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Soft Rough Neutrosophic Influence Graphs with Application

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Abstract: In this paper, we apply the notion of soft rough neutrosophic sets to graph theory. We develop certain new concepts, including soft rough neutrosophic graphs, soft rough neutrosophic influence graphs, soft rough neutrosophic influence cycles and soft rough neutrosophic influence trees. We illustrate these concepts with examples, and investigate some of their properties. We solve the decision-making problem by using our proposed algorithm.

Keywords: soft rough neutrosophic graphs; soft rough neutrosophic influence graphs; soft rough neutrosophic influence cycles; soft rough neutrosophic influence trees

1. Introduction

Smarandache [1] introduced neutrosophic sets as a generalization of fuzzy sets and intuitionistic fuzzy sets. A neutrosophic set has three constituents: truth-membership, indeterminacy-membership and falsity-membership, in which each membership value is a real standard or non-standard subset of $]0^-, 1^+[$. In real-life problems, neutrosophic sets can be applied more appropriately by using the single-valued neutrosophic sets defined by Smarandache [1] and Wang et al. [2]. Ye [3,4] and Peng et al. [5] further extended the study of neutrosophic sets. Soft set theory [6] was proposed by Molodtsov in 1999 to deal with uncertainty in a parametric manner. Babitha and Sunil discussed the concept of soft set relation [7]. On the other hand, Pawlak [8] proposed the notion of rough sets. It is a rigid appearance of modeling and processing partial information. It has been effectively connected to decision analysis, machine learning, inductive reasoning, intelligent systems, pattern recognition, signal analysis, expert systems, knowledge discovery, image processing, and many other fields [9–12]. In literature, rough theory has been applied in different field of mathematics [13–16]. Dubois and Prade [17] developed two concepts called rough fuzzy sets and fuzzy rough sets and concluded that these two theories are different approaches to handle vagueness. Feng et al. [18] combined soft sets with fuzzy sets and rough sets. Meng et al. [19] dealt with soft rough fuzzy sets and soft fuzzy rough sets. Broumi et al. [20] studied rough neutrosophic sets. Yang et al. [21] proposed single-valued neutrosophic rough sets, and established an algorithm for decision-making problem based on single- valued neutrosophic rough sets on two universes.

A graph is a convenient way of representing information involving relationship between objects. The objects are represented by vertices and relations by edges. When there is vagueness in the description of the objects or in its relationships or in both, it is natural that we need to design a fuzzy graph model. Fuzzy models has vital role as their aspiration in decreasing the irregularity between the traditional numerical models used in engineering and sciences and the symbolic models used in expert systems. The fuzzy graph theory as a generalization of Euler's graph theory was first introduced by

Kaufmann [22]. Later, Rosenfeld [23] considered fuzzy graphs and obtained analogs of several graph theoretical concepts. Mordeson and Peng [24] defined some operations on fuzzy graphs. Mathew and Sunitha [25,26] presented some new concepts on fuzzy graphs. Gani et al. [27–30] discussed several concepts, including size, order, degree, regularity and edge regularity in fuzzy graphs and intuitionistic fuzzy graphs. Parvathi and Karunambigai [31] described some operation on intuitionistic fuzzy graph. Recently, Akram et al. [32–36] has introduced several extensions of fuzzy graphs with applications. Denish [37] considered the idea of fuzzy incidence graph. Fuzzy incidence graphs were further studied in [38,39]. Due to the limitation of humans knowledge to understand the complex problems, it is very difficult to apply only a single type of uncertainty method to deal with such problems. Therefore, it is necessary to develop hybrid models by incorporating the advantages of many other different mathematical models dealing uncertainty. Recently, new hybrid models, including rough fuzzy graphs [40,41], fuzzy rough graphs [42], intuitionistic fuzzy rough graphs [43,44], rough neutrosophic graphs [45] and neutrosophic soft rough graphs [46] are constructed. For other notations and definitions, the readers are refereed to [47–51]. In this paper, we apply the notion of soft rough neutrosophic sets to graph theory. We develop certain new concepts, including soft rough neutrosophic graphs, soft rough neutrosophic influence graphs, soft rough neutrosophic influence cycles and soft rough neutrosophic influence trees. We illustrate these concepts with examples, and investigate some of their properties. We solve decision-making problem by using our proposed algorithm.

This paper is organized as follows. In Section 2, some definitions and some properties of soft rough neutrosophic graphs are given. In Section 3, soft rough neutrosophic influence graphs, soft rough neutrosophic influence cycles and soft rough neutrosophic influence trees are discussed. In Section 4, an application is presented. Finally, we conclude our contribution with a summary in Section 5 and an outlook for the further research.

2. Soft Rough Neutrosophic Graphs

Definition 1. *Let B be Boolean set and A a set of attributes. For an arbitrary full soft set S over B such that $S_s(a)\subset B$, for some $a\in A$, where $S_s{:}A\to\mathcal{P}(B)$ is a set-valued function defined as $S_s(a)=\{b\in B|(a,b)\in S\}$, for all $a\in A$. Let (B,S) be a full soft approximation space. For any neutrosophic set $N=\{(b,T_N(b),I_N(b),F_N(b))|\ b\in B\}\in\mathcal{N}(B)$, where $\mathcal{N}(B)$ is neutrosophic power set of set B. The upper and lower soft rough neutrosophic approximations of N* w.r.t *(B,S), denoted by $\overline{S}(N)$ and $\underline{S}(N)$, respectively, are defined as follows:*

$$\overline{S}(N)=\{(b,T_{\overline{S}(N)}(b),I_{\overline{S}(N)}(b),F_{\overline{S}(N)}(b))\mid b\in B\},$$
$$\underline{S}(N)=\{(b,T_{\underline{S}(N)}(b),I_{\underline{S}(N)}(b),F_{\underline{S}(N)}(b))\mid b\in B\},$$

where

$$\begin{aligned}
T_{\overline{S}(N)}(b)&=\bigwedge_{b\in S_s(a)}\ \bigvee_{t\in S_s(a)} T_N(t),\ \ T_{\underline{S}(N)}(b)=\bigvee_{b\in S_s(a)}\ \bigwedge_{t\in S_s(a)} T_N(t),\\
I_{\overline{S}(N)}(b)&=\bigvee_{b\in S_s(a)}\ \bigwedge_{t\in S_s(a)} I_N(t),\ \ I_{\underline{S}(N)}(b)=\bigwedge_{b\in S_s(a)}\ \bigvee_{t\in S_s(a)} I_N(t),\\
F_{\overline{S}(N)}(b)&=\bigvee_{b\in S_s(a)}\ \bigwedge_{t\in S_s(a)} F_N(t),\ \ F_{\underline{S}(N)}(b)=\bigwedge_{b\in S_s(a)}\ \bigvee_{t\in S_s(a)} F_N(t).
\end{aligned}\tag{1}$$

In other words,

$$T_{\overline{S}(N)}(b)=\bigwedge_{a\in A}\Big((1-S(a,b))\vee\Big(\bigvee_{t\in B}(S(a,t)\wedge T_N(t))\Big)\Big),$$
$$T_{\underline{S}(N)}(b)=\bigvee_{a\in A}\Big(S(a,b)\wedge\Big(\bigwedge_{t\in B}((1-S(a,t))\vee T_N(t))\Big)\Big),$$

$$I_{\overline{S}(N)}(b) = \bigvee_{a \in A} \Big(S(a,b) \wedge \Big(\bigwedge_{t \in B} ((1 - S(a,t)) \vee I_N(t)) \Big) \Big),$$
$$I_{\underline{S}(N)}(b) = \bigwedge_{a \in A} \Big((1 - S(a,b)) \vee \Big(\bigvee_{t \in B} (S(a,t) \wedge I_N(t)) \Big) \Big),$$
$$F_{\overline{S}(N)}(b) = \bigvee_{a \in A} \Big(S(a,b) \wedge \Big(\bigwedge_{t \in B} ((1 - S(a,t)) \vee F_N(t)) \Big) \Big),$$
$$F_{\underline{S}(N)}(b) = \bigwedge_{a \in A} \Big((1 - S(a,b)) \vee \Big(\bigvee_{t \in B} (S(a,t) \wedge F_N(t)) \Big) \Big).$$

The pair $(\underline{S}(N), \overline{S}(N))$ *is called soft rough neutrosophic set (SRNS) of* N w.r.t (B,S).

Example 1. *Suppose* $N=\{(b_1,0.8,0.3,0.16),(b_2,0.85,0.24,0.2),(b_3,0.79,0.2,0.2),(b_4,0.85,0.36,0.25),(b_5,0.82,0.25,0.25)\}$ *is a neutrosophic set on the universal set* $B=\{b_1,b_2,b_3,b_4,b_5\}$ *under consideration. Let* $A=\{a_1,a_2,a_3\}$ *be a set of parameter on B. A full soft set over B, denoted by S, is defined in Table 1.*

Table 1. Full soft set S.

S	b_1	b_2	b_3	b_4	b_5
a_1	0	0	1	0	1
a_2	1	0	1	0	0
a_3	0	1	1	1	1

A set-valued function $S_s{:}A \to \mathcal{P}(B)$ *is defined as* $S_s(a_1)=\{b_3,b_5\}, S_s(a_2)=\{b_1,b_3\}, S_s(a_3)=\{b_2,b_3,b_4,b_5\}$. *From Equation (1) of Definition 1, we have*

$$T_{\overline{S}(A)}(b_1) = \bigvee_{y \in S_s(a_2)} N(y) = \vee\{0.8, 0.79\} = 0.80,$$
$$I_{\overline{S}(N)}(b_1) = \bigwedge_{y \in S_s(a_2)} N(y) = \wedge\{0.3, 0.2\} = 0.20,$$
$$F_{\overline{S}(N)}(b_1) = \bigwedge_{y \in S_s(a_2)} N(y) = \wedge\{0.16, 0.2\} = 0.16;$$
$$T_{\underline{S}(N)}(b_1) = \bigwedge_{y \in S_s(a_2)} N(y) = \wedge\{0.8, 0.79\} = 0.79,$$
$$I_{\underline{S}(N)}(b_1) = \bigvee_{y \in S_s(a_2)} N(y) = \vee\{0.3, 0.2\} = 0.30,$$
$$F_{\underline{S}(N)}(b_1) = \bigvee_{y \in S_s(a_2)} N(y) = \vee\{0.16, 0.2\} = 0.20.$$

Similarly,

$$T_{\overline{S}(N)}(b_2) = 0.85, \quad I_{\overline{S}(N)}(b_2) = 0.20, \quad F_{\overline{S}(N)}(b_2) = 0.20,$$
$$T_{\overline{S}(N)}(b_3) = 0.80, \quad I_{\overline{S}(N)}(b_3) = 0.20, \quad F_{\overline{S}(N)}(b_3) = 0.20,$$
$$T_{\overline{S}(N)}(b_4) = 0.85, \quad I_{\overline{S}(N)}(b_4) = 0.20, \quad F_{\overline{S}(N)}(b_4) = 0.20,$$
$$T_{\overline{S}(N)}(b_5) = 0.82, \quad I_{\overline{S}(N)}(b_5) = 0.20, \quad F_{\overline{S}(N)}(b_5) = 0.20;$$
$$T_{\underline{S}(N)}(b_2) = 0.79, \quad I_{\underline{S}(N)}(b_2) = 0.36, \quad F_{\underline{S}(N)}(b_2) = 0.25,$$
$$T_{\underline{S}(N)}(b_3) = 0.79, \quad I_{\underline{S}(N)}(b_3) = 0.25, \quad F_{\underline{S}(N)}(b_3) = 0.20,$$
$$T_{\underline{S}(N)}(b_4) = 0.79, \quad I_{\underline{S}(N)}(b_4) = 0.36, \quad F_{\underline{S}(N)}(b_4) = 0.25,$$

$$T_{\underline{S}(N)}(b_5) = 0.79, \quad I_{\underline{S}(N)}(b_5) = 0.25, \quad F_{\underline{S}(N)}(b_5) = 0.25.$$

Thus, we obtain

$$\begin{aligned}
\overline{S}(N) = \{&(b_1, 0.80, 0.20, 0.16), (b_2, 0.85, 0.20, 0.20), (b_3, 0.80, 0.20, 0.20),\\
&(b_4, 0.85, 0.20, 0.20), (b_5, 0.82, 0.20, 0.20)\},\\
\underline{S}(N) = \{&(b_1, 0.79, 0.30, 0.20), (b_2, 0.79, 0.36, 0.25), (b_3, 0.79, 0.25, 0.20),\\
&(b_4, 0.79, 0.36, 0.25), (b_5, 0.79, 0.25, 0.25)\}.
\end{aligned}$$

Definition 2. *A soft rough neutrosophic relation(SRNR)* $(\underline{R}(M), \overline{R}(M))$ *on* $\tilde{B} = B \times B$ *is a soft rough neutrosophic set,* $R : \tilde{A}(A \times A) \to \mathcal{P}(\tilde{B})$ *is a full soft set on* $\tilde{B}$ *and defined by*

$$R(a_{kl}, b_{ij}) \leq \min\{S(a_k, b_i), S(a_l, b_j)\},$$

for all $(a_{kl}, b_{ij}) \in R$*, such that* $R_s(a_{kl}) \subset \tilde{B}$ *for some* $a_{kl} \in \tilde{A}$*, where* $R_s : \tilde{A} \to \mathcal{P}(\tilde{B})$ *is a set-valued function, for all* $a_{kl} \in \tilde{A}$*, defined by*

$$R_s(a_{kl}) = \{b_{ij} \in \tilde{B} \mid (a_{kl}, b_{ij}) \in R\}, \;\; b_{ij} \in \tilde{B}.$$

For any neutrosophic set $M \in \mathcal{N}(\tilde{B})$*, the upper and lower soft rough neutrosophic approximation of* M w.r.t $(\tilde{B}, R)$ *are defined as follows:*

$$\begin{aligned}
\overline{R}(M) &= \{(b_{ij}, T_{\overline{R}(M)}(b_{ij}), I_{\overline{R}(M)}(b_{ij}), F_{\overline{R}(M)}(b_{ij})) \mid b_{ij} \in \tilde{B}\},\\
\underline{R}(M) &= \{(b_{ij}, T_{\underline{R}(M)}(b_{ij}), I_{\underline{R}(M)}(b_{ij}), F_{\underline{R}(M)}(b_{ij})) \mid b_{ij} \in \tilde{B}\},
\end{aligned}$$

where

$$\begin{aligned}
T_{\overline{R}(M)}(b_{ij}) &= \bigwedge_{b_{ij} \in R_s(a_{kl})} \bigvee_{t_{ij} \in R_s(a_{kl})} T_M(t_{ij}), \;\; T_{\underline{R}(M)}(b_{ij}) = \bigvee_{b_{ij} \in R_s(a_{kl})} \bigwedge_{t_{ij} \in R_s(a_{kl})} T_M(t_{ij}),\\
I_{\overline{R}(M)}(b_{ij}) &= \bigvee_{b_{ij} \in R_s(a_{kl})} \bigwedge_{t_{ij} \in R_s(a_{kl})} I_M(t_{ij}), \;\; I_{\underline{R}(M)}(b_{ij}) = \bigwedge_{b_{ij} \in R_s(a_{kl})} \bigvee_{t_{ij} \in R_s(a_{kl})} I_M(t_{ij}),\\
F_{\overline{R}(M)}(b_{ij}) &= \bigvee_{b_{ij} \in R_s(a_{kl})} \bigwedge_{t_{ij} \in R_s(a_{kl})} F_M(t_{ij}), \;\; F_{\underline{R}(M)}(b_{ij}) = \bigwedge_{b_{ij} \in R_s(a_{kl})} \bigvee_{t_{ij} \in R_s(a_{kl})} F_M(t_{ij}).
\end{aligned} \tag{2}$$

In other words,

$$\begin{aligned}
T_{\overline{R}(M)}(b_{ij}) &= \bigwedge_{a_{kl} \in A} \Big((1 - R(a_{kl}, b_{ij})) \vee \Big(\bigvee_{t_{ij} \in B} (R(a_{kl}, t_{ij}) \wedge T_M(t_{ij})) \Big) \Big),\\
T_{\underline{R}(M)}(b_{ij}) &= \bigvee_{a_{kl} \in A} \Big(R(a_{kl}, b_{ij}) \wedge \Big(\bigwedge_{t_{ij} \in B} ((1 - R(a_{kl}, t_{ij})) \vee T_M(t_{ij})) \Big) \Big),\\
I_{\overline{R}(M)}(b_{ij}) &= \bigvee_{a_{kl} \in A} \Big(R(a_{kl}, b_{ij}) \wedge \Big(\bigwedge_{t_{ij} \in B} ((1 - R(a_{kl}, t_{ij})) \vee I_M(t_{ij})) \Big) \Big),\\
I_{\underline{R}(M)}(b_{ij}) &= \bigwedge_{a_{kl} \in A} \Big((1 - R(a_{kl}, b_{ij})) \vee \Big(\bigvee_{t_{ij} \in B} (R(a_{kl}, t_{ij}) \wedge I_M(t_{ij})) \Big) \Big),\\
F_{\overline{R}(M)}(b_{ij}) &= \bigvee_{a_{kl} \in A} \Big(R(a_{kl}, b_{ij}) \wedge \Big(\bigwedge_{t_{ij} \in B} ((1 - R(a_{kl}, t_{ij})) \vee F_M(t_{ij})) \Big) \Big),\\
F_{\underline{R}(M)}(b_{ij}) &= \bigwedge_{a_{kl} \in A} \Big((1 - R(a_{kl}, b_{ij})) \vee \Big(\bigvee_{t_{ij} \in B} (R(a_{kl}, t_{ij}) \wedge F_M(t_{ij})) \Big) \Big).
\end{aligned}$$

If $\overline{R}(M)=\underline{R}(M)$, then it is called induced soft rough neutrosophic relation on soft rough neutrosophic set, otherwise, soft rough neutrosophic relation.

Remark 1. *For a neutrosophic set M on $\tilde{B}$ and a neutrosophic set N on B, we have neutrosophic relation as follow*

$$T_M(b_{ij}) \leq \min_i\{T_N(b_i)\}, \quad I_M(b_{ij}) \leq \min_i\{I_N(b_i)\}, \quad F_M(b_{ij}) \leq \min_i\{F_N(b_i)\}.$$

From Definition 2, it follows that:

$$\begin{aligned}
&T_{\overline{R}(M)}(b_{ij}) \leq \min\{T_{\overline{S}(N)}(b_i), T_{\overline{S}(N)}(b_j)\}, \quad T_{\underline{R}(M)}(b_{ij}) \leq \min\{T_{\underline{S}(N)}(b_i), T_{\underline{S}(N)}(b_j)\},\\
&I_{\overline{R}(M)}(b_{ij}) \leq \max\{I_{\overline{S}(N)}(b_i), I_{\overline{S}(N)}(b_j)\}, \quad I_{\underline{R}(M)}(b_{ij}) \leq \max\{I_{\underline{S}(N)}(b_i), I_{\underline{S}(N)}(b_j)\},\\
&F_{\overline{R}(M)}(b_{ij}) \leq \max\{F_{\overline{S}(N)}(b_i), F_{\overline{S}(N)}(b_j)\}, \quad F_{\underline{R}(M)}(b_{ij}) \leq \max\{F_{\underline{S}(N)}(b_i), F_{\underline{S}(N)}(b_j)\}.
\end{aligned}$$

Definition 3. *In Definition 2 b_{ij} is called effective, if*

$$\begin{aligned}
&T_{\underline{R}(M)}(b_{ij}) = T_{\underline{S}(N)}(b_i) \wedge T_{\underline{S}N}(b_j), \; T_{\overline{R}(M)}(b_{ij}) = T_{\overline{S}(N)}(b_i) \wedge T_{\overline{S}N}(b_j),\\
&I_{\underline{R}(M)}(b_{ij}) = I_{\underline{S}(N)}(b_i) \vee I_{\underline{S}N}(b_j), \; I_{\overline{R}(M)}(b_{ij}) = I_{\overline{S}(N)}(b_i) \vee I_{\overline{S}N}(b_j),\\
&F_{\underline{R}(M)}(b_{ij}) = F_{\underline{S}(N)}(b_i) \vee F_{\underline{S}N}(b_j), \; F_{\overline{R}(M)}(b_{ij}) = F_{\overline{S}(N)}(b_i) \vee F_{\overline{S}N}(b_j).
\end{aligned}$$

Definition 4. *A soft rough neutrosophic influence (SRNI) is a relation from soft rough neutrosophic set to soft rough neutrosophic relation, denoted by $(\underline{X}(Q),\overline{X}(Q))$ on $\hat{B}=B\times\tilde{B}$, where $X{:}\hat{A}(A\times\tilde{A})\to\mathcal{P}(\hat{B})$ is a full soft set on $\hat{B}$ defined by*

$$X(a_l a_{mn}, b_i b_{jk}) \leq S(a_l, b_i) \wedge R(a_{mn}, b_{jk}),$$

for all $(a_l a_{mn}, b_i b_{jk}) \in X$ and for some $i \neq j \neq k$ and $l \neq m \neq n$. Let $X_s{:}\hat{A}\to\mathcal{P}(\hat{B})$ be a set-valued function defined by

$$X_s(a_l a_{mn}) = \{b_i b_{jk} \in \hat{B} | (a_l a_{mn}, (b_i, , b_{jk})) \in X\}, \; \forall (a_, l a_{mn}) \in \hat{A},$$

For any $Q \in \mathcal{N}(\hat{B})$, the upper and lower soft rough neutrosophic approximation of Q w.r.t *$(\hat{B},X)$, for all $b_i b_{jk} \in \hat{B}$, are defined as follows:*

$$\begin{aligned}
&\overline{X}(Q) = \{(b_i b_{jk}, T_{\overline{X}(Q)}(b_i b_{jk}), I_{\overline{X}(Q)}(b_i b_{jk}), F_{\overline{X}(Q)}(b_i b_{jk}))\},\\
&\underline{X}(Q) = \{(b_i b_{jk}, T_{\underline{X}(Q)}(b_i b_{jk}), I_{\underline{X}(Q)}(b_i b_{jk}), F_{\underline{X}(Q)}(b_i b_{jk}))\},
\end{aligned}$$

where

$$\begin{aligned}
T_{\overline{X}(Q)}(b_i b_{jk}) &= \bigwedge_{b_i b_{jk} \in X_s(a_l a_{mn})} \; \bigvee_{t_i t_{jk} \in X_s(a_l a_{mn})} T_Q(t_i t_{jk}),\\
T_{\underline{X}(Q)}(b_i b_{jk}) &= \bigvee_{b_i b_{jk} \in X_s(a_l a_{mn})} \; \bigwedge_{t_i t_{jk} \in X_s(a_l a_{mn})} T_Q(t_i t_{jk}),\\
I_{\overline{X}(Q)}(b_i b_{jk}) &= \bigvee_{b_i b_{jk} \in X_s(a_l a_{mn})} \; \bigwedge_{t_i t_{jk} \in X_s(a_l a_{mn})} I_Q(t_i t_{jk}),\\
I_{\underline{X}(Q)}(b_i b_{jk}) &= \bigwedge_{b_i b_{jk} \in X_s(a_l a_{mn})} \; \bigvee_{t_i t_{jk} \in X_s(a_l a_{mn})} I_Q(t_i t_{jk}),\\
F_{\overline{X}(Q)}(b_i b_{jk}) &= \bigvee_{b_i b_{jk} \in X_s(a_l a_{mn})} \; \bigwedge_{t_i t_{jk} \in X_s(a_l a_{mn})} F_Q(t_i t_{jk}),
\end{aligned} \tag{3}$$

$$F_{\underline{X}(Q)}(b_ib_{jk}) = \bigwedge_{b_ib_{jk}\in X_s(a_la_{mn})} \bigvee_{t_it_{jk}\in X_s(a_la_{mn})} F_Q(t_it_{jk}) .$$

In other words,

$$T_{\overline{X}(Q)}(b_ib_{jk}) = \bigwedge_{a_la_{mn}\in A}\Big((1-X(a_la_{mn},b_ib_{jk}))\vee\Big(\bigvee_{t_it_{jk}\in B}(X(a_la_{mn},t_it_{jk})\wedge T_Q(t_it_{jk}))\Big)\Big),$$

$$T_{\underline{X}(Q)}(b_ib_{jk}) = \bigvee_{a_la_{mn}\in A}\Big(X(a_la_{mn},b_ib_{jk})\wedge\Big(\bigwedge_{t_it_{jk}\in B}((1-X(a_la_{mn},t_it_{jk}))\vee T_Q(t_it_{jk}))\Big)\Big),$$

$$I_{\overline{X}(Q)}(b_ib_{jk}) = \bigvee_{a_la_{mn}\in A}\Big(X(a_la_{mn},b_ib_{jk})\wedge\Big(\bigwedge_{t_it_{jk}\in B}((1-X(a_la_{mn},t_it_{jk}))\vee I_Q(t_it_{jk}))\Big)\Big),$$

$$I_{\underline{X}(Q)}(b_ib_{jk}) = \bigwedge_{a_la_{mn}\in A}\Big((1-X(a_la_{mn},b_ib_{jk}))\vee\Big(\bigvee_{t_it_{jk}\in B}(X(a_la_{mn},t_it_{jk})\wedge I_Q(t_it_{jk}))\Big)\Big),$$

$$F_{\overline{X}(Q)}(b_ib_{jk}) = \bigvee_{a_la_{mn}\in A}\Big(X(a_la_{mn},b_ib_{jk})\wedge\Big(\bigwedge_{t_it_{jk}\in B}((1-X(a_la_{mn},t_it_{jk}))\vee F_Q(t_it_{jk}))\Big)\Big),$$

$$F_{\underline{X}(Q)}(b_ib_{jk}) = \bigwedge_{a_la_{mn}\in A}\Big((1-X(a_la_{mn},b_ib_{jk}))\vee\Big(\bigvee_{t_it_{jk}\in B}(X(a_la_{mn},t_it_{jk})\wedge F_Q(t_it_{jk}))\Big)\Big).$$

Remark 2. *For a neutrosophic set Q on $\hat{B}$ and a neutrosophic set N and M on B and $\tilde{B}$, respectively, we have neutrosophic relation as follow*

$$T_Q(b_ib_{jk}) \le \min_{jk}\{T_M(b_{jk})\},\quad I_Q(b_ib_{jk}) \le \min_{jk}\{I_M(b_{jk})\},\quad F_Q(b_ib_{jk}) \le \min_{jk}\{F_M(b_{jk})\}.$$

From Definition 4, we have

$$T_{\overline{X}(Q)}(b_ib_{jk}) \le \min\{T_{\overline{S}(N)}(b_i), T_{\overline{R}(M)}(b_{jk})\},\quad T_{\underline{X}(Q)}(b_ib_{jk}) \le \min\{T_{\underline{S}(N)}(b_i), T_{\underline{R}(M)}(b_{jk})\},$$
$$I_{\overline{X}(Q)}(b_ib_{jk}) \le \max\{I_{\overline{S}(N)}(b_i), I_{\overline{R}(M)}(b_{jk})\},\quad I_{\underline{X}(Q)}(b_ib_{jk}) \le \max\{I_{\underline{S}(N)}(b_i), I_{\underline{R}(M)}(b_{jk})\},$$
$$F_{\overline{X}(Q)}(b_ib_{jk}) \le \max\{F_{\overline{S}(N)}(b_i), F_{\overline{R}(M)}(b_{jk})\},\quad F_{\underline{X}(Q)}(b_ib_{jk}) \le \max\{F_{\underline{S}(N)}(b_i), F_{\underline{R}(M)}(b_{jk})\}.$$

Definition 5. *In Definition 4 b_ib_{jk} is called influence effective, if*

$$T_{\underline{X}(Q)}(b_ib_{jk}) = T_{\underline{S}(N)}(b_i) \wedge T_{\underline{R}M}(b_{ij}),\ T_{\overline{X}(Q)}(b_ib_{jk}) = T_{\overline{S}(N)}(b_i) \wedge T_{\overline{R}M}(b_{ij}),$$
$$I_{\underline{X}(Q)}(b_ib_{jk}) = I_{\underline{S}(N)}(b_i) \vee I_{\underline{R}M}(b_{ij}),\ I_{\overline{X}(Q)}(b_ib_{jk}) = I_{\overline{S}(N)}(b_i) \vee I_{\overline{R}M}(b_{ij}),$$
$$F_{\underline{X}(Q)}(b_ib_{jk}) = F_{\underline{S}(N)}(b_i) \vee F_{\underline{R}M}(b_{ij}),\ F_{\overline{X}(Q)}(b_ib_{jk}) = F_{\overline{S}(N)}(b_i) \vee F_{\overline{R}M}(b_{ij}).$$

Example 2. *Let a full soft set S on an universal set $B=\{b_1,b_2,b_3,b_4\}$ with $A=\{a_1,a_2,a_3\}$ a set of parameters can be defined in tabular form as Table 2 as follows:*

Table 2. Full soft set S.

S	b_1	b_2	b_3	b_4
a_1	1	1	0	1
a_2	0	0	1	1
a_3	1	1	1	1

Now, we can define set-valued function S_s such that

$$S_s(a_1) = \{b_1, b_2, b_4\}, S_s(a_2) = \{b_3, b_4\}, S_s(a_3) = \{b_1, b_2, b_3, b_4\}.$$

Let $N = \{(b_1,1.0,0.0,0.0),(b_2,0.8,0.0,0.1),(b_3,0.5,0.5,0.5),(b_4,0.4,0.7,0.3)\}$ *be a neutrosophic set on* B, *then by using Equation (1) of Definition 1, we have*

$$\overline{S}(N) = \{(b_1, 1.0, 0.0, 0.0), (b_2, 1.0, 0.0, 0.0), (b_3, 0.5, 0.5, 0.3), (b_4, 0.5, 0.5, 0.3)\},$$

$$\underline{S}(N) = \{(b_1, 0.4, 0.7, 0.3), (b_2, 0.4, 0.7, 0.3), (b_3, 0.4, 0.7, 0.5), (b_4, 0.4, 0.7, 0.3)\}.$$

Hence $(\underline{S}(N), \overline{S}(N))$ *is soft rough neutrosophic set. Let a full soft set* R *on* $C=\{b_{12},b_{22},b_{23},b_{32},b_{42}\}\subseteq\tilde{B}$ *with* $L=\{a_{13},a_{21},a_{32}\}\subseteq\tilde{A}$ *can be defined in Table 3 (from L to C) as follows:*

Table 3. Full soft set R.

R	b_{12}	b_{22}	b_{23}	b_{32}	b_{42}
a_{13}	1	1	1	0	1
a_{21}	0	0	0	1	0
a_{32}	0	0	1	0	0

Now, we can define set-valued function R_s *such that*

$$R_s(a_{13}) = \{b_{12}, b_{22}, b_{23}, b_{42}\}, R_s(a_{21}) = \{b_{32}\}, R_s(a_{32}) = \{b_{23}\}.$$

and $M = \{(b_{12},0.4,0.0,0.0),(b_{22},0.4,0.0,0.0),(b_{23},0.4,0.0,0.0),(b_{32},0.4,0.0,0.0),(b_{42},0.4,0.0,0.0)\}$ *a neutrosophic relation on* B, *then by using Equation (2) of Definition 2, we get*

$$\overline{R}(M)=\{(b_{12}, 0.4, 0.0, 0.0),(b_{22}, 0.4, 0.0, 0.0),(b_{23}, 0.4, 0.0, 0.0),(b_{32}, 0.4, 0.0, 0.0),(b_{42}, 0.4, 0.0, 0.0)\},$$
$$\underline{R}(M)=\{(b_{12}, 0.4, 0.0, 0.0),(b_{22}, 0.4, 0.0, 0.0),(b_{23}, 0.4, 0.0, 0.0),(b_{32}, 0.4, 0.0, 0.0),(b_{42}, 0.4, 0.0, 0.0)\}.$$

Hence $(\underline{R}(M), \overline{R}(M))$ *is an induced soft rough neutrosophic relation. Let a full soft set* X *on* $D=\{b_1b_{22},b_1b_{23},b_1b_{32},b_1b_{42},b_3b_{12},b_3b_{22},b_3b_{42},b_4b_{12},b_4b_{22},b_4b_{23},b_4b_{32}\}\subseteq\hat{B}$ *with* $P=\{a_{13},a_{21},a_{32}\}\subseteq\hat{A}$ *can be defined in Table 4 (from P to D) as follows:*

Table 4. Full soft set X.

X	b_1b_{22}	b_1b_{23}	b_1b_{32}	b_1b_{42}	b_3b_{12}	b_3b_{22}	b_3b_{42}	b_4b_{12}	b_4b_{22}	b_4b_{23}	b_4b_{32}
a_1a_{32}	0	1	0	0	0	0	0	0	0	1	0
a_2a_{13}	0	0	0	0	0	1	1	1	1	1	1
a_3a_{21}	0	1	0	0	0	0	0	0	0	0	1

Since X *is not full soft set on* D, *therefore, soft rough neutrosophic influence cannot be defined on* D.

Definition 6. *A soft rough neutrosophic graph on a nonempty* V *is a* 5*-ordered tuple* $G=(A,S,SN,R,RM)$ *such that*

(i) A *is a set of attributes,*
(ii) S *is an arbitrary full soft set over* V,
(iii) R *is an arbitrary full soft set over* $E \subseteq \tilde{V}$,
(vi) $SN=(\underline{S}(N),\overline{S}(N))$ *is a soft rough neutrosophic set of* V,
(v) $RM=(\underline{R}(M),\overline{R}(M))$ *is a soft rough neutrosophic set on* $E \subset \tilde{V}$,

In other words $G=(\underline{G},\overline{G})=(SN,RM)$ *is a soft rough neutrosophic graph(SRNG), where* $\underline{G}=(\underline{S}(N),\underline{R}(M))$ *and* $\overline{G}=(\overline{S}(N),\overline{R}(M))$ *are lower soft rough neutrosophic approximate graphs (LSRNAGs) and upper soft rough neutrosophic approximate graphs (USRNAGs), respectively, of* $G=(SN,RM)$.

Example 3. *Let $V=\{v_1,v_2,v_3,v_4,v_5,v_6\}$ be a vertex set and $A=\{a_1,a_2,a_3\}$ a set of parameters. A full soft set S from A on V can be defined in tabular form in Table 5 as follows:*

Table 5. Full soft set S.

S	v_1	v_2	v_3	v_4	v_5	v_6
a_1	1	1	1	1	1	0
a_2	0	0	1	1	1	1
a_3	1	1	0	0	1	1

Let $N=\{(v_1,0.8,0.6,0.4),(v_2,0.9,0.4,0.45),(v_3,0.7,0.4,0.35),(v_4,0.6,0.3,0.5),(v_5,0.4,0.7,0.6),(v_6,0.5,0.5,0.5)\}$ be a neutrosophic set on V. Then from Equation (1) of Definition 1, we have

$$\overline{S}(N)=\{(v_1,0.9,0.4,0.4),(v_2,0.9,0.4,0.4),(v_3,0.7,0.3,0.5),(v_4,0.7,0.3,0.5),(v_5,0.7,0.4,0.5),(v_6,0.7,0.4,0.5)\},$$
$$\underline{S}(N)=\{(v_1,0.4,0.7,0.6),(v_2,0.4,0.7,0.6),(v_3,0.4,0.7,1.0),(v_4,0.4,0.7,1.0),(v_5,0.4,0.7,0.6),(v_6,0.4,0.7,0.6)\}.$$

Hence, $SN=(\underline{S}(N),\overline{S}(N))$ is a soft rough neutrosophic set on V. Let $E=\{v_{11},v_{15},v_{16},v_{23},v_{25},v_{34},v_{41},v_{43},v_{56},v_{62},v_{63}\}\subseteq\tilde{V}$ and $L=\{a_{12},a_{13},a_{21},a_{23},a_{31}\}\subseteq\tilde{A}$. Then a full soft set R on E (from L to E) can be defined in Table 6 as follows:

Table 6. Full soft set R.

R	v_{11}	v_{15}	v_{16}	v_{23}	v_{25}	v_{34}	v_{41}	v_{43}	v_{56}	v_{62}	v_{63}
a_{12}	0	1	1	1	1	1	0	1	1	0	0
a_{13}	1	1	1	0	1	0	1	0	1	0	0
a_{21}	0	0	0	0	0	1	1	1	0	1	1
a_{23}	0	0	0	0	0	0	1	0	1	1	0
a_{31}	1	1	0	1	1	0	0	0	0	1	0

Let $M=\{(v_{11},0.4,0.3,0.35),(v_{15},0.3,0.3,0.2),(v_{16},0.3,0.2,0.25),(v_{23},0.4,0.1,0.1),(v_{25},0.4,0.2,0.0),(v_{34},0.3,0.1,0.3),(v_{41},0.2,0.1,0.2),(v_{43},0.4,0.28,0.2),(v_{56},0.4,0.3,0.3),(v_{62},0.35,0.25,0.32),(v_{63},0.4,0.12,0.34)\}$ be a neutrosophic set on E. Then from Equation (2) of Definition 2, we have

$$\overline{R}(M)=\{(v_{11},0.4,0.1,0.00),(v_{15},0.4,0.10,0.00),(v_{16},0.4,0.10,0.00),(v_{23},0.4,0.10,0.00),(v_{25},0.4,0.1,0.00),(v_{34},0.4,0.10,0.20),(v_{41},0.4,0.10,0.30),(v_{43},0.4,0.10,0.20),(v_{56},0.4,0.1,0.30),(v_{62},0.4,0.10,0.30),(v_{63},0.4,0.10,0.20)\},$$
$$\underline{R}(M)=\{(v_{11},0.3,0.3,0.35),(v_{15},0.3,0.30,0.35),(v_{16},0.3,0.30,1.00),(v_{23},0.3,0.30,0.35),(v_{25},0.3,0.3,0.35),(v_{34},0.3,0.28,0.34),(v_{41},0.2,0.28,0.32),(v_{43},0.3,0.28,0.34),(v_{56},0.3,0.3,0.32),(v_{62},0.3,0.28,0.32),(v_{63},0.2,0.28,0.34)\}.$$

Hence, $RM=(\underline{R}(M),\overline{R}(M))$ is soft rough neutrosophic set on E. Thus, $\underline{G}=(\underline{S}(N),\underline{R}(M))$ and $\overline{G}=(\overline{S}(N),\overline{R}(M))$ are LSRNAG and USRNAG, respectively, as shown in Figure 1.

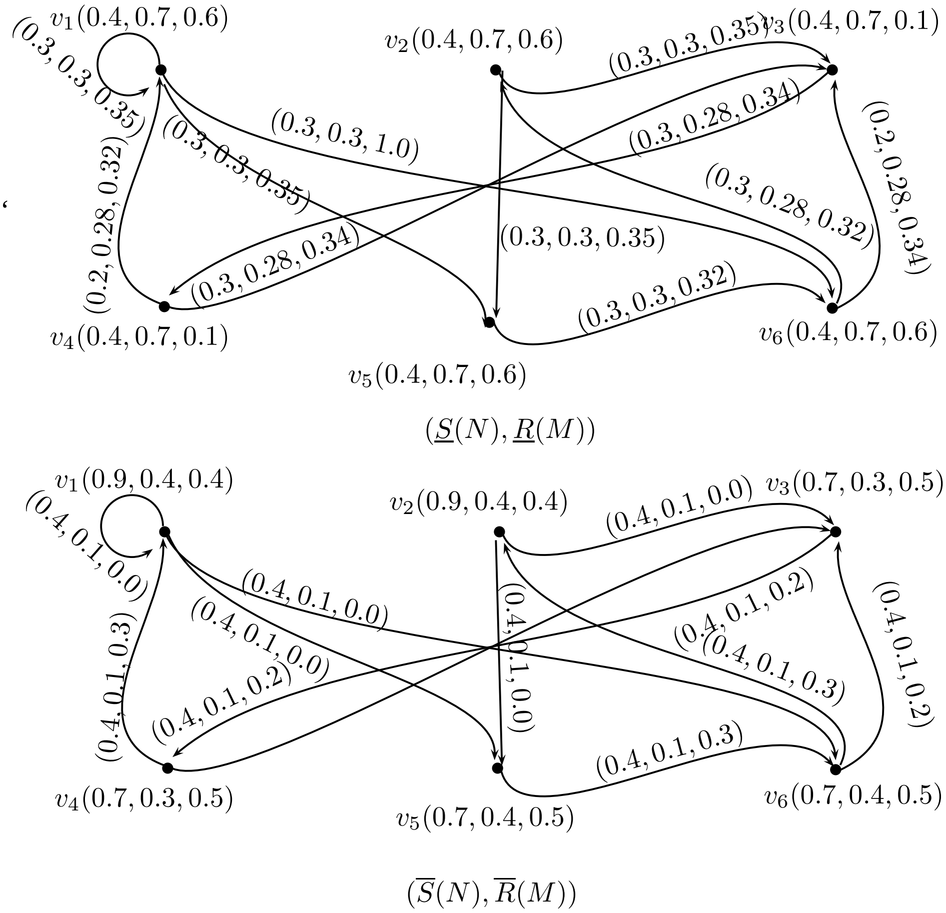

Figure 1. Soft rough neutrosophic graph $G = (\underline{G}, \overline{G})$.

Hence, $G=(\overline{G},\underline{G})$ *is SRNG.*

Definition 7. *An underlying graph(supporting graph)* $G^*=(\underline{G}^*,\overline{G}^*)$ *of a soft rough neutrosophic graph* $G=(\underline{G},\overline{G})$ *is of the form* $\underline{G}^*=(\underline{V}^*,\underline{E}^*)$ *and* $\overline{G}^*=(\overline{V}^*,\overline{E}^*)$,

$$\underline{V}^* = \textit{Lower Vertex Set} = \{v \in V | T_{\underline{S}(N)}(v) \neq 0, I_{\underline{S}(N)}(v) \neq 0, F_{\underline{S}(N)}(v) \neq 0\},$$
$$\overline{V}^* = \textit{Upper Vertex Set} = \{v \in V | T_{\overline{S}(N)}(v) \neq 0, I_{\overline{S}(N)}(v) \neq 0, F_{\overline{S}(N)}(v) \neq 0\},$$
$$\underline{E}^* = \textit{Lower Edge Set} = \{v_{ij} \in E | T_{\underline{R}(M)}(v_{ij}) \neq 0, I_{\underline{R}(M)}(v_{ij}) \neq 0, F_{\underline{R}(M)}(v_{ij}) \neq 0\},$$
$$\overline{E}^* = \textit{Upper Edge Set} = \{v_{ij} \in E | T_{\underline{R}(M)}(v_{ij}) \neq 0, I_{\underline{R}(M)}(v_{ij}) \neq 0, F_{\underline{R}(M)}(v_{ij}) \neq 0\}.$$

Definition 8. *A soft rough neutrosophic graph has a walk if each approximation graph has an alternative sequence of the form*

$$v_0, e_0, v_1, e_1, v_2, \cdots, v_{n-1}, e_{n-1}, v_n$$

such that

$$v_k \in \underline{V}^*,\ e_k \in \underline{E}^*,\ v_k \in \overline{V}^*,\ e_k \in \overline{E}^*$$

where $e_k = v_{k(k+1)} \in E$, $\forall k$=0,1,2,$\cdots$,$n-1$. *If* $v_0 = v_n$, *then it is called closed walk. If the edges are distinct, then it is called a soft rough neutrosophic trail (SRN trail). If the vertices are distinct, then it is called a soft rough neutrosophic path (SRN path). If a path in a SRNG is closed, then it is called a cycle.*

Definition 9. *A strength of soft rough neutrosophic graph, denoted by stren, is defined as*

$$stren=\Big(\big(\bigwedge_{v_{jk}\in\underline{E}^*} T_{\underline{R}(M)}(v_{jk})\big)\wedge\big(\bigwedge_{v_{jk}\in\overline{E}^*} T_{\overline{R}(M)}(v_{jk})\big),\big(\bigvee_{v_{jk}\in\underline{E}^*} I_{\underline{R}(M)}(v_{jk})\big)\vee$$
$$\big(\bigvee_{v_{jk}\in\overline{E}^*} I_{\overline{R}(M)}(v_{jk})\big),\big(\bigvee_{v_{jk}\in\underline{E}^*} F_{\underline{R}(M)}(v_{jk})\big)\vee\big(\bigvee_{v_{jk}\in\overline{E}^*} F_{\overline{R}(M)}(v_{jk})\big)\Big).$$

Definition 10. *A strongest path joining any two vertices v_i and v_k is the soft rough neutrosophic path which has maximum strength from v_i and v_k, denoted by $CONN_G(v_i, v_k)$ or $E^{\infty}(v_i, v_k)$, is called strength of connectedness from v_i and v_k.*

Definition 11. *A soft rough neutrosophic graph is a cycle if and only if the underlying graphs of each approximation is a cycle. A soft rough neutrosophic cycle is a soft rough neutrosophic graph if and only if the supporting graph of each approximation graph is a cycle and there exist $v_{lm}, v_{ij}\in\underline{E}^*, v_{lm}, v_{ij}\in\overline{E}^*$ and $v_{lm}\neq v_{ij}$ such that*

$$\underline{R}(M)(v_{ij}) = \bigwedge_{v_{lm}\in\underline{E}^*-v_{ij}} \underline{R}(M)(v_{lm}),\ \overline{R}(M)(v_{ij}) = \bigwedge_{v_{lm}\in\overline{E}^*-v_{ij}} \overline{R}(M)(v_{lm}).$$

Equivalently, each approximation graph is a cycle such that

$$\underline{R}(M)(v_{ij})=\Big(\bigwedge_{v_{lm}\in\underline{E}^*-v_{ij}} T_{\underline{R}(M)}(v_{lm}),\ \bigvee_{v_{lm}\in\underline{E}^*-v_{ij}} I_{\underline{R}(M)}(v_{lm}),\ \bigvee_{v_{lm}\in\underline{E}^*-v_{ij}} F_{\underline{R}(M)}(v_{lm})\Big),$$
$$\overline{R}(M)(v_{ij})=\Big(\bigwedge_{v_{lm}\in\overline{E}^*-v_{ij}} T_{\overline{R}(M)}(v_{lm}),\ \bigvee_{v_{lm}\in\overline{E}^*-v_{ij}} I_{\overline{R}(M)}(v_{lm}),\ \bigvee_{v_{lm}\in\overline{E}^*-v_{ij}} F_{\overline{R}(M)}(v_{lm})\Big).$$

Example 4. *Let $V=\{v_1,v_2,v_3,v_4\}$ be a vertex set and $A=\{a_1,a_2,a_3,a_4\}$ a set of parameters. A relation S over $A\times V$ can be defined in tabular form in Table 7 as follows:*

Table 7. Full soft set S.

S	v_1	v_2	v_3	v_4
a_1	1	1	1	1
a_2	0	1	0	1
a_3	1	0	1	1
a_4	1	0	1	0

Let $N=\{(v_1,0.3,0.4,0.6),(v_2,0.4,0.5,0.1),(v_3,0.9,0.6,0.4),(v_4,1.0,0.2,0.1)\}$ be a neutrosophic set on V. Then from Equation (1) of Definition 1, we have

$$\overline{S}(N) = \{(v_1, 0.9, 0.4, 0.4), (v_2, 1.0, 0.2, 0.1), (v_3, 0.9, 0.4, 0.4), (v_4, 1.0, 0.2, 0.1)\},$$
$$\underline{S}(N) = \{(v_1, 0.3, 0.6, 0.6), (v_2, 0.4, 0.5, 0.1), (v_3, 0.3, 0.6, 0.6), (v_4, 0.4, 0.5, 0.1)\}.$$

Hence, $SN = (\underline{S}(N), \overline{S}(N))$ is soft rough neutrosophic set on V. Let $E=\{v_{13},v_{32},v_{24},v_{41}\}\subseteq\tilde{V}$ and $L=\{a_{13},a_{32},a_{43}\}\subseteq\tilde{A}$. Then a full soft set R on E (from L to E) can be defined in Table 8 as follows:

Table 8. Full soft set R.

R	v_{13}	v_{32}	v_{24}	v_{41}
a_{13}	1	0	1	1
a_{32}	0	1	0	0
a_{43}	1	0	0	0

Let $M=\{(v_{13},0.3,0.2,0.1),(v_{32},0.2,0.1,0.1),(v_{24},0.3,0.2,0.1),(v_{41},0.3,0.1,0.1)\}$ *be a neutrosophic set on E. Then from Equation (2) of Definition 2, we have*

$$\overline{R}(M)=\{(v_{13},0.3,0.2,0.1),(v_{32},0.2,0.1,0.1),(v_{24},0.3,0.1,0.1),(v_{41},0.3,0.1,0.1)\},$$
$$\underline{R}(M)=\{(v_{13},0.3,0.2,0.1),(v_{32},0.2,0.1,0.1),(v_{24},0.3,0.2,0.1),(v_{41},0.2,0.1,0.1)\}.$$

Hence, $RM=(\underline{R}(M),\overline{R}(M))$ *is soft rough neutrosophic set on E. Thus,* $\underline{G}=(\underline{S}(N),\underline{R}(M))$ *and* $\overline{G}=(\overline{S}(N),\overline{R}(M))$ *are LSRNAG and USRNAG, respectively, as shown in Figure 2. Hence,* $G=(\underline{G},\overline{G})$ *is SRNG and it is also a soft rough neutrosophic cycle.*

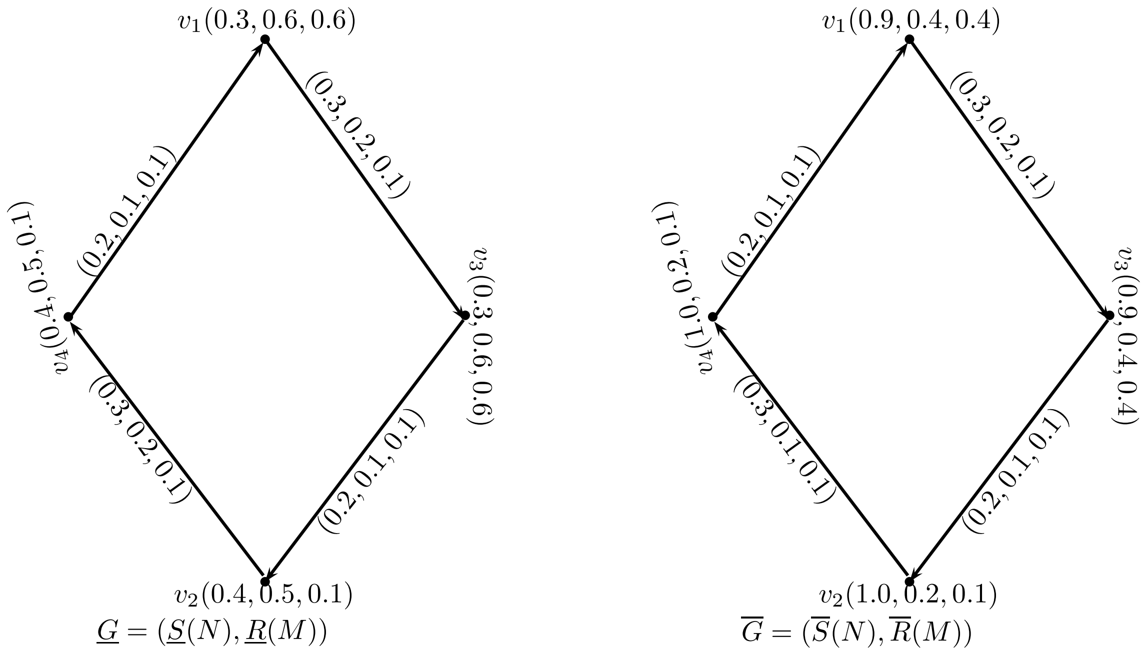

Figure 2. Soft rough neutrosophic cycle $G=(\underline{G},\overline{G})$.

Definition 12. *A soft rough neutrosophic subgraph* $H=(SN_2,RM_2)$ *of a soft rough neutrosophic graph* $G=(SN_1,RM_1)$, *if* $v\in H$ *such that*

$$T_{\underline{S}(N_2)}(v)\leq T_{\underline{S}(N_1)}(v),\ I_{\underline{S}(N_2)}(v)\geq I_{\underline{S}(N_1)}(v),\ F_{\underline{S}(N_2)}(v)\geq F_{\underline{S}(N_1)}(v),$$
$$T_{\overline{S}(N_2)}(v)\leq T_{\overline{S}(N_1)}(v),\ I_{\overline{S}(N_2)}(v)\geq I_{\overline{S}(N_1)}(v),\ F_{\overline{S}(N_2)}(v)\geq F_{\overline{S}(N_1)}(v),$$

and $v_{ij}\in H$,

$$T_{\underline{R}(M_2)}(v_{ij})\leq T_{\underline{R}(M_1)}(v_{ij}),\ I_{\underline{R}(M_2)}(v_{ij})\geq I_{\underline{R}(M_1)}(v_{ij}),\ F_{\underline{R}(M_2)}(v_{ij})\geq F_{\underline{R}(M_1)}(v_{ij}),$$
$$T_{\overline{R}(M_2)}(v_{ij})\leq T_{\overline{R}(M_1)}(v_{ij}),\ I_{\overline{R}(M_2)}(v_{ij})\geq I_{\overline{R}(M_1)}(v_{ij}),\ F_{\overline{R}(M_2)}(v_{ij})\geq F_{\overline{R}(M_1)}(v_{ij}).$$

Definition 13. *A* $H{=}(SN_2,RM_2)$ *is called soft rough neutrosophic spanning subgraph of a soft rough neutrosophic graph* $G{=}(SN_1,RM_1)$*, if H is a soft rough neutrosophic subgraph such that*

$$T_{\underline{S}(N_2)}(v){=}T_{\underline{S}(N_1)}(v),\ I_{\underline{S}(N_2)}(v){=}I_{\underline{S}(N_1)}(v),\ F_{\underline{S}(N_2)}(v){=}F_{\underline{S}(N_1)}(v),$$
$$T_{\overline{S}(N_2)}(v){=}T_{\overline{S}(N_1)}(v),\ I_{\overline{S}(N_2)}(v){=}I_{\overline{S}(N_1)}(v),\ F_{\overline{S}(N_2)}(v){=}F_{\overline{S}(N_1)}(v).$$

Definition 14. *A soft rough neutrosophic graph is a ditree if and only if each supporting approximation graph is a ditree. A soft rough neutrosophic graph* $G{=}(SN_1,RM_1)$ *is a soft rough neutrosophic ditree if and only if there exists a soft rough neutrosophic spanning subgraph* $H{=}(SN_1,RM_2)$ *is a ditree such that* $v_{ij}{\in}G{-}H$

$$T_{\underline{R}(M_1)}(v_{ij}){<}T_{CONN_{\underline{H}}}(v_i,v_j),\ I_{\underline{R}(M_1)}(v_{ij}){>}I_{CONN_{\underline{H}}}(v_i,v_j),\ F_{\underline{R}(M_1)}(v_{ij})\ {>}F_{CONN_{\underline{H}}}(v_i,v_j),$$
$$T_{\overline{R}(M_1)}(v_{ij}){<}T_{CONN_{\overline{H}}}(v_i,v_j),\ I_{\overline{R}(M_1)}(v_{ij}){>}I_{CONN_{\overline{H}}}(v_i,v_j),\ F_{\overline{R}(M_1)}(v_{ij})\ {>}F_{CONN_{\overline{H}}}(v_i,v_j).$$

Definition 15. *Let* $G{=}(SN,RM)$ *be a soft rough neutrosophic graph, an edge* v_{ij} *is a bridge if the edge* v_{ij} *is a bridge in both supporting graph of* $\underline{G}$ *and* $\overline{G}$*, that is the removal of* v_{ij} *disconnects both the* $\underline{G}$ *and* $\overline{G}$*. An edge* v_{ij} *is a soft rough neutrosophic bridge in a soft rough neutrosophic graph* $G{=}(SN,RM)$*, if* $v_{lm}{\in}G$

$$T_{CONN_{\underline{G}}-v_{ij}}(v_l,v_m){<}T_{CONN_{\underline{G}}}(v_l,v_m),\ T_{CONN_{\overline{G}}-v_{ij}}(v_l,v_m){<}T_{CONN_{\overline{G}}}(v_l,v_m),$$
$$I_{CONN_{\underline{G}}-v_{ij}}(v_l,v_m)\ {>}I_{CONN\underline{G}}(v_l,v_m),\ I_{CONN_{\overline{G}}-v_{ij}}(v_l,v_m)\ {>}I_{CONN\overline{G}}(v_l,v_m),$$
$$F_{CONN\underline{G}-v_{ij}}(v_l,v_m){>}F_{CONN_{\underline{G}}}(v_l,v_m),\ F_{CONN\overline{G}-v_{ij}}(v_l,v_m){>}F_{CONN_{\overline{G}}}(v_l,v_m).$$

Definition 16. *Let* $G{=}(SN_1,RM_1)$ *be a soft rough neutrosophic graph then a vertex* v_i *in G is a cutnode(cutvertex) if it is a cutnode in each supporting graph of* $\underline{G}$ *and* $\overline{G}$*. That is, the deletion of* v_i *from the supporting graphs of* $\underline{G}$ *and* $\overline{G}$ *increase the components in the supporting graphs. A vertex* v_i *is called soft rough neutrosophic cutnode(cutvertex) in a soft rough neutrosophic graph if the removal of* v_i *reduces the strength of the connectedness from nodes* $v_j tov_k{\in}\underline{V}^*,\overline{V}^*$

$$T_{CONN_{\underline{G}-v_i}}(v_j,v_k){<}T_{CONN_{\underline{G}}}(v_j,v_k),\ T_{CONN_{\overline{G}-v_i}}(v_j,v_k){<}T_{CONN_{\overline{G}}}(v_j,v_k),$$
$$I_{CONN_{\underline{G}-v_i}}(v_j,v_k)\ {>}I_{CONN_{\underline{G}}}(v_j,v_k),\ \ I_{CONN_{\overline{G}-v_i}}(v_j,v_k)\ {>}I_{CONN_{\overline{G}}}(v_j,v_k),$$
$$F_{CONN_{\underline{G}-v_i}}(v_j,v_k){>}F_{CONN_{\underline{G}}}(v_j,v_k),\ F_{CONN_{\overline{G}-v_i}}(v_j,v_k){>}F_{CONN_{\overline{G}}}(v_j,v_k).$$

Definition 17. *An edge* v_{ij} *in soft rough neutrosophic graph G is called strong soft rough neutrosophic edge if*

$$T_{\underline{R}(M)}(v_{ij}){\geq}T_{CONN_{\underline{G}-v_{ij}}}(v_i,v_j),\ \ T_{\overline{R}(M)}(v_{ij})\ {\geq}T_{CONN_{\overline{G}-v_{ij}}}(v_i,v_j),$$
$$I_{\underline{R}(M)}(v_{ij})\ {\leq}I_{CONN_{\underline{G}-v_{ij}}}(v_i,v_j)\ ,\ \ I_{\overline{R}(M)}(v_{ij})\ \leq I_{CONN_{\overline{G}-v_{ij}}}(v_i,v_j),$$
$$F_{\underline{R}(M)}(v_{ij}){\leq}F_{CONN_{\underline{G}-v_{ij}}}(v_i,v_j),\ \ F_{\overline{R}(M)}(v_{ij})\ {\leq}F_{CONN_{\overline{G}-v_{ij}}}(v_i,v_j).$$

Definition 18. *An edge* v_{ij} *in soft rough neutrosophic graph G is called* $\alpha{-}$*strong soft rough neutrosophic edge if*

$$T_{\underline{R}(M)}(v_{ij}){>}T_{CONN_{\underline{G}-v_{ij}}}(v_i,v_j),\ \ T_{\overline{R}(M)}(v_{ij})\ {>}T_{CONN_{\overline{G}-v_{ij}}}(v_i,v_j),$$
$$I_{\underline{R}(M)}(v_{ij})\ {<}I_{CONN_{\underline{G}-v_{ij}}}(v_i,v_j)\ ,\ \ I_{\overline{R}(M)}(v_{ij})\ <I_{CONN_{\overline{G}-v_{ij}}}(v_i,v_j),$$
$$F_{\underline{R}(M)}(v_{ij}){<}F_{CONN_{\underline{G}-v_{ij}}}(v_i,v_j),\ \ F_{\overline{R}(M)}(v_{ij})\ {<}F_{CONN_{\overline{G}-v_{ij}}}(v_i,v_j).$$

Definition 19. *An edge v_{ij} in soft rough neutrosophic graph G is called $\beta-$strong soft rough neutrosophic edge if*

$$T_{\underline{R}(M)}(v_{ij})=T_{CONN_{\underline{G}-v_{ij}}}(v_i,v_j),\ T_{\overline{R}(M)}(v_{ij})=T_{CONN_{\overline{G}-v_{ij}}}(v_i,v_j),$$
$$I_{\underline{R}(M)}(v_{ij})=I_{CONN_{\underline{G}-v_{ij}}}(v_i,v_j),\ I_{\overline{R}(M)}(v_{ij})=I_{CONN_{\overline{G}-v_{ij}}}(v_i,v_j),$$
$$F_{\underline{R}(M)}(v_{ij})=F_{CONN_{\underline{G}-v_{ij}}}(v_i,v_j),\ F_{\overline{R}(M)}(v_{ij})=F_{CONN_{\overline{G}-v_{ij}}}(v_i,v_j).$$

Definition 20. *An edge v_{ij} in soft rough neutrosophic graph G is called $\delta-$strong soft rough neutrosophic edge if*

$$T_{\underline{R}(M)}(v_{ij})<T_{CONN_{\underline{G}-v_{ij}}}(v_i,v_j),\ T_{\overline{R}(M)}(v_{ij})<T_{CONN_{\overline{G}-v_{ij}}}(v_i,v_j),$$
$$I_{\underline{R}(M)}(v_{ij})>I_{CONN_{\underline{G}-v_{ij}}}(v_i,v_j),\ I_{\overline{R}(M)}(v_{ij})>I_{CONN_{\overline{G}-v_{ij}}}(v_i,v_j),$$
$$F_{\underline{R}(M)}(v_{ij})>F_{CONN_{\underline{G}-v_{ij}}}(v_i,v_j),\ F_{\overline{R}(M)}(v_{ij})>F_{CONN_{\overline{G}-v_{ij}}}(v_i,v_j).$$

Example 5. *Let $V=\{v_1,v_2,v_3,v_4\}$ be a vertex set and $A=\{a_1,a_2,a_3,a_4\}$ a set of parameters. A relation S over $A\times V$ can be defined in tabular form in Table 9 as follows:*

Table 9. Full soft set S.

S	v_1	v_2	v_3	v_4
a_1	1	1	1	1
a_2	0	1	0	1
a_3	1	0	1	1
a_4	1	0	1	0

Let $N=\{(v_1,0.3,0.4,0.6),(v_2,0.4,0.5,0.1),(v_3,0.9,0.6,0.4),(v_4,1.0,0.2,0.1)\}$ be a neutrosophic set on V. Then from Equation (1) of Definition 1, we have

$$\overline{S}(N)=\{(v_1,0.9,0.4,0.4),(v_2,1.0,0.2,0.1),(v_3,0.9,0.4,0.4),(v_4,1.0,0.2,0.1)\},$$
$$\underline{S}(N)=\{(v_1,0.3,0.6,0.6),(v_2,0.4,0.5,0.1),(v_3,0.3,0.6,0.6),(v_4,0.4,0.5,0.1)\}.$$

Hence, $SN=(\underline{S}(N),\overline{S}(N))$ is soft rough neutrosophic set on V. Let $E=\{v_{13},v_{32},v_{43}\}\subseteq\tilde{V}$ and $L=\{a_{12},a_{24},a_{34}\}\subseteq\tilde{A}$. Then a full soft set R on E (from L to E) can be defined in Table 10 as follows:

Table 10. Full soft set R.

R	v_{13}	v_{32}	v_{43}
a_{12}	0	1	0
a_{24}	1	0	1
a_{34}	0	0	1

Let $M=\{(v_{13},0.3,0.2,0.0),(v_{32},0.3,0.0,0.1),(v_{43},0.3,0.2,0.1)\}$ be a neutrosophic set on E. Then from Equation (2) of Definition 2, we have

$$\overline{R}(M)=\{(v_{13},0.3,0.2,0.0),(v_{32},0.3,0.0,0.1),(v_{43},0.3,0.2,0.1)\},$$
$$\underline{R}(M)=\{(v_{13},0.3,0.2,0.1),(v_{32},0.3,0.0,0.1),(v_{43},0.3,0.2,0.1)\}.$$

Hence, $RM=(\underline{R}(M),\overline{R}(M))$ *is soft rough neutrosophic set on E. Thus,* $\underline{G}=(\underline{S}(N),\underline{R}(M))$ *and* $\overline{G}=(\overline{S}(N),\overline{R}(M))$ *are LSRNAG and USRNAG, respectively, as shown in Figure 3. Hence,* $G=(\underline{G},\overline{G})$ *is SRNG and a tree.* v_{13} *is a bridge and* v_3 *is a cute node*

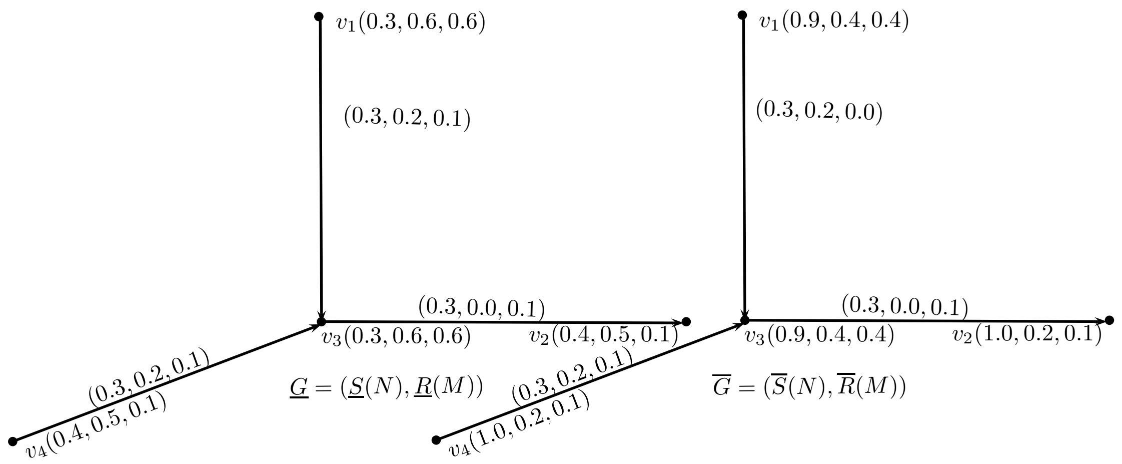

Figure 3. Soft rough neutrosophic graph $G=(\underline{G},\overline{G})$.

We state the following Theorems without their proofs.

Theorem 1. *Let* $G=(SN_1,RM_1)$ *be a soft rough neutrosophic graph tree. An edge* v_{ij} *is the strongest edge if* v_{ij} *is an edge of its subgraph* $H=(SN_1,RM_2)$.

Theorem 2. *If v is a common node of at least two soft rough neutrosophic bridges, then v is a soft rough neutrosophic cutnode.*

Theorem 3. *If* v_{ij} *is a soft rough neutrosophic bridge of G, then*

$$T_{\underline{R}(M)}(v_{ij})=T_{CONN_{\underline{G}-v_{ij}}}(v_i,v_j),\ T_{\overline{R}(M)}(v_{ij})=T_{CONN_{\overline{G}-v_{ij}}}(v_i,v_j),$$
$$I_{\underline{R}(M)}(v_{ij})=I_{CONN_{\underline{G}-v_{ij}}}(v_i,v_j),\ I_{\overline{R}(M)}(v_{ij})=I_{CONN_{\overline{G}-v_{ij}}}(v_i,v_j),$$
$$F_{\underline{R}(M)}(v_{ij})=F_{CONN_{\underline{G}-v_{ij}}}(v_i,v_j),\ F_{\overline{R}(M)}(v_{ij})=F_{CONN_{\overline{G}-v_{ij}}}(v_i,v_j).$$

3. Soft Rough Neutrosophic Influence Graphs

Definition 21. *A soft rough neutrosophic influence graph* G *on a nonempty set V is a 7-ordered tuple* (A,S,SN,R,RM,X,XQ) *such that*

(i) *A is a set of parameters,*
(ii) *S is an arbitrary full soft set over* V,
(iii) *R is an arbitrary full soft set over* $E\subseteq V\times V$,
(iv) *X is an arbitrary full soft set over* $I\subseteq V\times E$,
(v) $SN=(\underline{S}(N),\overline{S}(N))$ *is a soft rough neutrosophic set on* V,
(vi) $RM=(\underline{R}(M),\overline{R}(M))$ *is a soft rough neutrosophic set on* E,
(vii) $XQ=(\underline{X}(Q),\overline{X}(Q))$ *is a soft rough neutrosophic set on* I,

Thus, $G=(\underline{G},\overline{G})=(SN,RM,XQ)$ *is a soft rough neutrosophic influence graph (SRNIG), where* $\underline{G}=(\underline{S}(N),\underline{R}(M),\underline{X}(Q))$ *and* $\overline{G}=(\overline{S}(N),\overline{R}(M),\overline{X}(Q))$ *are lower and upper soft rough neutrosophic influence approximation graphs (LSRNIAGs) and (USRNIAGs), respectively, of* G.

Example 6. *Let $V=\{v_1,v_2,v_3,v_4,v_5,v_6\}$ be a vertex set and $A=\{a_1,a_2,a_3,a_4\}$ a set of parameters. A full soft set S over $A \times V$ can be defined in tabular form in Table 11 as follows:*

Table 11. Full soft set S.

S	v_1	v_2	v_3	v_4	v_5	v_6
a_1	1	1	1	0	0	1
a_2	0	1	0	0	1	1
a_3	1	0	1	1	1	1
a_4	1	1	1	1	1	1

Let $N=\{(v_1,1.0,0.4,0.7),(v_2,0.9,0.6,0.55),(v_3,0.7,0.9,0.5),(v_4,0.6,0.5,0.6),(v_5,0.65,0.8,0.65),(v_6,0.8,0.7,0.8)\}$ be a neutrosophic set on V. Then from Equation (1) of Definition 1, we have

$$\overline{S}(N)=\{(v_1,1.0,0.4,0.50),(v_2,0.9,0.6,0.55),(v_3,1.0,0.4,0.5),(v_4,1.0,0.4,0.5),(v_5,0.9,0.6,0.55),(v_6,0.9,0.6,0.55)\},$$
$$\underline{S}(N)=\{(v_1,0.7,0.9,0.80),(v_2,0.7,0.8,0.80),(v_3,0.7,0.9,0.8),(v_4,0.6,0.9,0.8),(v_5,0.65,0.8,0.8)(v_6,0.7,0.8,0.8)\}.$$

Hence, $SN=(\underline{S}(N),\overline{S}(N))$ is soft rough neutrosophic set on V. Let $E=\{v_{12},v_{24},v_{32},v_{42},v_{52},v_{62},\}\subseteq\tilde{V}$ and $L=\{a_{13},a_{24},a_{34},a_{41}\}\subseteq\tilde{A}$. Then a full soft set R on E (from L to E) can be defined in Table 12 as follows:

Table 12. Full soft set R.

R	v_{12}	v_{24}	v_{32}	v_{42}	v_{52}	v_{62}
a_{13}	0	1	0	0	0	1
a_{24}	0	1	0	0	1	1
a_{34}	1	0	1	1	1	1
a_{41}	1	1	1	1	1	1

Let $M=\{(v_{12},0.6,0.3,0.4),(v_{24},0.58,0.38,0.46),(v_{32},0.56,0.37,0.47),(v_{42},0.54,0.34,0.38),(v_{52},0.52,0.32,0.5),(v_{62},0.5,0.4,0.45)\}$ be a neutrosophic set on E. Then from Equation (2) of Definition 2, we have

$$\overline{R}(M)=\{(v_{12},0.60,0.30,0.38),(v_{24},0.58,0.38,0.45),(v_{32},0.60,0.30,0.38),(v_{42},0.60,0.30,0.38),(v_{52},0.58,0.32,0.45),(v_{62},0.58,0.38,0.45)\},$$
$$\underline{R}(M)=\{(v_{12},0.50,0.40,0.50),(v_{24},0.50,0.40,0.46),(v_{32},0.50,0.40,0.50),(v_{42},0.50,0.40,0.50),(v_{52},0.50,0.40,0.50),(v_{62},0.50,0.40,0.46)\}.$$

Hence, $RM=(\underline{R}(M),\overline{R}(M))$ is soft rough neutrosophic set on E. Thus, $\underline{G}=(\underline{S}(N),\underline{R}(M))$ and $\overline{G}=(\overline{S}(N),\overline{R}(M))$ are LSRNAG and USRNAG, respectively, as shown in Figure 4. Hence, $G=(\underline{G},\overline{G})$ is SRNG. Let $I=\{v_1v_{24},v_1v_{32},v_1v_{42},v_1v_{52},v_1v_{62},v_3v_{12},v_3v_{24},v_3v_{42},v_3v_{52},v_3v_{62},v_4v_{12},v_4v_{32},v_4v_{52},v_4v_{62},v_5v_{12},v_5v_{24},v_5v_{32},v_5v_{42},v_5v_{62},v_6v_{12},v_6v_{24},v_6v_{32},v_6v_{42},v_6v_{52}\}\subseteq V\times E$ and $P=\{a_1a_{24},a_1a_{34},a_2a_{13},a_2a_{34},a_2a_{41},a_3a_{24},a_3a_{41},a_4a_{13}\}\subseteq\hat{A}$. Then and Q a neutrosophic set on I and a full soft set X on I (from P to I) can be defined in Table 13, respectively as follows:

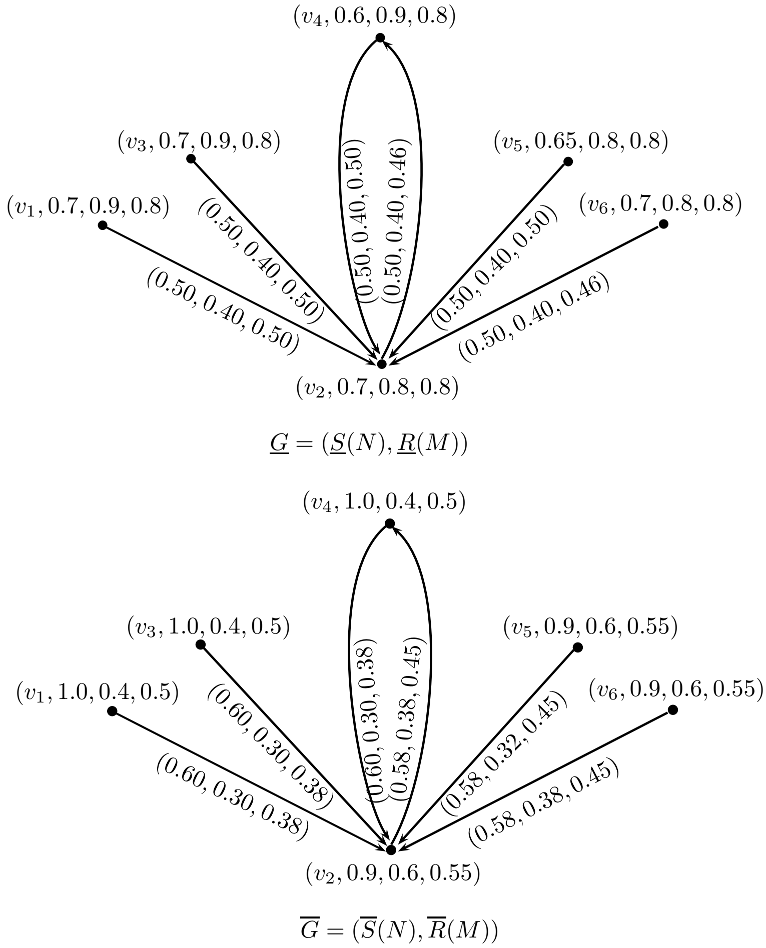

Figure 4. Soft rough neutrosophic graph $G = (\underline{G}, \overline{G})$.

Table 13. Full soft set X.

X	v_1v_{24} v_4v_{52}	v_1v_{32} v_4v_{62}	v_1v_{42} v_5v_{12}	v_1v_{52} v_5v_{24}	v_1v_{62} v_5v_{32}	v_3v_{12} v_5v_{42}	v_3v_{24} v_5v_{62}	v_3v_{42} v_6v_{12}	v_3v_{52} v_6v_{24}	v_3v_{62} v_6v_{32}	v_4v_{12} v_6v_{42}	v_4v_{32} v_6v_{52}
a_1a_{24}	1	1	1	1	1	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	1	0	0	1
a_1a_{34}	0	0	1	1	1	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	1	0	0	1	1
a_2a_{13}	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	1	0	0	1	0	1	0	1	0
a_2a_{34}	0	0	0	0	0	0	0	0	0	0	1	1
	1	1	1	0	1	1	1	1	0	1	1	1
a_2a_{41}	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	1	1	1	1	1	1	1	1	1	1
a_3a_{24}	1	0	0	1	1	0	0	0	0	0	0	0
	1	1	0	1	0	0	1	0	1	0	0	1
a_3a_{41}	1	1	1	1	1	1	1	1	1	1	1	1
	1	1	1	1	1	1	1	1	1	1	1	1
a_4a_{13}	0	0	0	0	1	0	1	0	0	1	0	0
	0	1	0	1	0	0	1	0	1	0	0	0

$$Q=\{(v_1v_{24},0.42,0.3,0.38),(v_1v_{32},0.43,0.28,0.37),(v_1v_{42},0.49,0.26,0.33),(v_1v_{52},0.47,0.29,0.32),\\ (v_1v_{62},0.46,0.28,0.36),(v_3v_{12},0.4,0.29,0.37),(v_3v_{24},0.45,0.24,0.36),(v_3v_{42},0.48,0.29,0.35),\\ (v_3v_{52},0.41,0.24,0.36),(v_3v_{62},0.42,0.26,0.34),(v_4v_{12},0.5,0.25,0.3),(v_4v_{32},0.44,0.27,0.32),\\ (v_4v_{52},0.45,0.23,0.31),(v_4v_{62},0.48,0.23,0.38),(v_5v_{12},0.46,0.24,0.3),(v_5v_{24},0.47,0.26,0.34),\\ (v_5v_{32},0.4,0.3,0.36),(v_5v_{42},0.48,0.29,0.38),(v_5v_{62},0.49,0.3,0.37),(v_6v_{12},0.49,0.3,0.37),\\ (v_6v_{24},0.4,0.28,0.35),(v_6v_{32},0.47,0.27,0.34),(v_6v_{42},0.46,0.29,0.33),(v_6v_{52},0.49,0.3,0.32)\}$$

Then the lower and upper soft rough neutrosophic approximation is directly calculated using Equation (3) of Definition 4, we have

$$\overline{X}(Q)=\{(v_1v_{24},0.49,0.26,0.32),(v_1v_{32},0.49,0.26,0.32),(v_1v_{42},0.49,0.26,0.32),(v_1v_{52},0.49,0.26,\\ 0.32),(v_1v_{62},0.49,0.26,0.34),(v_3v_{12},0.5,0.23,0.3),(v_3v_{24},0.49,0.23,0.34),(v_3v_{42},0.5,\\ 0.23,0.3),(v_3v_{52},0.5,0.23,0.3),(v_3v_{62},0.49,0.23,0.3),(v_4v_{12},0.5,0.23,0.38),(v_4v_{32},\\ 0.5,0.23,0.3),(v_4v_{52},0.49,0.23,0.31),(v_4v_{62},0.49,0.23,0.34),(v_5v_{12},0.49,0.24,0.3),\\ (v_5v_{24},0.49,0.26,0.34),(v_5v_{32},0.49,0.24,0.3),(v_5v_{42},0.49,0.24,0.3),(v_5v_{62},0.49,0.26,\\ 0.34),(v_6v_{12},0.49,0.26,0.32),(v_6v_{24},0.49,0.26,0.34),(v_6v_{32},0.49,0.24,0.3),(v_6v_{42},0.49,\\ 0.26,0.33),(v_6v_{52},0.49,0.26,0.32)\};$$

$$\underline{X}(Q)=\{(v_1v_{24},0.4,0.3,0.38),(v_1v_{32},0.4,0.3,0.38),(v_1v_{42},0.46,0.3,0.37),(v_1v_{52},0.46,0.3,0.37),\\ (v_1v_{62},0.46,0.3,0.37),(v_3v_{12},0.4,0.3,0.38),(v_3v_{24},0.4,0.3,0.38),(v_3v_{42},0.4,0.3,0.38),\\ (v_3v_{52},0.4,0.3,0.38),(v_3v_{62},0.4,0.3,0.38),(v_4v_{12},0.4,0.3,0.38),(v_4v_{32},0.4,0.3,0.38),\\ (v_4v_{52},0.4,0.3,0.38),(v_4v_{62},0.4,0.3,0.38),(v_5v_{12},0.4,0.3,0.38),(v_5v_{24},0.4,0.3,0.37),\\ (v_5v_{32},0.4,0.3,0.38),(v_5v_{42},0.4,0.3,0.38),(v_5v_{62},0.4,0.3,0.37),(v_6v_{12},0.46,0.3,0.37),\\ (v_6v_{24},0.4,0.3,0.37),(v_6v_{32},0.4,0.3,0.38),(v_6v_{42},0.46,0.3,0.37),(v_6v_{52},0.46,0.3,0.37)\}.$$

Thus, $\underline{G}=(\underline{S}(N),\underline{R}(M),\underline{X}(Q))$ *and* $\overline{G}=(\overline{S}(N),\overline{R}(M),X(Q))$ *are LSRNIAG and USRNIAG, respectively, as shown in Figures 5 and 6. Hence,* $G=(\overline{G},\underline{G})$ *is SRNIG.*

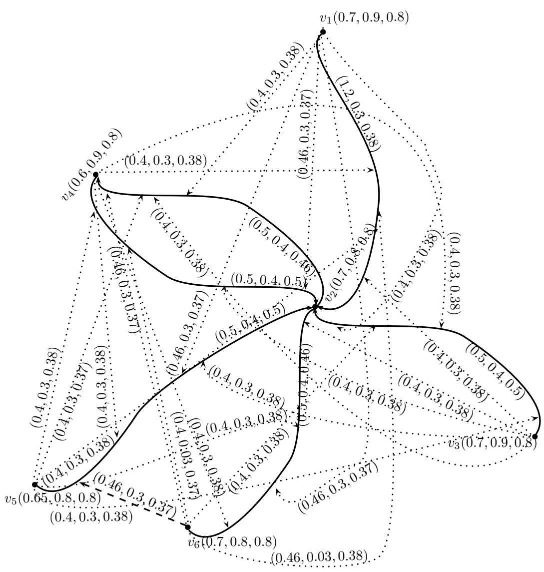

Figure 5. Lower Soft rough neutrosophic graph $\underline{G}=(\underline{S}(N),\underline{R}(M),\underline{X}(Q))$.

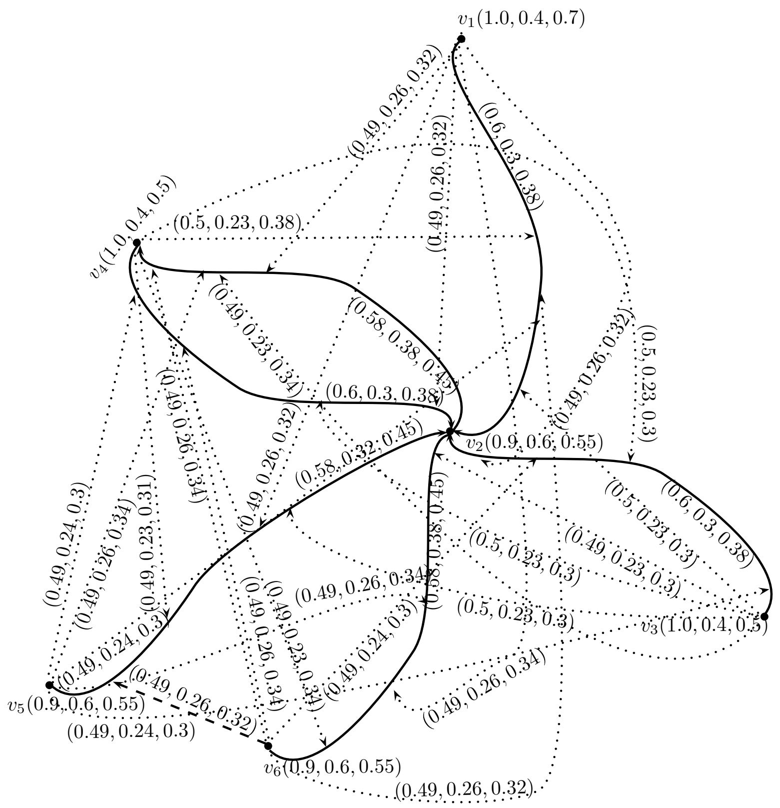

Figure 6. Upper Soft rough neutrosophic graph $\overline{G} = (\overline{S}(N), \overline{R}(M), \overline{X}(Q))$.

Definition 22. *An underlying influence graph(supporting influence graph)* $G^*=(\underline{G}^*,\overline{G}^*)$ *of a soft rough neutrosophic influence graph* $G=(\underline{G},\overline{G})$ *is of the form* $\underline{G}^*=(\underline{V}^*,\underline{E}^*,\underline{I}^*)$ *and* $\overline{G}^*=(\overline{V}^*,\overline{E}^*,\overline{I}^*)$*, where*

$$\begin{aligned}
\underline{V}^* &= \textit{Lower Vertex Set} = \{v \in V | T_{\underline{S}(N)}(v) \neq 0, I_{\underline{S}(N)}(v) \neq 0, F_{\underline{S}(N)}(v) \neq 0\},\\
\overline{V}^* &= \textit{Upper Vertex Set} = \{v \in V | T_{\overline{S}(N)}(v) \neq 0, I_{\overline{S}(N)}(v) \neq 0, F_{\overline{S}(N)}(v) \neq 0\},\\
\underline{E}^* &= \textit{Lower Edge Set} \ = \{v_{ij} \in E | T_{\underline{R}(M)}(v_{ij}) \neq 0, I_{\underline{R}(M)}(v_{ij}) \neq 0, F_{\underline{R}(M)}(v_{ij}) \neq 0\},\\
\overline{E}^* &= \textit{Upper Edge Set} \ = \{v_{ij} \in E | T_{\underline{R}(M)}(v_{ij}) \neq 0, I_{\underline{R}(M)}(v_{ij}) \neq 0, F_{\underline{R}(M)}(v_{ij}) \neq 0\},\\
\underline{I}^* &= \textit{Lower Influence} \ = \{v_i v_{jk} \in I | T_{\underline{X}(Q)}(v_i v_{jk}) \neq 0, I_{\underline{X}(Q)}(v_i v_{jk}) \neq 0, F_{\underline{X}(Q)}(v_i v_{jk}) \neq 0\},\\
\overline{I}^* &= \textit{Upper Influence} \ = \{v_i v_{jk} \in I | T_{\overline{X}(Q)}(v_i v_{jk}) \neq 0, I_{\overline{X}(Q)}(v_i v_{jk}) \neq 0, F_{\overline{X}(Q)}(v_i v_{jk}) \neq 0\}.
\end{aligned}$$

Definition 23. *If* $v_{ij} \in \underline{E}^*$ $(\overline{E}^*)$*, then* v_{ij} *is a lower edge (upper edge) of the soft rough neutrosophic influence graph. If* $v_i v_{jk} \in \underline{I}^*$ $(\overline{I}^*)$*, then* $v_i v_{jk}$ *is lower pairs (upper pair). If* $v_{jk} \in \underline{E}^* (\overline{E}^*)$ *and* $v_i v_{jk}$*, is not lower pairs (upper pairs), then it is a lower non-influence edge (upper non-influence edge).*

Definition 24. *A soft rough neutrosophic influence graph has a walk if each approximation graph has an alternative sequence of the form*

$$v_0, i_0, e_0, i'_0, v_1, \cdots, v_{n-1}, i_{n-1}, e_{n-1}, i'_{n-1}, v_n$$

such that

$$v_k \in \underline{V}^*,\ e_k \in \underline{E}^*,\ i_k, i'_k \in \underline{I}^*,$$
$$v_k \in \overline{V}^*,\ e_k \in \overline{E}^*,\ i_k, i'_k \in \overline{I}^*.$$

where $i_k=(v_k uv)$, $e_k=uv$, $i'_k=(vwv_{k+1})$ and $\forall k=0,1,2,\cdots,n-1$. If $v_0 = v_n$, then it is called closed. If the pairs are distinct, then it is called a soft rough neutrosophic influence trail (SRNI trail). If the edges are distinct, then it is called a soft rough neutrosophic trail (SRN trail). If the vertices are distinct in SRN trail, then it is called a soft rough neutrosophic path (SRN path). If the vertices, edge and pairs are distinct in a walk of SRNIG, then it is called a soft rough neutrosophic influence path (SRNI path). A path is a trail and an influence trail. If a path in a soft rough neutrosophic influence graph is closed, then it is called a cycle.

Definition 25. *A strength of soft rough neutrosophic influence graph, denoted by stren, is defined as*

$$stren=\Big((\bigwedge_{v_{jk}\in\underline{E}^*} T_{\underline{R}(M)}(v_{jk})) \wedge (\bigwedge_{v_{jk}\in\overline{E}^*} T_{\overline{R}(M)}(v_{jk})), (\bigvee_{v_{jk}\in\underline{E}^*} I_{\underline{R}(M)}(v_{jk}))\vee (\bigvee_{v_{jk}\in\overline{E}^*} I_{\overline{R}(M)}(v_{jk})), (\bigvee_{v_{jk}\in\underline{E}^*} F_{\underline{R}(M)}(v_{jk})) \vee (\bigvee_{v_{jk}\in\overline{E}^*} F_{\overline{R}(M)}(v_{jk}))\Big).$$

An influence strength of soft rough neutrosophic influence graph, denoted by In stren, is defined as

$$In\ stren=\Big((\bigwedge_{v_iv_{jk}\in\underline{I}^*} T_{\underline{X}(Q)}(v_iv_{jk}) \wedge \bigwedge_{v_iv_{jk}\in\overline{I}^*} T_{\overline{X}(Q)}(v_iv_{jk})), (\bigvee_{(v_iv_{jk})\in\underline{I}^*} I_{\underline{R}(M)}(v_iv_{jk})\vee \bigvee_{v_iv_{jk}\in\overline{I}^*} I_{\overline{R}(M)}(v_iv_{jk})), (\bigvee_{v_iv_{jk}\in\underline{I}^*} F_{\underline{R}(M)}(v_iv_{jk}) \vee \bigvee_{v_iv_{jk}\in\overline{E}^*} F_{\overline{R}(M)}(v_iv_{jk}))\Big).$$

Definition 26. *In a soft rough neutrosophic influence graph* G*, if in each approximation graph*

$$CONN_{\underline{G}}(v_i,v_k) = \underline{E}^{\infty}(v_i,v_k) = \vee_{\alpha}\{\underline{E}^{\alpha}(v_i,v_k)\},\ CONN_{\overline{G}}(v_i,v_k) = \overline{E}^{\infty}(v_i,v_k) = \vee_{\alpha}\{\overline{E}^{\alpha}(v_i,v_k)\}.$$

where

$$\underline{E}^{\alpha}(v_i,v_k) = (\underline{E}^{\alpha-1}\circ\underline{E})(v_i,v_k),\ \overline{E}^{\alpha}(v_i,v_k) = (\overline{E}^{\alpha-1}\circ\overline{E})(v_i,v_k),$$

and

$$(\underline{E}\circ\underline{E})(v_i,v_k)=\Big(\bigvee_{v_j\in\underline{V}^*}(T_{\underline{R}(M)}(v_{ij})\wedge T_{\underline{R}(M)}(v_{jk})),\ \bigwedge_{v_j\in\underline{V}^*}(I_{\underline{R}(M)}(v_{ij})\vee I_{\underline{R}(M)}(v_{jk})),\ \bigwedge_{v_j\in\underline{V}^*}(F_{\underline{R}(M)}(v_{ij})\vee F_{\underline{R}(M)}(v_{jk}))\Big),$$

$$(\overline{E}\circ\overline{E})(v_i,v_k)=\Big(\bigvee_{v_j\in\overline{V}^*}(T_{\overline{R}(M)}(v_{ij})\wedge T_{\overline{R}(M)}(v_{jk})),\ \bigwedge_{v_j\in\overline{V}^*}(I_{\overline{R}(M)}(v_{ij})\vee I_{\overline{R}(M)}(v_{jk})),\ \bigwedge_{v_j\in\overline{V}^*}(F_{\overline{R}(M)}(v_{ij})\vee F_{\overline{R}(M)}(v_{jk}))\Big).$$

Thus it is the strength of strongest path from v_i to v_k in G*.*

In a soft rough neutrosophic influence graph G*, if in each approximation graph*

$$ICONN_{\underline{G}}(v_i,v_k) = \underline{I}^{\infty}(v_i,v_k) = \vee_{\alpha}\{\underline{I}^{\alpha}(v_i,v_k)\},\ ICONN_{\overline{G}}(v_i,v_k) = \overline{I}^{\infty}(v_i,v_k) = \vee_{\alpha}\{\overline{I}^{\alpha}(v_i,v_k)\}.$$

where

$$\underline{I}^{\alpha}(v_i,v_k) = (\underline{I}^{\alpha-1}\circ\underline{I})(v_i,v_k),\ \overline{I}^{\alpha}(v_i,v_k) = (\overline{I}^{\alpha-1}\circ\overline{I})(v_i,v_k),$$

and

$$(\underline{\mathrm{I}} \circ \underline{\mathrm{I}})(v_i, v_k) = \Big(\bigvee_{v_m \in \underline{\mathrm{V}}^*} (T_{\underline{X}(Q)}(v_i v_{lm}) \wedge T_{\underline{X}(Q)}(v_m v_{pk})), \bigwedge_{v_m \in \underline{\mathrm{V}}^*} (I_{\underline{X}(Q)}(v_i v_{lm}) \vee I_{\underline{X}(Q)}(v_m v_{pk})), \bigwedge_{v_m \in \underline{\mathrm{V}}^*} (F_{\underline{X}(Q)}(v_i v_{lm}) \vee F_{\underline{X}(Q)}(v_m v_{pk})) \Big),$$

$$(\overline{\mathrm{I}} \circ \overline{\mathrm{I}})(v_i, v_k) = \Big(\bigvee_{v_m \in \overline{\mathrm{V}}^*} (T_{\overline{X}(Q)}(v_i v_{lm}) \wedge T_{\overline{X}(Q)}(v_m v_{pk})), \bigwedge_{v_m \in \overline{\mathrm{V}}^*} (I_{\overline{X}(Q)}(v_i v_{lm}) \vee I_{\overline{X}(Q)}(v_m v_{pk})), \bigwedge_{v_m \in \overline{\mathrm{V}}^*} (F_{\overline{X}(Q)}(v_i v_{lm}) \vee F_{\overline{X}(Q)}(v_m v_{pk})) \Big).$$

Thus it is the strength of strongest path from v_i *to* v_k *in* G.

Definition 27. *A SRNIG is called connected if each two vertex* v_j *and* v_k *are joined by a SRN (SRNI) path. Maximal connected partial subgraphs in each approximation subgraph are called component.*

Definition 28. *A soft rough neutrosophic influence graph is a cycle if and only if the underlying graphs of each approximation is a cycle. A soft rough neutrosophic influence graph is a soft rough neutrosophic cycle if and only if the underlying graphs of each approximations is a cycle and there exist* $v_{lm}, v_{ij} \in \underline{E}^*, v_{lm}, v_{ij} \in \overline{E}^*$ *and* $v_{lm} \neq v_{ij}$*, such that*

$$\underline{R}(M)(v_{ij}) = \Big(\bigwedge_{v_{lm} \in \underline{\mathrm{E}}^* - v_{ij}} T_{\underline{R}(M)}(v_{lm}), \bigvee_{v_{lm} \in \underline{\mathrm{E}}^* - v_{ij}} I_{\underline{R}(M)}(v_{lm}), \bigvee_{v_{lm} \in \underline{\mathrm{E}}^* - v_{ij}} F_{\underline{R}(M)}(v_{lm}) \Big),$$

$$\overline{R}(M)(v_{ij}) = \Big(\bigwedge_{v_{lm} \in \overline{\mathrm{E}}^* - v_{ij}} T_{\overline{R}(M)}(v_{lm}), \bigvee_{v_{lm} \in \overline{\mathrm{E}}^* - v_{ij}} I_{\overline{R}(M)}(v_{lm}), \bigvee_{v_{lm} \in \overline{\mathrm{E}}^* - v_{ij}} F_{\overline{R}(M)}(v_{lm}) \Big).$$

A soft rough neutrosophic influence graph is a soft rough neutrosophic influence cycle if and only if the graphs is soft rough neutrosophic cycle and there exist $v_l v_{mn}, v_i v_{jk} \in \underline{I}^*, v_l v_{mn}, v_i v_{jk} \in \overline{I}^*$ *and* $v_l v_{mn} \neq v_i v_{jk}$*, such that*

$$\underline{X}(Q)(v_i v_{jk}) = \Big(\bigwedge_{v_l v_{mn} \in \underline{\mathrm{I}}^* - v_i v_{jk}} T_{\underline{X}(Q)}(v_l v_{mn}), \bigvee_{v_l v_{mn} \in \underline{\mathrm{I}}^* - v_i v_{jk}} I_{\underline{X}(Q)}(v_l v_{mn}), \bigvee_{v_l v_{mn} \in \underline{\mathrm{I}}^* - v_i v_{jk}} F_{\underline{X}(Q)}(v_l v_{mn}) \Big),$$

$$\overline{X}(Q)(v_i v_{jk}) = \Big(\bigwedge_{v_l v_{mn} \in \overline{\mathrm{I}}^* - v_i v_{jk}} T_{\overline{X}(Q)}(v_l v_{mn}), \bigvee_{v_l v_{mn} \in \overline{\mathrm{I}}^* - v_i v_{jk}} I_{\overline{X}(Q)}(v_l v_{mn}), \bigvee_{v_l v_{mn} \in \overline{\mathrm{I}}^* - v_i v_{jk}} F_{\overline{X}(Q)}(v_l v_{mn}) \Big).$$

Example 7. *Considering Example 4. Let* $I = \{v_1 v_{32}, v_1 v_{24}, v_2 v_{13}, v_3 v_{24}, v_3 v_{41}, v_4 v_{13}, v_4 v_{32}\} \subseteq \hat{V}$ *and* $P = \{a_1 a_{32}, a_2 a_{43}, a_4 a_{13}\} \subseteq \hat{A}$*. Then a full soft set X on I (from P to I) can be defined in Table 14 as follows:*

Table 14. Full soft set X.

X	v_1v_{32}	v_1v_{24}	v_2v_{13}	v_3v_{24}	v_3v_{41}	v_4v_{13}	v_4v_{32}
a_1a_{32}	1	0	0	0	0	0	1
a_2a_{43}	0	0	1	0	0	1	0
a_4a_{13}	0	1	0	1	1	0	0

Let $Q=\{(v_1v_{32},0.2,0.1,0.0),(v_1v_{24},0.1,0.0,0.1),(v_2v_{13},0.2,0.1,0.0),(v_3v_{24},0.2,0.1,0.0),(v_4v_{13},0.1,0.1,0.0),(v_4v_{32},0.0,0.1,0.0)\}$ *be a neutrosophic set on* I*. Then from Equation (3) of Definition 4, we have*

$$\overline{X}(Q)=\{(v_1v_{32},0.2,0.1,0.0),(v_1v_{24},0.1,0.0,0.0),(v_2v_{13},0.2,0.1,0.0),(v_3v_{24},0.1,0.0,0.0),(v_3v_{41},0.1,0.0,0.0),(v_4v_{13},0.1,0.1,0.0),(v_4v_{32},0.2,0.1,0.0)\},$$
$$\underline{X}(Q)=\{(v_1v_{32},0.0,0.1,0.0),(v_1v_{24},0.1,0.1,0.1),(v_2v_{13},0.1,0.1,0.1),(v_3v_{24},0.0,0.1,0.1),(v_3v_{41},0.1,0.1,0.1),(v_4v_{13},0.1,0.1,0.1),(v_4v_{32},0.0,0.1,0.0)\}.$$

Thus, $\underline{G}=(\underline{S}(N),\underline{R}(M),\underline{X}(Q))$ *and* $\overline{G}=(\overline{S}(N),\overline{R}(M),X(Q))$ *are LSRNIAG and USRNIAG, respectively, as shown in Figure 7. Hence,* $G=(\overline{G},\underline{G})$ *is SRNIG. The underlying graph* $G^*=(\underline{G}^*,\overline{G}^*)$ *such that* $\underline{G}^*=(\underline{V}^*,\underline{E}^*,\underline{I}^*)$, $\overline{G}^*=(\overline{V}^*,\overline{E}^*,\overline{I}^*)$ *where* $\underline{V}^*=V=\overline{V}^*$, $\underline{E}^*=E=\overline{E}^*$ *and* $\underline{I}^*=I=\overline{I}^*$. $v_{13},v_{32},v_{24},v_{41}$ *are the lower edge and upper edge and* v_1v_{32},v_4v_{32} *is a lower pair and upper pair.* v_2v_{41} *is both lower and upper non-influence edge.* $P(v_1,v_4)$ *is a path of the sequence of the form* $v_1,v_1v_{24},v_{24},v_2v_{13},v_3,v_3v_{41},v_{41},v_1v_{32},v_2,v_2v_{13},v_3v_{24},v_4$*. By direct calculations, the strength and influence strength of this path are* $(0.2,0.2,0.1)$ *and* $(0.0,0.1,0.1)$*, respectively.* G *is cycle, soft rough neutrosophic cycle and soft rough neutrosophic influence cycle.*

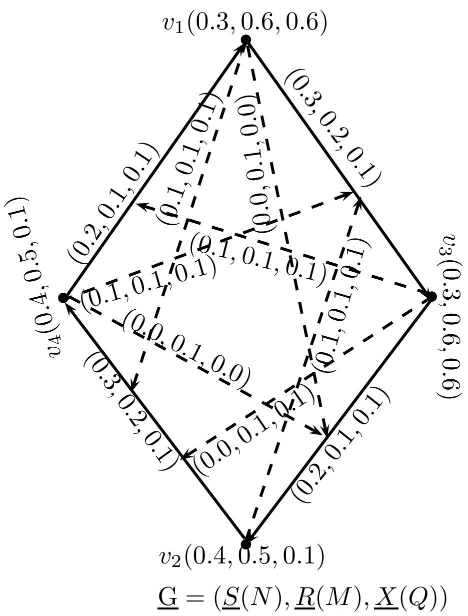

Figure 7. Soft rough neutrosophic influence graph $G=(\underline{G},\overline{G})$.

Definition 29. *A soft rough neutrosophic influence subgraph* $H=(SN_2,RM_2,XQ_2)$ *of a soft rough neutrosophic influence graph* $G=(SN_1,RM_1,XQ_1)$*, if* $v\in H$ *such that*

$$T_{\underline{S}(N_2)}(v)\leq T_{\underline{S}(N_1)}(v),\ I_{\underline{S}(N_2)}(v)\geq I_{\underline{S}(N_1)}(v),\ F_{\underline{S}(N_2)}(v)\geq F_{\underline{S}(N_1)}(v),$$
$$T_{\overline{S}(N_2)}(v)\leq T_{\overline{S}(N_1)}(v),\ I_{\overline{S}(N_2)}(v)\geq I_{\overline{S}(N_1)}(v),\ F_{\overline{S}(N_2)}(v)\geq F_{\overline{S}(N_1)}(v),$$

$v_{ij}\in H$,

$$T_{\underline{R}(M_2)}(v_{ij})\leq T_{\underline{R}(M_1)}(v_{ij}),\ I_{\underline{R}(M_2)}(v_{ij})\geq I_{\underline{R}(M_1)}(v_{ij}),\ F_{\underline{R}(M_2)}(v_{ij})\geq F_{\underline{R}(M_1)}(v_{ij}),$$
$$T_{\overline{R}(M_2)}(v_{ij})\leq T_{\overline{R}(M_1)}(v_{ij}),\ I_{\overline{R}(M_2)}(v_{ij})\geq I_{\overline{R}(M_1)}(v_{ij}),\ F_{\overline{R}(M_2)}(v_{ij})\geq F_{\overline{R}(M_1)}(v_{ij}),$$

and $v_iv_{jk}\in H$,

$$T_{\underline{X}(Q_2)}(v_iv_{jk})\leq T_{\underline{X}(Q_1)}(v_iv_{jk}),\ I_{\underline{X}(Q_2)}(v_iv_{jk})\geq I_{\underline{X}(Q_1)}(v_iv_{jk}),\ F_{\underline{X}(Q_2)}(v_{ij})\geq F_{\underline{X}(Q_1)}(v_iv_{jk}),$$

$$T_{\overline{X}(Q_2)}(v_iv_{jk}){\le}T_{\overline{X}(Q_1)}(v_iv_{jk}),\ I_{\overline{X}(Q_2)}(v_iv_{jk}){\ge}I_{\overline{X}(Q_1)}(v_iv_{jk}),\ F_{\overline{X}(Q_2)}(v_{ij}){\ge}F_{\overline{X}(Q_1)}(v_iv_{jk}).$$

Definition 30. *A* $\mathrm{H}{=}(SN_2{,}RM_2{,}XQ_2)$ *is called soft rough neutrosophic influence spanning subgraph of a soft rough neutrosophic influence graph* $\mathrm{G}{=}(SN_1{,}RM_1{,}XQ_1)$*, if H is a soft rough neutrosophic influence subgraph such that*

$$T_{\underline{S}(N_2)}(v){=}T_{\underline{S}(N_1)}(v),\ I_{\underline{S}(N_2)}(v){=}I_{\underline{S}(N_1)}(v),\ F_{\underline{S}(N_2)}(v){=}F_{\underline{S}(N_1)}(v),$$
$$T_{\overline{S}(N_2)}(v){=}T_{\overline{S}(N_1)}(v),\ I_{\overline{S}(N_2)}(v){=}I_{\overline{S}(N_1)}(v),\ F_{\overline{S}(N_2)}(v){=}F_{\overline{S}(N_1)}(v).$$

Definition 31. *A soft rough neutrosophic influence graph is a forest if and only if each supporting approximation graph is a forest. A soft rough neutrosophic influence graph* $G{=}(SN_1{,}RM_1{,}XQ_1)$ *is a soft rough neutrosophic forest if and only if there exist a soft rough neutrosophic spanning subgraph* $\mathrm{H}{=}(SN_1{,}RM_2{,}XQ_2)$ *is a forest such that* $v_{ij}{\in}\mathrm{G}{-}\mathrm{H}$

$$T_{\underline{R}(M_1)}(v_{ij}){<}T_{CONN_{\underline{\mathrm{H}}}}(v_i,v_j),\ I_{\underline{R}(M_1)}(v_{ij}){>}I_{CONN_{\underline{\mathrm{H}}}}(v_i,v_j),\ F_{\underline{R}(M_1)}(v_{ij})\ {>}F_{CONN_{\underline{\mathrm{H}}}}(v_i,v_j),$$
$$T_{\overline{R}(M_1)}(v_{ij}){<}T_{CONN_{\overline{\mathrm{H}}}}(v_i,v_j),\ I_{\overline{R}(M_1)}(v_{ij}){>}I_{CONN_{\overline{\mathrm{H}}}}(v_i,v_j),\ F_{\overline{R}(M_1)}(v_{ij})\ {>}F_{CONN_{\overline{\mathrm{H}}}}(v_i,v_j).$$

A soft rough neutrosophic influence graph $\mathrm{G}{=}(SN_1{,}RM_1{,}XQ_1)$ *is a soft rough neutrosophic influence forest if and only if there exist a soft rough neutrosophic spanning subgraph* $\mathrm{H}{=}(SN_1{,}RM_1{,}XQ_2)$ *is a forest such that* $v_iv_{jk}{\in}\mathrm{G}{-}\mathrm{H}$

$$T_{\underline{X}(Q_1)}(v_iv_{jk}){<}T_{ICONN_{\underline{\mathrm{H}}}}(v_i,v_k),\ T_{\overline{X}(Q_1)}(v_iv_{jk}){<}T_{ICONN_{\overline{\mathrm{H}}}}(v_i,v_k),$$
$$I_{\underline{X}(Q_1)}(v_iv_{jk})\ {>}I_{ICONN_{\underline{\mathrm{H}}}}(v_i,v_k),\ I_{\overline{X}(Q_1)}(v_iv_{jk})\ {>}I_{ICONN_{\overline{\mathrm{H}}}}(v_i,v_k),$$
$$F_{\underline{X}(Q_1)}(v_iv_{jk}){>}F_{ICONN_{\underline{\mathrm{H}}}}(v_i,v_k),\ F_{\overline{X}(Q_1)}(v_iv_{jk}){>}F_{ICONN_{\overline{\mathrm{H}}}}(v_i,v_k).$$

Definition 32. *A soft rough neutrosophic influence graph is a tree if and only if each supporting approximation graph is a tree. A soft rough neutrosophic influence graph* $G{=}(SN_1{,}RM_1{,}XQ_1)$ *is a soft rough neutrosophic tree if and only if there exist a soft rough neutrosophic spanning subgraph* $\mathrm{H}{=}(SN_1{,}RM_2{,}XQ_2)$ *is a tree such that* $v_{ij}{\in}\mathrm{G}{-}\mathrm{H}$

$$T_{\underline{R}(M_1)}(v_{ij}){<}T_{CONN_{\underline{\mathrm{H}}}}(v_i,v_j),\ I_{\underline{R}(M_1)}(v_{ij}){>}I_{CONN_{\underline{\mathrm{H}}}}(v_i,v_j),\ F_{\underline{R}(M_1)}(v_{ij})\ {>}F_{CONN_{\underline{\mathrm{H}}}}(v_i,v_j),$$
$$T_{\overline{R}(M_1)}(v_{ij}){<}T_{CONN_{\overline{\mathrm{H}}}}(v_i,v_j),\ I_{\overline{R}(M_1)}(v_{ij}){>}I_{CONN_{\overline{\mathrm{H}}}}(v_i,v_j),\ F_{\overline{R}(M_1)}(v_{ij})\ {>}F_{CONN_{\overline{\mathrm{H}}}}(v_i,v_j).$$

A soft rough neutrosophic influence graph $\mathrm{G}{=}(SN_1{,}RM_1{,}XQ_1)$ *is a soft rough neutrosophic influence tree if and only if there exist a soft rough neutrosophic spanning subgraph* $\mathrm{H}{=}(SN_1{,}RM_1{,}XQ_2)$ *is a tree such that* $v_iv_{jk}{\in}\mathrm{G}{-}\mathrm{H}$

$$T_{\underline{X}(Q_1)}(v_iv_{jk}){<}T_{ICONN_{\underline{\mathrm{H}}}}(v_i,v_k),\ T_{\overline{X}(Q_1)}(v_iv_{jk}){<}T_{ICONN_{\overline{\mathrm{H}}}}(v_i,v_k),$$
$$I_{\underline{X}(Q_1)}(v_iv_{jk})\ {>}I_{ICONN_{\underline{\mathrm{H}}}}(v_i,v_k),\ I_{\overline{X}(Q_1)}(v_iv_{jk})\ {>}I_{ICONN_{\overline{\mathrm{H}}}}(v_i,v_k),$$
$$F_{\underline{X}(Q_1)}(v_iv_{jk}){>}F_{ICONN_{\underline{\mathrm{H}}}}(v_i,v_k),\ F_{\overline{X}(Q_1)}(v_iv_{jk}){>}F_{ICONN_{\overline{\mathrm{H}}}}(v_i,v_k).$$

Definition 33. *Let* $\mathrm{G}{=}(SN{,}RM{,}XQ)$ *be a soft rough neutrosophic influence graph, an edge* v_{ij} *is a bridge if edge* v_{ij} *is a bridge in both underlying graphs of* $\underline{\mathrm{G}}$ *and* $\overline{\mathrm{G}}$*. Let* $\mathrm{G}{=}(SN{,}RM{,}XQ)$ *be a soft rough neutrosophic influence graph, an edge* v_{ij} *is a soft rough neutrosophic bridge if* $v_{lm}{\in}\mathrm{G}$

$$T_{CONN_{\underline{G}-v_{ij}}}(v_l,v_m)<T_{CONN_{\underline{G}}}(v_l,v_m),\ T_{CONN_{\overline{G}-v_{ij}}}(v_l,v_m)<T_{CONN_{\overline{G}}}(v_l,v_m),$$
$$I_{CONN_{\underline{G}-v_{ij}}}(v_l,v_m)>I_{CONN_{\underline{G}}}(v_l,v_m),\ I_{CONN_{\overline{G}-v_{ij}}}(v_l,v_m)>I_{CONN_{\overline{G}}}(v_l,v_m),$$
$$F_{CONN_{\underline{G}-v_{ij}}}(v_l,v_m)>F_{CONN_{\underline{G}}}(v_l,v_m),\ F_{CONN_{\overline{G}-v_{ij}}}(v_l,v_m)>F_{CONN_{\overline{G}}}(v_l,v_m),$$

Let $\mathrm{G}=(SN,RM,XQ)$ *be a soft rough neutrosophic influence graph, an edge* v_{ij} *is an soft rough neutrosophic influence bridge if* $v_{lm}\in\mathrm{G}$

$$T_{ICONN_{\underline{G}-v_{ij}}}(v_l,v_m)<T_{ICONN_{\underline{G}}}(v_l,v_m),\ T_{ICONN_{\overline{G}-v_{ij}}}(v_l,v_m)<T_{ICONN_{\overline{G}}}(v_l,v_m),$$
$$I_{ICONN_{\underline{G}-v_{ij}}}(v_l,v_m)>I_{ICONN_{\underline{G}}}(v_l,v_m),\ ;I_{ICONN_{\overline{G}-v_{ij}}}(v_l,v_m)>I_{ICONN_{\overline{G}}}(v_l,v_m),$$
$$F_{ICONN_{\underline{G}-v_{ij}}}(v_l,v_m)>F_{ICONN_{\underline{G}}}(v_l,v_m),\ F_{ICONN_{\overline{G}-v_{ij}}}(v_l,v_m)>F_{ICONN_{\overline{G}}}(v_l,v_m),$$

Definition 34. *Let* $\mathrm{G}=(SN,RM,XQ)$ *be a soft rough neutrosophic influence graph, a vertex is a cutnode if a vertex* v_i *is a cutnode in underlying graphs of* $\underline{\mathrm{G}}$ *and* $\overline{\mathrm{G}}$*. Let* $\mathrm{G}=(SN,RM,XQ)$ *be a soft rough neutrosophic influence graph then a vertex* v_i *in* G *is a soft rough neutrosophic cutnode if the deletion of* v_i *from* G *reduces the strength of the connectedness from nodes* $v_j\to v_k\in\underline{\mathrm{V}}^*,\overline{\mathrm{V}}^*$

$$T_{CONN_{\underline{G}-v_i}}(v_j,v_k)<T_{CONN_{\underline{G}}}(v_j,v_k),\ T_{CONN_{\overline{G}-v_i}}(v_j,v_k)<T_{CONN_{\overline{G}}}(v_j,v_k),$$
$$I_{CONN_{\underline{G}-v_i}}(v_j,v_k)>I_{CONN_{\underline{G}}}(v_j,v_k),\ I_{CONN_{\overline{G}-v_i}}(v_j,v_k)>I_{CONN_{\overline{G}}}(v_j,v_k),$$
$$F_{CONN_{\underline{G}-v_i}}(v_j,v_k)>F_{CONN_{\underline{G}}}(v_j,v_k),\ F_{CONN_{\overline{G}-v_i}}(v_j,v_k)>F_{CONN_{\overline{G}}}(v_j,v_k).$$

Let $\mathrm{G}=(SN,RM,XQ)$ *be a soft rough neutrosophic influence graph then a vertex* v_i *in* G *is an neutrosophic influence cutnode if the deletion of* v_i *from* G *reduces the influence strength of the connectedness from* $v_j\to v_k\in\underline{\mathrm{V}}^*,\overline{\mathrm{V}}^*$

$$T_{ICONN_{\underline{G}-v_i}}(v_j,v_k)<T_{ICONN_{\underline{G}}}(v_j,v_k),\ T_{ICONN_{\overline{G}-v_i}}(v_j,v_k)<T_{ICONN_{\overline{G}}}(v_j,v_k),$$
$$I_{ICONN_{\underline{G}-v_i}}(v_j,v_k)>I_{ICONN_{\underline{G}}}(v_j,v_k),\ I_{ICONN_{\overline{G}-v_i}}(v_j,v_k)>I_{ICONN_{\overline{G}}}(v_j,v_k),$$
$$F_{ICONN_{\underline{G}-v_i}}(v_j,v_k)>F_{ICONN_{\underline{G}}}(v_j,v_k),\ F_{ICONN_{\overline{G}-v_i}}(v_j,v_k)>F_{ICONN_{\overline{G}}}(v_j,v_k),$$

Definition 35. *Let* $\mathrm{G}=(SN,RM,XQ)$ *be a soft rough neutrosophic influence graph. A pair* v_iv_{jk} *is called a cutpair if and only if* v_iv_{jk} *is a cutpair in both supporting influence graph of* $\underline{\mathrm{G}}$ *and* $\overline{\mathrm{G}}$*. That is after removing the pair* v_iv_{jk} *there is no path from* v_i *to* v_k *in both supporting influence graph of* $\underline{\mathrm{G}}$ *and* $\overline{\mathrm{G}}$*. Let* $\mathrm{G}=(SN,RM,XQ)$ *be a soft rough neutrosophic influence graph. A pair* v_iv_{jk} *is called a soft rough neutrosophic cutpair if and only if if deleting the pair* v_iv_{jk} *reduces the connectedness from* v_i *to* v_k *in both graph* $\underline{\mathrm{G}}$ *and* $\overline{\mathrm{G}}$*. That is,*

$$T_{CONN_{\underline{G}-v_iv_{jk}}}(v_i,v_k)<T_{CONN_{\underline{G}}}(v_i,v_k),\ T_{ICONN_{\overline{G}-v_iv_{jk}}}(v_i,v_k)<T_{ICONN_{\overline{G}}}(v_i,v_k),$$
$$I_{CONN_{\underline{G}-v_iv_{jk}}}(v_i,v_k)>I_{CONN_{\underline{G}}}(v_i,v_k),\ I_{ICONN_{\overline{G}-v_iv_{jk}}}(v_i,v_k)>I_{ICONN_{\overline{G}}}(v_i,v_k),$$
$$F_{CONN_{\underline{G}-v_iv_{jk}}}(v_i,v_k)>F_{CONN_{\underline{G}}}(v_i,v_k),\ F_{ICONN_{\overline{G}-v_iv_{jk}}}(v_i,v_k)>F_{ICONN_{\overline{G}}}(v_i,v_k),$$

A soft rough neutrosophic influence cutpair v_iv_{jk} is a pair in a soft rough neutrosophic influence graph G=(SN,RM,XQ) *if there is spanning influence subgraph* $\mathrm{H} = G - v_iv_{jk}$ *reduces the strength of the influence connectedness from v_i to v_k. That is,*

$$T_{ICONN_{\underline{G}-v_iv_{jk}}}(v_i,v_k)<T_{ICONN_{\underline{G}}}(v_i,v_k),\ T_{ICONN_{\overline{G}-v_iv_{jk}}}(v_i,v_k)<T_{ICONN_{\overline{G}}}(v_i,v_k),$$
$$I_{ICONN_{\underline{G}-v_iv_{jk}}}(v_i,v_k)>I_{ICONN_{\underline{G}}}(v_i,v_k),\ I_{ICONN_{\overline{G}-v_iv_{jk}}}(v_i,v_k)>I_{ICONN_{\overline{G}}}(v_i,v_k),$$
$$F_{ICONN_{\underline{G}-v_iv_{jk}}}(v_i,v_k)>F_{ICONN_{\underline{G}}}(v_i,v_k),\ F_{ICONN_{\overline{G}-v_iv_{jk}}}(v_i,v_k)>F_{ICONN_{\overline{G}}}(v_i,v_k),$$

Definition 36. *An edge v_{ij} in soft rough neutrosophic influence graph* G *is called strong soft rough neutrosophic edge if*

$$T_{\underline{R}(M)}(v_{ij})\geq T_{CONN_{\underline{G}-v_{ij}}}(v_i,v_j),\ T_{\overline{R}(M)}(v_{ij})\geq T_{CONN_{\overline{G}-v_{ij}}}(v_i,v_j),$$
$$I_{\underline{R}(M)}(v_{ij})\leq I_{CONN_{\underline{G}-v_{ij}}}(v_i,v_j),\ I_{\overline{R}(M)}(v_{ij})\leq I_{CONN_{\overline{G}-v_{ij}}}(v_i,v_j),$$
$$F_{\underline{R}(M)}(v_{ij})\leq F_{CONN_{\underline{G}-v_{ij}}}(v_i,v_j),\ F_{\overline{R}(M)}(v_{ij})\leq F_{CONN_{\overline{G}-v_{ij}}}(v_i,v_j).$$

A pair v_iv_{jk} in soft rough neutrosophic influence graph G *is called strong pair if*

$$T_{\underline{X}(Q)}(v_iv_{jk})\geq T_{ICONN_{\underline{G}-v_iv_{jk}}}(v_i,v_k),\ T_{\overline{X}(Q)}(v_iv_{jk})\geq T_{ICONN_{\overline{G}-v_iv_{jk}}}(v_i,v_k),$$
$$I_{\underline{X}(Q)}(v_iv_{jk})\leq I_{ICONN_{\underline{G}-v_iv_{jk}}}(v_i,v_k),\ I_{\overline{X}(Q)}(v_iv_{jk})\leq I_{ICONN_{\overline{G}-v_iv_{jk}}}(v_i,v_k),$$
$$F_{\underline{X}(Q)}(v_iv_{jk})\leq F_{ICONN_{\underline{G}-v_iv_{jk}}}(v_i,v_k),\ F_{\overline{X}(Q)}(v_iv_{jk})\leq F_{ICONN_{\overline{G}-v_iv_{jk}}}(v_i,v_k).$$

Definition 37. *An edge v_{ij} in soft rough neutrosophic influence graph* G *is called $\alpha-$strong soft rough neutrosophic edge if*

$$T_{\underline{R}(M)}(v_{ij})>T_{CONN_{\underline{G}-v_{ij}}}(v_i,v_j),\ T_{\overline{R}(M)}(v_{ij})>T_{CONN_{\overline{G}-v_{ij}}}(v_i,v_j),$$
$$I_{\underline{R}(M)}(v_{ij})<I_{CONN_{\underline{G}-v_{ij}}}(v_i,v_j),\ I_{\overline{R}(M)}(v_{ij})<I_{CONN_{\overline{G}-v_{ij}}}(v_i,v_j),$$
$$F_{\underline{R}(M)}(v_{ij})<F_{CONN_{\underline{G}-v_{ij}}}(v_i,v_j),\ F_{\overline{R}(M)}(v_{ij})<F_{CONN_{\overline{G}-v_{ij}}}(v_i,v_j).$$

A pair v_iv_{jk} in soft rough neutrosophic influence graph G *is called $\alpha-$strong pair if*

$$T_{\underline{X}(Q)}(v_iv_{jk})>T_{ICONN_{\underline{G}-v_iv_{jk}}}(v_i,v_k),\ T_{\overline{X}(Q)}(v_iv_{jk})>T_{ICONN_{\overline{G}-v_iv_{jk}}}(v_i,v_k),$$
$$I_{\underline{X}(Q)}(v_iv_{jk})<I_{ICONN_{\underline{G}-v_iv_{jk}}}(v_i,v_k),\ I_{\overline{X}(Q)}(v_iv_{jk})<I_{ICONN_{\overline{G}-v_iv_{jk}}}(v_i,v_k),$$
$$F_{\underline{X}(Q)}(v_iv_{jk})<F_{ICONN_{\underline{G}-v_iv_{jk}}}(v_i,v_k),\ F_{\overline{X}(Q)}(v_iv_{jk})<F_{ICONN_{\overline{G}-v_iv_{jk}}}(v_i,v_k).$$

Definition 38. *An edge v_{ij} in soft rough neutrosophic influence graph* G *is called $\beta-$strong soft rough neutrosophic edge if*

$$T_{\underline{R}(M)}(v_{ij})=T_{CONN_{\underline{G}-v_{ij}}}(v_i,v_j),\ T_{\overline{R}(M)}(v_{ij})=T_{CONN_{\overline{G}-v_{ij}}}(v_i,v_j),$$
$$I_{\underline{R}(M)}(v_{ij})=I_{CONN_{\underline{G}-v_{ij}}}(v_i,v_j),\ I_{\overline{R}(M)}(v_{ij})=I_{CONN_{\overline{G}-v_{ij}}}(v_i,v_j),$$
$$F_{\underline{R}(M)}(v_{ij})=F_{CONN_{\underline{G}-v_{ij}}}(v_i,v_j),\ F_{\overline{R}(M)}(v_{ij})=F_{CONN_{\overline{G}-v_{ij}}}(v_i,v_j).$$

A pair v_iv_{jk} in soft rough neutrosophic influence graph G *is called $\beta-$strong pair if*

$$T_{\underline{X}(Q)}(v_iv_{jk})=T_{ICONN_{\underline{G}-v_iv_{jk}}}(v_i,v_k),\ T_{\overline{X}(Q)}(v_iv_{jk})=T_{ICONN_{\overline{G}-v_iv_{jk}}}(v_i,v_k),$$
$$I_{\underline{X}(Q)}(v_iv_{jk})=I_{ICONN_{\underline{G}-v_iv_{jk}}}(v_i,v_k),\ I_{\overline{X}(Q)}(v_iv_{jk})=I_{ICONN_{\overline{G}-v_iv_{jk}}}(v_i,v_k),$$
$$F_{\underline{X}(Q)}(v_iv_{jk})=F_{ICONN_{\underline{G}-v_iv_{jk}}}(v_i,v_k),\ F_{\overline{X}(Q)}(v_iv_{jk})=F_{ICONN_{\overline{G}-v_iv_{jk}}}(v_i,v_k).$$

Definition 39. *An edge v_{ij} in soft rough neutrosophic influence graph* G *is called $\delta-$strong soft rough neutrosophic edge if*

$$T_{\underline{R}(M)}(v_{ij})<T_{CONN_{\underline{G}-v_{ij}}}(v_i,v_j),\ T_{\overline{R}(M)}(v_{ij})<T_{CONN_{\overline{G}-v_{ij}}}(v_i,v_j),$$
$$I_{\underline{R}(M)}(v_{ij})>I_{CONN_{\underline{G}-v_{ij}}}(v_i,v_j),\ I_{\overline{R}(M)}(v_{ij})>I_{CONN_{\overline{G}-v_{ij}}}(v_i,v_j),$$
$$F_{\underline{R}(M)}(v_{ij})>F_{CONN_{\underline{G}-v_{ij}}}(v_i,v_j),\ F_{\overline{R}(M)}(v_{ij})>F_{CONN_{\overline{G}-v_{ij}}}(v_i,v_j).$$

A pair v_iv_{jk} in soft rough neutrosophic influence graph G *is called $\delta-$strong pair if*

$$T_{\underline{X}(Q)}(v_iv_{jk})<T_{ICONN_{\underline{G}-v_iv_{jk}}}(v_i,v_k),\ T_{\overline{X}(Q)}(v_iv_{jk})<T_{ICONN_{\overline{G}-v_iv_{jk}}}(v_i,v_k),$$
$$I_{\underline{X}(Q)}(v_iv_{jk})>I_{ICONN_{\underline{G}-v_iv_{jk}}}(v_i,v_k),\ I_{\overline{X}(Q)}(v_iv_{jk})>I_{ICONN_{\overline{G}-v_iv_{jk}}}(v_i,v_k),$$
$$F_{\underline{X}(Q)}(v_iv_{jk})>F_{ICONN_{\underline{G}-v_iv_{jk}}}(v_i,v_k),\ F_{\overline{X}(Q)}(v_iv_{jk})>F_{ICONN_{\overline{G}-v_iv_{jk}}}(v_i,v_k).$$

Definition 40. *A $\delta-$strong soft rough neutrosophic edge v_{ij} is called a δ^*-strong soft rough neutrosophic edge if*

$$T_{\underline{R}(M)}(v_{ij})>\bigwedge_{v_{lm}\in\underline{E}^*}T_{\underline{R}(M)}(v_{lm}),\ T_{\overline{R}(M)}(v_{ij})>\bigwedge_{v_{lm}\in\overline{E}^*}T_{\overline{R}(M)}(v_{lm}),$$
$$I_{\underline{R}(M)}(v_{ij})<\bigwedge_{v_{lm}\in\underline{E}^*}I_{\underline{R}(M)}(v_{lm}),\ I_{\overline{R}(M)}(v_{ij})<\bigwedge_{v_{lm}\in\overline{E}^*}I_{\overline{R}(M)}(v_{lm}),$$
$$F_{\underline{R}(M)}(v_{ij})<\bigwedge_{v_{lm}\in\underline{E}^*}F_{\underline{R}(M)}(v_{lm}),\ F_{\overline{R}(M)}(v_{ij})<\bigwedge_{v_{lm}\in\overline{E}^*}F_{\overline{R}(M)}(v_{lm}).$$

A $\delta-$strong pair v_iv_{jk} is called a δ^*-strong pair if

$$T_{\underline{R}(M)}(v_{ij})>\bigwedge_{v_{lm}\in\underline{E}^*}T_{\underline{R}(M)}(v_{lm}),\ T_{\overline{R}(M)}(v_{ij})>\bigwedge_{v_{lm}\in\overline{E}^*}T_{\overline{R}(M)}(v_{lm}),$$
$$I_{\underline{R}(M)}(v_{ij})<\bigwedge_{v_{lm}\in\underline{E}^*}I_{\underline{R}(M)}(v_{lm}),\ I_{\overline{R}(M)}(v_{ij})<\bigwedge_{v_{lm}\in\overline{E}^*}I_{\overline{R}(M)}(v_{lm}),$$
$$F_{\underline{R}(M)}(v_{ij})<\bigwedge_{v_{lm}\in\underline{E}^*}F_{\underline{R}(M)}(v_{lm}),\ F_{\overline{R}(M)}(v_{ij})<\bigwedge_{v_{lm}\in\overline{E}^*}F_{\overline{R}(M)}(v_{lm}).$$

A $\delta-$strong pair v_iv_{jk} is called a δ^*-strong pair if $v_iv_{jk}\neq v_lv_{mn}$

$$T_{\underline{X}(Q)}(v_iv_{jk})>\bigwedge_{v_lv_{mn}\in\underline{I}^*}T_{\underline{X}(Q)}(v_lv_{mn}),\ T_{\overline{X}(Q)}(v_iv_{jk})>\bigwedge_{v_lv_{mn}\in\overline{I}^*}T_{\overline{X}(Q)}(v_lv_{mn}),$$
$$I_{\underline{X}(Q)}(v_iv_{jk})<\bigwedge_{v_lv_{mn}\in\underline{I}^*}I_{\underline{X}(Q)}(v_lv_{mn}),\ I_{\overline{X}(Q)}(v_iv_{jk})<\bigwedge_{v_lv_{mn}\in\overline{I}^*}I_{\overline{X}(Q)}(v_lv_{mn}),$$
$$F_{\underline{X}(Q)}(v_iv_{jk})<\bigwedge_{v_lv_{mn}\in\underline{I}^*}F_{\underline{X}(Q)}(v_lv_{mn}),\ F_{\overline{X}(Q)}(v_iv_{jk})<\bigwedge_{v_lv_{mn}\in\underline{I}^*}F_{\overline{X}(Q)}(v_lv_{mn}).$$

Definition 41. *A soft rough neutrosophic influence graph is said to be a soft rough neutrosophic influence block if it has no soft rough neutrosophic influence cutnodes.*

Example 8. *Consider Example 5 Let* $I=\{v_1v_{32}, v_1v_{43}, v_2v_{13}, v_3v_{32}, v_4v_{13}\}\subseteq\hat{V}$ *and* $P=\{a_1a_{34}, a_3a_{24}, a_4a_{12}\}\subseteq\hat{A}$*. Then a full soft set X on I (from P to I) can be defined in Table 15 as follows:*

Table 15. Full soft set X.

X	v_1v_{32}	v_1v_{43}	v_2v_{13}	v_3v_{32}	v_4v_{13}
a_1a_{34}	0	1	1	0	1
a_3a_{24}	0	1	0	0	0
a_4a_{12}	1	0	0	1	0

Let $Q=\{(vv_{32},0.3,0.0,0.0),(vv_{43},0.2,0.0,0.0),(v_2v_{13},0.1,0.0,0.0),(v_3v_{32},0.2,0.0,0.0),(v_4v_{13},0.3,0.0,0.0\}$ *be a neutrosophic set on I. Then from Equation (3) of Definition 4, we have*

$$\overline{X}(Q)=\{(v_1v_{32},0.3,0.0,0.0),(v_1v_{43},0.2,0.0,0.0),(v_2v_{13},0.3,0.0,0.0),(v_3v_{32},0.3,0.0,0.0),(v_4v_{13},0.3,0.0,0.0)\},$$

$$\underline{X}(Q)=\{(v_1v_{32},0.2,0.0,0.0),(v_1v_{43},0.2,0.0,0.0),(v_2v_{13},0.1,0.0,0.0),(v_3v_{32},0.2,0.0,0.0),(v_4v_{13},0.1,0.0,0.0)\}.$$

Thus, $\underline{G}=(\underline{S}(N),\underline{R}(M),\underline{X}(Q))$ *and* $\overline{G}=(\overline{S}(N),\overline{R}(M),X(Q))$ *are LSRNIAG and USRNIAG, respectively, as shown in Figure 8. Hence,* $G=(\overline{G},\underline{G})$ *is SRNIG. Hence G is a tree,* v_3 *is a cutvertex,* v_{13} *is a bridge,* v_3v_{32} *is a cutpair.*

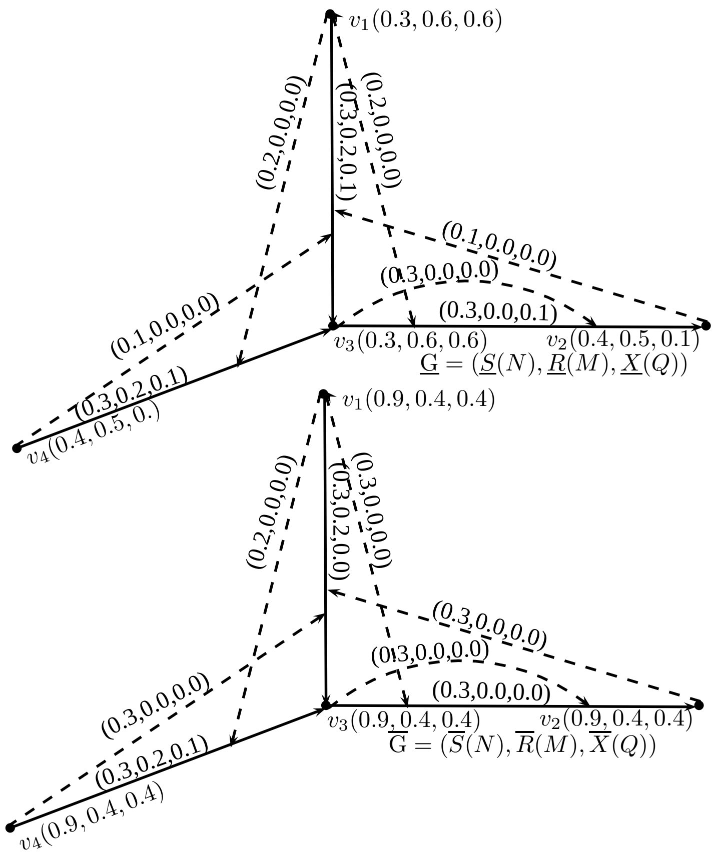

Figure 8. Soft rough neutrosophic influence graph $G=(\underline{G},\overline{G})$.

Theorem 4. *G is a soft rough neutrosophic influence forest if and only if in any cycle of G, there is a pair v_iv_{jk} such that*

$$T_{\underline{X}(Q)}(v_iv_{jk})<T_{ICONN_{\underline{G}-v_iv_{jk}}}(v_i,v_k),\ T_{\overline{X}(Q)}(v_iv_{jk})<T_{ICONN_{\overline{G}-v_iv_{jk}}}(v_i,v_k),$$
$$I_{\underline{X}(Q)}(v_iv_{jk})>I_{ICONN_{\underline{G}-v_iv_{jk}}}(v_i,v_k),\ I_{\overline{X}(Q)}(v_iv_{jk})>I_{ICONN_{\overline{G}-v_iv_{jk}}}(v_i,v_k),$$
$$F_{\underline{X}(Q)}(v_iv_{jk})>F_{ICONN_{\underline{G}-v_iv_{jk}}}(v_i,v_k),\ F_{\overline{X}(Q)}(v_iv_{jk})>F_{ICONN_{\overline{G}-v_iv_{jk}}}(v_i,v_k).$$

Proof. The proof is obvious. □

Theorem 5. *A soft rough neutrosophic graph G is a soft rough neutrosophic influence forest if there is at most one path with the most influence strength.*

Proof. Let G be not a soft rough neutrosophic influence forest. Then by Theorem 4, there exist a cycle C in G such that

$$T_{\underline{X}(Q)}(v_iv_{jk})\geq T_{ICONN_{\underline{G}-v_iv_{jk}}}(v_i,v_k),\ T_{\overline{X}(Q)}(v_iv_{jk})\geq T_{ICONN_{\overline{G}-v_iv_{jk}}}(v_i,v_k),$$
$$I_{\underline{X}(Q)}(v_iv_{jk})\leq I_{ICONN_{\underline{G}-v_iv_{jk}}}(v_i,v_k),\ I_{\overline{X}(Q)}(v_iv_{jk})\leq I_{ICONN_{\overline{G}-v_iv_{jk}}}(v_i,v_k),$$
$$F_{\underline{X}(Q)}(v_iv_{jk})\leq F_{ICONN_{\underline{G}-v_iv_{jk}}}(v_i,v_k),\ F_{\overline{X}(Q)}(v_iv_{jk})\leq F_{ICONN_{\overline{G}-v_iv_{jk}}}(v_i,v_k),$$

for every pair v_iv_{jk} of C.

Therefore, v_iv_{jk} is the path within the most influence strength from v_i to v_k. Let v_iv_{jk} be a pair such that

$$T_{\underline{X}(Q)}(v_iv_{jk})>\bigwedge_{v_lv_{mn}\in\underline{I}^*}T_{\underline{X}(Q)}(v_lv_{mn}),\ T_{\overline{X}(Q)}(v_iv_{jk})>\bigwedge_{v_lv_{mn}\in\overline{I}^*}T_{\overline{X}(Q)}(v_lv_{mn}),$$
$$I_{\underline{X}(Q)}(v_iv_{jk})<\bigwedge_{v_lv_{mn}\in\underline{I}^*}I_{\underline{X}(Q)}(v_lv_{mn}),\ I_{\overline{X}(Q)}(v_iv_{jk})<\bigwedge_{v_lv_{mn}\in\underline{I}^*}I_{\overline{X}(Q)}(v_lv_{mn}),$$
$$F_{\underline{X}(Q)}(v_iv_{jk})<\bigwedge_{v_lv_{mn}\in\underline{I}^*}F_{\underline{X}(Q)}(v_lv_{mn}),\ F_{\overline{X}(Q)}(v_iv_{jk})<\bigwedge_{v_lv_{mn}\in\underline{I}^*}F_{\overline{X}(Q)}(v_lv_{mn}),$$

in C. Then remaining part of C is a path with the most influence strength from v_i to v_{jk}. This is a contradiction to the the fact there is at most one path with the most influence strength. Hence, G is a soft rough neutrosophic influence forest. □

Theorem 6. *Assume that G is a cycle. Then G is not a soft rough neutrosophic influence tree if and only if G is a soft rough neutrosophic influence cycle.*

Proof. Let G=(SN,RM,XQ_1) be a soft rough neutrosophic influence cycle. Then there exist at least two distinct edge and two distinct pair such that

$$\begin{aligned}
\underline{R}(M)(v_{ij}) &= \Big(\bigwedge_{v_{lm}\in \underline{E}^*-v_{ij}} T_{\underline{R}(M)}(v_{lm}), \bigvee_{v_{lm}\in \underline{E}^*-v_{ij}} I_{\underline{R}(M)}(v_{lm}), \bigvee_{v_{lm}\in \underline{E}^*-v_{ij}} F_{\underline{R}(M)}(v_{lm}) \Big),\\
\overline{R}(M)(v_{ij}) &= \Big(\bigwedge_{v_{lm}\in \overline{E}^*-v_{ij}} T_{\overline{R}(M)}(v_{lm}), \bigvee_{v_{lm}\in \overline{E}^*-v_{ij}} I_{\overline{R}(M)}(v_{lm}), \bigvee_{v_{lm}\in \overline{E}^*-v_{ij}} F_{\overline{R}(M)}(v_{lm}) \Big),\\
\underline{X}(Q)(v_iv_{jk}) &= \Big(\bigwedge_{v_lv_{mn}\in \underline{I}^*-v_iv_{jk}} T_{\underline{X}(Q)}(v_lv_{mn}), \bigvee_{v_lv_{mn}\in \underline{I}^*-v_iv_{jk}} I_{\underline{X}(Q)}(v_lv_{mn}), \bigvee_{v_lv_{mn}\in \underline{I}^*-v_iv_{jk}} F_{\underline{X}(Q)}(v_lv_{mn}) \Big),\\
\overline{X}(Q)(v_iv_{jk}) &= \Big(\bigwedge_{v_lv_{mn}\in \overline{I}^*-v_iv_{jk}} T_{\overline{X}(Q)}(v_lv_{mn}), \bigvee_{v_lv_{mn}\in \overline{I}^*-v_iv_{jk}} I_{\overline{X}(Q)}(v_lv_{mn}), \bigvee_{v_lv_{mn}\in \overline{I}^*-v_iv_{jk}} F_{\overline{X}(Q)}(v_lv_{mn}) \Big).
\end{aligned}$$

Let H=(SN,RM,XQ_2) be a spanning soft rough neutrosophic influence tree in G. Then there exists a path from v_i to v_k not involving v_iv_{jk} such that $E_1^*-E_2^*=\{(v_iv_{jk})\}$. Hence there does not exist a path in H from v_i to v_k such that

$$\begin{aligned}
&T_{\underline{X}(Q_2)}(v_iv_{jk}) \le T_{ICONN_{\underline{G}}}(v_i,v_k),\ T_{\overline{X}(Q_2)}(v_iv_{jk}) \le T_{ICONN_{\overline{G}}}(v_i,v_k),\\
&I_{\underline{X}(Q_2)}(v_iv_{jk}) \ge I_{ICONN_{\underline{G}}}(v_i,v_k),\ I_{\overline{X}(Q_2)}(v_iv_{jk}) \ge I_{ICONN_{\overline{G}}}(v_i,v_k),\\
&F_{\underline{X}(Q_2)}(v_iv_{jk}) \ge F_{ICONN_{\underline{G}}}(v_i,v_k),\ F_{\overline{X}(Q_2)}(v_iv_{jk}) \ge F_{ICONN_{\overline{G}}}(v_i,v_k).
\end{aligned}$$

Thus G is not a soft rough neutrosophic influence tree.

Conversely, suppose that G is not a soft rough neutrosophic influence tree. Since, G is a soft rough neutrosophic influence cycle. So for all $v_iv_{jk}\in\underline{I}^*$ and $v_iv_{jk}\in\overline{I}^*$, we have a soft rough neutrosophic spanning influence subrgraph H=(SN,RM,XQ_2) which is tree and $\underline{X}(Q_2)(v_iv_{jk})$=0, $\overline{X}(Q_2)(v_iv_{jk})$=0 such that $\forall v_iv_{lm} \neq v_lv_{mn}$

$$\begin{aligned}
&T_{\underline{X}(Q_2)}(v_iv_{jk}) \le T_{ICONN_{\underline{H}}}(v_i,v_k),\ T_{\overline{X}(Q_2)}(v_iv_{jk}) \le T_{ICONN_{\overline{G}}}(v_i,v_k),\\
&I_{\underline{X}(Q_2)}(v_iv_{jk}) \ge I_{ICONN_{\underline{G}}}(v_i,v_k),\ I_{\overline{X}(Q_2)}(v_iv_{jk}) \ge I_{ICONN_{\overline{G}}}(v_i,v_k),\\
&F_{\underline{X}(Q_2)}(v_iv_{jk}) \ge F_{ICONN_{\underline{G}}}(v_i,v_k),\ F_{\overline{X}(Q_2)}(v_iv_{jk}) \ge F_{ICONN_{\overline{G}}}(v_i,v_k),
\end{aligned}$$

$\forall v_lv_{mn} \in \underline{I}^* - v_iv_{jk}$ and $v_lv_{mn} \in \overline{I}^* - v_iv_{jk}$

$$\begin{aligned}
&T_{\underline{X}(Q_2)}(v_iv_{jk}) = \bigwedge_{v_lv_{mn}\in\underline{I}^*} T_{\underline{X}(Q_1)}(v_lv_{mn}),\ T_{\overline{X}(Q_2)}(v_iv_{jk}) = \bigwedge_{v_lv_{mn}\in\overline{I}^*} T_{\overline{X}(Q_1)}(v_lv_{mn}),\\
&I_{\underline{X}(Q_2)}(v_iv_{jk}) = \bigwedge_{v_lv_{mn}\in\underline{I}^*} I_{\underline{X}(Q_1)}(v_lv_{mn}),\ I_{\overline{X}(Q_2)}(v_iv_{jk}) = \bigwedge_{v_lv_{mn}\in\underline{I}^*} I_{\overline{X}(Q_1)}(v_lv_{mn}),\\
&F_{\underline{X}(Q_2)}(v_iv_{jk}) = \bigwedge_{v_lv_{mn}\in\underline{I}^*} F_{\underline{X}(Q_1)}(v_lv_{mn}),\ F_{\overline{X}(Q_2)}(v_iv_{jk}) = \bigwedge_{v_lv_{mn}\in\underline{I}^*} F_{\overline{X}(Q_1)}(v_lv_{mn}).
\end{aligned}$$

Therefore,

$$\begin{aligned}
\underline{X}(Q)(v_iv_{jk}) &= \Big(\bigwedge_{v_lv_{mn}\in \underline{I}^*-v_iv_{jk}} T_{\underline{X}(Q)}(v_lv_{mn}), \bigvee_{v_lv_{mn}\in \underline{I}^*-v_iv_{jk}} I_{\underline{X}(Q)}(v_lv_{mn}), \bigvee_{v_lv_{mn}\in \underline{I}^*-v_iv_{jk}} F_{\underline{X}(Q)}(v_lv_{mn}) \Big),\\
\overline{X}(Q)(v_iv_{jk}) &= \Big(\bigwedge_{v_lv_{mn}\in \overline{I}^*-v_iv_{jk}} T_{\overline{X}(Q)}(v_lv_{mn}), \bigvee_{v_lv_{mn}\in \overline{I}^*-v_iv_{jk}} I_{\overline{X}(Q)}(v_lv_{mn}), \bigvee_{v_lv_{mn}\in \overline{I}^*-v_iv_{jk}} F_{\overline{X}(Q)}(v_lv_{mn}) \Big).
\end{aligned}$$

where $v_iv_{jk} \neq v_lv_{mn}$ not uniquely. Therefore G is a soft rough neutrosophic influence cycle. □

Theorem 7. *If*

$$
\begin{aligned}
&T_{\underline{X}(Q)}(v_iv_{jk})>T_{ICONN_{\underline{G}-v_iv_{jk}}}(v_i,v_k),\ \ T_{\overline{X}(Q)}(v_iv_{jk})>T_{ICONN_{\overline{G}-v_iv_{jk}}}(v_i,v_k),\\
&I_{\underline{X}(Q)}(v_iv_{jk})<I_{ICONN_{\underline{G}-v_iv_{jk}}}(v_i,v_k),\ \ I_{\overline{X}(Q)}(v_iv_{jk})<I_{ICONN_{\overline{G}-v_iv_{jk}}}(v_i,v_k),\\
&F_{\underline{X}(Q)}(v_iv_{jk})<F_{ICONN_{\underline{G}-v_iv_{jk}}}(v_i,v_k),\ \ F_{\overline{X}(Q)}(v_iv_{jk})<F_{ICONN_{\overline{G}-v_iv_{jk}}}(v_i,v_k),
\end{aligned}
$$

in a soft rough neutrosophic graph. Then v_iv_{jk} is a cutpair in soft rough neutrosophic influence graph G.

Proof. Suppose v_iv_{jk} is not a cutapir in soft rough neutrosophic influence graph, then

$$
\begin{aligned}
&T_{ICONN_{\underline{G}-v_i,v_k}}(v_i,v_k)=T_{ICONN_{\underline{G}}}(v_i,v_k),\ T_{ICONN_{\overline{G}-v_i,v_k}}(v_i,v_k)=T_{ICONN_{\overline{G}}}(v_lv_{mn}),\\
&I_{ICONN_{\underline{G}-v_i,v_k}}(v_i,v_k)=I_{ICONN_{\underline{G}}}(v_i,v_k),\ I_{ICONN_{\overline{G}-v_i,v_k}}(v_i,v_k)=I_{ICONN_{\overline{G}}}(v_lv_{mn}),\\
&F_{ICONN_{\underline{G}-v_i,v_k}}(v_i,v_k)=F_{ICONN_{\underline{G}}}(v_i,v_k),\ F_{ICONN_{\overline{G}-v_i,v_k}}(v_i,v_k)=F_{ICONN_{\overline{G}}}(v_lv_{mn}).
\end{aligned}
$$

Since,

$$
\begin{aligned}
&T_{\underline{X}(Q)}(v_i,v_k)\leq T_{ICONN_{\underline{G}}}(v_i,v_k),\ T_{\overline{X}(Q)}(v_i,v_k)\leq T_{ICONN_{\overline{G}}}(v_lv_{mn}),\\
&I_{\underline{X}(Q)}(v_i,v_k)\geq I_{ICONN_{\underline{G}}}(v_i,v_k),\ I_{\overline{X}(Q)}(v_i,v_k)\geq I_{ICONN_{\overline{G}}}(v_lv_{mn}),\\
&F_{\underline{X}(Q)}(v_i,v_k)\geq F_{ICONN_{\underline{G}}}(v_i,v_k),\ F_{\overline{X}(Q)}(v_i,v_k)\geq F_{ICONN_{\overline{G}}}(v_lv_{mn}).
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
&T_{ICONN_{\underline{G}-v_i,v_k}}(v_i,v_k)\geq T_{\underline{X}(Q)}(v_i,v_k),\ T_{ICONN_{\overline{G}-v_i,v_k}}(v_i,v_k)\geq T_{\overline{X}(Q)}((v_i,v_k)),\\
&I_{ICONN_{\underline{G}-v_i,v_k}}(v_i,v_k)\leq I_{\underline{X}(Q)}(v_i,v_k),\ I_{ICONN_{\overline{G}-v_i,v_k}}(v_i,v_k)\leq I_{\overline{X}(Q)}((v_i,v_k)),\\
&F_{ICONN_{\underline{G}-v_i,v_k}}(v_i,v_k)\leq F_{\underline{X}(Q)}(v_i,v_k),\ F_{ICONN_{\overline{G}-v_i,v_k}}(v_i,v_k)\leq F_{\overline{X}(Q)}((v_i,v_k)),
\end{aligned}
$$

which is a contradiction. Hence, it is proved. □

Theorem 8. *If*

$$
\begin{aligned}
&T_{\underline{X}(Q)}(v_iv_{jk})>T_{\underline{X}(Q)}(v_lv_{mn}),\ \ T_{\underline{X}(Q)}(v_iv_{jk})>T_{\underline{X}(Q)}(v_lv_{mn}),\\
&I_{\underline{X}(Q)}(v_iv_{jk})<I_{\underline{X}(Q)}(v_lv_{mn}),\ \ I_{\underline{X}(Q)}(v_iv_{jk})<I_{\underline{X}(Q)}(v_lv_{mn}),\\
&F_{\underline{X}(Q)}(v_iv_{jk})<F_{\underline{X}(Q)}(v_lv_{mn}),\ \ F_{\underline{X}(Q)}(v_iv_{jk})<F_{\underline{X}(Q)}(v_lv_{mn}),
\end{aligned}
$$

for some v_iv_{jk} among all cycles in soft rough neutrosophic influence graph G. *Then*

$$
\begin{aligned}
&T_{\underline{X}(Q)}(v_iv_{jk})>T_{ICONN_{\underline{G}-v_iv_{jk}}}(v_i,v_k),\ \ T_{\overline{X}(Q)}(v_iv_{jk})>T_{ICONN_{\overline{G}-v_iv_{jk}}}(v_i,v_k),\\
&I_{\underline{X}(Q)}(v_iv_{jk})<I_{ICONN_{\underline{G}-v_iv_{jk}}}(v_i,v_k),\ \ I_{\overline{X}(Q)}(v_iv_{jk})<I_{ICONN_{\overline{G}-v_iv_{jk}}}(v_i,v_k),\\
&F_{\underline{X}(Q)}(v_iv_{jk})<F_{ICONN_{\underline{G}-v_iv_{jk}}}(v_i,v_k),\ \ F_{\overline{X}(Q)}(v_iv_{jk})<F_{ICONN_{\overline{G}-v_iv_{jk}}}(v_i,v_k).
\end{aligned}
$$

Proof. Since

$$T_{ICONN_{\underline{G}-v_iv_{jk}}}(v_iv_{jk})\geq T_{ICONN_{\underline{G}}}(v_iv_{jk}),\ T_{ICONN_{\overline{G}-v_iv_{jk}}}(v_iv_{jk})\geq T_{ICONN_{\overline{G}}}((v_iv_{jk})),$$
$$I_{ICONN_{\underline{G}-v_iv_{jk}}}(v_iv_{jk})\leq I_{ICONN_{\underline{G}}}(v_iv_{jk}),\ I_{ICONN_{\overline{G}-v_iv_{jk}}}(v_iv_{jk})\leq I_{ICONN_{\overline{G}}}((v_iv_{jk})),$$
$$F_{ICONN_{\underline{G}-v_iv_{jk}}}(v_iv_{jk})\leq F_{ICONN_{\underline{G}}}(v_iv_{jk}),\ F_{ICONN_{\overline{G}-v_iv_{jk}}}(v_iv_{jk})\leq F_{ICONN_{\overline{G}}}((v_iv_{jk})).$$

Therefore, there exists a path from v_i to v_k not involving (v_iv_{jk}) such that

$$T_{ICONN_{\underline{G}-v_iv_{jk}}}(v_iv_{jk})\geq T_{\underline{X}(Q)}(v_iv_{jk}),\ T_{ICONN_{\overline{G}-v_iv_{jk}}}(v_iv_{jk})\geq T_{\overline{X}(Q)}((v_iv_{jk})),$$
$$I_{ICONN_{\underline{G}-v_iv_{jk}}}(v_iv_{jk})\leq I_{\underline{X}(Q)}(v_iv_{jk}),\ I_{ICONN_{\overline{G}-v_iv_{jk}}}(v_iv_{jk})\leq I_{\overline{X}(Q)}((v_iv_{jk})),$$
$$F_{ICONN_{\underline{G}-v_iv_{jk}}}(v_iv_{jk})\leq F_{\underline{X}(Q)}(v_iv_{jk}),\ F_{ICONN_{\overline{G}-v_iv_{jk}}}(v_iv_{jk})\leq F_{\overline{X}(Q)}((v_iv_{jk})),$$

This along with v_iv_{jk} is a cycle and v_iv_{jk} is least value. □

Theorem 9. *If v_iv_{jk} is a soft rough neutrosophic influence cutpair in soft rough neutrosophic influence graph* G. *Then*

$$T_{\underline{X}(Q)}(v_iv_{jk})>T_{\underline{X}(Q)}(v_lv_{mn}),\ T_{\underline{X}(Q)}(v_iv_{jk})>T_{\underline{X}(Q)}(v_lv_{mn}),$$
$$I_{\underline{X}(Q)}(v_iv_{jk})<I_{\underline{X}(Q)}(v_lv_{mn}),\ I_{\underline{X}(Q)}(v_iv_{jk})<I_{\underline{X}(Q)}(v_lv_{mn}),$$
$$F_{\underline{X}(Q)}(v_iv_{jk})<F_{\underline{X}(Q)}(v_lv_{mn}),\ F_{\underline{X}(Q)}(v_iv_{jk})<F_{\underline{X}(Q)}(v_lv_{mn}),$$

for some v_iv_{jk} among all cycles of G.

Proof. Suppose on contrary in a cycle, we

$$T_{\underline{X}(Q)}(v_iv_{jk})>T_{\underline{X}(Q)}(v_lv_{mn}),\ T_{\underline{X}(Q)}(v_iv_{jk})>T_{\underline{X}(Q)}(v_lv_{mn}),$$
$$I_{\underline{X}(Q)}(v_iv_{jk})<I_{\underline{X}(Q)}(v_lv_{mn}),\ I_{\underline{X}(Q)}(v_iv_{jk})<I_{\underline{X}(Q)}(v_lv_{mn}),$$
$$F_{\underline{X}(Q)}(v_iv_{jk})<F_{\underline{X}(Q)}(v_lv_{mn}),\ F_{\underline{X}(Q)}(v_iv_{jk})<F_{\underline{X}(Q)}(v_lv_{mn}).$$

Then any path involving it can be converted into a path not involving it with influence strength greater than and equal to the value of XQ for previously deleted pairs. So v_iv_{jk} is not a cutpair. This is a contradiction to our assumption. Hence v_iv_{jk} is not a pair with the least value among all cycle. □

Theorem 10. *If* G$=(SN_1,RM_1,XQ_1)$ *is a soft rough neutrosophic forest, then the pairs of neutrosophic spanning subgraph $H=(SN_1,RM_1,XQ_2)$ such that*

$$T_{\underline{X}(Q_1)}(v_iv_{jk})<T_{ICONN_{\underline{H}}}(v_i,v_k),\ T_{\overline{X}(Q_1)}(v_iv_{jk})<T_{ICONN_{\overline{H}}}(v_i,v_k),$$
$$I_{\underline{X}(Q_1)}(v_iv_{jk})>I_{ICONN_{\underline{H}}}(v_i,v_k),\ I_{\overline{X}(Q_1)}(v_iv_{jk})>I_{ICONN_{\overline{H}}}(v_i,v_k),$$
$$F_{\underline{X}(Q_1)}(v_iv_{jk})>F_{ICONN_{\underline{H}}}(v_i,v_k),\ F_{\overline{X}(Q_1)}(v_iv_{jk})>F_{ICONN_{\overline{H}}}(v_i,v_k),$$

are exactly the cutpairs of G.

Theorem 11. *A soft rough neutrosophic influence graph* G *is a cycle. Then an edge v_{jk} is a soft rough neutrosophic influence bridge if and only if it is an edge common to atmost two cutpair.*

Theorem 12. *Let* G *be a soft rough neutrosophic influence graph. Then the following conditions are equivalent.*

1. *For a pair* $v_iv_{jk} \in \underline{\mathrm{I}}^* \cap \overline{\mathrm{I}}^*$

$$T_{\underline{X}(Q)}(v_iv_{jk}) > T_{ICONN_{\underline{G}-v_iv_{jk}}}(v_i, v_k), \ T_{\overline{X}(Q)}(v_iv_{jk}) > T_{ICONN_{\overline{G}-v_iv_{jk}}}(v_i, v_k),$$
$$I_{\underline{X}(Q)}(v_iv_{jk}) < I_{ICONN_{\underline{G}-v_iv_{jk}}}(v_i, v_k), \ I_{\overline{X}(Q)}(v_iv_{jk}) < I_{ICONN_{\overline{G}-v_iv_{jk}}}(v_i, v_k),$$
$$F_{\underline{X}(Q)}(v_iv_{jk}) < F_{ICONN_{\underline{G}-v_iv_{jk}}}(v_i, v_k), \ F_{\overline{X}(Q)}(v_iv_{jk}) < F_{ICONN_{\overline{G}-v_iv_{jk}}}(v_i, v_k).$$

2. v_iv_{jk} *is an influence cutpair*

4. Application to Decision-Making

Decision making is a process that plays an important role in our daily lives. Decision making process can help us make more purposeful, thoughtful decisions by systemizing relevant information step by step. The process of decision making involves making a choice among different alternatives, it starts when we do not know what to do.

The selection of the right path for transferring goods from one state to another states illegally. Every state has different polices within or out side the state, there are several factors to take into consideration for selecting the right path. Whether the economy of a country is good, having job opportunity or a safety.

Suppose a trader wants to extend his business to the countries C_1,C_2,C_3,C_4,C_5 and C_6. Initially, he takes C_1 and extends his business one by one. Assume A is set of the parameters, consisting of element a_1 = job, a_2 = economy above average, a_3 = safety, a_4 = other.

Let S be a full soft set from A to parameter set V, as shown in Table 16.

Table 16. Soft Neutrosophic Set S.

S	C_1	C_2	C_3	C_4	C_5	C_6
a_1	1	1	1	0	1	1
a_2	0	0	1	1	1	1
a_3	1	1	1	0	0	1
a_3	1	1	1	1	1	1

Suppose N={(C_1,0.8,0.6, 0.7),(C_2,0.9,0.5,0.65),(C_3,0.75,0.6,0.65),(C_4,1.0,0.55,0.85),(C_5,0.95,0.63,0.8), (C_6,0.85,0.65,0.95)} is most favorable object describes membership of suitable countries foreign polices corresponding to the boolean set V, which is a neutrosophic set on the set V under consideration.

$SN = (\underline{S}(N), \overline{S}(N))$ is a full soft rough set in full soft approximation space (V, S) where

$$\overline{S}(N) = \{(C_1, 0.90, 0.50, 0.65), (C_2, 0.90, 0.50, 0.65), (C_3, 0.90, 0.55, 0.65), (C_4, 1.00, 0.55, 0.65), (C_5, 0.95, 0.55, 0.65), (C_6, 0.9, 0.55, 0.65)\},$$
$$\underline{S}(N) = \{(C_1, 0.75, 0.65, 0.95), (C_2, 0.75, 0.65, 0.95), (C_3, 0.75, 0.65, 0.95), (C_4, 0.75, 0.65, 0.95), (C_5, 0.75, 0.65, 0.95), (C_6, 0.75, 0.65, 0.95)\}.$$

Let E={C_{12},C_{14},C_{15},C_{23},C_{26},C_{34},C_{35},C_{45},C_{46},C_{56}}$\subseteq \tilde{V} = V \times V$ and L={$a_{14}, a_{21}, a_{34}, a_{42}$}$\subseteq \tilde{A} = A \times A$. A full soft relation R on E (from L to E) can be defined as shown in Table 17.

Table 17. Full soft set R.

R	C_{12}	C_{14}	C_{15}	C_{23}	C_{26}	C_{34}	C_{35}	C_{45}	C_{46}	C_{56}
a_{14}	1	1	1	1	1	1	1	0	0	1
a_{21}	0	0	0	0	0	0	1	1	1	1
a_{34}	1	1	1	1	1	1	1	0	0	0
a_{42}	0	1	1	1	1	1	1	1	1	1

Let M={(C_{12},0.74,0.5,0.62),(C_{14},0.75,0.45,0.63),(C_{15},0.74,0.54,0.61),(C_{23},0.72,0.48,0.65),(C_{26},0.71, 0.49,0.64),(C_{34},0.72,0.53,0.64),(C_{35},0.73,0.52,0.63),(C_{45},0.7,0.51,0.61),(C_{46},0.74,0.55,0.6),(C_{56},0.73,0.47, 0.64)} be most favorable object describes membership of countries foreign polices toward others countries corresponding to the boolean set E, which is a neutrosophic set on the set V under consideration.

$RM = (\underline{R}M, \overline{R}M)$ is a soft neutrosophic rough relation, where

$$\overline{R}M=\{(C_{12},0.75,0.45,0.61),(C_{14},0.75,0.45,0.61),(C_{15},0.75,0.45,0.61),(C_{23},0.75,0.45,0.61),\\(C_{26},0.75,0.45,0.61),(C_{34},0.75,0.45,0.61),(C_{35},0.74,0.47,0.61),(C_{45},0.74,0.47,0.6),\\(C_{46},0.74,0.47,0.6),(C_{56},0.74,0.47,0.61)\},$$

$$\underline{R}M=\{(C_{12},0.71,0.54,0.65),(C_{14},0.71,0.54,0.65),(C_{15},0.71,0.54,0.65),(C_{23},0.71,0.54,0.65),\\(C_{26},0.71,0.54,0.65),(C_{34},0.71,0.54,0.65),(C_{35},0.71,0.54,0.64),(C_{45},0.70,0.55,0.64),\\(C_{46},0.70,0.55,0.64),(C_{56},0.71,0.54,0.64)\}.$$

Let I={C_1C_{15},C_1C_{23},C_1C_{35},C_2C_{34},C_3C_{14},C_3C_{26},C_3C_{45},C_4C_{23},C_4C_{45},C_4C_{46},C_5C_{23},C_5C_{34}, C_5C_{46}, C_6C_{12},C_6C_{15}}$\subseteq \hat{V} = V \times E$ and F={a_1a_{42},a_2a_{14},a_3a_{34},a_4a_{21},a_4a_{42}}$\subseteq \hat{A} = A \times L$.

A full soft relation X on I (from F to I) can be defined in Table 18 as follows:

Table 18. Full soft set X.

X	C_1C_{15} C_4C_{45}	C_1C_{23} C_4C_{46}	C_1C_{35} C_5C_{23}	C_2C_{34} C_5C_{34}	C_3C_{14} C_5C_{46}	C_3C_{26} C_6C_{12}	C_3C_{45}, C_6C_{15}	C_4C_{23}
e_1e_{42}	1 0	1 0	1 1	1 1	1 1	1 0	1 1	0
e_2e_{14}	0 0	0 0	0 1	0 1	1 0	1 1	0 1	0
e_2e_{34}	0 0	0 0	0 0	0 0	0 0	0 0	0 0	1
e_3e_{34}	1 0	1 0	1 0	1 0	1 0	1 1	0 1	0
e_4e_{21}	0 1	0 1	1 0	0 0	0 1	0 0	1 0	0
e_4e_{42}	1 1	1 1	1 1	1 1	1 1	1 0	1 1	0

Let $Q = \{(C_1C_{15}, 0.7, 0.43, 0.58), (C_1C_{23}, 0.65, 0.39, 0.54), (C_1C_{35}, 0.66, 0.37, 0.56), (C_2C_{34}, 0.68, 0.38, 0.59), (C_3C_{14}, 0.6, 0.4, 0.6), (C_3C_{26}, 0.62, 0.42, 0.58), (C_3C_{45}, 0.64, 0.45, 0.54), (C_4C_{23}, 0.7, 0.45, 0.60), (C_4C_{45}, 0.7, 0.36, 0.48), (C_4C_{46}, 0.68, 0.35, 0.5), (C_5C_{23}, 0.69, 0.45, 0.54), (C_5C_{34}, 0.65, 0.42, 0.58), (C_5C_{46}, 0.64, 0.41, 0.59), (C_6C_{12}, 0.63, 0.4, 0.6), (C_6C_{15}, 0.62, 0.39, 0.5)\}$ be most favorable object describes membership of countries impact toward others countries regarding trade corresponding to the boolean set I, which is a neutrosophic set on the set I under consideration.

$XQ = (\underline{X}Q, \overline{X}Q)$ is a soft neutrosophic rough influence, where

$$\overline{X}Q = \{(C_1C_{15}, 0.70, 0.37, 0.50), (C_1C_{23}, 0.70, 0.37, 0.50), (C_1C_{35}, 0.70, 0.37, 0.50), (C_2C_{34}, 0.70, 0.37, 0.50), (C_3C_{14}, 0.69, 0.39, 0.50), (C_3C_{26}, 0.69, 0.39, 0.50), (C_3C_{45}, 0.70, 0.37, 0.50), (C_4C_{23}, 0.7, 0.45, 0.60), (C_4C_{45}, 0.70, 0.35, 0.48), (C_4C_{46}, 0.70, 0.35, 0.48), (C_5C_{23}, 0.69, 0.39, 0.50), (C_5C_{34}, 0.69, 0.39, 0.50), (C_5C_{46}, 0.70, 0.37, 0.50), (C_6C_{12}, 0.69, 0.39, 0.50), (C_6C_{15}, 0.69, 0.39, 0.50)\},$$

$$\underline{X}Q = \{(C_1C_{15}, 0.60, 0.43, 0.60), (C_1C_{23}, 0.60, 0.43, 0.60), (C_1C_{35}, 0.64, 0.43, 0.59), (C_2C_{34}, 0.60, 0.43, 0.60), (C_3C_{14}, 0.60, 0.43, 0.60), (C_3C_{26}, 0.60, 0.43, 0.60), (C_3C_{45}, 0.64, 0.45, 0.59), (C_4C_{23}, 0.7, 0.45, 0.60), (C_4C_{45}, 0.64, 0.45, 0.59), (C_4C_{46}, 0.64, 0.45, 0.59), (C_5C_{23}, 0.60, 0.45, 0.60), (C_5C_{34}, 0.60, 0.45, 0.60), (C_5C_{46}, 0.64, 0.45, 0.59), (C_6C_{12}, 0.60, 0.43, 0.60), (C_6C_{15}, 0.60, 0.43, 0.60)\}.$$

Thus, $\mathrm{G} = (\underline{\mathrm{G}}, \overline{\mathrm{G}})$ is a soft neutrosophic rough influence graph as shown in Figure 9. He finds the strength of each path from C_1 to C_6. The paths are

$$P_1 : C_1, C_5, C_2, C_3, C_6,$$

$$P_2 : C_1, C_4, C_5, C_6,$$

$$P_3 : C_1, C_3, C_5, C_2, C_6$$

with their influence strength as $(0.6, 0.45, 0.5)$, respectively.

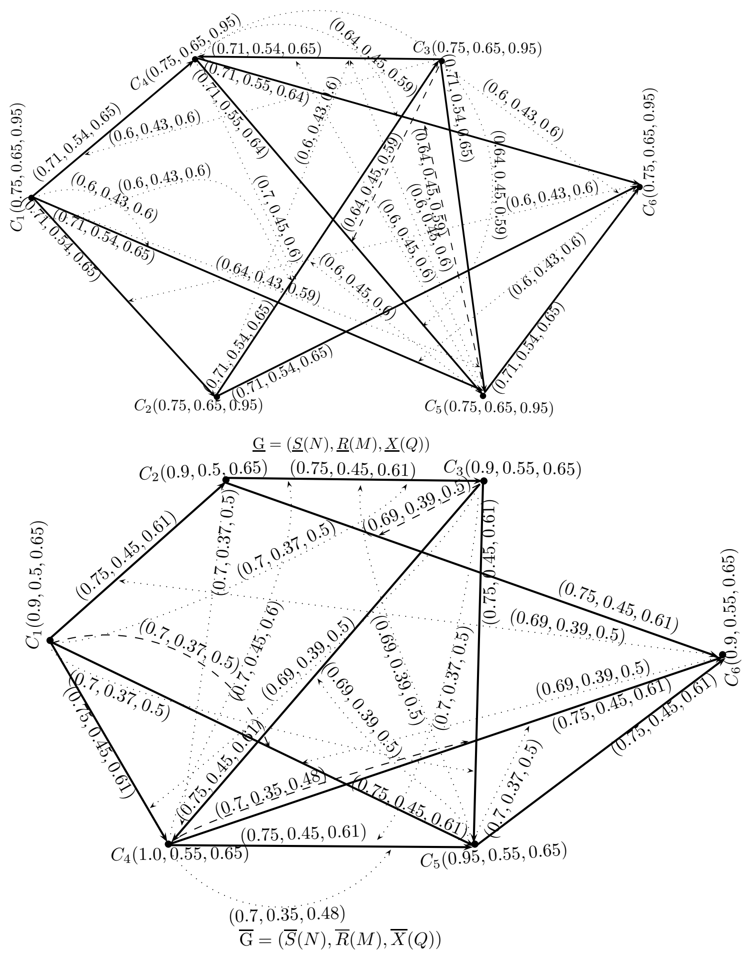

Figure 9. Soft rough neutrosophic influence graph $\mathrm{G} = (\underline{\mathrm{G}}, \overline{\mathrm{G}})$.

Since, there is more than one path, therefore, the trader calculates the score function which is formulated in Equation (4):

$$\text{Score Function}(C_i)=\Big(T_{\underline{S}(N)}(C_i)+T_{\overline{S}(N)}(C_i)+T_{\underline{R}(M)}(C_{ij})+T_{\overline{R}(M)}(C_{ij})+T_{\underline{X}(Q)}(C_iC_{jk})+ T_{\overline{X}(Q)}(C_iC_{jk}), I_{\underline{S}(N)}(C_i)I_{\overline{S}(N)}(C_i)+I_{\underline{R}(M)}(C_{ij})I_{\overline{R}(M)}(C_{ij})+ I_{\underline{X}(Q)}(C_iC_{jk})I_{\overline{X}(Q)}(C_iC_{jk}), F_{\underline{S}(N)}(C_i)F_{\overline{S}(N)}(C_i)+ F_{\underline{R}(M)}(C_{ij})F_{\overline{R}(M)}(C_{ij})+F_{\underline{X}(Q)}(C_iC_{jk})F_{\overline{X}(Q)}(C_iC_{jk})\Big). \quad (4)$$

For each C_i, the score values of C_i is calculated directly and as shown in Table 19.

Table 19. Score Function.

V	Score Values
C_1	(9.97,1.054,2.702)
C_2	(5.87,1.2979,1.7105)
C_3	(8.48,1.3562,2.2994)
C_4	(6.73,1.392,2.3119)
C_5	(7.07,1.3673,1.9029)
C_6	(4.23,0.6929,1.2175)

So, he chooses the path P_3:C_1,C_3,C_5,C_2,C_6. The Algorithm 1 of the application is also be given in Algorithm 1. The flow chart is given in Figure 10.

Algorithm 1: Influence strength of each path in rough neutrosophic influence graph

1. Input the universal sets C and P.
2. Input the full soft set S and neutrosophic set N on V.
3. Calculate the Soft rough neutrosophic sets on V.
4. Input the universal sets E and L.
5. Input the full soft set R and neutrosophic set M on E.
6. Calculate the Soft rough neutrosophic sets on E.
7. Input the universal sets I and F.
8. Input the full soft set X and neutrosophic set Q on I.
9. Calculate the Soft rough neutrosophic sets on I.
10. Find the number of path and calculate their influence strength of each path from C_1 to C_n.
11. Choose that path which has maximum membership, minimum indeterminacy and falsity value. If $i > 1$, than calculate the score values of each C_i, choose that C_i which has maximum membership and come immediately after C_1 in one of the paths.

5. Conclusions

Graph theory has been applied widely in various areas of engineering, computer science, database theory, expert systems, neural networks, artificial intelligence, signal processing, pattern recognition, robotics, computer networks, and medical diagnosis. Present research has shown that two or more theories can be combined into a more flexible and expressive framework for modeling and processing incomplete information in information systems. Various mathematical models that combine rough sets, soft sets and neutrosophic sets have been introduced. A soft rough neutrosophic set is a hybrid tool for handling indeterminate, inconsistent and uncertain information that exist in real life. We have applied this concept to graph theory. We have presented certain concepts, including soft rough neutrosophic graphs, soft rough neutrosophic influence graphs, soft rough neutrosophic influence cycles, soft rough neutrosophic influence trees. We also have considered an application of soft rough neutrosophic influence graph in decision-making to illustrate the best path in the business. In the future, we will study, (1) Neutrosophic rough hypergraphs, (2) Bipolar neutrosophic rough hypergraphs, (3) Neutrosophic soft rough hypergraphs, (4) Decision support systems based on soft rough neutrosophic information.

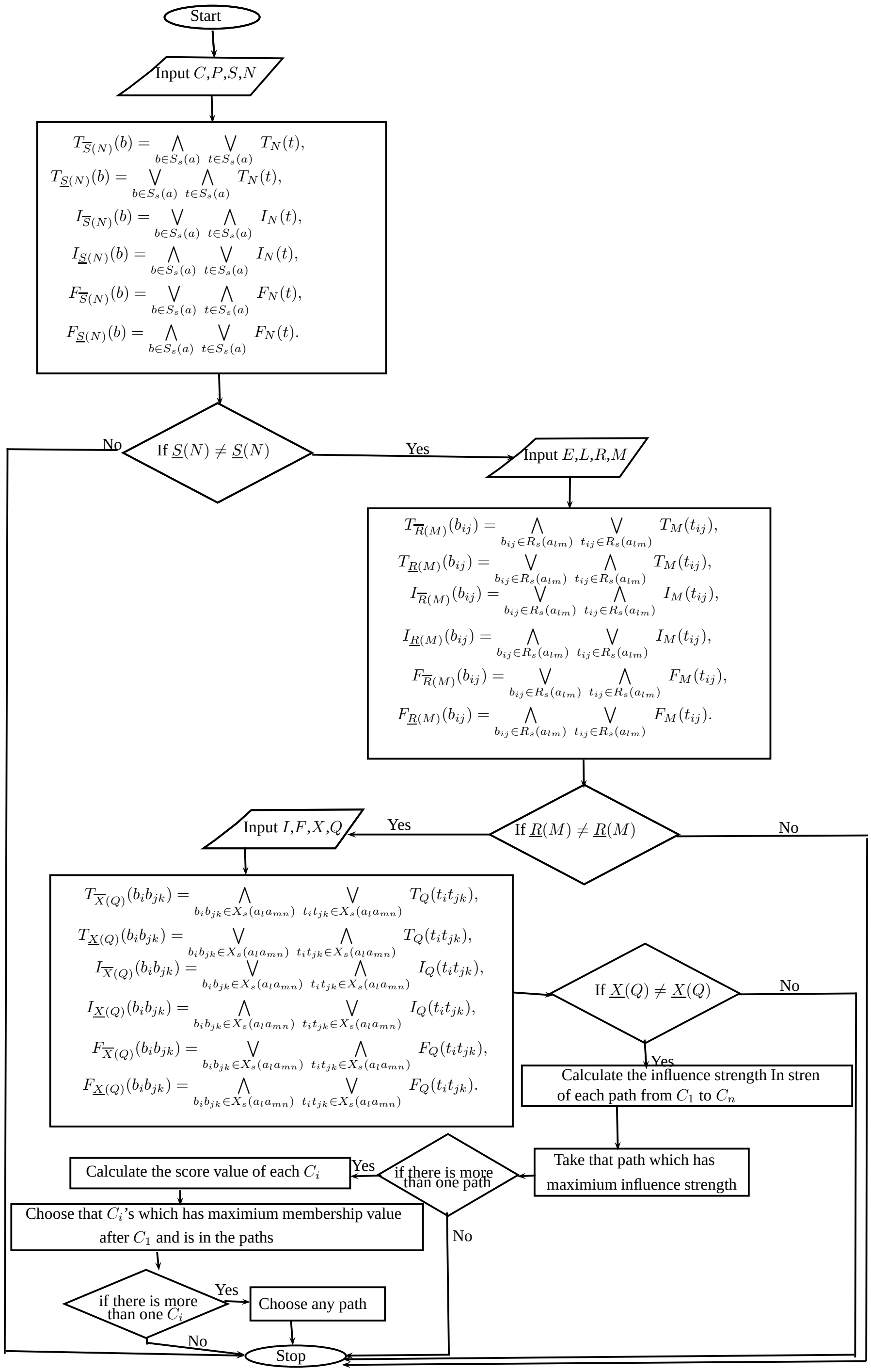

Figure 10. The flow chart of the application.

Author Contributions: H.M.M., M.A. and F.S. conceived and designed the experiments; M.A. and F.S. analyzed the data; H.M.M. wrote the paper.

Conflicts of Interest: The authors declare no conflict of interest.

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