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Fixed Point Results for α_*-ψ-Dominated Multivalued Contractive Mappings Endowed with Graphic Structure

Tahair Rasham [1,*], Abdullah Shoaib [2], Badriah A. S. Alamri [3,4], Awais Asif [1] and Muhammad Arshad [1]

1 Department of Mathematics, International Islamic University, H-10, Islamabad 44000, Pakistan; awais.asif21@gmail.com (A.A.); marshadzia@iiu.edu.pk (M.A.)
2 Department of Mathematics and Statistics, Riphah International University, Islamabad 44000, Pakistan; abdullahshoaib15@yahoo.com
3 Department of Mathematics, King Abdulaziz University, P.O. Box 80203, Jeddah 21589, Saudi Arabia; baalamri@kau.edu.sa
4 Mathematics Department, Faculty of Science, University of Jeddah, P.O. Box 80327, Jeddah 21589, Saudi Arabia
* Correspondence: tahir_resham@yahoo.com; Tel.: +9-231-4531-5045

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Abstract: The purpose of this paper is to establish fixed point results for a pair α_*-dominated multivalued mappings fulfilling generalized locally new α_*-ψ-Ćirić type rational contractive conditions on a closed ball in complete dislocated metric spaces. Examples and applications are given to demonstrate the novelty of our results. Our results extend several comparable results in the existing literature.

Keywords: fixed point; complete dislocated metric space; α_*-dominated multivalued mapping; α_*-ψ-Ćirić type rational contraction; graphic contraction; closed ball

1. Introduction and Preliminaries

Let $H : S \longrightarrow S$ be a mapping. A point $w \in S$ is called a fixed point of S if $w = Sw$. In literature, there are many fixed point results for contractive mappings defined on the whole space. It is possible that $H : S \longrightarrow S$ is not a contractive mapping but $H : Y \longrightarrow S$ is a contraction. Shoaib et al. [1], proved the result related with intersection of an iterative sequence on closed ball with graph. Recently Rasham et al. [2], proved fixed point results for a pair of multivalued mappings on closed ball for new rational type contraction in dislocated metric spaces. Further fixed point results on closed ball can be observed in [3–6].

Many authors proved fixed point theorems in complete dislocated metric space. The idea of dislocated topologies have useful applications in the context of logic programming semantics (see [7]). Dislocated metric space [8] is a generalization of partial metric space [9], which has applications in computer sciences. Nadler [10], started the research of fixed point results for the multivalued mappings. Asl et al. [11] gave the idea of α_*-ψ contractive multifunctions, α_*-admissible mapping and got some fixed point conclusions for these multifunctions. Further results in this direction can be seen in [12–15]). Recently, Senapati and Dey [16], introduced the concept of a pair of multi β_*-admissible mapping and established some common fixed point theorems for multivalued β_*-ψ-contractive mappings. Recently, Alofi et al. [17] introduced the concept of α-dominated multivalued mappings and established some fixed point results for such mappings on a closed ball in complete dislocated quasi b-metric spaces.

In this paper, we establish common fixed point of α-dominated multivalued mappings for new Ćirić type rational multivalued contractions on a closed ball in complete dislocated metric spaces. Interesting new results in metric space and partial metric space can be obtained as corollaries of our theorems. As an application is derived in the setting of an ordered dislocated metric space for multi $\preceq$-dominated mappings. The notion of multi graph dominated mapping is introduced. Also some new fixed point results with graphic contractions on closed ball for multi graph dominated mappings on dislocated metric space are established. New definition and results for singlevalued mappings are also given. Examples are given to show the superiority of our result. Our results generalize several comparable results in the existing literature.We give the following concepts which will be helpful to understand the paper.

Definition 1. *Let M be a nonempty set and let $d_l : M \times M \to [0, \infty)$ be a function, called a dislocated metric (or simply d_l-metric), if for any $c, g, z \in M$, the following conditions satisfy:*

(i) If $d_l(c, g) = 0$, then $c = g$;
(ii) $d_l(c, g) = d_l(g, c)$;
(iii) $d_l(c, g) \leq d_l(c, z) + d_l(z, g) - d_l(z, z)$.

The pair (M, d_l) is called a dislocated metric space. It is clear that if $d_l(c, g) = 0$, then from (i), $c = g$. But if $c = g$, $d_l(c, g)$ may not be 0. For $c \in M$ and $\varepsilon > 0$, $\overline{B(c, \varepsilon)} = \{g \in M : d_l(c, g) \leq \varepsilon\}$ is a closed ball in (M, d_l).We use D.L. space instead by dislocated metric space.

Example 1. *[3] If $M = R^+ \cup \{0\}$, then $d_l(c, g) = c + g$ defines a dislocated metric d_l on M.*

Definition 2. *[3] Let (M, d_l) be a D.L. space.*

(i) A sequence $\{c_n\}$ in (M, d_l) is called Cauchy sequence if given $\varepsilon > 0$, there corresponds $n_0 \in N$ such that for all $n, m \geq n_0$ we have $d_l(c_m, c_n) < \varepsilon$ or $\lim_{n,m\to\infty} d_l(c_n, c_m) = 0$.
(ii) A sequence $\{c_n\}$ dislocated-converges (for short d_l -converges) to c if $\lim_{n\to\infty} d_l(c_n, c) = 0$. In this case c is called a d_l-limit of $\{c_n\}$.
(iii) (M, d_l) is called complete if every Cauchy sequence in M converges to a point $c \in M$ such that $d_l(c, c) = 0$.

Definition 3. *[1] Let K be a nonempty subset of D.L. space M and let $c \in M$. An element $g_0 \in K$ is called a best approximation in K if*

$$d_l(c, K) = d_l(c, g_0), \text{ where } d_l(c, K) = \inf_{g \in K} d_l(c, g).$$

If each $c \in M$ has at least one best approximation in K, then K is called a proximinal set.

We denote $CP(M)$ be the set of all closed proximinal subsets of M. Let Ψ denote the family of all nondecreasing functions $\psi : [0, +\infty) \to [0, +\infty)$ such that $\sum_{n=1}^{+\infty} \psi^n(t) < +\infty$ for all $t > 0$, where ψ^n is the n^{th} iterate of ψ. if $\psi \in \Psi$, then $\psi(t) < t$ for all $t > 0$.

Definition 4. *[16] Let $S, T : M \to P(M)$ be the closed valued mulifunctions and $\beta : M \times M \to [0, +\infty)$ be a function. We say that the pair (S, T) is $\beta_\star$-admissible if for all $c, g \in M$*

$$\beta(c, g) \geq 1 \Rightarrow \beta_\star(Sc, Tg) \geq 1, \text{ and } \beta_\star(Tc, Sg) \geq 1,$$

where $\beta_\star(Tc, Sg) = \inf\{\beta(a, b) : a \in Tc, b \in Sg\}$. When $S = T$, then we obtain the definition of α_*-admissible mapping given in [11].

Definition 5. *Let (M, d_l) be a D.M. space, $S, T : M \to P(M)$ be multivalued mappings and $\alpha : M \times M \to [0, +\infty)$. Let $A \subseteq M$, we say that the S is α_*-dominated on A, whenever $\alpha_*(c, Sc) \geq 1$ for* *all $c \in A$, where $\alpha_*(c, Sc) = \inf\{\alpha(c,b) : b \in Sc\}$. If $A = M$, then we say that the S is α_*-dominated on M. If $S, T : M \to M$ be self mappings, then S is α-dominated on A, whenever $\alpha(c, Sc) \geq 1$ for all $c \in A$.*

Definition 6. *[1] The function $H_{d_l} : P(M) \times P(M) \to R^+$, defined by*

$$H_{d_l}(A,B) = \max\{\sup_{a \in A} d_l(a,B),\ \sup_{b \in B} d_l(A,b)\}$$

is called dislocated Hausdorff metric on $P(M)$.

Lemma 1. *[1] Let (M, d_l) be a D.L. space. Let $(P(M), H_{d_l})$ is a dislocated Hausdorff metric space on $P(M)$. Then for all $H, U \in P(M)$ and for each $h \in H$ there exists $u_h \in U$ satisfies $d_l(h, U) = d_l(h, u_h)$ then $H_{d_l}(H, U) \geq d_l(h, u_h)$.*

Example 2. *Let $M = \mathbb{R}$. Define the mapping $\alpha : M \times M \to [0, \infty)$ by*

$$\alpha(c,g) = \left\{ \begin{array}{c} 1 \text{ if } c > g \\ \frac{1}{2} \text{ otherwise} \end{array} \right\}.$$

Define the multivalued mappings $S, T : M \to P(M)$ by

$$Sc = \{[c-4, c-3] \text{ if } c \in M\}$$

and,

$$Tg = \{[g-2, g-1] \text{ if } g \in M\}.$$

Suppose $c = 3$ and $g = 2$. As $3 > 2$, then $\alpha(3,2) \geq 1$. Now, $\alpha_\star(S3, T2) = \inf\{\alpha(a,b) : a \in S3, b \in T2\} = \frac{1}{2} \ngeq 1$, this means $\alpha_\star(S3, T2) < 1$, that is, the pair (S, T) is not $\alpha_\star$-admissible. Also, $\alpha_\star(S3, S2) \ngeq 1$ and $\alpha_\star(T3, T2) \ngeq 1$. This implies S and T are not $\alpha_\star$-admissible individually. As, $\alpha_\star(c, Sc) = \inf\{\alpha(c,b) : b \in Sc\} \geq 1$, for all $c \in M$. Hence S is $\alpha_\star$-dominated mapping. Similarly $\alpha_\star(g, Tg) = \inf\{\alpha(g,b) : b \in Tg\} \geq 1$. Hence it is clear that S and T are $\alpha_\star$-dominated but not $\alpha_\star$-admissible.

2. Main Result

Let (M, d_l) be a $D.L.$ space, $c_0 \in M$ and $S, T : M \to P(M)$ be the multifunctions on M. Let $c_1 \in Sc_0$ be an element such that $d_l(c_0, Sc_0) = d_l(c_0, c_1)$. Let $c_2 \in Tc_1$ be such that $d_l(c_1, Tc_1) = d_l(c_1, c_2)$. Let $c_3 \in Sc_2$ be such that $d_l(c_2, Sc_2) = d_l(c_2, c_3)$. Continuing this process, we construct a sequence c_n of points in M such that $c_{2n+1} \in Sc_{2n}$ and $c_{2n+2} \in Tc_{2n+1}$, where $n = 0, 1, 2, \ldots$. Also $d_l(c_{2n}, Sc_{2n}) = d_l(c_{2n}, c_{2n+1})$, $d_l(c_{2n+1}, Tc_{2n+1}) = d_l(c_{2n+1}, c_{2n+2})$. We denote this iterative sequence by $\{TS(c_n)\}$. We say that $\{TS(c_n)\}$ is a sequence in M generated by c_0.

Theorem 1. *Let (M, d_l) be a complete D.L. space. Suppose there exist a function $\alpha : M \times M \to [0, \infty)$. Let, $r > 0$, $c_0 \in \overline{B_{d_l}(c_0, r)}$ and $S, T : M \to P(M)$ be a α_*-dominated mappings on $\overline{B_{d_l}(c_0, r)}$. Assume that for some $\psi \in \Psi$ and*

$$D_l(c,g) = \max\{d_l(c,g), \frac{d_l(c,Tg) + d_l(g,Sc)}{2}, \frac{d_l(c,Sc).d_l(g,Tg)}{a + d_l(c,g)}, d_l(c,Sc), d_l(g,Tg)\},$$

where $a > 0$ the following hold:

$$H_{d_l}(Sc, Tg) \leq \psi(D_l(c,g)), \tag{1}$$

for all $c, g \in \overline{B_{d_l}(c_0, r)} \cap \{TS(c_n)\}$, with either $\alpha(c,g) \geq 1$ or $\alpha(g,c) \geq 1$. Also

$$\sum_{i=0}^{n} \psi^i(d_l(c_0, Sc_0)) \leq r \text{ for all } n \in N \cup \{0\}. \tag{2}$$

Then $\{TS(c_n)\}$ *is a sequence in* $\overline{B_{d_l}(c_0,r)}$, $\alpha(c_n,c_{n+1}) \geq 1$ *for all* $n \in N \cup \{0\}$ *and* $\{TS(c_n)\} \to c^* \in \overline{B_{d_l}(c_0,r)}$. *Also if* $\alpha(c_n,c^*) \geq 1$ *or* $\alpha(c^*,c_n) \geq 1$ *for all* $n \in N \cup \{0\}$ *and the inequality (1) holds for* c^* *also. Then* S *and* T *have common fixed point* c^* *in* $\overline{B_{d_l}(c_0,r)}$.

Proof. Consider a sequence $\{TS(c_0)\}$. From Equation (2), we get

$$d_l(c_0,c_1) \leq \sum_{i=0}^{n} \psi^i(d_l(c_0,c_1)) \leq r.$$

It follows that □

$$c_1 \in \overline{B_{d_l}(c_0,r)}.$$

Let $c_3,\cdots,c_j \in \overline{B_{d_l}(c_0,r)}$ for some $j \in N$. If $j = 2i+1$, where $i = 0,1,2,\ldots,\frac{j-1}{2}$. Since $S,\ T\ :\ M \to P(M)$ be a α_*-dominated mappings on $\overline{B_{d_l}(c_0,r)}$, so $\alpha_*(c_{2i},Sc_{2i}) \geq 1$ and $\alpha_*(c_{2i+1},Tc_{2i+1}) \geq 1$. Now by using Lemma 1, we obtain,

$$\begin{aligned}
d_l(c_{2i+1},c_{2i+2}) &\leq H_{d_l}(Sc_{2i},Tc_{2i+1}) \\
&\leq \psi(\max\{d_l(c_{2i},c_{2i+1}),\frac{d_l\,(c_{2i},Tc_{2i+1})+d_l\,(c_{2i+1},Sc_{2i})}{2}, \\
&\quad \frac{d_l\,(c_{2i},Sc_{2i})\,.d_l\,(c_{2i+1},Tc_{2i+1})}{a+d_l\,(c_{2i},c_{2i+1})},d_l(c_{2i},Sc_{2i}),d_l(c_{2i+1},Tc_{2i+1})\} \\
&\leq \psi(\max\{d_l(c_{2i},c_{2i+1}),\frac{d_l\,(c_{2i},c_{2i+2})+d_l\,(c_{2i+1},c_{2i+1})}{2}, \\
&\quad \frac{d_l\,(c_{2i},c_{2i+1})\,.d_l\,(c_{2i+1},c_{2i+2})}{a+d_l\,(c_{2i},c_{2i+1})},d_l(c_{2i},c_{2i+1}),d_l(c_{2i+1},c_{2i+2})\} \\
&\leq \psi(\max\{d_l(c_{2i},c_{2i+1}), \\
&\quad \frac{d_l\,(c_{2i},c_{2i+1})+d_l\,(c_{2i+1},c_{2i+2})-d_l\,(c_{2i+1},c_{2i+1})+d_l\,(c_{2i+1},c_{2i+1})}{2}, \\
&\quad \frac{d_l\,(c_{2i},c_{2i+1})\,.d_l\,(c_{2i+1},c_{2i+2})}{a+d_l\,(c_{2i},c_{2i+1})},d_l(c_{2i},c_{2i+1}),d_l(c_{2i+1},c_{2i+2})\} \\
&\leq \psi(\max\{d_l(c_{2i},c_{2i+1}),\frac{d_l\,(c_{2i},c_{2i+1})+d_l\,(c_{2i+1},c_{2i+2})}{2}, \\
&\quad \frac{d_l\,(c_{2i},c_{2i+1})\,.d_l\,(c_{2i+1},c_{2i+2})}{a+d_l\,(c_{2i},c_{2i+1})},d_l(c_{2i},c_{2i+1}),d_l(c_{2i+1},c_{2i+2})\} \\
&\leq \psi\left(\max\left\{d_l(c_{2i},c_{2i+1}),d_l(c_{2i+1},c_{2i+2})\right\}\right).
\end{aligned}$$

If $\max\{d_l(c_{2i},c_{2i+1}),d_l(c_{2i+1},c_{2i+2})\} = d_l(c_{2i+1},c_{2i+2})$, then $d_l(c_{2i+1},c_{2i+2}) \leq \psi\,(d_l(c_{2i+1},c_{2i+2}))$. This is the contradiction to the fact that $\psi(t) < t$ for all $t > 0$. So $\max\{d_l(c_{2i},c_{2i+1}),d_l(c_{2i+1},c_{2i+2})\} = d_l(c_{2i},c_{2i+1})$. Hence, we obtain

$$d_l(c_{2i+1},c_{2i+2}) \leq \psi(d_l(c_{2i},c_{2i+1})). \tag{3}$$

As $\alpha_*(c_{2i}, Sc_{2i}) \geq 1$ and $c_{2i+1} \in Sc_{2i}$, so $\alpha(c_{2i}, c_{2i+1}) \geq 1$. Similarly we can get $\alpha_*(c_{2i-1}, Tc_{2i-1}) \geq 1$ and $c_{2i-1} \in Tc_{2i-1}$, so $\alpha(c_{2i-1}, c_{2i}) \geq 1$. Now by using inequality (1), and Lemma 1, we have

$$\begin{aligned} d_l(c_{2i}, c_{2i+1}) &\leq H_{d_l}(Tc_{2i-1}, Sc_{2i}) = H_{d_l}(Sc_{2i}, Tc_{2i-1}) \\ &\leq \psi(\max\{d_l(c_{2i}, c_{2i-1}), \frac{d_l\,(c_{2i}, Tc_{2i-1}) + d_l\,(c_{2i-1}, Sc_{2i})}{2}, \\ &\quad \frac{d_l\,(c_{2i}, Sc_{2i})\,.d_l\,(c_{2i-1}, Tc_{2i-1})}{a + d_l\,(c_{2i}, c_{2i-1})}, d_l(c_{2i}, Sc_{2i}), d_l(c_{2i-1}, Tc_{2i-1})\} \\ &\leq \psi(\max\{d_l(c_{2i}, c_{2i-1}), \frac{d_l\,(c_{2i}, c_{2i}) + d_l\,(c_{2i-1}, c_{2i+1})}{2}, \\ &\quad \frac{d_l\,(c_{2i}, c_{2i+1})\,.d_l\,(c_{2i-1}, c_{2i})}{a + d_l\,(c_{2i}, c_{2i-1})}, d_l(c_{2i}, c_{2i+1}), d_l(c_{2i-1}, c_{2i})\} \\ &\leq \psi(\max\{d_l(c_{2i}, c_{2i-1}), d_l(c_{2i}, c_{2i+1}), d_l(c_{2i-1}, c_{2i})\}). \end{aligned}$$

If $\max\{d_l(c_{2i}, c_{2i-1}), d_l(c_{2i}, c_{2i+1})\} = d_l(c_{2i}, c_{2i+1})$, then

$$d_l(c_{2i}, c_{2i+1}) \leq \psi(d_l(c_{2i}, c_{2i+1})).$$

This is the contradiction to the fact that $\psi(t) < t$ for all $t > 0$. If

$$\max\{d_l(c_{2i}, c_{2i-1}), d_l(c_{2i-1}, c_{2i})\} = d_l(c_{2i-1}, c_{2i}),$$

then

$$d_l(c_{2i}, c_{2i+1})) \leq \psi(d_l(c_{2i-1}, c_{2i})).$$

As ψ is nondecreasing function, so

$$\psi(d_l(c_{2i}, c_{2i+1})) \leq \psi^2(d_l(c_{2i-1}, c_{2i})),$$

by using the above inequality in inequality (3), we obtain

$$d_l(c_{2i+1}, c_{2i+2}) \leq \psi^2(d_l(c_{2i-1}, c_{2i})),$$

continuing in this way, we obtain

$$d_l(c_{2i+1}, c_{2i+2}) \leq \psi^{2i+1}(d_l(c_0, c_1)). \tag{4}$$

Now, if $j = 2i$, where $i = 1, 2, \ldots \frac{j}{2}$. Then, similarly, we have

$$d_l(c_{2i}, c_{2i+1}) \leq \psi^{2i}(d_l(c_1, c_0)). \tag{5}$$

Now, by combining inequalities (4) and (5), we obtain

$$d_l(c_j, c_{j+1}) \leq \psi^j(d_l(c_1, c_0)) \text{ for some } j \in N. \tag{6}$$

Now,

$$\begin{aligned} d_l(c_0, c_{j+1}) &\leq d_l(c_0, c_1) + ... + d_l(c_j, c_{j+1}) \\ &\leq d_l(c_0, c_1) + ... + \psi^j(d_l(c_0, c_1)), \quad \text{by (6)} \\ d_l(c_0, c_{j+1}) &\leq \sum_{i=0}^{j} \psi^i(d_l(c_0, c_1)) \leq r. \quad \text{by (2)} \end{aligned}$$

Thus $c_{j+1} \in \overline{B_{d_l}(c_0,r)}$. Hence $c_n \in \overline{B_{d_l}(c_0,r)}$ for all $n \in N$ therefore $\{TS(c_n)\}$ is a sequence in $\overline{B_{d_l}(c_0,r)}$. As $S,T : M \to P(M)$ be a semi α_*-dominated mappings on $\overline{B_{d_l}(c_0,r)}$, so $\alpha_*(c_n, Sc_n) \geq 1$ and $\alpha_*(c_n, Tc_n) \geq 1$, for all $n \in N$. Now inequality (6) can be written as

$$d_l(c_n, c_{n+1}) \leq \psi^n(d_l(c_0, c_1)), \text{ for all } n \in N. \tag{7}$$

Fix $\varepsilon > 0$ and let $n(\varepsilon) \in N$ such that $\sum\limits_{k \geq n(\varepsilon)} \psi^k(d_l(c_0, c_1)) < \varepsilon$. Let $n, m \in N$ with $m > n > n(\varepsilon)$, then, we obtain,

$$\begin{aligned} d_l(c_n, c_m) &\leq \sum_{i=n}^{m-1} d_l(c_i, c_{i+1}) \leq \sum_{i=n}^{m-1} \psi^i(d_l(c_0, c_1)) \\ &\leq \sum_{k \geq n(\varepsilon)} \psi^k(d_l(c_0, c_1)) < \varepsilon. \end{aligned}$$

Thus we proved that $\{TS(c_0)\}$ is a Cauchy sequence in $(\overline{B_{d_l}(c_0,r)}, d_l)$. As every closed ball in a complete $D.L.$ space is complete, so there exists $c^* \in \overline{B_{d_l}(c_0,r)}$ such that $\{TS(c_n)\} \to c^*$, that is

$$\lim_{n \to \infty} d_l(c^*, c_n) = 0. \tag{8}$$

By assumption, if $\alpha(c^*, c_{2n+1}) \geq 1$ for all $n \in N \cup \{0\}$. Since $\alpha_*(c^*, Sc^*) \geq 1$ and $\alpha_*(c_{2n+1}, Tc_{2n+1}) \geq 1$. Now by using Lemma 1 and inequality Equation (1), we have

$$\begin{aligned} d_l(c^*, Sc^*) &\leq d_l(c^*, c_{2n+2}) + d_l(c_{2n+2}, Sc^*) \\ &\leq d_l(c^*, c_{2n+2}) + H_{d_l}(Tc_{2n+1}, Sc^*) \\ &\leq d_l(c^*, c_{2n+2}) + H_{d_l}(Sc^*, Tc_{2n+1}) \\ &\leq d_l(c^*, c_{2n+2}) + \psi(D_l(c^*, c_{2n+1})) \\ &\leq d_l(c^*, c_{2n+2}) + \psi(\max\{d_l(c^*, c_{2n+1}), \frac{d_l\,(c^*, c_{2n+2}) + d_l\,(c_{2n+1}, Sc^*)}{2}, \\ &\quad \frac{d_l\,(c^*, Sc^*)\,.d_l\,(c_{2n+1}, c_{2n+2})}{a + d_l\,(c^*, c_{2n+1})}, d_l(c^*, Sc^*), d_l(c_{2n+1}, c_{2n+2})\}. \end{aligned}$$

Letting $n \to \infty$, and using the inequalities (7) and (8), we can easily get that $d_l(c^*, Sc^*) \leq \psi\,(d_l(c^*, Sc^*))$ and hence $d_l(c^*, Sc^*) \leq 0$ or $c^* \in Sc^*$. Similarly, by using,

$$d_l(c^*, Tc^*) \leq d_l(c^*, c_{2n+1}) + d_l(c_{2n+1}, Tc^*),$$

we can show that $c^* \in Tc^*$. Hence S and T have a common fixed point c^* in $\overline{B_{d_l}(c_0,r)}$. Since $\alpha_*(c^*, Sc^*) \geq 1$ and (S,T) be the pair of sub α_*-dominated multifunction on $\overline{B_{d_l}(c_0,r)}$, we have $\alpha_*(c^*, Tc^*) \geq 1$ so $\alpha(c^*, c^*) \geq 1$. Now,

$$\begin{aligned} d_l(c^*, c^*) &\leq d_l(c^*, Tc^*) \leq H_{d_l}(Sc^*, Tc^*) \\ &\leq \psi(\max\{d_l(c^*, c^*), \frac{d_l\,(c^*, Tc^*) + d_l\,(c^*, Sc^*)}{2}, \\ &\quad \frac{d_l\,(c^*, Sc^*)\,.d_l\,(c^*, Tc^*)}{a + d_l\,(c^*, c^*)}, d_l(c^*, Sc^*), d_l(c^*, Tc^*)\}. \end{aligned}$$

This implies that $d_l(c^*, c^*) = 0$.

Theorem 2. *Let (M, d_l) be a complete D.L. space. Suppose there exist a function $\alpha : M \times M \to [0, \infty)$. Let, $r > 0$, $c_0 \in \overline{B_{d_l}(c_0, r)}$ and $S, T : M \to P(M)$ be the semi α_*-dominated mappings on $\overline{B_{d_l}(c_0, r)}$. Assume that for some $\psi \in \Psi$ and $D_l(c, g) = \max\{d_l(c, g), d_l(c, Sc), d_l(g, Tg)\}$, the following hold:*

$$H_{d_l}(Sc, Tg)\} \leq \psi(D_l(c, g)), \tag{9}$$

for all $c, g \in \overline{B_{d_l}(c_0, r)} \cap \{TS(c_n)\}$ with either $\alpha(c, g) \geq 1$ or $\alpha(g, c) \geq 1$. Also

$$\sum_{i=0}^{n} \psi^i(d_l(c_0, c_1)) \leq r \text{ for all } n \in N \cup \{0\}.$$

Then $\{TS(c_n)\}$ is a sequence in $\overline{B_{d_l}(c_0, r)}$ and $\{TS(c_n)\} \to c^* \in \overline{B_{d_l}(c_0, r)}$. Also, if the inequality (9) holds for c^* and either $\alpha(c_n, c^*) \geq 1$ or $\alpha(c^*, c_n) \geq 1$ for all $n \in N \cup \{0\}$. Then S and T have a common fixed point c^* in $\overline{B_{d_l}(c_0, r)}$ and $d_l(c^*, c^*) = 0$.

Theorem 3. *Let (M, d_l) be a complete D.L. space. Suppose there exist a function $\alpha : M \times M \to [0, \infty)$. Let, $r > 0$, $c_0 \in \overline{B_{d_l}(c_0, r)}$ and $S : M \to P(M)$ be a semi α_*-dominated mappings on $\overline{B_{d_l}(c_0, r)}$. Assume that for some $\psi \in \Psi$ and*

$$D_l(c, g) = \max\{d_l(c, g), \frac{d_l\,(c, Sg) + d_l\,(g, Sc)}{2}, \frac{d_l\,(c, Sc)\,.d_l\,(g, Sg)}{a + d_l\,(c, g)}, d_l(c, Sc), d_l(g, Sg)\},$$

where $a > 0$ the following hold:

$$H_{d_l}(Sc, Sg) \leq \psi(D_l(c, g)), \tag{10}$$

for all $c, g \in \overline{B_{d_l}(c_0, r)} \cap \{S(c_n)\}$ with $\alpha(c, g) \geq 1$. Also

$$\sum_{i=0}^{n} \psi^i(d_l(c_0, c_1)) \leq r \text{ for all } n \in N \cup \{0\}.$$

Then $\{S(c_n)\}$ is a sequence in $\overline{B_{d_l}(c_0, r)}$ and $\{S(c_n)\} \to c^* \in \overline{B_{d_l}(c_0, r)}$. Also, if the inequality (10) holds for c^* and either $\alpha(c_n, c^*) \geq 1$ or $\alpha(c^*, c_n) \geq 1$ for all $n \in N \cup \{0\}$. Then S has a fixed point c^* in $\overline{B_{d_l}(c_0, r)}$ and $d_l(c^*, c^*) = 0$.

Definition 7. *Let M be a nonempty set, $\preceq$ is a partial order on M and $A \subseteq M$. We say that $a \preceq B$ whenever for all $b \in B$, we have $a \preceq b$. A mapping $S : M \to P(M)$ is said to be semi dominated on A if $a \preceq Sa$ for each $a \in A \subseteq M$. If $A = M$, then $S : M \to P(M)$ is said to be dominated.*

Theorem 4. *Let $(M, \preceq, d_l)$ be an ordered complete D.L. space. Let, $r > 0$, $c_0 \in \overline{B_{d_l}(c_0, r)}$ and $S, T : M \to P(M)$ be a semi dominated mappings on $\overline{B_{d_l}(c_0, r)}$. Assume that for some $\psi \in \Psi$ and*

$$D_l(c, g) = \max\{d_l(c, g), \frac{d_l\,(c, Tg) + d_l\,(g, Sc)}{2}, \frac{d_l\,(c, Sc)\,.d_l\,(g, Tg)}{a + d_l\,(c, g)}, d_l(c, Sc), d_l(g, Tg)\},$$

where $a > 0$ the following hold:

$$H_{d_l}(Sc, Tg) \leq \psi(D_l(c, g)) \tag{11}$$

for all $c, g \in \overline{B_{d_l}(c_0, r)} \cap \{TS(c_n)\}$ with either $c \preceq g$ or $g \preceq c$. Also

$$\sum_{i=0}^{n} \psi^i(d_l(c_1, c_0)) \leq r \text{ for all } n \in N \cup \{0\}. \tag{12}$$

Then $\{TS(c_n)\}$ is a sequence in $\overline{B_{d_l}(c_0,r)}$ and $\{TS(c_n)\} \to c^* \in \overline{B_{d_l}(c_0,r)}$. Also if the inequality (11) holds for c^* and either $c_n \preceq c^*$ or $c^* \preceq c_n$ for all $n \in N \cup \{0\}$. Then S and T have a common fixed point c^* in $\overline{B_{d_l}(c_0,r)}$ and $d_l(c^*,c^*) = 0$.

Proof. Let $\alpha : M \times M \to [0,+\infty)$ be a mapping defined by $\alpha(c,g) = 1$ for all $c \in \overline{B_{d_l}(c_0,r)}$ with either $c \preceq g$, and $\alpha(c,g) = 0$ for all other elements $c,g \in M$. As S and T are the semi dominated mappings on $\overline{B_{d_l}(c_0,r)}$, so $c \preceq Sc$ and $c \preceq Tc$ for all $c \in \overline{B_{d_l}(c_0,r)}$. This implies that $c \preceq b$ for all $b \in Sc$ and $c \preceq e$ for all $c \in Tc$. So, $\alpha(c,b) = 1$ for all $b \in Sc$ and $\alpha(c,e) = 1$ for all $c \in Tc$. This implies that $\inf\{\alpha(c,g) : g \in Sc\} = 1$, and $\inf\{\alpha(c,g) : g \in Tc\} = 1$. Hence $\alpha_*(c,Sc) = 1$, $\alpha_*(c,Tc) = 1$ for all $c \in \overline{B_{d_l}(c_0,r)}$. So, $S,T : M \to P(M)$ are the semi α_*-dominated mapping on $\overline{B_{d_l}(c_0,r)}$. Moreover, inequality (11) can be written as

$$H_{d_l}(Sc,Tg) \le \psi(D_l(c,g)),$$

for all elements c,g in $\overline{B_{d_l}(c_0,r)} \cap \{TS(c_n)\}$, with either $\alpha(c,g) \ge 1$ or $\alpha(g,c) \ge 1$. Also, inequality (12) holds. Then, by Theorem 1, we have $\{TS(c_n)\}$ is a sequence in $\overline{B_{d_l}(c_0,r)}$ and $\{TS(c_n)\} \to c^* \in \overline{B_{d_l}(c_0,r)}$. Now, $c_n, c^* \in \overline{B_{d_l}(c_0,r)}$ and either $c_n \preceq c^*$ or $c^* \preceq c_n$ implies that either $\alpha(c_n,c^*) \ge 1$ or $\alpha(c^*,c_n) \ge 1$. So, all the conditions of Theorem 1 are satisfied. Hence, by Theorem 1, S and T have a common fixed point c^* in $\overline{B_{d_l}(c_0,r)}$ and $d_l(c^*,c^*) = 0$. □

Example 3. *Let $M = Q^+ \cup \{0\}$ and let $d_l : M \times M \to M$ be the complete dislocated metric on M defined by*

$$d_l(c,g) = c + g \text{ for all } c,g \in M.$$

Define the multivalued mapping, $S,T : M \times M \to P(M)$ by,

$$Sc = \left\{ \begin{array}{c} [\frac{c}{3}, \frac{2}{3}c] \text{ if } c \in [0,7] \cap M \\ [c, c+1] \text{ if } c \in (7,\infty) \cap M \end{array} \right\}$$

and,

$$Tc = \left\{ \begin{array}{c} [\frac{c}{4}, \frac{3}{4}c] \text{ if } c \in [0,7] \cap M \\ [c+1, c+3] \text{ if } c \in (7,\infty) \cap M \end{array} \right\}.$$

Considering, $c_0 = 1, r = 8$, then $\overline{B_{d_l}(c_0,r)} = [0,7] \cap M$. Now we have $d_l(c_0,Sc_0) = d_l(1,S1) = d_l(1,\frac{1}{3}) = \frac{4}{3}$. So we obtain a sequence $\{TS(c_n)\} = \{1, \frac{1}{3}, \frac{1}{12}, \frac{1}{36}, ...\}$ in M generated by c_0.Let $\psi(t) = \frac{4t}{5}$, $a = 1$ and,

$$\alpha(c,g) = \left\{ \begin{array}{ll} 1 & \text{if } c,g \in [0,7] \\ \frac{3}{2} & \text{otherwise.} \end{array} \right.$$

Now take $8,9 \in M$, then, we have

$$H_{d_l}(S8,T9) = 20 > \psi(D_l(M,g)) = 19.$$

So, the contractive condition does not hold on whole space M. Now for all $c, g \in \overline{B_{d_q}(c_0, r)} \cap \{TS(c_n)\}$ *with either* $\alpha(c, g) \geq 1$ *or* $\alpha(g, c) \geq 1$*, we have*

$$\begin{aligned}
H_{d_l}(Sc, Tg) &= \max\left\{\sup_{a\in Sc} d_l\left(a, Tg\right), \sup_{b\in Tg} d_l(Sc, b)\right\} \\
&= \max\left\{\sup_{a\in Sc} d_l\left(a, \left[\frac{g}{4}, \frac{3g}{4}\right]\right), \sup_{b\in Tg} d_l\left(\left[\frac{c}{3}, \frac{2c}{3}\right], b\right)\right\} \\
&= \max\left\{d_l\left(\frac{2c}{3}, \left[\frac{g}{4}, \frac{3g}{4}\right]\right), d_l\left(\left[\frac{c}{3}, \frac{2c}{3}\right], \frac{3g}{4}\right)\right\} \\
&= \max\left\{d_l\left(\frac{2c}{3}, \frac{g}{4}\right), d_l\left(\frac{c}{3}, \frac{3g}{4}\right)\right\} \\
&= \max\left\{\frac{2c}{3} + \frac{g}{4}, \frac{c}{3} + \frac{3g}{4}\right\} \\
&\leq \psi\left(\max\left\{c + g, \frac{5cg}{3(1 + c + g)}, \frac{16c + 15g}{24}, \frac{4c}{3}, \frac{5g}{4}\right\}\right) = \psi(D_l(c, g)).
\end{aligned}$$

So, the contractive condition holds on $\overline{B_{d_q}(c_0, r)} \cap \{TS(c_n)\}$*. Also,*

$$\sum_{i=0}^{n} \psi^i(d_l(c_0, c_1)) = \frac{4}{3}\sum_{i=0}^{n}(\frac{4}{5})^i < 8 = r.$$

Hence, all the conditions of Theorem 1 are satisfied. Now, we have $\{TS(c_n)\}$ *is a sequence in* $\overline{B_{d_l}(c_0, r)}$*,* $\alpha(c_n, c_{n+1}) \geq 1$ *and* $\{TS(c_n)\} \to 0 \in \overline{B_{d_l}(c_0, r)}$*. Also,* $\alpha(c_n, 0) \geq 1$ *or* $\alpha(0, c_n) \geq 1$ *for all* $n \in N \cup \{0\}$*. Moreover, 0 is a common fixed point of S and T.*

3. Fixed Point Results for Graphic Contractions

In this section we presents an application of Theorem 3 in graph theory. Jachymski [18], proved the result concerning for contraction mappings on metric space with a graph. Hussain et al. [19], introduced the fixed points theorem for graphic contraction and gave an application. A graph K is connected if there is a path between any two different vertices (see for detail [20,21]).

Definition 8. *Let M be a nonempty set and* $K = (V(K), F(K))$ *be a graph such that* $V(K) = M$*,* $A \subseteq M$*. A mapping* $S : M \to P(M)$ *is said to be multi graph dominated on A if* $(c, g) \in F(K)$*, for all* $g \in Sc$ *and* $c \in A$*.*

Theorem 5. *Let* (M, d_l) *be a complete D.L. space endowed with a graph K. Suppose there exist a function* $\alpha : M \times M \to [0, \infty)$*. Let,* $r > 0$*,* $c_0 \in \overline{B_{d_l}(c_0, r)}$ *,* $S, T : M \to P(M)$ *and let for a sequence* $\{TS(c_n)\}$ *in M generated by* c_0*, with* $(c_{0,}\ c_1) \in F(K)$*. Suppose that the following satisfy:*

(i) *S and T are graph dominated for all* $c, g \in \overline{B_{d_l}(c_0, r)} \cap \{TS(c_n)\}$*;*
(ii) *there exists* $\psi \in \Psi$ *and*

$$D_l(c, g) = \max\left\{d_l(c, g), \frac{d_l\left(c, Tg\right) + d_l\left(g, Sc\right)}{2}, \frac{d_l\left(c, Sc\right).d_l\left(g, Tg\right)}{a + d_l\left(c, g\right)}, d_l(c, Sc), d_l(g, Tg)\right\},$$

where $a > 0$*, such that*

$$H_{d_l}(Sc, Tg) \leq \psi(D_l(c, g)), \quad (13)$$

for all $c, g \in \overline{B_{d_l}(c_0, r)} \cap \{TS(c_n)\}$*, and* $(c,g) \in F(K)$ *or* $(g,c) \in F(K)$*;*
(iii) $\sum_{i=0}^{n} \psi^i(d_l(c_0, Sc_0)) \leq r$ *for all* $n \in N \cup \{0\}$*.*

Then, $\{TS(c_n)\}$ *is a sequence in* $\overline{B_{d_l}(c_0,r)}$, $(c_n,c_{n+1}) \in F(K)$ *as the sequence* $\{TS(c_n)\} \to c^*$. *Also, if* $(c_n,c^*) \in F(K)$ *or* $(c^*,c_n) \in F(K)$ *for all* $n \in N\cup\{0\}$ *and the inequality (13) holds for all* $c,g \in \overline{B_{d_l}(c_0,r)} \cap \{TS(c_n)\}\cup\{c^*\}$.*Then* S *and* T *have common fixed point* c^* *in* $\overline{B_{d_l}(c_0,r)}$.

Proof. Define, $\alpha : M\times M \to [0,\infty)$ by

$$\alpha(c,g) = \begin{cases} 1, \text{ if } c,g \in F(K) \\ 0, \quad \text{otherwise.} \end{cases}$$

As $\{TS(c_n)\}$ is a sequence in c generated by c_0 with $(c_{0,}\ c_1) \in F(K)$, we have $\alpha(c_0,c_1) \geq 1$. Let, $\alpha(c,g) \geq 1$, then $(c,g) \in F(K)$. From (i) we have $(c,Sc) \in F(K)$ for all $g \in Sc$ this implies that $\alpha(c,g) = 1$ for all $g \in Sc$. This further implies that $\inf\{\alpha(c,g) : g \in Sc\} = 1$. Thus S is a α_*-dominated multifunction on $\overline{B_{d_l}(c_0,r)}$. Also if $(c,g) \in F(K)$, we have $\alpha(c,g) = 1$ and hence $\alpha_*(c,Sc) = 1$. Similarly it can be proved $\alpha_*(g,Tg) = 1$. Now,condition (ii) can be written as

$$H_{d_l}(Sc,Tg) \leq \psi(D_l(c,g)),$$

for all $c,g \in \overline{B_{d_l}(c_0,r)} \cap \{TS(c_n)\}$ with either $\alpha(c,g) \geq 1$ or $\alpha(g,c) \geq 1$. By including condition (iii), we obtain all the conditions of Theorem 1. Now, by Theorem 1, we have $\{TS(c_n)\}$ is a sequence in $\overline{B_{d_l}(c_0,r)}$, $\alpha(c_n,c_{n+1}) \geq 1$, that is $(c_n,c_{n+1}) \in F(K)$ and $\{TS(c_n)\} \to c^* \in \overline{B_{d_l}(c_0,r)}$. Also, if $(c_n,c^*) \in F(K)$ or $(c^*,c_n) \in F(K)$ for all $n \in N\cup\{0\}$ and the inequality (13) holds for all $c,g \in \overline{B_{d_l}(c_0,r)} \cap \{TS(c_n)\}\cup\{c^*\}$. Then, we have $\alpha(c_n,c^*) \geq 1$ or $\alpha(c^*,c_n) \geq 1$ for all $n \in N\cup\{0\}$ and the inequality (1) holds for all $c,g \in \overline{B_{d_l}(c_0,r)} \cap \{TS(c_n)\}\cup\{c^*\}$. Again, by Theorem 1, S and T have common fixed point c^* in $\overline{B_{d_l}(c_0,r)}$. □

4. Fixed Point Results for Singlevalued Mapping

In this section, we will give some new definition and results without proof for single-valued mappings which can easily be proved as corollaries of our theorems. Recently, Arshad et al. [22] has given the following definition for dislocated quasi metric space.

Definition 9. *Let* (M,d_l) *be a D.L. space,* $T : M \to M$ *be a self mapping,* $A \subseteq M$ *and* $\alpha : M\times M \to [0,+\infty)$ *be a function. We say that*

(i) T *is* α*-dominated mapping on* A*, if* $\alpha(c,Tc) \geq 1$ *for all* $c \in A$.
(ii) (M,d_l) *is* α*-regular on* A *if for any sequence* $\{c_n\}$ *in* A *such that* $\alpha(c_n,c_{n+1}) \geq 1$ *for all* $n \geq 0$ *and* $c_n \to u \in A$

as $n\to\infty$ *we have* $\alpha(c_n,u) \geq 1$ *for all* $n \geq 0$.

Theorem 6. *Let* (M,d_l) *be a complete D.L. space. Suppose there exist a function* $\alpha : M\times M \to [0,\infty)$. *Let,* $r > 0$, $c_0 \in \overline{B_{d_l}(c_0,r)}$ *and* $S,T : M \to M$ *be two* α*-dominated mappings on* $\overline{B_{d_l}(c_0,r)}$. *Assume that for some* $\psi \in \Psi$ *and*

$$D_l(c,g) = \max\{d_l(c,g), \frac{d_l(c,Tg)+d_l(g,Sc)}{2}, \frac{d_l\,(c,Sc)\,.d_l\,(g,Tg)}{a+d_l\,(c,g)}, d_l(c,Sc), d_l(g,Tg)\},$$

where $a > 0$*, the following hold:*

$$d_l(Sc,Tg) \leq \psi(D_l(c,g)),$$

for all $c,g \in \overline{B_{d_l}(c_0,r)}$ *with either* $\alpha(c,g) \geq 1$ *or* $\alpha(g,c) \geq 1$. *Also*

$$\sum_{i=0}^{n} \psi^i(d_l(c_0,Sc_0))\} \leq r, \text{ for all } n \in N\cup\{0\}.$$

If (M, d_l) is α-regular on $\overline{B_{d_l}(c_0, r)}$, then there exists a common fixed point c^* of S and T in $\overline{B_{d_l}(c_0, r)}$ and $d_l(c^*, c^*) = 0$.

By putting $D_l(c, g) = d_l(c, g)$, we obtain the following result of [22] as a corollary of Theorem 7.

Theorem 7. *[22] Let (M, d_l) be a complete D.L. space. Suppose there exist a function $\alpha : M \times M \to [0, \infty)$. Let, $r > 0$, $c_0 \in \overline{B_{d_l}(c_0, r)}$ and $S, T : M \to M$ be two α-dominated mappings on $\overline{B_{d_l}(c_0, r)}$. Assume that for some $\psi \in \Psi$, the following hold:*

$$d_l(Sc, Tg) \leq \psi(d_l(c, g)),$$

for all $c, g \in \overline{B_{d_l}(c_0, r)}$ with either $\alpha(c, g) \geq 1$ or $\alpha(g, c) \geq 1$. Also

$$\sum_{i=0}^{n} \psi^i(d_l(c_0, Sc_0)) \leq r \text{ for all } n \in N \cup \{0\}.$$

If (M, d_l) is α-regular on $\overline{B_{d_l}(c_0, r)}$, then there exists a common fixed point c^* of S and T in $\overline{B_{d_l}(c_0, r)}$ and $d_l(c^*, c^*) = 0$.

We have the following new result without closed ball in complete $D.L.$ space for α-dominated mapping. Also we write the result only for one singlevalued mapping.

Theorem 8. *Let (M, d_l) be a complete D.L. space. Suppose there exist a function $\alpha : M \times M \to [0, \infty)$, $S : M \to M$ be a α-dominated mappings on M. Assume that for some $\psi \in \Psi$, the following hold for either $\alpha(c, g) \geq 1$ or $\alpha(g, c) \geq 1$:*

$$d_l(Sc, Sg) \leq \psi(D_l(c, g)),$$

If (M, d_l) is α-regular on M, then there exists a fixed point c^* of S in $\overline{B_{d_l}(c_0, r)}$ and $d_l(c^*, c^*) = 0$.

Recall that [3] if $(M, \preceq)$ be a partially ordered set. A self mapping f on M is called dominated if $fc \preceq c$ for each c in M. Two elements $c, g \in M$ are called comparable if $c \preceq g$ or $g \preceq c$ holds.

Theorem 9. *Let $(M, \preceq, d_l)$ be a an ordered complete D.L. space, $S, T : M \to M$ be dominated maps and c_0 be an arbitrary point in M. Suppose that for some $\psi \in \Psi$ and for $S \neq T$, we have,*

$$d_l(Sc, Tg) \leq \psi(D_l(c, g)) \text{ for all comparable elements } c, g \text{ in } \overline{B(c_0, r)},$$

Also

$$\sum_{i=0}^{n} \psi^i(d_l(c_0, Sc_0))\} \leq r \text{ for all } n \in N \cup \{0\}.$$

If for a nonincreasing sequence $\{c_n\}$ in $\overline{B(c_0, r)}$, $\{c_n\} \to u$ implies that $u \preceq c_n$. Then there exists $c^* \in \overline{B(c_0, r)}$ such that $d_l(c^*, c^*) = 0$ and $c^* = Sc^* = Tc^*$.

By putting $D_l(c, g) = d_l(c, g)$ and $\psi(t) = kt$, we obtain the main result Theorem 3 of [3] as a corollary of Theorem 10.

Corollary 1. *[4] Let $(M, \preceq, d_l)$ be a an ordered complete D.L. space, $S, T : M \to M$ be dominated maps and c_0 be an arbitrary point in M. Suppose that for $k \in [0, 1)$ and for $S \neq T$, we have,*

$$d_l(Sc, Tg) \leq k d_l(c, g) \text{ for all comparable elements } c, g \text{ in } \overline{B(c_0, r)},$$

$$\text{and } d_l(c_0, Sc_0) \leq (1 - k)r.$$

If for a non-increasing sequence $\{c_n\}$ *in* $\overline{B(c_0,r)}$, $\{c_n\} \to u$ *implies that* $u \preceq c_n$. *Then there exists* $c^* \in \overline{B(c_0,r)}$ *such that* $d_l(c^*,c^*) = 0$ *and* $c^* = Sc^* = Tc^*$.

Definition 10. *Let* M *be a nonempty set and* $K = (V(K), F(K))$ *be a graph such that* $V(K) = M$, $A \subseteq M$. *A mapping* $S : M \to M$ *is said to be graph dominated on* A *if* $(c, Sc) \in F(K)$, *for all* $c \in A$.

Definition 11. *Let* (M, d_l) *be a complete D.L. space endowed with a graph* K *and* $S, T : M \to M$ *be two graph dominated mappings on* $\overline{B_{d_l}(c_0,r)}$, *for any* $r > 0$, c_0 *be any arbitrary point in* M. *Let* $\{c_n\}$ *be a Picard sequence in* M *with initial guess* c_0, $\psi \in \Psi$ *and*

$$D_l(c,g) = \max\{d_l(c,g), \frac{d_l(c,Tg) + d_l(g,Sc)}{2}, \frac{d_l(c,Sc).d_l(g,Tg)}{a + d_l(c,g)}, d_l(c,Sc), d_l(g,Tg)\},$$

where $a > 0$. *If the following condition holds:*

$$d_l(Sc,Tg) \le \psi(D_l(c,g)), \tag{14}$$

for all $c, g \in \overline{B_{d_l}(c_0,r)} \cap \{c_n\}$ *with either* $(c,g) \in F(K)$ *or* $(g,c) \in F(K)$. *Then the mappings* $S, T : M \to M$ *are called Ćirić type rational* ψ*-graphic contractive mappings on* $\overline{B_{d_l}(c_0,r)} \cap \{c_n\}$. *If* $\psi(t) = k(t)$ *for some* $k \in [0,1)$, *then we say that* $S, T : M \to M$ *are Ciric type rational G-contractive mappings on* $(\overline{B_{d_l}(M_0,r)} \cap \{c_n\})$.

Theorem 10. *Let* (M, d_l) *be a complete D.L. space endowed with a graph* K *and* $S, T : M \to M$ *are the Ćirić type rational* ψ*-graphic contractive mappings on* $\overline{B_{d_l}(c_0,r)} \cap \{c_n\}$. *Suppose that* $(c_0, c_1) \in F(K)$ *and*

$$\sum_{i=0}^{h} \psi^i(d_l(c_0,Sc_0)) \le r, \text{ for all } h \in N \cup \{0\}.$$

Then, $\{c_n\}$ *is a sequence in* $\overline{B_{d_l}(c_0,r)}$, $(c_n, c_{n+1}) \in F(K)$ *and* $\{c_n\} \to c^*$. *Also, if* $(c_n, c^*) \in F(K)$ *or* $(c^*, c_n) \in F(K)$ *for all* $n \in N \cup \{0\}$ *and the inequality (4.1) also holds for* c^*. *Then,* S *and* T *have a common fixed point* c^* *in* $\overline{B_{d_l}(c_0,r)}$.

Theorem 11. *Let* (M, d_l) *be a complete D.L. space endowed with a graph* K *and* $S, T : M \to M$ *are the Ćirić type rational K-contractive mappings on* $\overline{B_{d_l}(c_0,r)} \cap \{c_n\}$. *Suppose that* $(c_0, c_1) \in F(K)$ *and*

$$\sum_{i=0}^{h} k^i(d_l(c_0,Sc_0)) \le r, \text{ for all } h \in N \cup \{0\}.$$

Then, $\{c_n\}$ *is a sequence in* $\overline{B_{d_l}(c_0,r)}$, $(c_n, c_{n+1}) \in F(K)$ *and* $\{c_n\} \to c^*$. *Also, if* $(c_n, c^*) \in F(K)$ *or* $(c^*, c_n) \in F(K)$ *for all* $n \in N \cup \{0\}$ *and the contraction also holds for* c^*. *Then,* S *and* T *have a common fixed point* c^* *in* $\overline{B_{d_l}(c_0,r)}$.

Theorem 12. *Let* (c, d_l) *be a complete D.L. space endowed with a graph* K. *Let,* $r > 0$, $c_0 \in \overline{B_{d_l}(c_0,r)}$ *and* $S, T : M \to M$. *Suppose that the following satisfy:*

(i) S *and* T *are graph dominated on* $\overline{B_{d_l}(c_0,r)}$;
(ii) *there exists* $\psi \in \Psi$, *such that*

$$d_l(Sc,Tg) \le \psi(d_l(c,g)),$$

for all $c, g \in \overline{B_{d_l}(c_0,r)}$ *and* $(c,g) \in F(K)$ *or* $(g,c) \in F(K)$;
(iii) $\sum_{i=0}^{n} \psi^i(d_l(c_0,Sc_0)) \le r$ *for all* $n \in N \cup \{0\}$.

Then, there exist a sequence $\{c_n\}$ in $\overline{B_{d_l}(c_0, r)}$ such that $(c_n, c_{n+1}) \in F(K)$ and $\{c_n\} \to c^* \in \overline{B_{d_l}(c_0, r)}$. Also, if $(c_n, c^*) \in F(K)$ or $(c^*, c_n) \in F(K)$ for all $n \in N \cup \{0\}$, then S and T have common fixed point c^* in $\overline{B_{d_l}(c_0, r)}$ and $d_l(c^*, c^*) = 0$.

Theorem 13. *Let (M, d_l) be a complete D.L. space endowed with a graph K and $S : M \to M$ be a mapping. Suppose that the following satisfy:*

(i) *S is a graph dominated on M;*
(ii) *there exists $\psi \in \Psi$ such that*

$$d_l(Sc, Sg) \leq \psi(d_l(c, g)),$$

for all $c, g \in M$ and $(c,g) \in F(K)$ or $(g,c) \in F(K)$.

Then, there exist a sequence $\{c_n\}$ such that $(c_n, c_{n+1}) \in F(K)$ and $\{c_n\} \to c^*$. Also, if $(c_n, c^*) \in F(K)$ or $(c^*, c_n) \in F(K)$ for all $n \in N \cup \{0\}$, then S has a fixed point c^* in M and $d_l(c^*, c^*) = 0$.

Now, we present only one new result in metric space. Many other results can be derived as corollaries of our previous results.

Theorem 14. *Let (M, d) be a complete metric space endowed with a graph K and $S : M \to M$ be a mapping. Suppose that the following satisfy:*

(i) *S is a graph dominated on M;*
(ii) *there exists $k \in [0, 1)$ such that*

$$d(Sc, Sg) \leq kd(c, g),$$

for all $c, g \in M$ and $(c,g) \in F(K)$ or $(g,c) \in F(K)$.

Then, there exist a sequence $\{c_n\}$ such that $(c_n, c_{n+1}) \in F(K)$ and $\{c_n\} \to c^*$. Also, if $(c_n, c^*) \in F(K)$ or $(c^*, c_n) \in F(K)$ for all $n \in N \cup \{0\}$, then S has a fixed point c^* in M.

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