086	7.307	7.028	7.585

4.3. The Effect of Top Persuaders on Consensus Reaching

In Section 4.2, we observe the significant effect of social persuasion in promoting the consensus-reaching process. However, the specific role of top persuaders remains unclear. This section focuses on the initial top persuaders identified in the first round, denoted as TP^*. Specifically, we propose three indicators to examine their performance throughout the consensus-reaching process.

First, we examine whether each $v_k \in TP^*$ remains a top persuader in subsequent CRP rounds. This analysis helps determine if these initial TPs consistently influence the decision-making process. Specifically, we define $TPR^{k,t}$ to indicate v_k's retention in TP^t in round t, i.e., if $v_k \in TP^t$, then $TPR^{k,t} = 1$, otherwise $TPR^{k,t} = 0$. The results are presented in Figure 9.

Second, we examine whether each $v_k \in TP^*$ maintains their initial preferences in the face of group suggestion and social persuasion from others. Let $x_{(1)}^{k,t}$ be the best alternative of their current preference $P^{k,t}$, and let $x_{(1)}^{k,0}$ be the best alternative of their initial preference $P^{k.0}$. We define $AS^{k,t}$ as the alternative stability of v_k in round t, i.e., if $x_{(1)}^{k,t} = x_{(1)}^{k,0}$, then $AS^{k,t} = 1$, otherwise $AS^{k,t} = 0$. The results are presented in Figure 10.

Third, for each $v_k \in TP^*$, we examine whether their initial preferences align with the collective preference in subsequent CRP rounds. Let $x_{(1)}^{c,t}$ be the best alternative of current collective preference $P^{c,t}$. We define $AC^{k,t}$ as the alternative consistency of v_k in round

t, i.e., if $x_{(1)}^{c,t} = x_{(1)}^{k,0}$, then $AC^{k,t} = 1$, otherwise $AC^{k,t} = 0$. The results are presented in Figure 11.

We conduct simulation experiment IV (Appendix A) with $m = 25$, $n = 4$, $T = 15$, $\theta = 0.85$, $b = 0.03$, $\rho = 0.2$, $\eta = 0.3$, and in-degree centrality. Then, we set different values for K and run 10,000 times to obtain $TPR^{k,t}$, $AS^{k,t}$, and $AC^{k,t}$ in each round t. We particularly focus on the top 5 persuaders identified in the initial rounds of each experiment. From Figure 9-11, we can obtain the following observations.

(1) The retention probability TPR of the top 1 persuader consistently remains the highest, maintaining a level of 80% throughout the entire process. This suggests a persistent influence of the top 1 persuader in the decision-making process. For persuaders ranked beyond the top 2, there is a notable downward trend in the first five rounds. This indicates that it becomes increasingly challenging for these persuaders to maintain their top positions over time.

(2) The alternative stability AS of the top 1 persuader gradually decreases over time but remains higher than other persuaders. This highlights the dominant stability of the top 1 persuader's preferences throughout the decision-making process. For persuaders ranked beyond the top 2, they are more susceptible to external influence from group suggestion and social persuasion from others.

(3) The alternative consistency AC of the top 1 persuader is consistently the highest and remains stable around 0.45 throughout the entire process. In contrast, the AC for persuaders ranked from top 2 to top 5 is lower and exhibits a relatively stable pattern compared to the top 1 persuader. This suggests that the top 1 persuader has a greater influence on the collective preference and exhibits higher predictive ability.

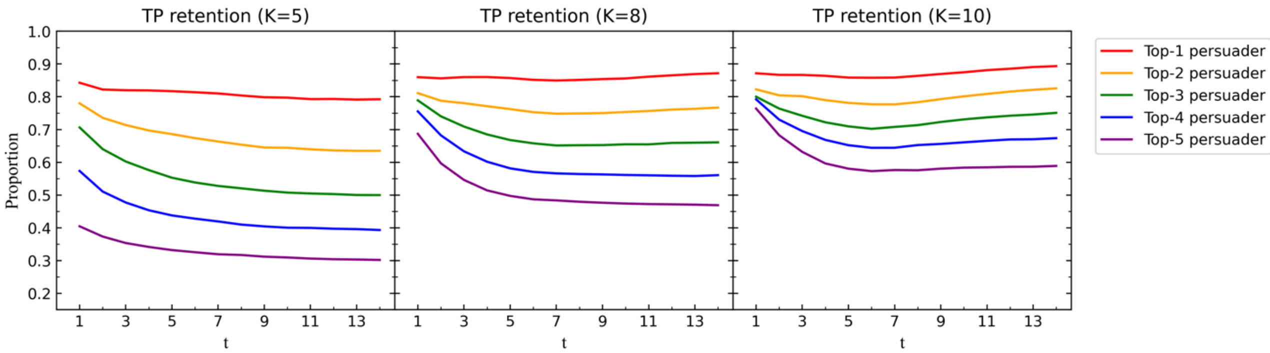

Figure 9. TPR under different parameters K values in simulation experiment IV.

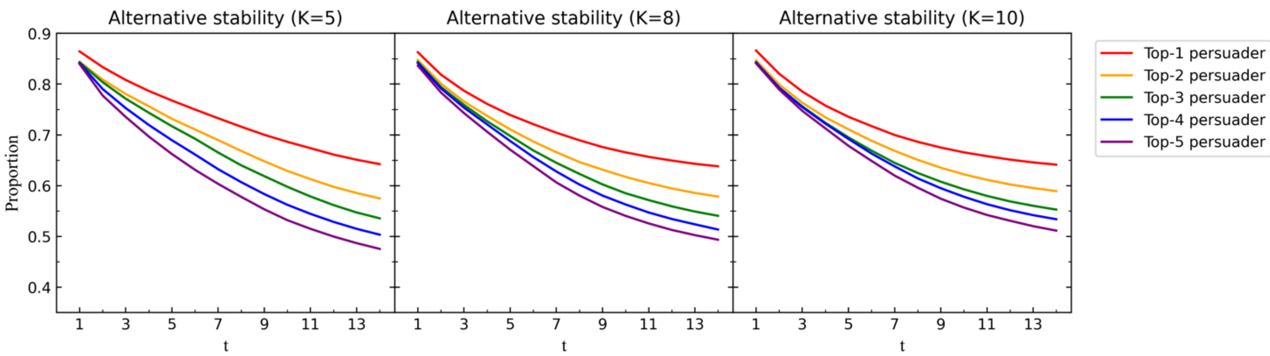

Figure 10. AS under different parameters K values in simulation experiment IV.

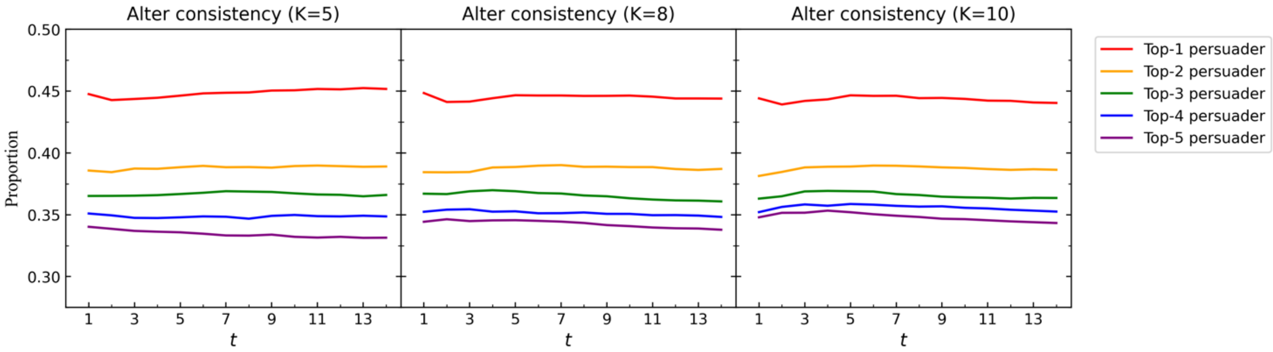

Figure 11. AC under different parameters K values in simulation experiment IV.

5. Discussion

5.1. Theoretical and Practical Implications

This study contributes to the growing field of SNGDM by offering a novel perspective on top persuaders and their role in shaping consensus reaching. By integrating social influence with social status, the proposed model deepens the theoretical understanding of how social persuasion dynamically operates within social networks. Additionally, the incorporation of the "limited attention" phenomenon enriches the existing literature, emphasizing that decision makers selectively allocate cognitive resources across trust relationships rather than distributing them evenly. This perspective sheds new light on the evolution of trust relationships over time and lays the groundwork for future research on dynamic, feedback-driven CRPs.

From a practical standpoint, identifying and strategically utilizing TPs offers substantial benefits in a variety of real-world contexts. In corporate or governmental decision-making, guiding key influencers can accelerate consensus formation, reducing both the time and costs associated with large-scale negotiations. In social media marketing, identifying TPs enables highly targeted campaigns that effectively leverage opinion leaders to shape public sentiment. Furthermore, in emergency or crisis management, where swift and accurate consensus is critical, understanding which individuals wield disproportionate influence within a network becomes vital for timely and effective interventions.

However, beyond merely enhancing efficiency, it is essential to ensure the quality and reliability of the final decision by preventing the voices of non-TP members from being overshadowed by a few highly influential individuals. Although TPs can expedite the consensus reaching process, their disproportionate influence raises concerns about potentially suppressing diverse perspectives. Consequently, future research should explore mechanisms that balance the benefits of TPs with the need to preserve inclusiveness and decision quality.

The heightened risk of over-influence by TPs raises important questions about the ultimate quality of collective outcomes. If TPs dominate the decision-making process, the final consensus may reflect a narrower perspective, undermining the richness and diversity of group insights. To address this issue, future studies could incorporate safeguards such as minimum adjustment thresholds or multi-criteria consensus metrics, ensuring that minority opinions are adequately considered throughout the process. Another promising avenue is the development of adaptive reliability assessments, which dynamically adjust TPs' weights based on the alignment and variability of group feedback. These mechanisms would prevent any single viewpoint from becoming overly dominant while maintaining the efficiency advantages provided by TPs.

5.2. Future Research Directions

Several limitations warrant further discussion. First, our framework relies on specific assumptions about the trust formation mechanism—particularly the integration of social influence and social status—which may oversimplify real-world dynamics in more complex network structures. Second, the simulations were conducted using synthetic datasets and controlled experimental settings. In large-scale, real-world social networks, user behavior is often more heterogeneous, and external noise factors, such as misinformation or evolving social contexts, may exert significant influence on the consensus process. Third, while our study focuses on top persuaders, it does not extensively address non-cooperative or adversarial behaviors, such as intentional opinion manipulation, which could hinder or disrupt consensus in certain scenarios.

Looking ahead, several promising research directions emerge. First, future work could incorporate dynamic and time-varying trust relationships to better capture the evolving nature of persuasion in rapidly changing social environments. Second, exploring the role of adversarial agents or malicious persuaders in shaping trust evolution would provide insights into developing robust defense mechanisms for consensus-based models. Third, employing more data-driven approaches—such as leveraging real-time user interactions, sentiment analysis, and machine learning techniques—could deepen our understanding of how top persuaders emerge and propagate their influence in diverse and large-scale networks. Furthermore, applying our proposed framework to specific real-world contexts (e.g., emergency decision-making, crowdsourcing innovation, or large-scale online policymaking) would offer concrete evidence of its scalability and practical utility, encouraging broader adoption in academic and applied domains.

6. Conclusions

In this study, we tackled the challenge of identifying and utilizing top persuaders (TPs)—individuals who wield disproportionately high influence on others—to enhance the efficiency and reduce the costs associated with consensus reaching. By leveraging social network theories, we integrated social influence (i.e., network centrality measures) with social status (i.e., alignment with the collective opinion) to develop a comprehensive social persuasion model. This model was subsequently incorporated into a novel CRP framework, providing fresh insights into trust degrees, feedback mechanisms, and trust relationship dynamics in SNGDM. Our simulations and comparative analyses show that (1) increasing the number of TPs substantially reduces the iterations required to achieve consensus; (2) establishing trust relationships between TPs and other individuals accelerates the consensus process; and (3) TPs retain a high and stable level of influence throughout the entire CRP rounds. By providing an integrated framework that captures individuals' persuasive power, this study offers actionable insights for optimizing decision making processes in digitally connected environments.

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Appendix A. Simulation Experiments

Simulation Experiment I

Input: The number of individuals m, the number of alternatives n, the established maximum round T, the number of top persuaders K, the consensus threshold μ, and the parameters θ, b, ρ, and η.

Output: The consensus degree CD^t and iteration number z.

Step 1: Generate initial data: (1) Generate a directed ER graph $G(V,E)$ with probability b, and let $A=(a_{kl})_{m\times m}$ be the adjacent matrix; (2) Generate individual preferences $P^k=(p_{ij}^k)_{n\times n}$, where p_{ij}^k is uniformly and randomly from interval $[0,1]$ for $i\geq j$, $p_{ii}^k=0.5$, and $p_{ij}^k=1-p_{ji}^k$ for $i<j$. Generate individual weights $\pi_k=1/m\ (k=1,2,\dots,m)$; 3) Generate self-confidence degree β_k uniformly and randomly from interval $[0,1]$.

Step 2: Let $t=0$, $z=T$, $P^{k,t}=P^k$, $G^t(V,E^t)=G(V,E)$, and $A^t=A$.

Step 3: Compute $P^{c,t}=\left(p_{ij}^{c,t}\right)_{n\times n}$ same as step 2 in Algorithm 1.

Step 4: Compute CD^t same as step 3 in Algorithm 1. If $CD^t\geq\mu$ and $z=T$, let $z=t+1$. If $t=T$, go to step 10; otherwise, go to the next step.

Step 5: First, compute social persuasion $SP^t=(sp_{kl}^t)_{m\times m}$ same as step 4(c) in Algorithm 1. Then, identify $TP^{A,t}$, $TP^{U,t}$, and $RP^{U,t}$ same as step 5(a) in Algorithm 1.

Step 6: (1) Let γ_{kl} be the restricted social persuasion sp'_{kl} from v_l to v_k, such as in Equation (24); (2) Let $\gamma_{k(m+1)}\in[\min_l(\gamma_{kl}),\max_l(\gamma_{kl})]$ be the weight v_k assigns to the group suggestion. Then we can obtain $\bar{\omega}_{kl}=\left(\gamma_{kl}\cdot(1-\beta_k)\right)/\sum_{y=1,y\neq k}^{m+1}\gamma_{ky}$ and $\omega_{kl}=\left(\gamma_{kl}\cdot(1-\beta_k)\right)/\sum_{y=1,y\neq k}^{m}\gamma_{ky}$.

Step 7: TP-based preference adjustment. Let $v_k\in TP^{U,t}$ accept the group suggestion with probability ρ. If $v_k\in TP^{U,t}$ and they accept the preference adjustment, compute the preference $p_{ij}^{k,t+1}$ based on Equation (34). For other individuals ($v_k\in TP^{U,t}$ who do not accept the group suggestion and non-TP individuals), compute the preference $p_{ij}^{k,t+1}$ based on Equation (35).

Step 8: TP-based trust relationships improvement. Let $v_k\in RP^{U,t}$ accept the recommendation to trust another individual $v_l\in TP^{A,t}$ with probability η. Once accepted, set $a_{kl}^t=1$.

Step 9: Update trust relationships $G^{t+1}(D,E^{t+1})$ and A^{t+1}. Let $t=t+1$, then go to Step 3.

Step 10: Output CD^t $(t=1,2,\dots,T)$ and z.

Simulation experiment II

To analyze the effectiveness of SIC, we replace step 5 and 6 in simulation experiment I with step 5′ and 6′, which are presented below.

Step 5′: First, compute social influence matrix $SI^t=(s_{kl}^t)_{m\times m}$ same as step 4(a) in Algorithm 1. Then, identify the top persuaders TP^t and resistant persuadees RP^t in round t based on $SP_l^t=\sum_{k=1,k\neq l}^{m}s_{kl}^t$ and $RP_k^t=1/\sum_{l=1,l\neq k}^{m}s_{kl}^t$.

Step 6′: (1) Let γ_{kl} be the restricted social influence s'_{kl} from v_l to v_k, such as in Equation (24); (2) Let $\gamma_{k(m+1)}\in[\min_l(\gamma_{kl}),\max_l(\gamma_{kl})]$ be the weight v_k assigns to the group suggestion. Then, we can obtain $\bar{\omega}_{kl}=\left(\gamma_{kl}\cdot(1-\beta_k)\right)/\sum_{y=1,y\neq k}^{m+1}\gamma_{ky}$ and $\omega_{kl}=\left(\gamma_{kl}\cdot(1-\beta_k)\right)/\sum_{y=1,y\neq k}^{m}\gamma_{ky}$.

Simulation experiment III

Input: The number of individuals m, the number of alternatives n, the established maximum round T, the consensus threshold μ, and the parameters b, ρ.

Output: The consensus degree CD^t and iteration number z.

Step 1–4: Same as step 1–4 in Simulation experiment I.

Step 5: let γ_{kl} be a trust value from v_k to v_l. If $a_{kl}=0$, then set $\gamma_{kl}=0$; otherwise, generate γ_{kl} uniformly and randomly from interval $(0,1]$. Let $\gamma_{k(m+1)}\in(0,1]$ be the trust value v_k assigns to the group suggestion. Then, we can obtain $\bar{\omega}_{kl} = \left(\gamma_{kl} \cdot (1-\beta_k)\right) / \sum_{y=1, y\neq k}^{m+1} \gamma_{ky}$ and $\omega_{kl} = \left(\gamma_{kl} \cdot (1-\beta_k)\right) / \sum_{y=1, y\neq k}^{m} \gamma_{ky}$.

Step 6: All individuals accept the group suggestion with probability ρ. If they accept the preference adjustment, compute the preference $p_{ij}^{k,t+1}$ based on Equation (34); otherwise, compute the preference $p_{ij}^{k,t+1}$ based on Equation (35).

Step 7: Let $t = t + 1$, then go to Step 3.

Step 8: Output CD^t $(t = 1,2,\ldots,T)$ and z.

Simulation experiment IV

To analyze the effect of top persuaders on consensus reaching, we replace step 4, 5 and 10 in simulation experiment I with step 4′, 5′ and 10′, which are presented below.

Output′: The retention probability TPR^t, alternative stability AS^t, and alternative consistency AC^t.

Step 4′: If $t = T$, go to step 10; otherwise, go to the next step.

Step 5′: (1) Compute social persuasion $SP^t = \left(sp_{ij}^t\right)_{m\times m}$ same as step 4(c) in Algorithm 1. Then, identify TP^t, $TP^{A,t}$, $TP^{U,t}$, and $RP^{U,t}$ same as step 5(a). (2) If $t = 0$, let $TP^* = TP^t$, and for each individual v_k, let $x_{(1)}^{k,0}$ be the initial best alternative of their preference $P^{k,0}$. (3) If $t > 0$, let $x_{(1)}^{c,t}$ and $x_{(1)}^{k,t}$ be the best alternative of the current collective preference $P^{c,t}$ and individual preference $P^{k,t}$. For each individual $v_k \in TP^*$, compute the following indicators: (a) the retention probability TPR, if $v_k \in TP^t$, then $TPR^{k,t} = 1$, otherwise $TPR^{k,t} = 0$; (b) alternative stability AS^t, if $x_{(1)}^{k,t} = x_{(1)}^{k,0}$, then $AS^{k,t} = 1$, otherwise $AS^{k,t} = 0$; (c) alternative consistency AC, if $x_{(1)}^{c,t} = x_{(1)}^{k,0}$, then $AC^{k,t} = 1$, otherwise $AC^{k,t} = 0$.

Step 10′: Output TPR^t, AS^t, and AC^t $(t = 1,2,\ldots,T)$.

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