

Article

Extreme value index estimation by means of an inequality curve – Supplement

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1. Montecarlo experiments

This Supplement provides tables and graphs to give further numerical evidence to the discussions in the main paper.

1.1. Asymptotic distribution of $\hat{\gamma}$

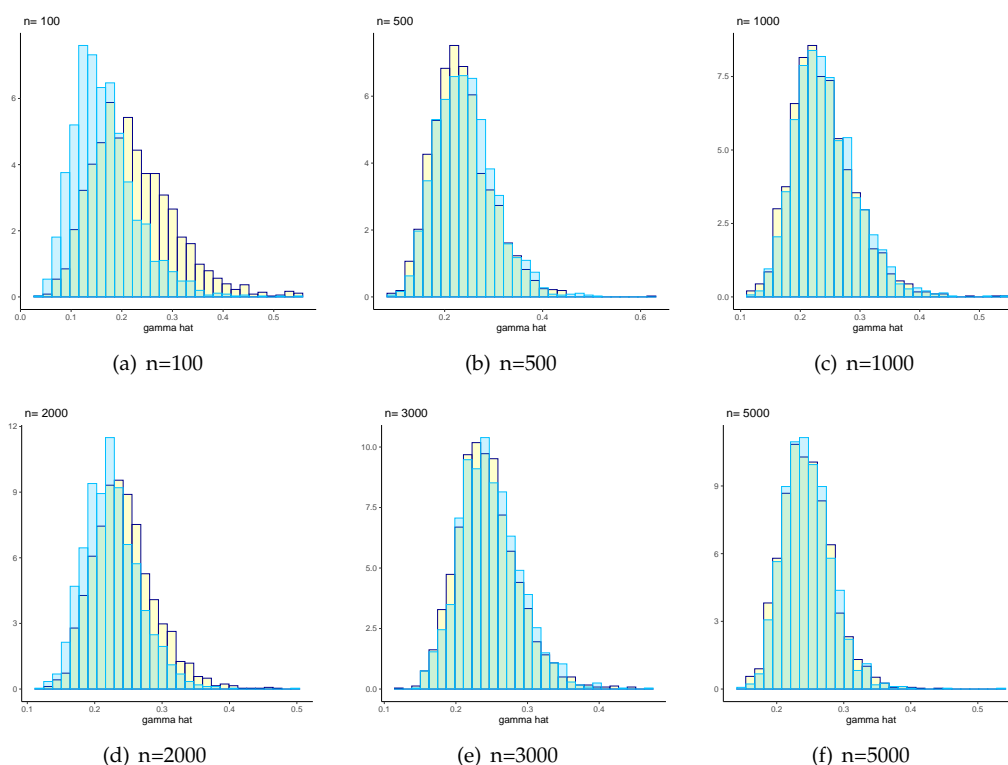


Figure 1. Histograms of the empirical distribution of $\hat{\gamma}_k$ for selected sample sizes; $k = n^{0.5}$; data samples are generated from a Pareto(4) distribution. Yellow: values obtained by Montecarlo simulations (2000 iterations); blue: values obtained by Algorithm 1 (2000 iterations). The value of γ used has been selected randomly from a pool of estimated values.

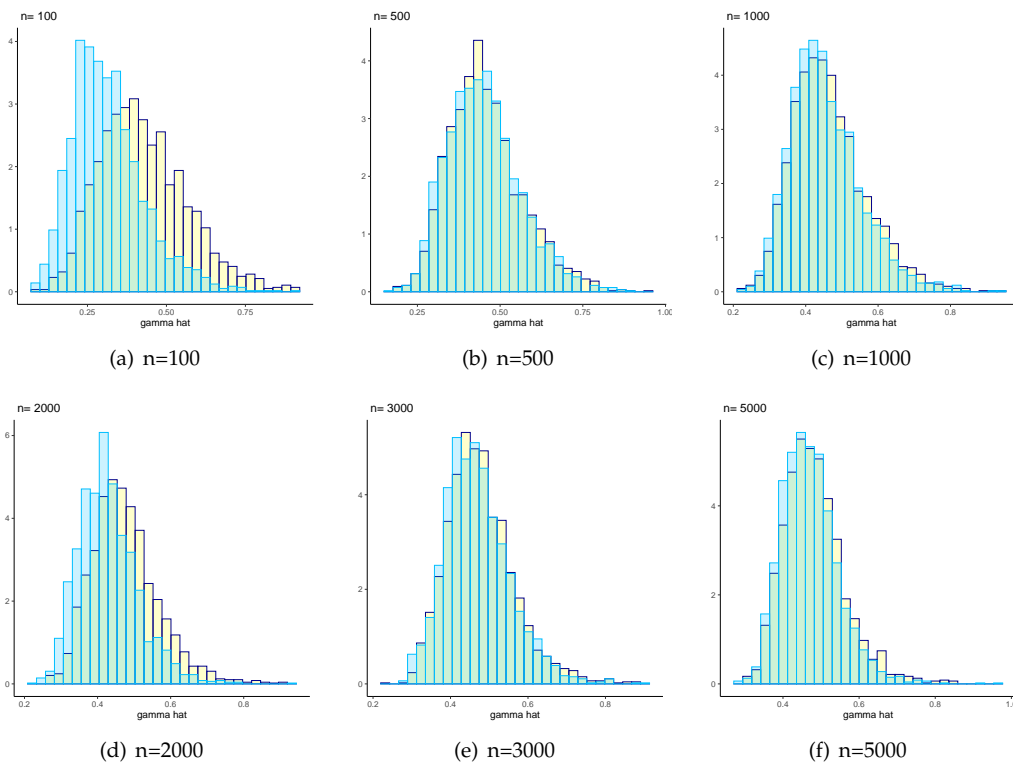


Figure 2. Histograms of the empirical distribution of $\hat{\gamma}_k$ for selected sample sizes; $k = n^{0.5}$; data samples are generated from a Burr(2) distribution. Yellow: values obtained by Montecarlo simulations (2000 iterations); blue: values obtained by Algorithm 1 (2000 iterations). The value of γ used has been selected randomly from a pool of estimated values.

5 1.2. Estimation of $\hat{\gamma}$

6 Tables 1 to 13 report further evidence on the performance of the estimators of γ for different
7 distributions. Refer to the main paper for a detailed description of the Montecarlo experiments,
8 notation, distributions and alternative estimation strategies used.

n	$\hat{\gamma}_{Opt}$	$\hat{\gamma}_{All}$	$\hat{\gamma}_{0.7}$	$\hat{\gamma}_{0.5}$	$\hat{\gamma}_{0.3}$	Hill	$MP_{0.5}$	MP_1	GH	MM
50	0.199	0.175	0.193	0.213	0.248	1.068	0.681	0.481	9.681	430.889
100	0.163	0.148	0.163	0.178	0.207	0.891	0.514	0.377	3.407	268572.411
300	0.131	0.115	0.125	0.135	0.155	1.032	0.703	0.517	20.787	288.311
500	0.123	0.104	0.113	0.121	0.137	0.908	0.634	0.451	10.400	18.750
1000	0.102	0.090	0.096	0.103	0.116	0.578	0.401	0.295	2.750	58.359

Table 1. RMSE of the estimators for the Pareto(1.1) distribution; 1000 Montecarlo replications.

n	$\hat{\gamma}_{Opt}$	$\hat{\gamma}_{All}$	$\hat{\gamma}_{0.7}$	$\hat{\gamma}_{0.5}$	$\hat{\gamma}_{0.3}$	Hill	$MP_{0.5}$	MP_1	GH	MM
50	0.132	0.112	0.125	0.140	0.167	0.783	0.560	0.415	7.098	48.484
100	0.106	0.092	0.103	0.115	0.137	0.653	0.420	0.317	2.493	5938.290
300	0.081	0.066	0.073	0.081	0.096	0.757	0.567	0.441	15.222	64.691
500	0.076	0.058	0.064	0.071	0.083	0.666	0.511	0.393	7.624	5.347
1000	0.061	0.048	0.053	0.058	0.068	0.424	0.321	0.252	2.016	42.133

Table 2. RMSE of the estimators for the Pareto(1.5) distribution; 1000 Montecarlo replications.

n	$\hat{\gamma}_{Opt}$	$\hat{\gamma}_{All}$	$\hat{\gamma}_{0.7}$	$\hat{\gamma}_{0.5}$	$\hat{\gamma}_{0.3}$	<i>Hill</i>	$MP_{0.5}$	MP_1	<i>GH</i>	<i>MM</i>
50	0.112	0.093	0.105	0.118	0.140	0.653	0.492	0.380	5.915	15.052
100	0.089	0.077	0.086	0.096	0.116	0.544	0.369	0.288	2.074	992.011
300	0.067	0.052	0.058	0.066	0.080	0.631	0.494	0.398	12.671	21.491
500	0.062	0.045	0.050	0.056	0.068	0.555	0.445	0.357	6.351	2.215
1000	0.048	0.036	0.040	0.045	0.054	0.353	0.279	0.227	1.680	4.405

Table 3. RMSE of the estimators for the Pareto(1.8) distribution; 1000 Montecarlo replications.

n	$\hat{\gamma}_{Opt}$	$\hat{\gamma}_{All}$	$\hat{\gamma}_{0.7}$	$\hat{\gamma}_{0.5}$	$\hat{\gamma}_{0.3}$	<i>Hill</i>	$MP_{0.5}$	MP_1	<i>GH</i>	<i>MM</i>
50	0.102	0.084	0.095	0.107	0.127	0.588	0.455	0.359	5.323	8.512
100	0.080	0.068	0.077	0.087	0.105	0.490	0.342	0.272	1.865	399.072
300	0.060	0.045	0.051	0.058	0.071	0.568	0.455	0.374	11.396	8.850
500	0.054	0.038	0.043	0.049	0.059	0.499	0.410	0.336	5.715	1.089
1000	0.042	0.029	0.034	0.038	0.046	0.318	0.257	0.213	1.512	2.718

Table 4. RMSE of the estimators for the Pareto(2) distribution; 1000 Montecarlo replications.

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n	$\hat{\gamma}_{Opt}$	$\hat{\gamma}_{All}$	$\hat{\gamma}_{0.7}$	$\hat{\gamma}_{0.5}$	$\hat{\gamma}_{0.3}$	Hill	$MP_{0.5}$	MP_1	GH	MM
50	0.069	0.055	0.063	0.072	0.087	0.392	0.329	0.280	3.548	1.287
100	0.052	0.043	0.050	0.057	0.071	0.327	0.250	0.210	1.238	23.178
300	0.037	0.025	0.030	0.035	0.045	0.379	0.325	0.283	7.569	1.510
500	0.033	0.021	0.024	0.028	0.035	0.333	0.292	0.256	3.807	1.221
1000	0.022	0.014	0.017	0.020	0.026	0.212	0.183	0.161	1.007	0.602

Table 5. RMSE of the estimators for the Pareto(3) distribution; 1000 Montecarlo replications.

n	$\hat{\gamma}_{Opt}$	$\hat{\gamma}_{All}$	$\hat{\gamma}_{0.7}$	$\hat{\gamma}_{0.5}$	$\hat{\gamma}_{0.3}$	Hill	$MP_{0.5}$	MP_1	GH	MM
50	0.099	0.122	0.092	0.097	0.120	0.139	0.139	0.136	0.128	13.262
100	0.081	0.122	0.081	0.080	0.095	0.106	0.106	0.106	0.099	4.458
300	0.064	0.118	0.069	0.060	0.066	0.071	0.072	0.075	0.068	0.540
500	0.056	0.117	0.064	0.051	0.052	0.056	0.057	0.060	0.055	0.427
1000	0.049	0.117	0.063	0.049	0.047	0.044	0.046	0.052	0.044	0.322

Table 6. RMSE of the estimators for the Fréchet(2) distribution; 1000 Montecarlo replications.

n	$\hat{\gamma}_{Opt}$	$\hat{\gamma}_{All}$	$\hat{\gamma}_{0.7}$	$\hat{\gamma}_{0.5}$	$\hat{\gamma}_{0.3}$	Hill	$MP_{0.5}$	MP_1	GH	MM
50	0.074	0.112	0.074	0.072	0.085	0.093	0.093	0.093	0.079	0.360
100	0.060	0.110	0.065	0.059	0.067	0.070	0.070	0.071	0.065	0.363
300	0.044	0.106	0.055	0.043	0.044	0.047	0.048	0.049	0.046	0.365
500	0.037	0.105	0.052	0.038	0.035	0.037	0.037	0.038	0.038	0.366
1000	0.031	0.104	0.051	0.035	0.029	0.030	0.030	0.031	0.030	0.367

Table 7. RMSE of the estimators for the Fréchet(3) distribution; 1000 Montecarlo replications.

n	$\hat{\gamma}_{Opt}$	$\hat{\gamma}_{All}$	$\hat{\gamma}_{0.7}$	$\hat{\gamma}_{0.5}$	$\hat{\gamma}_{0.3}$	Hill	$MP_{0.5}$	MP_1	GH	MM
50	0.058	0.094	0.059	0.056	0.065	0.070	0.070	0.070	0.063	0.403
100	0.047	0.093	0.052	0.045	0.050	0.053	0.053	0.053	0.054	0.418
300	0.034	0.089	0.044	0.034	0.033	0.036	0.036	0.036	0.039	0.426
500	0.028	0.089	0.042	0.030	0.026	0.028	0.028	0.028	0.032	0.428
1000	0.023	0.088	0.041	0.028	0.022	0.022	0.022	0.023	0.026	0.430

Table 8. RMSE of the estimators for the Fréchet(4) distribution; 1000 Montecarlo replications.

n	$\hat{\gamma}_{Opt}$	$\hat{\gamma}_{All}$	$\hat{\gamma}_{0.7}$	$\hat{\gamma}_{0.5}$	$\hat{\gamma}_{0.3}$	Hill	$MP_{0.5}$	MP_1	GH	MM
50	0.095	0.232	0.119	0.090	0.086	0.084	0.083	0.082	0.069	0.604
100	0.084	0.230	0.112	0.081	0.073	0.076	0.075	0.075	0.069	0.369
300	0.055	0.226	0.105	0.070	0.053	0.056	0.056	0.057	0.051	0.353
500	0.046	0.225	0.103	0.066	0.046	0.047	0.047	0.048	0.043	0.353
1000	0.037	0.224	0.102	0.064	0.040	0.037	0.037	0.038	0.033	0.354

Table 9. RMSE of the estimators for the Burr(3) distribution; 1000 Montecarlo replications.

n	$\hat{\gamma}_{Opt}$	$\hat{\gamma}_{All}$	$\hat{\gamma}_{0.7}$	$\hat{\gamma}_{0.5}$	$\hat{\gamma}_{0.3}$	Hill	$MP_{0.5}$	MP_1	GH	MM
50	0.078	0.208	0.100	0.074	0.068	0.063	0.062	0.062	0.062	0.394
100	0.068	0.205	0.093	0.066	0.057	0.057	0.057	0.057	0.058	0.405
300	0.043	0.202	0.087	0.057	0.041	0.042	0.042	0.042	0.042	0.412
500	0.035	0.200	0.086	0.054	0.035	0.035	0.035	0.036	0.034	0.414
1000	0.028	0.199	0.084	0.052	0.031	0.028	0.028	0.028	0.026	0.417

Table 10. RMSE of the estimators for the Burr(4) distribution; 1000 Montecarlo replications.

n	$\hat{\gamma}_{Opt}$	$\hat{\gamma}_{All}$	$\hat{\gamma}_{0.7}$	$\hat{\gamma}_{0.5}$	$\hat{\gamma}_{0.3}$	Hill	$MP_{0.5}$	MP_1	GH	MM
50	0.335	0.214	0.307	0.336	0.359	0.400	0.395	0.394	0.373	3.807
100	0.308	0.197	0.287	0.310	0.323	0.363	0.355	0.351	0.345	2.251
300	0.289	0.182	0.267	0.284	0.286	0.336	0.321	0.308	0.325	0.075
500	0.272	0.174	0.256	0.270	0.268	0.323	0.305	0.287	0.314	0.114
1000	0.254	0.166	0.244	0.255	0.249	0.311	0.288	0.263	0.305	0.028

Table 11. RMSE of the estimators for the Stable(1.3) distribution; 1000 Montecarlo replications.

n	$\hat{\gamma}_{Opt}$	$\hat{\gamma}_{All}$	$\hat{\gamma}_{0.7}$	$\hat{\gamma}_{0.5}$	$\hat{\gamma}_{0.3}$	Hill	$MP_{0.5}$	MP_1	GH	MM
50	0.310	0.149	0.270	0.310	0.339	0.383	0.379	0.377	0.344	0.386
100	0.292	0.137	0.260	0.293	0.313	0.356	0.350	0.346	0.327	0.135
300	0.291	0.129	0.250	0.279	0.290	0.334	0.326	0.317	0.317	0.045
500	0.280	0.124	0.243	0.270	0.278	0.321	0.311	0.299	0.307	0.044
1000	0.266	0.121	0.237	0.261	0.265	0.309	0.297	0.281	0.300	0.045

Table 12. RMSE of the estimators for the Stable(1.5) distribution; 1000 Montecarlo replications.

n	$\hat{\gamma}_{Opt}$	$\hat{\gamma}_{All}$	$\hat{\gamma}_{0.7}$	$\hat{\gamma}_{0.5}$	$\hat{\gamma}_{0.3}$	Hill	$MP_{0.5}$	MP_1	GH	MM
50	0.282	0.056	0.220	0.283	0.333	0.352	0.353	0.354	0.290	0.091
100	0.280	0.047	0.221	0.281	0.326	0.356	0.356	0.355	0.304	0.104
300	0.321	0.039	0.224	0.282	0.324	0.367	0.365	0.363	0.329	0.113
500	0.333	0.039	0.224	0.280	0.321	0.367	0.364	0.360	0.334	0.118
1000	0.329	0.039	0.224	0.279	0.318	0.365	0.360	0.354	0.339	0.123

Table 13. RMSE of the estimators for the Stable(1.9) distribution; 1000 Montecarlo replications.