mathematics

MDPI

Article

Set-Valued Interpolative Hardy–Rogers and Set-Valued Reich–Rus–Ćirić-Type Contractions in *b*-Metric Spaces

Pradip Debnath [1] **and Manuel de La Sen** [2,*]

[1] Department of Applied Science and Humanities, Assam University, Silchar, Cachar, Assam 788011, India; pradip.debnath@aus.ac.in or debnath.pradip@yahoo.com

[2] Institute of Research and Development of Processes, University of the Basque Country, Campus of Leioa, 48940 Leioa (Bizakaia), Spain

* Correspondence: manuel.delasen@ehu.eus

Received: 7 August 2019; Accepted: 10 September 2019; Published: 14 September 2019

Abstract: In this paper, using an interpolative approach, we investigate two fixed point theorems in the framework of a b-metric space whose all closed and bounded subsets are compact. One of the theorems is for set-valued Hardy–Rogers-type and the other one is for set-valued Reich–Rus–Ćirić-type contractions. Examples are provided to validate the results.

Keywords: Fixed point; b-metric space; set-valued map; contraction map

---

1. Introduction

After Banach proved the celebrated contraction principle [1] in 1922, numerous researchers have tried to improve or generalize it. The generalizations were mainly done in two directions—either the contractive condition was replaced by some more general ones, or new metric spaces were defined by incorporating additional conditions. In the former direction, many significant and improved results appeared, like Kannan's [2], Chatterjea's [3], Ćirić's [4], Meir-Keeler's [5], Boyd-Wong's [6], etc. The later direction produced introduction of semimetric, quasimetric, symmetric, partial metric, b-metric, fuzzy metric, and many other generalized classes of metric spaces. However, not all of such attempts of generalizations were useful in applications, which was the main motive of such investigations. Some of them were not even true generalizations as they appeared to be equivalent to already existing ones. For some recent relevant work we refer to the works by the authors of [7–9].

Among the many generalized versions of metric spaces, the one due to Bakhtin [10] and Czerwik [11], called b-metric space, has drawn the attention of researchers worldwide, due to its importance and ease of applicability in many fields. The initial motivation for the introduction of b-metric space by both Bakhtin and Czerwik was to study the issue of convergence of measurable functions in regard to measure. Since then, immense development of fixed point theory in the framework of b-metric space has taken place [12–21].

In the current paper, our aim is to study two significant set valued interpolative contractions in the setting of a b-metric space whose all closed and bounded subsets are compact.

2. Preliminaries

First, we list some important definitions and relevant theorems, which are useful in our main results. Throughout the paper $\mathbb{N}$, $\mathbb{R}$, and $\mathbb{R}_+$ denote the set of natural numbers, set of real numbers, and set of non-negative real numbers, respectively.

Definition 1. *[10,11] Let Ω be a nonempty set and the mapping $\delta : \Omega \times \Omega \to [0, \infty)$ satisfies:*
(1) $\delta(u, v) = 0$ if and only if $u = v$;
(2) $\delta(u, v) = \delta(v, u)$ for all $u, v \in \Omega$;
(3) there exists a real number $s \geq 1$ such that $\delta(u, z) \leq s[\delta(u, v) + \delta(v, z)]$ for all $u, v, z \in \Omega$.
Then δ is called a b-metric on Ω and (Ω, δ, s) is called a b-metric space with coefficient s.

The definitions of convergent sequence, Cauchy sequence, and complete *b*-metric space are exactly same as that of usual metric spaces.

Recently, some interesting fixed point results for interpolative contractions for single valued maps have been studied by Debnath et al. [22] and Karapinar et al. [23,24]. Our current results are generalized versions of those results for set-valued case.

Fixed point results for set-valued mappings play a fundamental role in nonlinear analysis. Fixed point theory for multivalued operators is also an important topic for set-valued analysis. The theory of set-valued mappings has applications in control theory, convex optimization, differential inclusions, and economics. The study of fixed point theorems for set-valued mappings was initiated by Markin [25] and Nadler [26]. In 1969, via Hausdorff concept of a distance between two arbitrary sets, Nadler proved some breakthrough fixed point results for set-valued maps.

The Pompeiu–Hausdorff metric $\mathcal{PH}$ is defined in the following manner in a complete metric space (Ω, δ).

If $CB(\Omega)$ denotes the class of all nonempty closed and bounded subsets of Ω, then for $\mathcal{A}, \mathcal{B} \in CB(\Omega)$, we define

$$\mathcal{PH}(\mathcal{A}, \mathcal{B}) = \max\{\sup_{\xi \in \mathcal{B}} \Delta(\xi, \mathcal{A}), \sup_{\eta \in \mathcal{A}} \Delta(\eta, \mathcal{B})\},$$

where $\Delta(\eta, \mathcal{B}) = \inf_{\xi \in \mathcal{B}} \delta(\eta, \xi)$. $(CB(\Omega), \mathcal{PH})$ is called the Pompeiu–Hausdorff metric space induced by δ.

Definition 2. *[26] The point $x \in \Omega$ is said to be a fixed point of the set-valued map $T : \Omega \to CB(\Omega)$ if $x \in Tx$.*

Theorem 1. *[26] Let (Ω, δ) be a complete metric space and $T : \Omega \to CB(\Omega)$ is a mapping such that for all $u, v \in \Omega$*

$$\mathcal{PH}(Tu, Tv) \leq \alpha\delta(u, v);$$

where $0 \leq \alpha < 1$. Then T has a unique fixed point.

Some other results that will be used in the sequel are also stated below.

Lemma 1. *[27] Suppose that (Ω, δ) is a metric space and $Cl(\Omega)$ be the class of all nonempty closed subsets of Ω. Let $\mathcal{B} \in Cl(\Omega)$. Then for each $u \in \Omega$ and $s > 1$, there exists an element $\xi \in \mathcal{B}$ such that $\delta(u, \xi) \leq s \cdot \Delta(u, \mathcal{B})$.*

Remark 1. *[27] If $\mathcal{B}$ is a compact subset of a metric space (Ω, δ), then there exists an element $\xi \in \mathcal{B}$ such that $\delta(u, \xi) = \delta(u, \mathcal{B})$.*

Lemma 2. *[28] Let (Ω, δ) be a metric space. Let $\mathcal{A}, \mathcal{B} \subset \Omega$ and $s > 1$. Then, for every $\eta \in \mathcal{A}$, there exists $\xi \in \mathcal{B}$ such that $\delta(\eta, \xi) \leq s \cdot \mathcal{PH}(\mathcal{A}, \mathcal{B})$.*

Remark 2. *[26] If $\mathcal{A}, \mathcal{B} \in CB(\Omega)$ and let $\eta \in \mathcal{A}$, then for $\alpha > 0$, there exists $\xi \in \mathcal{B}$, such that*

$$\delta(\eta, \xi) \leq \mathcal{PH}(\mathcal{A}, \mathcal{B}) + \alpha.$$

However, there may not be a point $\xi \in \mathcal{B}$ *such that*

$$\delta(\eta, \xi) \leq \mathcal{PH}(\mathcal{A}, \mathcal{B}).$$

If $\mathcal{B}$ *is compact, then such a point* ξ *does exists, i.e.* $\delta(\eta, \xi) \leq \mathcal{PH}(\mathcal{A}, \mathcal{B})$.

3. Main Results

First, we define set-valued Hardy–Rogers-type contraction in a b-metric space and discuss the corresponding fixed point theorem.

Definition 3. *Let* (Ω, δ, s) *be a b-metric space. The map* $T : \Omega \to CB(\Omega)$ *is called a set-valued interpolative Hardy–Rogers-type contraction if there exist* $\lambda \in [0, 1)$ *and* $p, q, r \in (0, 1)$ *with* $p + q + r < 1$ *such that*

$$\mathcal{PH}(Tu, Tv) \leq \lambda[\delta(u, v)]^q [\Delta(u, Tu)]^p [\Delta(v, Tv)]^r [\frac{1}{2s}(\Delta(u, Tv) + \Delta(v, Tu))]^{1-p-q-r} \tag{1}$$

for all $u, v \in \Omega \setminus Fix(T)$.

Theorem 2. *Let* (Ω, δ, s) *be a complete b-metric space whose all closed and bounded subsets are compact and* T *be a set-valued interpolative Hardy–Rogers-type contraction. Then,* T *has a fixed point.*

Proof. Let $u_0 \in \Omega$ and choose $u_1 \in Tu_0$. By Lemma 2, we can select $u_2 \in Tu_1$ such that $\delta(u_2, u_1) \leq \mathcal{PH}(Tu_1, Tu_0)$. Similarly we may choose $u_3 \in Tu_2$ such that $\delta(u_3, u_2) \leq \mathcal{PH}(Tu_2, Tu_1)$. Continuing in this manner we construct a sequence $\{u_n\}$ satisfying $u_n + 1 \in Tu_n$ such that $\delta(u_n + 1, u_n) \leq \mathcal{PH}(Tu_n, Tu_{n-1})$.

Now if there exists $n_0 \in \mathbb{N}$, such that $u_{n_0} \in Tu_{n_0}$, then u_{n_0} becomes a fixed point of T and the proof is complete. Thus, assume that $u_n \notin Tu_n$ for all $n \geq 0$. We show that the sequence $\{u_n\}$ above is a Cauchy sequence.

Replacing u by u_n and v by u_{n-1} in (1), we have

$$\begin{aligned} &\delta(u_{n+1}, u_n) \leq \mathcal{PH}(Tu_n, Tu_{n-1}) \\ &\leq \lambda[\delta(u_n, u_{n-1})]^q [\Delta(u_n, Tu_n)]^p [\Delta(u_{n-1}, Tu_{n-1})]^r [\tfrac{1}{2s}(\Delta(u_n, Tu_{n-1}) + \Delta(u_{n-1}, Tu_n))]^{1-p-q-r} \\ &\leq \lambda[\delta(u_n, u_{n-1})]^q [\delta(u_n, u_{n+1})]^p [\delta(u_{n-1}, u_n)]^r [\tfrac{1}{2s} \cdot \delta(u_{n-1}, u_{n+1})]^{1-p-q-r} \\ &\leq \lambda[\delta(u_n, u_{n-1})]^q [\delta(u_n, u_{n+1})]^p [\delta(u_{n-1}, u_n)]^r [\tfrac{1}{2s} \cdot s(\delta(u_{n-1}, u_n) + \delta(u_n, u_{n+1}))]^{1-p-q-r}. \end{aligned} \tag{2}$$

Assume that $\delta(u_{n-1}, u_n) < \delta(u_n, u_{n+1})$ for some $n \geq 1$. Then $\frac{1}{2}[\delta(u_{n-1}, u_n) + \delta(u_n, u_{n+1})] \leq \delta(u_n, u_{n+1})$.

Thus, from Equation (2), we have

$$\begin{aligned} \delta(u_{n+1}, u_n) &\leq \lambda[\delta(u_n, u_{n-1})]^q [\delta(u_n, u_{n+1})]^p [\delta(u_{n-1}, u_n)]^r [(\delta(u_n, u_{n+1})]^{1-p-q-r} \\ &= \lambda[\delta(u_n, u_{n-1})]^{q+r} [\delta(u_n, u_{n+1})]^{1-q-r}. \end{aligned}$$

This implies

$$[\delta(u_n, u_{n+1})]^{q+r} \leq \lambda[\delta(u_{n-1}, u_n)]^{q+r}. \tag{3}$$

So, we must have $\delta(u_n, u_{n+1}) \leq \delta(u_{n-1}, u_n)$, which is a contradiction to the previous assumption. Thus,

$$\delta(u_n, u_{n+1}) \leq \delta(u_{n-1}, u_n) \text{ for all } n \geq 1. \tag{4}$$

Now, from (2) we have

$$\begin{aligned}\delta(u_{n+1},u_n) &\leq \lambda[\delta(u_n,u_{n-1})]^q[\delta(u_n,u_{n+1})]^p[\delta(u_{n-1},u_n)]^r[\delta(u_{n-1},u_n))]^{1-p-q-r}\\ &= \lambda[\delta(u_{n-1},u_n)]^{1-p}[\delta(u_n,u_{n+1})]^p.\end{aligned}$$

This implies

$$[\delta(u_n,u_{n+1})]^{1-p} \leq \lambda[\delta(u_{n-1},u_n)]^{1-p} \text{ for all } n \geq 1. \tag{5}$$

Combining (4) and (5) we conclude that

$$\delta(u_n,u_{n+1}) \leq \lambda\delta(u_{n-1},u_n) \text{ for all } n \geq 1. \tag{6}$$

It has been proved in the work by the authors of [29] that every sequence $\{u_n\}$ in a b-metric space (Ω,δ,s) having the property that there exists $\lambda \in [0,1)$, such that $\delta(u_n,u_{n+1}) \leq \lambda\delta(u_{n-1},u_n)$ for all $n \geq 1$ is Cauchy.

Thus from (6), we conclude that $\{u_n\}$ is Cauchy. Again (Ω,δ,s) being complete, there exists $l \in \Omega$ such that $\lim_{n\to\infty} u_n = l$.

Since $u_n \notin Tu_n$ for all $n \geq 0$, replacing u by u_n and v by l in (1), we have

$$\begin{aligned}\Delta(u_{n+1},Tl) &\leq \mathcal{PH}(Tu_n,Tl)\\ &\leq \lambda[\delta(u_n,l)]^q[\Delta(u_n,Tu_n)]^p[\Delta(l,Tl)]^r[\frac{1}{2s}(\Delta(u_n,Tl)+\Delta(l,Tu_n))]^{1-p-q-r}\end{aligned} \tag{7}$$

Taking limit as $n \to \infty$ in (7), we have $\Delta(l,Tl) = 0$, which implies that $l \in Tl$.
□

Example 1. *Let $\Omega = [0,\infty)$ and $\delta(u,v) = (u-v)^2$. Then $(\Omega,\delta,2)$ is a complete b-metric space whose every closed and bounded subset is compact.*

Define $T : \Omega \to CB(\Omega)$ by

$$Tu = \begin{cases} \{0\}, & \text{if } u \in [0,1) \\ \{u,u+1\}, & \text{if } u \geq 1. \end{cases}$$

Let $u,v \in \Omega \setminus Fix(T)$. Then obviously $u,v \in (0,1)$. Now $\mathcal{PH}(Tu,Tv) = \mathcal{PH}(\{0\},\{0\}) = 0$, i.e., T is a set valued interpolative Hardy–Rogers-type contraction and the inequality (1) holds. Therefore, all hypotheses of Theorem 2 are true and thus T has a fixed point. Here, it is easy to see that T has infinitely many fixed points.

Next we prove a fixed point theorem for set-valued Reich–Rus–Ćirić-type contraction in b-metric spaces.

Definition 4. *Let (Ω,δ,s) be a b-metric space. A map $T : \Omega \to CB(\Omega)$ is said to be a set valued interpolative Reich–Rus–Ćirić-type contraction if there are constants $\lambda \in [0,1)$ and $A,B \in (0,1)$ with $A+B < 1$, such that*

$$\mathcal{PH}(Tu,Tv) \leq \lambda[\delta(u,v)]^B[\Delta(u,Tu)]^A[(\Delta(v,Tv)]^{1-A-B} \tag{8}$$

for all $u,v \in \Omega \setminus Fix(T)$.

Theorem 3. *Let (Ω,δ,s) be a complete b-metric space whose all closed and bounded subsets are compact. If $T : \Omega \to CB(\Omega)$ is an interpolative Reich–Rus–Ćirić-type contraction, then T has a fixed point in Ω.*

Proof. Let $u_0 \in \Omega$ and choose $u_1 \in Tu_0$. By Lemma 2, we can select $u_2 \in Tu_1$ such that $\delta(u_2,u_1) \leq \mathcal{PH}(Tu_1,Tu_0)$. Similarly we may choose $u_3 \in Tu_2$ such that $\delta(u_3,u_2) \leq \mathcal{PH}(Tu_2,Tu_1)$. Continuing in this manner we construct a sequence $\{u_n\}$ satisfying $u_n + 1 \in Tu_n$ $\delta(u_n+1,u_n) \leq \mathcal{PH}(Tu_n,Tu_{n-1})$.

Now if there exists $n_0 \in \mathbb{N}$ such that $u_{n_0} \in Tu_{n_0}$, then u_{n_0} becomes a fixed point of T and the proof is complete. Hence assume that $u_n \notin Tu_n$ for all $n \geq 0$. We show that the sequence $\{u_n\}$ above is a Cauchy sequence.

Substituting u by u_n and v by u_{n-1} in (8), we have

$$\begin{aligned}\delta(u_{n+1}, u_n) &\leq \mathcal{PH}(Tu_n, Tu_{n-1}) \\ &\leq \lambda[\delta(u_n, u_{n-1})]^B[\Delta(u_n, Tu_n)]^A[\Delta(u_{n-1}, Tu_{n-1})]^{1-A-B} \\ &\leq \lambda[\delta(u_n, u_{n-1})]^B[\delta(u_n, u_{n+1})]^A[h(u_{n-1}, u_n)]^{1-A-B} \\ &= \lambda[\delta(u_n, u_{n-1})]^{1-A}[\delta(u_n, u_{n+1})]^A.\end{aligned} \tag{9}$$

From the above, we obtain

$$[\delta(u_n, u_{n+1})]^{1-A} \leq \lambda[\delta(u_n, u_{n-1})]^{1-A}, \tag{10}$$

which implies that

$$\delta(u_n, u_{n+1}) \leq \delta(u_n, u_{n-1}) \text{ for all } n \geq 0. \tag{11}$$

Using (10) and (11), we have

$$\delta(u_n, u_{n+1}) \leq \lambda^{\frac{1}{1-A}}\delta(u_{n-1}, u_n) \text{ for all } n \geq 1. \tag{12}$$

However, we know from the work by the authors of [29] that every sequence $\{u_n\}$ in a b-metric space (Ω, δ, s) satisfying the property (12) is Cauchy.

Thus, we conclude that as $\{u_n\}$ is a Cauchy sequence and (Ω, δ, s) is complete, there exists $\xi \in \Omega$ such that $\lim_{n\to\infty} u_n = \xi$.

Next we show that ξ is a fixed point of T. Suppose that $\xi \notin T\xi$ so that $\Delta(T\xi, \xi) > 0$. Also, our assumption is that $u_n \notin Tu_n$ for all $n \geq 0$.

By substituting u by u_n and v by ξ in (8), we have

$$\begin{aligned}\Delta(\xi, T\xi) &\leq s[\delta(\xi, u_{n+1}) + \Delta(u_{n+1}, T\xi)] \\ &\leq s[\delta(\xi, u_{n+1}) + \mathcal{PH}(Tu_n, T\xi)] \\ &\leq sh(\xi, u_{n+1}) + \lambda s[\delta(u_n, \xi)]^B[\Delta(u_n, Tu_n)]^A[\Delta(\xi, T\xi)]^{1-A-B} \\ &= s\delta(\xi, u_{n+1}) + \lambda s[\delta(u_n, \xi)]^B[\delta(u_n, u_{n+1})]^A[\Delta(\xi, T\xi)]^{1-A-B}.\end{aligned} \tag{13}$$

Taking limit as $n \to \infty$ in (13), we have $\Delta(\xi, T\xi) = 0$, which is a contradiction to our last hypothesis. Hence $\xi \in T\xi$.

□

It is to be noted that in the previous theorem the role of coefficient s from the definition of b-metric space is not so visible except for the last part of the proof due to the fact that the definition of set-valued Reich–Rus–Ćirić-type contraction in b-metric spaces is almost analogous to its single-valued counterpart.

Below we give an example of Theorem 3.

Example 2. *Let $\Omega = \{0, 1, 2\}$ and $\delta : \Omega \times \Omega \to [0, \infty)$ be defined as $\delta(u, v) = 0$, $\delta(u, v) = \delta(v, u)$ for all $u, v \in \Omega$, $\delta(0, 1) = 1$, $\delta(0, 2) = 2.2$ and $\delta(1, 2) = 1.1$. Then we can verify that $(\Omega, \delta, \frac{22}{21})$ is a complete b-metric space (but it is not a metric space) whose every closed and bounded subset is compact.*

Define the map $T : \Omega \to CB(\Omega)$ on Ω by

$$Tu = \begin{cases} \{0\}, & \text{if } u \neq 2 \\ \{0, 1\}, & \text{if } u = 2. \end{cases}$$

Further we can see that

$$\mathcal{PH}(Tu, Tv) = \begin{cases} \mathcal{PH}(\{0\}, \{0\}) = 0, & \text{if } u \neq 2, v \neq 2 \\ \mathcal{PH}(\{0,1\}, \{0\}) = 1, & \text{if } u = 2, v \neq 2 \\ \mathcal{PH}(\{0\}, \{0,1\}) = 1, & \text{if } u \neq 2, v = 2 \\ \mathcal{PH}(\{0,1\}, \{0,1\}) = 1, & \text{if } u = 2, v = 2. \end{cases}$$

Let $u, v \in \Omega \setminus Fix(T)$. Then clearly the maximum value of $\mathcal{PH}(Tu, Tv)$ is 1, i.e., Inequality (8) and all hypotheses of Theorem 3 hold if we choose $\lambda = \frac{3}{100}$, $A = \frac{1}{3}$, $B = \frac{1}{5}$. Thus, T is a set-valued Reich–Rus–Ćirić-type contraction and has a (unique) fixed point $0 \in T0$.

4. Conclusions

In this paper, we considered two types of set valued interpolative contractions in b-metric spaces. We have imposed a strong condition on the b-metric space that all its closed and bounded subsets are compact. It would be an interesting future study to see if this condition can be dropped. These new interpolative approaches helped us to establish the existence of fixed points for those set-valued contractions. Study of the uniqueness of their fixed points is also a suggested future work.

Author Contributions: Author P.D. contributed in Conceptualization, Investigation, Methodology and Writing the original draft; Author M.d.L.S. contributed in Investigation, Writing and Editing, Funding Acquisition and Project administration.

Funding: This research received no external funding.

Acknowledgments: Research of the first author (P. Debnath) was supported by UGC (Ministry of HRD, Govt. of India) through UGC-BSR Start-Up Grant vide letter No. F.30-452/2018(BSR) dated 12 Feb 2019. The second author (M. de La Sen) expresses gratitude to the Basque Government for the support of this work through Grant IT 1207-19. The authors are immensely grateful to the learned referees for their excellent suggestions and remarks towards improvement of the paper.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Banach, S. Sur les opérations dans les ensembles abstraits et leur applications aux tequations integrales. *Fund. Math.* **1922**, *3*, 133–181. [CrossRef]
2. Kannan, R. Some results on fixed points. *Bull. Calc. Math. Soc.* **1968**, *60*, 71–77.
3. Chatterjea, S.K. Fixed point theorems. *C. R. Acad. Bulgare Sci.* **1972**, *25*, 727–730. [CrossRef]
4. Ćirić, L. Fixed point theorems for multivalued contractions in complete metric spaces. *J. Math. Anal. Appl.* **2008**, *348*, 499–507. [CrossRef]
5. Meir, A.; Keeler, E. A theorem on contraction mappings. *J. Math. Anal. Appl.* **1969**, *28*, 326–329. [CrossRef]
6. Boyd, D.W.; Wong, J.S. On nonlinear contractions. *Proc. Am. Math. Soc.* **1969**, *20*, 458–464. [CrossRef]
7. Park, S. Some general fixed point theorems on topological vector spaces. *Appl. Set-Valued Anal. Optim.* **2019**, *1*, 19–28.
8. Petrusel, A. Local fixed point results for graphic contractions. *J. Nonlinear Var. Anal.* **2019**, *3*, 141–1448.
9. Zaslavski, A.J. Two fixed point results for a class of mappings of contractive type. *J. Nonlinear Var. Anal.* **2018**, *2*, 113–119.
10. Bakhtin, I.A. The contraction mapping principle in almost metric spaces. *Funct. Anal.* **1989**, *30*, 26–37.
11. Czerwik, S. Contraction mappings in b-metric spaces. *Acta Math. Univ. Ostrav.* **1993**, *1*, 5–11.
12. Alharbi, N.; Aydi, H.; Felhi, A.; Ozel, C.; Sahmim, S. α-contractive mappings on rectangular b-metric spaces and an application to integral equations. *J. Math. Anal.* **2018**, *9*, 47–60.
13. Ali, M.U.; Kamran, T.; Postolache, M. Solution of Volterra integgral inclusion in b-metric spaces via new fixed point theorem. *Nonlinear Anal. Modell. Control* **2017**, *22*, 17–30. [CrossRef]
14. Aydi, H.; Chen, C.M.; Karapinar, E. Interpolative Ćirić-Reich-Rus type contractions via the Branciari distance. *Mathematics* **2019**, *7*, 84. [CrossRef]
15. Aydi, H.; Karapinar, E.; Hierro, A.F.R. ω-Interpolative Ćirić-Reich-Rus-type contractions. *Mathematics* **2019**, *7*, 57. [CrossRef]

16. Collaco, P.; Silva, J.C. A complete comparison of 25 contraction conditions. *Nonlinear Anal. Theory Methods Appl.* **1997**, *30*, 471–476. [CrossRef]
17. Ege, O. Complex valued rectangular b-metric spaces and an application to linear equations. *J. Nonlinear Sci. Appl.* **2015**, *8*, 1014–1021. [CrossRef]
18. Jeong, G.S.; Rhoades, B.E. Maps for which $F(T) = F(T^n)$. *Fixed Point Theory Appl.* **2005**, *6*, 71–105.
19. Kamran, T.; Postolache, M.; Ali, M.U.; Kiran, Q. Feng and Liu type F-contraction in b-metric spaces with application to integral equations. *J. Math. Anal.* **2016**, *7*, 18–27.
20. Rhoades, B.E. A comparison of various definitions of contractive mappings. *Trans. Am. Math. Soc.* **1977**, *226*, 257–290. [CrossRef]
21. Shatanawi, W.; Pitea, A.; Lazović, R. Contraction conditions using comparison functions on b-metric spaces. *Fixed Point Theory Appl.* **2014**, *2014*, 135. [CrossRef]
22. Debnath, P.; Mitrović, Z.; Radenović, S. Interpolative Hardy-Rogers and Reich-Rus-Ćirić type contractions in b-metric spaces and rectangular b-metric spaces. *Math. Vesnik.* **2019**, to appear.
23. Karapinar, E.; Agarwal, R.P.; Aydi, H. Interpolative Reich-Rus-Ćirić type contractions on partial metric spaces. *Mathematics* **2018**, *6*, 256. [CrossRef]
24. Karapinar, E.; Alahtani, O.; Aydi, H. On interpolative Hardy-Rogers type contractions. *Symmetry* **2018**, *11*, 8. [CrossRef]
25. Markin, J.T. A fixed point theorem for set- valued mappings. *Bull. Am. Math. Soc.* **1968**, *74*, 639–640. [CrossRef]
26. Nadler, S.B. Multi-valued contraction mappings. *Pac. J. Math.* **1969**, *30*, 475–488. [CrossRef]
27. Kamran, T. Mizoguchi-Takahashi's type fixed point theorem. *Comput. Math. Appl.* **2009**, *57*, 507–511. [CrossRef]
28. Berinde, M.; Berinde, V. On a general class of multivalued weakly picard mappings. *J. Math. Anal. Appl.* **2007**, *326*, 772–782. [CrossRef]
29. Miculescu, R.; Mihail, A. New fixed point theorems for set-valued contractions in b-metric spaces. *J. Fixed Point Theory Appl.* **2017**, *19*, 2153–2163. [CrossRef]