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Computing Eccentricity Based Topological Indices of Octagonal Grid O_n^m

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Abstract: Graph theory is successfully applied in developing a relationship between chemical structure and biological activity. The relationship of two graph invariants, the eccentric connectivity index and the eccentric Zagreb index are investigated with regard to anti-inflammatory activity, for a dataset consisting of 76 pyrazole carboxylic acid hydrazide analogs. The eccentricity ε_v of vertex v in a graph G is the distance between v and the vertex furthermost from v in a graph G. The distance between two vertices is the length of a shortest path between those vertices in a graph G. In this paper, we consider the Octagonal Grid O_n^m. We compute Connective Eccentric index $C^{\xi}(G) = \sum_{v \in V(G)} d_v/\varepsilon_v$, Eccentric Connective Index $\xi(G) = \sum_{v \in V(G)} d_v \varepsilon_v$ and eccentric Zagreb index of Octagonal Grid O_n^m, where d_v represents the degree of the vertex v in G.

Keywords: eccentric connective index; connective eccentric index; eccentric Zagreb index; the octagonal grid O_n^m

MSC: 05C12, 05C90

1. Introduction

Chemical graph theory is broadly utilized in the branch of scientific science and a few people say that chemical graph hypothesis and this hypothesis are connected with the commonsense utilizations of chart hypothesis for tackling the atomic issues. In Arithmetic, a model of synthetic framework depicts a substance chart of arrangements to clarify the relations between its fragments, for example, its particles, bonds between iotas, bunch of particles or atoms.

An associated basic graph $G = (V(G) \cup E(G))$ is a chart comprising of n nodes and m joint line segment in which there is path between any of two distinct nodes. A system is simply an associated diagram comprising of no numerous edges and circles. The *degree of a node* q in G is the quantity of line segment which is occurrence to the node q and spoken to by d_q . In a chart G, if there is no reiteration of vertices in $(p-q)$ path then such sort of path is called $(p-q)$ way. The quantity of line segment in $(p-q)$ way is called its length.

The separation $d(w,y)$ from node w to node y is the length of a briefest $(w-y)$ way in a diagram G where $w, y \in G$. In a associated diagram G, the eccentricity ε_v of a node w is the separation amongst w and a node uttermost from w in G. Along these lines, $\varepsilon_w = \max_{w \in V(G)} d(w,y)$. Along these lines, the greatest capriciousness over all nodes of G is the width of G which is meant by $D(G)$.

A diagram can be perceived by an alternate sort of a number, a polynomial, an arrangement of nodes or a grid. A topological list is a real number that is related with a chart which describes the topology of diagram and is investigated under diagram automorphism. From last two, topological lists, for example, Wiener record, Balaban's list [1–3], Hosoya file [4,5], Randić index [6] et cetera, have been considered widely; as of late, the exploration here has increased exponentially.

There are some real classes of topological lists, for example, distance based topological lists, unusualness based topological files, degree based topological lists and tallying related polynomials and files of charts. In this article, we consider the eccentricity based indices. In [7], the creators presented the *total eccentricity* of the chart G, which is characterized as add up of eccentricities of all nodes of a given diagram G and meant by $\zeta(G)$. It is anything but difficult to see that for a k-standard diagram G one has $\zeta(G) = k\zeta(G)$.

Another very relevant and special eccentricity based topological index is Connective Eccentric index $C^{\xi}(G)$ that was proposed by Gupta et al. [8]. The Connective Eccentric index is defined as:

$$C^{\xi}(G) = \sum_{v \in V(G)} \frac{d_v}{\varepsilon_v} \tag{1}$$

where d_v denotes degree of the vertex v and ε_v denotes the eccentricity of the vertex v. The readers are referred to [9,10] for more information on Connective Eccentric index.

A very important eccentricity based topological index of a graph G is the *eccentric connectivity index* $\xi(G)$ which is defined as:

$$\xi(G) = \sum_{v \in V(G)} d_v \varepsilon_v \tag{2}$$

This topological index is used as a mathematical model to predict biological activities of diverse nature [11,12]. For more mathematical properties of this index, we encourage the reader to refer to [13–16].

In 2012, Ghorbani and Hosseinzadeh [17] defined new versions of eccentric Zagreb indices, one of which is as follows:

$$M_1^{**}(G) = \sum_{u \in V(G)} (\epsilon_u)^2 \tag{3}$$

The *eccentric connectivity polynomial* is the polynomial version of the eccentric-connectivity index which is introduced in [18] and $\xi(G)$ is defined as:

$$ECP(G, x) = \sum_{v \in V(G)} d_v x^{\varepsilon_v} \tag{4}$$

In this article, we consider G to be a connected graph with vertex set $V(G)$ and edge set $E(G)$, and compute the $C^{\xi}(G)$, $\xi(G)$ and $M_1^{**}(G)$ of the Octagonal grid O_n^m.

2. Methods

For the computation of our results, we utilized a strategy for combinatorial registering, a vertex partition strategy, an edge partition technique, graph hypothetical instruments, scientific systems, a degree-counting strategy, and a degrees of neighbors strategy.

3. The Octagonal Grid O_n^m

In [19,20], Diudea et al. constructed a C_4C_8 net as a trivalent decoration made by alternating squares C_4 and octagons C_8 in two different ways. One is by alternating squares C_4 and octagons C_8 in different ways denoted by $C_4C_8(S)$ and the other is by alternating rhombus and octagons in different ways denoted by $C_4C_8(R)$. We denote $C_4C_8(R)$ by O_n^m (see Figure 1). In [21,22], they also called it the Octagonal grid.

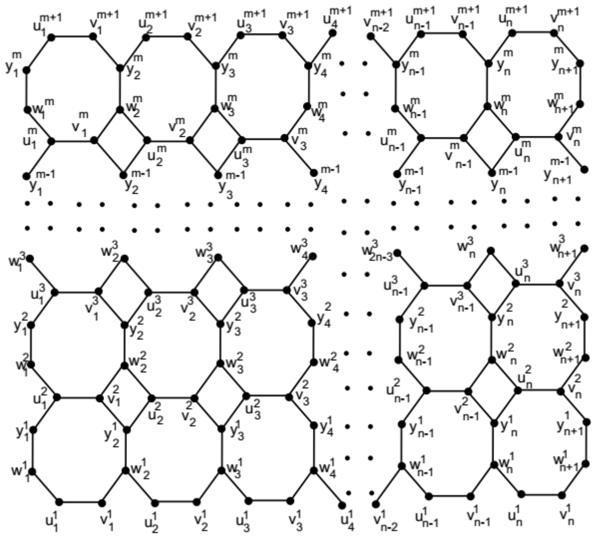

Figure 1. The Octagonal grid O_n^m.

For $n, m \geq 3$, the Octagonal grid O_n^m is the grid with m rows and n columns of octagons. The symbols $V(O_n^m)$ and $E(O_n^m)$ denote the vertex set and the edge set of O_n^m, respectively.

$$\begin{aligned} V(O_n^m) &= \{u_s^t : 1 \leq s \leq n,\ 1 \leq t \leq m+1\} \cup \{v_s^t : 1 \leq s \leq n;\ 1 \leq t \leq m+1\} \\ &\cup \ \{w_s^t : 1 \leq s \leq n+1,\ 1 \leq t \leq m\} \cup \{y_s^t : 1 \leq s \leq n+1,\ 1 \leq t \leq m\}. \end{aligned}$$

$$\begin{aligned} E(O_n^m) &= \{u_s^t v_s^t :\ 1 \leq s \leq n,\ 1 \leq t \leq m+1\} \cup \{u_s^t w_s^t : 1 \leq s \leq n,\ 1 \leq t \leq m\} \\ &\cup \ \{w_s^t y_s^t : 1 \leq s \leq n+1,\ 1 \leq t \leq m\} \cup \{v_s^t w_{s+1}^t : 1 \leq s \leq n,\ 1 \leq t \leq m\} \\ &\cup \ \{v_s^t y_{s+1}^{t-1} : 1 \leq s \leq n,\ 2 \leq t \leq m+1\} \cup \{u_s^{t+1} y_s^t :\ 1 \leq s \leq n,\ 2 \leq t \leq m\}. \end{aligned}$$

In this paper, we consider O_n^m with $n = m$. and we compute Connective Eccentric index $C^{\xi}(G)$, Eccentric connectivity index $\xi(G)$ and eccentric Zagreb indices of O_n^m. For this, we discuss two cases of n, when $n \equiv 0(mod2)$ and when $n \equiv 1(mod2)$.

Theorem 1. *For every $n \geq 4$ and $n \equiv 0(mod2)$, consider the graph of $G \cong O_n^m$, with $n = m$. Then, the Connective Eccentric index of G is equal to:*

$$\begin{aligned} C^{\xi}(G) &= 32\left[\frac{14n+1}{7n(7n+1)}\right] + 16\sum_{s=1}^{\frac{n}{2}-1}\left[\ \frac{1}{4n+1-s} + \frac{2}{7n+2s}\ \right] \\ &+ \ \sum_{s=1}^{\frac{n}{2}}\left[\ \sum_{t=s+1}^{\frac{n}{2}+1}\frac{12}{4n-3(s-1)-t}\ \right] + \sum_{s=2}^{\frac{n}{2}}\left[\ \sum_{t=2}^{s}\frac{12}{4(n+1)-s-3t}\ \right] \\ &+ \ \sum_{s=\frac{n}{2}+1}^{n-1}\left[\ \sum_{t=2}^{n-s+1}\frac{12}{3(n-t)+s+2}\ \right] + \sum_{s=\frac{n}{2}+2}^{n}\left[\ \sum_{t=n-s+2}^{\frac{n}{2}+1}\frac{12}{n+3s-t-1}\ \right] \\ &+ \ \sum_{s=1}^{\frac{n}{2}-1}\left[\ \sum_{t=\frac{n}{2}+2}^{n-s+1}\frac{12}{3(n-s)+t+1}\ \right] + \sum_{s=2}^{\frac{n}{2}}\left[\ \sum_{t=n+2-s}^{n}\frac{12}{n-s+3t-2}\ \right] \\ &+ \ \sum_{s=\frac{n}{2}+2}^{n}\left[\ \sum_{t=\frac{n}{2}+2}^{s}\frac{12}{4s+t-n-3}\ \right] + \sum_{s=\frac{n}{2}+1}^{n-1}\left[\ \sum_{t=s+1}^{n}\frac{12}{s+3t-4}\ \right]. \end{aligned}$$

Proof. Let G be the graph of O_n^m. Note that graph of O_n^m is a symmetric about reflection and rotation at right angles. Thus, the eccentricities $\varepsilon_{u_s^t} = \varepsilon_{v_{n+1-s}^t}$ and from the symmetry at right angles we can obtain that the eccentricities $\varepsilon_{y_s^t} = \varepsilon_{u_t^s}$, $\varepsilon_{w_s^t} = \varepsilon_{v_t^s}$. Therefore, from Equation (1), the Connective Eccentric index of O_n^m is equal to

$$C^{\xi}(O_n^m) = 4 \sum_{u_s^t \in V(G)} \frac{d_{u_s^t}}{\varepsilon_{u_s^t}}$$

By using the values in Table 1, we get

$$\begin{aligned} C^{\xi}(O_n^m) &= 4\left[2\sum_{s=1}^{\frac{n}{2}+1} \frac{2}{4n+1-s} + 2\sum_{s=\frac{n}{2}+2}^{n} \frac{2}{3n+s-1}\right] + 4\left[\sum_{s=1}^{\frac{n}{2}} \left[\sum_{t=s+1}^{\frac{n}{2}+1} \frac{3}{4n-3(s-1)-t}\right]\right. \\ &+ \sum_{s=2}^{\frac{n}{2}} \left[\sum_{t=2}^{s} \frac{3}{4(n+1)-s-3t}\right] + \sum_{s=\frac{n}{2}+1}^{n-1} \left[\sum_{t=2}^{n-s+1} \frac{3}{3(n-t)+s+2}\right] \\ &+ \sum_{s=1}^{\frac{n}{2}-1} \left[\sum_{t=\frac{n}{2}+2}^{n-s+1} \frac{3}{3(n-s)+t+1}\right] + \sum_{s=2}^{\frac{n}{2}} \left[\sum_{t=n+2-s}^{n} \frac{3}{n-s+3t-2}\right] \\ &+ \sum_{s=\frac{n}{2}+2}^{n} \left[\sum_{t=\frac{n}{2}+2}^{s} \frac{3}{4s+t-n-3}\right] + \left.\sum_{s=\frac{n}{2}+1}^{n-1} \left[\sum_{t=s+1}^{n} \frac{3}{s+3t-4}\right]\right] \\ &+ \sum_{s=\frac{n}{2}+2}^{n} \left[\sum_{t=n-s+2}^{\frac{n}{2}+1} \frac{3}{n+3s-t-1}\right]. \end{aligned}$$

Table 1. Partition of vertices of the type u_s^t of O_n^m based on degree and eccentricity of each vertex when $n \equiv 0(mod2)$.

Representative	Degree	Eccentricity	Range	Frequency
u_s^t	2	$4n-s+1$	$t = 1,\ n+1$, $1 \le s \le \frac{n}{2}+1$	$2(\frac{n+2}{2})$
u_s^t	2	$3n+s-1$	$t = 1,\ n+1$, $\frac{n}{2}+2 \le s \le n$	$2(\frac{n-2}{2})$
u_s^t	3	$4n-3(s-1)-t$	$1 \le s \le \frac{n}{2}$, $s+1 \le t \le \frac{n}{2}+1$	$\frac{n}{4}(\frac{n}{2}+1)$
u_s^t	3	$4(n+1)-s-3t$	$2 \le s \le \frac{n}{2}$, $2 \le t \le s$	$\frac{n}{4}(\frac{n}{2}+1)$
u_s^t	3	$3n+s-3t+2$	$\frac{n}{2}+1 \le s \le n-1$, $2 \le t \le n+1-s$	$\frac{n}{4}(\frac{n}{2}-1)$
u_s^t	3	$n+3s-t-1$	$\frac{n}{2}+2 \le s \le n$, $n-s+2 \le t \le \frac{n}{2}+1$	$\frac{n}{4}(\frac{n}{2}-1)$
u_s^t	3	$3(n-s)+t+1$	$1 \le s \le \frac{n}{2}-1$, $\frac{n}{2}+2 \le t \le n-s+1$	$\frac{1}{2}(\frac{n}{2}-2)(\frac{n}{2}-1)$
u_s^t	3	$n-s+3t-2$	$2 \le s \le \frac{n}{2}$, $n-s+2 \le t \le n$	$\frac{1}{2}(\frac{n}{2}-2)(\frac{n}{2}-1)$
u_s^t	3	$4s-n+t-3$	$\frac{n}{2}+2 \le s \le n$, $\frac{n}{2}+2 \le t \le s$	$\frac{n}{4}(\frac{n}{2}-1)$
u_s^t	3	$s+3t-4$	$\frac{n}{2}+1 \le s \le n-1$, $s+1 \le t \le n$	$\frac{n}{4}(\frac{n}{2}-1)$

After an easy computation, we get

$$
\begin{aligned}
C^{\xi}(G) &= 32\left[\frac{14n+1}{7n(7n+1)}\right]+16\sum_{s=1}^{\frac{n}{2}-1}\left[\frac{1}{4n+1-s}+\frac{2}{7n+2s}\right] \\
&+ \sum_{s=1}^{\frac{n}{2}}\left[\sum_{t=s+1}^{\frac{n}{2}+1}\frac{12}{4n-3(s-1)-t}\right] + \sum_{s=2}^{\frac{n}{2}}\left[\sum_{t=2}^{s}\frac{12}{4(n+1)-s-3t}\right] \\
&+ \sum_{s=\frac{n}{2}+1}^{n-1}\left[\sum_{t=2}^{n-s+1}\frac{12}{3(n-t)+s+2}\right]+\sum_{s=\frac{n}{2}+2}^{n}\left[\sum_{t=n-s+2}^{\frac{n}{2}+1}\frac{12}{n+3s-t-1}\right] \\
&+ \sum_{s=1}^{\frac{n}{2}-1}\left[\sum_{t=\frac{n}{2}+2}^{n-s+1}\frac{12}{3(n-s)+t+1}\right]+\sum_{s=2}^{\frac{n}{2}}\left[\sum_{t=n+2-s}^{n}\frac{12}{n-s+3t-2}\right] \\
&+ \sum_{s=\frac{n}{2}+2}^{n}\left[\sum_{t=\frac{n}{2}+2}^{s}\frac{12}{4s+t-n-3}\right]+\sum_{s=\frac{n}{2}+1}^{n-1}\left[\sum_{t=s+1}^{n}\frac{12}{s+3t-4}\right].
\end{aligned}
$$

□

Theorem 2. *For every $n \geq 3$ and $n \equiv 1(mod2)$, consider the graph of $G \cong O_n^m$, with $n = m$. Then, the Connective Eccentric index of G is equal to*

$$
\begin{aligned}
C^{\xi}(G) &= \frac{32}{7n+1}+16\sum_{s=1}^{\frac{n+1}{2}-1}\left[\frac{1}{4n+1-s}+\frac{2}{5n-2s-3}\right] \\
&+ \sum_{s=1}^{\frac{n+1}{2}-1}\left[\sum_{t=s+1}^{\frac{n+1}{2}}\frac{12}{4n-3(s-1)-t}\right]+\sum_{s=2}^{\frac{n+1}{2}}\left[\sum_{t=2}^{s}\frac{12}{4(n+1)-s-3t}\right] \\
&+ \sum_{s=\frac{n+1}{2}+1}^{n-1}\left[\sum_{t=2}^{n-s+1}\frac{12}{3(n-t)+s+2}\right]+\sum_{s=\frac{n+1}{2}+1}^{n}\left[\sum_{t=n-s+2}^{\frac{n+1}{2}}\frac{12}{n+3s-t-1}v\right] \\
&+ \sum_{s=1}^{\frac{n+1}{2}-1}\left[\sum_{t=\frac{n+1}{2}+1}^{n-s+1}\frac{12}{3(n-s)+t+1}\right]+\sum_{s=2}^{\frac{n+1}{2}}\left[\sum_{t=n+2-s}^{n}\frac{12}{n-s+3t-2}\right] \\
&+ \sum_{s=\frac{n+1}{2}+1}^{n}\left[\sum_{t=\frac{n+1}{2}+1}^{s}\frac{12}{4s+t-n-3}\right]+\sum_{s=\frac{n+1}{2}+1}^{n-1}\left[\sum_{t=s+1}^{n}\frac{12}{s+3t-4}\right].
\end{aligned}
$$

Proof. Let G be the graph of O_n^m and $n \geq 3$ is odd. As above noted, the graph of O_n^m is a symmetric about reflection and rotation at right angles. Thus, the eccentricities $\varepsilon_{u_s^t} = \varepsilon_{v^t_{n+1-s}}$ and from the symmetry at right angles we can obtain that the eccentricities $\varepsilon_{y_s^t} = \varepsilon_{u_t^s}$, $\varepsilon_{w_s^t} = \varepsilon_{v_t^s}$. Therefore, from Equation (1), the Connective Eccentric index of O_n^m is equal to

$$C^{\xi}(O_n^m) = 4\sum_{u_s^t \in V(G)} \frac{d_{u_s^t}}{\varepsilon_{u_s^t}}$$

By using the values in Table 2, we get

$$
\begin{aligned}
C^{\xi}(G) &= C^{\xi}(O_n^m) = 4\Bigg[\ 2\sum_{s=1}^{\frac{n+1}{2}} \frac{2}{4n+1-s} + 2\sum_{s=\frac{n+1}{2}+1}^{n} \frac{2}{3n+s-1}\ \Bigg] \\
&+ \quad 4\Bigg[\ \sum_{s=1}^{\frac{n+1}{2}-1}\Bigg[\sum_{t=s+1}^{\frac{n+1}{2}} \frac{3}{4n-3(s-1)-t}\Bigg] + \sum_{s=2}^{\frac{n+1}{2}}\Bigg[\ \sum_{t=2}^{s} \frac{3}{4(n+1)-s-3t}\ \Bigg] \\
&+ \quad \sum_{s=\frac{n+1}{2}+1}^{n-1}\Bigg[\ \sum_{t=2}^{n-s+1} \frac{3}{3(n-t)+s+2}\Bigg] + \sum_{s=\frac{n+1}{2}+1}^{n}\Bigg[\ \sum_{t=n-s+2}^{\frac{n+1}{2}} \frac{3}{n+3s-t-1}\ \Bigg] \\
&+ \quad \sum_{s=1}^{\frac{n+1}{2}-1}\Bigg[\ \sum_{t=\frac{n+1}{2}+1}^{n-s+1} \frac{3}{3(n-s)+t+1}\Bigg] + \sum_{s=2}^{\frac{n+1}{2}}\Bigg[\sum_{t=n+2-s}^{n} \frac{3}{n-s+3t-2}\Bigg] \\
&+ \quad \sum_{s=\frac{n+1}{2}+1}^{n}\Bigg[\ \sum_{t=\frac{n+1}{2}+1}^{s} \frac{3}{4s+t-n-3}\Bigg] + \sum_{s=\frac{n+1}{2}+1}^{n-1}\Bigg[\ \sum_{t=s+1}^{n} \frac{3}{s+3t-4}\ \Bigg]\Bigg].
\end{aligned}
$$

Table 2. Partition of vertices of the type u_s^t of O_n^m based on degree and eccentricity of each vertex when $n \equiv 1(mod2)$.

Representative	Degree	Eccentricity	Range	Frequency
u_s^t	2	$4n-s+1$	$t=1,\ n+1$, $1 \le s \le \frac{n+1}{2}$	$2(\frac{n+1}{2})$
u_s^t	2	$3n+s-1$	$t=1,\ n+1,$ $\frac{n+1}{2}+1 \le s \le n$	$2(\frac{n-1}{2})$
u_s^t	3	$4n-3(s-1)-t$	$1 \le s \le \frac{n+1}{2}-1$, $s+1 \le t \le \frac{n+1}{2}$	$\frac{n+1}{4}(\frac{n+1}{2}-1)$
u_s^t	3	$4(n+1)-s-3t$	$2 \le s \le \frac{n+1}{2}$, $2 \le t \le s$	$\frac{n+1}{4}(\frac{n+1}{2}-1)$
u_s^t	3	$3n+s-3t+2$	$\frac{n+1}{2}+1 \le s \le n-1$, $2 \le t \le n+1-s$	$\frac{n-1}{4}(\frac{n-1}{2}-1)$
u_s^t	3	$n+3s-t-1$	$\frac{n+1}{2}+1 \le s \le n$, $n-s+2 \le t \le \frac{n+1}{2}$	$\frac{n-1}{4}(\frac{n-1}{2}+1)$
u_s^t	3	$3(n-s)+t+1$	$1 \le s \le \frac{n+1}{2}-1,$ $\frac{n+1}{2}+1 \le t \le n-s+1$	$\frac{n+1}{4}(\frac{n+1}{2}-1)$
u_s^t	3	$n-s+3t-2$	$2 \le s \le \frac{n+1}{2}$, $n-s+2 \le t \le n$	$\frac{n+1}{4}(\frac{n+1}{2}-1)$
u_s^t	3	$4s-n+t-3$	$\frac{n+1}{2}+1 \le s \le n$, $\frac{n+1}{2}+1 \le t \le s$	$\frac{n-1}{4}(\frac{n+1}{2})$
u_s^t	3	$s+3t-4$	$\frac{n+1}{2}+1 \le s \le n-1$, $s+1 \le t \le n$	$\frac{n-1}{4}(\frac{n-1}{2}-1)$

After an easy computation, we get

$$
\begin{aligned}
C^{\xi}(G) &= \frac{32}{7n+1} + 16\sum_{s=1}^{\frac{n+1}{2}-1}\left[\frac{1}{4n+1-s} + \frac{2}{5n-2s-3}\right] \\
&+ \sum_{s=1}^{\frac{n+1}{2}-1}\left[\sum_{t=s+1}^{\frac{n+1}{2}}\frac{12}{4n-3(s-1)-t}\right] + \sum_{s=2}^{\frac{n+1}{2}}\left[\sum_{t=2}^{s}\frac{12}{4(n+1)-s-3t}\right] \\
&+ \sum_{s=\frac{n+1}{2}+1}^{n-1}\left[\sum_{t=2}^{n-s+1}\frac{12}{3(n-t)+s+2}\right] + \sum_{s=\frac{n+1}{2}+1}^{n}\left[\sum_{t=n-s+2}^{\frac{n+1}{2}}\frac{12}{n+3s-t-1}\right] \\
&+ \sum_{s=1}^{\frac{n+1}{2}-1}\left[\sum_{t=\frac{n+1}{2}+1}^{n-s+1}\frac{12}{3(n-s)+t+1}\right] + \sum_{s=2}^{\frac{n+1}{2}}\left[\sum_{t=n+2-s}^{n}\frac{12}{n-s+3t-2}\right] \\
&+ \sum_{s=\frac{n+1}{2}+1}^{n}\left[\sum_{t=\frac{n+1}{2}+1}^{s}\frac{12}{4s+t-n-3}\right] + \sum_{s=\frac{n+1}{2}+1}^{n-1}\left[\sum_{t=s+1}^{n}\frac{12}{s+3t-4}\right].
\end{aligned}
$$

□

Theorem 3. *For every $n \geq 4$ and $n \equiv 0(mod2)$, consider the graph of $G \cong O_n^m$, with $n = m$. Then, the Eccentric Connectivity index of G is equal to*

$$
\begin{aligned}
\xi(G) &= 60n^2 + \sum_{s=1}^{\frac{n}{2}}\left[\sum_{t=s+1}^{\frac{n}{2}+1}12\{4n-3(s-1)-t\}\right] + \sum_{s=2}^{\frac{n}{2}}\left[\sum_{t=2}^{s}12\{4(n+1)-s-3t\}\right] \\
&+ \sum_{s=\frac{n}{2}+1}^{n-1}\left[\sum_{t=2}^{n-s+1}12\{3(n-t)+s+2\}\right] + \sum_{s=\frac{n}{2}+2}^{n}\left[\sum_{t=n-s+2}^{\frac{n}{2}+1}12\{n+3s-t-1\}\right] \\
&+ \sum_{s=1}^{\frac{n}{2}-1}\left[\sum_{t=\frac{n}{2}+2}^{n-s+1}12\{3(n-s)+t+1\}\right] + \sum_{s=2}^{\frac{n}{2}}\left[\sum_{t=n+2-s}^{n}12\{n-s+3t-2\}\right] \\
&+ \sum_{s=\frac{n}{2}+2}^{n}\left[\sum_{t=\frac{n}{2}+2}^{s}12\{4s+t-n-3\}\right] + \sum_{s=\frac{n}{2}+1}^{n-1}\left[\sum_{t=s+1}^{n}12\{s+3t-4\}\right].
\end{aligned}
$$

Proof. By using the arguments in the proof of Theorem 1, the values in Table 1 and Equation (2), we get

$$
\begin{aligned}
\xi(G) &= \xi(O_n^m) = 4\left[2\sum_{s=1}^{\frac{n}{2}+1}2\{4n+1-s\} + 2\sum_{s=\frac{n}{2}+2}^{n}2\{3n+s-1\}\right] \\
&+ 4\left[\sum_{s=1}^{\frac{n}{2}}\left[\sum_{t=s+1}^{\frac{n}{2}+1}3\{4n-3(s-1)-t\}\right] + \sum_{s=2}^{\frac{n}{2}}\left[\sum_{t=2}^{s}3\{4(n+1)-s-3t\}\right]\right. \\
&+ \sum_{s=\frac{n}{2}+1}^{n-1}\left[\sum_{t=2}^{n-s+1}3\{3(n-t)+s+2\}\right] + \sum_{s=\frac{n}{2}+2}^{n}\left[\sum_{t=n-s+2}^{\frac{n}{2}+1}3\{n+3s-t-1\}\right] \\
&+ \sum_{s=1}^{\frac{n}{2}-1}\left[\sum_{t=\frac{n}{2}+2}^{n-s+1}3\{3(n-s)+t+1\}\right] + \sum_{s=2}^{\frac{n}{2}}\left[\sum_{t=n+2-s}^{n}3\{n-s+3t-2\}\right] \\
&+ \left.\sum_{s=\frac{n}{2}+2}^{n}\left[\sum_{t=\frac{n}{2}+2}^{s}3\{4s+t-n-3\}\right] + \sum_{s=\frac{n}{2}+1}^{n-1}\left[\sum_{t=s+1}^{n}3\{s+3t-4\}\right]\right].
\end{aligned}
$$

After some easy calculations, we get

$$\begin{aligned}
\xi(G) &= 60n^2 + \sum_{s=1}^{\frac{n}{2}} \Bigg[\sum_{t=s+1}^{\frac{n}{2}+1} 12\{4n-3(s-1)-t\} \Bigg] + \sum_{s=2}^{\frac{n}{2}} \Bigg[\sum_{t=2}^{s} 12\{4(n+1)-s-3t\} \Bigg] \\
&+ \sum_{s=\frac{n}{2}+1}^{n-1} \Bigg[\sum_{t=2}^{n-s+1} 12\{3(n-t)+s+2\} \Bigg] + \sum_{s=\frac{n}{2}+2}^{n} \Bigg[\sum_{t=n-s+2}^{\frac{n}{2}+1} 12\{n+3s-t-1\} \Bigg] \\
&+ \sum_{s=1}^{\frac{n}{2}-1} \Bigg[\sum_{t=\frac{n}{2}+2}^{n-s+1} 12\{3(n-s)+t+1\} \Bigg] + \sum_{s=2}^{\frac{n}{2}} \Bigg[\sum_{t=n+2-s}^{n} 12\{n-s+3t-2\} \Bigg] \\
&+ \sum_{s=\frac{n}{2}+2}^{n} \Bigg[\sum_{t=\frac{n}{2}+2}^{s} 12\{4s+t-n-3\} \Bigg] + \sum_{s=\frac{n}{2}+1}^{n-1} \Bigg[\sum_{t=s+1}^{n} 12\{s+3t-4\} \Bigg].
\end{aligned}$$

□

Theorem 4. *For every $n \geq 3$ and $n \equiv 1(mod2)$, consider the graph of $G \cong O_n^m$, with $n = m$. Then, the Eccentric Connectivity index of G is equal to*

$$\begin{aligned}
\xi(G) &= 60n^2 - 32n + 10 \\
&+ \sum_{s=1}^{\frac{n+1}{2}-1} \Bigg[\sum_{t=s+1}^{\frac{n+1}{2}} 12\{4n-3(s-1)-t\} \Bigg] + \sum_{s=2}^{\frac{n+1}{2}} \Bigg[\sum_{t=2}^{s} 12\{4(n+1)-s-3t\} \Bigg] \\
&+ \sum_{s=\frac{n+1}{2}+1}^{n-1} \Bigg[\sum_{t=2}^{n-s+1} 12\{3(n-t)+s+2\} \Bigg] + \sum_{s=\frac{n+1}{2}+1}^{n} \Bigg[\sum_{t=n-s+2}^{\frac{n+1}{2}} 12\{n+3s-t-1\} \Bigg] \\
&+ \sum_{s=1}^{\frac{n+1}{2}-1} \Bigg[\sum_{t=\frac{n+1}{2}+1}^{n-s+1} 12\{3(n-s)+t+1\} \Bigg] + \sum_{s=2}^{\frac{n+1}{2}} \Bigg[\sum_{t=n+2-s}^{n} 12\{n-s+3t-2\} \Bigg] \\
&+ \sum_{s=\frac{n+1}{2}+1}^{n} \Bigg[\sum_{t=\frac{n+1}{2}+1}^{s} 12\{4s+t-n-3\} \Bigg] + \sum_{s=\frac{n+1}{2}+1}^{n-1} \Bigg[\sum_{t=s+1}^{n} 12\{s+3t-4\} \Bigg].
\end{aligned}$$

Proof. By using the arguments in the proof of Theorem 2, the values in Table 2 and Equation (2), we get

$$\begin{aligned}
\xi(G) &= \xi(O_n^m) = 4\Bigg[2\sum_{s=1}^{\frac{n+1}{2}} 2\{4n+1-s\} + 2\sum_{s=\frac{n+1}{2}+1}^{n} 2\{3n+s-1\} \Bigg] \\
&+ 4\Bigg[\sum_{s=1}^{\frac{n+1}{2}-1} \Bigg[\sum_{t=s+1}^{\frac{n+1}{2}} 3\{4n-3(s-1)-t\} \Bigg] + \sum_{s=2}^{\frac{n+1}{2}} \Bigg[\sum_{t=2}^{s} 3\{4(n+1)-s-3t\} \Bigg] \\
&+ \sum_{s=\frac{n+1}{2}+1}^{n-1} \Bigg[\sum_{t=2}^{n-s+1} 3\{3(n-t)+s+2\} \Bigg] + \sum_{s=\frac{n+1}{2}+1}^{n} \Bigg[\sum_{t=n-s+2}^{\frac{n+1}{2}} 3\{n+3s-t-1\} \Bigg] \\
&+ \sum_{s=1}^{\frac{n+1}{2}-1} \Bigg[\sum_{t=\frac{n+1}{2}+1}^{n-s+1} 3\{3(n-s)+t+1\} \Bigg] + \sum_{s=2}^{\frac{n+1}{2}} \Bigg[\sum_{t=n+2-s}^{n} 3\{n-s+3t-2\} \Bigg] \\
&+ \sum_{s=\frac{n+1}{2}+1}^{n} \Bigg[\sum_{t=\frac{n+1}{2}+1}^{s} 3\{4s+t-n-3\} \Bigg] + \sum_{s=\frac{n+1}{2}+1}^{n-1} \Bigg[\sum_{t=s+1}^{n} 3\{s+3t-4\} \Bigg] \Bigg].
\end{aligned}$$

After some easy calculations, we get

$$
\begin{aligned}
\xi(G) &= 60n^2 - 32n + 10 \\
&+ \sum_{s=1}^{\frac{n+1}{2}-1} \left[\sum_{t=s+1}^{\frac{n+1}{2}} 12\{4n - 3(s-1) - t\} \right] + \sum_{s=2}^{\frac{n+1}{2}} \left[\sum_{t=2}^{s} 12\{4(n+1) - s - 3t\} \right] \\
&+ \sum_{s=\frac{n+1}{2}+1}^{n-1} \left[\sum_{t=2}^{n-s+1} 12\{3(n-t) + s + 2\} \right] + \sum_{s=\frac{n+1}{2}+1}^{n} \left[\sum_{t=n-s+2}^{\frac{n+1}{2}} 12\{n + 3s - t - 1\} \right] \\
&+ \sum_{s=1}^{\frac{n+1}{2}-1} \left[\sum_{t=\frac{n+1}{2}+1}^{n-s+1} 12\{3(n-s) + t + 1\} \right] + \sum_{s=2}^{\frac{n+1}{2}} \left[\sum_{t=n+2-s}^{n} 12\{n - s + 3t - 2\} \right] \\
&+ \sum_{s=\frac{n+1}{2}+1}^{n} \left[\sum_{t=\frac{n+1}{2}+1}^{s} 12\{4s + t - n - 3\} \right] + \sum_{s=\frac{n+1}{2}+1}^{n-1} \left[\sum_{t=s+1}^{n} 12\{s + 3t - 4\} \right].
\end{aligned}
$$

□

Theorem 5. *For every $n \geq 4$ and $n \equiv 0(mod2)$, consider the graph of $G \cong O_n^m$, with $n = m$. Then, the eccentric Zagreb index of G is equal to*

$$
\begin{aligned}
M_1^{**}(O_n^m) &= \frac{338}{3}n^3 + \frac{4n}{3} + 4\Bigg[\sum_{s=1}^{\frac{n}{2}} \left[\sum_{t=s+1}^{\frac{n}{2}+1} (4n - 3(s-1) - t)^2 \right] + \sum_{s=2}^{\frac{n}{2}} \left[\sum_{t=2}^{s} (4(n+1) - s - 3t)^2 \right] \\
&+ \sum_{s=\frac{n}{2}+1}^{n-1} \left[\sum_{t=2}^{n-s+1} (3(n-t) + s + 2)^2 \right] + \sum_{s=\frac{n}{2}+2}^{n} \left[\sum_{t=n-s+2}^{\frac{n}{2}+1} (n + 3s - t - 1)^2 \right] \\
&+ \sum_{s=1}^{\frac{n}{2}-1} \left[\sum_{t=\frac{n}{2}+2}^{n-s+1} (3(n-s) + t + 1)^2 \right] + \sum_{s=2}^{\frac{n}{2}} \left[\sum_{t=n+2-s}^{n} (n - s + 3t - 2)^2 \right] \\
&+ \sum_{s=\frac{n}{2}+2}^{n} \left[\sum_{t=\frac{n}{2}+2}^{s} (4s + t - n - 3)^2 \right] + \sum_{s=\frac{n}{2}+1}^{n-1} \left[\sum_{t=s+1}^{n} (s + 3t - 4)^2 \right] \Bigg].
\end{aligned}
$$

Proof. By using the arguments in the proof of Theorem 1, the values in Table 1 and the following equation, we get

$$M_1^{**}(G) = \sum_{u \in V(G)} (\epsilon_u)^2, \tag{5}$$

$$
\begin{aligned}
M_1^{**}(O_n^m) &= 4 \sum_{u_s^t \in V(G)} (\varepsilon_{u_s^t})^2 = 4\left[2\sum_{s=1}^{\frac{n}{2}+1} (4n + 1 - s)^2 + 2 \sum_{s=\frac{n}{2}+2}^{n} (3n + s - 1)^2 \right] \\
&+ 4\Bigg[\sum_{s=1}^{\frac{n}{2}} \left[\sum_{t=s+1}^{\frac{n}{2}+1} (4n - 3(s-1) - t)^2 \right] + \sum_{s=2}^{\frac{n}{2}} \left[\sum_{t=2}^{s} (4(n+1) - s - 3t)^2 \right] \\
&+ \sum_{s=\frac{n}{2}+1}^{n-1} \left[\sum_{t=2}^{n-s+1} (3(n-t) + s + 2)^2 \right] + \sum_{s=\frac{n}{2}+2}^{n} \left[\sum_{t=n-s+2}^{\frac{n}{2}+1} (n + 3s - t - 1)^2 \right] \\
&+ \sum_{s=1}^{\frac{n}{2}-1} \left[\sum_{t=\frac{n}{2}+2}^{n-s+1} (3(n-s) + t + 1)^2 \right] + \sum_{s=2}^{\frac{n}{2}} \left[\sum_{t=n+2-s}^{n} (n - s + 3t - 2)^2 \right] \\
&+ \sum_{s=\frac{n}{2}+2}^{n} \left[\sum_{t=\frac{n}{2}+2}^{s} (4s + t - n - 3)^2 \right] + \sum_{s=\frac{n}{2}+1}^{n-1} \left[\sum_{t=s+1}^{n} (s + 3t - 4)^2 \right] \Bigg].
\end{aligned}
$$

After some easy calculations, we get

$$
\begin{aligned}
M_1^{**}(O_n^m) &= \frac{338}{3}n^3 + \frac{4n}{3} + 4\Bigg[\sum_{s=1}^{\frac{n}{2}}\Bigg[\sum_{t=s+1}^{\frac{n}{2}+1}(4n-3(s-1)-t)^2\Bigg] + \sum_{s=2}^{\frac{n}{2}}\Bigg[\sum_{t=2}^{s}(4(n+1)-s-3t)^2\Bigg] \\
&+ \sum_{s=\frac{n}{2}+1}^{n-1}\Bigg[\sum_{t=2}^{n-s+1}(3(n-t)+s+2)^2\Bigg] + \sum_{s=\frac{n}{2}+2}^{n}\Bigg[\sum_{t=n-s+2}^{\frac{n}{2}+1}(n+3s-t-1)^2\Bigg] \\
&+ \sum_{s=1}^{\frac{n}{2}-1}\Bigg[\sum_{t=\frac{n}{2}+2}^{n-s+1}(3(n-s)+t+1)^2\Bigg] + \sum_{s=2}^{\frac{n}{2}}\Bigg[\sum_{t=n+2-s}^{n}(n-s+3t-2)^2\Bigg] \\
&+ \sum_{s=\frac{n}{2}+2}^{n}\Bigg[\sum_{t=\frac{n}{2}+2}^{s}(4s+t-n-3)^2\Bigg] + \sum_{s=\frac{n}{2}+1}^{n-1}\Bigg[\sum_{t=s+1}^{n}(s+3t-4)^2\Bigg]\Bigg].
\end{aligned}
$$

□

Theorem 6. *For every $n \geq 3$ and $n \equiv 1(mod 2)$, consider the graph of $G \cong O_n^m$, with $n = m$. Then, the eccentric Zagreb index of G is equal to*

$$
\begin{aligned}
M_1^{**}(O_n^m) &= \frac{338}{3}n^3 + \frac{46n}{3} + 4\Bigg[\sum_{s=1}^{\frac{n+1}{2}-1}\Bigg[\sum_{t=s+1}^{\frac{n+1}{2}}(4n-3(s-1)-t)^2\Bigg] + \sum_{s=2}^{\frac{n+1}{2}}\Bigg[\sum_{t=2}^{s}(4(n+1)-s-3t)^2\Bigg] \\
&+ \sum_{s=\frac{n+1}{2}+1}^{n-1}\Bigg[\sum_{t=2}^{n-s+1}(3(n-t)+s+2)^2\Bigg] + \sum_{s=\frac{n+1}{2}+1}^{n}\Bigg[\sum_{t=n-s+2}^{\frac{n+1}{2}}(n+3s-t-1)^2\Bigg] \\
&+ \sum_{s=1}^{\frac{n+1}{2}-1}\Bigg[\sum_{t=\frac{n+1}{2}+1}^{n-s+1}(3(n-s)+t+1)^2\Bigg] + \sum_{s=2}^{\frac{n+1}{2}}\Bigg[\sum_{t=n+2-s}^{n}(n-s+3t-2)^2\Bigg] \\
&+ \sum_{s=\frac{n+1}{2}+1}^{n}\Bigg[\sum_{t=\frac{n+1}{2}+1}^{s}(4s+t-n-3)^2\Bigg] + \sum_{s=\frac{n+1}{2}+1}^{n-1}\Bigg[\sum_{t=s+1}^{n}(s+3t-4)^2\Bigg]\Bigg]
\end{aligned}
$$

Proof. By using the arguments in the proof of Theorem 1, the values in Table 2 and the following equation, we get

$$M_1^{**}(G) = \sum_{u \in V(G)} (\epsilon_u)^2,$$

$$
\begin{aligned}
M_1^{**}(O_n^m) &= 4\sum_{u_s^t \in V(G)}(\varepsilon_{u_s^t})^2 = 4\Bigg[2\sum_{s=1}^{\frac{n+1}{2}}(4n+1-s)^2 + 2\sum_{s=\frac{n+1}{2}+1}^{n}(3n+s-1)^2\Bigg] \\
&+ 4\Bigg[\sum_{s=1}^{\frac{n+1}{2}-1}\Bigg[\sum_{t=s+1}^{\frac{n+1}{2}}(4n-3(s-1)-t)^2\Bigg] + \sum_{s=2}^{\frac{n+1}{2}}\Bigg[\sum_{t=2}^{s}(4(n+1)-s-3t)^2\Bigg] \\
&+ \sum_{s=\frac{n+1}{2}+1}^{n-1}\Bigg[\sum_{t=2}^{n-s+1}(3(n-t)+s+2)^2\Bigg] + \sum_{s=\frac{n+1}{2}+1}^{n}\Bigg[\sum_{t=n-s+2}^{\frac{n+1}{2}}(n+3s-t-1)^2\Bigg] \\
&+ \sum_{s=1}^{\frac{n+1}{2}-1}\Bigg[\sum_{t=\frac{n+1}{2}+1}^{n-s+1}(3(n-s)+t+1)^2\Bigg] + \sum_{s=2}^{\frac{n+1}{2}}\Bigg[\sum_{t=n+2-s}^{n}(n-s+3t-2)^2\Bigg] \\
&+ \sum_{s=\frac{n+1}{2}+1}^{n}\Bigg[\sum_{t=\frac{n+1}{2}+1}^{s}(4s+t-n-3)^2\Bigg] + \sum_{s=\frac{n+1}{2}+1}^{n-1}\Bigg[\sum_{t=s+1}^{n}(s+3t-4)^2\Bigg]\Bigg].
\end{aligned}
$$

After some easy calculations, we get

$$\begin{aligned} M_1^{**}(O_n^m) &= \frac{338}{3}n^3+\frac{46n}{3}+4\Bigg[\sum_{s=1}^{\frac{n+1}{2}-1}\Big[\sum_{t=s+1}^{\frac{n+1}{2}}(4n-3(s-1)-t)^2\Big]+\sum_{s=2}^{\frac{n+1}{2}}\Big[\sum_{t=2}^{s}(4(n+1)-s-3t)^2\Big] \\ &+ \sum_{s=\frac{n+1}{2}+1}^{n-1}\Big[\sum_{t=2}^{n-s+1}(3(n-t)+s+2)^2\Big]+\sum_{s=\frac{n+1}{2}+1}^{n}\Big[\sum_{t=n-s+2}^{\frac{n+1}{2}}(n+3s-t-1)^2\Big] \\ &+ \sum_{s=1}^{\frac{n+1}{2}-1}\Big[\sum_{t=\frac{n+1}{2}+1}^{n-s+1}(3(n-s)+t+1)^2\Big]+\sum_{s=2}^{\frac{n+1}{2}}\Big[\sum_{t=n+2-s}^{n}(n-s+3t-2)^2\Big] \\ &+ \sum_{s=\frac{n+1}{2}+1}^{n}\Big[\sum_{t=\frac{n+1}{2}+1}^{s}(4s+t-n-3)^2\Big]+\sum_{s=\frac{n+1}{2}+1}^{n-1}\Big[\sum_{t=s+1}^{n}(s+3t-4)^2\Big]\Bigg]. \end{aligned}$$

□

Theorem 7. *For every $n \geq 4$ and $n \equiv 0(mod2)$, consider the graph of $G \cong O_n^m$, with $n = m$. Then, the Eccentric Connectivity Polynomial of G is equal to*

$$\begin{aligned} ECP(O_n^m,x) &= \frac{1}{(x-1)}\Big(-16x^{(\frac{7n}{2}+1)}+16x^{(4n+1)}+\big(-16(\frac{1}{x})^{\frac{n}{2}}+16\big)x^{4n}\Big) \\ &+ 4\Bigg[\sum_{s=1}^{\frac{n}{2}}\Big[\sum_{t=s+1}^{\frac{n}{2}+1}3x^{(4n-3(s-1)-t)}\Big]+\sum_{s=2}^{\frac{n}{2}}\Big[\sum_{t=2}^{s}3x^{(4(n+1)-s-3t)}\Big] \\ &+ \sum_{s=\frac{n}{2}+1}^{n-1}\Big[\sum_{t=2}^{n-s+1}3x^{(3(n-t)+s+2)}\Big]+\sum_{s=\frac{n}{2}+2}^{n}\Big[\sum_{t=n-s+2}^{\frac{n}{2}+1}3x^{(n+3s-t-1)}\Big] \\ &+ \sum_{s=1}^{\frac{n}{2}-1}\Big[\sum_{t=\frac{n}{2}+2}^{n-s+1}3x^{(3(n-s)+t+1)}\Big]+\sum_{s=2}^{\frac{n}{2}}\Big[\sum_{t=n+2-s}^{n}3x^{(n-s+3t-2)}\Big] \\ &+ \sum_{s=\frac{n}{2}+2}^{n}\Big[\sum_{t=\frac{n}{2}+2}^{s}3x^{(4s+t-n-3)}\Big]+\sum_{s=\frac{n}{2}+1}^{n-1}\Big[\sum_{t=s+1}^{n}3x^{(s+3t-4)}\Big]\Bigg]. \end{aligned}$$

Proof. By using the arguments in the proof of Theorem 1, the values in Table 1 and Equation (4) we get

$$\begin{aligned} ECP(O_n^m,x) &= 4\Bigg[2\sum_{s=1}^{\frac{n}{2}+1}2x^{(4n+1-s)}+2\sum_{s=\frac{n}{2}+2}^{n}2x^{(3n+s-1)}\Bigg] \\ &+ 4\Bigg[\sum_{s=1}^{\frac{n}{2}}\Big[\sum_{t=s+1}^{\frac{n}{2}+1}3x^{(4n-3(s-1)-t)}\Big]+\sum_{s=2}^{\frac{n}{2}}\Big[\sum_{t=2}^{s}3x^{(4(n+1)-s-3t)}\Big] \\ &+ \sum_{s=\frac{n}{2}+1}^{n-1}\Big[\sum_{t=2}^{n-s+1}3x^{(3(n-t)+s+2)}\Big]+\sum_{s=\frac{n}{2}+2}^{n}\Big[\sum_{t=n-s+2}^{\frac{n}{2}+1}3x^{(n+3s-t-1)}\Big] \\ &+ \sum_{s=1}^{\frac{n}{2}-1}\Big[\sum_{t=\frac{n}{2}+2}^{n-s+1}3x^{(3(n-s)+t+1)}\Big]+\sum_{s=2}^{\frac{n}{2}}\Big[\sum_{t=n+2-s}^{n}3x^{(n-s+3t-2)}\Big] \\ &+ \sum_{s=\frac{n}{2}+2}^{n}\Big[\sum_{t=\frac{n}{2}+2}^{s}3x^{(4s+t-n-3)}\Big]+\sum_{s=\frac{n}{2}+1}^{n-1}\Big[\sum_{t=s+1}^{n}3x^{(s+3t-4)}\Big]\Bigg]. \end{aligned}$$

After some easy calculations, we get

$$\begin{aligned}
ECP(O_n^m, x) &= \frac{1}{(x-1)}\Big(-16x^{(\frac{7n}{2}+1)} + 16x^{(4n+1)} + \big(-16\big(\frac{1}{x}\big)^{\frac{n}{2}} + 16\big)x^{4n}\Big) \\
&+ 4\Bigg[\sum_{s=1}^{\frac{n}{2}}\Big[\sum_{t=s+1}^{\frac{n}{2}+1} 3x^{(4n-3(s-1)-t)}\Big] + \sum_{s=2}^{\frac{n}{2}}\Big[\sum_{t=2}^{s} 3x^{(4(n+1)-s-3t)}\Big] \\
&+ \sum_{s=\frac{n}{2}+1}^{n-1}\Big[\sum_{t=2}^{n-s+1} 3x^{(3(n-t)+s+2)}\Big] + \sum_{s=\frac{n}{2}+2}^{n}\Big[\sum_{t=n-s+2}^{\frac{n}{2}+1} 3x^{(n+3s-t-1)}\Big] \\
&+ \sum_{s=1}^{\frac{n}{2}-1}\Big[\sum_{t=\frac{n}{2}+2}^{n-s+1} 3x^{(3(n-s)+t+1)}\Big] + \sum_{s=2}^{\frac{n}{2}}\Big[\sum_{t=n+2-s}^{n} 3x^{(n-s+3t-2)}\Big] \\
&+ \sum_{s=\frac{n}{2}+2}^{n}\Big[\sum_{t=\frac{n}{2}+2}^{s} 3x^{(4s+t-n-3)}\Big] + \sum_{s=\frac{n}{2}+1}^{n-1}\Big[\sum_{t=s+1}^{n} 3x^{(s+3t-4)}\Big]\Bigg].
\end{aligned}$$

□

Theorem 8. *For every $n \geq 3$ and $n \equiv 1(mod 2)$, consider the graph of $G \cong O_n^m$, with $n = m$. Then, the Eccentric Connectivity Polynomial of G is equal to*

$$\begin{aligned}
ECP(O_n^m, x) &= \frac{1}{(x-1)}\Big(-32x^{(\frac{7n+1}{2})} + 16x^{(4n+1)} + 16x^{(4n)}\Big) \\
&+ 4\Bigg[\sum_{s=1}^{\frac{n+1}{2}-1}\Big[\sum_{t=s+1}^{\frac{n+1}{2}} 3x^{(4n-3(s-1)-t)}\Big] + \sum_{s=2}^{\frac{n+1}{2}}\Big[\sum_{t=2}^{s} 3x^{(4(n+1)-s-3t)}\Big] \\
&+ \sum_{s=\frac{n+1}{2}+1}^{n-1}\Big[\sum_{t=2}^{n-s+1} 3x^{(3(n-t)+s+2)}\Big] + \sum_{s=\frac{n+1}{2}+1}^{n}\Big[\sum_{t=n-s+2}^{\frac{n+1}{2}} 3x^{(n+3s-t-1)}\Big] \\
&+ \sum_{s=1}^{\frac{n+1}{2}-1}\Big[\sum_{t=\frac{n+1}{2}+1}^{n-s+1} 3x^{(3(n-s)+t+1)}\Big] + \sum_{s=2}^{\frac{n+1}{2}}\Big[\sum_{t=n+2-s}^{n} 3x^{(n-s+3t-2)}\Big] \\
&+ \sum_{s=\frac{n+1}{2}+1}^{n}\Big[\sum_{t=\frac{n+1}{2}+1}^{s} 3x^{(4s+t-n-3)}\Big] + \sum_{s=\frac{n+1}{2}+1}^{n-1}\Big[\sum_{t=s+1}^{n} 3x^{(s+3t-4)}\Big]\Bigg].
\end{aligned}$$

Proof. By using the arguments in the proof of Theorem 1, the values in Table 2 and Equation (4), we get

$$\begin{aligned} ECP(O_n^m,x) &= 4\Bigg[2\sum_{s=1}^{\frac{n+1}{2}} 2x^{(4n+1-s)} + 2\sum_{s=\frac{n+1}{2}+1}^{n} 2x^{(3n+s-1)}\Bigg] \\ &+ 4\Bigg[\sum_{s=1}^{\frac{n+1}{2}-1}\Bigg[\sum_{t=s+1}^{\frac{n+1}{2}} 3x^{(4n-3(s-1)-t)}\Bigg] + \sum_{s=2}^{\frac{n+1}{2}}\Bigg[\sum_{t=2}^{s} 3x^{(4(n+1)-s-3t)}\Bigg] \\ &+ \sum_{s=\frac{n+1}{2}+1}^{n-1}\Bigg[\sum_{t=2}^{n-s+1} 3x^{(3(n-t)+s+2)}\Bigg] + \sum_{s=\frac{n+1}{2}+1}^{n}\Bigg[\sum_{t=n-s+2}^{\frac{n+1}{2}} 3x^{(n+3s-t-1)}\Bigg] \\ &+ \sum_{s=1}^{\frac{n+1}{2}-1}\Bigg[\sum_{t=\frac{n+1}{2}+1}^{n-s+1} 3x^{(3(n-s)+t+1)}\Bigg] + \sum_{s=2}^{\frac{n+1}{2}}\Bigg[\sum_{t=n+2-s}^{n} 3x^{(n-s+3t-2)}\Bigg] \\ &+ \sum_{s=\frac{n+1}{2}+1}^{n}\Bigg[\sum_{t=\frac{n+1}{2}+1}^{s} 3x^{(4s+t-n-3)}\Bigg] + \sum_{s=\frac{n+1}{2}+1}^{n-1}\Bigg[\sum_{t=s+1}^{n} 3x^{(s+3t-4)}\Bigg]\Bigg]. \end{aligned}$$

After some easy calculations, we get

$$\begin{aligned} ECP(O_n^m,x) &= \frac{1}{(x-1)}\Bigg(-32x^{(\frac{7n+1}{2})} + 16x^{(4n+1)} + 16x^{(4n)}\Bigg) \\ &+ 4\Bigg[\sum_{s=1}^{\frac{n+1}{2}-1}\Bigg[\sum_{t=s+1}^{\frac{n+1}{2}} 3x^{(4n-3(s-1)-t)}\Bigg] + \sum_{s=2}^{\frac{n+1}{2}}\Bigg[\sum_{t=2}^{s} 3x^{(4(n+1)-s-3t)}\Bigg] \\ &+ \sum_{s=\frac{n+1}{2}+1}^{n-1}\Bigg[\sum_{t=2}^{n-s+1} 3x^{(3(n-t)+s+2)}\Bigg] + \sum_{s=\frac{n+1}{2}+1}^{n}\Bigg[\sum_{t=n-s+2}^{\frac{n+1}{2}} 3x^{(n+3s-t-1)}\Bigg] \\ &+ \sum_{s=1}^{\frac{n+1}{2}-1}\Bigg[\sum_{t=\frac{n+1}{2}+1}^{n-s+1} 3x^{(3(n-s)+t+1)}\Bigg] + \sum_{s=2}^{\frac{n+1}{2}}\Bigg[\sum_{t=n+2-s}^{n} 3x^{(n-s+3t-2)}\Bigg] \\ &+ \sum_{s=\frac{n+1}{2}+1}^{n}\Bigg[\sum_{t=\frac{n+1}{2}+1}^{s} 3x^{(4s+t-n-3)}\Bigg] + \sum_{s=\frac{n+1}{2}+1}^{n-1}\Bigg[\sum_{t=s+1}^{n} 3x^{(s+3t-4)}\Bigg]\Bigg]. \end{aligned}$$

□

4. Conclusions

In this paper, we discuss the Octagonal Grid and compute the eccentric connectivity index, eccentricity Zagreb index, connective eccentric index and eccentric connectivity polynomial of the Octagonal grid of O_n^m. Since eccentricity based indices have various applications in nanomedicine and materials science, these theoretical results could have applications in medical science.

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