mathematics

MDPI

Article

Neutrosophic Portfolios of Financial Assets. Minimizing the Risk of Neutrosophic Portfolios

Marcel-Ioan Boloș [1], Ioana-Alexandra Bradea [2] and Camelia Delcea [2,*]

[1] Department of Finance and Banks, University of Oradea, 410087 Oradea, Romania; marcel.bolos@softscape.ro
[2] Department of Informatics and Cybernetics, Bucharest University of Economic Studies, 010552 Bucharest, Romania; ioana.bradea@softscape.ro
* Correspondence: camelia.delcea@csie.ase.ro; Tel.: +40-7695-432-813

Received: 16 September 2019; Accepted: 24 October 2019; Published: 3 November 2019

Abstract: This paper studies the problem of neutrosophic portfolios of financial assets as part of the modern portfolio theory. Neutrosophic portfolios comprise those categories of portfolios made up of financial assets for which the neutrosophic return, risk and covariance can be determined and which provide concomitant information regarding the probability of achieving the neutrosophic return, both at each financial asset and portfolio level and also information on the probability of manifestation of the neutrosophic risk. Neutrosophic portfolios are characterized by two fundamental performance indicators, namely: the neutrosophic portfolio return and the neutrosophic portfolio risk. Neutrosophic portfolio return is dependent on the weight of the financial assets in the total value of the portfolio but also on the specific neutrosophic return of each financial asset category that enters into the portfolio structure. The neutrosophic portfolio risk is dependent on the weight of the financial assets that enter the portfolio structure but also on the individual risk of each financial asset. Within this scientific paper was studied the minimum neutrosophic risk at the portfolio level, respectively, to establish what should be the weight that the financial assets must hold in the total value of the portfolio so that the risk is minimum. These financial assets weights, after calculations, were found to be dependent on the individual risk of each financial asset but also on the covariance between two financial assets that enter into the portfolio structure. The problem of the minimum risk that characterizes the neutrosophic portfolios is of interest for the financial market investors. Thus, the neutrosophic portfolios provide complete information about the probabilities of achieving the neutrosophic portfolio return but also of risk manifestation probability. In this context, the innovative character of the paper is determined by the use of the neutrosophic triangular fuzzy numbers and by the specific concepts of financial assets, in order to substantiating the decisions on the financial markets.

Keywords: financial assets; neutrosophicportfolio; neutrosophic portfolio return; neutrosophic portfolio risk; neutrosophic covariance

1. Introduction

The portfolios of financial assets have been the subject of numerous researches in the specialized literature, the main concern of the specialists being to identify a solution for the portfolio risk management, known being the fact that the capital market can generate huge losses if no solution is identified against the losses generated by the manifestation of the financial risk. In a first stage, to solve this sensitive problem, solutions were identified at the level of each financial asset, determining a set of three financial performance indicators that characterize the financial assets, namely: financial return, financial risk and covariance.

In order to quantify the financial asset's return, we took into consideration the profit realized by the investors, both from the price fluctuations of the financial assets and from the dividends obtained. The methods of calculating the financial return were different; starting from the return on financial assets based on time series recorded in previous time periods, to the returns determined based on estimates for future time periods. Subsequently, the foundations of another performance indicator of financial assets known as financial risk were made, determined with the help of the statistical indicators. These statistical indicators were—the square deviation from the mean and the variance that measures the deviation of the return of financial assets from its average value. The greater this deviation, the greater the risk associated with financial assets is.

In terms of covariance, the third indicator used for evaluating the performance of financial assets, measures the intensity of the link between the return of two financial assets; simultaneously being introduced also the correlation coefficient, which, depending on the recorded value, provides information on the return of the financial assets evolution. A positive correlation coefficient indicates that the return on financial assets increases or decreases as appropriate. A negative value of the correlation coefficient indicates that the evolutions of the return of the financial assets are of opposite sign, respectively while the return of one financial asset increases, the return of the other financial asset may decrease and vice versa.

Financial performance indicators have been a step forward in evaluating the performance of financial assets but not enough. To these was added the modern portfolio theory which lays the foundation of the correlation between profitability and risk at the level of the financial assets portfolio. The mathematical model for correlating the relationship between return and risk is known as the Markowitz efficient frontier, which essentially shows that the portfolio risk of financial assets increases in proportion to the value of the portfolio's return or, on the contrary, the portfolio risk decreases in proportion to the return value; between these two variables being a direct proportionality relation. Moreover, Markowitz's frontier theory demonstrates that risk management of a financial assets portfolio is much more efficient if capital market investments are made in a diversified portfolio of financial assets [1].

Despite all the progress made by introducing the relationship between return and risk but also by diversifying the risk making investments in diversified portfolios, not enough information is provided to investors in the capital market regarding the probability of achieving the return on financial assets or financial risk. This category of information is necessary to properly substantiate the investment decision on the capital market.

To solve this problem caused by the lack of information regarding the probabilities of achieving the financial performance indicators, fuzzy neutrosophic numbers were introduced to model the performance indicators of the financial assets. The use of neutrosophic fuzzy numbers in modelling the performance indicators of financial assets brings several advantages over the existing theory so far, as:

- Modelling the financial performance indicators taking into account the probabilities of their achievement;
- Clustering, respectively, modelling the value of financial performance indicators using the linguistic values that characterize the recorded values;
- Funding the investment decisions on the capital market by selecting value ranges and probabilities of achieving the financial performance indicators desired by investors;

In order to complete the modelling of financial performance indicators with the help of fuzzy neutrosophic numbers, the present paper bases two fundamental concepts in the portfolio theory literature, namely: it introduces a new category of portfolios, respectively the neutrosophic portfolios of financial assets and bases the algorithm for minimizing the risk of the neutrosophic portfolio of financial assets. Regarding the neutrosophic portfolios of financial assets, they will provide information on the risk and return of the portfolio, together with the probabilities of their achievement, with the mention that the probability of achievement is influenced by the risk and return of each financial asset that enters into the portfolio structure.

The risk minimization algorithm of the neutrosophic portfolio of financial assets provides solutions to the investor when it seeks to minimize the risk, respectively, sets the value of the investments that the investor will make in each of the financial assets that enter the portfolio structure, so that the risk is minimal.

The paper is organized as follows: Section 2 deals with the state of the art in the area of neutrosophic theory, while stating the main characteristics and assumptions related to the structure of the financial assets. Section 3 presents the neutrosophic portfolios concept, by highlighting some of the specific notions, structure and formation related to the neutrosophic portfolios theory. Two numerical examples are provided with Section 3 in order to better explain the introduced concepts. Section 4 deals with the neutrosophic portfolio equations. Both the analytical and matrix form are discussed within this section. Sections 5 and 6 deal with the minimizing the risk of the neutrosophic portfolio consisting of two or more financial assets, while Section 7 presents the limitations of the study and draws the main concluding remarks.

2. State of the Art

2.1. The Classical Theory of Financial Asset Portfolios. A New Approach

The structure of a portfolio is based on one or more financial assets $(A_1, A_2, A_3, \ldots A_n)$ or $\left(A_i, i = \overline{1, n}\right)$. Each of the financial assets that enter into the portfolio structure is characterized by an average financial return $\left(\overline{R}_{A_i}\right)$, a financial risk of the form $\left(\sigma_{A_i}\right)$ but also of the covariance $cov\left(A_i, A_j\right)$ between the asset $\left(A_i, i = \overline{1, n}\right)$ and $\left(A_j, j = \overline{1, m}\right)$. The covariance measures the intensity of the link between the returns of the two assets. Thus, for the modern portfolio theory, the financial asset (A_i) will have the characteristic performance indicators of $A_i : \begin{cases} \overline{R}_{A_i} \\ \sigma_{A_i} \\ x_{A_i} \end{cases}$, respectively the average financial return, the financial risk and x_{A_i}, which represents the weight of a financial asset (VA_i) in the total value of the portfolio $\left(\sum_{i=1}^{n} VA_i\right)$: $x_{A_i} = \frac{VA_i}{\sum_{i=1}^{n} VA_i} \times 100[\%]$.

The calculations regarding the performance indicators of financial assets are already known in the literature. The average return of a financial asset is determined either in the form of historical yields using the arithmetic mean $\overline{R}_{A_i} = \frac{1}{N}\sum_{i=1}^{n} R_{A_i}$ or using the geometric mean according to the formula: $\overline{R}_{A_i} = \left[\prod_{i=1}^{n}\left(1 + R_{A_i}\right)\right]^{\frac{1}{n}} - 1$, either in the form of expected returns using the probabilities assigned by investors (p_i) for each evolution scenario (S) of the financial asset expected return of the form: $\overline{R}_{A_i} = \sum_{i=1}^{S} p_i \times R_{A_i}$. If the variable represented by the return on the financial asset is continuous, a normal distribution can be used, with $\overline{R}_{A_i} = 0, \sigma_{A_i} = 1$ and the distribution density function of the form: $f\left(R_{A_i}, \overline{R}_{A_i}, \sigma_{A_i}\right) = \frac{1}{\sqrt{2\pi\sigma_{A_i}}} e^{-\frac{1}{2}\frac{\left(R_{A_i} - \overline{R}_{A_i}\right)^2}{\sigma_{A_i}{}^2}}$, for which the probability of occurrence the expected return on the financial asset A_i will be $P\left(-1.96 \leq \overline{R}_{A_i} \leq +1.96\right) = 95\%$ or $\left(-1.645 \leq \overline{R}_{A_i} \leq +1.645\right) = 90\%$. If the variable R_{A_i} has $\overline{R}_{A_i} \neq 0$ and $\sigma_{A_i} \neq 1$, then the expected average return $\overline{R}_{A_i}$ can be transformed into a variable of the form: $\overline{R}_{A_i}(z) = \frac{R_{A_i} - \overline{R}_{A_i}}{\sigma_{A_i}}$ for which $P\left(\overline{R}_{A_i} - 1.96\sigma_{A_i} \leq \overline{R}_{A_i}(z) \leq \overline{R}_{A_i} + 1.96\sigma_{A_i}\right) = 95\%$ and $P\left(\overline{R}_{A_i} - 1.645\sigma_{A_i} \leq \overline{R}_{A_i}(z) \leq \overline{R}_{A_i} + 1.645\sigma_{A_i}\right) = 90\%$.

The financial risk, as financial assets performance indicator studied in the specialized literature, is determined with the help of the squared deviations from the mean, by using the calculation formula $\sigma_{A_i}^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(R_{A_i} - \overline{R}_{A_i}\right)^2$, as well as using the statistical indicator known as variance using the calculation formula $\sigma_{A_i} = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}\left(R_{A_i} - \overline{R}_{A_i}\right)^2}$. Regardless how the financial risk is calculated, it measures the deviation of the financial asset return R_{A_i} from its average return $\overline{R}_{A_i}$. The greater the deviation, the greater the financial asset risk is, otherwise the smaller the deviation, the smaller the financial risk, between the magnitude of the deviation and the size of the financial risk being a directly proportional relationship.

The third statistical indicator used in the portfolio theory is the covariance between two assets (A_i) and respectively $\left(A_j\right)$, established using the calculation formula: $cov\left(A_i.A_j\right) = \frac{1}{N-1}\sum_{i,j=1}^{n}\left(R_{A_i} - \overline{R}_{A_i}\right)\left(R_{A_j} - \overline{R}_{A_j}\right)$, which, as mentioned above, measures the intensity of the connections, respectively the dependency or how two assets return mutual influence each other. The correlation coefficient was introduced in the portfolio literature, as: $\rho_{i,j} = \sigma_{A_iA_j}/\sigma_{A_i}\sigma_{A_j}$ with values between $\rho_{i,j} = [-1,+1]$. If $\rho_{i,j} = -1$, the returns of the two financial assets evolve in the opposite direction, respectively when one increases the other decreases and vice versa. If $\rho_{i,j} = 0$, the returns of the two financial assets do not influence each other. If $\rho_{i,j} = +1$, the returns of the two financial assets increase or, as the case may be, they decrease simultaneously.

As mentioned previously, the performance indicators presented above, respectively the average return of the financial asset $\left(\overline{R}_{A_i}\right)$, the financial risk $\left(\sigma_{A_i}\right)$ and the covariance between two financial assets $cov\left(A_i.A_j\right)$ are specific to the financial assets which are part of a portfolio structure.

The modern theory of the financial asset's portfolio has devoted notions specific to the portfolio such as the portfolio return $\left(R_p\right)$ and the portfolio risk $\left(\sigma_P^2\right)$, in order to mathematically quantify the relationship between return and risk. The portfolio return $\left(R_p\right)$ determined by the existence of N financial assets in the portfolio is mathematically quantified as the sum of the products between the weight $\left(x_{A_i}\right)$ of each asset (A_i) in the total value of the portfolio and the average return specific to each asset $\left(\overline{R}_{A_i}\right)$, of form:

$$R_p = x_{A_1}R_{A_1} + x_{A_2}R_{A_2} + \cdots + x_{A_n}R_{A_n} = \sum_{i=1}^{N} x_{A_i}R_{A_i} \tag{1}$$

The above expression can be written in matrix form as follows:

$$R_p = \left(x_{A_1}x_{A_2}\ldots x_{A_n}\right)\begin{pmatrix} R_{A_1} \\ R_{A_2} \\ \cdots \\ R_{A_n} \end{pmatrix} = x_A^T R_A \tag{2}$$

The portfolio risk $\left(\sigma_P^2\right)$ also made up of N financial assets is determined by squared deviations from the mean and is influenced by the weight held by each financial asset in the total portfolio $\left(x_{A_i}\right)$, as well as by the individual risk of each asset entering the portfolio structure $\left(\sigma_{A_i}^2\right)$, respectively the covariance between two assets $cov\left(A_i.A_j\right)$, according to an expression of the form:

$$\begin{aligned} \sigma_{\mathrm{P}}^2 = \mathrm{x}_{\mathrm{A}_1}^2\sigma_{\mathrm{A}_1}^2 + \mathrm{x}_{\mathrm{A}_2}^2\sigma_{\mathrm{A}_2}^2 + \cdots + \mathrm{x}_{\mathrm{A}_n}^2\sigma_{\mathrm{A}_n}^2 + 2\mathrm{x}_{\mathrm{A}_1}\mathrm{x}_{\mathrm{A}_2}\sigma_{\mathrm{A}_1\mathrm{A}_2} + 2\mathrm{x}_{\mathrm{A}_1}\mathrm{x}_{\mathrm{A}_3}\sigma_{\mathrm{A}_1\mathrm{A}_3} + \cdots \\ +2\mathrm{x}_{\mathrm{A}_1}\mathrm{x}_{\mathrm{A}_n}\sigma_{\mathrm{A}_1\mathrm{A}_n} + 2\mathrm{x}_{\mathrm{A}_2}\mathrm{x}_{\mathrm{A}_1}\sigma_{\mathrm{A}_2\mathrm{A}_1} + 2\mathrm{x}_{\mathrm{A}_2}\mathrm{x}_{\mathrm{A}_3}\sigma_{\mathrm{A}_2\mathrm{A}_3} + \cdots \\ +2\mathrm{x}_{\mathrm{A}_2}\mathrm{x}_{\mathrm{A}_n}\sigma_{\mathrm{A}_2\mathrm{A}_n} + 2\mathrm{x}_{\mathrm{A}_n}\mathrm{x}_{\mathrm{A}_1}\sigma_{\mathrm{A}_n\mathrm{A}_1} + 2\mathrm{x}_{\mathrm{A}_n}\mathrm{x}_{\mathrm{A}_2}\sigma_{\mathrm{A}_n\mathrm{A}_2} + \cdots \\ +2\mathrm{x}_{\mathrm{A}_n}\mathrm{x}_{\mathrm{A}_{n-1}}\sigma_{\mathrm{A}_n\mathrm{A}_{n-1}} \end{aligned} \tag{3}$$

$$\sigma_P^2 = \sum_{i=1}^{n} x_{A_i}^2\sigma_{A_i}^2 + 2\sum_{i=1}^{n}\sum_{j=1}^{n} x_{A_i}x_{A_j}\sigma_{A_iA_j} \tag{4}$$

The portfolio risk in matrix form can be written as:

$$\sigma_P^2 = (x_{A_1} x_{A_2} \ldots x_{A_n}) \begin{pmatrix} \sigma_{A_1A_1} & \sigma_{A_1A_2} & \cdots & \sigma_{A_1A_n} \\ \sigma_{A_2A_1} & \sigma_{A_2A_2} & \cdots & \sigma_{A_2A_n} \\ \cdots & \cdots & \cdots & \cdots \\ \sigma_{A_nA_1} & \sigma_{A_nA_2} & \cdots & \sigma_{A_nA_n} \end{pmatrix} \begin{pmatrix} x_{A_1} \\ x_{A_2} \\ \cdots \\ x_{A_n} \end{pmatrix} = x_A^T \Omega x_A \tag{5}$$

In the specialized literature, starting with the modern portfolio theory, the relationship between the portfolio return and portfolio risk was established and also the concept of an optimal portfolio has been stipulated. According to this theory, financial asset portfolios are considered optimal if the portfolio return $R_p = \rho$, in which ρ has a fixed level, while the portfolio risk $\sigma_P^2 \to min$. The equations of an optimal portfolio will be of the form:

$$\begin{cases} R_p = \sum_{i=1}^{N} x_{A_i} R_{A_i} = \rho \\ \sigma_P^2 = \sum_{i=1}^{n} x_{A_i}^2 \sigma_{A_i}^2 + 2\sum_{i=1}^{n} \sum_{j=1}^{n} x_{A_i} x_{A_j} \sigma_{A_i A_j} \to min \\ \sum_{i=1}^{n} x_{A_i} = 1 \end{cases} \tag{6}$$

The mathematical model for quantifying the relationship between the portfolio return $\left(R_p\right)$ and its risk $\left(\sigma_P^2\right)$ is known as Markowitz's frontier and has the following form:

$$\sigma_P^2 = \frac{1}{D}\left(AR_P^2 - 2BR_p + C\right) \tag{7}$$

where the coefficients A, B, C, D have the following calculation formulas (results from the literature): $A = e\Omega^{-1}e$; $B = e\Omega^{-1}R_A^T = e^T\Omega^{-1}R_A$; $C = R_A\Omega^{-1}R_A^T$. From the Markowitz's frontier relation, it emerged that there is a direct proportionality relation between risk and return, respectively, the higher the portfolio's return, the higher the risk. All the investment portfolios located on the Markowitz's frontier (the upper branch of hyperbole) are considered to be efficient portfolios. Any portfolio located for example below the Markowitz's frontier will have an equivalent portfolio located on the frontier which will have the same risk and a higher return.

Regardless the popularity of the portfolio theory and how advanced the research in the field of capital markets is, any portfolio, regardless of the number of financial assets, has a certain degree of certainty/uncertainty $(G_r(R_P))$ to realize the portfolio return and to produce the risk. This degree of portfolio returns and risk is divided into three categories:

The first category: certain degree for the portfolio return and risk $\mu(G_r(\sigma_P, R_P))$, corresponding to that situation where the portfolio return and risk have an achievement degree, estimated using professional judgment, around the value of 50%. Each portfolio constitutes a specific degree of achievement for return and risk of each portfolio.

The second category: very poor or almost null degree for the portfolio return and risk $\vartheta(G_r(\sigma_P, R_P))$, corresponding to that situation where the return and risk of a portfolio have an estimated degree of achievement of 10–20%. The causes that can lead to such situations are numerous: the assumption of a certain level of return and risk by investors, the poor ability to pay financial assets, the negative influence of national macroeconomic factors.

Third category: uncertain degree for the portfolio return and risk, noted as $\lambda(G_r(\sigma_P, R_P))$ representing the situation where the degree of return and risk is quite uncertain, estimated based on professional reasoning at 20–30%.

The introduction of these measuring degrees for the financial asset portfolios return and risk allows the creation of neutrosophic portfolios, modelled using triangular fuzzy numbers. These portfolios meet the real needs of investors on the financial market. Thus, if a portfolio will have a high degree of return and risk, the investors will have a degree of certainty that they will obtain the expected returns from the financial market. It is worth mentioning that each financial asset that constitutes the portfolio has in turn a certain return and a specific risk which will determine a certain influence on the portfolio return and risk.

The introduction of these ways of measuring the degree of portfolio return and the degree of producing the portfolio risk creates the basis for the formation of the neutrosophic portfolios of financial assets, as mentioned, modelled using the neutrosophic triangular fuzzy numbers. Neutrosophic portfolios have as performance indicators the neutrosophic portfolio return but also the neutrosophic portfolio risk.

2.2. Literature Review

Regarding the studies in the area of neutrosophic theory, it can be underline the fact that the neutrosophic theory and its derivates has been extensively applied in the last two decades various economic and social fields such as—decision making [2–13], supply chain management [14], best product selection [15], management [16] forecasting [17], sentiment analysis [18,19] and so forth.

As for the portfolio theory, there are only few studies who have tried to use the advantages of the neutrosophic theory. Islam and Ray [20] propose a multi-objective portfolio selection which is used through a neutrosophic optimization technique. The authors introduce a new objective function based on entropy and generalize the portfolio selection problem with diversification (GPSPD), stating that, as the proposed method is general, it can easily be applied to other areas in the engineering sciences or operations research. Pamucar et al. [21] propose a multicriteria decision making model in which the authors have considered the linguistic neutrosophic numbers for the purpose of eliminating the subjectivity which derives from the qualitative assessment and the assumptions made by the decision-making in complex situations.

The problem of project selection has been addressed through the use of the neutrosophic set theory by Abdel-Basset et al. [22]. The authors state in the paper the importance of a proper identification of the important criteria based on which the project selection had to be done and propose a model base on TOPSIS and DEMATEL for selection of the best project alternative. Villegas Alava et al. [23] used the single value neutrosophic numbers for project selection. In the paper, the authors present a case study for information technology project selection for proving the applicability of the proposed approach.

In the area of project management, Saleh Al-Subhi et al. [24] use the neutrosophic sets and propose a new decision making model based on neutrosophic cognitive maps and compare the proposed approach with a traditional model in order to prove its efficiency and efficacy. Perez Pupo et al. [25] use the neutrosophic theory for project management decisions, while Su et al. [26] develop a project procurement method selection model under an interval neutrosophic environment. The results gathered in the papers are compared with exiting methods and the results are encouraging.

The project risk assessment in the area of construction engineering is addressed in Reference [27] through the use of 2-tuple linguistic neutrosophic hamy mean operators. The authors provide both the theoretical background and an applicable example for better explain the proposed approach.

Regarding the identified problem within this paper, the modern portfolio theory currently quantifies with the help of Markowitz model the relationship between return and risk. The main disadvantage of this financial asset portfolio theory is that it does not provide sufficient information to investors regarding the probability of realizing the return and of producing the portfolio risk. In addition, the risk and return of the portfolio is influenced by the risk and return of each financial asset that makes up the portfolio. Under these conditions, the substantiation of the investment decision on the financial market is not based on complete information that would also include the probability of achieving the portfolio return and risk and could have as an impact a risk decrease assumed by investors.

The proposed solution in the present research paper is to use the neutrosophic triangular fuzzy numbers, that use the aforementioned categories of information, regarding the degree of achieving the return and producing the portfolio risk. At the same time, the neutrosophic triangular fuzzy numbers allow the stratification of the values recorded by each financial asset for the return and risk specific to each asset. The information resulting from neutrosophic fuzzy modelling has a much more detailed character and allows the financial market investors to more rigorously base their financial decisions. In addition, as a way of solving the problem, are proposed the concepts of neutrosophic return, neutrosophic risk and neutrosophic covariance specific to financial assets.

The innovative character of the paper is determined by the use of the neutrosophic triangular fuzzy numbers but also by the specific concepts of financial assets, namely: neutrosophic return, neutrosophic risk and/or neutrosophic covariance. The information for substantiating the decisions on the financial market is based on neutrosophic fuzzy modelling as a way to improve the decision-making process on the market.

3. The Neutrosophic Portfolios Concept. Specific Notions, Structure and Formation

The theory of neutrosophic fuzzy numbers of the form: $\widetilde{A} = \{\langle\widetilde{a}, \mu_{\widetilde{a}}, \vartheta_{\widetilde{a}}, \lambda_{\widetilde{a}}\rangle / a \in A\}$ has the characteristic that besides to the specific membership functions related to the fuzzy numbers of the form: $\mu_{\widetilde{a}} : \widetilde{A} \to [0,1]$; $\vartheta_{\widetilde{a}} : \widetilde{A} \to [0,1]$ and $\lambda_{\widetilde{a}} : \widetilde{A} \to [0,1]$, also contain the achievement degree of fuzzy numbers of the form: $(w\widetilde{A}, u\widetilde{A}, y\widetilde{A})$, with the following meanings: $w\widetilde{A}$-certainty degree for the achievement of the fuzzy number, $u\widetilde{A}$-indeterminacy degree for the achievement of the fuzzy number and $y\widetilde{A}$-falsity degree for the achievement of the fuzzy number. The membership functions for the neutrosophic fuzzy numbers of the form $\widetilde{A} = \{\langle\widetilde{a}, \mu_{\widetilde{a}}, \vartheta_{\widetilde{a}}, \lambda_{\widetilde{a}}\rangle / a \in A\}$ are determined according to their achievement degrees [28]:

The membership function for the neutrosophic numbers with truth value, the truth membership $(\mu\widetilde{A}_{(x)})$ is of the form [28]:

$$\mu\widetilde{A}_{(x)} = \begin{cases} \frac{w_{\widetilde{A}}(\widetilde{A}_x - \widetilde{A}_{a1})}{\widetilde{A}_{b1} - \widetilde{A}_{a1}} \; for \; \widetilde{A}_{a1} \le \widetilde{A}_x \le \widetilde{A}_{b1} \\ w_{\widetilde{A}} \; for \; \widetilde{A}_x = \widetilde{A}_{b1} \\ \frac{w_{\widetilde{A}}(\widetilde{A}_{c1} - \widetilde{A}_x)}{\widetilde{A}_{c1} - \widetilde{A}_{b1}} \; for \; \widetilde{A}_{b1} \le \widetilde{A}_x \le \widetilde{A}_{c1} \\ 0, \; for \; any \; other \; value \; out \; of \; range \left[\widetilde{A}_{c1}; \widetilde{A}_{a1}\right] \end{cases} \quad (8)$$

The membership function for the neutrosophic numbers with uncertain achievement degree, the indeterminacy membership $(\vartheta\widetilde{A}_{(x)})$ is of the form [28]:

$$\vartheta\widetilde{A}_{(x)} = \begin{cases} \frac{u_{\widetilde{A}}(\widetilde{A}_x - \widetilde{A}_{a1}) + \widetilde{A}_{b1} - \widetilde{A}_x}{\check{A}_{b1} - \widetilde{A}_{a1}} \; for \; \widetilde{A}_{a1} \le \widetilde{A}_x \le \widetilde{A}_{b1} \\ u_{Ra} \; for \; \widetilde{A}_x = \widetilde{A}_{b1} \\ \frac{u_{\widetilde{A}}(\widetilde{A}_{c1} - \widetilde{A}_x) + \widetilde{A}_x - \widetilde{A}_{b1}}{\widetilde{A}_{c1} - \widetilde{A}_{b1}} \; for \; \widetilde{A}_{b1} \le \widetilde{A}_x \le \widetilde{A}_{c1} \\ 0, \; for \; any \; other \; value \; out \; of \; range \left[\widetilde{A}_{c1}; \widetilde{A}_{a1}\right] \end{cases} \quad (9)$$

The membership function for the neutrosophic numbers with false achievement degree, the falsity membership $(\lambda\widetilde{Ra}_{(x)})$ is of the form [28]:

$$\lambda\widetilde{A}_{(x)} = \begin{cases} \frac{y_{\widetilde{A}}(\widetilde{A}_x - \widetilde{A}_{a1}) + \widetilde{A}_{b1} - \widetilde{A}_x}{\widetilde{A}_{b1} - \widetilde{A}_{a1}} \; for \; \widetilde{A}_{a1} \le \widetilde{A}_x \le \widetilde{A}_{b1} \\ \lambda_{\widetilde{A}} \; for \; \widetilde{A}_x = \widetilde{A}_{b1} \\ \frac{y_{\widetilde{A}}(\widetilde{A}_{c1} - \widetilde{A}_x) + \widetilde{A}_x - \widetilde{A}_{b1}}{\widetilde{A}_{c1} - \widetilde{A}_{b1}} \; for \; \widetilde{A}_{b1} \le \widetilde{A}_x \le \widetilde{A}_{c1} \\ 0, \; for \; any \; other \; value \; out \; of \; range \left[\widetilde{A}_{c1}; \widetilde{A}_{a1}\right] \end{cases} \quad (10)$$

The neutrosophic fuzzy number theory, helps to obtain complete information about fuzzy numbers, by taking into account the achievement degrees, namely: the degree of truth, uncertainty (indeterminacy) degree or falsity degree, that are extremely useful in substantiating decisions on the capital market.

Neutrosophic portfolios can consist of two or more financial assets. Let N be the number of financial assets denoted by $(A_1, A_2, A_3, \ldots A_n)$ or $\left(A_i, i = \overline{1,n}\right)$. Their characteristic is that the financial asset performance indicators, noted with A_i that enter into a neutrosophic portfolio structure are:

The neutrosophic return: $\widetilde{R_{Ai}} = \langle(\widetilde{R_{Aai}}, \widetilde{R_{Abi}}, \widetilde{R_{Aci}}); w\widetilde{R_A}, u\widetilde{R_A}, y\widetilde{R_A}\rangle$;

The neutrosophic risk: $\widetilde{\sigma}_{Ai} = \langle(\widetilde{\sigma}_{Aai}, \widetilde{\sigma}_{Abi}, \widetilde{\sigma}_{Aci}); w\widetilde{\sigma}_A, u\widetilde{\sigma}_A, y\widetilde{\sigma}_A\rangle$;

The neutrosophic covariance: $cov(\widetilde{R_{A1}}, \widetilde{R_{A2}})$;

The neutrosophic triangular fuzzy numbers that underlie the financial assets performance indicators, of the form: $\widetilde{A} = \{\langle\widetilde{a}, \mu_{\widetilde{a}}, \vartheta_{\widetilde{a}}, \ \lambda_{\widetilde{a}}\rangle / a \in A\}$, were defined in Bolos et al. [28] and are characterized by the membership functions of the form $\mu_{\widetilde{a}} : A \to [0,1]$; $\vartheta_{\widetilde{a}} : A \to [0,1]$ and $\lambda_{\widetilde{a}} : A \to [0,1]$ and by the achievement degree of the performance indicators, of the form: $(w\widetilde{A}, u\widetilde{A}, y\widetilde{A})$, with the following meanings: $w\widetilde{A}$-certain achievement degree for the performance indicators, $u\widetilde{A}$-indeterminate achievement degree for the performance indicators and $y\widetilde{A}$-falsity achievement degree for the performance indicators.

Definition 1. *Is defined the neutrosophic average return* $\langle E_f(\widetilde{R_{Ai}}); w\widetilde{R_A}, u\widetilde{R_A}, y\widetilde{R_A}\rangle$ *for the neutrosophic triangular fuzzy number* $\widetilde{R_{Ai}} = \langle(\widetilde{R_{Aai}}, \widetilde{R_{Abi}}, \widetilde{R_{Aci}}); w\widetilde{R_A}, u\widetilde{R_A}, y\widetilde{R_A}\rangle$, *specific for the financial asset* (A_i) *and component part of the neutrosophic portfolio* $(\widetilde{P}; w\widetilde{P}, u\widetilde{P}, y\widetilde{P},)$ *any value of the financial asset return appreciated after the achievement degree, using the following coefficients:* $w\widetilde{R_A} \in [0,1]$ *for certain achievement degree,* $u\widetilde{R_A} \in [0,1]$ *for indeterminate achievement degree and* $y\widetilde{R_A} \in [0,1]$ *for falsity achievement degree; determined by the calculation formula:*

$$\langle E_f(\widetilde{R_A}); w\widetilde{R_A}, u\widetilde{R_A}, y\widetilde{R_A}\rangle = \langle\left(\frac{1}{6}(\widetilde{R_{Aa1}} + \widetilde{R_{Ac1}}) + \frac{2}{3}\widetilde{R_{Ab1}}\right); w\widetilde{R_A}, u\widetilde{R_A}, y\widetilde{R_A}\rangle \tag{11}$$

Note 1: The formula for neutrosophic average return was demonstrated in Boloș et.al. [28].

Definition 2. *Is defined the neutrosophic risk* $\langle\sigma f^2_{A_i}; w\widetilde{\sigma}_A, u\widetilde{\sigma}_A, y\widetilde{\sigma}_A\rangle$ *for the neutrosophic triangular fuzzy number* $\widetilde{\sigma}_{Ai} = \langle(\widetilde{\sigma}_{Aai}, \widetilde{\sigma}_{Abi}, \widetilde{\sigma}_{Aci}); w\widetilde{\sigma}_A, u\widetilde{\sigma}_A, y\widetilde{\sigma}_A\rangle$ *determined for the financial asset* (A_i) *and component part of the neutrosophic portfolio* $(\widetilde{P}; w\widetilde{P}, u\widetilde{P}, y\widetilde{P},)$ *any value of the financial asset risk appreciated after the achievement degree, using the following coefficients:* $w\widetilde{\sigma}_A \in [0,1]$ *for certain achievement degree,* $u\widetilde{\sigma}_A \in [0,1]$ *for indeterminate achievement degree and* $y\widetilde{\sigma}_A \in [0,1]$ *for falsity achievement degree; determined by the calculation formula:*

$$\begin{aligned}\langle\sigma f^2_{A_i}; w\widetilde{\sigma}_A, u\widetilde{\sigma}_A, y\widetilde{\sigma}_A\rangle &= \langle\tfrac{1}{4}\left[(\widetilde{R_{Ab1}} - \widetilde{R_{Aa1}})^2 + (\widetilde{R_{Ac1}} - \widetilde{R_{Ab1}})^2\right]; w\widetilde{R_A}, u\widetilde{R_A}, y\widetilde{R_A}\rangle \\ &+ \langle\tfrac{2}{3}\left[\widetilde{R_{Aa1}}(\widetilde{R_{Ab1}} - \widetilde{R_{Aa1}}) - \widetilde{R_{Ac1}}(\widetilde{R_{Ac1}} - \widetilde{R_{Ab1}})\right]; w\widetilde{R_A}, u\widetilde{R_A}, y\widetilde{R_A}\rangle \\ &+ \langle\tfrac{1}{2}(\widetilde{R_{Aa1}}^2 + \widetilde{R_{Ac1}}^2); w\widetilde{R_A}, u\widetilde{R_A}, y\widetilde{R_A}\rangle - \langle\tfrac{1}{2}E^2_f(\widetilde{Ra_i}); w\widetilde{R_A}, u\widetilde{R_A}, y\widetilde{R_A}\rangle\end{aligned} \tag{12}$$

Note 2: The formula for neutrosophic risk was demonstrated in Boloș et al. [28].

Definition 3. *Is defined the neutrosophic covariance* $\langle cov(\widetilde{R_{A1}}, \widetilde{R_{A2}}); w\widetilde{R_{A1}}, u\widetilde{R_{A1}}, y\widetilde{R_{A1}};\ w\widetilde{R_{A2}}, u\widetilde{R_{A2}}, y\widetilde{R_{A2}}\rangle$ *for two neutrosophic triangular fuzzy numbers* $\widetilde{R_{A1}} = \langle(\widetilde{R_{Aa1}}, \widetilde{R_{Ab1}}, \widetilde{R_{Ac1}}); w\widetilde{R_{A1}}, u\widetilde{R_{A1}}, y\widetilde{R_{A1}}\rangle$ *and respectively* $\widetilde{R_{A2}} = \langle(\widetilde{R_{Aa2}}, \widetilde{R_{Ab2}}, \widetilde{R_{Ac2}}); w\widetilde{R_{A2}}, u\widetilde{R_{A2}}, y\widetilde{R_{A2}}\rangle$ *characterizing two financial assets* (A_1, A_2) *and component parts of the neutrosophic portfolio* $(\widetilde{P}; w\widetilde{P}, u\widetilde{P}, y\widetilde{P},)$ *any value of the financial asset covariance appreciated after the achievement degree, using the following coefficients: for certain achievement degree,* $u\widetilde{R_{A1}}, u\widetilde{R_{A2}} \in [0,1]$ *for indeterminate achievement degree and for falsity achievement degree; determined by the calculation formula:*

$$
\begin{aligned}
&\langle cov(\widetilde{R_{A1}},\widetilde{R_{A2}});w\widetilde{R_{A1}},u\widetilde{R_{A1}},y\widetilde{R_{A1}};\ w\widetilde{R_{A2}},u\widetilde{R_{A2}},y\widetilde{R_{A2}}\rangle = \\
&\langle\Big(\tfrac{1}{4}\Big[(\widetilde{R_{Ab11}}-\widetilde{R_{Aa11}})(\widetilde{R_{Ab21}}-\widetilde{R_{Aa21}})+(\widetilde{R_{Ac11}}-\widetilde{R_{Ab11}})(\widetilde{R_{Ac21}}-\widetilde{R_{Ab21}})\Big] \\
&\quad+\tfrac{1}{3}\Big\{\Big[\widetilde{R_{Aa21}}(\widetilde{R_{Ab11}}-\widetilde{R_{Aa11}})+\widetilde{R_{Aa11}}(\widetilde{R_{Ab21}}-\widetilde{R_{Aa21}})\Big] \\
&\quad-\Big[\widetilde{R_{Ac11}}(\widetilde{R_{Ac21}}-\widetilde{R_{Ab21}})+\widetilde{R_{Ac21}}(\widetilde{R_{Ac11}}-\widetilde{R_{Ab11}})\Big]\Big\} \\
&\quad+\tfrac{1}{2}(\widetilde{R_{Aa11}}\widetilde{R_{Aa21}}+\widetilde{R_{Ac11}}\widetilde{R_{Ac21}}) \\
&\quad+\tfrac{1}{2}E_f(\widetilde{R_{A1}})E_f(\widetilde{R_{A2}})\Big);w\widetilde{R}_{A_1}\wedge\ w\widetilde{R_{A2}},u\widetilde{R_{A1}}\vee u\widetilde{R_{A2}},y\widetilde{R_{A1}}\vee y\widetilde{R_{A2}}\rangle
\end{aligned}
\tag{13}
$$

Note 3: The formula for neutrosophic covariance was demonstrated in Boloș et al. [28]. Upon these demonstrations we will no longer return.

Definition 4. *Any portfolio P is called a neutrosophic portfolio of financial assets and is denoted $\langle\widetilde{P};w\widetilde{P},u\widetilde{P},y\widetilde{P}\rangle$ if it cumulativelysatisfies two conditions:*

- contains in its structure financial assets marked with $(A_i); i = \overline{2,n}$ which have as performance indicators: the neutrosophic return $\langle E_f(\widetilde{R_{Ai}});w\widetilde{R_A},u\widetilde{R_A},y\widetilde{R_A}\rangle$, the neutrosophic risk $\langle\sigma f^2_{A_i};w\widetilde{\sigma}_A,u\widetilde{\sigma}_A,y\widetilde{\sigma}_A\rangle$ and the neutrosophic covariance that characterizes the intensity of the links between the neutrosophic returns of two financial assets $\langle cov(\widetilde{R_{A1}},\widetilde{R_{A2}});w\widetilde{R_{A1}},u\widetilde{R_{A1}},y\widetilde{R_{A1}};\ w\widetilde{R_{A2}},u\widetilde{R_{A2}},y\widetilde{R_{A2}}\rangle$;
- allows to calculate the return of the neutrosophic portfolio $\langle\widetilde{R}_P;w\widetilde{Rp},u\widetilde{Rp},y\widetilde{Rp}\rangle$ and the neutrosophic portfolio risk $\langle\widetilde{\sigma^2_P};w\widetilde{\sigma p},u\widetilde{\sigma p},y\widetilde{\sigma p}\rangle$ as fundamental variables that characterize any neutrosophic portfolio $\langle\widetilde{P};w\widetilde{P},u\widetilde{P},y\widetilde{P}\rangle$.

Proposition 1. *The neutrosophic portfolio return $\langle\widetilde{R}_P;w\widetilde{Rp},u\widetilde{Rp},y\widetilde{Rp}\rangle$ modeled using neutrosophic triangular fuzzy numbers of the form: $\widetilde{R_{Ai}}=\langle(\widetilde{R_{Aai}},\widetilde{R_{Abi}},\widetilde{R_{Aci}});w\widetilde{R_A},u\widetilde{R_A},y\widetilde{R_A}\rangle$* is a fundamental variable that characterizes the neutrosophic portfolio and is determined by the formula:

$$\langle\widetilde{R}_P;w\widetilde{Rp},u\widetilde{Rp},y\widetilde{Rp}\rangle=\sum_{i=1}^{n}\langle x_{A_i}\Big(\frac{1}{6}\Big(\widetilde{R}_{Aai}+\widetilde{R}_{Aci}\Big)+\frac{2}{3}\widetilde{R}_{Abi}\Big);w\widetilde{R}_{A_i},u\widetilde{R}_{A_i},y\widetilde{R}_{A_i}\rangle \tag{14}$$

Demonstration: From the calculation relation of the neutrosophic portfolio return made up of N financial assets we know that:

$$
\begin{aligned}
&\langle\widetilde{R}_P;w\widetilde{Rp},u\widetilde{Rp},y\widetilde{Rp}\rangle \\
&\quad=\langle x_{A_1}\widetilde{R}_{A_1};w\widetilde{R}_{A_1},u\widetilde{R}_{A_1},y\widetilde{R}_{A_1}\rangle+\langle x_{A_2}\widetilde{R}_{A_2};w\widetilde{R}_{A_2},u\widetilde{R}_{A_2},y\widetilde{R}_{A_2}\rangle+\cdots \\
&\quad+\langle x_{A_n}\widetilde{R}_{A_n};w\widetilde{R}_{A_n},u\widetilde{R}_{A_n},y\widetilde{R}_{A_n}\rangle
\end{aligned}
\tag{15}
$$

The above relationship can be written as follows:

$$\langle\widetilde{R}_P;w\widetilde{Rp},u\widetilde{Rp},y\widetilde{Rp}\rangle=\sum_{i=1}^{n}\langle x_{A_i}\widetilde{R}_{A_i};w\widetilde{R}_{A_i},u\widetilde{R}_{A_i},y\widetilde{R}_{A_i}\rangle \tag{16}$$

From the definition no.1 we know that the average neutrosophic return specific to a financial asset is of the form:

$$\langle E_f(\widetilde{R_{Ai}});w\widetilde{R_A},u\widetilde{R_A},y\widetilde{R_A}\rangle=\langle\Big(\frac{1}{6}(\widetilde{R_{Aa1}}+\widetilde{R_{Ac1}})+\frac{2}{3}\widetilde{R_{Ab1}}\Big);w\widetilde{R_A},u\widetilde{R_A},y\widetilde{R_A}\rangle \tag{17}$$

Substituting the expression of the average neutrosophic return of a financial asset in the calculation formula of the neutrosophic portfolio return is obtained:

$$\langle\widetilde{R}_P;w\widetilde{Rp},u\widetilde{Rp},y\widetilde{Rp}\rangle=\sum_{i=1}^{n}\langle x_{A_i}\Big(\frac{1}{6}(\widetilde{R}_{Aai}+\widetilde{R}_{Aci})+\frac{2}{3}\widetilde{R}_{Abi}\Big);w\widetilde{R}_{A_i},u\widetilde{R}_{A_i},y\widetilde{R}_{A_i}\rangle \tag{18}$$

where $\widetilde{R}_{Aai}$; $\widetilde{R}_{Abi}$; $\widetilde{R}_{Aci}$ represents the financial asset return values, component part of the neutrosophic triangular fuzzy number determined according to the calculation relationships known in the specialized literature.

Example 1. *There are considered three financial assets (A_1, A_2, A_3) to which three triangular neutrosophic numbers are specified for the financial assets return, of the form:*

$$\begin{aligned} \widetilde{R}_{A1} &= \langle (0.2\ 0.3\ 0.5); 0.5,\ 0.2,\ 0.3 \rangle \text{for} \widetilde{R}_A \in \\ \widetilde{R}_{A2} &= \langle (0.1\ 0.2\ 0.3); 0.6,\ 0.3,\ 0.2 \rangle for \widetilde{R}_A \in [0,1; 0,3] \\ \widetilde{R}_{A3} &= \langle (0.3\ 0.4\ 0.6); 0.4,\ 0.3,\ 0.3 \rangle \text{for} \widetilde{R}_A \in [0,3; 0,6] \end{aligned} \tag{19}$$

The weights held by the three financial assets in the total portfolio are determined according to the value of each financial asset and the total value of the portfolio and have the values: $x_{A_1} = 0,4$; $x_{A_2} = 0,3$ și $x_{A_3} = 0,3$. In order to establish the neutrosophic portfolio return, from proposition1 it is known that:

$$\langle \widetilde{R}_P; w\widetilde{Rp}, u\widetilde{Rp}, y\widetilde{Rp} \rangle = \sum_{i=1}^{n} \langle x_{A_i} \Big(\frac{1}{6} \Big(\widetilde{R}_{Aai} + \widetilde{R}_{Aci} \Big) + \frac{2}{3} \widetilde{R}_{Abi} \Big); w\widetilde{R}_{A_i}, u\widetilde{R}_{A_i}, y\widetilde{R}_{A_i} \rangle \tag{20}$$

By replacing in the above expression is obtained:

$$\begin{aligned} &\langle \widetilde{R}_P; w\widetilde{Rp}, u\widetilde{Rp}, y\widetilde{Rp} \rangle \\ &\quad = \langle 0,4 \big(\tfrac{1}{6}(0.2 + 0.5) + \tfrac{2}{3} \times 0.3 \big); 0.5,\ 0.2,\ 0.3 \rangle \\ &\quad + \langle 0,3 \big(\tfrac{1}{6}(0.1 + 0.3) + \tfrac{2}{3} \times 0.2 \big); 0.6,\ 0.3,\ 0.2 \rangle \\ &\quad + \langle 0,3 \big(\tfrac{1}{6}(0.3 + 0.6) + \tfrac{2}{3} \times 0.4 \big); 0.4,\ 0.3,\ 0.3 \rangle \end{aligned} \tag{21}$$

$$\begin{aligned} &\langle \widetilde{R}_P; w\widetilde{Rp}, u\widetilde{Rp}, y\widetilde{Rp} \rangle \\ &\quad = \langle 0,4 \big(\tfrac{1}{6} 0.7 + \tfrac{2}{3} 0.3 \big); 0.5,\ 0.2,\ 0.3 \rangle \\ &\quad + \langle 0,3 \big(\tfrac{1}{6} 0.4 + \tfrac{2}{3} 0.2 \big); 0.6,\ 0.3,\ 0.2 \rangle \\ &\quad + \langle 0,3 \big(\tfrac{1}{6} 0.9 + \tfrac{2}{3} 0.4 \big); 0.4,\ 0.3,\ 0.3 \rangle \end{aligned} \tag{22}$$

$$\begin{aligned} &\langle \widetilde{R}_P; w\widetilde{Rp}, u\widetilde{Rp}, y\widetilde{Rp} \rangle \\ &\quad = \langle 0.4 \times 0.316; 0.5,\ 0.2,\ 0.3 \rangle + \langle 0.3 \times 0.199; 0.5,\ 0.2,\ 0.3 \rangle \\ &\quad + \langle 0.3 \times 0.416; 0.4,\ 0.3,\ 0.3 \rangle \end{aligned} \tag{23}$$

$$\begin{aligned} &\langle \widetilde{R}_P; w\widetilde{Rp}, u\widetilde{Rp}, y\widetilde{Rp} \rangle \\ &= \langle 0.1264; 0.5,\ 0.2,\ 0.3 \rangle + \langle 0.0597;\ 0.5,\ 0.2,\ 0.3 \rangle \\ &+ \langle 0.1248; 0.4,\ 0.3,\ 0.3 \rangle \\ &\quad \langle \widetilde{R}_P; w\widetilde{Rp}, u\widetilde{Rp}, y\widetilde{Rp} \rangle = \langle 0.3109; 0.5,\ 0.2,\ 0.3 \rangle \end{aligned} \tag{24}$$

Result interpretation: The average neutrosophic portfolio return has a value of 31.09% with a degree of certainty of 50%, a degree of uncertainty of 20% and a degree of falsification of 30%. In order to obtain the neutrosophic portfolio return, the addition rule for two triangular neutrosophic numbers was applied according to which:

$$\widetilde{R}_{A1} + \widetilde{R}_{A2} = \langle \begin{pmatrix} \widetilde{R}_{Aa1} + \widetilde{R}_{Aa2}, \widetilde{R}_{Ab1} + \widetilde{R}_{Ab2}, \\ \widetilde{R}_{Ac1} + \widetilde{R}_{Ac2} \end{pmatrix}; wR_{A_1} \wedge\ w\widetilde{R}_{A2}, u\widetilde{R}_{A1} \vee u\widetilde{R}_{A2}, y\widetilde{R}_{A1} \vee y\widetilde{R}_{A2} \rangle \tag{25}$$

Proposition 2. *The neutrosophic portfolio risk noted with* $\langle\widetilde{\sigma_P^2}; w\widetilde{\sigma p}, u\widetilde{\sigma p}, y\widetilde{\sigma p}\rangle$ *modeled using the fuzzy neutrosophic numbers of the form:* $\widetilde{\sigma}_{Ai} = \langle(\widetilde{\sigma}_{Aai}, \widetilde{\sigma}_{Abi}, \widetilde{\sigma}_{Aci}); w\widetilde{\sigma}_A, u\widetilde{\sigma}_A, y\widetilde{\sigma}_A\rangle$ *is also a fundamental variable of the neutrosophic portfolio that is determined by the calculation formula:*

$$\begin{aligned}
&\langle\widetilde{\sigma_P^2}; w\widetilde{\sigma p}, u\widetilde{\sigma p}, y\widetilde{\sigma p}\rangle \\
&= \sum_{i=1}^{n} x_{A_i}^2 \langle \tfrac{1}{4}\Big[\big(\widetilde{R_{Abi}} - \widetilde{R_{Aai}}\big)^2 + \big(\widetilde{R_{Aci}} - \widetilde{R_{Abi}}\big)^2\Big]; w\widetilde{R_{Ai}}, u\widetilde{R_{Ai}}, y\widetilde{R_{Ai}}\rangle \\
&+\langle \tfrac{2}{3}\Big[\widetilde{R_{Aai}}(\widetilde{R_{Abi}} - \widetilde{R_{Aai}}) - \widetilde{R_{Aci}}(\widetilde{R_{Aci}} - \widetilde{R_{Abi}})\Big]; w\widetilde{R_{Ai}}, u\widetilde{R_{Ai}}, y\widetilde{R_{Ai}}\rangle \\
&+\langle \tfrac{1}{2}(\widetilde{R_{Aai}}^2 + \widetilde{R_{Aci}}^2); w\widetilde{Ra_i}, u\widetilde{Ra_i}, y\widetilde{Ra_i}\rangle \\
&-\langle \tfrac{1}{2}E_f^2(\widetilde{R_{Ai}}); w\widetilde{R_{Ai}}, u\widetilde{R_{Ai}}, y\widetilde{R_{Ai}}\rangle \\
&+2\sum_{i=1}^{n}\sum_{j=1}^{n} x_{A_i}x_{A_j}\langle\Big(\tfrac{1}{4}\Big[(\widetilde{R_{Abi1}} - \widetilde{R_{Aai1}})(\widetilde{R_{Abj1}} - \widetilde{R_{Aaj1}}) \\
&+(\widetilde{R_{Aci1}} - \widetilde{R_{Abi1}})(\widetilde{R_{Acj1}} - \widetilde{R_{Abj1}})\Big] \\
&+\tfrac{1}{3}\Big\{\Big[\widetilde{R_{Aaj1}}(\widetilde{R_{Abi1}} - \widetilde{R_{Aai1}}) + \widetilde{R_{Aai1}}(\widetilde{R_{Abj1}} - \widetilde{R\widetilde{R_A}a_{aj1}})\Big] \\
&-\Big[\widetilde{R_{Aci1}}(\widetilde{R_{Acj1}} - \widetilde{R_{Abj1}}) + \widetilde{R_{Acj1}}(\widetilde{R_{Aci1}} - \widetilde{R_{Abi1}})\Big]\Big\} \\
&+\tfrac{1}{2}(\widetilde{R_{Aai1}}\widetilde{R_{Aaj1}} + \widetilde{R_{Aci1}}\widetilde{R_{Acj1}}) \\
&+\tfrac{1}{2}E_f(\widetilde{R_{Ai}})E_f(\widetilde{R_{Aj}})\Big); w\widetilde{R_{A_i}} \wedge w\widetilde{R_{Aj}}, u\widetilde{R_{Ai}} \vee u\widetilde{R_{Aj}}, y\widetilde{R_{Ai}} \vee y\widetilde{R_{Aj}}\rangle
\end{aligned} \tag{26}$$

Demonstration: It is known that the neutrosophic portfolio risk made up of N financial assets is of the form:

$$\begin{aligned}
&\langle\widetilde{\sigma_P^2}; w\widetilde{\sigma p}, u\widetilde{\sigma p}, y\widetilde{\sigma p}\rangle \\
&= \langle x_{A_1}^2\widetilde{\sigma}_{A_1}^2; w\widetilde{\sigma}_{A_1}, u\widetilde{\sigma}_{A_1}, y\widetilde{\sigma}_{A_1}\rangle + \langle x_{A_2}^2\widetilde{\sigma}_{A_2}^2; w\widetilde{\sigma}_{A_2}, u\widetilde{\sigma}_{A_2}, y\widetilde{\sigma}_{A_2}\rangle + \cdots \\
&+\langle x_{A_n}^2\widetilde{\sigma}_{A_n}^2; w\widetilde{\sigma}_{A_n}, u\widetilde{\sigma}_{A_n}, y\widetilde{\sigma}_{A_n}\rangle \\
&+\langle 2x_{A_1}x_{A_2}\widetilde{\sigma}_{A_1A_2}; w\widetilde{\sigma}_{A_1} \wedge w\widetilde{\sigma}_{A_2}, u\widetilde{\sigma}_{A_1} \vee u\widetilde{\sigma}_{A_2}, y\widetilde{\sigma}_{A_1} \vee y\widetilde{\sigma}_{A_2}\rangle \\
&+\langle 2x_{A_1}x_{A_3}\widetilde{\sigma}_{A_1A_3}; w\widetilde{\sigma}_{A_1} \wedge w\widetilde{\sigma}_{A_3}, u\widetilde{\sigma}_{A_1} \vee u\widetilde{\sigma}_{A_3}, y\widetilde{\sigma}_{A_1} \vee y\widetilde{\sigma}_{A_3}\rangle + \cdots \\
&+\langle 2x_{A_1}x_{A_n}\widetilde{\sigma}_{A_1A_n}; w\widetilde{\sigma}_{A_1} \wedge w\widetilde{\sigma}_{A_n}, u\widetilde{\sigma}_{A_1} \vee u\widetilde{\sigma}_{A_n}, y\widetilde{\sigma}_{A_1} \vee y\widetilde{\sigma}_{A_n}\rangle \\
&+\langle 2x_{A_2}x_{A_1}\widetilde{\sigma}_{A_2A_1}; w\widetilde{\sigma}_{A_2} \wedge w\widetilde{\sigma}_{A_1}, u\widetilde{\sigma}_{A_2} \vee u\widetilde{\sigma}_{A_1}, y\widetilde{\sigma}_{A_2} \vee y\widetilde{\sigma}_{A_1}\rangle \\
&+\langle 2x_{A_2}x_{A_3}\widetilde{\sigma}_{A_2A_3}; w\widetilde{\sigma}_{A_2} \wedge w\widetilde{\sigma}_{A_3}, u\widetilde{\sigma}_{A_2} \vee u\widetilde{\sigma}_{A_3}, y\widetilde{\sigma}_{A_2} \vee y\widetilde{\sigma}_{A_3}\rangle + \cdots \\
&+\langle 2x_{A_2}x_{A_n}\widetilde{\sigma}_{A_2A_n}; w\widetilde{\sigma}_{A_2} \wedge w\widetilde{\sigma}_{A_n}, u\widetilde{\sigma}_{A_2} \vee u\widetilde{\sigma}_{A_n}, y\widetilde{\sigma}_{A_2} \vee y\widetilde{\sigma}_{A_n}\rangle \\
&+\langle 2x_{A_n}x_{A_1}\widetilde{\sigma}_{A_nA_1}; w\widetilde{\sigma}_{A_n} \wedge w\widetilde{\sigma}_{A_1}, u\widetilde{\sigma}_{A_n} \vee u\widetilde{\sigma}_{A_1}, y\widetilde{\sigma}_{A_n} \vee y\widetilde{\sigma}_{A_1}\rangle \\
&+\langle 2x_{A_n}x_{A_2}\widetilde{\sigma}_{A_nA_2}; w\widetilde{\sigma}_{A_n} \wedge w\widetilde{\sigma}_{A_2}, u\widetilde{\sigma}_{A_n} \vee u\widetilde{\sigma}_{A_2}, y\widetilde{\sigma}_{A_n} \vee y\widetilde{\sigma}_{A_2}\rangle + \cdots
\end{aligned} \tag{27}$$

The analytical relation above can be written as follows:

$$\begin{aligned}
&\langle\widetilde{\sigma_P^2}; w\widetilde{\sigma p}, u\widetilde{\sigma p}, y\widetilde{\sigma p}\rangle \\
&= \sum_{i=1}^{n}\langle x_{A_i}^2\widetilde{\sigma}_{A_i}^2; w\widetilde{\sigma}_{A_i}, u\widetilde{\sigma}_{A_i}, y\widetilde{\sigma}_{A_i}\rangle \\
&+2\sum_{i=1}^{n}\sum_{j=1}^{n}\langle x_{A_i}x_{A_j}\widetilde{\sigma}_{A_iA_j}; w\widetilde{\sigma}_{A_i} \wedge w\widetilde{\sigma}_{A_j}, u\widetilde{\sigma}_{A_i} \vee u\widetilde{\sigma}_{A_j}, y\widetilde{\sigma}_{A_i} \vee y\widetilde{\sigma}_{A_j}\rangle
\end{aligned} \tag{28}$$

In the neutrosophic portfolio risk relation, we substitute the expression for the determination of the mean square deviation according to the Definition 2 and the expression for the covariance according to the Definition 3, established for a financial asset and we obtain the calculation relation for determining the risk size of the portfolio according to the weight of the financial asset in the total value of the portfolio x_{A_i} but also of the individual financial asset risk $\langle\widetilde{\sigma}_{A_i}^2; w\widetilde{\sigma}_{A_i}, u\widetilde{\sigma}_{A_i}, y\widetilde{\sigma}_{A_i}\rangle$ and the covariance between two financial assets $\langle\widetilde{\sigma}_{A_iA_j}; w\widetilde{\sigma}_{A_i} \wedge w\widetilde{\sigma}_{A_j}, u\widetilde{\sigma}_{A_i} \vee u\widetilde{\sigma}_{A_j}, y\widetilde{\sigma}_{A_i} \vee y\widetilde{\sigma}_{A_j}\rangle$:

$$\begin{aligned}
&\langle \widetilde{\sigma_P^2}; w\widetilde{\sigma p}, u\widetilde{\sigma p}, y\widetilde{\sigma p}\rangle \\
&= \sum_{i=1}^{n} x_{A_i}^2 \langle \tfrac{1}{4}\Big[\big(\widetilde{R_{Abi}} - \widetilde{R_{Aai}}\big)^2 + \big(\widetilde{R_{Aci}} - \widetilde{R_{Abi}}\big)^2\Big]; w\widetilde{R_{Ai}}, u\widetilde{R_{Ai}}, y\widetilde{R_{Ai}}\rangle \\
&+\langle \tfrac{2}{3}\Big[\widetilde{R_{Aai}}(\widetilde{R_{Abi}} - \widetilde{R_{Aai}}) - \widetilde{R_{Aci}}(\widetilde{R_{Aci}} - \widetilde{R_{Abi}})\Big]; w\widetilde{R_{Ai}}, u\widetilde{R_{Ai}}, y\widetilde{R_{Ai}}\rangle \\
&+\langle \tfrac{1}{2}(\widetilde{R_{Aai}}^2 + \widetilde{R_{Aci}}^2); w\widetilde{Ra_i}, u\widetilde{Ra_i}, y\widetilde{Ra_i}\rangle \\
&-\langle \tfrac{1}{2}E_f^2(\widetilde{R_{Ai}}); w\widetilde{R_{Ai}}, u\widetilde{R_{Ai}}, y\widetilde{R_{Ai}}\rangle \\
&+2\sum_{i=1}^{n}\sum_{j=1}^{n} x_{A_i}x_{A_j}\langle\Big(\tfrac{1}{4}\Big[(\widetilde{R_{Abi1}} - \widetilde{R_{Aai1}})(\widetilde{R_{Abj1}} - \widetilde{R_{Aaj1}}) \\
&+(\widetilde{R_{Aci1}} - \widetilde{R_{Abi1}})(\widetilde{R_{Acj1}} - \widetilde{R_{Abj1}})\Big] \\
&+\tfrac{1}{3}\Big\{\Big[\widetilde{R_{Aaj1}}(\widetilde{R_{Abi1}} - \widetilde{R_{Aai1}}) + \widetilde{R_{Aai1}}(\widetilde{R_{Abj1}} - \widetilde{R\widetilde{R_A}a_{aj1}})\Big] \\
&-\Big[\widetilde{R_{Aci1}}(\widetilde{R_{Acj1}} - \widetilde{R_{Abj1}}) + \widetilde{R_{Acj1}}(\widetilde{R_{Aci1}} - \widetilde{R_{Abi1}})\Big]\Big\} \\
&+\tfrac{1}{2}(\widetilde{R_{Aai1}}\widetilde{R_{Aaj1}} + \widetilde{R_{Aci1}}\widetilde{R_{Acj1}}) \\
&+\tfrac{1}{2}E_f(\widetilde{R_{Ai}})E_f(\widetilde{R_{Aj}})\Big); w\widetilde{R_{A_i}} \wedge w\widetilde{R_{Aj}}, u\widetilde{R_{Ai}} \vee u\widetilde{R_{Aj}}, y\widetilde{R_{Ai}} \vee y\widetilde{R_{Aj}}\rangle
\end{aligned} \tag{29}$$

Example 2. *There are considered three financial assets* (A_1, A_2, A_3) *to which three triangular neutrosophic numbers are specified for the financial assets return, of the form:*

$$\widetilde{R_{A1}} = \langle (0.2\ 0.3\ 0.5); 0.5,\ 0.2,\ 0.3\rangle \text{ pentru valori ale } \widetilde{R_A} \in [0,2; 0,5]$$

$$\widetilde{R_{A2}} = \langle (0.1\ 0.2\ 0.3); 0.6,\ 0.3,\ 0.2\rangle \text{ pentru valori ale } \widetilde{R_A} \in [0,1; 0,3] \tag{30}$$

$$\widetilde{R_{A3}} = \langle (0.3\ 0.4\ 0.6); 0.4,\ 0.3,\ 0.3\rangle \text{ pentru valori ale } \widetilde{R_A} \in [0,3; 0,6]$$

The weights held by the three financial assets in the total portfolio are determined according to the value of each financial asset and the total value of the portfolio and have the values: $x_{A_1} = 0,4$; $x_{A_2} = 0,3$ și $x_{A_3} = 0,3$. In order to establish the neutrosophic portfolio risk, from proposition2 it is known that:

$$\begin{aligned}
&\langle \widetilde{\sigma_P^2}; w\widetilde{\sigma p}, u\widetilde{\sigma p}, y\widetilde{\sigma p}\rangle \\
&= \sum_{i=1}^{n} \langle x_{A_i}^2 \widetilde{\sigma}_{A_i}^2; w\widetilde{\sigma}_{A_i}, u\widetilde{\sigma}_{A_i}, y\widetilde{\sigma}_{A_i}\rangle \\
&+2\sum_{i=1}^{n}\sum_{j=1}^{n} \langle x_{A_i}x_{A_j}\widetilde{\sigma}_{A_iA_j}; w\widetilde{\sigma}_{A_i} \wedge\ w\widetilde{\sigma}_{A_j}, u\widetilde{\sigma}_{A_i} \vee u\widetilde{\sigma}_{A_j}, y\widetilde{\sigma}_{A_i} \vee y\widetilde{\sigma}_{A_j}\rangle
\end{aligned} \tag{31}$$

The values of the neutrosophic risk for a financial asset are determined:

$$\begin{aligned}
\widetilde{\sigma}_{f_{A_1}}^2 = &\langle \tfrac{1}{4}[(0.3-0.2)^2 + (0.5-0.3)^2]; 0.5,\ 0.2,\ 0.3\rangle \\
&+\langle \tfrac{2}{3}(0.2(0.3-0.2) - 0.5(0.5-0.2)); 0.5,\ 0.2,\ 0.3\rangle \\
&+\langle \tfrac{1}{2}\Big(0.2^2 + 0.5^2\Big); 0.5,\ 0.2,\ 0.3\rangle - \langle \tfrac{1}{2}(0.316)^2; 0.5,\ 0.2,\ 0.3\rangle
\end{aligned} \tag{32}$$

$$\widetilde{\sigma}_{f_{A_1}}^2 = \langle 0.0225; 0.5,\ 0.2,\ 0.3\rangle \tag{33}$$

Proceeding in the same manner, we get the following results for $\widetilde{\sigma}_{f_{A_2}}^2$ and $\widetilde{\sigma}_{f_{A_3}}^2$:

$$\widetilde{\sigma}_{f_{A_2}}^2 = \langle 0.0180; 0.6,\ 0.3,\ 0.2\rangle \tag{34}$$

$$\widetilde{\sigma}_{f_{A_3}}^2 = \langle 0.0925; 0.4,\ 0.3,\ 0.3\rangle \tag{35}$$

We establish the covariance between financial assets according to the Definition 3 as follows:

$$\begin{aligned}\sigma_{A_1A_2} = \langle & \tfrac{1}{4}[(0.3-0.2)(0.2-0.1)+(0.5-0.3)(0.3-0.2)] \\ & +\tfrac{1}{3}[0.1(0.3-0.2)+0.2(0.2-0.1)] \\ & -[0.5(0.3-0.2)+0.3(0.5-0.3)]+\tfrac{1}{2}(0.2*0.1+0.5*0.3) \\ & +\tfrac{1}{2}0.316*0.199; 0.5\wedge 0.6,\ 0.2\vee 0.3, 0.3\vee 0.2\rangle\end{aligned}$$

$$\sigma_{A_1A_2} = \langle 0.0705; 0.6,\ 0.2,\ 0.2\rangle \tag{36}$$

In the same way, we get:

$$\sigma_{A_1A_3} = \langle 0.1914; 0.5,\ 0.2,\ 0.3\rangle \tag{37}$$

$$\sigma_{A_2A_3} = \langle 0.0805;\ 0.6,\ 0.3,\ 0.2\rangle \tag{38}$$

$$\begin{aligned}&\langle\sigma_p^2; w\widetilde{\sigma p}, u\widetilde{\sigma p}, y\widetilde{\sigma p}\rangle \\ &= \langle x_{A_1}^2\widetilde{\sigma}_{A_1}^2; w\widetilde{\sigma}_{A_1}, u\widetilde{\sigma}_{A_1}, y\widetilde{\sigma}_{A_1}\rangle + \langle x_{A_2}^2\widetilde{\sigma}_{A_2}^2; w\widetilde{\sigma}_{A_2}, u\widetilde{\sigma}_{A_2}, y\widetilde{\sigma}_{A_2}\rangle \\ &+\langle x_{A_3}^2\widetilde{\sigma}_{A_3}^2; w\widetilde{\sigma}_{A_3}, u\widetilde{\sigma}_{A_3}, y\widetilde{\sigma}_{A_3}\rangle \\ &+2x_{A_1}x_{A_2}\widetilde{\sigma}_{A_1A_2}; w\widetilde{\sigma}_{A_1}\wedge\ w\widetilde{\sigma}_{A_2}, u\widetilde{\sigma}_{A_1}\vee u\widetilde{\sigma}_{A_2}, y\widetilde{\sigma}_{A_1}\vee y\widetilde{\sigma}_{A_2} \\ &+\langle 2x_{A_1}x_{A_3}\widetilde{\sigma}_{A_1A_3}; w\widetilde{\sigma}_{A_1}\wedge\ w\widetilde{\sigma}_{A_3}, u\widetilde{\sigma}_{A_1}\vee u\widetilde{\sigma}_{A_3}, y\widetilde{\sigma}_{A_1}\vee y\widetilde{\sigma}_{A_3}\rangle \\ &+\langle 2x_{A_2}x_{A_1}\widetilde{\sigma}_{A_2A_1}; w\widetilde{\sigma}_{A_2}\wedge\ w\widetilde{\sigma}_{A_1}, u\widetilde{\sigma}_{A_2}\vee u\widetilde{\sigma}_{A_1}, y\widetilde{\sigma}_{A_2}\vee y\widetilde{\sigma}_{A_1}\rangle \\ &+\langle 2x_{A_2}x_{A_3}\widetilde{\sigma}_{A_2A_3}; w\widetilde{\sigma}_{A_2}\wedge\ w\widetilde{\sigma}_{A_3}, u\widetilde{\sigma}_{A_2}\vee u\widetilde{\sigma}_{A_3}, y\widetilde{\sigma}_{A_2}\vee y\widetilde{\sigma}_{A_3}\rangle \\ &+\langle 2x_{A_3}x_{A_1}\widetilde{\sigma}_{A_3A_1}; w\widetilde{\sigma}_{A_3}\wedge\ w\widetilde{\sigma}_{A_1}, u\widetilde{\sigma}_{A_3}\vee u\widetilde{\sigma}_{A_1}, y\widetilde{\sigma}_{A_3}\vee y\widetilde{\sigma}_{A_1}\rangle \\ &+\langle 2x_{A_3}x_{A_2}\widetilde{\sigma}_{A_3A_2}; w\widetilde{\sigma}_{A_3}\wedge\ w\widetilde{\sigma}_{A_2}, u\widetilde{\sigma}_{A_3}\vee u\widetilde{\sigma}_{A_2}, y\widetilde{\sigma}_{A_3}\vee y\widetilde{\sigma}_{A_2}\rangle\end{aligned} \tag{39}$$

$$\begin{aligned}&\langle\sigma_p^2; w\widetilde{\sigma p}, u\widetilde{\sigma p}, y\widetilde{\sigma p}\rangle \\ &= \langle 0.16\times 0.0225; 0.5,\ 0.2,\ 0.3\rangle + \langle 0.09\times 0.0180; 0.6,\ 0.3,\ 0.2\rangle \\ &+\langle 0.09\times 0.0925; 0.4,\ 0.3,\ 0.3\rangle + \langle 2\times 0.12\times 0.0705; 0.6,\ 0.2,\ 0.2\rangle \\ &+\langle 2\times 0.12\times 0.1914; 0.5,\ 0.2,\ 0.3\rangle + \langle 2\times 0.12\times 0.0705; 0.6,\ 0.2,\ 0.2\rangle \\ &+\langle 2\times 0.09\times 0.0805;\ 0.6,\ 0.3,\ 0.2\rangle \\ &+\langle 2\times 0,12\times 0.1914; 0.5,\ 0.2,\ 0.3\rangle \\ &+\langle 2\times 0.09\times 0.0805;\ 0.6,\ 0.3,\ 0.2\rangle\end{aligned} \tag{40}$$

$$\langle\sigma_p; w\widetilde{\sigma p}, u\widetilde{\sigma p}, y\widetilde{\sigma p}\rangle = \sqrt{\langle 0,168237; 0.6,\ 0.2,\ 0.2\rangle} = \langle 0,41016; 0.6,\ 0.2,\ 0.2\rangle \tag{41}$$

Interpretation: The neutrosophic portfolio return was previously determined $\langle\widetilde{R}_P; w\widetilde{Rp}, u\widetilde{Rp}, y\widetilde{Rp}\rangle = \langle 0,3109; 0.5,\ 0.2,\ 0.3\rangle$. For this neutrosophic portfolio return value corresponds a high risk $\langle\sigma_p; w\widetilde{\sigma p}, u\widetilde{\sigma p}, y\widetilde{\sigma p} = 0,41016; 0.6,\ 0.2,\ 0.2\rangle$ which confirms that between return and risk there is a directly proportional relationship. The probabilities for risk manifestation is about 60%, while the probability that the risk is certain/uncertain is 20% and the probability that the risk does not occur is quite small and has a value of 20%.

4. Neutrosophic Portfolio Equations. The Analytical and Matrix Form

Neutrosophic portfolios of the form $\langle\widetilde{P}; w\widetilde{p}, u\widetilde{p}, y\widetilde{p}\rangle$ have the characteristic that each of the financial assets they contain can be modelled using the neutrosophic performance indicators such as: neutrosophic return $\langle E_f(\widetilde{R_A}); w\widetilde{R_A}, u\widetilde{R_A}, y\widetilde{R_A}\rangle$, the neutrosophic risk $\langle\sigma f_{A_i}^2; w\widetilde{\sigma}_A, u\widetilde{\sigma}_A, y\widetilde{\sigma}_A\rangle$, the neutrosophic covariance $\langle cov(\widetilde{R_{A1}}, \widetilde{R_{A2}}); w\widetilde{R_{A1}}, u\widetilde{R_{A1}}, y\widetilde{R_{A1}};\ w\widetilde{R_{A2}}, u\widetilde{R_{A2}}, y\widetilde{R_{A2}}\rangle$.

With these neutrosophic performance indicators specific to each financial asset, are determined the two fundamental variables of the neutrosophic portfolios, namely: neutrosophic portfolio return $\langle\widetilde{R}_P; w\widetilde{Rp}, u\widetilde{Rp}, y\widetilde{Rp}\rangle$ and the neutrosophic portfolio risk $\langle\widetilde{\sigma}_p^2; w\widetilde{\sigma p}, u\widetilde{\sigma p}, y\widetilde{\sigma p}\rangle$. The neutrosophic portfolio return, according to sentence no.1 can be written in analytical form as follows:

$$\langle\widetilde{R}_P; w\widetilde{Rp}, u\widetilde{Rp}, y\widetilde{Rp}\rangle = \sum_{i=1}^{n}\langle x_{A_i}\widetilde{R}_{A_i}; w\widetilde{R}_{A_i}, u\widetilde{R}_{A_i}, y\widetilde{R}_{A_i}\rangle \tag{42}$$

The neutrosophic portfolio risk can be written in analytical form as follows:

$$\begin{aligned} &\langle \widetilde{\sigma_P^2}; w\widetilde{\sigma p}, u\widetilde{\sigma p}, y\widetilde{\sigma p} \rangle \\ &\quad = \sum_{i=1}^{n} \langle x_{A_i}^2 \widetilde{\sigma}_{A_i}^2; w\widetilde{\sigma}_{A_i}, u\widetilde{\sigma}_{A_i}, y\widetilde{\sigma}_{A_i} \rangle \\ &\quad + 2\sum_{i=1}^{n} \sum_{j=1}^{n} \langle x_{A_i} x_{A_j} \widetilde{\sigma}_{A_i A_j}; w\widetilde{\sigma}_{A_i} \wedge\ w\widetilde{\sigma}_{A_j}, u\widetilde{\sigma}_{A_i} \vee u\widetilde{\sigma}_{A_j}, y\widetilde{\sigma}_{A_i} \vee y\widetilde{\sigma}_{A_j} \rangle \end{aligned} \tag{43}$$

In order to form the system of equations that characterize the neutrosophic portfolios of financial assets, it should be mentioned that these portfolios are made up of financial assets whose weight in the total value of the portfolio is 100% which can be mathematically quantified by the formula:

$$\sum_{i=1}^{n} x_{A_i} = 100\% \tag{44}$$

Under these conditions, the system of equations of the neutrosophicportfolio of financial assets in analytical form will be written as follows:

$$\begin{gathered} \langle \widetilde{R_P}; w\widetilde{Rp}, u\widetilde{Rp}, y\widetilde{Rp} \rangle = \sum_{i=1}^{n} \langle x_{A_i} \widetilde{R}_{A_i}; w\widetilde{R}_{A_i}, u\widetilde{R}_{A_i}, y\widetilde{R}_{A_i} \rangle \\ \langle \widetilde{\sigma_P^2}; w\widetilde{\sigma p}, u\widetilde{\sigma p}, y\widetilde{\sigma p} \rangle = \sum_{i=1}^{n} \langle x_{A_i}^2 \widetilde{\sigma}_{A_i}^2; w\widetilde{\sigma}_{A_i}, u\widetilde{\sigma}_{A_i}, y\widetilde{\sigma}_{A_i} \rangle + \\ +2\sum_{i=1}^{n} \sum_{j=1}^{n} \langle x_{A_i} x_{A_j} \widetilde{\sigma}_{A_i A_j}; w\widetilde{\sigma}_{A_i} \wedge\ w\widetilde{\sigma}_{A_j}, u\widetilde{\sigma}_{A_i} \vee u\widetilde{\sigma}_{A_j}, y\widetilde{\sigma}_{A_i} \vee y\widetilde{\sigma}_{A_j} \rangle \\ \sum_{i=1}^{n} x_{A_i} = 100\% \end{gathered} \tag{45}$$

In matrix form the equations of the neutrosophicportfolio made up of Nfinancial assets will be written as follows:

$$\langle \widetilde{R_P}; w\widetilde{Rp}, u\widetilde{Rp}, y\widetilde{Rp} \rangle = (x_{A_1} x_{A_2} \ldots x_{A_n}) \begin{pmatrix} \langle \widetilde{R}_{A_1}; w\widetilde{R}_{A_1}, u\widetilde{R}_{A_1}, y\widetilde{R}_{A_1} \rangle \\ \langle \widetilde{R}_{A_2}; w\widetilde{R}_{A_2}, u\widetilde{R}_{A_2}, y\widetilde{R}_{A_2} \rangle \\ \ldots \\ \langle \widetilde{R}_{A_n}; w\widetilde{R}_{A_n}, u\widetilde{R}_{A_n}, y\widetilde{R}_{A_n} \rangle \end{pmatrix} \tag{46}$$

We note: $X_A^T = \left(x_{A_1} x_{A_2} \ldots x_{A_n} \right)$ and

$$\langle \widetilde{R}_A; w\widetilde{R}_A, u\widetilde{R}_A, y\widetilde{R}_A \rangle = \begin{pmatrix} \langle \widetilde{R}_{A_1}; w\widetilde{R}_{A_1}, u\widetilde{R}_{A_1}, y\widetilde{R}_{A_1} \rangle \\ \langle \widetilde{R}_{A_2}; w\widetilde{R}_{A_2}, u\widetilde{R}_{A_2}, y\widetilde{R}_{A_2} \rangle \\ \ldots \\ \langle \widetilde{R}_{A_n}; w\widetilde{R}_{A_n}, u\widetilde{R}_{A_n}, y\widetilde{R}_{A_n} \rangle \end{pmatrix} \tag{47}$$

Under these conditions, the equation of the neutrosophic portfolio return will be written in matrix form as follows:

$$\langle \widetilde{R_p}; w\widetilde{Rp}, u\widetilde{Rp}, y\widetilde{Rp} \rangle = X_A^T \langle \widetilde{R}_A; w\widetilde{R}_A, u\widetilde{R}_A, y\widetilde{R}_A \rangle \tag{48}$$

The portfolio risk equation above can be written in matrix form as follows:

$$\begin{aligned} &\langle \widetilde{\sigma_P^2}; w\widetilde{\sigma p}, u\widetilde{\sigma p}, y\widetilde{\sigma p} \rangle \\ &= (x_{A_1} x_{A_2} \ldots x_{A_n}) \begin{pmatrix} \langle \widetilde{\sigma}_{A_{11}}; w\widetilde{\sigma}_{A_{11}}, u\widetilde{\sigma}_{A_{11}}, y\widetilde{\sigma}_{A_{11}} \rangle & \ldots & \langle \widetilde{\sigma}_{A_{1n}}; w\widetilde{\sigma}_{A_{1n}}, u\widetilde{\sigma}_{A_{1n}}, y\widetilde{\sigma}_{A_{1n}} \rangle \\ \langle \widetilde{\sigma}_{A_{21}}; w\widetilde{\sigma}_{A_{21}}, u\widetilde{\sigma}_{A_{21}}, y\widetilde{\sigma}_{A_{21}} \rangle & \ldots & \langle \widetilde{\sigma}_{A_{2n}}; w\widetilde{\sigma}_{A_{2n}}, u\widetilde{\sigma}_{A_{2n}}, y\widetilde{\sigma}_{A_{2n}} \rangle \\ \ldots & \ldots & \ldots \\ \langle \widetilde{\sigma}_{A_{n1}}; w\widetilde{\sigma}_{A_{n1}}, u\widetilde{\sigma}_{A_{n1}}, y\widetilde{\sigma}_{A_{n1}} \rangle & \ldots & \langle \widetilde{\sigma}_{A_{nn}}; w\widetilde{\sigma}_{A_{nn}}, u\widetilde{\sigma}_{A_{nn}}, y\widetilde{\sigma}_{A_{nn}} \rangle \end{pmatrix} \begin{pmatrix} x_{A_1} \\ x_{A_2} \\ \ldots \\ x_{A_n} \end{pmatrix} \end{aligned} \tag{49}$$

In the matrix equation of the neutrosophic portfolio risk above we note:

$$X_A^T = \left(x_{A_1} x_{A_2} \ldots x_{A_n}\right)$$

and

$$\langle\widetilde{\Omega}; w\widetilde{\sigma}_A, u\widetilde{\sigma}_A, y\widetilde{\sigma}_A\rangle = \begin{pmatrix} \langle\widetilde{\sigma}_{A_{11}}; w\widetilde{\sigma}_{A_{11}}, u\widetilde{\sigma}_{A_{11}}, y\widetilde{\sigma}_{A_{11}}\rangle & \ldots & \langle\widetilde{\sigma}_{A_{1n}}; w\widetilde{\sigma}_{A_{1n}}, u\widetilde{\sigma}_{A_{1n}}, y\widetilde{\sigma}_{A_{1n}}\rangle \\ \langle\widetilde{\sigma}_{A_{21}}; w\widetilde{\sigma}_{A_{21}}, u\widetilde{\sigma}_{A_{21}}, y\widetilde{\sigma}_{A_{21}}\rangle & \ldots & \langle\widetilde{\sigma}_{A_{2n}}; w\widetilde{\sigma}_{A_{2n}}, u\widetilde{\sigma}_{A_{2n}}, y\widetilde{\sigma}_{A_{2n}}\rangle \\ \ldots & \ldots & \ldots \\ \langle\widetilde{\sigma}_{A_{n1}}; w\widetilde{\sigma}_{A_{n1}}, u\widetilde{\sigma}_{A_{n1}}, y\widetilde{\sigma}_{A_{n1}}\rangle & \ldots & \langle\widetilde{\sigma}_{A_{nn}}; w\widetilde{\sigma}_{A_{nn}}, u\widetilde{\sigma}_{A_{nn}}, y\widetilde{\sigma}_{A_{nn}}\rangle \end{pmatrix} \quad (50)$$

Under these conditions the matrix equation of the neutrosophic portfolio risk becomes:

$$\langle\widetilde{\sigma}_p^2; w\widetilde{\sigma p}, u\widetilde{\sigma p}, y\widetilde{\sigma p}\rangle = X_A^T\langle\widetilde{\Omega}; w\widetilde{\sigma}_A, u\widetilde{\sigma}_A, y\widetilde{\sigma}_A\rangle X_A \quad (51)$$

The equation of the financial assets weight in the total value of the neutrosophic portfolio can be written as:

$$\left(x_{A_1} x_{A_2} \ldots x_{A_n}\right)\begin{pmatrix} 1 \\ 1 \\ \ldots \\ 1 \end{pmatrix} = 1 \quad (52)$$

In matrix form, the equation of weights will be written as follows: $X_A^T e = 1$.

The system of equations of the neutrosophic portfolio in matrix form will be written as follows:

$$\begin{cases} \langle\widetilde{R_p}; w\widetilde{Rp}, u\widetilde{Rp}, y\widetilde{Rp}\rangle = X_A^T\langle\widetilde{R}_A; w\widetilde{R}_A, u\widetilde{R}_A, y\widetilde{R}_A\rangle \\ \langle\widetilde{\sigma}_p^2; w\widetilde{\sigma p}, u\widetilde{\sigma p}, y\widetilde{\sigma p}\rangle = X_A^T\langle\widetilde{\Omega}; w\widetilde{\sigma}_A, u\widetilde{\sigma}_A, y\widetilde{\sigma}_A\rangle X_A \\ X_A^T e = 1 \end{cases} \quad (53)$$

Neutrosophic portfolio equations in analytical or matrix form will be used for risk minimization calculations or for determining the optimal portfolio structure depending on the needs.

5. Minimizing the Risk of the Neutrosophic Portfolio

5.1. Minimizing the Risk of the Neutrosophic Portfolio Consisting of Two Financial Assets

The purpose of this section is to determine that structure of the neutrosophic portfolio $x_{A_1} x_{A_2} \ldots x_{A_n}$ for which the risk is minimal. The financial assets that enter in the structure of the neutrosophic portfolio allow the determination of the performance indicators using the neutrosophic triangular fuzzy numbers according to theorem no.1. For this we write the equations of the neutrosophic portfolio consisting of two financial assets A_1, A_2 as follows:

$$\begin{cases} \langle\widetilde{R_p}; w\widetilde{Rp}, u\widetilde{Rp}, y\widetilde{Rp}\rangle = \langle x_{A_1}\widetilde{R}_{A_1}; w\widetilde{R}_{A1}, u\widetilde{R}_{A1}, y\widetilde{R}_{A1}\rangle + \langle x_{A_2}\widetilde{R}_{A_2}; w\widetilde{R}_{A2}, u\widetilde{R}_{A2}, y\widetilde{R}_{A2}\rangle \\ \langle\widetilde{\sigma}_p^2; w\widetilde{\sigma p}, u\widetilde{\sigma p}, y\widetilde{\sigma p}\rangle = \langle x_{A_1}^2\widetilde{\sigma}_{A_1}^2; w\widetilde{\sigma}_{A_1}, u\widetilde{\sigma}_{A_1}, y\widetilde{\sigma}_{A_1}\rangle + \langle x_{A_2}^2\widetilde{\sigma}_{A_2}^2; w\widetilde{\sigma}_{A_2}, u\widetilde{\sigma}_{A_2}, y\widetilde{\sigma}_{A_2}\rangle + \\ +\langle 2x_{A_1}x_{A_2}\sigma_{A_1A_2}; w\sigma_A, u\sigma_A, y\sigma_A\rangle \\ x_{A_1} + x_{A_2} = 1 \end{cases} \quad (54)$$

In order to establish the structure of the neutrosophic portfolio for which its risk is minimal, are imposed the minimum conditions for the portfolio risk resulted from the cancellation of the first order derivative of the neutrosophic portfolio risk:

$$\begin{cases} \frac{\partial\langle\widetilde{\sigma}_p^2; w\widetilde{\sigma p}, u\widetilde{\sigma p}, y\widetilde{\sigma p}\rangle}{\partial x_{A_1}} = 0 \\ \frac{\partial\langle\widetilde{\sigma}_p^2; w\widetilde{\sigma p}, u\widetilde{\sigma p}, y\widetilde{\sigma p}\rangle}{\partial x_{A_2}} = 0 \end{cases} \quad (55)$$

Theorem 1. *Let be two financial assets A_1, A_2 for which the neutrosophic return can be determined $\langle \widetilde{R}_{A_1}; w\widetilde{R}_{A1}, u\widetilde{R}_{A1}, y\widetilde{R}_{A1}\rangle$ and $\langle \widetilde{R}_{A_2}; w\widetilde{R}_{A2}, u\widetilde{R}_{A2}, y\widetilde{R}_{A2}\rangle$. Also, the specific neutrosophic risk can be determined for each financial asset $\langle \widetilde{\sigma}^2_{A_1}; w\widetilde{\sigma}_{A_1}, u\widetilde{\sigma}_{A_1}, y\widetilde{\sigma}_{A_1}\rangle$ and $\langle \widetilde{\sigma}^2_{A_2}; w\widetilde{\sigma}_{A_2}, u\widetilde{\sigma}_{A_2}, y\widetilde{\sigma}_{A_2}\rangle$. These two financial assets form a neutrosophic portfolio of the form: $\langle \widetilde{P}; w\widetilde{P}, u\widetilde{P}, y\widetilde{P}\rangle$. The risk of the neutrosophic portfolio has the minimum value for x_{A_1}, respectively x_{A_2} of the form:*

$$x_{A_1} = \frac{\langle \widetilde{\sigma}_{A_1A_2}; w\widetilde{\sigma}_{A}, u\widetilde{\sigma}_{A}, y\widetilde{\sigma}_{A}\rangle}{\langle \widetilde{\sigma}_{A_1A_2}; w\widetilde{\sigma}_{A}, u\widetilde{\sigma}_{A}, y\widetilde{\sigma}_{A}\rangle - \langle \widetilde{\sigma}^2_{A_1}; w\widetilde{\sigma}_{A_1}, u\widetilde{\sigma}_{A_1}, y\widetilde{\sigma}_{A_1}\rangle} \tag{56}$$

$$x_{A_2} = \frac{\langle \widetilde{\sigma}^2_{A_1}; w\widetilde{\sigma}_{A_1}, u\widetilde{\sigma}_{A_1}, y\widetilde{\sigma}_{A_1}\rangle}{\langle \widetilde{\sigma}^2_{A_1}; w\widetilde{\sigma}_{A_1}, u\widetilde{\sigma}_{A_1}, y\widetilde{\sigma}_{A_1}\rangle - \langle \widetilde{\sigma}_{A_1A_2}; w\widetilde{\sigma}_{A}, u\widetilde{\sigma}_{A}, y\widetilde{\sigma}_{A}\rangle} \tag{57}$$

with the condition that $\langle \widetilde{\sigma}_{A_1A_2}; w\widetilde{\sigma}_{A}, u\widetilde{\sigma}_{A}, y\widetilde{\sigma}_{A}\rangle \neq \langle \widetilde{\sigma}^2_{A_1}; w\widetilde{\sigma}_{A_1}, u\widetilde{\sigma}_{A_1}, y\widetilde{\sigma}_{A_1}\rangle$.

Demonstration: We know that the equations of the neutrosophic portfolio in analytical form can be written according to the above equations:

$$\begin{gathered}\langle \widetilde{R_p}; w\widetilde{Rp}, u\widetilde{Rp}, y\widetilde{Rp}\rangle = \langle x_{A_1}\widetilde{R}_{A_1}; w\widetilde{R}_{A1}, u\widetilde{R}_{A1}, y\widetilde{R}_{A1}\rangle + \langle x_{A_2}\widetilde{R}_{A_2}; w\widetilde{R}_{A2}, u\widetilde{R}_{A2}, y\widetilde{R}_{A2}\rangle \\ \langle \widetilde{\sigma}^2_P; w\widetilde{\sigma p}, u\widetilde{\sigma p}, y\widetilde{\sigma p}\rangle = \langle x^2_{A_1}\widetilde{\sigma}^2_{A_1}; w\widetilde{\sigma}_{A_1}, u\widetilde{\sigma}_{A_1}, y\widetilde{\sigma}_{A_1}\rangle + \langle x^2_{A_2}\widetilde{\sigma}^2_{A_2}; w\widetilde{\sigma}_{A_2}, u\widetilde{\sigma}_{A_2}, y\widetilde{\sigma}_{A_2}\rangle + \\ + \langle 2x_{A_1}x_{A_2}\widetilde{\sigma}_{A_1A_2}; w\widetilde{\sigma}_{A}, u\widetilde{\sigma}_{A}, y\widetilde{\sigma}_{A}\rangle \\ x_{A_1} + x_{A_2} = 1\end{gathered} \tag{58}$$

We set the minimum conditions for the neutrosophic portfolio risk and obtain:

$$\begin{cases} \frac{\partial \langle \widetilde{\sigma}^2_P; w\widetilde{\sigma p}, u\widetilde{\sigma p}, y\widetilde{\sigma p}\rangle}{\partial x_{A_1}} = 0 \\ \frac{\partial \langle \widetilde{\sigma}^2_P; w\widetilde{\sigma p}, u\widetilde{\sigma p}, y\widetilde{\sigma p}\rangle}{\partial x_{A_2}} = 0 \end{cases} \tag{59}$$

Based on these conditions, will obtain:

$$\left\{\left\{\begin{array}{c} \langle 2x_{A_1}\widetilde{\sigma}^2_{A_1}; w\widetilde{\sigma}_{A_1}, u\widetilde{\sigma}_{A_1}, y\widetilde{\sigma}_{A_1}\rangle + \langle 2x_{A_2}\widetilde{\sigma}_{A_1A_2}; w\widetilde{\sigma}_{A}, u\widetilde{\sigma}_{A}, y\widetilde{\sigma}_{A}\rangle = 0 \\ \langle 2x_{A_2}\widetilde{\sigma}^2_{A_2}; w\widetilde{\sigma}_{A_2}, u\widetilde{\sigma}_{A_2}, y\widetilde{\sigma}_{A_2}\rangle + \langle 2x_{A_1}\widetilde{\sigma}_{A_1A_2}; w\widetilde{\sigma}_{A}, u\widetilde{\sigma}_{A}, y\widetilde{\sigma}_{A}\rangle = 0 \\ x_{A_1} + x_{A_2} = 1 \end{array}\right.\right. \tag{60}$$

From the last equation of the above system results that $x_{A_2} = 1 - x_{A_1}$ and replacing in the first equation we will have:

$$\langle 2x_{A_1}\widetilde{\sigma}^2_{A_1}; w\widetilde{\sigma}_{A_1}, u\widetilde{\sigma}_{A_1}, y\widetilde{\sigma}_{A_1}\rangle + \langle 2\left(1 - x_{A_1}\right)\widetilde{\sigma}_{A_1A_2}; w\widetilde{\sigma}_{A}, u\widetilde{\sigma}_{A}, y\widetilde{\sigma}_{A}\rangle = 0 \tag{61}$$

$$\langle 2x_{A_1}\widetilde{\sigma}^2_{A_1}; w\widetilde{\sigma}_{A_1}, u\widetilde{\sigma}_{A_1}, y\widetilde{\sigma}_{A_1}\rangle + \langle 2\widetilde{\sigma}_{A_1A_2}; w\widetilde{\sigma}_{A}, u\widetilde{\sigma}_{A}, y\widetilde{\sigma}_{A}\rangle - \langle 2x_{A_1}\widetilde{\sigma}_{A_1A_2}; w\widetilde{\sigma}_{A}, u\widetilde{\sigma}_{A}, y\widetilde{\sigma}_{A}\rangle = 0 \tag{62}$$

$$\langle x_{A_1}\widetilde{\sigma}^2_{A_1}; w\widetilde{\sigma}_{A_1}, u\widetilde{\sigma}_{A_1}, y\widetilde{\sigma}_{A_1}\rangle - \langle x_{A_1}\widetilde{\sigma}_{A_1A_2}; w\widetilde{\sigma}_{A}, u\widetilde{\sigma}_{A}, y\widetilde{\sigma}_{A}\rangle = -\langle 2\widetilde{\sigma}_{A_1A_2}; w\widetilde{\sigma}_{A}, u\widetilde{\sigma}_{A}, y\widetilde{\sigma}_{A}\rangle \tag{63}$$

$$x_{A_1}\left(\langle \widetilde{\sigma}_{A_1A_2}; w\widetilde{\sigma}_{A}, u\widetilde{\sigma}_{A}, y\widetilde{\sigma}_{A}\rangle - \langle \widetilde{\sigma}^2_{A_1}; w\widetilde{\sigma}_{A_1}, u\widetilde{\sigma}_{A_1}, y\widetilde{\sigma}_{A_1}\rangle\right) = \langle \widetilde{\sigma}_{A_1A_2}; w\widetilde{\sigma}_{A}, u\widetilde{\sigma}_{A}, y\widetilde{\sigma}_{A}\rangle \tag{64}$$

$$x_{A_1} = \frac{\langle \widetilde{\sigma}_{A_1A_2}; w\widetilde{\sigma}_{A}, u\widetilde{\sigma}_{A}, y\widetilde{\sigma}_{A}\rangle}{\langle \widetilde{\sigma}_{A_1A_2}; w\widetilde{\sigma}_{A}, u\widetilde{\sigma}_{A}, y\widetilde{\sigma}_{A}\rangle - \langle \widetilde{\sigma}^2_{A_1}; w\widetilde{\sigma}_{A_1}, u\widetilde{\sigma}_{A_1}, y\widetilde{\sigma}_{A_1}\rangle} \tag{65}$$

With the condition that: $\langle \widetilde{\sigma}_{A_1A_2}; w\widetilde{\sigma}_{A}, u\widetilde{\sigma}_{A}, y\widetilde{\sigma}_{A}\rangle \neq \langle \widetilde{\sigma}^2_{A_1}; w\widetilde{\sigma}_{A_1}, u\widetilde{\sigma}_{A_1}, y\widetilde{\sigma}_{A_1}\rangle$.

By substituting the formula for x_{A_1} in the expression $x_{A_2} = 1 - x_{A_1}$ it is obtained:

$$x_{A_2} = 1 - \frac{\langle \widetilde{\sigma}_{A_1A_2}; w\widetilde{\sigma}_A, u\widetilde{\sigma}_A, y\widetilde{\sigma}_A \rangle}{\langle \widetilde{\sigma}_{A_1A_2}; w\widetilde{\sigma}_A, u\widetilde{\sigma}_A, y\widetilde{\sigma}_A \rangle - \langle \widetilde{\sigma}^2_{A_1}; w\widetilde{\sigma}_{A_1}, u\widetilde{\sigma}_{A_1}, y\widetilde{\sigma}_{A_1} \rangle} \quad (66)$$

$$x_{A_2} = \frac{\langle \widetilde{\sigma}^2_{A_1}; w\widetilde{\sigma}_{A_1}, u\widetilde{\sigma}_{A_1}, y\widetilde{\sigma}_{A_1} \rangle}{\langle \widetilde{\sigma}^2_{A_1}; w\widetilde{\sigma}_{A_1}, u\widetilde{\sigma}_{A_1}, y\widetilde{\sigma}_{A_1} \rangle - \langle \widetilde{\sigma}_{A_1A_2}; w\widetilde{\sigma}_A, u\widetilde{\sigma}_A, y\widetilde{\sigma}_A \rangle} \quad (67)$$

With the same condition: $\langle \widetilde{\sigma}_{A_1A_2}; w\widetilde{\sigma}_A, u\widetilde{\sigma}_A, y\widetilde{\sigma}_A \rangle \neq \langle \widetilde{\sigma}^2_{A_1}; w\widetilde{\sigma}_{A_1}, u\widetilde{\sigma}_{A_1}, y\widetilde{\sigma}_{A_1} \rangle$.

5.2. Minimizing the Risk of the Neutrosophic Portfolio Consisting of N Financial Assets

The neutrosophic portfolio is composed of N financial assets and assumes that in the portfolio there are financial assets that allow the determination of performance indicators using neutrosophic triangular fuzzy numbers. As a result, each financial asset that is part of the portfolio allows the determination of the neutrosophic return $\langle \widetilde{R}_{A_i}; w\widetilde{R}_{Ai}, u\widetilde{R}_{Ai}, y\widetilde{R}_{Ai} \rangle$ and of the neutrosophic risk $\langle \widetilde{\sigma}^2_{A_i}; w\widetilde{\sigma}_{A_i}, u\widetilde{\sigma}_{A_i}, y\widetilde{\sigma}_{A_i} \rangle$. Each financial asset A_i holds a weight in the total value of the neutrosophic portfolio noted with x_{A_i}. According to the above mentioned, for each financial asset A_i we will have:

$$\begin{array}{l} A_1\colon x_{A_1}; \langle \widetilde{R}_{A_1}; w\widetilde{R}_{A1}, u\widetilde{R}_{A1}, y\widetilde{R}_{A1} \rangle;\ \langle \widetilde{\sigma}^2_{A_1}; w\widetilde{\sigma}_{A_1}, u\widetilde{\sigma}_{A_1}, y\widetilde{\sigma}_{A_1} \rangle \\ A_2\colon x_{A_2}; \langle \widetilde{R}_{A_2}; w\widetilde{R}_{A2}, u\widetilde{R}_{A2}, y\widetilde{R}_{A2} \rangle;\ \langle \widetilde{\sigma}^2_{A_2}; w\widetilde{\sigma}_{A_2}, u\widetilde{\sigma}_{A_2}, y\widetilde{\sigma}_{A_2} \rangle \\ \\ A_n\colon x_{A_n}; \langle \widetilde{R}_{A_n}; w\widetilde{R}_{An}, u\widetilde{R}_{An}, y\widetilde{R}_{An} \rangle;\ \langle \widetilde{\sigma}^2_{A_n}; w\widetilde{\sigma}_{A_n}, u\widetilde{\sigma}_{A_n}, y\widetilde{\sigma}_{A_n} \rangle \end{array} \quad (68)$$

The equations that describe the portfolio refer to the neutrosophic portfolio return and risk and are of the form:

$$\begin{cases} \langle \widetilde{R_p}; w\widetilde{Rp}, u\widetilde{Rp}, y\widetilde{Rp} \rangle = \sum\limits_{i=1}^{n} \langle x_{A_i}\widetilde{R}_{A_i}; w\widetilde{R}_{A_i}, u\widetilde{R}_{A_i}, y\widetilde{R}_{A_i} \rangle \\ \langle \widetilde{\sigma}^2_P; w\widetilde{\sigma p}, u\widetilde{\sigma p}, y\widetilde{\sigma p} \rangle = \\ = \sum\limits_{i=1}^{n} \langle x^2_{A_i}\widetilde{\sigma}^2_{A_i}; w\widetilde{\sigma}_{A_i}, u\widetilde{\sigma}_{A_i}, y\widetilde{\sigma}_{A_i} \rangle + 2\sum\limits_{i=1}^{n}\sum\limits_{j=1}^{n} \langle x_{A_i}x_{A_j}\widetilde{\sigma}_{A_iA_j}; w\widetilde{\sigma}_{A_i} \wedge\ w\widetilde{\sigma}_{A_j}, u\widetilde{\sigma}_{A_i} \vee u\widetilde{\sigma}_{A_j}, y\widetilde{\sigma}_{A_i} \vee y\widetilde{\sigma}_{A_j} \rangle \\ \sum\limits_{i=1}^{n} x_{A_i} = 100\% \end{cases} \quad (69)$$

The analytical equations for the neutrosophic portfolio return and risk are written as follows:

$$\begin{array}{l} \langle \widetilde{R}_P; w\widetilde{Rp}, u\widetilde{Rp}, y\widetilde{Rp} \rangle \\ \qquad = \langle x_{A_1}\widetilde{R}_{A_1}; w\widetilde{R}_{A_1}, u\widetilde{R}_{A_1}, y\widetilde{R}_{A_1} \rangle + \langle x_{A_2}\widetilde{R}_{A_2}; w\widetilde{R}_{A_2}, u\widetilde{R}_{A_2}, y\widetilde{R}_{A_2} \rangle + \cdots \\ \qquad + \langle x_{A_n}\widetilde{R}_{A_n}; w\widetilde{R}_{A_n}, u\widetilde{R}_{A_n}, y\widetilde{R}_{A_n} \rangle \end{array} \quad (70)$$

And respectively:

$$
\begin{aligned}
&\langle \widetilde{\sigma_P^2}; w\widetilde{\sigma p}, u\widetilde{\sigma p}, y\widetilde{\sigma p}\rangle \\
&\quad = \langle x_{A_1}^2\widetilde{\sigma}_{A_1}^2; w\widetilde{\sigma}_{A_1}, u\widetilde{\sigma}_{A_1}, y\widetilde{\sigma}_{A_1}\rangle + \langle x_{A_2}^2\widetilde{\sigma}_{A_2}^2; w\widetilde{\sigma}_{A_2}, u\widetilde{\sigma}_{A_2}, y\widetilde{\sigma}_{A_2}\rangle + \cdots \\
&\quad + \langle x_{A_n}^2\widetilde{\sigma}_{A_n}^2; w\widetilde{\sigma}_{A_n}, u\widetilde{\sigma}_{A_n}, y\widetilde{\sigma}_{A_n}\rangle \\
&\quad + \langle 2x_{A_1}x_{A_2}\widetilde{\sigma}_{A_1A_2}; w\widetilde{\sigma}_{A_1} \wedge w\widetilde{\sigma}_{A_2}, u\widetilde{\sigma}_{A_1} \vee u\widetilde{\sigma}_{A_2}, y\widetilde{\sigma}_{A_1} \vee y\widetilde{\sigma}_{A_2}\rangle \\
&\quad + \langle 2x_{A_1}x_{A_3}\widetilde{\sigma}_{A_1A_3}; w\widetilde{\sigma}_{A_1} \wedge w\widetilde{\sigma}_{A_3}, u\widetilde{\sigma}_{A_1} \vee u\widetilde{\sigma}_{A_3}, y\widetilde{\sigma}_{A_1} \vee y\widetilde{\sigma}_{A_3}\rangle + \cdots \\
&\quad + \langle 2x_{A_1}x_{A_n}\widetilde{\sigma}_{A_1A_n}; w\widetilde{\sigma}_{A_1} \wedge w\widetilde{\sigma}_{A_n}, u\widetilde{\sigma}_{A_1} \vee u\widetilde{\sigma}_{A_n}, y\widetilde{\sigma}_{A_1} \vee y\widetilde{\sigma}_{A_n}\rangle \\
&\quad + \langle 2x_{A_2}x_{A_1}\widetilde{\sigma}_{A_2A_1}; w\widetilde{\sigma}_{A_2} \wedge w\widetilde{\sigma}_{A_1}, u\widetilde{\sigma}_{A_2} \vee u\widetilde{\sigma}_{A_1}, y\widetilde{\sigma}_{A_2} \vee y\widetilde{\sigma}_{A_1}\rangle \\
&\quad + \langle 2x_{A_2}x_{A_3}\widetilde{\sigma}_{A_2A_3}; w\widetilde{\sigma}_{A_2} \wedge w\widetilde{\sigma}_{A_3}, u\widetilde{\sigma}_{A_2} \vee u\widetilde{\sigma}_{A_3}, y\widetilde{\sigma}_{A_2} \vee y\widetilde{\sigma}_{A_3}\rangle + \cdots \\
&\quad + \langle 2x_{A_2}x_{A_n}\widetilde{\sigma}_{A_2A_n}; w\widetilde{\sigma}_{A_2} \wedge w\widetilde{\sigma}_{A_n}, u\widetilde{\sigma}_{A_2} \vee u\widetilde{\sigma}_{A_n}, y\widetilde{\sigma}_{A_2} \vee y\widetilde{\sigma}_{A_n}\rangle \\
&\quad + \langle 2x_{A_n}x_{A_1}\widetilde{\sigma}_{A_nA_1}; w\widetilde{\sigma}_{A_n} \wedge w\widetilde{\sigma}_{A_1}, u\widetilde{\sigma}_{A_n} \vee u\widetilde{\sigma}_{A_1}, y\widetilde{\sigma}_{A_n} \vee y\widetilde{\sigma}_{A_1}\rangle \\
&\quad + \langle 2x_{A_n}x_{A_2}\widetilde{\sigma}_{A_nA_2}; w\widetilde{\sigma}_{A_n} \wedge w\widetilde{\sigma}_{A_2}, u\widetilde{\sigma}_{A_n} \vee u\widetilde{\sigma}_{A_2}, y\widetilde{\sigma}_{A_n} \vee y\widetilde{\sigma}_{A_2}\rangle + \cdots \\
&\quad + \langle 2x_{A_n}x_{A_{n-1}}\widetilde{\sigma}_{A_nA_{n-1}}; w\widetilde{\sigma}_{A_n} \wedge w\widetilde{\sigma}_{A_{n-1}}, u\widetilde{\sigma}_{A_n} \vee u\widetilde{\sigma}_{A_{n-1}}, y\widetilde{\sigma}_{A_n} \vee y\widetilde{\sigma}_{A_{n-1}}\rangle
\end{aligned}
\tag{71}
$$

Theorem 2. *There are considered N financial assets $A_1, A_2, \ldots A_n$ for which the neutrosophic return can be determined $\langle \widetilde{R}_{A_1}; w\widetilde{R}_{A1}, u\widetilde{R}_{A1}, y\widetilde{R}_{A1}\rangle$, $\langle \widetilde{R}_{A_2}; w\widetilde{R}_{A2}, u\widetilde{R}_{A2}, y\widetilde{R}_{A2}\rangle$, ..., $\langle \widetilde{R}_{A_n}; w\widetilde{R}_{An}, u\widetilde{R}_{An}, y\widetilde{R}_{An}\rangle$. It is also possible to determine the neutrosophic risk specific to each financial asset that is part of the portfolio $\langle \widetilde{\sigma}_{A_1}^2; w\widetilde{\sigma}_{A_1}, u\widetilde{\sigma}_{A_1}, y\widetilde{\sigma}_{A_1}\rangle$, $\langle \widetilde{\sigma}_{A_2}^2; w\widetilde{\sigma}_{A_2}, u\widetilde{\sigma}_{A_2}, y\widetilde{\sigma}_{A_2}\rangle$ and $\langle \widetilde{\sigma}_{A_n}^2; w\widetilde{\sigma}_{A_n}, u\widetilde{\sigma}_{A_n}, y\widetilde{\sigma}_{A_n}\rangle$. These N financial assets form a neutrosophic portfolio of the form: $\langle \widetilde{P}; w\widetilde{P}, u\widetilde{P}, y\widetilde{P}\rangle$. The risk of the neutrosophic portfolio has the minimum value for $x_{A_1}, x_{A_2}, \ldots, x_{A_n}$ generalized using the weight x_{A_k} of the form:*

$$
x_{A_k} = \frac{\begin{pmatrix} \langle \widetilde{\sigma}_{A_{k2}}; w\widetilde{\sigma}_{A_{k2}}, u\widetilde{\sigma}_{A_{k2}}, y\widetilde{\sigma}_{A_{k2}}\rangle \\ \langle \widetilde{\sigma}_{A_{k3}}; w\widetilde{\sigma}_{A_{k3}}, u\widetilde{\sigma}_{A_{k3}}, y\widetilde{\sigma}_{A_{k3}}\rangle \\ \cdots \\ \langle \widetilde{\sigma}_{A_{kn}}; w\widetilde{\sigma}_{A_{kn}}, u\widetilde{\sigma}_{A_{kn}}, y\widetilde{\sigma}_{A_{kn}}\rangle \end{pmatrix} - \begin{pmatrix} \langle \sigma_{A_{kk}}; w\widetilde{\sigma}_{A_{kk}}, u\widetilde{\sigma}_{A_{kk}}, y\widetilde{\sigma}_{A_{kk}}\rangle \\ \langle \sigma_{A_{kk}}; w\widetilde{\sigma}_{A_{kk}}, u\widetilde{\sigma}_{A_{kk}}, y\widetilde{\sigma}_{A_{kk}}\rangle \\ \cdots \\ \langle \sigma_{A_{kk}}; w\widetilde{\sigma}_{A_{kk}}, u\widetilde{\sigma}_{A_{kk}}, y\widetilde{\sigma}_{A_{kk}}\rangle \end{pmatrix}}{\begin{pmatrix} \langle \widetilde{\sigma}_{A_{k2}}; w\widetilde{\sigma}_{A_{k2}}, u\widetilde{\sigma}_{A_{k2}}, y\widetilde{\sigma}_{A_{k2}}\rangle \\ \langle \widetilde{\sigma}_{A_{k3}}; w\widetilde{\sigma}_{A_{k3}}, u\widetilde{\sigma}_{A_{k3}}, y\widetilde{\sigma}_{A_{k3}}\rangle \\ \cdots \\ \langle \widetilde{\sigma}_{A_{kn}}; w\widetilde{\sigma}_{A_{kn}}, u\widetilde{\sigma}_{A_{kn}}, y\widetilde{\sigma}_{A_{kn}}\rangle \end{pmatrix}}
\tag{72}
$$

with the condition that: $\begin{pmatrix} \langle \widetilde{\sigma}_{A_{k2}}; w\widetilde{\sigma}_{A_{k2}}, u\widetilde{\sigma}_{A_{k2}}, y\widetilde{\sigma}_{A_{k2}}\rangle \\ \langle \widetilde{\sigma}_{A_{k3}}; w\widetilde{\sigma}_{A_{k3}}, u\widetilde{\sigma}_{A_{k3}}, y\widetilde{\sigma}_{A_{k3}}\rangle \\ \cdots \\ \langle \widetilde{\sigma}_{A_{kn}}; w\widetilde{\sigma}_{A_{kn}}, u\widetilde{\sigma}_{A_{kn}}, y\widetilde{\sigma}_{A_{kn}}\rangle \end{pmatrix} \neq 0.$

Demonstration: The equations of the neutrosophic portfolio in analytical form have been written previously and refer to those portfolios that contain a number of N financial assets. The conditions for minimizing the risk of the neutrosophic portfolio are obtained at the points where the first order derivative of the neutrosophic portfolio risk is null, respectively from the equations:

$$
\begin{cases}
\frac{\partial \langle \widetilde{\sigma_P^2}; w\widetilde{\sigma p}, u\widetilde{\sigma p}, y\widetilde{\sigma p}\rangle}{\partial x_{A_1}} = 0 \\
\frac{\partial \langle \widetilde{\sigma_P^2}; w\widetilde{\sigma p}, u\widetilde{\sigma p}, y\widetilde{\sigma p}\rangle}{\partial x_{A_2}} = 0 \\
\cdots \\
\frac{\partial \langle \widetilde{\sigma_P^2}; w\widetilde{\sigma p}, u\widetilde{\sigma p}, y\widetilde{\sigma p}\rangle}{\partial x_{A_n}} = 0
\end{cases}
\tag{73}
$$

Based on the condition for minimizing the neutrosophic portfolio risk will be obtained:

$$\left\{\begin{array}{c} \langle 2x_{A_1}\widetilde{\sigma}^2_{A_1}; w\widetilde{\sigma}_{A_1}, u\widetilde{\sigma}_{A_1}, y\widetilde{\sigma}_{A_1}\rangle + \langle 2x_{A_2}\widetilde{\sigma}_{A_1A_2}; w\widetilde{\sigma}_{A_1} \wedge w\widetilde{\sigma}_{A_2}, u\widetilde{\sigma}_{A_1} \vee u\widetilde{\sigma}_{A_2}, y\widetilde{\sigma}_{A_1} \vee y\widetilde{\sigma}_{A_2}\rangle + \\ + \cdots + \langle 2x_{A_n}\widetilde{\sigma}_{A_1A_n}; w\widetilde{\sigma}_{A_1} \wedge w\widetilde{\sigma}_{A_n}, u\widetilde{\sigma}_{A_1} \vee u\widetilde{\sigma}_{A_n}, y\widetilde{\sigma}_{A_1} \vee y\widetilde{\sigma}_{A_n}\rangle = 0 \\ \langle 2x_{A_2}\widetilde{\sigma}^2_{A_2}; w\widetilde{\sigma}_{A_1}, u\widetilde{\sigma}_{A_1}, y\widetilde{\sigma}_{A_1}\rangle + \langle 2x_{A_1}\widetilde{\sigma}_{A_2A_1}; w\widetilde{\sigma}_{A_2} \wedge w\widetilde{\sigma}_{A_1}, u\widetilde{\sigma}_{A_2} \vee u\widetilde{\sigma}_{A_1}, y\widetilde{\sigma}_{A_2} \vee y\widetilde{\sigma}_{A_1}\rangle \\ + \cdots + \langle 2x_{A_n}\widetilde{\sigma}_{A_2A_n}; w\widetilde{\sigma}_{A_2} \wedge w\widetilde{\sigma}_{A_n}, u\widetilde{\sigma}_{A_2} \vee u\widetilde{\sigma}_{A_n}, y\widetilde{\sigma}_{A_2} \vee y\widetilde{\sigma}_{A_n}\rangle = 0 \\ \langle 2x_{A_1}\widetilde{\sigma}_{A_nA_1}; w\widetilde{\sigma}_{A_n} \wedge w\widetilde{\sigma}_{A_1}, u\widetilde{\sigma}_{A_n} \vee u\widetilde{\sigma}_{A_1}, y\widetilde{\sigma}_{A_n} \vee y\widetilde{\sigma}_{A_1}\rangle + \\ + \langle 2x_{A_n}\widetilde{\sigma}_{A_nA_1}; w\widetilde{\sigma}_{A_n} \wedge w\widetilde{\sigma}_{A_2}, u\widetilde{\sigma}_{A_n} \vee u\widetilde{\sigma}_{A_2}, y\widetilde{\sigma}_{A_n} \vee y\widetilde{\sigma}_{A_2}\rangle + \cdots + 2x_{A_n}\widetilde{\sigma}^2_{A_n}; w\widetilde{\sigma}_{A_n}, u\widetilde{\sigma}_{A_n}, y\widetilde{\sigma}_{A_n} = 0 \\ x_{A_1} + x_{A_2} + \cdots + x_{A_n} = 1 \end{array}\right. \tag{74}$$

In matrix form the equations above, ordered according to the neutrosophic variance are written as follows:

$$2\langle x_{A_1}\widetilde{\sigma}_{A_{11}}; w\widetilde{\sigma}_{A_{11}}, u\widetilde{\sigma}_{A_{11}}, y\widetilde{\sigma}_{A_{11}}\rangle + 2(x_{A_2}\ldots x_{A_n})\begin{pmatrix} \langle\widetilde{\sigma}_{A_{12}}; w\widetilde{\sigma}_{A_{12}}, u\widetilde{\sigma}_{A_{12}}, y\widetilde{\sigma}_{A_{12}}\rangle \\ \langle\widetilde{\sigma}_{A_{13}}; w\widetilde{\sigma}_{A_{13}}, u\widetilde{\sigma}_{A_{13}}, y\widetilde{\sigma}_{A_{13}}\rangle \\ \cdots \\ \langle\widetilde{\sigma}_{A_{1n}}; w\widetilde{\sigma}_{A_{1n}}, u\widetilde{\sigma}_{A_{1n}}, y\widetilde{\sigma}_{A_{1n}}\rangle \end{pmatrix} = 0 \tag{75}$$

$$2\langle x_{A_2}\widetilde{\sigma}_{A_{22}}; w\widetilde{\sigma}_{A_{22}}, u\widetilde{\sigma}_{A_{22}}, y\widetilde{\sigma}_{A_{22}}\rangle + 2(x_{A_1}\ldots x_{A_n})\begin{pmatrix} \widetilde{\sigma}_{A_{21}}; w\widetilde{\sigma}_{A_{21}}, u\widetilde{\sigma}_{A_{21}}, y\widetilde{\sigma}_{A_{21}} \\ \widetilde{\sigma}_{A_{23}}; w\widetilde{\sigma}_{A_{23}}, u\widetilde{\sigma}_{A_{23}}, y\widetilde{\sigma}_{A_{23}} \\ \cdots \\ \widetilde{\sigma}_{A_{2n}}; w\widetilde{\sigma}_{A_{2n}}, u\widetilde{\sigma}_{A_{2n}}, y\widetilde{\sigma}_{A_{2n}} \end{pmatrix} = 0 \tag{76}$$

$$\begin{array}{l} 2\langle x_{A_n}\widetilde{\sigma}_{A_{nn}}; w\widetilde{\sigma}_{A_{nn}}, u\widetilde{\sigma}_{A_{nn}}, y\widetilde{\sigma}_{A_{nn}}\rangle \\ \qquad + \;+ 2(x_{A_1}\ldots x_{A_{n-1}})\begin{pmatrix} \langle\widetilde{\sigma}_{A_{n1}}; w\widetilde{\sigma}_{A_{n1}}, u\widetilde{\sigma}_{A_{n1}}, y\widetilde{\sigma}_{A_{n1}}\rangle \\ \langle\widetilde{\sigma}_{A_{n3}}; w\widetilde{\sigma}_{A_{n3}}, u\widetilde{\sigma}_{A_{n3}}, y\widetilde{\sigma}_{A_{n3}}\rangle \\ \cdots \\ \langle\widetilde{\sigma}_{A_{nn-1}}; w\widetilde{\sigma}_{A_{nn-1}}, u\widetilde{\sigma}_{A_{nn-1}}, y\widetilde{\sigma}_{A_{nn-1}}\rangle \end{pmatrix} = 0 \end{array} \tag{77}$$

The last neutrosophic portfolio equation $x_{A_1} + x_{A_2} + \cdots + x_{A_n} = 1$, written in the matrix form will be:

$$\left\{ x_{A_1} + \left(x_{A_2}\ldots x_{A_n}\right)\begin{pmatrix} 1 \\ \ldots \\ 1 \end{pmatrix} = 1 \right. \tag{78}$$

Resulting that:

$$x_{A_2}\ldots x_{A_n} = \left(1 - x_{A_1}\right)e^{-1} \tag{79}$$

By replacing the obtained expression for $x_{A_2}\ldots x_{A_n}$ in the first relationship, it is obtained that:

$$2\langle x_{A_1}\sigma_{A_{11}}; w\widetilde{\sigma}_{A_{11}}, u\widetilde{\sigma}_{A_{11}}, y\widetilde{\sigma}_{A_{11}}\rangle + 2(1 - x_{A_1})e^{-1}\begin{pmatrix} \langle\widetilde{\sigma}_{A_{12}}; w\widetilde{\sigma}_{A_{12}}, u\widetilde{\sigma}_{A_{12}}, y\widetilde{\sigma}_{A_{12}}\rangle \\ \langle\widetilde{\sigma}_{A_{13}}; w\widetilde{\sigma}_{A_{13}}, u\widetilde{\sigma}_{A_{13}}, y\widetilde{\sigma}_{A_{13}}\rangle \\ \cdots \\ \langle\widetilde{\sigma}_{A_{1n}}; w\widetilde{\sigma}_{A_{1n}}, u\widetilde{\sigma}_{A_{1n}}, y\widetilde{\sigma}_{A_{1n}}\rangle \end{pmatrix} = 0 \tag{80}$$

$$\begin{array}{c} x_{A_1}\left[\begin{pmatrix} \langle\widetilde{\sigma}_{A_{12}}; w\widetilde{\sigma}_{A_{12}}, u\widetilde{\sigma}_{A_{12}}, y\widetilde{\sigma}_{A_{12}}\rangle \\ \langle\widetilde{\sigma}_{A_{13}}; w\widetilde{\sigma}_{A_{13}}, u\widetilde{\sigma}_{A_{13}}, y\widetilde{\sigma}_{A_{13}}\rangle \\ \cdots \\ \langle\widetilde{\sigma}_{A_{1n}}; w\widetilde{\sigma}_{A_{1n}}, u\widetilde{\sigma}_{A_{1n}}, y\widetilde{\sigma}_{A_{1n}}\rangle \end{pmatrix} - \begin{pmatrix} \langle\sigma_{A_{11}}; w\widetilde{\sigma}_{A_{11}}, u\widetilde{\sigma}_{A_{11}}, y\widetilde{\sigma}_{A_{11}}\rangle \\ \langle\sigma_{A_{11}}; w\widetilde{\sigma}_{A_{11}}, u\widetilde{\sigma}_{A_{11}}, y\widetilde{\sigma}_{A_{11}}\rangle \\ \cdots \\ \langle\sigma_{A_{11}}; w\widetilde{\sigma}_{A_{11}}, u\widetilde{\sigma}_{A_{11}}, y\widetilde{\sigma}_{A_{11}}\rangle \end{pmatrix}\right] \\ = \begin{pmatrix} \langle\widetilde{\sigma}_{A_{12}}; w\widetilde{\sigma}_{A_{12}}, u\widetilde{\sigma}_{A_{12}}, y\widetilde{\sigma}_{A_{12}}\rangle \\ \langle\widetilde{\sigma}_{A_{13}}; w\widetilde{\sigma}_{A_{13}}, u\widetilde{\sigma}_{A_{13}}, y\widetilde{\sigma}_{A_{13}}\rangle \\ \cdots \\ \langle\widetilde{\sigma}_{A_{1n}}; w\widetilde{\sigma}_{A_{1n}}, u\widetilde{\sigma}_{A_{1n}}, y\widetilde{\sigma}_{A_{1n}}\rangle \end{pmatrix} \end{array} \tag{81}$$

$$x_{A_1} = \frac{\begin{pmatrix} \langle\widetilde{\sigma}_{A_{12}}; w\widetilde{\sigma}_{A_{12}}, u\widetilde{\sigma}_{A_{12}}, y\widetilde{\sigma}_{A_{12}}\rangle \\ \langle\widetilde{\sigma}_{A_{13}}; w\widetilde{\sigma}_{A_{13}}, u\widetilde{\sigma}_{A_{13}}, y\widetilde{\sigma}_{A_{13}}\rangle \\ \cdots \\ \langle\widetilde{\sigma}_{A_{1n}}; w\widetilde{\sigma}_{A_{1n}}, u\widetilde{\sigma}_{A_{1n}}, y\widetilde{\sigma}_{A_{1n}}\rangle \end{pmatrix} - \begin{pmatrix} \langle\sigma_{A_{11}}; w\widetilde{\sigma}_{A_{11}}, u\widetilde{\sigma}_{A_{11}}, y\widetilde{\sigma}_{A_{11}}\rangle \\ \langle\sigma_{A_{11}}; w\widetilde{\sigma}_{A_{11}}, u\widetilde{\sigma}_{A_{11}}, y\widetilde{\sigma}_{A_{11}}\rangle \\ \cdots \\ \langle\sigma_{A_{11}}; w\widetilde{\sigma}_{A_{11}}, u\widetilde{\sigma}_{A_{11}}, y\widetilde{\sigma}_{A_{11}}\rangle \end{pmatrix}}{\begin{pmatrix} \langle\widetilde{\sigma}_{A_{12}}; w\widetilde{\sigma}_{A_{12}}, u\widetilde{\sigma}_{A_{12}}, y\widetilde{\sigma}_{A_{12}}\rangle \\ \langle\widetilde{\sigma}_{A_{13}}; w\widetilde{\sigma}_{A_{13}}, u\widetilde{\sigma}_{A_{13}}, y\widetilde{\sigma}_{A_{13}}\rangle \\ \cdots \\ \langle\widetilde{\sigma}_{A_{1n}}; w\widetilde{\sigma}_{A_{1n}}, u\widetilde{\sigma}_{A_{1n}}, y\widetilde{\sigma}_{A_{1n}}\rangle \end{pmatrix}} \tag{82}$$

For the weight x_{A_k} of the asset A_k it will be obtained:

$$x_{A_k} = \frac{\begin{pmatrix} \langle\widetilde{\sigma}_{A_{k2}}; w\widetilde{\sigma}_{A_{k2}}, u\widetilde{\sigma}_{A_{k2}}, y\widetilde{\sigma}_{A_{k2}}\rangle \\ \langle\widetilde{\sigma}_{A_{k3}}; w\widetilde{\sigma}_{A_{k3}}, u\widetilde{\sigma}_{A_{k3}}, y\widetilde{\sigma}_{A_{k3}}\rangle \\ \cdots \\ \langle\widetilde{\sigma}_{A_{kn}}; w\widetilde{\sigma}_{A_{kn}}, u\widetilde{\sigma}_{A_{kn}}, y\widetilde{\sigma}_{A_{kn}}\rangle \end{pmatrix} - \begin{pmatrix} \langle\sigma_{A_{kk}}; w\widetilde{\sigma}_{A_{kk}}, u\widetilde{\sigma}_{A_{kk}}, y\widetilde{\sigma}_{A_{kk}}\rangle \\ \langle\sigma_{A_{kk}}; w\widetilde{\sigma}_{A_{kk}}, u\widetilde{\sigma}_{A_{kk}}, y\widetilde{\sigma}_{A_{kk}}\rangle \\ \cdots \\ \langle\sigma_{A_{kk}}; w\widetilde{\sigma}_{A_{kk}}, u\widetilde{\sigma}_{A_{kk}}, y\widetilde{\sigma}_{A_{kk}}\rangle \end{pmatrix}}{\begin{pmatrix} \langle\widetilde{\sigma}_{A_{k2}}; w\widetilde{\sigma}_{A_{k2}}, u\widetilde{\sigma}_{A_{k2}}, y\widetilde{\sigma}_{A_{k2}}\rangle \\ \langle\widetilde{\sigma}_{A_{k3}}; w\widetilde{\sigma}_{A_{k3}}, u\widetilde{\sigma}_{A_{k3}}, y\widetilde{\sigma}_{A_{k3}}\rangle \\ \cdots \\ \langle\widetilde{\sigma}_{A_{kn}}; w\widetilde{\sigma}_{A_{kn}}, u\widetilde{\sigma}_{A_{kn}}, y\widetilde{\sigma}_{A_{kn}}\rangle \end{pmatrix}} \tag{83}$$

Respecting the condition that:

$$\begin{pmatrix} \langle\widetilde{\sigma}_{A_{k2}}; w\widetilde{\sigma}_{A_{k2}}, u\widetilde{\sigma}_{A_{k2}}, y\widetilde{\sigma}_{A_{k2}}\rangle \\ \langle\widetilde{\sigma}_{A_{k3}}; w\widetilde{\sigma}_{A_{k3}}, u\widetilde{\sigma}_{A_{k3}}, y\widetilde{\sigma}_{A_{k3}}\rangle \\ \cdots \\ \langle\widetilde{\sigma}_{A_{kn}}; w\widetilde{\sigma}_{A_{kn}}, u\widetilde{\sigma}_{A_{kn}}, y\widetilde{\sigma}_{A_{kn}}\rangle \end{pmatrix} \neq 0 \tag{84}$$

6. Numerical Applications

6.1. Numerical Application for the Case of the Neutrosophic Portfolio Consisting of Two Financial Assets

Two financial assets (A_1, A_2) are considered to which two triangular neutrosophic fuzzy numbers are specified for the financial assets return of the form:

$$\widetilde{R_{A1}} = \langle(0.2\ 0.3\ 0.5); 0.5,\ 0.2,\ 0.3\rangle \text{ for } \widetilde{R_A} \in [0.2; 0.5]$$

$$\widetilde{R_{A2}} = \langle(0.1\ 0.2\ 0.3); 0.6,\ 0.3,\ 0.2\rangle \text{ for } \widetilde{R_A} \in [0.1; 0.3]$$

And we aim at:

(a) determining the variance-covariance matrix;

(b) calculating the rentability and the variance for a given portfolio P formed by the two financial assets, having the structure $P = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ and with $x_2 = 1 - x_1$;

(c) determining the structure of a random portfolio P for which the risk is minimum (the value of the minimum risk is also required).

Starting from Example 2, the following elements are known:

- the neutrosophic risk for asset A_1 :

$$\widetilde{\sigma_f}^2 A_1 = \langle 0.0225; 0.5,\ 0.2,\ 0.3\rangle$$

- the neutrosophic risk for asset A_2 :

$$\widetilde{\sigma}_{f_{A_2}}^2 = \langle 0.0180; 0.6,\ 0.3,\ 0.2\rangle$$

Also, from Example 2 we know that the covariance between asset A_1 and A_2 has the value:

$$\sigma_{A_1A_3} = \langle 0.1914; 0.5,\ 0.2,\ 0.3\rangle$$

(a) In this context, the variance-covariance matrix has the following form:

$$\Omega = \begin{pmatrix} \widetilde{\sigma}_{fA_1A_1} & \widetilde{\sigma}_{fA_1A_3} \\ \widetilde{\sigma}_{fA_2A_1} & \widetilde{\sigma}_{fA_2A_2} \end{pmatrix} = \begin{pmatrix} \langle 0.0225; 0.5,\ 0.2,\ 0.3\rangle & \langle 0,0705; 0.6,\ 0.2,\ 0.2\rangle \\ \langle 0.0705; 0.6,\ 0.2,\ 0.2\rangle & \langle 0.0180; 0.6,\ 0.3,\ 0.2\rangle \end{pmatrix}$$

(b) It is known that the rentability of the neutrosophic portfolio P has the following form:

$$\langle \widetilde{R}_P; w\widetilde{Rp}, u\widetilde{Rp}, y\widetilde{Rp}\rangle = \langle x_{A_1}\widetilde{R}_{A_1}; w\widetilde{R}_{A_1}, u\widetilde{R}_{A_1}, y\widetilde{R}_{A_1}\rangle + \langle x_{A_2}\widetilde{R}_{A_2}; w\widetilde{R}_{A_2}, u\widetilde{R}_{A_2}, y\widetilde{R}_{A_2}\rangle$$

Also, it is given that: $x_2 = 1 - x_1$ and that the P portfolio has the following form: $P = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$;

By replacing these expressions in the formula for the portfolio rentability $\langle \widetilde{R}_P; w\widetilde{Rp}, u\widetilde{Rp}, y\widetilde{Rp}\rangle$ we get that:

$$\begin{aligned}\langle \widetilde{R}_P; w\widetilde{Rp}, u\widetilde{Rp}, y\widetilde{Rp}\rangle &= \langle x_{A_1}\widetilde{R}_{A_1}; w\widetilde{R}_{A_1}, u\widetilde{R}_{A_1}, y\widetilde{R}_{A_1}\rangle + \langle x_{A_2}\widetilde{R}_{A_2}; w\widetilde{R}_{A_2}, u\widetilde{R}_{A_2}, y\widetilde{R}_{A_2}\rangle \\ &= \langle x_{A_1}(0.2\ 0.3\ 0.5); 0.5,\ 0.2,\ 0.3\rangle + \langle \left(1 - x_{A_1}\right)(0.1\ 0.2\ 0.3); 0.6,\ 0.3,\ 0.2\rangle\end{aligned}$$

$$\langle \widetilde{R}_P; w\widetilde{Rp}, u\widetilde{Rp}, y\widetilde{Rp}\rangle = \langle x_{A_1}(0.1\ 0.1\ 0.2); 0.6,\ 0.2,\ 0.2\rangle - (\langle 0.1\ 0.2\ 0.3); 0.6,\ 0.3,\ 0.2\rangle$$

As for the portfolio risk, it will be of the following form: $\langle \widetilde{\sigma}_P^2; w\widetilde{\sigma p}, u\widetilde{\sigma p}, y\widetilde{\sigma p}\rangle = \langle x_{A_1}^2\widetilde{\sigma}_{A_1}^2; w\widetilde{\sigma}_{A_1}, u\widetilde{\sigma}_{A_1}, y\widetilde{\sigma}_{A_1}\rangle + \langle x_{A_2}^2\widetilde{\sigma}_{A_2}^2; w\widetilde{\sigma}_{A_2}, u\widetilde{\sigma}_{A_2}, y\widetilde{\sigma}_{A_2}\rangle + \langle 2x_{A_1}x_{A_2}\sigma_{A_1A_2}; w\sigma_A, u\sigma_A, y\sigma_A\rangle$

By replacing it in the equations for x_1 and $x_2 = 1 - x_1$ we get that:

$$\begin{aligned}&\langle \widetilde{\sigma}_P^2; w\widetilde{\sigma p}, u\widetilde{\sigma p}, y\widetilde{\sigma p}\rangle \\ &= \langle x_{A_1}^2\widetilde{\sigma}_{A_1}^2; w\widetilde{\sigma}_{A_1}, u\widetilde{\sigma}_{A_1}, y\widetilde{\sigma}_{A_1}\rangle + \langle (1 - x_{A_1})^2\widetilde{\sigma}_{A_2}^2; w\widetilde{\sigma}_{A_2}, u\widetilde{\sigma}_{A_2}, y\widetilde{\sigma}_{A_2}\rangle \\ &+\langle 2x_{A_1}(1 - x_{A_1})\sigma_{A_1A_2}; w\sigma_A, u\sigma_A, y\sigma_A\rangle\end{aligned}$$

$$\begin{aligned}&\langle \widetilde{\sigma}_P^2; w\widetilde{\sigma p}, u\widetilde{\sigma p}, y\widetilde{\sigma p}\rangle \\ &= \langle x_{A_1}^2 0.0225; 0.5,\ 0.2,\ 0.3\rangle + \langle (1 - 2x_{A_1} + x_{A_1}^2)0.0180; 0.6,\ 0.3,\ 0.2\rangle \\ &+\langle (2x_{A_1} - 2x_{A_1}^2)0.0705; 0.6,\ 0.2,\ 0.2\rangle\end{aligned}$$

$$\begin{aligned}&\langle \widetilde{\sigma}_P^2; w\widetilde{\sigma p}, u\widetilde{\sigma p}, y\widetilde{\sigma p}\rangle \\ &= -\langle x_{A_1}^2 0.1005; 0.6,\ 0.2,\ 0.2\rangle - \langle 0.105x_{A_1}; 0.6,\ 0.2,\ 0.2\rangle + \langle 0.141; 0.6,\ 0.2,\ 0.2\rangle\end{aligned}$$

(c) Further on, we know from Example 1 that the covariance between A_1 and A_2 has the following form:

$$\widetilde{\sigma}_{fA_1A_2} = \langle 0.0705; 0.6,\ 0.2,\ 0.2\rangle$$

By replacing the values obtained in the expression of x_{A_1} and x_{A_2} it is obtained that:

$$\widetilde{\sigma}_{fA_1A_2} = \langle 0.0705; 0.6,\ 0.2,\ 0.2\rangle$$

$$x_{A_1} = \frac{\langle \widetilde{\sigma}_{A_1A_2}; w\widetilde{\sigma}_A, u\widetilde{\sigma}_A, y\widetilde{\sigma}_A \rangle}{\langle \widetilde{\sigma}_{A_1A_2}; w\widetilde{\sigma}_A, u\widetilde{\sigma}_A, y\widetilde{\sigma}_A \rangle - \langle \widetilde{\sigma}^2_{A_1}; w\widetilde{\sigma}_{A_1}, u\widetilde{\sigma}_{A_1}, y\widetilde{\sigma}_{A_1} \rangle}$$

$$x_{A_1} = \frac{\langle 0,1914; 0.5,\ 0.2,\ 0.3 \rangle}{\langle 0,1914; 0.5,\ 0.2,\ 0.3 \rangle - \langle 0.0225; 0.5,\ 0.2,\ 0.3 \rangle} \times 100$$

$$x_{A_1} = \frac{\langle 0,1914; 0.5,\ 0.2,\ 0.3 \rangle}{\langle 0,1689; 0.5,\ 0.2,\ 0.3 \rangle} \times 100$$

$$x_{A_1} = 112.72\%$$

$$x_{A_2} = \frac{\langle \widetilde{\sigma}^2_{A_1}; w\widetilde{\sigma}_{A_1}, u\widetilde{\sigma}_{A_1}, y\widetilde{\sigma}_{A_1} \rangle}{\langle \widetilde{\sigma}^2_{A_1}; w\widetilde{\sigma}_{A_1}, u\widetilde{\sigma}_{A_1}, y\widetilde{\sigma}_{A_1} \rangle - \langle \widetilde{\sigma}_{A_1A_2}; w\widetilde{\sigma}_A, u\widetilde{\sigma}_A, y\widetilde{\sigma}_A \rangle}$$

$$x_{A_2} = \frac{\langle 0.0225; 0.5,\ 0.2,\ 0.3 \rangle}{\langle 0.0225; 0.5,\ 0.2,\ 0.3 - 0,1914; 0.5,\ 0.2,\ 0.3 \rangle} \times 100$$

$$x_{A_2} = -\frac{\langle 0.0225; 0.5,\ 0.2,\ 0.3 \rangle}{\langle 0,1689; 0.5,\ 0.2,\ 0.3 \rangle} \times 100$$

$$x_{A_2} = -13.250\%$$

The portfolio risk will be given by the relationship:

$$\begin{aligned} &\langle \widetilde{\sigma}^2_P; w\widetilde{\sigma p}, u\widetilde{\sigma p}, y\widetilde{\sigma p} \rangle \\ &\quad = \langle x^2_{A_1} \widetilde{\sigma}^2_{A_1}; w\widetilde{\sigma}_{A_1}, u\widetilde{\sigma}_{A_1}, y\widetilde{\sigma}_{A_1} \rangle + \langle x^2_{A_2} \widetilde{\sigma}^2_{A_2}; w\widetilde{\sigma}_{A_2}, u\widetilde{\sigma}_{A_2}, y\widetilde{\sigma}_{A_2} \rangle \\ &\quad + \langle 2x_{A_1} x_{A_2} \sigma_{A_1A_2}; w\sigma_A, u\sigma_A, y\sigma_A \rangle \end{aligned}$$

$$\begin{aligned} &\langle \widetilde{\sigma}^2_P; w\widetilde{\sigma p}, u\widetilde{\sigma p}, y\widetilde{\sigma p} \rangle \\ &\quad = (1.1272)^2 \langle 0.0225; 0.5,\ 0.2,\ 0.3 \rangle \\ &\quad + (-0.13250)^2 \langle 0.0180; 0.6,\ 0.3,\ 0.2 \rangle \\ &\quad + 2(1.1272)(-0.13250) \langle 0.1914; 0.5,\ 0.2,\ 0.3 \rangle \end{aligned}$$

$$\begin{aligned} &\langle \widetilde{\sigma}^2_P; w\widetilde{\sigma p}, u\widetilde{\sigma p}, y\widetilde{\sigma p} \rangle \\ &\quad = \langle 0.0285; 0.5,\ 0.2,\ 0.3 \rangle + \langle 0.0033; 0.5,\ 0.2,\ 0.3 \rangle \\ &\quad + \langle 0.05717; 0.5,\ 0.2,\ 0.3 \rangle \end{aligned}$$

$$\langle \widetilde{\sigma}^2_P; w\widetilde{\sigma p}, u\widetilde{\sigma p}, y\widetilde{\sigma p} \rangle = \langle 0.08897; 0.5,\ 0.2,\ 0.3 \rangle$$

$$\langle \sigma_p; w\widetilde{\sigma p}, u\widetilde{\sigma p}, y\widetilde{\sigma p} \rangle = \sqrt{\langle \widetilde{\sigma}^2_P; w\widetilde{\sigma p}, u\widetilde{\sigma p}, y\widetilde{\sigma p} \rangle} = \sqrt{\langle 0,08897; 0.5,\ 0.2,\ 0.3 \rangle} = 0.2982$$

Conclusion: The neutrosophic portfolio risk is minimal and registers the value of 29.82%.

6.2. Numerical Application for the Case of the Neutrosophic Portfolio Consisting of N Financial Assets

For three financial assets held by three listed companies (A_1, A_2, A_3), the financial return was determined according to the information provided by the stock exchange website. The financial returns were fuzzified with the help of three triangular neutrosophic numbers and the following values were obtained:

$$\widetilde{R_{A1}} = \langle (0.3\ 0.4\ 0.6); 0.5,\ 0.2,\ 0.3 \rangle, \text{ for } \widetilde{R_A} \in [0.3; 0.6]$$

$$\widetilde{R_{A2}} = \langle (0.15\ 0.25\ 0.35); 0.6,\ 0.3,\ 0.2 \rangle, \text{ for } \widetilde{R_A} \in [0.15; 0.35]$$

$$\widetilde{R_{A3}} = \langle (0.25\ 0.45\ 0.65); 0.4,\ 0.3,\ 0.3 \rangle, \text{ for } \widetilde{R_A} \in [0.25; 0.65]$$

In order to establish:

- The variance-covariance matrix of the neutrosophic portfolio;

- The weight of financial assets $x_{A_1}, x_{A_2}, x_{A_3}$ in the total value of the neutrosophic portfolio so that the neutrosophic portfolio risk is minimal;
- The value of the neutrosophic portfolio return and risk.

The followings are undertaken:

For computing the variance-covariance matrix, the financial returns of the three assets are established in a first stage according to the calculation formula:

$$\langle \widetilde{R}_{A_i}; w\widetilde{R}_{A_i}, u\widetilde{R}_{A_i}, y\widetilde{R}_{A_i} \rangle = \langle \left(\frac{1}{6}\left(\widetilde{R}_{Aai} + \widetilde{R}_{Aci} \right) + \frac{2}{3}\widetilde{R}_{Abi} \right); w\widetilde{R}_{A_i}, u\widetilde{R}_{A_i}, y\widetilde{R}_{A_i} \rangle$$

Thus:

$$\begin{aligned}\langle \widetilde{R}_{A_1}; w\widetilde{R}_{A_1}, u\widetilde{R}_{A_1}, y\widetilde{R}_{A_1} \rangle &= \langle \left(\tfrac{1}{6}(0.3 + 0.6) + \tfrac{2}{3} \times 0.4 \right); 0.5,\ 0.2,\ 0.3 \rangle \\ &= \langle 0.166 \times 0.9 + 0.666 \times 0.4; 0.5,\ 0.2,\ 0.3 \rangle \\ &= \langle 0.149 + 0.266; 0.5,\ 0.2,\ 0.3 \rangle = \langle 0.415; 0.5,\ 0.2,\ 0.3 \rangle\end{aligned}$$

Proceeding in the same way, we get the following results:

$$\langle \widetilde{R}_{A_2}; w\widetilde{R}_{A_2}, u\widetilde{R}_{A_2}, y\widetilde{R}_{A_2} \rangle = \langle 0.249; 0.6,\ 0.3,\ 0.2 \rangle$$

$$\langle \widetilde{R}_{A_3}; w\widetilde{R}_{A_3}, u\widetilde{R}_{A_3}, y\widetilde{R}_{A_3} \rangle = \langle 0.448; 0.4,\ 0.3,\ 0.3 \rangle$$

The following calculation formula is used to determine the variance of the three financial assets:

$$\begin{aligned}\widetilde{\sigma_f A_i} = \langle \tfrac{1}{4}\left[\left(\widetilde{Ra}_{bi} - \widetilde{Ra}_{ai} \right)^2 + \left(\widetilde{Ra}_{ci} - \widetilde{Ra}_{bi} \right)^2 \right]; w\widetilde{Ra}, u\widetilde{Ra}, y\widetilde{Ra} \rangle \\ + \langle \tfrac{2}{3}\left[\widetilde{Ra}_{ai}\left(\widetilde{Ra}_{bi} - \widetilde{Ra}_{ai} \right) - \widetilde{Ra}_{ci}\left(\widetilde{Ra}_{ci} - \widetilde{Ra}_{bi} \right) \right]; w\widetilde{Ra}, u\widetilde{Ra}, y\widetilde{Ra} \rangle \\ + \langle \tfrac{1}{2}\left(\widetilde{Ra}_{ai}^2 + \widetilde{Ra}_{ci}^2 \right); w\widetilde{Ra}, u\widetilde{Ra}, y\widetilde{Ra} \rangle - \langle \tfrac{1}{2} E_f^2\left(\widetilde{Ra}_i \right); w\widetilde{Ra}, u\widetilde{Ra}, y\widetilde{Ra} \rangle\end{aligned}$$

By replacing the data in the above expression, we will have:

$$\begin{aligned}&\widetilde{\sigma}_f a_1 = \langle \tfrac{1}{4}[(0.4 - 0.3)^2 + (0.6 - 0.4)^2]; 0.5,\ 0.2,\ 0.3 \rangle + \langle \tfrac{2}{3}(0.3(0.4 - 0.3) - \\ &0.6(0.6 - 0.4)); 0.5,\ 0.2,\ 0.3 \rangle + \langle \tfrac{1}{2}(0.3^2 + 0.6^2); 0.5,\ 0.2,\ 0.3 \rangle - \langle \tfrac{1}{2}(0.415)^2; 0.5,\ 0.2,\ 0.3 \rangle \\ &= \langle 0.0925; 0.5,\ 0.2,\ 0.3 \rangle\end{aligned}$$

For the remaining of the values we will obtain:

$$\widetilde{\sigma}_f a_2 = \langle 0.033; 0.6,\ 0.3,\ 0.2 \rangle$$

$$\widetilde{\sigma}_f a_3 = \langle 0.0910; 0.4,\ 0.3,\ 0.3 \rangle$$

In order to establish the covariance between these three assets, respectively to measure the intensity of the connection between the financial assets, will be applied the following calculation formula:

$$\begin{aligned}cov(\widetilde{Ra}_i, \widetilde{Ra}_j) = \langle \Big(\tfrac{1}{4}\Big[(\widetilde{Ra}_{bii} - \widetilde{Ra}_{aii})(\widetilde{Ra}_{bji} - \widetilde{Ra}_{aji}) \\ + (\widetilde{Ra}_{cii} - \widetilde{Ra}_{bii})(\widetilde{Ra}_{cji} - \widetilde{Ra}_{bji}) \Big] \\ + \tfrac{1}{3}\Big\{ \Big[\widetilde{Ra}_{aji}(\widetilde{Ra}_{bii} - \widetilde{Ra}_{aii}) + \widetilde{Ra}_{aii}(\widetilde{Ra}_{bji} - \widetilde{Ra}_{aji}) \Big] \\ - \Big[\widetilde{Ra}_{cii}(\widetilde{Ra}_{cji} - \widetilde{Ra}_{bji}) + \widetilde{Ra}_{cji}(\widetilde{Ra}_{cii} - \widetilde{Ra}_{bii}) \Big] \Big\} \\ + \tfrac{1}{2}(\widetilde{Ra}_{aii}\widetilde{Ra}_{aji} + \widetilde{Ra}_{cii}\widetilde{Ra}_{cji}) \\ + \tfrac{1}{2}E_f(\widetilde{Ra}_i)E_f(\widetilde{Ra}_j) \Big); w\widetilde{Ra}_i \wedge\ w\widetilde{Ra}_j, u\widetilde{Ra}_i \vee u\widetilde{Ra}_j, y\widetilde{Ra}_i \vee y\widetilde{Ra}_j \rangle\end{aligned}$$

$$\sigma_{A_1A_2} = \langle 0.160; 0.6,\ 0.2,\ 0.2 \rangle$$

Proceeding in the same manner, the following results are obtained:

$$\sigma_{A_1A_3} = \langle 0.284; 0.6,\ 0.2,\ 0.2\rangle$$

$$\sigma_{A_2A_3} = \langle 0.171; 0.6,\ 0.2,\ 0.2\rangle$$

The variance-covariance matrix will be of the form:

$$\Omega = \begin{pmatrix} \widetilde{\sigma}_{fA11} & \widetilde{\sigma}_{fA12} & \widetilde{\sigma}_{fA13} \\ \widetilde{\sigma}_{fA21} & \widetilde{\sigma}_{fA22} & \widetilde{\sigma}_{fA23} \\ \widetilde{\sigma}_{fA31} & \widetilde{\sigma}_{fA32} & \widetilde{\sigma}_{fA33} \end{pmatrix}$$

By replacing the above values, we will have:

$$\Omega = \begin{pmatrix} \langle 0.0925; 0.5,\ 0.2,\ 0.3\rangle & \langle 0.160; 0.6,\ 0.2,\ 0.2\rangle & \langle 0.284; 0.6,\ 0.2,\ 0.2\rangle \\ \langle 0.160; 0.6,\ 0.2,\ 0.2\rangle & \langle 0.033; 0.6,\ 0.3,\ 0.2\rangle & \langle 0.171; 0.6,\ 0.2,\ 0.2\rangle \\ \langle 0.284; 0.6,\ 0.2,\ 0.2\rangle & \langle 0.171; 0.6,\ 0.2,\ 0.2\rangle & \langle 0.0910; 0.4,\ 0.3,\ 0.3\rangle \end{pmatrix}$$

According to Theorem 2 the weight of the financial asset x_{A_1} in the total value of the portfolio will be given by the relation:

$$x_{A_1} = \frac{\begin{pmatrix} \langle \widetilde{\sigma}_{A_{12}}; w\widetilde{\sigma}_{A_{12}}, u\widetilde{\sigma}_{A_{12}}, y\widetilde{\sigma}_{A_{12}}\rangle \\ \langle \widetilde{\sigma}_{A_{13}}; w\widetilde{\sigma}_{A_{13}}, u\widetilde{\sigma}_{A_{13}}, y\widetilde{\sigma}_{A_{13}}\rangle \end{pmatrix} - \begin{pmatrix} \langle \sigma_{A_{11}}; w\widetilde{\sigma}_{A_{11}}, u\widetilde{\sigma}_{A_{11}}, y\widetilde{\sigma}_{A_{11}}\rangle \\ \langle \sigma_{A_{11}}; w\widetilde{\sigma}_{A_{11}}, u\widetilde{\sigma}_{A_{11}}, y\widetilde{\sigma}_{A_{11}}\rangle \end{pmatrix}}{\begin{pmatrix} \langle \widetilde{\sigma}_{A_{12}}; w\widetilde{\sigma}_{A_{12}}, u\widetilde{\sigma}_{A_{12}}, y\widetilde{\sigma}_{A_{12}}\rangle \\ \langle \widetilde{\sigma}_{A_{13}}; w\widetilde{\sigma}_{A_{13}}, u\widetilde{\sigma}_{A_{13}}, y\widetilde{\sigma}_{A_{13}}\rangle \end{pmatrix}}$$

Through the calculations the following value is obtained:

$$x_{A_1} = 0.2376$$

For the weight of the financial asset x_{A_2} and x_{A_3} in the total value of the neutrosophic portfolio, the same calculation formula will be used and the results are the following:

$$x_{A_2} = 0.6808$$

$$x_{A_3} = 0.0816$$

Conclusion: In order to mitigate the neutrosophic portfolio risk, it is necessary to invest: in the first financial asset (A_1) a weight of $x_{A_1} = 0.2376$, in the second asset (A_2) a weight of $x_{A_2} = 0.6808$ and respectively in the third financial asset (A_3) a weight $x_{A_3} = 0.0816$.

The neutrosophic portfolio return will be:

$$\begin{aligned} &\langle \widetilde{R}_p; w\widetilde{\sigma p}, u\widetilde{\sigma p}, y\widetilde{\sigma p}\rangle \\ &\quad = \langle x_{A_1}\widetilde{R}_{A_1}; w\widetilde{\sigma}_{A_1}, u\widetilde{\sigma}_{A_1}, y\widetilde{\sigma}_{A_1}\rangle + \langle x_{A_2}\widetilde{R}_{A_2}; w\widetilde{\sigma}_{A_2}, u\widetilde{\sigma}_{A_2}, y\widetilde{\sigma}_{A_2}\rangle \\ &\quad + \langle x_{A_3}\widetilde{R}_{A_3}; w\widetilde{\sigma}_{A_3}, u\widetilde{\sigma}_{A_3}, y\widetilde{\sigma}_{A_3}\rangle \end{aligned}$$

By replacing in the formula, will be obtained:

$$\begin{aligned} &\langle \widetilde{R}_p; w\widetilde{\sigma p}, u\widetilde{\sigma p}, y\widetilde{\sigma p}\rangle \\ &\quad = \langle 0.2376 \times 0.415; 0.5,\ 0.2,\ 0.3\rangle + \langle 0.6808 \times 0.249; 0.6,\ 0.3,\ 0.2\rangle \\ &\quad + \langle 0.0816 \times 0.448; 0.4,\ 0.3,\ 0.3\rangle \end{aligned}$$

$$\langle \widetilde{R}_p; w\widetilde{\sigma p}, u\widetilde{\sigma p}, y\widetilde{\sigma p}\rangle = \langle 0.0986; 0.5,\ 0.2,\ 0.3\rangle + \langle 0.1849; 0.6,\ 0.3,\ 0.2\rangle + \langle 0.0365; 0.4,\ 0.3,\ 0.3\rangle$$

After performing the neutrosophic calculations we obtain:

$$\langle \widetilde{R_p}; w\widetilde{\sigma p}, u\widetilde{\sigma p}, y\widetilde{\sigma p}\rangle = \langle 0.3200; 0.6,\ 0.2,\ 0.2\rangle$$

The neutrosophic portfolio risk will be as follows:

$$\begin{aligned}
&\langle \sigma_p^2; w\widetilde{\sigma p}, u\widetilde{\sigma p}, y\widetilde{\sigma p}\rangle \\
&\quad = \langle x_{A_1}^2 \widetilde{\sigma}_{A_1}^2; w\widetilde{\sigma}_{A_1}, u\widetilde{\sigma}_{A_1}, y\widetilde{\sigma}_{A_1}\rangle + \langle x_{A_2}^2 \widetilde{\sigma}_{A_2}^2; w\widetilde{\sigma}_{A_2}, u\widetilde{\sigma}_{A_2}, y\widetilde{\sigma}_{A_2}\rangle \\
&\quad + \langle x_{A_3}^2 \widetilde{\sigma}_{A_3}^2; w\widetilde{\sigma}_{A_3}, u\widetilde{\sigma}_{A_3}, y\widetilde{\sigma}_{A_3}\rangle \\
&\quad + 2x_{A_1}x_{A_2}\widetilde{\sigma}_{A_1A_2}; w\widetilde{\sigma}_{A_1} \wedge\ w\widetilde{\sigma}_{A_2}, u\widetilde{\sigma}_{A_1} \vee u\widetilde{\sigma}_{A_2}, y\widetilde{\sigma}_{A_1} \vee y\widetilde{\sigma}_{A_2} \\
&\quad + \langle 2x_{A_1}x_{A_3}\widetilde{\sigma}_{A_1A_3}; w\widetilde{\sigma}_{A_1} \wedge\ w\widetilde{\sigma}_{A_3}, u\widetilde{\sigma}_{A_1} \vee u\widetilde{\sigma}_{A_3}, y\widetilde{\sigma}_{A_1} \vee y\widetilde{\sigma}_{A_3}\rangle \\
&\quad + \langle 2x_{A_2}x_{A_1}\widetilde{\sigma}_{A_2A_1}; w\widetilde{\sigma}_{A_2} \wedge\ w\widetilde{\sigma}_{A_1}, u\widetilde{\sigma}_{A_2} \vee u\widetilde{\sigma}_{A_1}, y\widetilde{\sigma}_{A_2} \vee y\widetilde{\sigma}_{A_1}\rangle \\
&\quad + \langle 2x_{A_2}x_{A_3}\widetilde{\sigma}_{A_2A_3}; w\widetilde{\sigma}_{A_2} \wedge\ w\widetilde{\sigma}_{A_3}, u\widetilde{\sigma}_{A_2} \vee u\widetilde{\sigma}_{A_3}, y\widetilde{\sigma}_{A_2} \vee y\widetilde{\sigma}_{A_3}\rangle \\
&\quad + \langle 2x_{A_3}x_{A_1}\widetilde{\sigma}_{A_3A_1}; w\widetilde{\sigma}_{A_3} \wedge\ w\widetilde{\sigma}_{A_1}, u\widetilde{\sigma}_{A_3} \vee u\widetilde{\sigma}_{A_1}, y\widetilde{\sigma}_{A_3} \vee y\widetilde{\sigma}_{A_1}\rangle \\
&\quad + \langle 2x_{A_3}x_{A_2}\widetilde{\sigma}_{A_3A_2}; w\widetilde{\sigma}_{A_3} \wedge\ w\widetilde{\sigma}_{A_2}, u\widetilde{\sigma}_{A_3} \vee u\widetilde{\sigma}_{A_2}, y\widetilde{\sigma}_{A_3} \vee y\widetilde{\sigma}_{A_2}\rangle + \langle \sigma_p^2; w\widetilde{\sigma p}, u\widetilde{\sigma p}, y\widetilde{\sigma p}\rangle \\
&\quad = 49.42\%
\end{aligned}$$

Conclusion: In order to minimize the neutrosophic portfolio risk, the investments in financial assets must have the following weights: $x_{A_1} = 23.76\%$, $x_{A_2} = 68.08\%$ and $x_{A_3} = 8.16\%$. For these weights respective for the investments in financial assets A_1, A_2, A_3, the profitability of the neutrosophic portfolio will be of the form:

$$\langle \widetilde{R_p}; w\widetilde{\sigma p}, u\widetilde{\sigma p}, y\widetilde{\sigma p}\rangle = \langle 0.3200; 0.6,\ 0.2,\ 0.2\rangle$$

The neutrosophic portfolio risk will be minimal, respectively will have the value of $\langle \sigma_p; w\widetilde{\sigma p}, u\widetilde{\sigma p}, y\widetilde{\sigma p}\rangle = 49.42\%$. The risk was determined based on the high values of the return. The obtained results validate the risk minimization model for the neutrosophic portfolio, in the sense that for a financial return of 32%, the portfolio risk is high, reaching the value of 49.42%.

7. General Conclusions and Limitation

The neutrosophic portfolios are made up of financial assets for which it is possible to determine the financial performance indicators respectively: the neutrosophic return $\langle E_f(\widetilde{R_A}); w\widetilde{R_A}, u\widetilde{R_A}, y\widetilde{R_A}\rangle$ specific for the financial asset A_i, the neutrosophic risk $\langle \sigma f_{A_i}^2; w\widetilde{\sigma}_A, u\widetilde{\sigma}_A, y\widetilde{\sigma}_A\rangle$, specific to the same financial asset and the neutrosophic covariance $\langle cov(\widetilde{R_{A1}}, \widetilde{R_{A2}}); w\widetilde{R_{A1}}, u\widetilde{R_{A1}}, y\widetilde{R_{A1}};\ w\widetilde{R_{A2}}, u\widetilde{R_{A2}}, y\widetilde{R_{A2}}\rangle$ between two financial assets A_1 and A_2, which measures the intensity of the links between the neutrosophic returns specific to the two financial assets. Such a portfolio made up of financial assets for which financial performance indicators can be determined is called a neutrosophic portfolio.

For the neutrosophic portfolio $\langle \widetilde{P}; w\widetilde{p}, u\widetilde{p}, y\widetilde{p}\rangle$ can be determined: the neutrosophicreturn of the portfolio $\langle \widetilde{R_P}; w\widetilde{Rp}, u\widetilde{Rp}, y\widetilde{Rp}\rangle$ and the neutrosophic portfolio risk $\langle \widetilde{\sigma_p^2}; w\widetilde{\sigma p}, u\widetilde{\sigma p}, y\widetilde{\sigma p}\rangle$. These two performance indicators are fundamental indicators that characterize the neutrosophic portfolios.

Thus, the neutrosophic portfolio return is dependent on the weight held by the financial assets in the total value of the neutrosophic portfolio x_{A_i}, as well as on the neutrosophic return of each financial asset that makes up the portfolio $\langle \widetilde{R}_{A_i}; w\widetilde{R}_{A_i}, u\widetilde{R}_{A_i}, y\widetilde{R}_{A_i}\rangle$. At the same time, the neutrosophic portfolio risk is dependent on the weight held by the financial assets in the total value of the portfolio x_{A_i}, as well as on the neutrosophic risk of each financial asset of the form: $\langle \sigma f_{A_i}^2; w\widetilde{\sigma}_A, u\widetilde{\sigma}_A, y\widetilde{\sigma}_A\rangle$ and the covariance between two financial assets $\langle \widetilde{\sigma}_{A_iA_j}; w\widetilde{\sigma}_{A_i} \wedge\ w\widetilde{\sigma}_{A_j}, u\widetilde{\sigma}_{A_i} \vee u\widetilde{\sigma}_{A_j}, y\widetilde{\sigma}_{A_i} \vee y\widetilde{\sigma}_{A_j}\rangle$.

Also the neutrosophic portfolio risk, consisting of N financial assets admits a minimum value at the point where the first order derivative of the neutrosophic portfolio risk is zero $\frac{\partial\langle \widetilde{\sigma_p^2}; w\widetilde{\sigma p}, u\widetilde{\sigma p}, y\widetilde{\sigma p}\rangle}{\partial x_{A_i}} = 0$. Thus, it can be determined what the weight of every financial assets should be in the total value of the neutrosophic portfolio x_{A_i}, so that the portfolio risk is minimal $\langle \widetilde{\sigma_p^2}; w\widetilde{\sigma p}, u\widetilde{\sigma p}, y\widetilde{\sigma p}\rangle \rightarrow min$. From calculations but also from studying this risk category, it was found that the financial assets weight in the total value of the portfolio is dependent on the individual neutrosophic risk of each financial asset but also on the covariance between two financial assets.

The neutrosophic portfolios enables the access to complete information for the financial market investors, in order to substantiate investment decisions. This information provided by the neutrosophic portfolios refers to the probability of realizing the neutrosophic portfolio return, which in turn is influenced by the individual probabilities of achieving the desired return for each financial asset that enters the portfolio structure. Also, the neutrosophic portfolios provide information regarding the probability of producing the neutrosophic portfolio risk, which depends on the probability of producing the neutrosophic risk for each financial asset that enters the portfolio. These categories of information are stratified by means of linguistic variables, so that we will distinguish: the probability of obtaining the return and/or the production of the portfolio risk almost certainly, the probability that the return/production of the portfolio risk will not be realized and the probability that the return/production of the portfolio risk to be uncertain.

Obtaining concomitant information regarding risk and return at the level of the neutrosophic portfolio, as well as the probability of producing the risk and return for the neutrosophic portfolio as well as for the financial assets confer a strong innovative approach for this research paper. Neutrosophic portfolios also have certain limits which mainly refer to the determination of the probability of producing the risk and/or of realizing the return, both at the level of each financial asset as well as at the level of the neutrosophic portfolio as a whole.

Author Contributions: Conceptualization, M.-I.B., I.-A.B.; Data curation, C.D.; Formal analysis, M.-I.B. and I.-A.B.; Investigation, M.-I.B., I.-A.B. and C.D.; Methodology, M.-I.B. and I.-A.B.; Supervision, M.-I.B.; Validation, I.-A.B.; Visualization, C.D.; Writing—original draft, M.-I.B. and I.-A.B.; Writing—review & editing, C.D.

Funding: This research received no external funding.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Markowitz, H. Portfolio Selection. *J. Financ.* **1952**, *7*, 77.
2. Bo, C.; Zhang, X.; Shao, S. Non-dual multi-granulation neutrosophic rough set with applications. *Symmetry* **2019**, *11*, 910. [CrossRef]
3. Jamil, M.; Abdullah, S.; Yaqub Khan, M.; Smarandache, F.; Ghani, F. Application of the bipolar neutrosophic hamacher averaging aggregation operators to group decision making: An illustrative example. *Symmetry* **2019**, *11*, 698. [CrossRef]
4. Shao, S.; Zhang, X.; Zhao, Q. Multi-attribute decision making based on probabilistic neutrosophic hesitant fuzzy choquet aggregation operators. *Symmetry* **2019**, *11*, 623. [CrossRef]
5. Al-Quran, A.; Hassan, N.; Alkhazaleh, S. Fuzzy parameterized complex neutrosophic soft expert set for decision under uncertainty. *Symmetry* **2019**, *11*, 382. [CrossRef]
6. Shi, L.; Yuan, Y. Hybrid weighted arithmetic and geometric aggregation operator of neutrosophic cubic sets for MADM. *Symmetry* **2019**, *11*, 278. [CrossRef]
7. Cao, C.; Zeng, S.; Luo, D. A single-valued neutrosophic linguistic combined weighted distance measure and its application in multiple-attribute group decision-making. *Symmetry* **2019**, *11*, 275. [CrossRef]
8. Jiang, W.; Zhang, Z.; Deng, X. Multi-attribute decision making method based on aggregated neutrosophic set. *Symmetry* **2019**, *11*, 267. [CrossRef]
9. Khan, M.; Gulistan, M.; Yaqoob, N.; Khan, M.; Smarandache, F. Neutrosophic cubic einstein geometric aggregation operators with application to multi-criteria decision making method. *Symmetry* **2019**, *11*, 247. [CrossRef]
10. Abu Qamar, M.; Hassan, N. An approach toward a Q-neutrosophic soft set and its application in decision making. *Symmetry* **2019**, *11*, 139. [CrossRef]
11. Jana, C.; Pal, M. A robust single-valued neutrosophic soft aggregation operators in multi-criteria decision making. *Symmetry* **2019**, *11*, 110. [CrossRef]

12. Fan, C.; Feng, S.; Hu, K. Linguistic neutrosophic numbers einstein operator and its application in decision making. *Mathematics* **2019**, *7*, 389. [CrossRef]
13. Fan, C.; Ye, J.; Feng, S.; Fan, E.; Hu, K. Multi-criteria decision-making method using heronian mean operators under a bipolar neutrosophic environment. *Mathematics* **2019**, *7*, 97. [CrossRef]
14. Abdel-Basset, M.; Mohamed, R.; Zaied, A.E.-N.H.; Smarandache, F. A hybrid plithogenic decision-making approach with quality function deployment for selecting supply chain sustainability metrics. *Symmetry* **2019**, *11*, 903. [CrossRef]
15. Abdel-Basset, M.; Mohamed, M.; Chang, V.; Smarandache, F. IoT and its impact on the electronics market: A powerful decision support system for helping customers in choosing the best product. *Symmetry* **2019**, *11*, 611. [CrossRef]
16. Ahmad, F.; Adhami, A.Y.; Smarandache, F. Neutrosophic optimization model and computational algorithm for optimal shale gas water management under uncertainty. *Symmetry* **2019**, *11*, 544. [CrossRef]
17. Abdel-Basset, M.; Chang, V.; Mohamed, M.; Smarandche, F. A refined approach for forecasting based on neutrosophic time series. *Symmetry* **2019**, *11*, 457. [CrossRef]
18. Colhon, M.; Vlăduţescu, Ş.; Negrea, X. How objective a neutral word is? A neutrosophic approach for the objectivity degrees of neutral words. *Symmetry* **2017**, *9*, 280. [CrossRef]
19. Smarandache, F.; Colhon, M.; Vlăduţescu, Ş.; Negrea, X. Word-level neutrosophic sentiment similarity. *Appl. Soft Comput.* **2019**, *80*, 167–176. [CrossRef]
20. Islam, S.; Ray, P. Multi-objective portfolio selection model with diversification by neutrosophic optimization technique. *Neutrosophic Sets Syst.* **2018**, *21*, 74–83.
21. Pamučar, D.; Badi, I.; Sanja, K.; Obradović, R. A novel approach for the selection of power-generation technology using a linguistic neutrosophic CODAS method: A case study in libya. *Energies* **2018**, *11*, 2489. [CrossRef]
22. Abdel-Basset, M.; Atef, A.; Smarandache, F. A hybrid neutrosophic multiple criteria group decision making approach for project selection. *Cogn. Syst. Res.* **2019**, *57*, 216–227. [CrossRef]
23. Villegas Alava, M.; Delgado Figueroa, S.P.; Blum Alcivar, H.M.; Leyva Vazquez, M. Single valued neutrosophic numbers and analytic hierarchy process for project selection. *Neutrosophic Sets Syst.* **2018**, *21*, 122–130.
24. Saleh Al-Subhi, S.H.; Perez Pupo, I.; Garcia Vacacela, R.; Pinero Perez, P.Y.; Leyva Vazquez, M.Y. A new neutrosophic cognitive map with neutrosophic sets on connections, application in project management. *Neutrosophic Sets Syst.* **2018**, *22*, 63–75.
25. Perez Pupo, D.; Pinero Perez, P.Y.; Garcia Vacacela, R.; Bello, R.; Santos, O.; Leyva Vazquez, M.Y. Extensions to linguistic summaries indicators based on neutrosophic theory, applications in project management decisions. *Neutrosophic Sets Syst.* **2018**, *22*, 87–100.
26. Su, L.; Wang, T.; Wang, L.; Li, H.; Cao, Y. Project procurement method selection using a multi-criteria decision-making method with interval neutrosophic sets. *Information* **2019**, *10*, 201. [CrossRef]
27. Wu, S.; Wang, J.; Wei, G.; Wei, Y. Research on construction engineering project risk assessment with some 2-tuple linguistic neutrosophic hamy mean operators. *Sustainability* **2018**, *10*, 1536. [CrossRef]
28. Bolos, M.-I.; Bradea, I.-A.; Delcea, C. Modeling the performance indicators of financial assets with neutrosophic fuzzy numbers. *Symmetry* **2019**, *11*, 1021. [CrossRef]