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On (G, G')-Prešić–Ćirić Operators in Graphical Metric Spaces

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Abstract: The purpose of this paper is to introduce a new type of operators in graphical metric spaces and to prove some fixed point results for these operators. Several known results are generalized and extended in this new setting of graphical metric spaces. The results are illustrated and justified with examples.

Keywords: graphical metric space; fixed point; Prešić type mapping

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1. Introduction

Recently, Shukla et al. [1] generalized the notion of metric spaces for sets equipped with a graphical structure and presented the notion of graphical metric spaces, which are equipped with a weaker type of triangular inequality satisfied by a graphical metric for only those points which are situated on a path in a graph associated with the space. By introducing a new type of mappings associated with the graphical metric space, Shukla et al. [1] generalized the famous Banach contraction principle (BCP) and extended several known results of Ran and Reurings [2], Kirk et al. [3], Edelstein [4] and Jachymski [5].

On the other hand, Prešić [6,7] generalized the BCP in product spaces.

Theorem 1. *Let (X, d) be a complete metric space and $T\colon X^k \to X$ ($k > 0$ an integer). Assume there are nonnegative constants $q_1, q_2, \ldots, q_k$ with $q_1 + q_2 + \cdots + q_k < 1$ such that*

$$d(T(\xi_1, \xi_2, \ldots, \xi_k), T(\xi_2, \xi_3, \ldots, \xi_{k+1})) \leq \sum_{i=1}^{k} q_i d(\xi_i, \xi_{i+1}) \tag{1}$$

for all $\xi_1, \xi_2, \ldots, \xi_{k+1} \in X$. Then, there is a unique point $x \in X$ such that $T(x, x, \ldots, x) = x$. Furthermore, for $\xi_1, \xi_2, \ldots, \xi_k \in X$ with $\xi_{n+k} = T(\xi_n, \xi_{n+1}, \ldots, \xi_{n+k-1})$ ($n \in \mathbb{N}$), the sequence $\{\xi_n\}$ converges and $\lim_{n\to\infty} \xi_n = T\left(\lim_{n\to\infty} \xi_n, \lim_{n\to\infty} \xi_n, \ldots, \lim_{n\to\infty} \xi_n\right)$.

The map satisfying Label (1) is said to be a Prešić operator.

Ćirić and Prešić [8] considered a weaker contractive condition than the condition (1).

Theorem 2. *Let (X,d) be a complete metric space and $T\colon X^k \to X$ ($k > 0$ an integer). Assume that there is $\lambda \in [0,1)$ such that*

$$d(T(\xi_1,\xi_2,\ldots,\xi_k),T(\xi_2,\xi_3,\ldots,\xi_{k+1})) \le \lambda \max\{d(\xi_i,\xi_{i+1}) : 1 \le i \le k\},$$

for all $\xi_1,\xi_2,\ldots,\xi_{k+1} \in X$. Then, there is a point ξ in X so that $T(\xi,\xi,\ldots,\xi) = \xi$. Moreover, for arbitrary $\xi_1,\xi_2,\ldots,\xi_k \in X$ and for $n \in \mathbb{N}$, $\xi_{n+k} = T(\xi_n,\xi_{n+1},\ldots,\xi_{n+k-1})$, the sequence $\{\xi_n\}$ converges and $\lim_{n\to\infty} \xi_n = T\left(\lim_{n\to\infty} \xi_n, \lim_{n\to\infty} \xi_n,\ldots, \lim_{n\to\infty} \xi_n\right)$. If in addition, we have on the diagonal $\triangle \subset X^k$, $d(T(\rho,\rho,\ldots,\rho),T(v,v,\ldots,v)) < d(\rho,v)$ for $\rho,v \in X$, with $\rho \ne v$, then ξ is the unique element so that $\xi = T(\xi,\xi,\ldots,\xi)$.

Prešić type operators have several applications to solve problems in applied mathematics—see, for example, [6,7,9–14]. Recently, many authors worked on the result of Prešić in various directions—see [8,15–23]. Ran and Reurings [2] and Nieto and Lopez [24,25] gave fixed point results in metric spaces via a partial order. These results were generalized by Malhotra et al. [17] (see also [15,26]) and Shukla et al. [22] in product spaces. The cyclic operator was considered by Kirk et al. [3]. Shukla and Abbas [23] extended the result of Kirk et al. [3] by defining the class of cyclic-Prešić operators. An interesting generalization of Banach's result in the spaces endowed with graphs was given by Jachymski [5] which unifies the results of Ran and Reurings [2], Nieto and Lopez [24,25] and Edelstein [4]. Recently, Shukla and Shahzad [27] and Shahzad and Shukla [10] extended these results for single-valued and set-valued mappings in product spaces endowed with a graph—see also [28–32].

Here, we initiate the notion of (G,G')-Prešić–Ćirić operators in graphical metric spaces and prove some related fixed point theorems. The results of this paper generalize and unify the theorems of Prešić [8], Prešić and Ćirić [6,7], Luong and Thuan [15], Ran and Reurings [2], Nieto and Lopez [24,25], Kirk et al. [3], Shukla and Abbas [23], Shukla and Shahzad [27] and Shukla et al. [1] in graphical metric spaces. We also give examples illustrating and justifying the presented results.

2. Preliminaries

Given a nonempty set X, we define: $\Delta = \{(\eta,\eta)\colon \eta \in X\}$. Consider a directed graph G where $V(G)$ (resp. $E(G)$) is the set of vertices (resp. edges) such that $V(G) = X$, $E(G) \supseteq \Delta$ and G has no parallel edges, hence X is endowed with $G = (V(G),E(G))$. The conversion of the graph G is G^{-1}. In addition, $V(G^{-1}) = V(G)$ and $E(G^{-1}) = \{(x,y) \in X \times X\colon (y,x) \in E(G)\}$. $\widetilde{G}$ is denoted as the undirected graph. Consider $V(\widetilde{G}) = V(G)$ and $E(\widetilde{G}) = E(G) \cup E(G^{-1})$.

If $\xi,\eta \in V(G)$, then a path in G from ξ to η of length $l \in \mathbb{N}$ is $\{\xi_i\}_{i=0}^{l}$ of $l+1$ vertices so that $\xi_0 = \xi, \xi_l = \eta$ and $(\xi_{i-1},\xi_i) \in E(G)$ for $i = 1,2,\ldots,l$. A graph G is called connected if, there is a path between any two vertices. The graph G is weakly connected if, considering all of its edges as undirected, there is a path from each vertex to each other vertex. G is weakly connected if $\widetilde{G}$ is connected.

Consider: $(\xi P \eta)_G$ iff there is a directed path from ξ to η in G. $\mu \in (\xi P \eta)_G$ if μ is contained in some directed path from ξ to η in G. A sequence $\{\xi_n\}$ in X is called G-termwise connected if $(\xi_n P \xi_{n+1})_G$ for each $n \in \mathbb{N}$.

From now, we suppose that the graphs are directed where the sets of vertices and edges are nonempty.

Definition 1 (Shukla et al. [1])**.** *Let X be a nonempty set endowed with a graph G and $d_G\colon X \times X \to \mathbb{R}$ be a function such that for all $\xi,\eta,\mu \in X$,*

1. $d_G(\xi,\eta) \ge 0$;
2. $d_G(\xi,\eta) = 0$ *iff* $\xi = \eta$;
3. $d_G(\xi,\eta) = d_G(\eta,\xi)$;
4. $(\xi P \eta)_G$, $\mu \in (\xi P \eta)_G$ *implies* $d_G(\xi,\eta) \le d_G(\xi,\mu) + d_G(\mu,\eta)$.

Here, d_G is called a graphical metric on X.

There are several interesting examples and properties of graphical metric spaces—see [1].

Definition 2 (Shukla et al. [1])**.** *Let $\{\xi_n\}$ be a sequence in a graphical metric space (X, d_G). Then,*

1. *$\{\xi_n\}$ converges to $x \in X$ if, given $\varepsilon > 0$, there is $n_0 \in \mathbb{N}$ so that $d_G(\xi_n, x) < \varepsilon$ for each $n > n_0$. That is, $\lim\limits_{n\to\infty} d_G(\xi_n, x) = 0$.*
2. *$\{\xi_n\}$ is Cauchy sequence if, for $\varepsilon > 0$, there is $n_0 \in \mathbb{N}$ so that $d_G(\xi_n, \xi_m) < \varepsilon$ for all $n, m > n_0$. That is, $\lim\limits_{n,m\to\infty} d_G(\xi_n, \xi_m) = 0$.*
3. *(X, d_G) is complete if every Cauchy sequence in X is convergent in X. Let G' be another graph with $V(G') = X$, then (X, d_G) is G'-complete if each G'-termwise connected Cauchy sequence in X is convergent in X.*

Let X be a nonempty set and $T\colon X^k \to X$ ($k > 0$ an integer). An element $\xi \in X$ is a fixed point of T if $T(\xi, \xi, \ldots, \xi) = \xi$. Denote by $\mathrm{Fix}(T)$ the set of all fixed points of T. Let $\xi_1, \xi_2, \ldots, \xi_k \in X$. $\{\xi_n\}$ given as $\xi_{n+k} = T(\xi_n, \xi_{n+1}, \ldots, \xi_{n+k-1})$ is called a Prešić–Picard sequence (in short, a PP-sequence) with initial values $\xi_1, \xi_2, \ldots, \xi_k$, (see [27]).

3. Main Results

We first introduce a new class of operators in graphical metric spaces.

Suppose X is endowed with the graph G and $P_T^k(G)$ is the set of all paths $\{\xi_i\}_{i=1}^k$ of k vertices so that $(\xi_k, T(\xi_1, \xi_2, \ldots, \xi_k)) \in E(G)$, that is,

$$P_T^k(G) = \left\{ \{\xi_i\}_{i=1}^k \colon (\xi_i, \xi_{i+1}), (\xi_k, T(\xi_1, \xi_2, \ldots, \xi_k)) \in E(G),\ i = 1, 2, \ldots, k-1 \right\}.$$

We now define (G, G')-Prešić–Ćirić operators on a metric space endowed with a graph.

Definition 3. *Let (X, d_G) be a graphical metric space and $T\colon X^k \to X$ ($k > 0$ an integer). Let G' be a subgraph of G such that $E(G') \supseteq \Delta$. Suppose that there is $\lambda \in [0, 1)$ such that, for every path $\{\xi_i\}_{i=1}^{k+1}$ in G', the following holds:*

$$d_G(T(\xi_1, \xi_2, \ldots, \xi_k), T(\xi_2, \xi_3, \ldots, \xi_{k+1})) \leq \lambda \max_{1\leq i\leq 1} \{d_G(\xi_i, \xi_{i+1})\}. \tag{2}$$

Then, T is a (G, G')-Prešić–Ćirić operator.

We suppose that G' is a subgraph of G so that $E(G') \supseteq \triangle$.

Definition 4. *Let (X, d_G) be a graphical metric space and $T\colon X^k \to X$ ($k > 0$ an integer). Then, the mapping T is called a d_G-edge preserving in G' if, for every path $\{\xi_i\}_{i=1}^{k+1}$ in G' such that $d_G(T(\xi_1, \ldots, \xi_k), T(\xi_2, \ldots, \xi_{k+1})) < \max\limits_{1\leq i\leq k} \{d_G(\xi_i, \xi_{i+1})\}$, we have*

$$(T(\xi_1, \ldots, \xi_k), T(\xi_2, \ldots, \xi_{k+1})) \in E(G').$$

Lemma 1. *Let (X, d_G) be a graphical metric space, k a positive integer and $T\colon X^k \to X$ be a (G, G')-Prešić–Ćirić operator. If $\{\xi_i\}_{i=1}^k \in P_T^k(G')$ and T is d_G-edge preserving in G', then the PP-sequence $\{\xi_n\}$ with initial values $\xi_1, \xi_2, \ldots, \xi_k$ is a G'-termwise connected sequence.*

Proof. Suppose that $\{\xi_i\}_{i=1}^k \in P_T^k(G')$, then by definition of $P_T^k(G')$, we have

$$(\xi_i, \xi_{i+1}), (\xi_k, T(\xi_1, \xi_2, \ldots, \xi_k)) \in E(G') \text{ for } i = 1, 2, \ldots, k-1. \tag{3}$$

Now, consider the PP-sequence $\{\xi_n\}$ with initial values $\xi_1, \xi_2, \ldots, \xi_k$. Then, we know that the value of ξ_{k+1} is given by $\xi_{k+1} = T(\xi_1, \xi_2, \ldots, \xi_k)$, and so, by Label (3), we get $(\xi_k, \xi_{k+1}) \in E(G')$. Therefore, $\{\xi_i\}_{i=1}^{k+1}$ is a path in G'. Since T is a (G, G')-Prešić–Ćirić operator, we obtain by (2) that

$$d_G(T(\xi_1, \ldots, \xi_k), T(\xi_2, \ldots, \xi_{k+1})) \leq \lambda \max_{1\leq i\leq k}\{d_G(\xi_i, \xi_{i+1})\} < \max_{1\leq i\leq k}\{d_G(\xi_i, \xi_{i+1})\}.$$

As T is d_G-edge preserving in G', we obtain

$$(\xi_{k+1}, \xi_{k+2}) = (T(\xi_1, \ldots, \xi_k), T(\xi_2, \ldots, \xi_{k+1})) \in E(G').$$

The above inclusion shows that $\{\xi_i\}_{i=2}^{k+2}$ is a path in G', hence, again by (2), we obtain

$$d_G(T(\xi_2, \ldots, \xi_{k+1}), T(\xi_3, \ldots, \xi_{k+2})) \leq \lambda \max_{2\leq i\leq k+1}\{d_G(\xi_i, \xi_{i+1})\} < \max_{2\leq i\leq k+1}\{d_G(\xi_i, \xi_{i+1})\}.$$

As T is d_G-edge preserving in G', we obtain

$$(\xi_{k+2}, \xi_{k+3}) = (T(\xi_2, \ldots, \xi_{k+1}), T(\xi_3, \ldots, \xi_{k+2})) \in E(G').$$

By repeating the same arguments, the PP-sequence $\{\xi_n\}$ with initial values $\xi_1, \xi_2, \ldots, \xi_k$ is a G'-termwise connected sequence. □

We now prove the following theorem which ensures the convergence of a PP-sequence generated by (G, G')-Prešić–Ćirić operator in a graphical metric space.

Theorem 3. *Let (X, d_G) be a G'-complete graphical metric space, k a positive integer and $T\colon X^k \to X$ be a (G, G')-Prešić–Ćirić operator. Suppose that:*

(I) $P_T^k(G') \neq \emptyset$;

(II) *T is d_G-edge preserving in G';*

(III) *if a G'-termwise connected PP-sequence $\{\xi_n\}$ converges in X, then there exist $\mu \in X$ limit of $\{\xi_n\}$ and $n_0 \in \mathbb{N}$ such that $(\xi_n, \mu) \in E(G')$ or $(\mu, \xi_n) \in E(G')$ for each $n > n_0$.*

Then, for every path $\{\xi_i\}_{i=1}^k$ in $P_T^k(G')$, the PP-sequence with initial values $\xi_1, \xi_2, \ldots, \xi_k$ is G'-termwise connected and converges to both ρ and $T(\rho, \ldots, \rho)$ for some $\rho \in X$.

Proof. Since $P_T^k(G') \neq \emptyset$, suppose that $\{\xi_i\}_{i=1}^k \in P_T^k(G')$, then by Lemma 1 the PP-sequence $\{\xi_n\}$ with initial values $\xi_1, \xi_2, \ldots, \xi_k$ is a G'-termwise connected sequence, i.e., $(\xi_i, \xi_{i+1}) \in E(G')$ for all $i \in \mathbb{N}$.

Let $\delta_n = d_G(\xi_n, \xi_{n+1})$, $n \in \mathbb{N}$ and

$$\rho = \max\left\{\frac{\delta_1}{\lambda^{1/k}}, \frac{\delta_2}{\lambda^{2/k}}, \ldots, \frac{\delta_k}{\lambda}\right\}.$$

We claim that

$$\delta_n \leq \rho\lambda^{n/k} \text{ for all } n \in \mathbb{N}. \tag{4}$$

It will be done by mathematical induction. By the definition of ρ, our claim holds for $n = 1, 2, \ldots, k$. We now suppose the induction hypothesis:

$$\delta_{n+i} \leq \rho\lambda^{(n+i)/k}, i = 0, 1, \ldots, k-1. \tag{5}$$

The sequence $\{\xi_n\}$ is G'-termwise connected, for each $n \in \mathbb{N}$, so $\{\xi_{n+i}\}_{i=0}^{k}$ is a path in G'. By (2), we obtain

$$\begin{aligned}\delta_{n+k} &= d_{G}(\xi_{n+k}, \xi_{n+k+1}) \\ &= d_{G}(T(\xi_n, \xi_{n+1}, \ldots, \xi_{n+k-1}), T(\xi_{n+1}, \xi_{n+2}, \ldots, \xi_{n+k})) \\ &\leq \lambda \max_{0 \leq i \leq k-1} \{d_{G}(\xi_{n+i}, \xi_{n+i+1})\} \\ &= \lambda \max_{0 \leq i \leq k-1} \{\delta_{n+i}\}.\end{aligned}$$

Using (5), we get

$$\begin{aligned}\delta_{n+k} &\leq \lambda \max_{0 \leq i \leq k-1} \{\rho\lambda^{(n+i)/k}\} = \rho\lambda \max_{0 \leq i \leq k-1} \{\lambda^{(n+i)/k}\} \\ &\leq \rho\lambda \max_{0 \leq i \leq k-1} \{\lambda^{n/k}\} \\ &= \rho\lambda^{(n+k)/k}.\end{aligned}$$

Hence, (4) is proved.

We claim that $\{\xi_n\}$ is Cauchy. Let $n, m \in \mathbb{N}$ with $m > n$. Since $\{\xi_n\}$ is a G'-termwise connected sequence, we find from (GM4) and (4) that

$$\begin{aligned}d_{G}(\xi_n, \xi_m) &\leq d_{G}(\xi_n, \xi_{n+1}) + d_{G}(\xi_{n+1}, \xi_{n+2}) + \cdots + d_{G}(\xi_{m-1}, \xi_m) \\ &= \sum_{i=0}^{m-n-1} d_{G}(\xi_{n+i}, \xi_{n+i+1}) = \sum_{i=0}^{m-n-1} \delta_{n+i} \\ &\leq \sum_{i=0}^{m-n-1} \rho\lambda^{(n+i)/k} \\ &\leq \rho\lambda^{n/k} \sum_{i=0}^{\infty} \lambda^{i/k} \\ &= \frac{\rho\lambda^{n/k}}{1 - \lambda^{1/k}}.\end{aligned}$$

As k is fixed and $\lambda \in [0, 1)$, we obtain from the above inequality that

$$\lim_{n,m \to \infty} d_{G}(\xi_n, \xi_m) = 0.$$

Hence, $\{\xi_n\}$ is a G'-termwise connected Cauchy sequence. The G'-completeness of (X, d_{G}) yields that $\{\xi_n\}$ converges to some point in X. Using condition (III), there is $\rho \in X$ and $n_0 \in \mathbb{N}$ so that $(\xi_n, \rho) \in E(G')$ or $(\rho, \xi_n) \in E(G')$ for each $n > n_0$ and

$$\lim_{n \to \infty} d_{G}(\xi_n, \rho) = 0.$$

Suppose that $(\xi_n, u) \in E(G')$ for all $n > n_0$ (proof for the case $(u, \xi_n) \in E(G')$ will be same). Since $E(G') \supseteq \Delta$, $\{\xi_n\}$ is G'-termwise connected and $(\xi_n, \rho) \in E(G')$ for all $n > n_0$, the following sequences

$$\{\xi_n, \ldots, \xi_{n+k-1}, \rho\}, \{\xi_{n+1}, \ldots, \xi_{n+k-1}, \rho, \rho\}, \cdots, \{\xi_{n+k-1}, \rho, \ldots, \rho\}, \{\rho, \ldots, \rho, \rho\}$$

are the paths of length $k+1$ in G'. In addition, T is (G, G')-Prešić–Ćirić operator; therefore,

$$\begin{aligned}d_{G}(T(\xi_n, \ldots, \xi_{n+k-1}), T(\xi_{n+1}, \ldots, \xi_{n+k-1}, \rho)) &\leq \lambda \max_{n \leq i \leq n+k-2} \{d_{G}(\xi_i, \xi_{i+1}), d_{G}(\xi_{n+k-1}, \rho)\} \\ &< \max_{n \leq i \leq n+k-2} \{d_{G}(\xi_i, \xi_{i+1}), d_{G}(\xi_{n+k-1}, \rho)\}\end{aligned}$$

for each $n > n_0$. As T is d_G-edge preserving, we have

$$(T(\xi_n, \ldots, \xi_{n+k-1}), T(\xi_{n+1}, \ldots, \xi_{n+k-1}, \rho)) \in E(G') \text{ for each } n > n_0.$$

Similarly,

$$\begin{array}{l}(T(\xi_{n+1}, \ldots, \xi_{n+k-1}, \rho), T(\xi_{n+2}, \ldots, \xi_{n+k-1}, \rho, \rho)), \\ (T(\xi_{n+2}, \ldots, \xi_{n+k-1}, \rho, \rho), T(\xi_{n+3}, \ldots, \xi_{n+k-1}, \rho, \rho, \rho)), \\ \ldots, (T(\xi_{n+k-1}, \rho, \ldots, \rho), T(\rho, \ldots, \rho))\end{array}$$

are the members of $E(G')$ for each $n > n_0$. By (GM4), we obtain

$$\begin{aligned} d_G(\xi_{n+k}, T(\rho, \ldots, \rho)) &= d_G(T(\xi_n, \xi_{n+1}, \ldots, \xi_{n+k-1}), T(\rho, \ldots, \rho)) \\ &\leq d_G(T(\xi_n, \xi_{n+1}, \ldots, \xi_{n+k-1}), T(\xi_{n+1}, \ldots, \xi_{n+k-1}, \rho)) \\ &\quad + d_G(T(\xi_{n+1}, \ldots, \xi_{n+k-1}, \rho), T(\xi_{n+2}, \ldots, \xi_{n+k-1}, \rho, \rho)) \\ &\quad + \cdots + d_G(T(\xi_{n+k-1}, \rho, \ldots, \rho), T(\rho, \ldots, \rho)) \end{aligned}$$

for all $n > n_0$. Using (2), we find that

$$\begin{aligned} d_G(\xi_{n+k}, T(\rho, \ldots, \rho)) &\leq \lambda \max\{d_G(\xi_n, \xi_{n+1}), \ldots, d_G(\xi_{n+k-1}, \rho)\} \\ &\quad + \lambda \max\{d_G(\xi_{n+1}, \xi_{n+2}), \ldots, d_G(\xi_{n+k-1}, \rho)\} \\ &\quad + \cdots + \lambda d_G(\xi_{n+k-1}, \rho). \end{aligned}$$

As $\lim_{n,m\to\infty} d_G(\xi_n, \xi_m) = \lim_{n\to\infty} d_G(\xi_n, u) = 0$, we find that

$$\lim_{n\to\infty} d_G(\xi_{n+k}, T(\rho, \ldots, \rho)) = 0.$$

Thus, $\{\xi_n\}$ converges to both ρ and $T(\rho, \ldots, \rho)$. □

Theorem 3 ensures the convergence of a PP-sequence, but cannot ensure the existence of a fixed point of the (G, G')–Prešić–Ćirić operator. In the following, we explain this fact.

Example 1. *Let $X = \mathbf{2}_{\mathbb{N}} \cup \{0\}$, where $\mathbf{2}_{\mathbb{N}} = \left\{\frac{1}{2^n} : n \in \mathbb{N}\right\}$ and G and G' be the graphs given as $G = G'$, $V(G) = X$ and $E(G) = \Delta \cup \{(\xi, \eta) \in \mathbf{2}_{\mathbb{N}} \times \mathbf{2}_{\mathbb{N}} : \eta \leq \xi\}$.*

Consider $d_G : X \times X \to \mathbb{R}$ as

$$d_G(\xi, \eta) = \begin{cases} 0, & \text{if } \xi = \eta, \\ \xi\eta, & \text{if } \xi, \eta \in \mathbf{2}_{\mathbb{N}}, \xi \neq \eta, \\ \frac{1}{2}, & \text{otherwise.} \end{cases}$$

Note that (X, d_G) is a G-complete graphical metric space, but it not a metric space. Choose $T : X^2 \to X$ as

$$T(\xi, \eta) = \begin{cases} \xi\eta, & \text{if } \xi, \eta \in \mathbf{2}_{\mathbb{N}}, \\ \frac{1}{2}, & \text{otherwise.} \end{cases}$$

Notice that T is a (G, G')–Prešić–Ćirić operator with $\lambda = 1/4$.

If $p, q \in \mathbb{N}$ with $p \leq q$, we have $\left\{\frac{1}{2^p}, \frac{1}{2^q}\right\}$ is a path of length 2, and for this path we have $\left(\frac{1}{2^q}, T\left(\frac{1}{2^p}, \frac{1}{2^q}\right)\right) \in E(G)$. Hence, $P_T^2(G) \neq \emptyset$.

Note that any path $\{\xi_1, \xi_2, \xi_3\}$ *of length* 3 *in G must be one of the following form:*

$$\left\{\frac{1}{2^p}, \frac{1}{2^p}, \frac{1}{2^p}\right\}, \left\{\frac{1}{2^p}, \frac{1}{2^p}, \frac{1}{2^q}\right\}, \left\{\frac{1}{2^p}, \frac{1}{2^q}, \frac{1}{2^s}\right\},$$

where $p \leq q \leq s$. *In each case, we have* $(T(\xi_1, \xi_2), T(\xi_2, \xi_3)) \in E(G)$. *Hence, T is* d_G*-edge preserving.*

Finally, any G-termwise connected, PP-sequence $\{\xi_n\}$ *in X which converges to some* μ, *is either a constant sequence, or a subsequence of the sequence* $\left\{\frac{1}{2^p}\right\}$; *therefore, we have at least one value of z* $\left(=\frac{1}{2}\right) \in X$ *so that (III) of Theorem 3 is verified. Note that, for each path* $\{\xi_j\}_{j=1}^2$ *in* $P_T^2(G)$, *the PP-sequence with initial values* ξ_1, ξ_2 *is G-termwise connected and is convergent to* $\frac{1}{2}$ *and* $T\left(\frac{1}{2}, \frac{1}{2}\right)$; *however, T has no fixed point.*

The above example suggests that to prove the existence of fixed point of a (G, G')–Prešić–Ćirić operator in a graphical metric space, we must apply some additional condition to Theorem 3. Hence, inspired from Shukla et al. [1], we introduce the following property:

Definition 5. *Let* (X, d_G) *be a graphical metric space, k a positive integer and* $T\colon X^k \to X$ *be a mapping. Hence, the quadruple* (X, d_G, G', T) *has property* (S_k) *if:*

$$\begin{gathered}\text{whenever a } G'\text{-termwise connected } PP\text{-sequence } \{\xi_n\} \text{ has two limits } \rho \text{ and } v,\\ \text{where } \rho \in X, v \in T(X^k), \text{ then } \rho = v.\end{gathered} \tag{S_k}$$

Consider $\xi_T = \{\xi \in X\colon (\xi, T(\xi, \dots, \xi)) \in E(G')\}$.

Remark 1. *The property* (S_k) *is a k-dimensional version of the property (S) used by Shukla et al. [1]. In particular, the property* (S_1) *is equivalent to the property (S).*

The following result provides a sufficient condition on the existence of a fixed point of a (G, G')–Prešić–Ćirić operator in a graphical metric space.

Theorem 4. *Suppose that all the conditions of Theorem 3 are satisfied. If, in addition,* (X, d_G, G', T) *has the property* (S_k), *then T has a fixed point.*

Proof. It follows from Theorem 3 that there exists a PP-sequence $\{\xi_n\}$ with initial values $\xi_1, \xi_2, \dots, \xi_k$ and $\rho \in X$ such that $\{\xi_n\}$ is G'-termwise connected and is convergent to both ρ and $T(\rho, \dots, \rho)$. As $\rho \in X$, we have $T(\rho, \dots, \rho) \in T(X^k)$, hence by the property (S_k), we get $T(\rho, \dots, \rho) = \rho$. Thus, ρ is a fixed point of T. □

Example 2. *Consider* $X = [0, 1]$ *and G given by* $V(G) = X$ *and*

$$E(G) = \Delta \cup \{(\xi, \eta) \in X \times X\colon \xi, \eta \in (0, 1], \xi \leq \eta\}.$$

Let $G' = G$. *Take* $d_G\colon X \times X \to \mathbb{R}$ *as*

$$d_G(\xi, \eta) = \begin{cases} 0, & \text{if } \xi = \eta, \\ \ln\left(\frac{1}{\xi\eta}\right), & \text{if } \xi, \eta \in (0, 1], \xi \neq \eta, \\ 1, & \text{otherwise.} \end{cases}$$

Then, (X, d_G) is a G-complete graphical metric space. Put $k = 2$. Take $T\colon X^2 \to X$ as

$$T(\xi, \eta) = \max\left\{\sqrt{\xi}, \sqrt{\eta}\right\}.$$

Then, T is a (G, G')–Prešić–Ćirić operator with $\lambda = 1/2$. All conditions of Theorem 4 are fulfilled, and so, by Theorem 4, T must have a fixed point. Indeed, $Fix(T) = \{0, 1\}$.

T is not a G-Prešić operator (in the sense of Shukla and Shahzad [27]) with respect to the usual metric defined on X.

The fixed point of a (G, G')–Prešić–Ćirić operator satisfying conditions of Theorem 4 may not be unique. The above example verifies this fact.

Theorem 5. *Assume that all conditions of Theorem 4 hold. If, in addition, ξ_T is weakly connected (as a subgraph of G') and*

$$d(T(\xi, \dots, \xi), T(\eta, \dots, \eta)) < d(\xi, \eta) \text{ for all } (\xi, \eta) \in \left[(\xi_T \times \xi_T) \cap E(\widetilde{G'})\right] \setminus \Delta,$$

then T has a unique fixed point.

Proof. The existence of a fixed point ρ follows from Theorem 4. Suppose that v is a fixed point of T and $\rho \neq v$. Since $\rho, v \in$Fix(T), $E(G') \supseteq \Delta$ and ξ_T is weakly connected, we get $\rho, v \in \xi_T$ and $(\rho, v) \in E(\widetilde{G'})$, hence

$$d(\rho, v) = d(T(\rho, \dots, \rho), T(v, \dots, v)) < d(\rho, v).$$

This contradiction proves the result. □

Remark 2. *If in Theorem 5, Fix(T) is assumed weakly connected and the following*

$$d(T(\xi, \dots, \xi), T(\eta, \dots, \eta)) < d(\xi, \eta) \text{ for all } (\xi, \eta) \in \left[(Fix(T) \times Fix(T)) \cap E(\widetilde{G'})\right] \setminus \Delta$$

holds instead as we have assumed, then the conclusion remains the same.

Remark 3. *Let (X, d_G) be a metric space and $T\colon X^k \to X$. Let G' be a subgraph of G such that $E(G') \supseteq \Delta$. Then, T is called a (G, G')-Prešić operator if there are $q_i \geq 0$ such that $\sum_{i=1}^{k} q_i < 1$ and for each path $\{\xi_i\}_{i=1}^{k+1}$ in G', the following holds:*

$$d_G(T(\xi_1, \xi_2, \dots, \xi_k), T(\xi_2, \xi_3, \dots, \xi_{k+1})) \leq \sum_{i=1}^{k} q_i d_G(\xi_i, \xi_{i+1}). \tag{6}$$

Clearly, each (G, G')-Prešić operator is a (G, G')-Prešić-Ćirić operator. In addition, if T is a d_G-edge preserving in G', $(\rho, v) \in E(G')$, then, since $E(G') \supseteq \Delta$, we obtain from (GM4) that

$$\begin{aligned}
d_G(T(\rho, \dots, \rho), T(v, \dots, v)) &\leq d_G(T(\rho, \dots, \rho), T(\rho, \dots, \rho, v)) \\
&\quad + d_G(T(\rho, \dots, \rho, v), T(v, \dots, \rho, v, v)) + \cdots \\
&\quad + d_G(T(\rho, v, \dots, v), T(v, \dots, v)) \\
&\leq q_k d_G(\rho, v) + q_{k-1} d_G(\rho, v) + \cdots + q_1 d_G(\rho, v) \\
&= \sum_{i=1}^{k} q_i d_G(\rho, v) \\
&< d_G(\rho, v)
\end{aligned}$$

for all $\rho, v \in \xi_T$. *We conclude the same when* $(v, \rho) \in E(G')$.

We next derive generalizations and extensions of many known results.

The following is a graphical metric version of results of Shukla and Shahzad [27] and its proof follows directly using Remarks 2 and 3.

Corollary 1. *Let* (X, d_G) *be a* G'*-complete graphical metric space and* $T\colon X^k \to X$ *be a* (G, G')*-Prešić operator. Assume that*

(I) $P_T^k(G') \neq \emptyset$;

(II) T *is* d_G*-edge-preserving in* G';

(III) *if a* G'*-termwise connected* PP*-sequence* $\{\xi_n\}$ *is convergent in* X, *then there is* $\mu \in X$ *of* $\{\xi_n\}$ *and* $n_0 \in \mathbb{N}$ *so that* $(\xi_n, \mu) \in E(G')$ *or* $(\mu, \xi_n) \in E(G')$ *for each* $n > n_0$.

Then, for every path $\{\xi_i\}_{i=1}^k$ *in* $P_T^k(G')$, *the PP-sequence with initial values* $\xi_1, \xi_2, \ldots, \xi_k$ *is* G'*-termwise connected and is convergent to both* ρ *and* $T(\rho, \ldots, \rho)$, *for some* $\rho \in X$. *If in addition,* (X, d_G, G', T) *has the property* (S_k), *then* T *has a fixed point. Furthermore, if Fix(T) is weakly connected (a subgraph of* G'*), then such fixed point is unique.*

Corollary 2 (Ćirić and Prešić [8])**.** *Let* (X, d) *be a complete metric space and* $T : X^k \to X$ *be a mapping so that there is* $\lambda \in [0, 1)$ *so that*

$$d(T(\xi_1, \xi_2, \ldots, \xi_k), T(\xi_2, \xi_3, \ldots, \xi_{k+1})) \leq \lambda \max\{d(\xi_i, \xi_{i+1}) : 1 \leq i \leq k\},$$

for all $\xi_1, \xi_2, \ldots, \xi_{k+1} \in X$. *Then, there is* ξ *in* X *such that* $T(\xi, \xi, \ldots, \xi) = \xi$. *Moreover, for arbitrary* $\xi_1, \xi_2, \ldots, \xi_k \in X$ *and for* $n \in \mathbb{N}$, $\xi_{n+k} = T(\xi_n, \xi_{n+1}, \ldots, \xi_{n+k-1})$, $\{\xi_n\}$ *converges and* $\lim_{n\to\infty} \xi_n = T\left(\lim_{n\to\infty} \xi_n, \lim_{n\to\infty} \xi_n, \ldots, \lim_{n\to\infty} \xi_n\right)$. *If, in addition, on the diagonal* $\triangle \subset X^k$, $d(T(\rho, \rho, \ldots, \rho), T(v, v, \ldots, v)) < d(\rho, v)$ *for* $\rho, v \in X$, *with* $\rho \neq v$, *then* ξ *is the unique point satisfying* $\xi = T(\xi, \xi, \ldots, \xi)$.

Proof. Take the graphs G and G' as $G = G'$ where $V(G) = X$ and $E(G) = X \times X$. All the conditions of Theorem 5 hold, and the proof follows directly. □

Next, we give a result for cyclic contractions in product spaces (see [23]). The following definition generalizes the definition of cyclic-Prešić operator given by Shukla and Abbas [23].

Definition 6. *Let* X *be any nonempty set and* $T\colon X^k \to X$. *Take* $E_1, E_2, \ldots, E_p$ *nonempty subsets subsets of* X. *Then* $X = \bigcup_{j=1}^{p} E_j$ *is a cyclic representation of* X *with respect to* T *if*

1. E_j, $j = 1, 2, \ldots, p$ *are nonempty sets;*
2. $T(E_1 \times E_2 \times \cdots \times E_k) \subset E_{k+1}$, $T(E_2 \times E_3 \times \cdots \times E_{k+1}) \subset E_{k+2}, \ldots,$ $T(E_i \times E_{i+1} \times \cdots \times E_{i+k-1}) \subset E_{i+k}, \cdots$, *where* $E_{p+j} = E_j$ *for all* $j \in \mathbb{N}$.

If $Y \subseteq X$, *then* $T\colon Y^k \to Y$ *is said to be a cyclic-Prešić-Ćirić operator in the case that*

(CPC1) $Y = \bigcup_{j=1}^{p} E_j$ *is a cyclic representation of* Y *with respect to* T;

(CPC2) *there is* $\lambda \in [0, 1)$ *so that*

$$d(T(\xi_1, \xi_2, \ldots, \xi_k), T(\xi_2, \xi_3, \ldots, \xi_{k+1})) \leq \lambda \max_{1 \leq i \leq k}\{d(\xi_i, \xi_{i+1})\} \tag{7}$$

for all $\xi_1 \in E_j, \xi_2 \in E_{j+1}, \ldots, \xi_{k+1} \in E_{j+k}$, $(j = 1, 2, \ldots, p$ $E_{p+j} = E_j$ *for* $j \in \mathbb{N})$.

The following is a generalized form of the main result of Shukla and Abbas [23] (in view of (7)).

Corollary 3. *Let $E_1, E_2, \ldots, E_m$ be closed nonempty subsets of a complete metric space (X, d) and $Y = \bigcup_{j=1}^{p} E_j$. Let $T\colon Y^k \to Y$ be a cyclic–Prešić–Ćirić operator, then T has a fixed point $\rho \in \bigcap_{j=1}^{p} E_j$. If, in addition,*

$$d(T(\xi,\ldots,\xi), T(\eta,\ldots,\eta)) < d(\xi,\eta) \text{ for all } \xi,\eta \in \mathrm{Fix}(T) \text{ with } \xi \neq \eta$$

and $Fix(T) \subset \bigcap_{j=1}^{p} E_j$, then ρ is unique.

Proof. Take the graphs G and G' by $V(G) = X$, $E(G) = X^2$ and $V(G') = X$,

$$E(G') = \Delta \cup \{(\xi,\eta) \in X^2 \colon \xi \in E_i, \eta \in E_{i+1}, 1 \leq i \leq k\},$$

where $E_{p+j} = E_j$ for all $j \in \mathbb{N}$. Since each E_i is closed, Y is G'-complete. The condition (CPC2) shows that T is a (G, G')–Prešić–Ćirić operator, while (CPC1) ensures that T is d_G-edge preserving in G'. Since each E_i is nonempty, $P_T^k(G') \neq \emptyset$. Furthermore, since $E(G) = X^2$, (X, d_G, G', T) has the property (S_k). Proposition 2.1 of [23] shows that the condition (III) of Theorem 3 is satisfied. Hence, by Theorem 4, T has a fixed point $\rho \in Y$ as the limit of a G'-termwise connected sequence, and so, by Proposition 2.1 of [23], we have $\rho \in \bigcap_{j=1}^{p} E_j$. Finally, if $\mathrm{Fix}(T) \subset \bigcap_{j=1}^{p} E_j$, then Fix($T$) is weakly connected (a subgraph of G'), therefore the proof follows from Theorem 5. □

In the next definition and corollary, we generalize the results of Ran and Reurings [2] and Nieto and Lopez [24,25] in product spaces.

Definition 7. *Let X be a nonempty set equipped with a partial order "$\sqsubseteq$" and a metric d. A sequence $\{\xi_n\}$ in X is nondecreasing with respect to "$\sqsubseteq$" if $\xi_1 \sqsubseteq \xi_2 \sqsubseteq \ldots \sqsubseteq \xi_n \sqsubseteq \cdots$. A subset $A \subseteq X$ is called well-ordered if $\xi \sqsubseteq \eta$ or $\eta \sqsubseteq \xi$ for all $\xi, \eta \in A$. The map $T\colon X^k \to X$ is said nondecreasing with respect to "$\sqsubseteq$" if, for any finite nondecreasing sequence $\{\xi_i\}_{i=1}^{k+1}$, we have $T(\xi_1, \xi_2, \ldots, \xi_k) \sqsubseteq T(\xi_2, \xi_3, \ldots, \xi_{k+1})$. Such T is called an ordered Prešić–Ćirić type operator if:*

(OPC1) *T is nondecreasing with respect to "$\sqsubseteq$";*
(OPC2) *there is $\lambda \in [0, 1)$ so that*

$$d(f(\xi_1,\xi_2,\ldots,\xi_k), f(\xi_2,\xi_3,\ldots,\xi_{k+1})) \leq \lambda \max_{1\leq i\leq k}\{d(\xi_i,\xi_{i+1})\} \tag{8}$$

for all $\xi_1, \xi_2, \ldots, \xi_{k+1} \in X$ with $\xi_1 \sqsubseteq \xi_2 \sqsubseteq \cdots \sqsubseteq \xi_{k+1}$.

Corollary 4. *Let $(X, \sqsubseteq, d)$ be an ordered complete metric space and $T\colon X^k \to X$. Suppose that*

(A) *T is an ordered Prešić-Ćirić type contraction;*
(B) *there are $\xi_1, \xi_2, \ldots, \xi_k \in X$ so that $\xi_1 \sqsubseteq \xi_2 \sqsubseteq \cdots \sqsubseteq \xi_k \sqsubseteq T(\xi_1, \xi_2, \ldots, \xi_k)$;*
(C) *if a nondecreasing sequence $\{\xi_n\}$ is convergent to $\xi \in X$, then $\xi_n \sqsubseteq \xi$ or $\xi \sqsubseteq \xi_n$.*

Then, T has a fixed point $\rho \in X$. If, in addition,

$$d(T(\xi,\ldots,\xi), T(\eta,\ldots,\eta)) < d(\xi,\eta) \text{ for all } \xi,\eta \in \mathrm{Fix}(T) \text{ with } \xi \neq \eta,$$

then $Fix(T)$ is well-ordered iff ρ is unique.

Proof. Consider graphs G and G' such that $V(G) = X$, $E(G) = X \times X$ and $V(G') = X$,

$$E(G') = \{(x, y) \in X \times X \colon x \sqsubseteq y\}.$$

Then, X is G' complete. (OPC1) implies that T is a (G, G')–Prešić–Ćirić operator and (OPC2) shows that T is d_G-edge preserving in G'. Condition (B) ensures that $T_T^k(G') \neq \emptyset$. Since $E(G) = X \times X$, (X, d_G, G', T) has the property (S_k). Condition (C) shows that condition (III) of Theorem 3 holds. Hence, by Theorem 4, T has a fixed point $\rho \in X$. Finally, if $\mathrm{Fix}(T)$ is well-ordered, then $\mathrm{Fix}(T)$ is weakly connected (a subgraph of G',) so, from Theorem 5, the fixed point ρ is unique. If $\mathrm{Fix}(T)$ is a singleton, then it is well-ordered. □

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References

1. Shukla, S.; Radenović, S.; Vetro, C. Graphical metric space: A generalized setting in fixed point theory. *Revista de la Real Academia de Ciencias Exactas Físicas y Naturales Ser. A Matemáticas* **2017**, *111*, 641–655. [CrossRef]
2. Ran, A.C.M.; Reurings, M.C.B. A fixed point theorem in partially ordered sets and some application to matrix equations. *Proc. Am. Math. Soc.* **2004**, *132*, 1435–1443. [CrossRef]
3. Kirk, W.A.; Srinivasan, P.S.; Veeramani, P. Fixed points for mappings satisfying cyclical contractive conditions. *Fixed Point Theory* **2003**, *4*, 79–89.
4. Edelstein, M. An extension of Banach's contraction principle. *Proc. Am. Math. Soc.* **1961**, *12*, 7–10.
5. Jachymski, J. The contraction principle for mappings on a metric space with a graph. *Proc. Am. Math. Soc.* **2008**, *136*, 1359–1373. [CrossRef]
6. Prešić, S.B. Sur la convergence des suites. *Comptes Rendus de l'Acad. de Paris* **1965**, *260*, 3828–3830.
7. Prešić, S.B. Sur une classe d'inéquations aux differences finies et sur la convergence de certaines suites. *Publ. de l'Inst. Math. Belgrade* **1965**, *5*, 75–78.
8. Ćirić, L.B.; Prešić, S.B. On Prešić type generalisation of Banach contraction principle. *Acta Math. Univ. Com.* **2007**, *LXXVI*, 143–147
9. Khan, M.S.; Berzig, M.; Samet, B. Some convergence results for iterative sequences of Prešić type and applications. *Adv. Differ. Equ.* **2012**. [CrossRef]
10. Shahzad, N.; Shukla, S. Set-valued *G*-Prešić operators on metric spaces endowed with a graph and fixed point theorems. *Fixed Point Theory Appl.* **2015**, *2015*, 24. [CrossRef]
11. Shukla, S.; Gopal, D.; R-López, R. Fuzzy-Prešić–Ćirić operators and applications to certain nonlinear differential equations. *Math. Model. Anal.* **2016**, *21*, 811–835. [CrossRef]
12. Shukla, S.; Radenović, S. Some generalizations of Prešić type mappings and applications. *An. Ştiinţ. Univ. Al. I. Cuza Iaşi. Mat. (N.S.)* **2017**. [CrossRef]
13. Berinde, V.; Păcurar, M. Two elementary applications of some Prešić type fixed point theorems. *Creat. Math. Inform.* **2011**, *20*, 32–42.
14. Chen, Y.Z. A Prešić type contractive condition and its applications. *Nonlinear Anal.* **2009**, *71*, 2012–2017. [CrossRef]
15. Luong, N.V.; Thuan, N.X. Some fixed point theorems of Prešić–Ćirić type. *Acta Universitatis Apulensis* **2012**, *30*, 237–249.
16. George, R.; Reshma, K.P.; Rajagopalan, R. A generalised fixed point theorem of Prešić type in cone metric spaces and application to Morkov process. *Fixed Point Theory Appl.* **2011**, *2011*, 85. [CrossRef]

17. Malhotra, S.K.; Shukla, S.; Sen, R. A generalization of Banach contraction principle in ordered cone metric spaces. *J. Adv. Math. Stud.* **2012**, *5*, 59–67.
18. Shukla, S. Prešić type results in 2-Banach spaces. *Afrika Matematika* **2014**, *25*, 1043–1051. [CrossRef]
19. Shukla, S. Set-valued Prešić–Ćirić type contraction in 0-complete partial metric spaces. *Matematički Vesnik* **2014**, *66*, 178–189.
20. Shukla, S.; Sen, R. Set-valued Prešić-Reich type mappings in metric spaces. *Revista de la Real Academia de Ciencias Exactas Físicas y Naturales Serie A Matemáticas* **2014**, *108*, 431–440. [CrossRef]
21. Shukla, S.; Sen, R.; Radenović, S. Set-valued Prešić type contraction in metric spaces. *An. Ştiinţ. Univ. Al. I. Cuza Iaşi. Mat. (N.S.)* **2015**, *LXI*, f.2. [CrossRef]
22. Shukla, S.; Radojević, S.; Veljković, Z.A.; Radenović, S. Some coincidence and common fixed point theorems for ordered Prešić-Reich type contractions. *J. Inequ. Appl.* **2013**, *2013*, 520. [CrossRef]
23. Shukla, S.; Abbas, M. Fixed point results of cyclic contractions in product spaces. *Carpathian J. Math.* **2014**, *31*, 119–126.
24. Nieto, J.J.; Lopez, R.R. Contractive mapping theorems in partially ordered sets and applications to ordinary differential equations. *Order* **2005**, *22*, 223–239. [CrossRef]
25. Nieto, J.J.; Lopez, R.R. Existence and uniqueness of fixed point in partially ordered sets and applications to ordinary differential equations. *Acta Math. Sin. Engl. Ser.* **2007**, *23*, 2205–2212. [CrossRef]
26. Shukla, S.; Radenović, S. A generalization of Prešić type mappings in 0-Complete ordered partial metric spaces. *Chin. J. Math.* **2013**, *2013*, 859531. [CrossRef]
27. Shukla, S.; Shahzad, N. *G*-Prešić operators on metric spaces endowed with a graph and fixed point theorems. *Fixed Point Theory Appl.* **2014**, *2014*, 127. [CrossRef]
28. Felhi, A.; Aydi, H.; Zhang, D. Fixed points for α-admissible contractive mappings via simulation functions. *J. Nonlinear Sci. Appl.* **2016**, *9*, 5544–5560. [CrossRef]
29. Afshari, H.; Atapour, M.; Aydi, H. Generalized $\alpha - \psi$-Geraghty multivalued mappings on b-metric spaces endowed with a graph. *TWMS J. Appl. Eng. Math.* **2017**, *7*, 248–260.
30. Ameer, E.; Aydi, H.; Arshad, M.; Alsamir, H.; Noorani, M.S. Hybrid multivalued type contraction mappings in α_K-complete partial b-metric spaces and applications. *Symmetry* **2019**, *11*, 86. [CrossRef]
31. Aydi, H.; Felhi, A. Fixed points in modular spaces via α-admissible mappings and simulation functions. *J. Nonlinear Sci. Appl.* **2016**, *9*, 3686–3701. [CrossRef]
32. Souyah, N.; Aydi, H.; Abdeljawad, T.; Mlaiki, N. Best proximity point theorems on rectangular metric spaces endowed with a graph. *Axioms* **2019**, *8*, 17. [CrossRef]