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*Article*

# Risk Assessment for Failure Mode and Effects Analysis Using the Bonferroni Mean and TODIM Method

**Jianghong Zhu [1,*], Bin Shuai [1], Rui Wang [1] and Kwai-Sang Chin [2]**

[1] School of Transportation and Logistics, Southwest Jiaotong University, Chengdu 611756, China; shuaibin@home.swjtu.edu.cn (B.S.); wryuedi@my.swjtu.edu.cn (R.W.)
[2] Department of System Engineering and Engineering Management, City University of Hong Kong, Hong Kong, China; mekschin@cityu.edu.hk
* Correspondence: zhujianghong@my.swjtu.edu.cn

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**Abstract:** As a safety and reliability analysis technique, failure mode and effects analysis (FMEA) has been used extensively in several industries for the identification and elimination of known and potential failures. However, some shortcomings associated with the FMEA method have limited its applicability. This study aims at presenting a comprehensive FMEA model that could efficiently handle the preference interdependence and psychological behavior of experts in the process of failure modes ranking. In this model, a linguistic variable expressed by the interval-valued Pythagorean fuzzy number (IVPFN) is utilized by experts to provide preference information with regard to failure modes' evaluation and risk factors' weight. Then, to depict the interdependent relationships between experts' preferences, the Bonferroni mean operator is extended to IVPFN to aggregate the experts' preference. Subsequently, an extended TODIM approach in which the dominance degree of failure modes is calculated by grey relational analysis is utilized to determine the risk priority of failure modes. Finally, a practical example concerning the risk assessment of a nuclear reheat valve system is provided to demonstrate the effectiveness and feasibility of the presented method. In addition, a sensitivity analysis and comparison analysis are conducted, and the results show that the preference interdependence and psychological behavior of experts have an important effect on the risk priority of failure modes.

**Keywords:** failure mode and effects analysis; preference interdependence; Bonferroni mean; TODIM; risk assessment

## 1. Introduction

Failure mode and effects analysis (FMEA), which is a powerful engineering technique for risk management, has proven to be an effective methodology for accident prevention and risk analysis of a system/process to identify potential failure modes and assess the effects of different failure modes [1]. The formal FMEA method was originally proposed by the aerospace industry in the 1960s to satisfy the apparent requirements of reliability and safety [2]. The aim of implementing FMEA is to help identify failure modes that affect a system's functioning and to correct these failures by giving some suggestions. Currently, FMEA has been widely applied in various fields, such as semiconductor [3], manufacturing [4], and nuclear power [5].

In conventional FMEA, the risk priority number (RPN), which is calculated by the product of severity ($S$), occurrence ($O$), and detection ($D$), is used to obtain the risk priority of failure modes. Although the RPN method is simple, in practical applications, it also suffers some drawbacks, some of which are listed as follows [6–8]: (1) Different evaluations of risk factors may produce the same RPN values, but their risk implications are entirely dissimilar; (2) direct and indirect relationships among various failure modes and effects are not considered; (3) obtaining precise and complete evaluations for qualitative risk factors are difficult or even impossible; and (4) uncertainty, subjectivity, and vagueness of the judgments of FMEA team members cannot be managed appropriately. To overcome the above weakness of the RPN method, improved approaches of FMEA have been numerously presented in the past decade and applied to deal with practical problems [6,8–12]. More information with respect to improved FMEA methods is provided in recent surveys [13].

In these improved FMEA methods, FMEA can be considered as multi-criteria decision making (MCDM) problems [14] as it takes into account many risk factors. In order to avoid an individual bias, FMEA activity is usually performed based on a team and should be regarded as a group decision behavior [10]. Therefore, FMEA activity can be considered as multi-criteria group decision making (MCGDM) problems [15]. Generally, the implementation of group-based FMEA consists of three stages, which includes failure modes' evaluation, determination of the weights of risk factors, and failure modes ranking [16]. This study focuses on the following two stages: Failure modes evaluation and failure modes ranking.

Failure modes evaluation involves expression and aggregation expert preferences. To express precisely and completely experts' preferences, many alternative methods, which incorporate fuzzy set theory and its extended forms [7,15,17–19], have been introduced into FMEA to deal with the uncertainty and ambiguity of experts' judgment. However, no or little attention has been paid to the way of aggregating experts' preferences, which usually impact on the final ranking of failure modes. Generally, before the ranking of alternatives, experts' preferences are aggregated to collective preferences to prevent the loss of information [20], which indicates that the way of aggregation plays an important role in the aggregation preferences of MCGDM problems [21]. Therefore, it is a meaningful work to explore whether the method of aggregating preferences affects the final ranking results.

On the other hand, failure modes' ranking aims at using an MCDM technique to obtain the risk priority of failure modes. In reality, FMEA is essentially a multi-criteria group decision making problem [15], and many uncertainties exist in an FMEA team expert's subjective judgments and qualitative assessments due to different expertise and backgrounds. Meanwhile, many existing studies involving behavior experiments have indicated that the decision maker is not completely rational in a practical decision making process [22–24], and the psychological behavior of the decision maker, which plays an important role in decision analysis, should be taken into account in the decision making process [25]. However, most of the existing improved FMEA approaches based on the MCDM technique neglect the decision maker's psychological behavior in the process of failure modes ranking. Consequently, how to determine the risk priority of failure modes based on the decision maker's psychological behavior is a valuable research topic.

Based on the above analysis, the ranking results of failure modes will inevitably be influenced by the way preferences are aggregated and the decision maker's psychological behavior. This study presents an integrated method by combing the Bonferroni mean and TODIM (an acronym in Portuguese interactive and multi-criteria decision making) to deal with the ranking of failure modes. In addition, interval-valued Pythagorean fuzzy numbers are used to characterize the evaluation information. The contribution of this paper is to utilize the extended Bonferroni mean operator to handle expert preferences that their preferences are interdependent, and apply an improved TODIM method to deal with the failure modes ranking, which considers the expert's psychological behavior.

The rest of this paper is organized as follows. The relevant literature is reviewed in Section 2. The basic concepts and operational laws of an interval-valued Pythagorean fuzzy set are drawn in Section 3. The presented integrated FMEA risk assessment model is described in Section 4. A practical example is presented in Section 5 to illustrate the effectiveness and feasibility of the proposed method. Finally, Section 6 presents some conclusions and directions for future work.

## 2. Literature Review

### 2.1. Aggregation Expert Preferences

To form a collective assessment in FMEA, various information fusion operators have been utilized to aggregate expert preferences. For example, an arithmetic averaging operator, which assumes that the importance of each FMEA team expert is equal, is employed by many researchers to aggregate expert preferences [26–28]. In practice, FMEA team experts have different importance as they come from different departments and possess different backgrounds and expertise. Although the arithmetic averaging operator is extremely simple, it neglects the difference of expert weighting.

In order to overcome this problem, a weighted averaging (WA) operator and its extended form has been used by a number of researchers to construct a comprehensive assessment matrix in FMEA risk assessment [10,11,17,19]. On the other hand, Wang et al. [18] developed a hybrid MCDM model for evaluating the risk of failure modes, which applies an order weighted averaging (OWA) operator to fuse individual preferences under the interval-valued intuitionistic fuzzy context.

As is known, the main advantages of the WA and OWA operators consider the importance of the input argument itself and the location, respectively. To reflect the characteristics of the WA and OWA operators at the same time, Liu et al. [16] employed an interval 2-tuple hybrid averaging operator to construct a collective evaluation matrix in FMEA risk assessment. However, the above aggregation operators are only suitable to situations that input arguments that are independent, and they fail to reflect the interdependent relationships between expert preferences.

### 2.2. Bonferroni Mean

The Bonferroni mean (BM), initially developed by Bonferroni [29], is an effective tool for the aggregation of information of correlative arguments. The main advantage of the BM operator is its ability to capture the interrelationships among input arguments [30]. In practice, team experts may come from different departments or industries. Their subjective preferences usually indicate some interactive characteristic because these preferences are often influenced by their social status, power, knowledge, and other factors. Therefore, many studies in recent years have focused on the Bonferroni mean operator due to its ability to deal with interdependent input arguments. For example, Liu et al. [31] presented some BM operators of multi-valued neutrosophic numbers. Liu et al. [32] defined some new operational laws for an interval-valued 2-tuple linguistic and developed some new interval-valued 2-tuple linguistic BM operators based on these laws. He et al. [33] proposed two interval-valued hesitant fuzzy power BM operators by combining the BM with the power average operator. Fan et al. [34] presented linguistic neutrosophic numbers (LNNs) and a normalized weighted BM operator by merging the LNN and BM operator. Besides, the BM operator has also been extended to other fuzzy environment to aggregate various fuzzy information [35,36], such as hesitant fuzzy sets and linguistic intuitionistic fuzzy numbers.

### 2.3. Failure Mode Ranking

The risk priority of failure modes is a typical MCDM problem, so it is necessary to adopt an MCDM method to determine the ranking of failure modes [37]. In order to obtain a reasonable result, various MCDM methods have been applied in FMEA risk evaluation. These methods can be divided into two categories, including failure modes independence and failure modes interdependence.

For the first category, Franceschini and Galetto [14] developed an MCDM technique to determine the risk priority of failure modes in FMEA without requiring arbitrary and artificial numerical conversion. Since then, various MCDM methods have been introduced in traditional FMEA to improve its capability. For instance, Braglia et al. [9] proposed an MCDM method based on the TOPSIS (Technique for Order Preference by Similarity to an Ideal Solution) method, which obtains the prioritization of failure modes according to the descending order of the close coefficient. Chin et al. [10] presented a new FMEA model using the evidential reasoning method, which can capture the diversity opinions of FMEA team experts under different types of uncertainties well. Liu et al. [38] extended the VIKOR (VIsekriterijumska optimizacija i KOm-promisno Resenje) method, which employs the concepts of "maximum group utility" and "minimum individual regret" to determine the ranking of failure modes, thereby obtaining a compromise solution that can be accepted by most experts. Liu et al. [16] introduced the elimination and choice expressing reality (ELECTRE) to improve the ranking accuracy of failure modes in FMEA. The main characteristics of this method are that it is not compensative and it can deal with the uncertainty and fuzziness that affects the performance evaluation. Wang et al. [18] presented a new risk priority model by combing complex prorportational assessment (COPRAS) and an analytic network process to obtain the ranking of failure modes in FMEA. Huang et al. [11] developed a new FMEA model by integrating linguistic distribution assessments and a modified TODIM (an acronym in Portuguese for the interactive and MCDM) method.

On the other hand, many methods have been proposed to reflect the interdependent relationships among failure modes in the risk priority of failure modes. For instance, Xu et al. [39] developed a fuzzy-logic-based FMEA technique to solve interdependencies among various failure modes. Seyed-Hosseini et al. [40] first introduced the decision-making trial and evaluation laboratory (DEMATEL) method, which can deal with the interdependent relationships among failure modes well to rank the risk priority of failure modes. Since then, some new methods based on the DEMATEL, which is integrated with other methods, have been developed to rank the risk of failure modes. For example, Liu et al. [41] proposed a new method for the risk priority of failure modes by combining the fuzzy weighted averaging with the DEMATEL method. Motivated by the advantages of the TOPSIS method, Chang et al. [6] presented a new method that integrates TOPSIS and DEMATEL to determine the risk priority of failure modes. Liu et al. [42] combined the VIKOR, DEMATEL, and AHP method to analyze the risk of failure modes, thereby improving the assessment capability of the conventional FMEA.

### *2.4. TODIM Method*

The TODIM method was originally proposed by Gomes and Lima [43], which is the first MCDM method based on prospect theory [24]. The prominent characteristic of the TODIM method is that it can determine the gain and loss from the reference point and also reflect the fact that the decision maker is more sensitive to loss compared to gain. Therefore, it was demonstrated to be an effective and powerful tool for handing the MCDM problems and considers the expert's psychological behavior. Recently, the TODIM method has been widely applied to obtain the optimal ranking of alternatives in various MCDM problems. For instance, Qin et al. [44] developed an extended TODIM method based on interval type-2 fuzzy sets and applied it to deal with the green supplier selection problem. Zhang and Xu [45] presented a hesitant fuzzy TODIM method, which can capture the influences of experts' psychological factors on evaluation results, to evaluate the sustainable water management efficiency. Furthermore, the TODIM method has also been extended to other fuzzy environments to solve MCDM problems [46–48], such as the multi-valued neutrosophic set, multi granulatrity linguistic terms set, and Pythagorean fuzzy set.

The literature review above indicate that (1) some aggregation operators are used to fuse the expert preferences, but no or little attention has been paid to the interdependent relationship of preferences, and (2) various MCDM methods are applied to obtain the risk priority of failure modes, but only Huang et al. [11] focused on the expert's psychological behavior. However, Huang et al. [11] failed to consider the interdependent relationship between expert preferences and neglected the correlation between failure modes. Therefore, this study presents a risk priority model based on the extended Bonferroni mean and improved TODIM method, where the risk priority of failure modes can be determined. In this model, on the one hand, we used the extended Bonferroni mean operator to aggregate the expert preferences; on the other hand, we used an improved TODIM method to obtain the ranking of failure modes. Besides, a linguistic variable expressed by interval-valued Pythagorean fuzzy numbers is employed to represent the uncertainty and fuzziness of the evaluation information of experts.

## 3. Preliminaries

In this section, some basic concepts related to interval-valued Pythagorean fuzzy sets (IVPFS) are introduced briefly as follows.

**Definition 1.** [49] *Let $A = \{a_1, a_2, \cdots, a_n\}$ be an ordinary finite nonempty set and an IVPFS in A is an expression, P, shown as follows:*

$$P = \left\{ < a, \mu_p(a), \nu_p(a) >, a \in A \right\}, \tag{1}$$

*where $\mu_p(a) = [\underline{\mu}_p(a), \overline{\mu}_p(a)]$, $\mu_p(a) \subseteq [0,1]$ and $\nu_p(a) = [\underline{\nu}_p(a), \overline{\nu}_p(a)]$, $\nu_p(a) \subseteq [0,1]$ are interval values, and $\underline{\mu}_p(a) \in [0,1]$, $\overline{\mu}_p(a) \in [0,1]$, $\underline{\nu}_p(a) \in [0,1]$ and $\overline{\nu}_p(a) \in [0,1]$, satisfying $\underline{\mu}_p^2(a) + \overline{\nu}_p^2(a) \leq 1$.*

For every $a \in A$, $\underline{\pi}_p(a) = \sqrt{1 - \overline{\mu}_p^2(a) - \overline{\nu}_p^2(a)}$ and $\overline{\pi}_p(a) = \sqrt{1 - \underline{\mu}_p^2(a) - \underline{\nu}_p^2(a)}$. For convenience, Peng and Yang [50] designated $P = ([\underline{\mu}_p, \overline{\mu}_p], [\underline{\nu}_p, \overline{\nu}_p])$ as an interval-valued Pythagorean fuzzy number (IVPFN), where $\underline{\mu}_p^2 + \overline{\nu}_p^2 \leq 1$. Notably, the IVPFS reduces into PFS when $\underline{\mu}_p = \overline{\mu}_p$ and $\underline{\nu}_p = \overline{\nu}_p$.

**Definition 2.** [49] *Let $P = ([\underline{\mu}_P, \overline{\mu}_P], [\underline{\nu}_P, \overline{\nu}_P])$, $P_1 = ([\underline{\mu}_{P_1}, \overline{\mu}_{P_1}], [\underline{\nu}_{P_1}, \overline{\nu}_{P_1}])$, and $P_2 = ([\underline{\mu}_{P_2}, \overline{\mu}_{P_2}], [\underline{\nu}_{P_2}, \overline{\nu}_{P_2}])$ be three IVPFNs and $\lambda > 0$. Then, the basic operational rules can be expressed as follows:*

(1) $P_1 \oplus P_2 = ([\sqrt{\underline{\mu}_{p_1}^2 + \underline{\mu}_{p_2}^2 - \underline{\mu}_{p_1}^2 \underline{\mu}_{p_2}^2}, \sqrt{\overline{\mu}_{p_1}^2 + \overline{\mu}_{p_2}^2 - \overline{\mu}_{p_1}^2 \overline{\mu}_{p_2}^2}], [\underline{\nu}_{p_1}\underline{\nu}_{p_2}, \overline{\nu}_{p_1}\overline{\nu}_{p_2}]);$

(2) $P_1 \otimes P_2 = ([\underline{\nu}_{p_1}\underline{\nu}_{p_2}, \overline{\nu}_{p_1}\overline{\nu}_{p_2}], [\sqrt{\underline{\mu}_{p_1}^2 + \underline{\mu}_{p_2}^2 - \underline{\mu}_{p_1}^2 \underline{\mu}_{p_2}^2}, \sqrt{\overline{\mu}_{p_1}^2 + \overline{\mu}_{p_2}^2 - \overline{\mu}_{p_1}^2 \overline{\mu}_{p_2}^2}]);$

(3) $\lambda P = ([\sqrt{1 - (1 - \underline{\mu}_p^2)^\lambda}, \sqrt{1 - (1 - \overline{\mu}_p^2)^\lambda}], [(\underline{\nu}_p)^\lambda, (\overline{\nu}_p)^\lambda]);$

(4) $P^\lambda = ([(\underline{\nu}_p)^\lambda, (\overline{\nu}_p)^\lambda], [\sqrt{1 - (1 - \underline{\mu}_p^2)^\lambda}, \sqrt{1 - (1 - \overline{\mu}_p^2)^\lambda}]).$

**Definition 3.** [49] *Let $P_1 = ([\underline{\mu}_{P_1}, \overline{\mu}_{P_1}], [\underline{\nu}_{P_1}, \overline{\nu}_{P_1}])$ and $P_2 = ([\underline{\mu}_{P_2}, \overline{\mu}_{P_2}], [\underline{\nu}_{P_2}, \overline{\nu}_{P_2}])$, which are two IVPFNs; then, the distance between them is defined as follows:*

$$d(P_1, P_2) = \frac{1}{4}(|\underline{\mu}_{p_1}^2 - \underline{\mu}_{p_2}^2| + |\overline{\mu}_{p_1}^2 - \overline{\mu}_{p_2}^2| + |\underline{\nu}_{p_1}^2 - \underline{\nu}_{p_2}^2| + |\overline{\nu}_{p_1}^2 - \overline{\nu}_{p_2}^2| + |\underline{\pi}_{p_1}^2 - \underline{\pi}_{p_2}^2| + |\overline{\pi}_{p_1}^2 - \overline{\pi}_{p_2}^2|). \tag{2}$$

**Definition 4.** [50] *Let $P = ([\underline{\mu}_P, \overline{\mu}_P], [\underline{\nu}_P, \overline{\nu}_P])$ be an IVPFN. Then, the score and accuracy functions of P is defined respectively as follows:*

$$s(P) = \frac{1}{2}(\underline{\mu}_P^2 + \overline{\mu}_P^2 - \underline{\nu}_P^2 - \overline{\nu}_P^2), \tag{3}$$

$$a(P) = \frac{1}{2}(\underline{\mu}_P^2 + \overline{\mu}_P^2 + \underline{\nu}_P^2 + \overline{\nu}_P^2). \tag{4}$$

*According to the above two equations, a ranking method for two IVPFNs can be introduced as follows:*

(1) *If $s(P_1) > s(P_2)$, then $P_1 \succ P_2$;*
(2) *If $s(P_1) = s(P_2)$, then:*

   (a) *If $a(P_1) > a(P_2)$, then $P_1 \succ P_2$;*
   (b) *If $a(P_1) = a(P_2)$, then $P_1 = P_2$.*

## 4. The Proposed FMEA Method

Generally, we assume that an FMEA team consists of $L$ cross-functional experts, $E_k(k = 1, 2, \cdots, L)$, that come from different departments and domains, and the FMEA team is responsible for the assessment of a finite set of $m$ potential failure modes, $FM_i(i = 1, 2, \cdots, m)$ (Failure modes), with respect to a finite set of $n$ risk factors, $RF_j(j = 1, 2, \cdots, n)$ (Risk factors). In practical risk analysis, the relative weights of FMEA team experts may not be equal because of their differing domain knowledge and experiences. Therefore, let $\lambda = (\lambda_1, \lambda_2, \cdots, \lambda_L)^T$ be the weight vector of the team experts, where $\lambda_k \in [0, 1]$ and $\sum_{k=1}^{L} \lambda_k = 1$. Furthermore, suppose that $L_k = (l_{ij}^k)_{m \times n}$ and $Z_k = (z_j^k)_{1 \times n}$ express the linguistic assessment matrix of the $k$th team expert, where $l_{ij}^k$ and $z_j^k$ are the linguistic ratings obtained from $E_k$ and express the assessment of $FM_i$ with respect to $RF_j$ and the relative importance of the risk factors, respectively.

Based on the assumptions and notations above, we developed an FMEA risk assessment model under the IVPFS environment. A flowchart of the proposed model is shown in Figure 1.

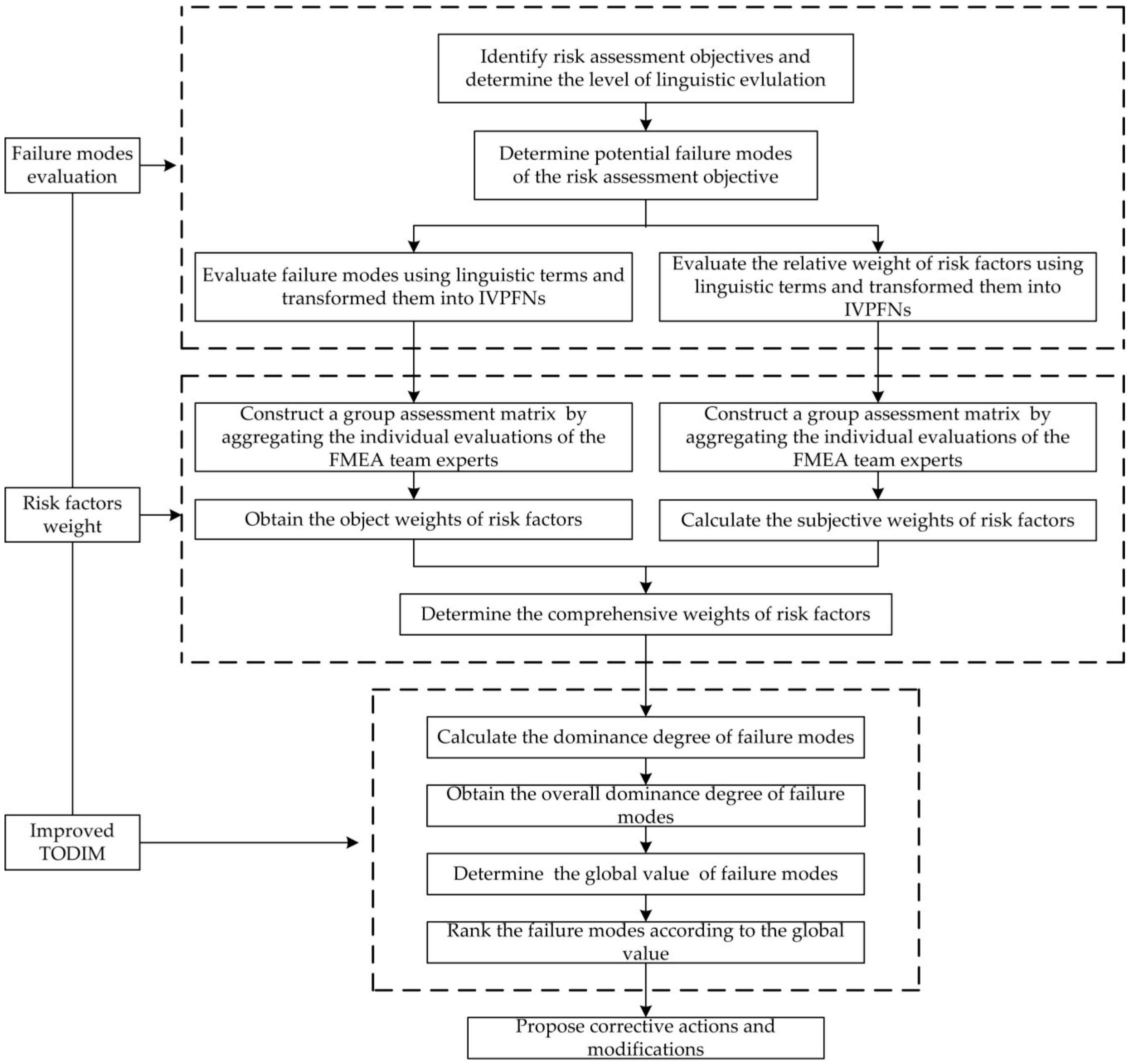


**Figure 1.** Flowchart of the proposed FMEA (Failure modes and effective analysis) model.

The main steps of the proposed method are explained as follows.

**Step 1.** Evaluate the failure modes' and risk factors' relative weights using linguistic terms.

In reality, FMEA team experts generally come from different departments and may have different practical experiences, knowledge structures, and evaluation levels [17]; the evaluation information provided by them may be certain or uncertain, complete or incomplete, and precise or imprecise [10]. Interval-valued Pythagorean fuzzy sets are a good tool to handle the imprecise and ambiguous information, and manage complex uncertainty in real world applications [49,51]. It is highly suitable for descripting the uncertainty and vagueness of experts' judgments in FMEA. Therefore, the ratings of failure modes and the relative importance of risk factors were evaluated by linguistic variables expressed in IVPFNs in Tables 1 and 2, respectively. Then, the linguistic evaluation matrices, $L_k$ and $Z_k$, can be transformed into the assessment matrices, $P_k = (p_{ij}^k)_{m\times n}$ and $W_k = (w_j^k)_{1\times n}$, respectively, according to Tables 1 and 2, where $p_{ij}^k$ and $w_j^k$ are IVPFNs.

**Table 1.** Linguistic terms for rating the failure modes [50].

| Linguistic Variables | Abbreviation | IVPFNs |
|---|---|---|
| Very high | VH | ([0.8000, 0.9000], [0.1000, 0.2000]) |
| High | H | ([0.7000, 0.8000], [0.2000, 0.3000]) |
| Medium high | MH | ([0.6000, 0.7000], [0.3000, 0.4000]) |
| Medium | M | ([0.5000, 0.6000], [0.4000, 0.5000]) |
| Medium low | ML | ([0.3000, 0.4000], [0.6000, 0.7000]) |
| Low | L | ([0.2000, 0.3000], [0.7000, 0.8000]) |
| Very low | VL | ([0.1000, 0.2000], [0.8000, 0.9000]) |

**Table 2.** Linguistic terms for rating the weights of risk factors [50].

| Linguistic Variables | Abbreviation | IVPFNs |
|---|---|---|
| Very high | VH | ([0.8000, 0.9000], [0.1000, 0.2000]) |
| High | H | ([0.7000, 0.8000], [0.2000, 0.3000]) |
| Medium | M | ([0.5000, 0.6000], [0.4000, 0.5000]) |
| Low | L | ([0.3000, 0.4000], [0.6000, 0.7000]) |
| Very low | VL | ([0.1000, 0.2000], [0.8000, 0.9000]) |

**Step 2.** Aggregate the preferences of the FMEA team experts into a group decision evaluation.

To consider the interdependent relationships between expert preferences, it is necessary to adopt a suitable aggregation operator to fuse expert preferences. The Bonferroni mean is a well-known operator because it is capable of handling interdependent input arguments. However, the conventional Bonferroni mean does not deal with IVPFN information. Hence, we extended the Bonferroni mean to the IVPFS environment to define a weighted interval-valued Pythagorean fuzzy BM operator, which is similar to the weighted interval-valued intuitionistic fuzzy BM operator [52].

**Definition 5.** *Let $x, y \geq 0$, where $x$ and $y$ do not simultaneously take the value of 0. Let $p_i = ([\underline{\mu}_{P_i}, \overline{\mu}_{P_i}], [\underline{\nu}_{P_i}, \overline{\nu}_{P_i}])(i = 1, 2, \cdots, n)$ be a collection of IVPFNs, and $w = (w_1, w_2, \cdots, w_n)^T$ be the weight vector of $p_i$,* where $w_i$ indicates the importance degree of $p_i$ and satisfies $w_i \in [0, 1]$ and $\sum_{i=1}^n w_i = 1$. If:

$$\mathrm{WIVPFBM}_w^{x,y}(p_1, p_2, \cdots, p_n) = \mathop{\oplus}_{i,j=1, i\neq j}^{n} \left(\frac{1}{n(n-1)} \times ((w_i p_i)^x \otimes (w_j p_j)^y)\right)^{\frac{1}{x+y}}, \tag{5}$$

*then $\mathrm{WIVPFBM}_w^{x,y}$ is regarded as a WIVPFBM (Weighted interval-value Pythagorean fuzzy Bonferroni mean) operator.*

**Theorem 1.** *Let $x, y \geq 0$, where $x$ and $y$ do not simultaneously take the value of 0. Let $p_i = ([\underline{\mu}_{P_i}, \overline{\mu}_{P_i}], [\underline{\nu}_{P_i}, \overline{\nu}_{P_i}])(i = 1, 2, \cdots, n)$ be a collection of IVPFNs, and $w = (w_1, w_2, \cdots, w_n)^T$ be* *the weight vector of $p_i$, where $w_i$ indicates the importance degree of $p_i$ and satisfies $w_i \in [0,1]$ and $\sum_{i=1}^{n} w_i = 1$. Then, the aggregated value using the WIVPFBM operator is also an IVPFN, and:*

$$\mathrm{WIVPFBM}_w^{x,y}(p_1, p_2, \cdots, p_n) = \left( \begin{array}{c} \left[ \begin{array}{c} \sqrt{\left(1 - \prod\limits_{\substack{i,j=1 \\ i \neq j}}^{n} \left(1 - (1 - (1 - \underline{\mu}_{P_i}^2)^{w_i})^x (1 - (1 - \underline{\mu}_{P_j}^2)^{w_j})^y\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{x+y}}}, \\ \sqrt{\left(1 - \prod\limits_{\substack{i,j=1 \\ i \neq j}}^{n} \left(1 - (1 - (1 - \overline{\mu}_{P_i}^2)^{w_i})^x (1 - (1 - \overline{\mu}_{P_j}^2)^{w_j})^y\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{x+y}}} \end{array} \right], \\ \left[ \sqrt{1 - \left(1 - \prod\limits_{\substack{i,j=1 \\ i \neq j}}^{n} \left(1 - \left(1 - \underline{\nu}_{P_i}^{2w_i}\right)^x \left(1 - \underline{\nu}_{P_j}^{2w_j}\right)^y\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{x+y}}}, \sqrt{1 - \left(1 - \prod\limits_{\substack{i,j=1 \\ i \neq j}}^{n} \left(1 - \left(1 - \overline{\nu}_{P_i}^{2w_i}\right)^x \left(1 - \overline{\nu}_{P_j}^{2w_j}\right)^y\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{x+y}}} \right] \end{array} \right). \tag{6}$$

According to the matrix, $P_k = (p_{ij}^k)_{m \times n}$, all individual assessments can be aggregated into a group assessment matrix, $P = (p_{ij})_{m \times n}$, by the WIVPFBM operator, which can depict the interdependent relationships of expert preferences.

$$P = \mathrm{WIVPFBM}_{\lambda}^{x,y}(p_{ij}^1, p_{ij}^2, \cdots, p_{ij}^L). \tag{7}$$

Similarly, a group assessment matrix, $W = (w_j)_{1 \times n}$, for the weights of risk factors can be calculated as follows:

$$W = \mathrm{WIVPFBM}_{\lambda}^{x,y}(w_{ij}^1, w_{ij}^2, \cdots, w_{ij}^L). \tag{8}$$

**Step 3.** Determine the combination weights of risk factors.

To sufficiently reflect experts' expertise and intrinsic information, this paper adopts subjective and objective weighting methods to determine the weights of risk factors. Numerous researchers have focused on the entropy method to obtain the objective weights of criteria [53] as it can handle the situation where conflict exists between the weight values, and eliminate the subjectivity of experts' judgments in the valuation of weights. Therefore, we defined an interval-valued Pythagorean fuzzy entropy, which is parallel to interval-valued intuitionistic fuzzy entropy [54], to calculate the objective weight of risk factors.

**Definition 6.** *Let P be an IVPFS defined in the universe of discourse A. An interval-valued Pythagorean fuzzy entropy (IVPFE) is given as follows:*

$$E(P) = \frac{1}{n} \sum_{i=1}^{n} \frac{2 - |\underline{\mu}_{P_i}^2 - \underline{\nu}_{P_i}^2| - |\overline{\mu}_{P_i}^2 - \overline{\nu}_{P_i}^2| + \underline{\pi}_{P_i}^2 + \overline{\pi}_{P_i}^2}{2 + |\underline{\mu}_{P_i}^2 - \underline{\nu}_{P_i}^2| + |\overline{\mu}_{P_i}^2 - \overline{\nu}_{P_i}^2| + \underline{\pi}_{P_i}^2 + \overline{\pi}_{P_i}^2}. \tag{9}$$

**Step 3.1.** Obtain the subjective weights of risk factors.

Based on the aggregated weights, $W = (w_j)_{1 \times n} (j = 1, 2, \cdots, n)$, of risk factors, the score values of weights of each risk factor can be calculated as follows:

$$s(w_j) = \frac{1}{2}(\underline{\mu}_{P_i}^2 + \overline{\mu}_{P_i}^2 - \underline{\nu}_{P_i}^2 - \overline{\nu}_{P_i}^2). \tag{10}$$

The normalized subjective weight of each risk factor can be obtained as:

$$w_j^s = \frac{s(w_j)}{\sum_{j=1}^{n} s(w_j)}. \tag{11}$$

**Step 3.2.** Obtain the objective weights of risk factors using the entropy method.

The objective weights, $w_j^o (j = 1, 2, \cdots, n)$, of risk factors can be determined through the entropy method. The entropy value of each failure mode with respect to the risk factor is calculated as follows:

$$E_{ij} = \frac{2 - |\underline{\mu}_{P_i}^2 - \underline{\nu}_{P_i}^2| - |\overline{\mu}_{P_i}^2 - \overline{\nu}_{P_i}^2| + \underline{\pi}_{P_i}^2 + \overline{\pi}_{P_i}^2}{2 + |\underline{\mu}_{P_i}^2 - \underline{\nu}_{P_i}^2| + |\overline{\mu}_{P_i}^2 - \overline{\nu}_{P_i}^2| + \underline{\pi}_{P_i}^2 + \overline{\pi}_{P_i}^2}. \tag{12}$$

Thus, the normalized objective weight of each risk factor can be obtained by the following equation:

$$w_j^o = \frac{\sum_{i=1}^{m} (1 - E_{ij})}{\sum_{j=1}^{n} \sum_{i=1}^{m} (1 - E_{ij})}. \tag{13}$$

**Step 3.3.** Calculate the comprehensive weights of risk factors.

It is significant to combine the subjective and objective weights to calculate the comprehensive weight of each risk factor, which can consider simultaneously the importance of risk factors and the experiences of the experts [28]. Therefore, based on the $w_j^s$ and $w_j^o$, the comprehensive weight is obtained as follows:

$$\hat{w}_j = \zeta w_j^s + (1 - \zeta) w_j^o, (j = 1, 2, \cdots, n), \tag{14}$$

where parameter $\zeta \in [0, 1]$ is the relative importance coefficient for the subjective weight. In this study, two types of weights were assumed to be equally important, that is, $\zeta = 0.5$.

**Step 4.** Obtain the ranking of failure modes by using the improved TODIM method.

TODIM, which is an effective behavior MCDM method based on prospect theory, was used by Huang et al. [11] to rank the risk priority of failure modes, but it cannot consider the interrelationships between failure modes when calculating the dominance degree of failure modes. In the real world, interdependent relationships may exist between failure modes because of the complexity of the failure system. Grey relational analysis is part of a grey system, which is suitable for handling problems with complicated interrelationships between multiple factors and variables [55]. Therefore, grey relational analysis is introduced in the TODIM method to compute the dominance degree of failure modes.

**Step 4.1.** Calculate the dominance degree of failure modes.

The dominance degree of $FM_i$ over $FM_s$ regarding the $RF_j$ can be obtained as follows:

$$\Phi_j(FM_i, FM_s) = \begin{cases} \sqrt{\frac{(\xi(p_{ij}, p_0) - \xi(p_{sj}, p_0))\hat{w}_{ij}}{\sum_{j=1}^{n} \hat{w}_{ij}}}, & if\ s(p_{ij}) > s(p_{sj}) \\ 0, & if\ s(p_{ij}) = s(p_{sj}) \\ -\frac{1}{\theta} \sqrt{\frac{(\xi(p_{ij}, p_0) - \xi(p_{sj}, p_0))\sum_{j=1}^{n} \hat{w}_{ij}}{\hat{w}_{ij}}}, & if\ s(p_{ij}) < s(p_{sj}) \end{cases}, \tag{15}$$

where $\Phi_j(FM_i, FM_s)$ represents the contribution of the risk factor, $RF_j$, to the function, $\Omega_j(FM_i, FM_s)$, when comparing $FM_s$ with $FM_i$. If $s(p_{ij}) > s(p_{sj})$, then a gain occurs. If $s(p_{ij}) < s(p_{sj})$, then a loss is incurred. Finally, if $s(p_{ij}) = s(p_{sj})$, then the result is zero. Moreover, $\hat{w}_{ij} = \hat{w}_j / \hat{w} (j = 1, 2, \cdots, n)$ denotes the relative weight of $RF_j$ to the reference, $RF_l$, and $\hat{w} = \max\{\hat{w}_j | j = 1, 2, \cdots, n\}$. Parameter $\theta$ represents the attenuation coefficient of the loss, which can be tuned according to the practical problem.

In this study, the value, $\theta$, was assumed to be 0.45. The term, $\xi(p_{ij}, p_0)$, denotes the grey relation coefficient between the group evaluation value and the standard value, which is calculated as follows:

$$\xi(p_{ij}, p_0) = \frac{\text{minmin}\left|s(p_{ij}) - s(p_0)\right| + \delta\text{maxmax}\left|s(p_{ij}) - s(p_0)\right|}{\left|s(p_{ij}) - s(p_0)\right| + \delta\text{maxmax}\left|s(p_{ij}) - s(p_0)\right|}. \tag{16}$$

**Step 4.2.** Obtain the overall dominance degree of failure modes.

The overall dominance degree of $FM_i$ over $FM_s$ can be calculated according to the following expression:

$$\Omega(FM_i, FM_s) = \sum_{j=1}^{n} \Phi_j(FM_i, FM_s), (i, s = 1, 2, \cdots, m). \tag{17}$$

**Step 4.3.** Calculate the global dominance degree of failure modes.

The global dominance degree of $FM_i$ can be obtained as:

$$\psi(FM_i) = \frac{\sum_{s=1}^{m} \Omega(FM_i, FM_s) - \min_{i\in m}\left\{\sum_{s=1}^{m} \Omega(FM_i, FM_s)\right\}}{\max_{i\in m}\left\{\sum_{s=1}^{m} \Omega(FM_i, FM_s)\right\} - \min_{i\in m}\left\{\sum_{s=1}^{m} \Omega(FM_i, FM_s)\right\}}. \tag{18}$$

**Step 4.4.** Rank the failure modes according to $\psi(FM_i)$.

## 5. Case Illustration

A nuclear reheat valve system in a nuclear steam turbine [28,56] was selected as the case study to demonstrate the applicability of the proposed method. This valve system is crucial to the operation of nuclear power stations, and its failure would decrease the reliability of the entire power station. Therefore, the reheat valve system must be quickly closed during abnormal operating conditions to cut off the steam entering the low-pressure cylinder and ensure system safety and reliability.

An FMEA team with four experts was formed to identify the most significant failure modes. The four team experts were given the following relative weights as 0.15, 0.30, 0.35, and 0.20 because of their different background knowledge and experiences. The team identified eight failure modes through brainstorming and selected three risk factors, namely, $S$, $O$, and $D$. The eight failure modes were a long closing time or no valve action, valve nor closing tightly, large leak around valve shaft, valve fluctuations, valve jam during operation, valve shaft fracture, fault of valve shaft support bearing, and abnormal noise from valve system.

### 5.1. Implementation of the Proposed Method

In this section, the proposed method was applied to solve the valve system risk assessment problem, and the implementation procedure is presented as follows:

**Step 1.** Evaluate the failure modes and risk factors' relative weights using linguistic terms.

The evaluations of the eight failure modes and the risk factors' relative weights provided by experts are shown Tables 3 and 4, respectively. Subsequently, the linguistic ratings were transformed into the corresponding IVPFNs.

**Table 3.** Assessment information on the eight failure modes by FMEA team experts.

| **Risk Factors** | **Severity ($S$)** | | | | **Occurrence ($O$)** | | | | **Detection ($D$)** | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Team Experts** | $E_1$ | $E_2$ | $E_3$ | $E_4$ | $E_1$ | $E_2$ | $E_3$ | $E_4$ | $E_1$ | $E_2$ | $E_3$ | $E_4$ |
| $FM_1$ | VH | VH | VH | H | ML | ML | M | L | H | H | MH | M |
| $FM_2$ | L | VL | VH | ML | ML | ML | M | ML | ML | M | M | ML |
| $FM_3$ | ML | ML | L | ML | MH | MH | M | M | ML | ML | L | M |
| $FM_4$ | MH | MH | M | M | ML | L | ML | ML | M | L | ML | ML |
| $FM_5$ | VH | VH | MH | H | ML | ML | M | M | ML | ML | L | ML |
| $FM_6$ | VH | VH | MH | VH | L | ML | L | ML | L | ML | ML | M |
| $FM_7$ | MH | MH | VH | H | M | M | MH | M | ML | L | ML | ML |
| $FM_8$ | M | M | H | MH | M | MH | M | MH | ML | M | ML | ML |

**Table 4.** Linguistic evaluations of risk factor weights.

| **Team Experts** | $E_1$ | $E_2$ | $E_3$ | $E_4$ |
|---|---|---|---|---|
| Occurrence | L | M | M | H |
| Severity | M | H | H | VH |
| Detection | M | H | M | H |

**Step 2.** Establish a group decision matrix by utilizing the WIVPFBM operator.

The aggregated fuzzy ratings of each failure mode were calculated by the WIVPFBM operator to establish the fuzzy collective evaluation matrix, $P = (p_{ij})_{8\times3}$, as shown in Table 5. Subsequently, the aggregated fuzzy weights vector of risk factors was calculated by Equation (8), as shown in Table 5.

**Table 5.** Fuzzy collective evaluation matrix and aggregated fuzzy weights of risk factors.

| **Risk Factors** | ***S*** | ***O*** | ***D*** |
|---|---|---|---|
| $FM_1$ | ([0.4517,0.5520], [0.6053,0.6972]) | ([0.1743,0.2292], [0.8708,0.9062]) | ([0.3416,0.4138], [0.7297,0.7875]) |
| $FM_2$ | ([0.1943, 0.2609], [0.8627, 0.9045]) | ([0.1847, 0.2403], [0.8620, 0.8982]) | ([0.2200, 0.2770], [0.8384, 0.8772]) |
| $FM_3$ | ([0.1357, 0.1884], [0.8936, 0.9264]) | ([0.2898, 0.3540], [0.7764, 0.8248]) | ([0.1619, 0.2163], [0.8768, 0.9123]) |
| $FM_4$ | ([0.2898, 0.3540], [0.7764, 0.8248]) | ([0.1371, 0.1897], [0.8931, 0.9258]) | ([0.1597, 0.2136], [0.8785, 0.9136]) |
| $FM_5$ | ([0.4077, 0.4978], [0.6539, 0.7330]) | ([0.2120, 0.2689], [0.8426, 0.8813]) | ([0.1357, 0.1884], [0.8936, 0.9264]) |
| $FM_6$ | ([0.4264, 0.5231], [0.6286, 0.7165]) | ([0.1268, 0.1791], [0.9000, 0.9321]) | ([0.1698, 0.2243], [0.8722, 0.9077]) |
| $FM_7$ | ([0.3818, 0.4630], [0.6928, 0.7586]) | ([0.2796, 0.3422], [0.7872, 0.8332]) | ([0.1371, 0.1897], [0.8931, 0.9258]) |
| $FM_8$ | ([0.3141, 0.3818], [0.7586, 0.8099]) | ([0.2924, 0.3568], [0.7745, 0.8231]) | ([0.1829, 0.2385], [0.8625, 0.8988]) |
| Weights | ([0.3736, 0.4573], [0.6817, 0.7526]) | ([0.2677, 0.3366], [0.7913, 0.8386]) | ([0.3177, 0.3873], [0.7421, 0.7980]) |

**Step 3.** Determine the comprehensive weights of risk factors.

According to Equations (10)–(13), the subjective and objective weights of risk factors were obtained as $w^s = (0.3935, 0.2767, 0.3298)^T$ and $w^o = (0.2563, 0.3635, 0.3802)^T$, respectively. Then, the comprehensive weights of risk factors were determined as $w = (0.3249, 0.3201, 0.3550)^T$.

**Step 4.** Use the improved TODIM method to rank the risk priority of failure modes.

**Step 4.1.** Calculate the dominance degree of the failure modes.

Applying Equations (15) (i.e., $\theta = 0.45$) and (16) (i.e., $\delta = 0.5$), the dominance degree matrices concerning the risk factor, $S$, $O$, and $D$, were obtained, respectively.

$$\Phi_S = \begin{pmatrix} 0 & -3.0274 & -2.5009 & -0.2140 & -0.8967 & -0.6298 & -1.2359 & -1.9135 \\ 0.4426 & 0 & 0.2494 & 0.3131 & 0.4228 & 0.4329 & 0.4041 & 0.3430 \\ 0.3656 & -1.7061 & 0 & 0.1892 & 0.3413 & 0.3539 & 0.3179 & 0.2354 \\ 0.3129 & -2.1414 & -1.2942 & 0 & 0.2841 & 0.2990 & 0.2554 & 0.1401 \\ 0.1311 & -2.8916 & -2.3346 & -1.9431 & 0 & 0.0933 & -0.8506 & -1.6904 \\ 0.0921 & -2.9612 & -2.4203 & -2.0452 & -0.6382 & 0 & -1.0634 & -1.8069 \\ 0.1807 & -2.7637 & -2.1742 & -1.7470 & 0.1244 & 0.1555 & 0 & -1.4608 \\ 0.2798 & -2.3460 & -1.6103 & -0.9582 & 0.2471 & 0.2642 & 0.2136 & 0 \end{pmatrix};$$

$$\Phi_O = \begin{pmatrix} 0 & -0.9524 & 0.2775 & 0.2016 & -1.2758 & 0.2250 & 0.2558 & 0.2815 \\ 0.1372 & 0 & 0.3095 & 0.2439 & -0.8488 & 0.2635 & 0.2903 & 0.3131 \\ -1.9262 & -2.1488 & 0 & -1.3232 & -2.3104 & -1.1275 & -0.7463 & 0.0473 \\ -1.3997 & -1.6930 & 0.1906 & 0 & -1.8939 & 0.0998 & 0.1574 & 0.1964 \\ 0.1838 & 0.1223 & 0.3328 & 0.2728 & 0 & 0.2905 & 0.3150 & 0.3361 \\ -1.5617 & -1.8292 & 0.1642 & -0.6927 & -2.0166 & 0 & 0.1217 & 0.1692 \\ -1.7758 & -2.0150 & 0.1075 & -1.0927 & -2.1865 & -0.8451 & 0 & 0.1175 \\ -1.9540 & -2.1738 & -0.3286 & -1.3634 & -2.3336 & -1.1744 & -0.8154 & 0 \end{pmatrix};$$

$$\Phi_D = \begin{pmatrix} 0 & -2.4641 & -2.1292 & -2.0989 & -1.8226 & -2.2266 & -1.8351 & -2.4234 \\ 0.3936 & 0 & 0.1981 & 0.2062 & 0.2649 & 0.1686 & 0.2627 & 0.0712 \\ 0.3401 & -1.2404 & 0 & 0.0571 & 0.1758 & -0.6514 & 0.1725 & -1.1574 \\ 0.3353 & -1.2909 & -0.3577 & 0 & 0.1663 & -0.7431 & 0.1627 & -1.2114 \\ 0.2912 & -1.6584 & -1.1007 & -1.0410 & 0 & -1.2790 & -0.2143 & -1.5972 \\ 0.3557 & -1.0556 & 0.1041 & 0.1187 & 0.2043 & 0 & 0.2014 & -0.9567 \\ 0.2932 & -1.6445 & -1.0797 & -1.0187 & 0.0342 & -1.2609 & 0 & -1.5828 \\ 0.3871 & -0.4460 & 0.1849 & 0.1935 & 0.2552 & 0.1528 & 0.2529 & 0 \end{pmatrix}.$$

**Step 4.2.** Compute the overall dominance degree of the failure modes.

The overall dominance degree matrix can be obtained based on Equation (17).

$$\Omega = \begin{pmatrix} 0 & -6.4440 & -4.3527 & -4.0373 & -3.9950 & -2.6315 & -2.8153 & -4.0555 \\ 0.9735 & 0 & 0.7571 & 0.7632 & -0.1611 & 0.8651 & 0.9570 & 0.7274 \\ -1.2204 & -5.0952 & 0 & -1.0769 & -1.7932 & -1.4250 & -0.2559 & -0.8746 \\ -0.7515 & -5.1254 & -1.4613 & 0 & -1.4435 & -0.3443 & 0.5756 & -0.8749 \\ 0.6060 & -4.4277 & -3.1026 & -2.7113 & 0 & -0.8952 & -0.7499 & -2.9515 \\ -1.1140 & -5.8460 & -2.1538 & -2.6192 & -2.4505 & 0 & -0.7402 & -2.5944 \\ -1.3019 & -6.4232 & -3.1464 & -3.8584 & -2.0279 & -1.9506 & 0 & -2.9262 \\ -1.2871 & -0.9658 & -1.7540 & -2.1281 & -1.8313 & -0.7574 & -0.3490 & 0 \end{pmatrix}.$$

**Step 4.3.** Determine the global dominance degree of each failure mode.

The global dominance of the failure mode, $FM_i$, was computed by Equation (18), namely, $\psi(FM_1) = 0.9795$, $\psi(FM_2) = 0$, $\psi(FM_3) = 0.6613$, $\psi(FM_4) = 0.6483$, $\psi(FM_5) = 0.7046$, $\psi(FM_6) = 0.8924$, $\psi(FM_7) = 1$, and $\psi(FM_8) = 0.7089$.

**Step 4.4.** Rank the failure modes.

Based on the global dominance degree of failure modes, the risk priority of failure modes is $FM_7 > FM_1 > FM_6 > FM_8 > FM_5 > FM_3 > FM_4 > FM_2$. According to the results, $FM_7$ is the most serious failure mode and should be given the top risk priority, followed by $FM_1$, $FM_6$, $FM_8$, $FM_5$, $FM_3$, $FM_4$, and $FM_2$.

## *5.2. Sensitivity Analysis*

A sensitivity analysis on the expert preferences' interdependence and failure modes' interdependence was conducted to illustrate their impact on the risk priority of failure modes. In the sensitivity analysis, we considered four methods as follows: (1) The proposed method in this paper (case 0); (2) GRA (Grey relational analysis)is not employed to compute the dominance degree of failure modes (case 1); (3) an interval-valued Pythagorean fuzzy weighted averaging (IVPFWA) operator is utilized to aggregate the expert preferences in Step 2 (case 2), and (4) the IVPFWA operator is used to aggregate the expert preferences and GRA is not applied to calculate the dominance degree of failure modes (case 3). The ranking results of the eight failure modes under the four cases are shown in Figure 2.

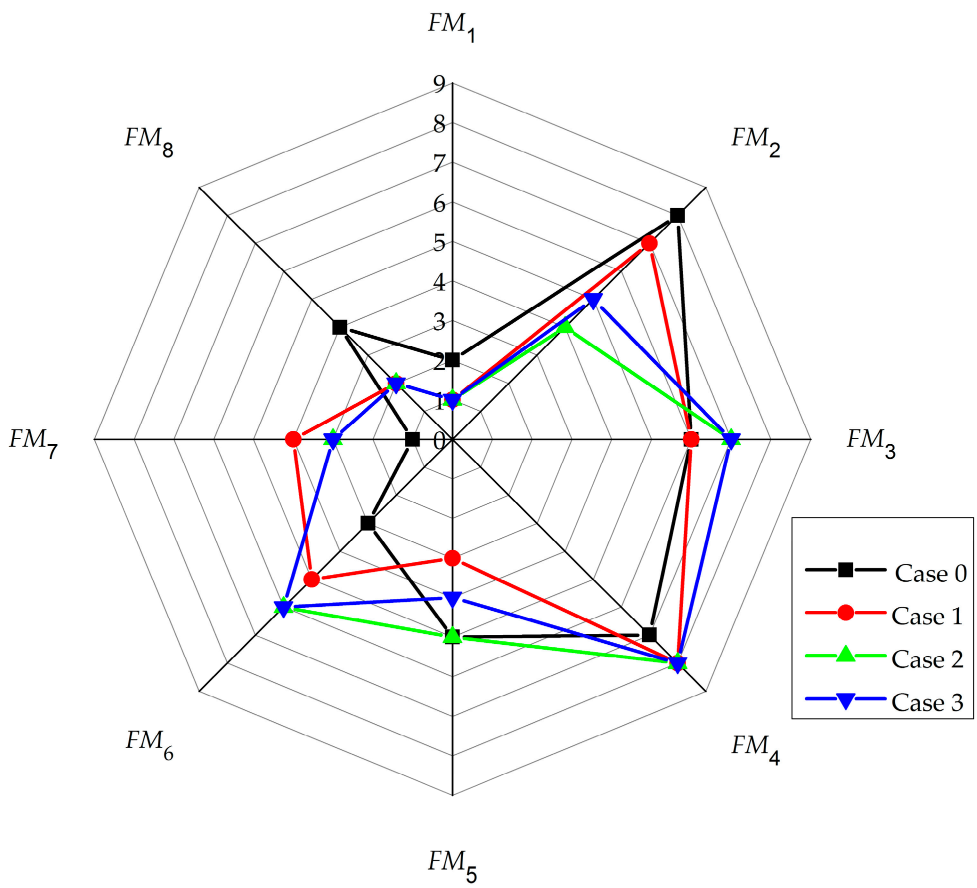


**Figure 2.** The ranking results of the sensitivity analysis.

As we can see in Figure 2, the risk priority of failure modes in case 0 is very different from the ranking results in the other three cases. This divergence can be demonstrated by the Spearman rank-correlation coefficients between the rankings of failure modes in case 0 and case 1, case 2, and case 3, and the correlation coefficients were 0.7143, 0.5714, and 0.6429, respectively. The ranking of failure modes in case 0 is different from those in case 1 except for $FM_3$. The largest difference between the two ranking results happens at $FM_7$, and $FM_7$ has a difference of three ranking places. The main reason for this difference is that the failure modes' interdependence was not considered in the calculation process of the dominance degree of failure modes in case 1. There is some difference between the ranking results in case 0 and case 2. In these two ranking results of the failure modes, $FM_2$ and $FM_6$ have a difference of four ranking places and three ranking places, respectively. The reason for this difference is that the IVPFWA operator was applied in case 2, rather than the WIVPFBM operator, to aggregate expert preferences, which neglects the interdependent relationship between the expert preferences. In addition, it can be clearly seen from Figure 1 that the eight failure modes have different risk priorities in case 0 and case 3. In case 3, the IVPFWA operator was utilized to aggregate expert preferences and GRA was not employed to compute the dominance degree of failure modes, which may lead to this difference. The sensitivity analysis shows that the expert preferences' interdependence and the failure modes' interdependence have a great influence on the final risk priority of failure modes.

### *5.3. Comparisons and Discussion*

To further demonstrate the effectiveness of the proposed method, comparisons of the results with the fuzzy TOPSIS [28], fuzzy VIKOR [12], and fuzzy digraph and matrix (FDM) [56] were made. Figure 3 shows the ranking results of all eight failure modes obtained by the four methods.

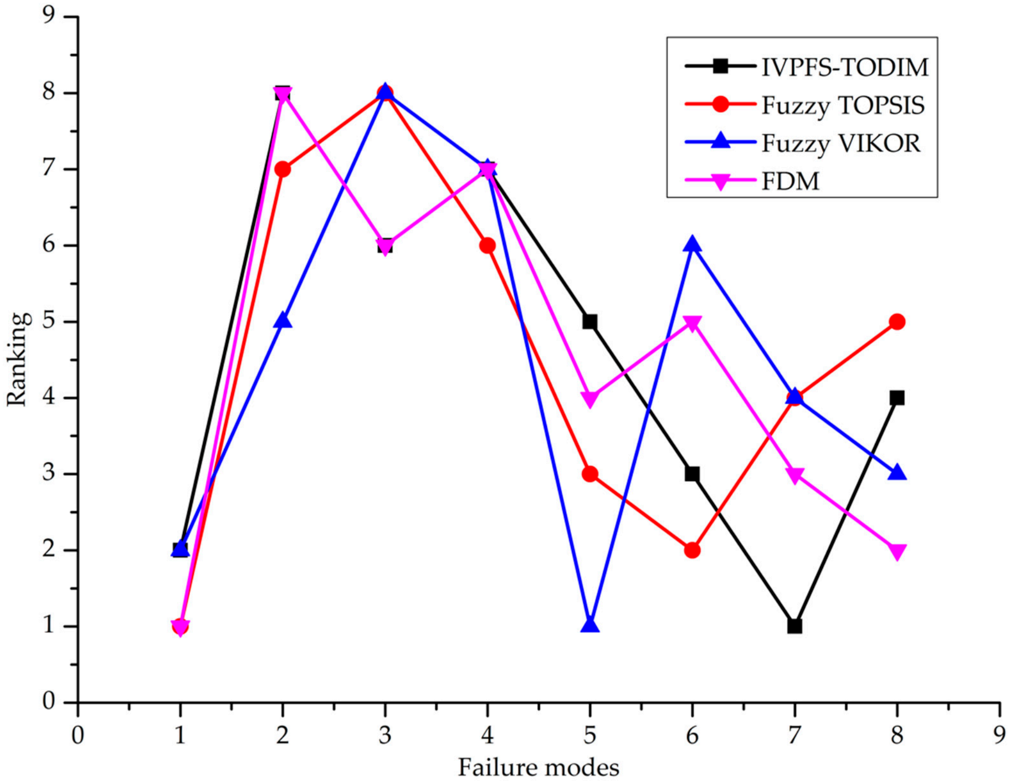


**Figure 3.** Comparative rankings of failure modes.

The rankings of failure modes obtained by the proposed method and fuzzy TOPSIS method show a notable difference. The ranking of $FM_7$ differs significantly for the fuzzy TOPSIS and the proposed method. $FM_7$ only ranked in fourth place in the former method, while it ranked first place in the latter method. Rankings determined by the fuzzy TOPSIS method may be irrational because each expert was assigned the same weight. In reality, experts usually come from several organizations and have different backgrounds, knowledge structures, and practical experience. Hence, they should be given different weights during risk assessments. In addition, the experts' psychological behavior, expert preferences' interdependence, and failure modes' interdependence were not considered in the fuzzy TOPSIS.

The ranking determined by the fuzzy VIKOR significantly differed from the ranking obtained by the proposed method. The Spearman rank-correlation coefficient between the two rankings of failure modes was 0.4286, which further validated this point. For example, $FM_5$ has a difference of four ranking places, and $FM_2$, $FM_6$, and $FM_7$ have a divergence of three ranking places. The reasons for this difference between the two ranking results are that the expert preferences' interdependence, experts' psychological behavior, and failure modes' interdependence were ignored in the fuzzy VIKOR methods. Furthermore, $FM_8$ ranks behind $FM_5$ with the fuzzy VIKOR method. However, the former is more important in reality; therefore, $FM_8$ merits a higher priority in comparison with $FM_5$.

The risk priority of failure modes determined by the FDM method are slightly different from the results yielded by the proposed method, except for $FM_2$, $FM_6$, and $FM_4$. The inconsistent ranking results may be explained by the following: The FDM method considered the interrelations between risk factors by a risk factors fuzzy digraph, while it neglected the expert preferences' interdependence and the experts' psychological behavior. Furthermore, the FDM method only considered the subjective weights of risk factors, which may also cause this difference between the two ranking results of failure modes.

Based on the comparative analysis above, compared with the fuzzy TOPSIS, fuzzy VIKOR, and FDM method, the advantages of the proposed method are summarized as follows: (1) The proposed method adopts the WIVPFBM operator to fuse the evaluations of experts into a group assessment, which reflects the interdependent relationships among expert preferences; (2) the risk priority of failure modes is obtained by applying the improved TODIM method, which considers the experts' psychological behavior and reflects the interdependence between failure modes.

## 6. Conclusions

In this study, we proposed a new risk assessment method of FMEA by combing the WIVPFBM operator and an improved TODIM method to evaluate and determine the risk priority of failure modes under the IVPFS environment. In this method, all evaluation information of FMEA team experts was initially provided in the form of linguistic terms represented by the IVPFNs. Then, the WIVPFBM operator was applied to aggregate the individual evaluation matrix into a group decision matrix. Subsequently, the weights of risk factors were obtained by the subjective and objective weighting method. Finally, the risk ranking of failure modes was obtained by the improved TODIM method. In addition, a sensitivity analysis indicated that the expert preferences' interdependence and failure modes' interdependence had a great influence on the final risk priority of failure modes.

The flexibility and effectiveness of the presented method were demonstrated by the application of the nuclear reheat valve system. The results indicate that the proposed method, which considered the team experts' psychological behaviors and the interrelations among expert preferences, is effective and flexible for real applications compared with other improved FMEA methods. As a direction for future research, extending the proposed method by considering the consensus reaching process in the risk assessment process is recommended.

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