



Article

# A Reputation-Enhanced Hybrid Approach for Supplier Selection with Intuitionistic Fuzzy Evaluation Information

Zhijia Yan <sup>1</sup>, Wenting Yang <sup>2</sup>, Xiaoling Huang <sup>3,\*</sup>, Xiangrong Shi <sup>2</sup>, Wenyu Zhang <sup>2</sup> and Shuai Zhang <sup>2</sup>

- School of Information Technology, Zhejiang Yuyin College of Vocational Technology, Hangzhou 310018, China; itt7061@126.com
- School of Information Technology, Zhejiang University of Finance and Economics, Hangzhou 310018, China; yangwt@zufe.edu.cn (W.Y.); sxr@zufe.edu.cn (X.S.); wyzhang@e.ntu.edu.sg (W.Z.); zhangshuai@zufe.edu.cn (S.Z.)
- School of International Education, Zhejiang University of Finance and Economics, Hangzhou 310018, China
- \* Correspondence: huangxl@zufe.edu.cn; Tel.: +86-571-8675-7487

Received: 28 January 2019; Accepted: 20 March 2019; Published: 24 March 2019



Abstract: Selecting optimal suppliers in fuzzy environments has become a major challenge for enterprises. Reputation plays an important role in the process of supplier selection because of its fuzziness, dynamicity, and transitivity. In this study, we first present a novel intuitionistic fuzzy sets (IFS)-hyperlink-induced topic search (HITS) method that combines the intuitionistic fuzzy set with the hyperlink-induced topic search (HITS) algorithm to extend the ability of processing fuzzy information in order to obtain post-propagated reputation values of suppliers. Then, we employ the dynamic intuitionistic fuzzy weighted average operator to gain dynamic reputation values and other evaluation attribute values. After that, intuitionistic fuzzy entropy weight method is adopted to acquire more accurate weights for each evaluation attribute. Finally, we employ the Vlsekriterijumska Optimizacija I Kompromisno Resenje method to acquire comprehensive evaluation values of candidate supplier to select optimal suppliers. Two groups of experiments for supplier selection are given to explain feasibility and practicality of the proposed method.

**Keywords:** supplier selection; HITS algorithm; intuitionistic fuzzy number; reputation propagation

# 1. Introduction

With the rapid development of information technologies, an increasing number of enterprises have developed and deployed electronic procurement systems to improve the efficiency and quality of procurement [1,2]. Supplier selection, a crucial component of electronic procurement, has been widely researched by scholars and many achievements have been obtained in recent decades. For example, Barbarosoglu et al. [3] applied the analytic hierarchy process (AHP) method to select an optimum supplier. Liu et al. [4] introduced data envelopment analysis (DEA) to improve the overall performance of a supplier. Dulmin et al. [5] presented a multi-objective decision aid method to improve procurement quality. Narasimhan et al. [6] constructed a multi-product, multi-criteria model product to optimize supplier selection with life-cycle considerations. These studies have focused mainly on leveraging exact evaluation values to select optimal suppliers. Because of the inherent complexity, uncertainty, and ambiguity of man-made evaluations, over the past decade, some researchers have focused on applying fuzzy theory to select optimal suppliers. For example, Boran et al. [7] incorporated the technique for order preference by similarity to ideal solution method into intuitionistic fuzzy sets (IFSs) to select an appropriate supplier. Jiang et al. [8] employed a fuzzy number to describe different attributes including

quality, service, warranty, delivery, reputation, and position to determine a supplier's selection. In our previous work [9], we combined the extended fuzzy AHP with fuzzy grey relational analysis to obtain optimal suppliers.

However, these studies have shown two shortcomings. Firstly, most of references either did not take supplier reputation into consideration or ignored the propagation characteristic of reputation. However, reputation is the important attribute to evaluate suppliers in the real life and it can be influenced by each other and propagated along the link of the supply chain network. Therefore, reputation computing has emerged as a difficult problem for supplier selection because of its uncertainty and transitivity. Secondly, most of existing works mainly use precise numbers or a group of attribute ratings to evaluate suppliers. However, it is difficult to accurately determine the attribute ratings using precise numbers or static numbers, because attribute ratings of suppliers always change over time, especially in fuzzy environment. Thus, fuzziness and dynamicity are important factors that influence the attribute ratings of suppliers. To address the aforementioned issues, we first present a novel IFS-HITS approach that combines IFSs with the hyperlink-induced topic search (HITS) algorithm to obtain post-propagated reputation values of suppliers in the fuzzy environment. Then, we employ the dynamic intuitionistic fuzzy weighted average (DIFWA) operator to obtain dynamic reputation values and other evaluation attribute values. Next, the intuitionistic fuzzy entropy weight (IFEW) method is adopted to acquire more accurate weights for each evaluation attribute. Finally, we apply the Vlsekriterijumska Optimizacija I Kompromisno Resenje (VIKOR) method to gain comprehensive evaluation values of candidate suppliers to select optimal suppliers.

The remainder of this paper is structured as follows. In Section 2, we review related work. In Section 3, we present a reputation-enhanced hybrid approach using the HITS algorithm, DIFWA operator, IFEW method, and VIKOR method for supplier selection with intuitionistic fuzzy information. Two groups of experiments are given to demonstrate the practicality and feasibility of the proposed approach in Section 4. In the final section, we conclude with our main contributions and present suggestions for future research.

## 2. Related Work

In this section, we will review the relevant literatures including supplier selection, intuitionistic fuzzy sets, and HITS.

### 2.1. Supplier Selection

Supplier selection is a key issue of the procurement process. An increasing number of scholars have focused on the study of supplier selection and proposed a variety of models and methods during the past decades.

Ebrahim et al. [10] designed a mathematical model that takes the different types of discounts to select an appropriate supplier into account. Mohammaditabar et al. [11] presented a model that considers the capacity-constrained, supplier-selection, and order-allocation problem to optimize supplier selection. Kuo et al. [12] developed a green supplier selection model that integrates an artificial neural network and two multi-attribute decision analysis methods to maintain sustainable development. Falagario et al. [13] extended a DEA-cross efficiency approach to improve the quality of government procurement. Wang et al. [14] proposed a cloud-based government procurement information integration platform to solve problems in government procurement information resources management such as low shared utilization, uncommunicated information, information risk, and other issues. Fong et al. [15] designed a web-based tendering system to improve the quality and transparency of procurement in the tendering process. Xu et al. [16] proposed a product semantic relevance model and developed an intelligent business-partner-locator recommendation system prototype to provide personalized government to business e-services. Wang et al. [17] proposed new and feasible approaches for supplier evaluation and selection in the food processing industry under a fuzzy environment to improve the efficiency of supplier selection in the food industry and other industries.

Mathematics **2019**, 7, 298 3 of 17

Nevertheless, these studies have two limitations that are obstacles for further development of supplier selection. Firstly, few studies concentrate on reputation propagation, which plays an important role during supplier selection. Secondly, describing the rating values by precise numbers or static numbers during supplier selection are not a favorable approach because of the inherent dynamicity, uncertainty, and ambiguity of the supplier evaluation.

#### 2.2. IFS

Fuzzy sets were first proposed by Zadeh [18] to tackle vague and uncertain data. Atanassov [19] extended fuzzy sets and proposed the IFS, which is characterized by a membership degree, non-membership degree, and hesitancy degree. The IFS has been applied in various fields including supplier selection [20], job shop scheduling [21], and manufacturing grids [22].

Next, we briefly introduce relevant concepts and definitions related to IFSs.

**Definition 1** [18]. Let X be a fixed set. An IFS can be defined by the following formula:

$$A = \{ \langle x, \mu_A(x), v_A(x) \rangle | x \in X \}, \tag{1}$$

where  $\mu_A: X \to [0,1]$  and  $v_A: X \to [0,1]$  are the degree of membership and degree of non-membership, respectively, of the element  $x \in X$  to A for any  $x \in X$ . Moreover,  $0 \le \mu_A(x) + v_A(x) \le 1$ .

Szmidt et al. [23] introduced a new element called the degree of indeterminacy or the degree of hesitancy of the element  $x \in X$  to A, i.e.,

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x).$$
 (2)

In particular, if  $\pi_A(x) = 1 - \mu_A(x) - [1 - \mu_A(x)] = 0$ , for any  $x \in X$ , then an IFS can be degraded into a fuzzy set. In fact, a smaller degree of hesitancy indicates that more accurate information is acquired. In the computation process, we often omit the degree of hesitancy to simplify the calculation procedure [23].

Chen et al. [24] and Hong et al. [25] defined the score function and the accurate function to compare the numeric size of intuitionistic fuzzy numbers (IFNs).

**Definition 2** [24,25]. Let s(a) and h(a) represent the score function and accuracy function, respectively, of the IFN a. These functions are defined according to the following formulas:

$$s(a) = \mu_a - \nu_a \tag{3}$$

$$h(a) = \mu_a + \nu_a,\tag{4}$$

where a greater s(a) implies a higher degree of score of the IFN a, and a greater h(a) implies a higher degree of accuracy of the IFN a.

Based on the score and accuracy functions, Xu et al. [26] defined an order relation between two IFNs.

**Definition 3** [26]. Let  $a = (\mu_a, v_a)$  and  $b = (\mu_b, v_b)$  be two IFNs,  $s(a) = \mu_a - v_a$  and  $s(b) = \mu_b - v_b$  be the scores of a and b, respectively, and  $h(a) = \mu_a + v_a$  and  $h(b) = \mu_b + v_b$  be the accuracies of a and b, respectively. Then,

- If s(a) < s(b), then a < b.
- *If* s(a) = s(b), then:
- If h(a) = h(b), then a = b.
- If h(a) < h(b), then a < b.

Mathematics **2019**, 7, 298 4 of 17

• If h(a) > h(b), then a > b.

Xu [27] later presented the following IFN operation laws.

**Definition 4** [27]. Let a and b be two IFNs,  $a = (\mu_a, v_a)$  and  $b = (\mu_b, v_b)$ , then:

$$a \oplus b = (\mu_a + \mu_b - \mu_a \mu_b, v_a v_b) \tag{5}$$

$$a \otimes b = (\mu_a \mu_b, v_a + v_b - v_a v_b) \tag{6}$$

$$\lambda a = (1 - (1 - \mu_a)^{\lambda}, \nu_a^{\lambda}), \lambda > 0. \tag{7}$$

#### 2.3. HITS

The HITS algorithm, which was proposed by Jon Kleinberg in 1999 [28], is a useful method for evaluating the quality of web pages by authority values and hub values on the internet. If a hub page with high quality is pointed to by many authority pages, these authority pages will be higher quality; if an authority page with high quality is pointed to by many hub pages, these hub pages will be higher quality [29]. The algorithm has been constantly extended and applied in numerous fields. Nomura et al. [30] modified the traditional HITS algorithm with the purpose of improving the service level of web communities. Deguchi et al. [31] adopted a weighted HITS algorithm to investigate the economic relationships of the world trade network from 1992 to 2012. Otsuka et al. [32] employed the HITS algorithm and PageRank algorithm to evaluate a reputation network. Chawla [33] used the HITS method to propagate reputation to improve the precision of personalized web searches. Ivanov et al. [34] constructed user trust models for automatic geotag propagation in images based on the HITS algorithm. In our previous work [35], we presented a novel approach to calculate the global and dynamic reputation values of enterprises by employing a time-aware HITS algorithm in the domain of manufacturing service.

In summary, HITS is a promising method for reputation evaluation because of its excellent propagation characteristic. However, the existing research results in HITS can only process the type of link where the value is a precise number and cannot process the type of link which value is fuzzy number. Describing the link value as a fuzzy number during supplier selection is a more favorable approach because of the inherent complexity, uncertainty, and ambiguity of the supplier evaluation. Since IFSs are powerful tools to concurrently represent positive, negative, and uncertain fuzzy information, we first apply IFNs to simultaneously express a positive rating (i.e., praise), a negative rating (i.e., criticism), and a hesitating rating (i.e., uncertainty) for the link of the supply chain in this study. Then, a novel IFS-HITS approach that combines IFSs with the HITS algorithm is presented to extend the ability of processing fuzzy information to perform fuzzy reputation propagation.

# 3. The Proposed Method

The proposed approach for supplier selection contains five subsections: (1) notations, (2) using the IFS-HITS method to obtain the post-propagated reputation, (3) using the DIFWA operator to obtain dynamic attribute ratings, (4) using the IFEW method to obtain attribute weights, and (5) using the VIKOR to select an optimal supplier.

### 3.1. Notations

To better illustrate our proposed method, the following notations are given. Indices and sets

- $E_i$  Index of enterprises, i = 1, ..., I, where I is the number of enterprises
- $P_i$  Index of products provided by suppliers, j = 1, ..., J, where J is the number of products
- $A_s$  Index of the sth year original vector of reputation values for different suppliers, s = 1, ..., S, where S is the number of years

Mathematics **2019**, 7, 298 5 of 17

*m* Index of suppliers, m = 1, ..., M, where M represents the number of suppliers Index of attribute weights, n = 1, ..., N, where N is the number of evaluation attributes and

 $w_n$   $\sum_{n=1}^N w_n = 1$ 

 $C_n$  Index of evaluation attributes, n = 1, ..., N, where N is the number of evaluation attributes

#### **Parameters**

t Iteration times

 $A_{ij}$  The number of enterprises  $E_i$  that have purchased product  $P_j$ 

 $A_{J1}$  The authority vector of product  $P_j$ 

 $\mathbf{H}_{I1}$  The hub vector of enterprise  $E_i$ 

The positive solution  $(f_n^+ = (\mu_n^+, \nu_n^+))$ , where  $\mu_n^+$  and  $v_n^+$  represent the degree of membership and degree of non-membership for positive solution, respectively

The negative solution  $(f_n^- = (\mu_n^-, \nu_n^-))$ , where  $\mu_n^-$  and  $v_n^-$  represent the degree of membership and degree of non-membership for negative solution, respectively

The current solution  $(f_{mn} = (\mu_{mn}, \nu_{mn}))$ , where  $\mu_{mn}$  and  $v_{mn}$  represent the degree of membership and degree of non-membership for current solution, respectively

d Hamming distance

 $Q_m$  The influence index of each solution, the smaller the influence index, the better the solution

#### Decision variables

 $\mathbf{M}_{II}$  The rating matrix of product  $P_i$  by enterprise  $E_i$ 

 $\mathbf{M}_{II}^{T}$  The transition matrix of product  $P_i$  by enterprise  $E_i$ 

 $\mu_{ij}$  The positive reputation rating given by  $E_i$  to  $P_j$ , which is expressed by membership

 $v_{ij}$  The negative reputation rating given by  $E_i$  to  $P_j$ , which is expressed by non-membership

The bth positive reputation rating given by  $E_i$  to  $P_i$ , which is expressed by membership,

 $\mu_{ij}^b$   $b = 1, ..., B_{ij}$ , where  $B_{ij}$  represents the number of times enterprise  $E_i$  has purchased product  $P_j$ 

The *b*th negative reputation rating given by  $E_i$  to  $P_j$ , which is expressed by non-membership,  $b = 1, ..., B_{ij}$ , where  $B_{ij}$  represents the number of times enterprise  $E_i$  has purchased product  $P_j$ 

 $\alpha_{i1}$  The positive authority value of product  $P_{i}$ , which is expressed by membership

 $\beta_{i1}$  The negative authority value of product  $P_i$ , which is expressed by non-membership

 $\chi_{i1}$  The positive hub value of enterprise  $E_i$ , which is expressed by membership

 $\delta_{i1}$  The negative hub value of enterprise  $E_i$ , which is expressed by non-membership

# 3.2. Using the IFS-HITS to Obtain the Post-Propagated Reputation

The HITS algorithm, which is famous for its excellent propagation characteristic, is often used to evaluate the quality of web pages [28]. The quality of hub pages and authority pages can be propagated along the link direction and are mutually influential on the internet. Similarly, the reputation of a supplier can also be propagated along the link direction of the supply chain and be mutually influential in the supply chain network. Consequently, we apply the HITS algorithm to conduct reputation propagation of supply chain networks to obtain the post-propagated reputation of suppliers in this study.

In a supply chain network, there are two types of nodes. One of them is represented as enterprise, and another is represented as product which is provided by suppliers. A consumer enterprise may point to one or more products which is provided by suppliers through a provision link or purchase link. Each link carries a corresponding rating. According to the link structure of the HITS algorithm, the authority score can be considered as a reputation value of a supplier. Figure 1 illustrates an example of a reputation-propagated supply chain network.

We assume that there is a directed network of supply chain G = (E, P, L), where E and P indicate the sets of nodes (including both nodes of consumer enterprise and product) and E represents a set of links (including provision and purchase links). We build a rating matrix E0, where E1 is the hub vector and E3 is the authority vector. Through iterative calculations, we obtain the hub value and authority

Mathematics 2019, 7, 298 6 of 17

value until the results converge (termination criterion). The iterative formulas of the traditional HITS algorithm are as follows [28]:

$$\mathbf{H}^{(t+1)} = \mathbf{M} \times \mathbf{A}^{(t)} \tag{8}$$

$$\mathbf{A}^{(t+1)} = \mathbf{M}^T \times \mathbf{H}^{(t)}. \tag{9}$$

Nevertheless, the traditional HITS algorithm can only process links in which the values are precise numbers and cannot process links in which the values are fuzzy numbers. Since an IFS has the powerful ability to express fuzzy information, we first apply IFNs to simultaneously express a positive rating (i.e., praise), a negative rating (i.e., criticism), and a hesitating rating (i.e., uncertainty) for every purchase link of the supply chain (provision links do not have ratings). Then, we incorporate IFNs into the HITS algorithm to extend the ability of processing fuzzy information to perform fuzzy reputation propagation.

For example, a purchase link from an enterprise to a product has a reputation rating  $(\mu_{ij}, v_{ij})$ , which is shown as RD = (0.8, 0.1, 0.1). The first value 0.8 indicates a positive rating (equivalent to the membership degree), the middle value 0.1 indicates a negative rating (equivalent to the non-membership degree), and the last value 0.1 indicates an uncertain rating (equivalent to the hesitancy degree). In order to simplify the calculation, we omit the third value according to Equation (2). Table 1 shows the rating matrix  $\mathbf{M}_1$  corresponding to the supply chain network in Figure 1.

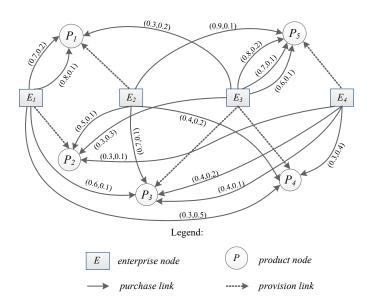


Figure 1. An example of a reputation-propagated supply chain network.

S	E	Edge	Rating	S	E	Edge	Rating
$E_1$	$P_1$	1	(0.7, 0.2)	E <sub>3</sub>	$P_2$	1	(0.3, 0.3)
$E_1$	$P_1$	2	(0.8, 0.1)	$E_3$	$P_3$	1	null
$E_1$	$P_2$	1	null	$E_3$	$P_4$	1	null
$E_1$	$P_3$	1	(0.6, 0.1)	$E_3$	$P_5$	1	(0.8, 0.2)
$E_1$	$P_4$	1	(0.3, 0.5)	$E_3$	$P_5$	2	(0.7, 0.1)
$E_1$	$P_5$	0	0	$E_3$	$P_5$	3	(0.6, 0.1)
$E_2$	$P_1$	1	null	$E_4$	$P_1$	0	0
$E_2$	$P_2$	1	(0.5, 0.1)	$E_4$	$P_2$	1	(0.3, 0.1)
$E_2$	$P_3$	1	(0.7, 0.1)	$E_4$	$P_3$	1	(0.4, 0.2)
F.	D.	1	(0.4, 0.2)	F.	D.	2	(0.4.0.1)

1

(0.9, 0.1)

(0.3, 0.2)

 $P_4$ 

1

(0.3, 0.4)

null

**Table 1.** The raw rating matrix  $M_1$ .

Mathematics **2019**, 7, 298 7 of 17

In Table 1, S denotes the start point, E denotes the end point, edge denotes the number of links from the start point to the end point, and rating denotes the reputation rating given by  $E_i$  to  $P_j$ . Because the HITS algorithm cannot be used directly to calculate the raw ratings under intuitionistic fuzzy environment, the original data must be preprocessed. Thus, the following three types of special ratings are discussed. (a) If enterprise  $E_i$  provides product  $P_j$ , then the matrix elements  $(\mu_{ij}, v_{ij})$  is equal to null because the enterprise cannot evaluate the products which can provide by themselves to avoid subjective rating. (b) If enterprise  $E_i$  has not purchased product  $P_j$  (i.e., rating is zero in Table 1), then the rating  $(\mu_{ij}, v_{ij})$  is equal to the average value of all purchase ratings from other enterprises to product  $P_j$ , i.e.,  $\frac{1}{\sum_{i=1}^{L} A_{ij}} \sum_{i=1}^{L} (\mu_{ij}, v_{ij})$  (inspired by the intuitionistic fuzzy average operator [27]). (c) If enterprise  $E_i$  purchases product  $P_j$  repeatedly, then the rating  $(\mu_{ij}, v_{ij})$  is equal to the average rating value of all purchase ratings from enterprise  $E_i$  to product  $P_j$ , i.e.,  $\frac{1}{B_{ij}} \sum_{b=1}^{B_{ij}} (\mu_{ij}^b, v_{ij}^b)$ . The above three types of special ratings can be summarized by the following equation:

$$(\mu_{ij}^{b}, v_{ij}^{b}) = \begin{cases} null & \text{if } E_{i} \text{ provide } P_{j} \\ \frac{1}{\sum_{i=1}^{I}} \sum_{A_{ij}}^{\sum_{i=1}^{I}} (\mu_{ij}, v_{ij}) & \text{if } E_{i} \text{ does not purchase } P_{j} \\ \frac{1}{B_{ij}} \sum_{b=1}^{B_{ij}} (\mu_{ij}^{b}, v_{ij}^{b}) & \text{if } E_{i} \text{ repeatedly purchases } P_{j}, \end{cases}$$

$$(10)$$

The three kinds of special ratings can be calculated using Equation (10); the final results are displayed in Table 2.

S	E	Rating	S	E	Rating
$E_1$	P <sub>1</sub>	(0.7551, 0.1414)	$E_3$	$P_1$	(0.3, 0.2)
$E_1$	$P_2$	null	$E_3$	$P_2$	(0.3, 0.3)
$E_1$	$P_3$	(0.6, 0.1)	$E_3$	$P_3$	null
$E_1$	$P_4$	(0.3, 0.5)	$E_3$	$P_4$	null
$E_1$	$P_5$	(0.6680, 0.2976)	$E_3$	$P_5$	(0.7116, 0.1260)
$E_2$	$P_1$	null	$E_4$	$P_1$	(0.6293, 0.3080)
$E_2$	$P_2$	(0.5, 0.1)	$E_4$	$P_2$	(0.3, 0.1)
$E_2$	$P_3$	(0.7, 0.1)	$E_4$	$P_3$	(0.4, 0.1414)
$E_2$	$P_4$	(0.4, 0.2)	$E_4$	$P_4$	(0.3, 0.4)
$E_2$	$P_5$	(0.9, 0.1)	$E_4$	$P_5$	null

Table 2. Rating matrix after preprocessing.

Considering the drawbacks of the traditional HITS algorithm, we propose a new method (i.e., IFS-HITS), which combines IFSs with the HITS algorithm to obtain post-propagated reputation values. The iterative computation process is shown in Equations (11)–(14):

$$\mathbf{H}_{I1}^{(t+1)} = \mathbf{M}_{IJ} \otimes \mathbf{A}_{J1}^{(t)}, \tag{11}$$

Mathematics **2019**, 7, 298 8 of 17

In addition, the formulation for the elements of  $M_{II}$  and  $A_{I1}$  is as follows:

$$\mathbf{H}_{\Pi}^{(t+1)} = \begin{bmatrix} (\mu_{11}, v_{11}) & \dots & (\mu_{1j}, v_{1j}) & \dots & (\mu_{1J}, v_{1J}) \\ \dots & \dots & \dots & \dots & \dots \\ (\mu_{1l}, v_{1l}) & \dots & (\mu_{ij}, v_{ij}) & \dots & (\mu_{ij}, v_{ij}) \\ \dots & \dots & \dots & \dots & \dots & \dots \\ (\mu_{1l}, v_{1l}) & \dots & (\mu_{lj}, v_{lj}) & \dots & (\mu_{lJ}, v_{lj}) \\ \dots & \dots & \dots & \dots & \dots & \dots \\ (\mu_{1l}, v_{1l}) & \dots & (\mu_{lj}, v_{lj}) & \dots & (\mu_{lJ}, v_{lj}) \end{bmatrix} \otimes \begin{bmatrix} (\alpha_{11}, \beta_{11}) \\ \dots & (\alpha_{J1}, \beta_{J1}) \\ \dots & \dots & \dots & \dots \\ (\alpha_{I1}, \beta_{J1}) \end{bmatrix}$$

$$= \begin{bmatrix} (\mu_{11} \times \alpha_{11}, v_{11} + \beta_{11} - v_{11} \times \beta_{11}) \oplus \dots \oplus (\mu_{lj} \times \alpha_{j1}, v_{1j} + \beta_{j1} - v_{1j} \times \beta_{j1}) \oplus \dots \oplus (\mu_{lJ} \times \alpha_{J1}, v_{1J} + \beta_{J1} - v_{iJ} \times \beta_{J1}) \\ \dots & \dots & \dots & \dots \\ (\mu_{l1} \times \alpha_{11}, v_{l1} + \beta_{11} - v_{l1} \times \beta_{11}) \oplus \dots \oplus (\mu_{lij} \times \alpha_{j1}, v_{lj} + \beta_{j1} - v_{lj} \times \beta_{j1}) \oplus \dots \oplus (\mu_{lJ} \times \alpha_{J1}, v_{lJ} + \beta_{J1} - v_{lJ} \times \beta_{J1}) \\ \dots & \dots & \dots & \dots \\ (\mu_{l1} \times \alpha_{11}, v_{l1} + \beta_{11} - v_{l1} \times \beta_{l1}) \oplus \dots \oplus (\mu_{lj} \times \alpha_{j1}, v_{lj} + \beta_{j1} - v_{lj} \times \beta_{j1}) \oplus \dots \oplus (\mu_{lJ} \times \alpha_{J1}, v_{lJ} + \beta_{J1} - v_{lJ} \times \beta_{J1}) \end{bmatrix}$$

$$= \begin{bmatrix} (\sum_{j=1}^{l} \mu_{1j} \times \alpha_{j1} - \prod_{j=1}^{l} \mu_{1j} \times \alpha_{j1}, \prod_{j=1}^{l} v_{1j} + \beta_{j1} - v_{1j} \times \beta_{j1}) \\ \dots & \dots & \dots & \dots \\ (\sum_{j=1}^{l} \mu_{lj} \times \alpha_{j1} - \prod_{j=1}^{l} \mu_{lj} \times \alpha_{j1}, \prod_{j=1}^{l} v_{lj} + \beta_{j1} - v_{lj} \times \beta_{j1}) \\ \dots & \dots & \dots & \dots & \dots \\ (\sum_{j=1}^{l} \mu_{lj} \times \alpha_{j1} - \prod_{j=1}^{l} \mu_{lj} \times \alpha_{j1}, \prod_{j=1}^{l} v_{lj} + \beta_{j1} - v_{lj} \times \beta_{j1}) \end{bmatrix}$$

According to Formulations (11) and (12), we can compute the hub values of enterprise node. In addition, we can use the following formulas to calculate the authority value.

$$\mathbf{A}_{I1}^{(t+1)} = \mathbf{M}_{IJ}^T \otimes \mathbf{H}_{I1}^{(t)} \tag{13}$$

The formulation for the elements of  $\mathbf{M}_{II}^T$  and  $\mathbf{H}_{I1}$  is as follows:

$$\mathbf{A}_{j_{1}}^{(t+1)} = \begin{bmatrix} (\mu_{11}, v_{11}) & \dots & (\mu_{i1}, v_{i1}) & \dots & (\mu_{I1}, v_{I1}) \\ \dots & \dots & \dots & \dots & \dots & \dots \\ (\mu_{Ij}, v_{1j}) & \dots & (\mu_{ij}, v_{ij}) & \dots & (\mu_{Ij}, v_{Ij}) \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ (\mu_{IJ}, v_{1J}) & \dots & (\mu_{ij}, v_{ij}) & \dots & (\mu_{IJ}, v_{IJ}) \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ (\mu_{II}, v_{1J}) & \dots & (\mu_{ij}, v_{ij}) & \dots & (\mu_{II}, v_{IJ}) \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ (\mu_{I1}, v_{11}, v_{11} + \delta_{11} - v_{11} \times \delta_{11}) \oplus \dots \oplus (\mu_{i1} \times \chi_{i1}, v_{i1} + \delta_{i1} - v_{i1} \times \delta_{i1}) \oplus \dots \oplus (\mu_{IJ} \times \chi_{II}, v_{IJ} + \delta_{II} - v_{IJ} \times \delta_{II}) \\ \dots & \dots & \dots & \dots & \dots \\ (\mu_{IJ} \times \chi_{11}, v_{1J} + \delta_{11} - v_{1J} \times \delta_{11}) \oplus \dots \oplus (\mu_{ij} \times \chi_{i1}, v_{ij} + \delta_{i1} - v_{ij} \times \delta_{i1}) \oplus \dots \oplus (\mu_{IJ} \times \chi_{II}, v_{IJ} + \delta_{II} - v_{IJ} \times \delta_{II}) \\ \dots & \dots & \dots & \dots \\ (\mu_{IJ} \times \chi_{11}, v_{IJ} + \delta_{11} - v_{IJ} \times \delta_{11}) \oplus \dots \oplus (\mu_{iJ} \times \chi_{i1}, v_{iJ} + \delta_{i1} - v_{iJ} \times \delta_{i1}) \oplus \dots \oplus (\mu_{IJ} \times \chi_{II}, v_{IJ} + \delta_{II} - v_{IJ} \times \delta_{II}) \end{bmatrix}$$

$$= \begin{bmatrix} (\sum_{i=1}^{I} \mu_{ii} \times \chi_{i1} - \prod_{i=1}^{I} \mu_{ii} \times \chi_{i1}, \prod_{i=1}^{I} v_{ij} + \delta_{i1} - v_{ij} \times \delta_{i1}) \\ \vdots & \vdots & \vdots & \vdots \\ (\sum_{i=1}^{I} \mu_{ij} \times \chi_{i1} - \prod_{i=1}^{I} \mu_{ij} \times \chi_{i1}, \prod_{i=1}^{I} v_{ij} + \delta_{i1} - v_{iJ} \times \delta_{i1}) \end{bmatrix}$$

$$= \begin{bmatrix} (\sum_{i=1}^{I} \mu_{ij} \times \chi_{i1} - \prod_{i=1}^{I} \mu_{ij} \times \chi_{i1}, \prod_{i=1}^{I} v_{ij} + \delta_{i1} - v_{ij} \times \delta_{i1}) \\ \vdots & \vdots & \vdots \\ (\sum_{i=1}^{I} \mu_{ij} \times \chi_{i1} - \prod_{i=1}^{I} \mu_{ij} \times \chi_{i1}, \prod_{i=1}^{I} v_{ij} + \delta_{i1} - v_{iJ} \times \delta_{i1}) \end{bmatrix}$$

According to Formulations (13) and (14), we can calculate the authority values of product node. The termination conditions are different from those of the standard HITS method. The termination conditions standard HITS algorithm is the result of last two iterations in approximation, i.e., the difference of  $\mathbf{H}^{(t+1)} - \mathbf{H}^{(t)}$  or  $\mathbf{A}^{(t+1)} - \mathbf{A}^{(t)}$  among all elements is less than a defined value. Herein,  $\mathbf{H}$  denotes hub vector,  $\mathbf{A}$  denotes authority vector, and t denotes the iteration times.

Because the IFN is different from the operation law of precise number, the termination condition is also different. Thus, we use Equation (4) to modify the termination condition. According to Equation (4), we firstly compute the accuracy function value of  $\mathbf{H}^{(t)}$  and  $\mathbf{A}^{(t)}$  for each vector element. Then, we can compute the accuracy function value of  $\mathbf{H}^{(t+1)}$  and  $\mathbf{A}^{(t+1)}$  for each vector element. Finally, if the difference of  $\mathbf{H}^{(t+1)} - \mathbf{H}^{(t)}$  or  $\mathbf{A}^{(t+1)} - \mathbf{A}^{(t)}$  among all elements is less than a defined value, then the termination condition is reached. In this study, we set that the difference of termination condition is  $10^{-6}$ . Because we need to obtain reputation values of products, the termination condition only computes the difference among the accuracy function values of each element for authority vector.

Based on IFS-HITS method, we take the five products nodes and four enterprise nodes as example to compute all the reputation values of suppliers. The original reputation rating matrix

Mathematics **2019**, 7, 298 9 of 17

and the preprocessing rating matrix are listed in the Tables 1 and 2. Let us assume that the original authority vector of the five product nodes is  $\mathbf{A}_1 = [(0.5, 0.5), (0.7, 0.3), (0.8, 0.2), (0.6, 0.4), (0.9, 0.1)]^T$ . We assume that original reputation values for product are regarded as the original reputation values for supplier. According to Equations (10)–(14), we can compute the reputation values of overall suppliers. The ultimate result, namely the reputation values of suppliers in the first year, can be obtained as follows (all values are rounded off to four decimal places):

$$\mathbf{A}[5]_1 = \left[ (0.9009, 0.0092), (0.7118, 0.0031), (0.9110, 0.0015), (0.6746, 0.0403), (0.9826, 0.0039) \right]^T$$

where  $A[5]_s$  (s = 1, ..., S) represents the sth year reputation value of five products which provided by five different suppliers. However, the computation rules of IFNs are different from those of precise numbers. In order to compare the trust values, we sort the values in decreasing order using accuracy and score functions (see Definition 3). Then, the maximum value that represents a supplier with the highest reputation value is chosen as the best one. Accordingly, we determined that supplier 5 is the best candidate supplier with respect to reputation.

#### 3.3. Using the DIFWA to Obtain the Attribute Ratings

It is difficult to accurately determine the attribute ratings using a group of static values. In fact, attribute ratings of suppliers always change over time. For example, despite a supplier having an extremely low score for their reputation degree in former years, it has constantly enhanced their service ability since then. Therefore, the reputation value of the supplier is improved in recent years. Dynamism is an important factor that influences the reputation degree of a supplier. It is reasonable to introduce the DIFWA operator to obtain dynamic attribute ratings from different periods (inspired by Reference [36]). Next, we describe the DIFWA operator.

In actual problems, some data use IFNs to demonstrate criteria ratings that are collected from different periods. Therefore, Xu et al. [37] proposed a novel operator called the DIFWA operator to aggregate information from different periods.

**Definition 5.** Let  $a_1, \ldots, a_s, \ldots, a_S$  be a collection of IFNs collected from S different periods  $s = 1, \ldots, S$ , and let  $\mathbf{W} = (\omega_1, \ldots, \omega_s, \ldots, \omega_s)^T$  be the weight vectors of different periods  $s = 1, \ldots, S$  with  $\omega_s \ge 0$ ,  $s = 1, \ldots, S$ ,  $\sum_{s=1}^{S} \omega_s = 1$ . Then, we call

$$DIFWA_{\mathbf{W}}(a_1,\ldots,a_s\ldots,a_s) = \omega_1 a_1 \oplus \ldots \oplus \omega_s a_s \oplus \ldots \oplus \omega_s a_s, \tag{15}$$

a dynamic DIFWA operator.

According to Definition 5, we can rewrite this equation as follows [37]:

$$DIFWA_{\mathbf{W}}(a_1, \dots a_s, \dots, a_s) = \left(1 - \prod_{s=1}^{S} (1 - \mu_{a_s})^{\omega_s}, \prod_{s=1}^{S} \nu_{a_s}^{\omega_s}, \prod_{s=1}^{S} (1 - \mu_{a_s}^{\omega_s} - \prod_{s=1}^{S} \nu_{a_s}^{\omega_s})\right).$$
(16)

The time weight and the length of time can be defined based on the real situation. As time passes, the time becomes closer, which means the weight increases.

For example, we consider three groups of values in the last three years with different time weights to better assess the reputation degree. Let  $\mathbf{W} = (0.2, 0.3, 0.5)^T$  be the dynamic time weights for the three years, the original authority vector of five suppliers are  $\mathbf{A}_2 = [(0.7, 0.3), (0.4, 0.6), (0.6, 0.4), (0.5, 0.5), (0.5, 0.5)]^T$  and  $\mathbf{A}_3 = [(0.9, 0.1), (0.6, 0.4), (0.8, 0.2), (0.5, 0.5), (0.4, 0.6)]^T$  for the

10 of 17 Mathematics 2019, 7, 298

remaining two years. Using the proposed IFS-HITS method, we can calculate the reputation values of the remaining two years for suppliers shown as follows:

$$\mathbf{A}[5]_2 = \left[ (0.9535, 0.0083), (0.8852, 0.0062), (0.9467, 0.0089), (0.8870, 0.0154), (0.8918, 0.0062) \right]^T$$

$$\mathbf{A}[5]_3 = \left[ (0.9767, 0.0041), (0.8697, 0.0124), (0.9539, 0.0031), (0.8982, 0.0087), (0.8295, 0.0107) \right]^T$$

Using Function (16), we acquire the following overall dynamic reputation ratings:

$$\mathbf{A}[5] = \left[ (0.9617, 0.0060), (0.8529, 0.0077), (0.9451, 0.0037), (0.8675, 0.0140), (0.9058, 0.0075) \right]^T$$

According to the final outcome, we determine that supplier 1 is the best candidate supplier with the highest reputation degree. In this study, other dynamic attribute ratings also can be acquired by the DIFWA operator in a similar manner.

# 3.4. Using the DIFWA to Obtain the Attribute Weights

Different services have different type of attributes. According to the characteristics of supplier selection, we consider suitable attributes [38] including reputation, quality, cost, satisfaction, and safety. In this section, we employ the IFEW method to compute attribute weights. The formula is as follows [39]:

$$w_n = \frac{1 - H_n}{N - \sum_{n=1}^{N} H_n},\tag{17}$$

where 
$$H_n = \frac{1}{M} \sum_{m=1}^{M} \pi_A(C_n)$$
 ( $m = 1, 2, ..., M$ ), and  $0 \le H_n \le 1$ .  
Herein,  $\pi_A$  represents the degree of hesitancy that is computed by Formula (2). Using Equation (17),

we can obtain the attribute weights.

# 3.5. Using VIKOR to Select Optimal Suppliers

Opricovic et al. [40] developed a method called the VIKOR method to determine a compromising solution by sorting a series of options. The main idea of this method should be as close to a positive solution and as far from a negative solution as possible. The primary steps of the VIKOR method are as follows [41]:

(1) Utilize the DIFWA operator. We first use the DIFWA operator (see Definition 5) to aggregate all the intuitionistic fuzzy ratings into an intuitionistic fuzzy decision matrix **D**, which is given by

$$\mathbf{D} = \begin{bmatrix} x_{11} & \cdots & x_{1N} \\ \vdots & \ddots & \vdots \\ x_{M1} & \cdots & x_{MN} \end{bmatrix}, \tag{18}$$

where M denotes the total number of suppliers and N denotes the total number of criteria.

(2) Determine a positive solution and a negative solution. As the name implies, the positive solution is the best value, and the negative solution is the worst value. If the attributes are benefit criteria, a greater preference value is better (see the below Formula (19)). If the attributes are cost criteria, a smaller preference value is better (see the below Formula (20)). Under the corresponding evaluation criterion  $C_n$ , max  $f_{mn}$  and min  $f_{mn}$  denote largest and lowest IFNs, respectively. The corresponding formulas shown as follows:

$$f_n^+ = \max f_{mn}, \qquad f_n^- = \min f_{mn}, \quad \text{(benefit criteria)}$$
 (19)  
 $f_n^+ = \min f_{mn}, \qquad f_n^- = \max f_{mn}. \quad \text{(cost criteria)}$  (20)

$$f_n^+ = \min f_{mn}, \qquad f_n^- = \max f_{mn}. \qquad (\cos t \text{ criteria})$$
 (20)

(3) Calculate the group utility value and the individual regret value. We can compute the group utility value  $S_m$  and the individual regret value  $R_m$  using the following formulas:

$$S_m = \sum_{n=1}^{N} w_n (f_n^+ - f_{mn}) / (f_n^+ - f_n^-)$$
 (21)

$$R_m = \max[w_n(f_n^+ - f_{mn})/(f_n^+ - f_n^-)]$$
(22)

According to the characteristics of IFNs, we modify the above formulas (inspired by Reference [36]) and choose the Hamming distance to calculate  $S_m$  and  $R_m$  as follows:

$$S_{m} = \sum_{n=1}^{N} w_{n} \times d(f_{n}^{+}, f_{mn})$$

$$= \frac{1}{2} \sum_{n=1}^{N} w_{n} (|\mu_{n}^{+} - \mu_{mn}| + |\nu_{n}^{+} - \nu_{mn}|)$$
(23)

$$R_{m} = \max_{1 \le n \le N} w_{n} \times d(f_{n}^{+}, f_{mn})$$

$$= \max_{1 \le n \le N} \frac{w_{n}(|\mu_{n}^{+} - \mu_{mn}| + |\nu_{n}^{+} - \nu_{mn}|)}{2},$$
(24)

(4) Compute the influence index  $Q_m$  of each solution.  $Q_m$  can be computed as follows:

$$Q_m = v \frac{(S_m - S^+)}{(S^- - S^+)} + (1 - v) \frac{(R_m - R^+)}{(R^- - R^+)},$$
(25)

where  $S^+ = \min S_m$ ,  $S^- = \max S_m$ ,  $R^+ = \min R_m$ , and  $R^- = \max R_m$ . Here, v denotes the weight of the maximum group utility; specifically, v > 0.5 denotes the decision of the majority of people, v < 0.5 denotes the minimization of individual regrets, and v = 0.5 denotes a consensus. Herein, we set the value to v = 0.5.

(5) Determine a compromising solution. We rank the compromising solutions in ascending order of the influence index  $Q_m$ . The enterprise with the lowest influence index is considered the optimum supplier.

Using the above methods, we select the optimum supplier.

# 4. Experimental Evaluation on Supplier Selection

In this section, we perform some experiments for supplier selection to validate the proposed approach in terms of feasibility and practicality.

# 4.1. Evaluating the Feasibility of Our Proposed Method through an Illustrative Example

To evaluate the feasibility of the proposed method, we illustrate an example on supplier selection to elaborate on the feasibility of our proposed method. By reviewing basic information of suppliers, there are five candidate suppliers  $X_i$  (i = 1, 2, 3, 4, 5) that meet the specified procurement requirements; the procurement committee determines the attributes including  $C_1$ : cost,  $C_2$ : quality,  $C_3$ : reputation,  $C_4$ : safety, and  $C_5$ : satisfaction to evaluate different suppliers. The corresponding ratings of different suppliers in the last three years s (s = 1, 2, 3) can be extracted from historical feedback. Let  $\mathbf{W} = (0.2, 0.3, 0.5)^T$  be the weights vector in the last three years.

Step 1: Using the IFS-HITS method (see Equations (10)–(14)) to obtain post-propagated reputation values.

According to the previous transaction records with other enterprises, the four enterprises  $E_i$  (i = 1, 2, 3, 4) give the reputation ratings to five different products  $P_j$  (j = 1, 2, 3, 4, 5) provided by different suppliers. The first year of reputation rating matrix has been showed in Table 1 and the remaining two years' values are listed in Tables 3 and 4. Using Equation (10), we can process three types of special

Mathematics 2019, 7, 298 12 of 17

situations (zero value, null value, and the repeat purchasing). Then, we apply Formulas (11)–(14) to obtain the post-propagation reputation values for three years in Table 5.

	$P_1$	$P_2$	$P_3$	$P_4$	$P_5$
$E_1$	(0.7, 0.2)	null	(0.7, 0.1) (0.7, 0.2)	(0.5, 0.5)	0
$E_2$	null	(0.5, 0.3)	(0.5, 0.3)	(0.5, 0.3)	(0.6, 0.2)
$E_3$	(0.4, 0.2)	(0.4, 0.1)	null	null	(0.5, 0.1)
$E_4$	(0.8, 0.2)	(0.7, 0.2)	(0.7, 0.2)	(0.6, 0.1)	null

**Table 3.** The raw reputation rating matrix  $M_2$ .

**Table 4.** The raw reputation rating matrix  $M_3$ .

	$P_1$	$P_2$	$P_3$	$P_4$	$P_5$
<i>E</i> <sub>1</sub>	(0.8, 0.1) (0.9, 0.1)	null	(0.8, 0.1) (0.7, 0.1)	(0.6, 0.2) (0.5, 0.1)	(0.5, 0.5)
$E_2$	null	(0.5, 0.3)	(0.6, 0.3)	(0.6, 0.3)	(0.5, 0.2)
$E_3$	(0.4, 0.2)	(0.3, 0.2)	null	null	(0.5, 0.1)
$E_4$	(0.8, 0.2)	(0.7, 0.2)	(0.6, 0.1)	(0.5, 0.2)	null

**Table 5.** The final reputation rating matrix **M**.

	$P_1$	$P_2$	$P_3$	$P_4$	$P_5$
1	(0.9008, 0.0092)	(0.7117, 0.0031)	(0.9110, 0.0015)	(0.6746, 0.0403)	(0.9826, 0.0039)
2	(0.9535, 0.0083)	(0.8852, 0.0062)	(0.9467, 0.0089)	(0.8870, 0.0154)	(0.8918, 0.0062)
3	(0.9767, 0.0041)	(0.8696, 0.0124)	(0.9539, 0.0087)	(0.8981, 0.0086)	(0.8295, 0.0107)

Step 2: Extracting the related attributes rating data at different time periods and using the DIFWA operator to aggregate all the intuitionistic fuzzy rating matrices  $\mathbf{D}_s$  into a collective intuitionistic fuzzy decision matrix  $\mathbf{D}$ .

The corresponding attributes rating for five candidate suppliers  $X_i$  (i = 1, 2, 3, 4, 5) in the last three years are listed in Tables 6–8. According to the DIFWA operator (see Equation (16)), all the intuitionistic fuzzy decision matrix  $\mathbf{D}_s$  can be aggregated into a final collective fuzzy decision matrix  $\mathbf{D}$  (see the Table 9).

**Table 6.** Intuitionistic fuzzy decision matrix  $D_1$ .

	<i>C</i> <sub>1</sub>	C <sub>2</sub>	C <sub>3</sub>	$C_4$	C <sub>5</sub>
$X_1$	(0.3, 0.6)	(0.7, 0.1)	(0.9008, 0.0092)	(0.7, 0.2)	(0.8, 0.1)
$X_2$	(0.7, 0.2)	(0.6, 0.1)	(0.7117, 0.0031)	(0.5, 0.1)	(0.6, 0.2)
$X_3$	(0.6, 0.3)	(0.7, 0.1)	(0.9110, 0.0015)	(0.7, 0.2)	(0.5, 0.3)
$X_4$	(0.5, 0.1)	(0.3, 0.2)	(0.6746, 0.0403)	(0.5, 0.1)	(0.8, 0.2)
$X_5$	(0.4, 0.6)	(0.7, 0.3)	(0.9826, 0.0039)	(0.6, 0.2)	(0.7, 0.1)

**Table 7.** Intuitionistic fuzzy decision matrix  $D_2$ .

	<i>C</i> <sub>1</sub>	C <sub>2</sub>	C <sub>3</sub>	C <sub>4</sub>	$C_5$
$X_1$	(0.2, 0.7)	(0.9, 0.1)	(0.9767, 0.0041)	(0.8, 0.2)	(0.7, 0.1)
$X_2$	(0.4, 0.6)	(0.7, 0.1)	(0.8696, 0.0124)	(0.7, 0.1)	(0.8, 0.2)
$X_3$	(0.6, 0.2)	(0.7, 0.1)	(0.9539, 0.0031)	(0.6, 0.2)	(0.6, 0.1)
$X_4$	(0.6, 0.1)	(0.5, 0.2)	(0.8981, 0.0086)	(0.6, 0.1)	(0.9, 0.1)
$X_5$	(0.5, 0.5)	(0.6, 0.4)	(0.8295, 0.0107)	(0.7, 0.2)	(0.6, 0.3)

Mathematics 2019, 7, 298 13 of 17

	$C_1$	$C_2$	C <sub>3</sub>	$C_4$	$C_5$
$X_1$	(0.2, 0.7)	(0.9, 0.1)	(0.9767, 0.0041)	(0.8, 0.2)	(0.7, 0.1)
$X_2$	(0.4, 0.6)	(0.7, 0.1)	(0.8696, 0.0124)	(0.7, 0.1)	(0.8, 0.2)
$X_3$	(0.6, 0.2)	(0.7, 0.1)	(0.9539, 0.0031)	(0.6, 0.2)	(0.6, 0.1)
$X_4$	(0.6, 0.1)	(0.5, 0.2)	(0.8981, 0.0086)	(0.6, 0.1)	(0.9, 0.1)
$X_5$	(0.5, 0.5)	(0.6, 0.4)	(0.8295, 0.0107)	(0.7, 0.2)	(0.6, 0.3)

**Table 8.** Intuitionistic fuzzy decision matrix  $D_3$ .

**Table 9.** The collective intuitionistic fuzzy decision matrix **D**.

	$C_1$	$C_2$	$C_3$	$C_4$	$C_5$
$X_1$	(0.25, 0.68)	(0.81, 0.10)	(0.9617, 0.0059)	(0.78, 0.20)	(0.70, 0.10)
$X_2$	(0.54, 0.35)	(0.68, 0.10)	(0.8529, 0.0077)	(0.64, 0.10)	(0.74, 0.20)
$X_3$	(0.60, 0.18)	(0.65, 0.10)	(0.9450, 0.0037)	(0.69, 0.20)	(0.53, 0.17)
$X_4$	(0.53, 0.10)	(0.47, 0.20)	(0.8675, 0.0140)	(0.58, 0.10)	(0.86, 0.14)
$X_5$	(0.48, 0.48)	(0.60, 0.38)	(0.9058, 0.0075)	(0.65, 0.20)	(0.60, 0.21)

Step 3: Determining both positive and negative solutions of each attribute.

According to Equations (19) and (20), both positive and negative solutions can be shown in Table 10.

**Table 10.** Positive and negative solutions.

	$C_1$	$C_2$	$C_3$	$C_4$	$C_5$
Positive Solution	(0.25, 0.68)	(0.81, 0.10)	(0.9617, 0.0059)	(0.78, 0.20)	(0.86, 0.14)
Negative Solution	(0.60, 0.18)	(0.47, 0.20)	(0.8296, 0.0042)	(0.58, 0.10)	(0.53, 0.17)

Step 4: Determining the criteria weights.

According to Equation (17), we acquire the optimal five criteria weights for the five candidate enterprises considered in this example. These weights are: "Cost 0.2153", "Quality 0.1922", "Reputation 0.2153", "Safety 0.1954", and "Satisfaction 0.2001", respectively.

Step 5: Calculating the influence index  $Q_m$ .

According to Equations (23)–(25), we can compute  $S_m$ ,  $R_m$ , and influence index  $Q_m$  for each supplier (see Table 11).

**Table 11.** The final evaluation result of candidate suppliers.

	$S_m$	$R_m$	$Q_m$
$X_1$	0.1119	0.0570	0.4037
$X_2$	0.1410	0.0524	0.7793
$X_3$	0.1328	0.0422	0.3584
$X_4$	0.1243	0.0503	0.4348
$X_5$	0.1269	0.0606	0.7566

Step 6: Ranking the order of candidate suppliers.

After calculating the closeness coefficient of each supplier, we can rank the order of candidate suppliers as follows:

$$X_3 \succ X_1 \succ X_4 \succ X_5 \succ X_2$$

Based on the above analysis, supplier 3 is ranked as the best supplier.

# 4.2. Evaluating the Practicality of Our Proposed Method through a Software Prototype Implementation

To evaluate the practicality of the proposed method, we study a case for supplier selection through a software prototype implementation. The software prototype was developed in the C#

programming language and the SQL Server database. The source code of the software prototype is provided in the Figshare database (Supplementary Materials). In the software prototype system, five graphical user interfaces (GUI) are developed, including GUI of enterprise information, GUI of time weights, GUI of criteria weights, GUI of expert ratings, GUI of criteria weights, and GUI of optimized enterprises. The illustrated case is to select an optimal one from 10 candidate suppliers, including "Aomiao", "Chengguang", "Haiqi", "Huaguang", "Jiannan", "Languang", "Leida", "Longchang", "Qianjiang", and "Yamei". Detailed information about these suppliers is provided in the Figshare database (Supplementary Materials).

Figure 2 partially shows the ratings of criteria including "Cost", "Quality", "Reputation", "Safety", and "Satisfaction" of these suppliers. In this figure, *u*1 denotes satisfaction (i.e., the degree of membership) and *v*1 denotes dissatisfaction (i.e., the degree of non-membership) in the first year, and likewise for the remaining two years. Using the DIFWA operator (see Function (16)), the overall ratings can be obtained, which denote the dynamic criteria ratings for the 10 candidate suppliers. In order to maintain consistency, all values are rounded to four decimal places.

<b>1</b>	Enterprises In	formation &	Time Weights	🧘 Experts Rating	gs 👰 Criteri	a Weights	Optimize Enterprise	5			•
	Enterprise	Criteria	Rat	ting 1	Re	iting 2	Ratin	ng 3	Over	all Rating	
	ancer prise	CITCEITA	ui	v1	u2	v2	u3	v3	u	v	
1		Cost	0.3000	0.6000	0.3000	0.7000	0.2000	0.7000	0.2517	0.6787	
1		Quality	0.7000	0.1000	0.6000	0.1000	0.9000	0.1000	0.8112	0.1000	
1	Aomiao	Reputation	0.9771	0.0021	0.9828	0.0042	0.9922	0.0007	0. 9878	0.0015	
1		Safety	0.7000	0.2000	0.8000	0.2000	0.8000	0.2000	0.7831	0.2000	
1		Satisfaction	0.8000	0.1000	0.6000	0.1000	0.7000	0.1000	0.6984	0.1000	
2		Cost	0.7000	0.2000	0.6000	0.2000	0.4000	0.6000	0.5375	0.3464	
2		Quality	0.6000	0.1000	0.7000	0.1000	0.7000	0.1000	0.6822	0.1000	
2	Chengguang	Reputation	0.9939	0.0002	0.9498	0.0054	0.9597	0.0054	0.9705	0.0028	
2		Safety	0.5000	0.1000	0.6000	0.1000	0.7000	0.1000	0.6378	0.1000	
2		Satisfaction	0.6000	0.2000	0.7000	0.2000	0.8000	0.2000	0.7405	0.2000	
3		Cost	0.9000	0.1000	0.8000	0.1000	0.7000	0.1000	0. 7868	0.1000	
3		Quality	0.3000	0.2000	0.5000	0. 2000	0.5000	0.2000	0.4652	0.2000	
3	Haiqi	Reputation	0.8584	0.0076	0.8198	0.0020	0.7894	0.0040	0.8144	0.0037	
3		Safety	0.5000	0.1000	0.6000	0.1000	0.6000	0.1000	0.5817	0.1000	
3		Satisfaction	0.5000	0.2000	0.5000	0.2000	0.5000	0.2000	0.5000	0.2000	_

Figure 2. Graphical user interfaces (GUI) of expert ratings.

According to Equation (17), the criteria weights of the 10 suppliers can be calculated as: "Cost 0.2053", "Quality 0.1781", "Reputation 0.2244", "Safety 0.1994", and "Satisfaction 0.1928", respectively. Figure 3 shows the experimental results with the 10 suppliers, indicating that the supplier "Qianjiang" has the lowest influence index "0.0571". Thus, "Qianjiang" is the best candidate for the procurement department requirements among the 10 suppliers.

	EntName	Influence index	EntTel	EntAddress	Edit	Delete
1	Qianjiang	0.0571	0571-87458962	589 Fengqi Rd., Jianggan District, Hangzhou	Edit	Delete
2	Yamei	0.0946	0571-88233375	72 XueYuan Rd., Jianggan District, Hangzhou	Edit	Delete
3	Haiqi	0.1182	0571-87414752	258 Qianjiang Rd., Shangcheng District, Hangzhou	Edit	Delete
4	Huaguang	0.1443	0571-89653562	154 Nueyuan Rd., Jianggan District, Hangzhou	Edit	Delete
5	Chengguang	0.2687	0571-89352144	895 Moganshan Rd., Shangcheng District, Hangzhou	Edit	Delete
6	Languang	0.6717	0571-87452366	186 Zhaowu Rd., Gongshu District, Hangzhou	Edit	Delete
7	Ji annan	0.7458	0571-86953562	51 Wuzhou Rd., Yuhang District, hangzhou	Edit	Delete
8	Aomiao	0.7509	0571-87456321	56 Gudun Rd., Gongshu District, Hangzhou	Edit	Delete
9	Longchang	0.8928	0571-89674586	589 Fengqi Rd., Jianggan District, Hangzhou	Edit	Delete
10	Lei da	0.9285	0571-85236985	25 JiGudun Rd., Gongshu District, Hangzhou	Edit	Delete

Figure 3. GUI of optimized enterprises.

Mathematics 2019, 7, 298 15 of 17

#### 5. Conclusions and Future Work

In this study, we presented a novel approach that can be represented as IFS-HITS, which incorporates IFNs into the traditional HITS algorithm to extend the ability of processing fuzzy information in order to obtain post-propagated reputation values of supplier. Then, we applied the IFS-HITS method, DIFWA operator, IFEW method, and VIKOR method to select optimal suppliers in the intuitionistic fuzzy environment. Finally, an example was provided to prove the feasibility and practicality of the proposed reputation-enhanced hybrid approach.

Despite these contributions, our approach still has some limitations, which will be addressed in the future research. For example, the current work only focuses on the theoretical contribution of a novel reputation-enhanced hybrid approach for supplier selection with intuitionistic fuzzy evaluation information through experiment, instead of presenting a comprehensive personalized supplier recommendation system. In the future, we will extend the proposed model to suit some new problem areas and further present a comprehensive personalized supplier recommendation system to provide a high-quality service of supplier selection. In addition, we mainly use simulated data on supplier selection to verify the practicality and feasibility of our proposed method. In the future, we will use the real data to verify the performance of the proposed method more comprehensively. The proposed IFS-HITS approach should be further enhanced to meet other types of fuzzy numbers such as triangular fuzzy numbers and trapezoidal fuzzy numbers. Moreover, the reputation transitivity can be further improved by combining a fuzzy number with other web page evaluating algorithms such as PageRank.

**Supplementary Materials:** All the data and the code of the experiment are displayed in the Figshare database (https://doi.org/10.6084/m9.figshare.7776683.v1).

**Author Contributions:** Conceptualization, Z.Y., W.Y., and X.H.; methodology, Z.Y. and W.Y.; validation, Z.Y. and W.Y.; formal analysis, Z.Y. and W.Y.; data curation, W.Y.; writing—original draft preparation, Z.Y. and W.Y.; writing—review and editing, X.H., X.S., S.Z., and W.Z.; supervision, X.S., S.Z., and W.Z.; project administration, S.Z.; funding acquisition, S.Z. and W.Z.

**Funding:** This work was supported by National Social Science Foundation of China (No. 17BGL047) and National Natural Science Foundation of China (No. 51875503 and No. 51475410).

**Acknowledgments:** The authors are deeply grateful for the comments provided by anonymous reviewers to improve this paper.

Conflicts of Interest: The authors declare that there is no conflict of interest regarding the publication of this paper.

#### References

- 1. Islam, M.S.; Dey, S.; Kundu, G.; Hoque, A.L. A solution to the security issues of an e-government procurement system. In Proceedings of the 2008 ICECE International Conference on Electrical and Computer Engineering, Dhaka, Bangladesh, 20–22 December 2008; pp. 659–664.
- 2. Du, T.C. Building an automatic e-tendering system on the Semantic Web. *Decis. Support Syst.* **2009**, 47, 13–21. [CrossRef]
- 3. Barbarosoglu, G.; Yazgac, T. An application of the analytic hierarchy process to the supplier selection problem. *Prod. Invent. Manag. J.* **1997**, *38*, 14–21.
- 4. Liu, J.; Ding, F.Y.; Lall, V. Using data envelopment analysis to compare suppliers for supplier selection and performance improvement. *Supply Chain Manag.* **2000**, *5*, 143–150. [CrossRef]
- 5. Dulmin, R.; Mininno, V. Supplier selection using a multi-criteria decision aid method. *J. Purch. Supply Manag.* **2003**, *9*, 177–187. [CrossRef]
- 6. Narasimhan, R.; Talluri, S.; Mahapatra, S.K. Multiproduct, multicriteria model for supplier selection with product life-cycle considerations. *Decis. Sci.* **2006**, *37*, 577–603. [CrossRef]
- 7. Boran, F.E.; Genç, S.; Kurt, M.; Akay, D. A multi-criteria intuitionistic fuzzy group decision making for supplier selection with TOPSIS method. *Expert Syst. Appl.* **2009**, *36*, 11363–11368. [CrossRef]
- 8. Jiang, H.; Lu, Z.G. A fuzzy programming model of supplier selection. *Trans. Tech. Publ.* **2012**, *468*, 668–673. [CrossRef]

Mathematics 2019, 7, 298 16 of 17

9. Wang, Y.; Xi, C.Y.; Zhang, S.; Yu, D.J.; Li, Y. A combination of extended fuzzy AHP and fuzzy GRA for government e-tendering in hybrid fuzzy environment. *Sci. World J.* **2014**. [CrossRef]

- 10. Ebrahim, R.M.; Razmi, J.; Haleh, H. Scatter search algorithm for supplier selection and order lot sizing under multiple price discount environment. *Adv. Eng. Softw.* **2009**, *40*, 766–776. [CrossRef]
- 11. Mohammaditabar, D.; Ghodsypour, S.H.; Hafezalkotob, A. A game theoretic analysis in capacity-constrained supplier-selection and cooperation by considering the total supply chain inventory costs. *Int. J. Prod. Econ.* **2016**, *181*, 87–97. [CrossRef]
- 12. Kuo, R.J.; Wang, Y.C.; Tien, F.C. Integration of artificial neural network and MADA methods for green supplier selection. *J. Clean. Prod.* **2010**, *18*, 1161–1170. [CrossRef]
- 13. Falagario, M.; Sciancalepore, F.; Costantino, N.; Pietroforte, P. Using a DEA-cross efficiency approach in public procurement tenders. *Eur. J. Op. Res.* **2012**, *218*, 523–529. [CrossRef]
- 14. Wang, P.Y.; Shen, J.; Guo, W.Q.; Zhang, C.; Zhang, B. Cloud-based government procurement information integration platform. *J. Dig. Inf. Manag.* **2015**, *13*, 147–155.
- 15. Fong, S.; Yan, Z. Design of a web-based tendering system for e-government procurement. In Proceedings of the 3rd International Conference on Theory and Practice of Electronic Governance, Orlando, FL, USA, 13–16 July 2009; pp. 321–328.
- 16. Xu, Y.; Shambour, Q.; Lin, Q.; Zhang, G. BizSeeker: A hybrid semantic recommendation system for personalized government-to-business e-services. *Internet Res.* **2010**, *20*, 342–365.
- 17. Wang, C.N.; Nguyen, V.T.; Thai, H.T.N.; Tran, N.N.A. Sustainable supplier selection process in edible oil production by a hybrid fuzzy analytical hierarchy process and green data envelopment analysis for the SMEs food processing industry. *Mathematics* **2018**, *6*, 302. [CrossRef]
- 18. Zadeh, L.A. Fuzzy sets. Inf. Control 1965, 8, 338–353. [CrossRef]
- 19. Atanassov, K.T. Intuitionistic fuzzy sets. Fuzzy Sets Syst. 1986, 20, 87–96. [CrossRef]
- 20. Khaleie, S.; Fasanghari, M.; Tavassoli, E. Supplier selection using a novel intuitionist fuzzy clustering approach. *Appl. Soft Comput.* **2012**, *12*, 1741–1754. [CrossRef]
- 21. Zhang, X.G.; Deng, Y.; Chan, F.T.S.; Xu, P.; Mahadevan, S.; Hu, Y. IFSJSP: A novel methodology for the job-shop scheduling problem based on intuitionistic fuzzy sets. *Int. J. Prod. Res.* **2013**, *51*, 5100–5119. [CrossRef]
- 22. Tao, F.; Zhao, D.M.; Zhang, L. Resource service optimal-selection based on intuitionistic fuzzy set and non-functionality QoS in manufacturing grid system. *Knowl. Inf. Syst.* **2010**, 25, 185–208. [CrossRef]
- 23. Szmidt, E.; Kacprzyk, J. Distances between intuitionistic fuzzy sets. *Fuzzy Sets Syst.* **2000**, *114*, 505–518. [CrossRef]
- 24. Chen, S.M.; Tan J., M. Handling multi-criteria fuzzy decision-making problems based on vague set theory. *Fuzzy Sets Syst.* **1994**, *67*, 163–172. [CrossRef]
- 25. Hong, D.H.; Choi, C.H. Multi-criteria fuzzy decision-making problems based on vague set theory. *Fuzzy Sets Syst.* **2000**, *114*, 103–113. [CrossRef]
- 26. Xu, Z.S.; Yager, R.R. Some geometric aggregation operators based on intuitionistic fuzzy sets. *Int. J. Gen. Syst.* **2006**, *35*, 417–433. [CrossRef]
- 27. Xu, Z.S. Intuitionistic fuzzy aggregation operators. IEEE Trans. Inf. Forensics Secur. 2007, 15, 1179–1187.
- 28. Kleinberg, J.M. Authoritative sources in a hyperlinked environment. J. ACM 1999, 46, 604–632. [CrossRef]
- 29. Zhang, W.Y.; Zhang, S.; Guo, S.S. A PageRank-based reputation model for personalised manufacturing service recommendation. *Enterp. Inf. Syst UK* **2017**, *11*, 672–693. [CrossRef]
- 30. Nomura, S.; Oyam, S.; Hayamizu, T.; Ishida, T. Analysis and improvement of HITS algorithm for detecting web communities. *Syst. Comput.* **2004**, *35*, 32–42. [CrossRef]
- 31. Deguchi, T.; Takahashi, K.; Takayasu, H.; Takayasu, M. Hubs and authorities in the world trade network using a weighted HITS algorithm. *PLoS ONE* **2014**. [CrossRef] [PubMed]
- 32. Otsuka, T.; Yoshimura, T.; Ito, T. Evaluation of the reputation network using realistic distance between facebook data. In Proceedings of the 2012 IEEE/WIC/ACM International Joint Conferences on Web Intelligence and Intelligent Agent Technology, Macau, China, 4–7 December 2012; pp. 383–388.
- 33. Chawla, S. Personalized web search using trust based hubs and authorities. *Int. J. Eng. Res. Appl.* **2014**, 4, 157–170.
- 34. Ivanov, I.; Vajda, P.; Korshunov, P.; Ebrahimi, T. Comparative study of trust modeling for automatic landmark tagging. *IEEE Trans. Inf. Forensic Secur.* **2013**, *8*, 911–923. [CrossRef]

35. Zhang, S.; Xu, S.; Zhang, W.Y.; Yu, D.J. A time-aware hyperlink-induced topic search-based reputation evaluation method for optimal manufacturing service recommendation in distributed peer-to-peer networks. *J. Algorithms Comput. Technol.* **2017**, *11*, 13–22. [CrossRef]

- 36. Jin, H.P.; Cho, H.J.; Kwun, Y.C. Extension of the VIKOR method to dynamic intuitionistic fuzzy multiple attribute decision making. *Comput. Math. Appl.* **2013**, *65*, 731–744.
- 37. Xu, Z.S.; Yager, R.R. Dynamic intuitionistic fuzzy multi-attribute decision making. *Int. J. Approx. Reason.* **2008**, 48, 246–262. [CrossRef]
- 38. Yu, Q.; Bouguettaya, A. Multi-attribute optimization in service selection. *World Wide Web* **2012**, *15*, 1–31. [CrossRef]
- 39. Ye, J. Fuzzy decision-making method based on the weighted correlation coefficient under intuitionistic fuzzy environment. *Eur. J. Oper. Res.* **2010**, 205, 202–204. [CrossRef]
- 40. Opricovic, S. *Multi-Criteria Optimization of Civil Engineering Systems*; Faculty of Civil Engineering, University of Belgrade: Belgrade, Serbia, 1998; Volume 2, pp. 5–21.
- 41. Opricovic, S.; Tzeng, G.H. Compromise solution by MCDM methods: A comparative analysis of VIKOR and TOPSIS. *Eur. J. Oper. Res.* **2004**, *156*, 445–455. [CrossRef]



© 2019 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (http://creativecommons.org/licenses/by/4.0/).