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Incorporating Outsourcing Strategy and Quality Assurance into a Multiproduct Manufacturer–Retailer Coordination Replenishing Decision

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Abstract: This study explores the multiproduct manufacturer-retailer coordination replenishing decision featuring outsourcing strategy and product quality assurance. Globalization has generated enormous opportunities. Consequently, transnational firms now face tough competition in global markets. To stay competitive, a firm should meet the client's multi-item and quality requirements under capacity constraints and optimize the intra-supply chain system to allow the timely distribution of finished goods under minimum system cost. The outsourcing option is considered to release machine loadings and reduce cycle time effectively. All items fabricated are screened for quality, and reworkable and scrap items are separated. Any reworked items that fail the quality reassurance screening are discarded, whereas all outsourced products are quality-guaranteed by the provider. A fixed-quantity multi-shipment plan is used when the whole finished lot is quality-ensured to help present-day transnational firms gain competitive advantage by making efficient and cost-effective multiproduct manufacturing and delivering decisions. Mathematical modeling is built to portray the system's characteristics, and conventional differential calculus is used to solve and derive the optimal operating policy for the proposed problem. Simultaneously, we find the optimal delivery frequency and common cycle time for the problem mentioned above. A simulated numerical example and sensitivity analysis demonstrate the research result's capability and applicability. Our precise analytical model can reveal/highlight the impact of deviations in quality- and outsourcing-related features on the optimal operating policy and several performance indicators that help managerial decision-making.

Keywords: optimization; multi-item system; manufacturer-retailer integrated system; outsourcing; quality guarantee; common cycle

1. Introduction

This study explores the multiproduct manufacturer-retailer coordination replenishing decision incorporating an outsourcing strategy and product quality assurance. Due to the growing tendency of the market's multi-item requirements in past decades, studies on multi-item fabrication planning and controlling have been broadly carried out. Lee et al. [1] employed goal programming to study a multi-period multiproduct fabrication scheduling problem. A model including three distinct fabrication lines and one inspection facility was built to deal with this multi-machine multiple objectives scheduling problem. An example was used to demonstrate how the proposed goal programming can handle the analytical and solution processes. Rosenblatt [2] examined a single-supplier multiproduct inventory problem. Two distinct ordering policies, namely the fixed-cycle and the basic cycle policies, were used to derive the cost functions. Dynamic programming and heuristic approaches were employed respectively to partition multiproduct into groups and resolve these problems with separate policies. The policies' effectiveness was explored through a simulation model, and the results were compared with the conventional economic order quantity policy. Kohli and Park [3] studied the coordination of multiproduct transactions between buyer and seller. To lower total transactions cost, the combined order policies for multiproduct were specifically analyzed for the case of a single-seller multi-buyers situation. As a result, the authors concluded that combined lot-size policies are dependent of the potential savings on buyer-seller transaction costs but independent of selling prices of multiproduct. Sambasivan and Schmidt [4] examined multi-period multi-item lot-sizing problems featuring multiple plants, inter-plant transfers, and under-capacity constraints. The authors first presented un-capacitated solutions to the problems using a heuristic approach. Then, they employed a smoothing technique to remove capacity violations from initial solutions. A number of experiments were conducted to demonstrate the accuracy of heuristic results through the mainframe computing environment. Taleizadeh et al. [5] studied a multiproduct economic production quantity (EPQ) problem featuring a single machine with limited capacity, discontinuous delivery plan, and under the common cycle policy. Different costs, including setup, unit fabrication, holding, and delivery costs, are associated with different end products. The authors developed a mixed-integer non-linear model to examine the problem. By employing the cutting plane, harmony search, particle swarm optimizing approaches, and numerical illustrations, the authors analyzed, solved, and evaluated characteristics of the problem. Other studies [6–10] examined diverse aspects of multi-item fabrication systems.

When facing the client's timely requirements and in-house capacity constraints, applying an outsourcing strategy can effectively release machine loadings and shorten fabrication cycle time. Prencipe [11] stated that the vertical integration of product-systems requires more understanding of a firm's core strategies on required technology, outsourcing, and research and development (R&D) activities to gain competitive advantages. The authors used a real case from Rolls-Royce as an example to depict empirical evidence for supporting their arguments. Kouvelis and Milner [12] investigated the dynamic interplay of uncertain supplies and demands in the two-stage supply-chain systems' capacity and outsourcing strategies. The stage one supply chain investment is for a firm's primary activities, whereas stage two is for the non-primary activities. Both investments aim to gain maximal multi-period profits. The effect of random demand on outsourcing decisions and impact of non-stationary supply on investment levels were studied. Optimal decisions were explored for both single- and multiple-period investments to study the effect of uncertain supplies and demands on outsourcing. As a result, a few managerial implications were revealed that can facilitate investment and/or outsourcing decisions. Wee et al. [13] conducted an empirical study via questionnaires from Taiwanese firms on supplier management's performance under various outsourcing strategies. The results indicated that different types of industries should select their appropriate outsourcing plans. The success of outsourcing implementation requires a good relationship between the firm and its outsourcer and other critical issues such as long-term relationship/contract, and outsourcer's capability on timely delivery, quality supplies, etc. Chraibi et al. [14] explored the risks and the chance of failure in the outsourcing plan. They examined an outsourcing model containing procurement activities with pre-contractual as well as post-contractual outsourcing issues. They proposed this exploratory outsourcing model, including seven significant implementing steps, generated according to benchmarking of leading enterprises, to aim at successful outsourcing. Additional studies explored various characteristics of outsourcing strategies in enterprises and manufacturing firms [15–19]. To maintain perfect product quality is always a crucial operating goal to a manufacturer for meeting a customer's satisfaction and allowing the firm to stay competitive in a turbulent market. However, due to many unforeseen factors in real fabrication processes, generating random defects is inevitable. The manufacturer must have the capability to identify these items. Consequently, they must perform repairs to them, or remove them from the quality-ensured finished lot. Dohi et al. [20] examined the economical manufacturing quantity (EMQ) models incorporating the Poisson machine failure rate and repairs. Models were constructed and formulated under two separate machine repairing policies to minimize the manufacturing operations' steady cost functions. Inderfurth et al. [21] explored a deterministic production planning problem wherein the regular fabrication and rework processes are scheduled on the same machine. The reworked items have a limited deterioration time while waiting to be repaired, and as waiting time gets longer the rework cost increases accordingly. The authors aimed to find the optimal regular lot sizes and amount of reworked items that keep total cost at a minimum. A proposed polynomial dynamic programming algorithm solved the problem. Extra studies [22–25] investigated different features of imperfect fabrication systems and their consequent actions.

Optimization of the intra-supply chain system (i.e., similar to the manufacturer-retailer integrated type of system) in current transnational enterprises will allow the timely distribution of their finished goods and minimize total system cost. Banerjee and Banerjee [26] examined a single product single-vendor multi-buyer inventory system that features order-less replenishment. They built a model to depict the problem's characteristics using a common cycle replenishing policy. It could also be computerized to enable real-time data interchange between trading parties. Viswanathan and Piplani [27] explored the benefit of a single product single-vendor multi-buyer coordinated supply-chain system under the common inventory replenishment periods. It was assumed that the vendor offered a price discount, so the vendor determines the common replenishing periods, and all buyers that follow these preset times to refill their stocks will receive the benefit through price discount. The objective was to jointly decide for the vendor the optimal replenishment times and the offering of a discounted price. Sancak and Salmann [28] studied a multiproduct dynamic lot-sizing problem featuring delayed delivery policy, wherein multiple items were purchased by a producer to meet its production needs, and the objective was to optimize the policies of ordering and inbound delivery that kept delivery and stock holding cost at a minimum. Regular delivery cost charge is assumed based on a full truckload. The authors explored using safety stocks to delay delivery to the following period with less than a full truckload. Real data from a transportation manufacturer was used to examine this delay delivery option's effect on service levels and system costs. As a result, the total delivery and stock holding cost were reduced without increasing the stock-out risk. Additional recent works [29–32] studied various natures of intra-supply chain or vendor-buyer integrated types of systems. The urgent need for a precise model to help managers of present-day transnational firms make efficient and cost-effective multiproduct manufacturing and delivering decisions. As few studies mainly focused on this specific area, this study aims to bridge the research gap by building a decision-support mathematical model to optimize the multi-item manufacturer- retailer integrated inventory system incorporating outsourcing and quality guarantee. This study's main contribution is that it can reveal/highlight the impact of deviations in quality- and outsourcing-related features on the optimal operating policy and several performance indicators that help managerial decision-making.

The rest of the paper includes the problem's description and mathematical modeling in Section 2 (containing notation, assumption, formulations, convexity, solution process, and prerequisite condition). Numerical example with sensitivity analyses in Section 3, and Conclusion in Section 4.

2. Description and Mathematical Modeling

2.1. Assumptions and Notations

This study optimizes a multi-item manufacturer–retailer integrated inventory system with outsourcing and quality guarantee. We consider an inventory system having a multiproduct fabrication plan on a single facility, under a rotation cycle discipline along with a partial outsourcing policy. Specifically, in each cycle a π_i proportion of batch size Q_i for each product i is provided by the outside contractor (where i = 1, 2, ... , L). As a part of the agreement, outsourced items must have perfect quality and be received right before product delivery time (see Figure 1). Accordingly, $K_{\pi i}$ and $C_{\pi i}$ denote the constant setup and unit purchase costs for outsourced items.

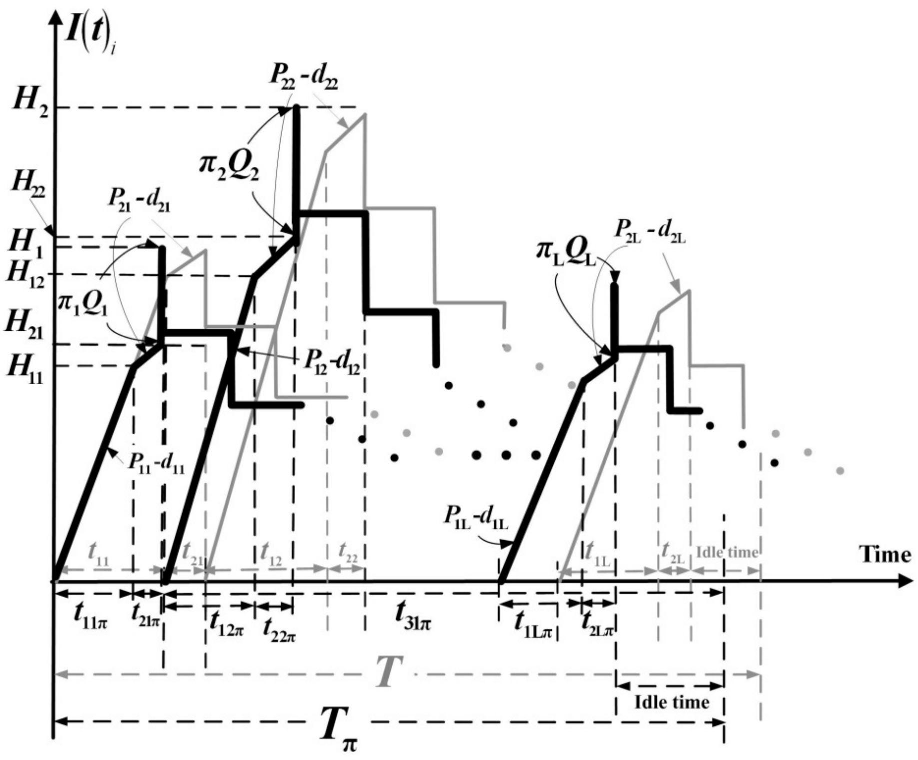

Figure 1. Inventory status of finished items at the manufacturer side of the proposed multi–item manufacturer–retailer integrated system with outsourcing and quality guarantee.

The other $(1 - \pi_i)$ portion of Q_i for each product is manufactured in-house at P_{1i} products per year. However, the in-house processes are not perfect. A random x_i portion of defective items are generated at the d_{1i} rate. Defective items are checked to separate a θ_{1i} portion of scrap from the other $(1 - \theta_{1i})$ re-workable (where $0 \leq \theta_{1i} \leq 1$). To avoid stock-out circumstances, the manufacturing rate P_{1i} has to satisfy $(P_{1i} - d_{1i} - \lambda_i) > 0$ (where λ_i represents product i's demand rate and d_{1i} equals to x_iP_{1i}). In each cycle, when the regular processes end, the reworking of each end product i is performed at the rate P_{2i} (Figure 1) with extra unit rework cost C_{Ri}. Figure 2 shows the on-hand inventory of defective products in the proposed multi-item manufacturer–retailer integrated system. In the rework, a θ_{2i} portion fails (where $0 \leq \theta_{2i} \leq 1$). So, the production rate of scrap d_{2i} is $\theta_{2i}P_{2i}$, and the maximum level of scrap items in a cycle is $\varphi_i\, x_i\, [(1 - \pi_i)\, Q_i]$, where φ_i is the sum of scrap rates among in-house defective items in $t_{1i\pi}$ and $t_{2i\pi}$ (so, $\varphi_i = [\theta_{1i} + \theta_{2i}(1 - \theta_{1i})]$).

Figure 3 exhibits the on-hand inventory of scraps in the proposed multi-item system. At the end of reworking, outsourced products are received in time to bring the stock level to H_i. Then, a fixed amount of multiple shipments of the quality-assured batch is shipped to the retailer at a fixed time interval $t_{ni\pi}$ (Figures 1 and 4).

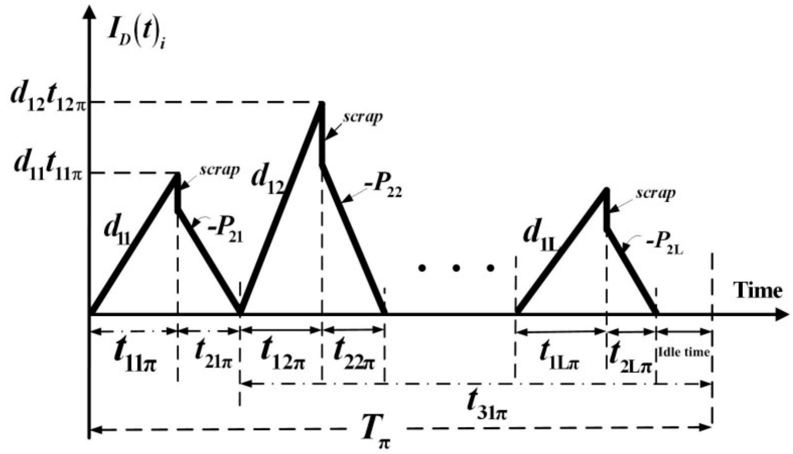

Figure 2. Inventory status of defective products at the manufacturer side of the proposed system.

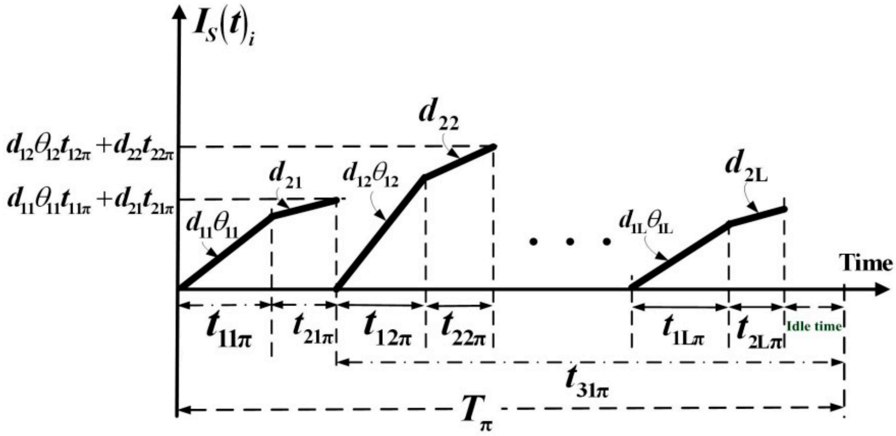

Figure 3. Inventory level of each scrap product i at the manufacturer side of the proposed system.

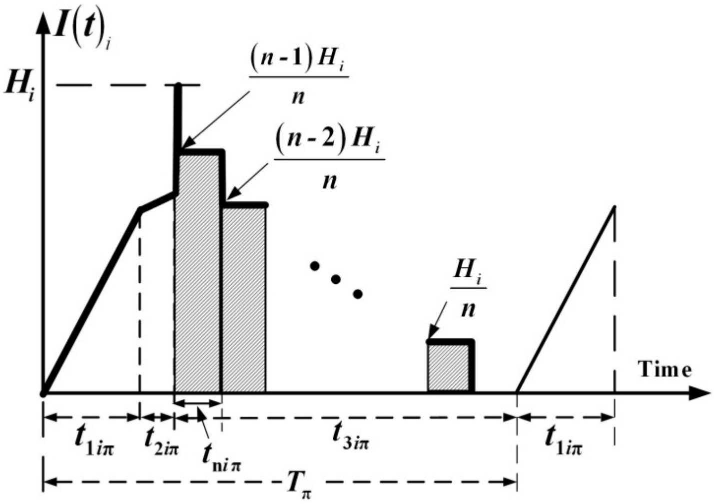

Figure 4. Inventory status in $t_{3i\pi}$ at the manufacturer side.

Extra notations used in the proposed multi-item manufacturer–retailer integrated system are listed in Table 1 below (where $i = 1, 2, \ldots, L$):

Table 1. Nomenclature.

T_π	rotation cycle time;
Q_i	batch size for product i,
K_i	in-house setup cost for product i,
C_i	unit in-house manufacturing cost for product i,
h_i	unit holding cost of product i,
h_{1i}	unit holding cost for reworked product i,
h_{2i}	unit holding cost in the retailer side,
C_{Si}	unit disposal cost,
$t_{1i\pi}$	uptime for product i,
$t_{2i\pi}$	rework time,
$t_{3i\pi}$	delivery time,
$t_{ni\pi}$	fixed interval of time between deliveries,
H_{1i}	inventory level when the uptime ends,
H_{2i}	inventory level when the rework time ends,
H_i	maximum inventory level in the beginning of delivery time (after receipt of outsourced items),
N	number of shipments per cycle − another decision variable,
K_{1i}	fixed delivery cost for product i,
C_{Ti}	unit delivery cost,
$I(t)_i$	stock level of finished items at time t,
$I_D(t)_i$	inventory level of defective items,
$I_S(t)_i$	inventory level of scrap,
$I_c(t)_i$	stock level of product i in the retailer's side at time t,
t_{1i}	uptime for the product i in the proposed system without outsourcing plan,
t_{2i}	rework time in a system without outsourcing,
t_{3i}	delivery time in a system without outsourcing,
T	rotation cycle time a system without outsourcing,
$TC(T_\pi, n)$	total cost per cycle,
$E[TCU(T_\pi, n)]$	the long-run average system cost per unit time,
$\overline{\pi}$	the average of π_i,
$\overline{x}$	the average of x_i,
$\overline{\varphi}$	the average of φ_i,
$\overline{\beta_1}$	the average of β_{1i},
$\overline{\beta_2}$	the average of β_{2i},

2.2. Formulations

From Figures 1–3, we observe the following formulas:

$$T_\pi = t_{1i\pi} + t_{2i\pi} + t_{3i\pi} \tag{1}$$

$$t_{1i\pi} = \frac{H_{1i}}{P_{1i} - d_{1i}} = \frac{(1-\pi_i)Q_i}{P_{1i}} \tag{2}$$

$$t_{2i\pi} = \frac{x_i[(1-\pi_i)Q_i](1-\theta_{1i})}{P_{2i}} \tag{3}$$

$$t_{3i\pi} = n \cdot t_{ni\pi} = T_\pi - (t_{1i\pi} + t_{2i\pi}) \tag{4}$$

$$Q_i = \frac{\lambda_i T_\pi}{[1 - \varphi_i x_i(1-\pi_i)]} \tag{5}$$

$$H_{1i} = t_{1i\pi}(P_{1i} - d_{1i}) \tag{6}$$

$$H_{2i} = H_{1i} + (P_{2i} - d_{2i})t_{2i\pi} \tag{7}$$

$$H_i = H_{2i} + \pi_i Q_i = \lambda_i \cdot T_\pi \tag{8}$$

$$d_{1i}t_{1i\pi} = x_i[(1-\pi_i)Q_i] = x_i P_{1i} t_{1i\pi}. \tag{9}$$

$$\varphi_i x_i[(1-\pi_i)Q_i] = [\theta_{1i} + \theta_{2i}(1-\theta_{1i})]x_i[(1-\pi_i)Q_i]. \tag{10}$$

Figure 4 exhibits the on-hand inventory status in $t_{3i\pi}$. Total inventories of product i are [33] as follows:

$$\left(\frac{1}{n^2}\right)\left(\sum_{i=1}^{n-1} i\right)H_i(t_{3i\pi}) = \left(\frac{1}{n^2}\right)\left[\frac{n(n-1)}{2}\right]H_i(t_{3i\pi}) = \left(\frac{n-1}{2n}\right)H_i(t_{3i\pi}) \tag{11}$$

Figure 5 depicts the stock status at the retailer's side. Because n fixed quantity shipments are transported to the retailer at a fixed $t_{\mathrm{n}i\pi}$ time period, the following formulas can be observed:

$$I_i = D_i - \lambda_i t_{\mathrm{n}i\pi} \tag{12}$$

$$t_{\mathrm{n}i\pi} = \frac{t_{3i\pi}}{n} \tag{13}$$

$$D_i = \frac{H_i}{n} \tag{14}$$

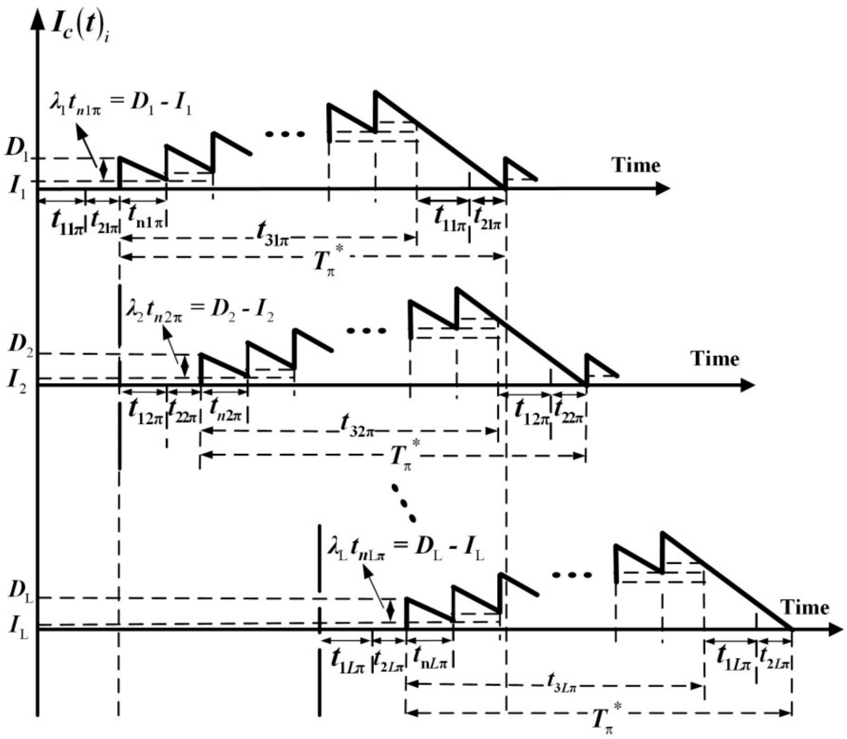

Figure 5. Inventory status at the retailer side.

The inventories of product i at the retailer side are (for details, refer to Equation (A3) in Appendix A):

$$\frac{1}{2}\left[\frac{H_i t_{3i\pi}}{n} + T_\pi(H_i - \lambda_i t_{3i\pi})\right] \tag{15}$$

$TC(T_\pi, n)$ for L distinct end products consists of the fixed and variable outsourcing and in-house fabrication costs, variable in-house rework and disposal costs, fixed and variable distribution costs, holding costs for reworked, finished, and defective items during T_π, and holding costs in the retailer side.

$$TC(T_\pi,\, n) = \sum_{i=1}^{L}\left\{\begin{array}{l} K_{\pi i} + (\pi_i Q_i)C_{\pi i} + K_i + (1-\pi_i)Q_i C_i + C_{\mathrm{R}i}x_i[(1-\pi_i)Q_i](1-\theta_{1i}) \\ +C_{\mathrm{S}i}\varphi_i x_i[(1-\pi_i)Q_i] + nK_{1i} + C_{\mathrm{T}i}Q_i[1-\varphi_i x_i(1-\pi_i)] + h_{1i}\frac{P_{2i}t_{2i\pi}}{2}(t_{2i\pi}) \\ +h_i\left[\frac{H_{1i}+d_{1i}t_{1i\pi}}{2}(t_{1i\pi}) + \frac{H_{1i}+H_{2i}}{2}(t_{2i\pi}) + \left(\frac{n-1}{2n}\right)H_i(t_{3i\pi})\right] \\ +\frac{h_{2i}}{2}\left[\frac{H_i t_{3i\pi}}{n} + T_\pi(H_i - \lambda_i t_{3i\pi})\right] \end{array}\right\} \tag{16}$$

Let β_{1i} be the linking factor between K_i and $K_{\pi i}$, and $K_{\pi i} = K_i\ (1 + \beta_{1i})$. Because the in-house setup cost K_i is often much greater than the fixed delivery cost $K_{\pi i}$, we assume that $-1 < \beta_{1i} < 0$. Also, let β_{2i} denote the linking factor between C_i and $C_{\pi i}$, and $C_{\pi i} = C_i\ (1 + \beta_{2i})$, since unit outsourcing price is more significant than unit in-house manufacturing cost, so we assume where $\beta_{2i} > 0$.

Apply expected values E[x_i] to deal with the randomness of x_i, substitute Equation (1) to (15) and the aforementioned linking parameters $K_{\pi i}$ and $C_{\pi i}$ in Equation (16), with extra derivations E[$TCU(T_\pi, n)$] are obtained as follows:

$$\begin{array}{c} E[TCU(T_\pi,n)] = \frac{E[TC(T_\pi,n)]}{E[T_\pi]} = \sum\limits_{i=1}^{L}\left\{\frac{K_i(1+\beta_{1i})}{T_\pi} + \frac{K_i}{T_\pi} + \frac{nK_{1i}}{T_\pi} + C_{Ti}\lambda_i\right\} \\ +\sum\limits_{i=1}^{L} E_{0i}\left\{\begin{array}{c} (1+\beta_{2i})C_i\pi_i + C_i(1-\pi_i) + C_{\mathrm{R}i}E_{2i} + C_{\mathrm{S}i}\varphi_i E[x_i](1-\pi_i) \\ +T_\pi E_{3i} + \frac{h_i T_\pi}{2E_{1i}}E_{4i} + \frac{h_{2i}\lambda_i T_\pi}{2}\left[\frac{(1-\pi_i)}{P_{1i}} + \frac{E_{2i}}{P_{2i}}\right] + T_\pi E_{5i} \end{array}\right\} \end{array} \tag{17}$$

where $E_{0i} = \frac{\lambda_i}{[1-\varphi_i E[x_i](1-\pi_i)]}$; $E_{1i} = [1-\varphi_i E[x_i](1-\pi_i)]$; $E_{2i} = E[x_i](1-\pi_i)(1-\theta_{1i})$

$$\begin{array}{l} E_{3i} = \frac{\lambda_i E[x_i]^2(1-\pi_i)^2(1-\theta_{1i})}{2P_{2i}E_{1i}}[h_{1i}(1-\theta_{1i}) - h_i]; \\ E_{4i} = \left[E_{1i}{}^2 + \frac{\lambda_i(1-\pi_i)}{P_{1i}}[\varphi_i E[x_i](1-\pi_i) - \pi_i] + \frac{\lambda_i E_{2i}}{P_{2i}}(1-2\pi_i)\right]; \\ E_{5i} = \frac{(h_{2i}-h_i)}{2}\left(\frac{1}{n}\right)\left[E_{1i} - \frac{\lambda_i(1-\pi_i)}{P_{1i}} - \frac{\lambda_i E_{2i}}{P_{2i}}\right]. \end{array}$$

2.3. Convexity and the Optimal Solution

Before deriving the optimal (T_π*, n*) solutions, we first verify that if E[$TCU(T_\pi, n)$] is convex. Applying the Hessian matrix equations (Rardin [34]), Equation (18) can be obtained (for details refer to Appendix B):

$$\begin{bmatrix} T_\pi & n \end{bmatrix} \cdot \begin{pmatrix} \frac{\partial^2 E[TCU(T_\pi,\ n)]}{\partial T_\pi{}^2} & \frac{\partial^2 E[TCU(T_\pi,\ n)]}{\partial T_\pi \partial n} \\ \frac{\partial^2 E[TCU(T_\pi,\ n)]}{\partial T_\pi \partial n} & \frac{\partial^2 E[TCU(T_\pi,\ n)]}{\partial n^2} \end{pmatrix} \cdot \begin{bmatrix} T_\pi \\ n \end{bmatrix} = 2\sum_{i=1}^{L}\left[\frac{K_i(1+\beta_{1i}) + K_i}{T_\pi}\right] > 0 \tag{18}$$

Equation (18) yields a positive result, since K_i, (1 + β_{1i}), and T_π are positive. Therefore, E[$TCU(T_\pi, n)$] is strictly convex for all n and T_π values other than zero, and a minimum for E[$TCU(T_\pi, n)$] exists.

In order to simultaneously decide T_π* and n*, we set the following first derivative of E[$TCU(T_\pi, n)$] concerning T_π and n both equal to zero, and then solve this linear system (i.e., Equations (19) and (20)).

$$\begin{array}{rl} \frac{\partial E[TCU(T_\pi,\ n)]}{\partial T_\pi} & = \sum\limits_{i=1}^{L}\left\{\frac{-K_i(1+\beta_{1i})}{T_\pi^2} - \frac{nK_{1i}}{T_\pi^2} - \frac{K_i}{T_\pi^2}\right\} \\ & +\sum\limits_{i=1}^{L} E_{0i}\left\{E_{3i} + \frac{h_i E_{4i}}{2E_{1i}} + \frac{h_{2i}\lambda_i}{2}\left[\frac{(1-\pi_i)}{P_{1i}} + \frac{E_{2i}}{P_{2i}}\right] + E_{5i}\right\} = 0 \end{array} \tag{19}$$

$$\frac{\partial E[TCU(T_\pi,\ n)]}{\partial n} = \sum_{i=1}^{L}\left[\frac{K_{1i}}{T_\pi}\right] + \sum_{i=1}^{L} E_{0i}\left\{-\left(\frac{1}{n^2}\right)\left[E_{1i} - \frac{\lambda_i(1-\pi_i)}{P_{1i}} - \frac{\lambda_i E_{2i}}{P_{2i}}\right]\frac{T_\pi(h_{2i}-h_i)}{2}\right\} = 0 \tag{20}$$

The following optimal T_π* and n* can be derived simultaneously with extra derivations:

$$T_\pi{}^* = \sqrt{\frac{2\sum\limits_{i=1}^{L}[K_i(2+\beta_{1i}) + nK_{1i}]}{\sum\limits_{i=1}^{L}\left\{E_{0i}\left[2E_{3i} + \frac{h_i E_{4i}}{E_{1i}} + h_{2i}\lambda_i\left[\frac{(1-\pi_i)}{P_{1i}} + \frac{E_{2i}}{P_{2i}}\right] + 2E_{5i}\right]\right\}}} \tag{21}$$

and

$$n* = \sqrt{\frac{\sum_{i=1}^{L}[K_i(2+\beta_{1i})] \cdot \sum_{i=1}^{L}\left\{(h_{2i}-h_i)E_{0i}\left[E_{1i} - \frac{\lambda_i(1-\pi_i)}{P_{1i}} - \frac{\lambda_i E_{2i}}{P_{2i}}\right]\right\}}{\sum_{i=1}^{L}(K_{1i}) \cdot \sum_{i=1}^{L}\left\{E_{0i}\left[2E_{3i} + h_{2i}\lambda_i\left[\frac{(1-\pi_i)}{P_{1i}} + \frac{E_{2i}}{P_{2i}}\right] + \frac{h_i E_{4i}}{E_{1i}}\right]\right\}}} \tag{22}$$

2.4. *The Prerequisite Condition of the Fabrication*

Sufficient capacity for the proposed multi-item fabrication and rework processes need to be guaranteed. Therefore, the following prerequisite formula must hold:

$$\sum_{i=1}^{L}\left[\left(\frac{(1-\pi_i)\lambda_i}{[1-\varphi_i E[x_i](1-\pi_i)]} \cdot \frac{1}{P_{1i}}\right) + \left(\frac{(1-\pi_i)\lambda_i E[x_i](1-\theta_{1i})}{[1-\varphi_i E[x_i](1-\pi_i)]} \cdot \frac{1}{P_{2i}}\right)\right] < 1 \tag{23}$$

If the summation of setup time S_i becomes significant to T_π, then Equation (24) also must hold:

$$\sum_{i=1}^{L}\left[S_i + \left(\frac{(1-\pi_i)Q_i}{P_{1i}}\right) + \left(\frac{(1-\pi_i)Q_i E[x_i](1-\theta_{1i})}{P_{2i}}\right)\right] < T \tag{24}$$

or, T_π must be larger than $T_{\min}$ as follows (refer to Appendix C for details):

$$T_\pi > \frac{\sum_{i=1}^{L}(S_i)}{1 - \sum_{i=1}^{L}\left[\left(\frac{(1-\pi_i)E_{0i}}{P_{1i}}\right) + \left(\frac{E_{0i}E_{2i}}{P_{2i}}\right)\right]} = T_{\min} \tag{25}$$

Therefore, when incorporating setup times into the proposed problem, one should select the maximum of T_π* (i.e., Equation (20)) or $T_{\min}$ (i.e., Equation (24)) (Nahmias [35]) as the optimal length.

3. Numerical Example

This section offers a simulated numerical example and the sensitivity analyses to illustrate our results' applicability. As exhibited in Table 2, these assumed parameters' values are for demonstration purposes.

Table 2. Values of system parameters of our example.

End Item No.	C_i	β_{2i}	$C_{\pi i}$	K_i	β_{1i}	$K_{\pi i}$	λ_i	π_i	P_{1i}	P_{2i}	
1	80	0.40	112.0	10,000	−0.60	4000	3000	0.4	58,000	2900	
2	90	0.35	121.5	11,000	−0.65	3850	3200	0.4	59,000	2950	
3	100	0.30	130.0	12,000	−0.70	3600	3400	0.4	60,000	3000	
4	110	0.25	137.5	13,000	−0.75	3250	3600	0.4	61,000	3050	
5	120	0.20	144.0	14,000	−0.80	2800	3800	0.4	62,000	3100	
End Item No.	x_i	C_{Ri}	C_{Si}	K_{1i}	C_{Ti}	h_i	h_{1i}	h_{2i}	θ_{1i}	θ_{2i}	φ_i
1	5%	50	20	2300	0.1	10	30	50	0.05	0.05	0.0975
2	10%	55	25	2400	0.2	15	35	55	0.10	0.10	0.1900
3	15%	60	30	2500	0.3	20	40	60	0.15	0.15	0.2775
4	20%	65	35	2600	0.4	25	45	65	0.20	0.20	0.3600
5	25%	70	40	2700	0.5	30	50	70	0.25	0.25	0.4375

3.1. *Optimal Cycle Time, Deliveries, and Critical Managerial Information*

Using the assumed values of variables (as shown in Table 2) to calculate Equations (21), (22) and (17), we obtain n* = 3, T_π* = 0.5982, $E[TCU(T_\pi*, n*)]$ = $2,390,389 (for $\overline{\pi}$ at 0.4; see Table A1 in

Appendix D). It is noted that the cost for quality guarantee in the proposed system is $70,423, that is 2.95% of $E[TCU(T_\pi{}^*, n^*)]$ (see Table A1 in Appendix D).

The effect of variations in average unit cost linking parameter $\overline{\beta_2}$ on total cost of each end product is analyzed, and its outcome is depicted in Figure 6. It indicates that as $\overline{\beta_2}$ increases, each item's total cost goes up accordingly because unit outsourcing cost is higher than unit in-house manufacturing cost.

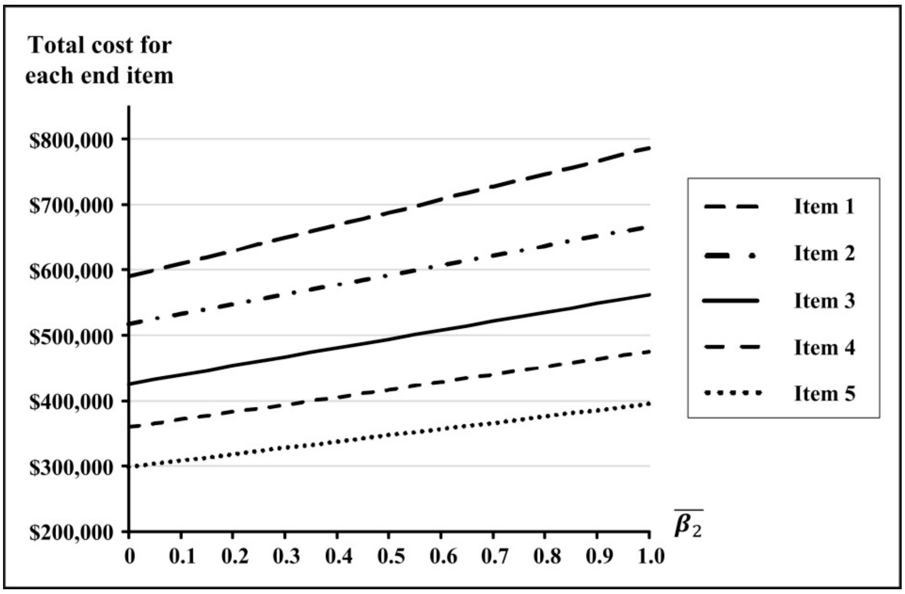

Figure 6. Effect of differences in $\overline{\beta_2}$ on each item's total cost.

Figure 7 exhibits the impact of differences in the average setup cost linking parameter $\overline{\beta_1}$ on the optimal average system costs $E[TCU(T_\pi{}^*, n^*)]$. It specifies that $E[TCU(T_\pi{}^*, n^*)]$ declines as $\overline{\beta_1}$ decreases because total outsourcing setup costs are reduced.

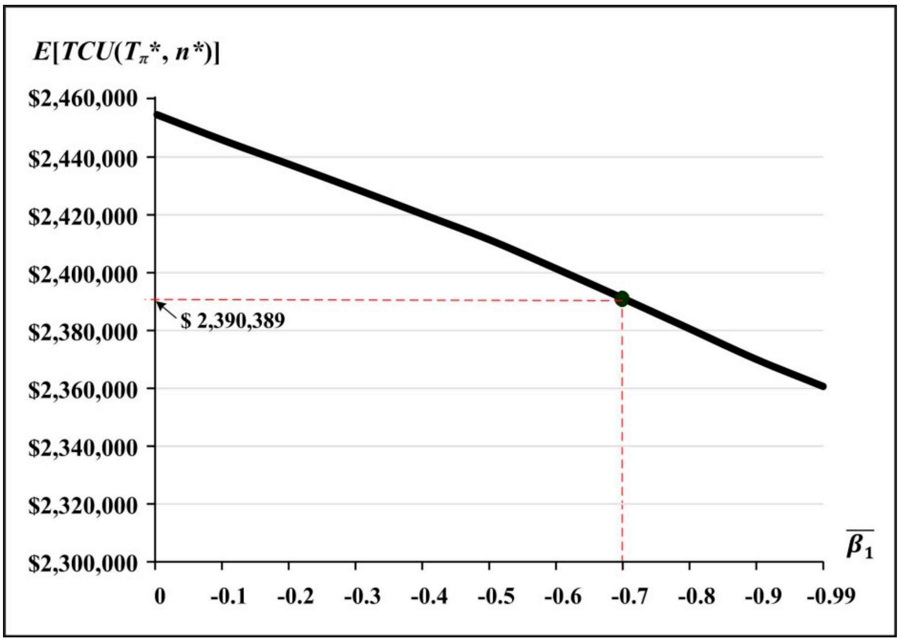

Figure 7. Impact of differences in $\overline{\beta_1}$ on $E[TCU(T_\pi{}^*, n^*)]$

The effect of variations in rotation cycle time T_π on different cost contributors of $E[TCU(T_\pi, n)]$ is explored and the outcomes are illustrated in Figure 8. It is noted that as cycle time T_π increases, the cost for quality assurance goes up accordingly, and both holding costs at customer and producer sides increase significantly. Conversely, annual delivery costs, and both in-house and outsourcing setup costs decrease notably. In Figure 8, as the cycle length T_π varies, the expected annual variable cost (λC_i) changes slightly. Because (λC_i) represents the annual variable cost, it is not directly/significantly affected by lot-size Q_i or cycle-time T_π.

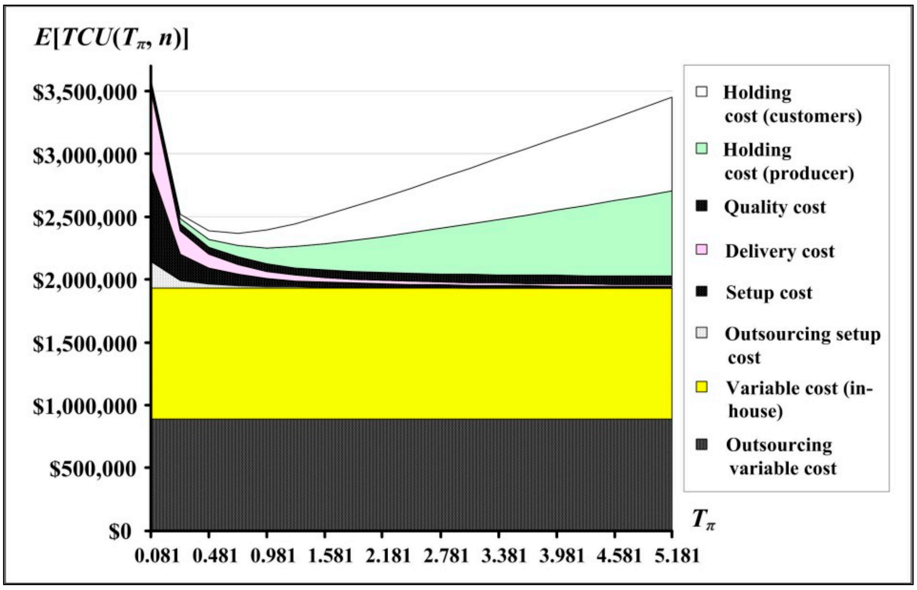

Figure 8. Effect of variations in T_π on different cost contributors of $E[TCU(T_\pi, n)]$.

Figure 9 displays the impact of changes in average outsourcing percentage $\overline{\pi}$ on each item's total cost. It reveals that as $\overline{\pi}$ becomes higher, each item's total cost increases accordingly, for outsourcing is a more expensive stock-replenishment policy.

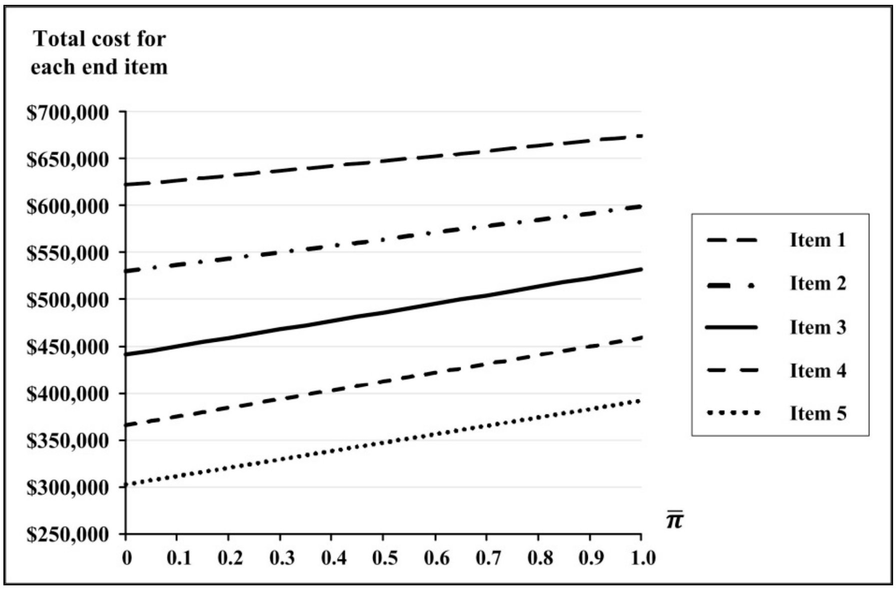

Figure 9. Effect of variations in $\overline{\pi}$ on each item's total cost.

The impact of differences in average outsourcing percentage $\overline{\pi}$ on overall machine utilization for multi-item manufacturing processes is studied, and the outcome is depicted in Figure 10. It shows that machine utilization declines significantly as $\overline{\pi}$ increases; and at $\overline{\pi}$ = 0.4 (in our example), machine utilization drops from 65.8% (refer to Table A2 in Appendix D) to 39.0%. This utilization drop is at the cost of 6.75% increase in $E[TCU(T_{\pi}*, n*)]$ (for the system cost goes up from $2,239,231 to $2,390,389, refer to Table A1). Moreover, Table A2 reveals real production uptime, rework time, and idle time.

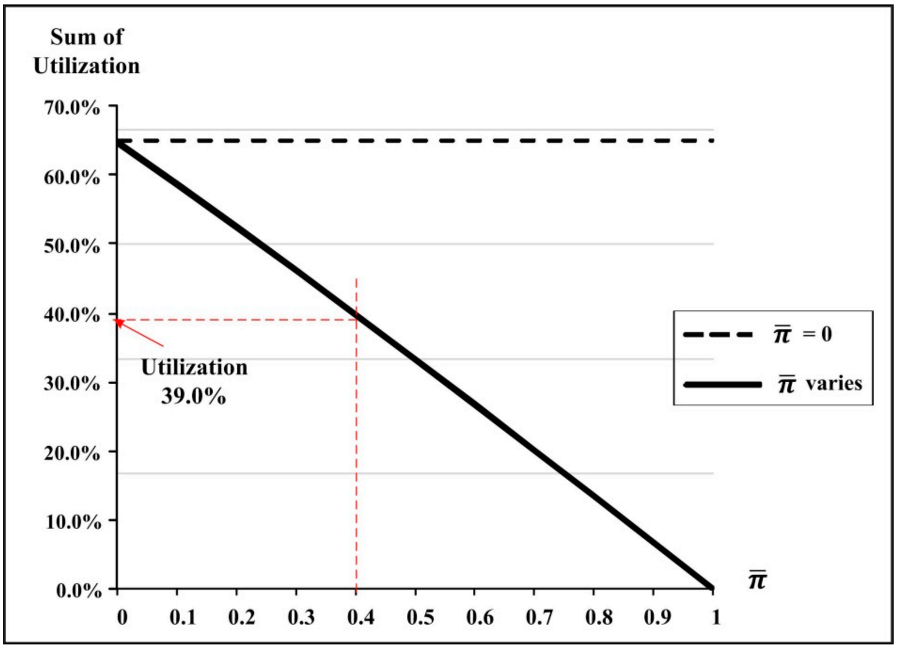

Figure 10. Impact of differences in $\overline{\pi}$ on overall machine utilization.

Moreover, our proposed model can reveal the critical ratio of $\overline{\pi}$ to support the make-or-buy decision (see Figure 11). It shows that as $\overline{\pi}$ goes up to 0.702 or higher, a 100% outsourcing option is more cost-effective (for details, please refer to Table A2, in Appendix D).

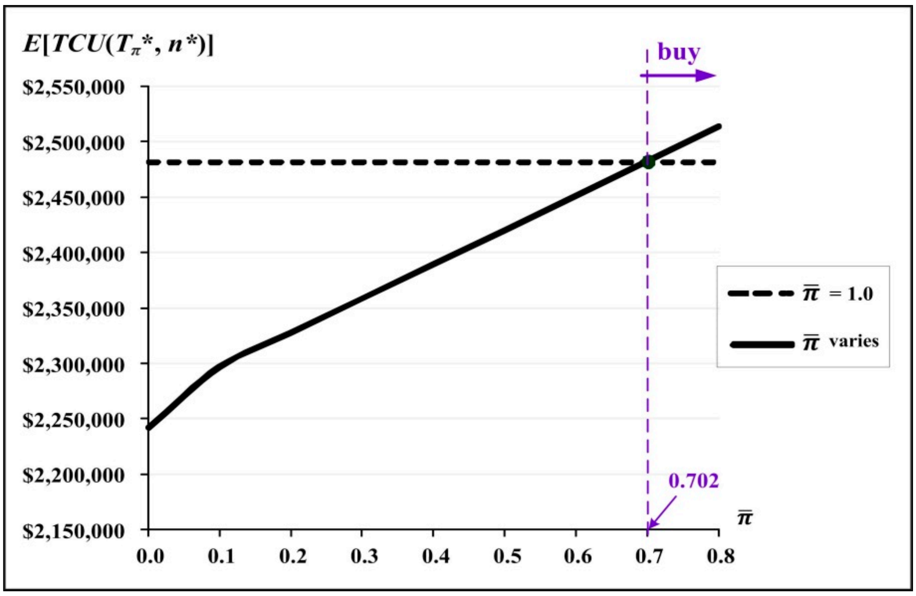

Figure 11. Effect of variations in $\overline{\pi}$ on $E[TCU(T_{\pi}*, n*)]$ for managerial make-or-buy decision.

Furthermore, Figure 12 illustrates the impact of changes in average scrap rate $\overline{\varphi}$ on $E[TCU(T_{\pi}*, n*)]$. It specifies that as $\overline{\varphi}$ rises, $E[TCU(T_{\pi}*, n*)]$ increases considerably, and at $\overline{\pi}$ = 0.4 and $\overline{x}$ = 0.3, if $\overline{\varphi}$ increases to 0.349 or higher, a 100% outsourcing plan (i.e., the 'buy' decision) are more economical.

Additionally, another critical ratio $\overline{\varphi}$ = 0.550 (at $\overline{\pi}$ = 0 and $\overline{\pi}$ = 0.3) is revealed by the proposed model to support managerial decision making. That is if the average scrap rate $\overline{\varphi}$ is greater than 0.550, then 'buy' is recommended.

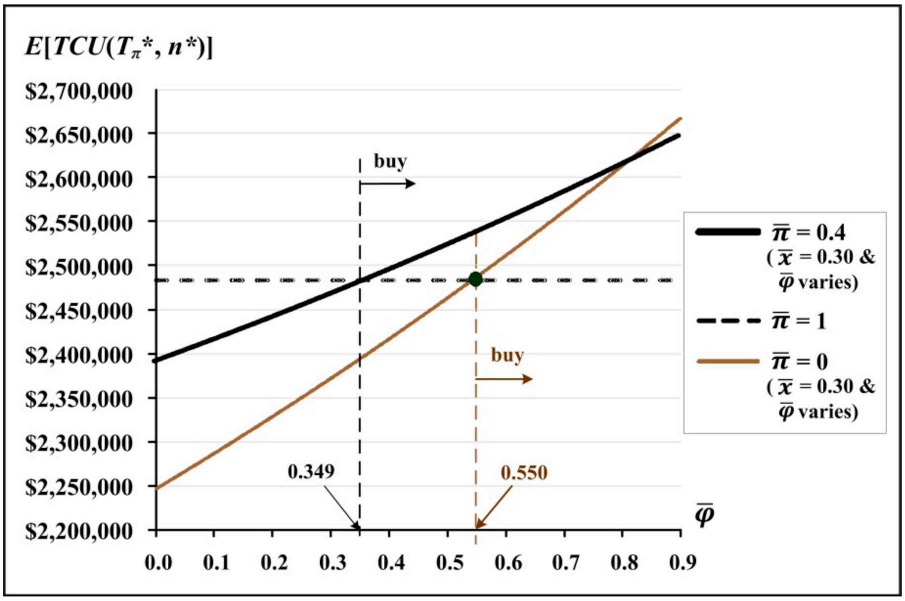

Figure 12. Impact of changes in $\overline{\varphi}$ on $E[TCU(T_\pi^*, n^*)]$ for managerial decision makings.

3.2. Joint Impacts from Combined System Factors

Looking into the quality guarantee matter in manufacturing processes, joint impacts of variations in average scrap rate $\overline{\varphi}$ and average defective rate $\overline{x}$ on $E[TCU(T_\pi^*, n^*)]$ are investigated. The results are presented in Figure 13. This specifies that $E[TCU(T_\pi^*, n^*)]$ raises drastically, as both $\overline{x}$ and $\overline{\varphi}$ increase.

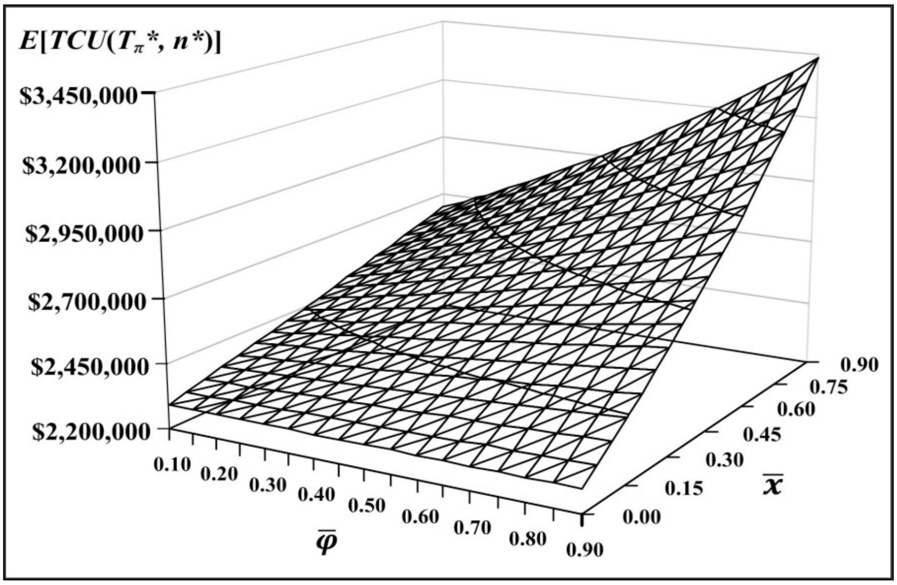

Figure 13. Joint impacts of changes in $\overline{\varphi}$ and $\overline{\pi}$ on $E[TCU(T_\pi^*, n^*)]$.

Figure 14 shows the analytical result of the joint effects of changes in rotation cycle time T_π and average unit cost linking parameter $\overline{\beta_2}$ on $E[TCU(T_\pi, n)]$. It indicates that $E[TCU(T_\pi, n)]$ raises considerably, as $\overline{\beta_2}$ goes up; and when T_π deviates from its optimal value 0.5982, $E[TCU(T_\pi, n)]$ increases significantly. Furthermore, the joint impacts of changes in $\overline{\pi}$ and $\overline{\varphi}$ on $E[TCU(T_\pi^*, n^*)]$ is analyzed, and the outcome is presented in Figure 15. It can be seen that (1) when $\overline{\pi}$ is smaller than 0.5, $E[TCU(T_{\pi}^{*}, n^{*})]$ boosts up considerably, as $\overline{\varphi}$ increases and; (2) quite the opposite, when $\overline{\varphi} > 0.65$, $E[TCU(T_{\pi}^{*}, n^{*})]$ declines notably, as $\overline{\pi}$ increases. However, (3) when $\overline{\varphi} < 0.4$, as $\overline{\pi}$ goes up, $E[TCU(T_{\pi}^{*}, n^{*})]$ increases accordingly.

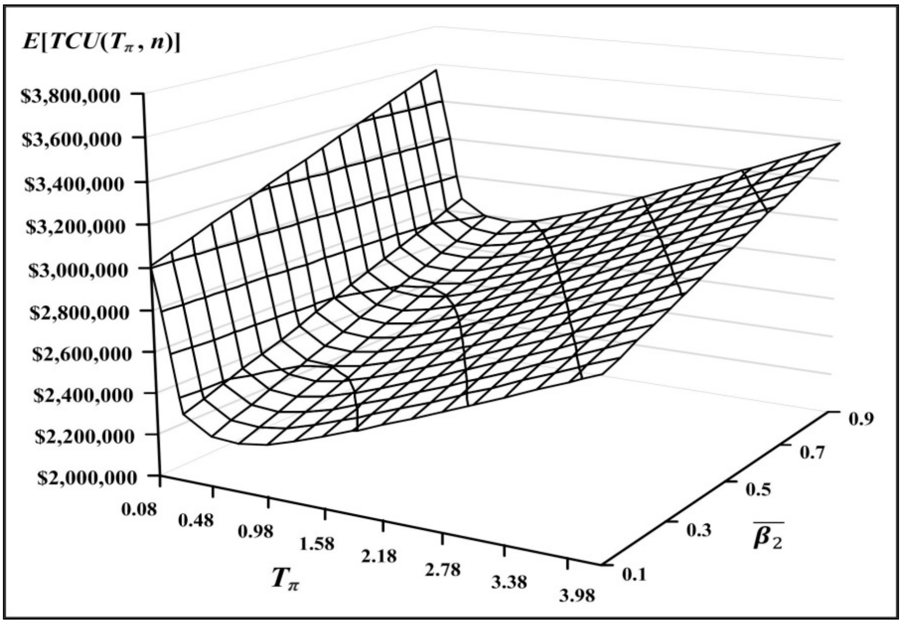

Figure 14. Joint effects of differences in T_{π} and $\overline{\beta_2}$ on $E[TCU(T_{\pi}, n)]$.

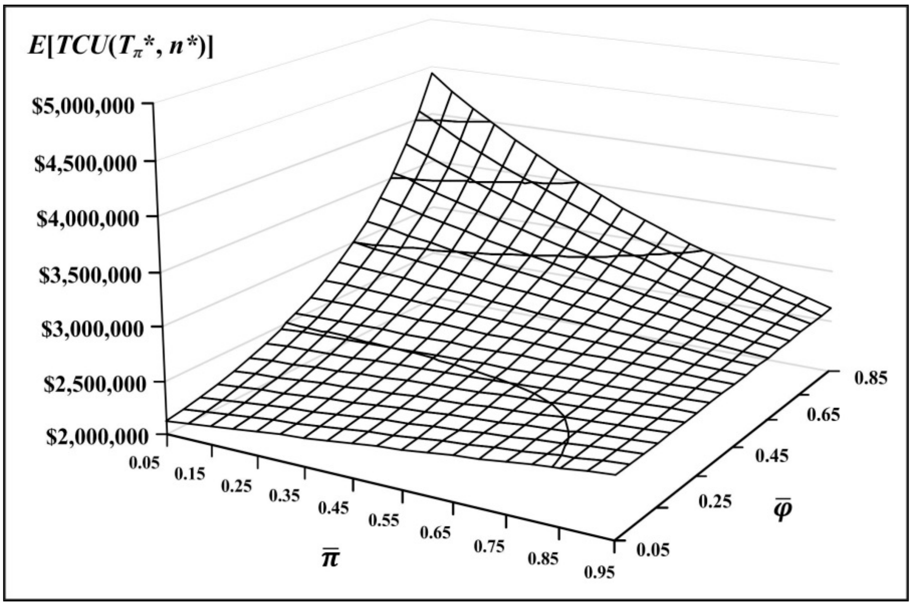

Figure 15. Joint impacts of changes in $\overline{\pi}$ and $\overline{\varphi}$ on $E[TCU(T_{\pi}^{*}, n^{*})]$.

The reasons for the situations mentioned above are as follows: (1) if the amount of outsourced items is less than that of in-house manufactured items, the impact of $\overline{\varphi}$ on $E[TCU(T_{\pi}^{*}, n^{*})]$ is significant; (2) in contrast, although $\overline{\varphi}$ is high, when the outsourced amount is much larger than in-house made amount, $E[TCU(T_{\pi}^{*}, n^{*})]$ decreases notably, as $\overline{\pi}$ goes up; and (3) the impact from $\overline{\varphi}$ (in terms of the in-house quality cost) does not exceed that from $\overline{\pi}$ (in terms of outsourcing added cost), hence, as $\overline{\pi}$ increases, $E[TCU(T_{\pi}^{*}, n^{*})]$ still raises accordingly.

4. Conclusions

To meet the client's multi-item and quality requirements under capacity constraints and to satisfy the timely distribution of finished goods under minimum system cost, a multi-item manufacturer–retailer integrated type of system incorporating outsourcing and quality guarantee is explored. All in-house fabricated/reworked products are screened to make sure of the desired quality, whereas the external provider guarantees the quality of outsourced items. A fixed-quantity multi-shipment plan starts when the entire lot is quality-ensured. Accordingly, we build a precise model to portray the system characteristics and use mathematical derivation and optimization approach to obtain the total system cost and optimal policy (in terms of common cycle time and frequency of deliveries).

This study's main contribution is that we developed a decision support model (please refer to Section 2) to enable production managers to explore such a specific multiproduct manufacturer–retailer coordination problem featuring outsourcing strategy and product quality assurance. Using the proposed optimization techniques, managers can simultaneously find the optimal delivery frequency and common cycle time for the problem (please see Section 2.3). This helps the managers in making efficient and cost-effective multiproduct manufacturing and delivering decisions. By taking advantage of our results, the diverse individual and collective impact of variations in a system's features on the proposed problem can now be revealed to facilitate managerial decision-making. For instance: (1) The effect of variations in outsourcing proportion, setup cost, or unit outsourcing add-up expense on the optimal operating policy, individual cost of each end product, utilization, and the total system cost (see Figures 6, 7, 9 and 10). (2) The impact of changes in the optimal cycle time on each cost contributors and the total system cost (refer to Figure 8). (3) The make-or-buy decision relating information based on outsourcing proportion or total in-house scrap rate (refer to Figures 11 and 12). (4) The collective influence of differences in system features on the total system cost (see Figures 13–15). The limitations of this study concerning fabrication capacity and setup times for producing multiproduct are shown in Equations (23) and (25) (refer to Section 2.4). Future studies may examine the influence of random demand or another uptime-reduction strategy, such as the expedited fabrication rate on the system's optimal operating policy. The results obtained can also be compared/verified with the results from artificial neural networks.

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Appendix A

Detailed calculations of Equation (15) are as follows:

Referring to Figure 5, the total inventories in the retailer's side are as follows:

$$\begin{aligned} &\left[\left(\frac{D_i+I_i}{2}\right)t_{\mathrm{n}i\pi}\right]+\left[\frac{(D_i+I_i)+[(D_i+I_i)-\lambda_i t_{\mathrm{n}i\pi}]}{2}(t_{\mathrm{n}i\pi})\right]+\left[\frac{(D_i+2I_i)+[(D_i+2I_i)-\lambda_i t_{\mathrm{n}i\pi}]}{2}(t_{\mathrm{n}i\pi})\right] \\ &+\ldots+\left[\frac{[D_i+(n-1)I_i]+[[D_i+(n-1)I_i]-\lambda_i t_{\mathrm{n}i\pi}]}{2}(t_{\mathrm{n}i\pi})\right]+\left(\frac{nI_i}{2}\right)(t_{1i\pi}+t_{2i\pi}) \end{aligned} \tag{A1}$$

or

$$\begin{aligned} \text{Total inventories} &= \Big(D_i - \tfrac{\lambda_i}{2} t_{\mathrm{n}i\pi}\Big) t_{\mathrm{n}i\pi} + \Big(D_i + I_i - \tfrac{\lambda_i}{2} t_{\mathrm{n}i\pi}\Big) t_{\mathrm{n}i\pi} + \Big(D_i + 2I_i - \tfrac{\lambda_i}{2} t_{\mathrm{n}i\pi}\Big) t_{\mathrm{n}i\pi} + \ldots \\ &+ \Big(D_i + (n-1)I_i - \tfrac{\lambda_i}{2} t_{\mathrm{n}i\pi}\Big) t_{ni\pi} + \Big(\tfrac{nI_i}{2}\Big)(t_{1i\pi} + t_{2i\pi}) \\ &= n\Big(D_i - \tfrac{\lambda_i}{2} t_{\mathrm{n}i\pi}\Big) t_{\mathrm{n}i\pi} + \tfrac{n(n-1)}{2} I t_{\mathrm{n}i\pi} + \tfrac{nI_i}{2}(t_{1i\pi} + t_{2i\pi}) \end{aligned} \tag{A2}$$

Substitute Equation (12) to (14) in Equation (A2) and with extra derivations, we have the total inventories as follows (i.e., Equation (15)):

$$\begin{aligned} &\text{Total inventories} = n\Big(\tfrac{H_i}{n} - \tfrac{\lambda_i}{2} t_{\mathrm{n}i\pi}\Big) t_{\mathrm{n}i\pi} + \tfrac{n(n-1)}{2}\Big(\tfrac{H_i}{n} - \lambda_i t_{\mathrm{n}i\pi}\Big) t_{\mathrm{n}i\pi} + \tfrac{n}{2}\Big(\tfrac{H_i}{n} - \lambda_i t_{\mathrm{n}i\pi}\Big)(t_{1i\pi} + t_{2i\pi}) \\ &= H_i t_{\mathrm{n}i\pi} - \tfrac{n\lambda_i}{2} t^2_{\mathrm{n}i\pi} + H_i t_{\mathrm{n}i\pi} \tfrac{(n-1)}{2} - \tfrac{n(n-1)}{2} \lambda_i t^2_{\mathrm{n}i\pi} + \tfrac{H_i}{2}(t_{1i\pi} + t_{2i\pi}) - \tfrac{n}{2}(\lambda_i t_{\mathrm{n}i\pi})(t_{1i\pi} + t_{2i\pi}) \\ &= \tfrac{H_i t_{3i\pi}}{n} - \tfrac{\lambda_i t^2_{3i\pi}}{2n} + \tfrac{H_i (n-1) t_{3i\pi}}{2n} - \tfrac{(n-1)\lambda_i t^2_{3i\pi}}{2n} + \tfrac{H_i}{2}(t_{1i\pi} + t_{2i\pi}) - \tfrac{\lambda_i t_{3i\pi}}{2}(t_{1i\pi} + t_{2i\pi}) \\ &= \tfrac{1}{2}\Big[\tfrac{H_i t_{3i\pi}}{n} + T_\pi (H_i - \lambda_i t_{3i\pi})\Big] \end{aligned} \tag{A3}$$

Appendix B

From Equation (17) the following can be obtained (Rardin [34]):

$$\begin{aligned} \frac{\partial E[TCU(T_\pi, n)]}{\partial T_\pi} &= \sum_{i=1}^{L} \Big\{ \tfrac{-K_i(1+\beta_{1i})}{T_\pi^2} - \tfrac{K_i}{T_\pi^2} - \tfrac{nK_{1i}}{T_\pi^2} \Big\} \\ &+ \sum_{i=1}^{L} E_{0i} \Big\{ E_{3i} + \tfrac{h_i E_{4i}}{2E_{1i}} + \tfrac{h_{2i}\lambda_i}{2}\Big[\tfrac{(1-\pi_i)}{P_{1i}} + \tfrac{E_{2i}}{P_{2i}}\Big] + E_{5i} \Big\} \end{aligned} \tag{A4}$$

$$\frac{\partial^2 E[TCU(T_\pi, n)]}{\partial {T_\pi}^2} = \sum_{i=1}^{L} 2\left[\frac{K_i(1+\beta_{1i}) + (K_i + nK_{1i})}{T_\pi^3}\right] \tag{A5}$$

$$\frac{\partial E[TCU(T_\pi, n)]}{\partial n} = \sum_{i=1}^{L}\left[\frac{K_{1i}}{T_\pi}\right] + \sum_{i=1}^{L} E_{0i}\left\{\frac{-T_\pi (h_{2i} - h_i)}{2}\left(\frac{1}{n^2}\right)\left[E_{1i} - \frac{\lambda_i(1-\pi_i)}{P_{1i}} - \frac{\lambda_i E_{2i}}{P_{2i}}\right]\right\} \tag{A6}$$

$$\frac{\partial^2 E[TCU(T_\pi, n)]}{\partial n^2} = \sum_{i=1}^{L} E_{0i}\left\{T_\pi (h_{2i} - h_i)\left(\frac{1}{n^3}\right)\left[E_{1i} - \frac{\lambda_i(1-\pi_i)}{P_{1i}} - \frac{\lambda_i E_{2i}}{P_{2i}}\right]\right\} \tag{A7}$$

$$\frac{\partial E[TCU(T_\pi, n)]}{\partial T_\pi \partial n} = \sum_{i=1}^{L}\left[\frac{-K_{1i}}{T_\pi^2}\right] + \sum_{i=1}^{L} E_{0i}\left\{\frac{-(h_{2i} - h_i)}{2}\left(\frac{1}{n^2}\right)\left[E_{1i} - \frac{\lambda_i(1-\pi_i)}{P_{1i}} - \frac{\lambda_i E_{2i}}{P_{2i}}\right]\right\} \tag{A8}$$

Substitute Equations (A5), (A7) and (A8) in the following Hessian matrix equations and, with extra derivation, we obtain Equation (A9) or Equation (18), as shown in the text.

$$\begin{bmatrix} T_\pi & n \end{bmatrix} \cdot \begin{pmatrix} \frac{\partial^2 E[TCU(T_\pi, n)]}{\partial {T_\pi}^2} & \frac{\partial^2 E[TCU(T_\pi, n)]}{\partial T_\pi \partial n} \\ \frac{\partial^2 E[TCU(T_\pi, n)]}{\partial T_\pi \partial n} & \frac{\partial^2 E[TCU(T_\pi, n)]}{\partial n^2} \end{pmatrix} \cdot \begin{bmatrix} T_\pi \\ n \end{bmatrix} = 2\sum_{i=1}^{L}\left[\frac{K_i(1+\beta_{1i}) + K_i}{T_\pi}\right] > 0 \tag{A9}$$

Appendix C

Detailed derivations of $T_{\min}$ in Equation (25) are presented as follows. From Equation (24) we have:

$$\sum_{i=1}^{L}\left\{\left[S_i + \left(\frac{(1-\pi_i)Q_i}{P_{1i}}\right) + \left(\frac{(1-\pi_i)Q_i E[x_i](1-\theta_{1i})}{P_{2i}}\right)\right]\right\} < T \tag{A10}$$

Since $Q_i = \lambda_i T_\pi [1 - \varphi_i E[x_i](1-\pi_i)]^{-1}$ (i.e., Equation (5)), so Equation (A10) becomes:

$$\sum_{i=1}^{L}(S_i) + T\sum_{i=1}^{L}\left[\left(\frac{(1-\pi_i)\lambda_i}{[1-\varphi_i E[x_i](1-\pi_i)]P_{1i}}\right) + \left(\frac{(1-\pi_i)\lambda_i E[x_i](1-\theta_{1i})}{[1-\varphi_i E[x_i](1-\pi_i)]P_{2i}}\right)\right] < T \tag{A11}$$

or

$$\sum_{i=1}^{L}(S_i) < T\left\{1-\sum_{i=1}^{L}\left[\left(\frac{(1-\pi_i)E_{0i}}{P_{1i}}\right)+\left(\frac{E_{0i}E_{2i}}{P_{2i}}\right)\right]\right\} \tag{A12}$$

or, the fabrication cycle length T must be greater than $T_{\min}$, as shown in Equation (25) or (A13) as follows:

$$T > \frac{\sum\limits_{i=1}^{L}(S_i)}{1-\sum\limits_{i=1}^{L}\left[\left(\frac{(1-\pi_i)E_{0i}}{P_{1i}}\right)+\left(\frac{E_{0i}E_{2i}}{P_{2i}}\right)\right]} = T_{\min} \tag{A13}$$

Appendix D

Table A1. Effects of differences in $\overline{\pi}$ on distinct cost categories in the proposed system.

$\overline{\pi}$	n^*	$T_\pi{}^*$	Annual System Cost E[$TCU(T_\pi{}^*, n^*)$] (1)	Outsourcing Relating Cost (2)	% (2)/(1)	Quality Guarantee Cost (3)	% (3)/(1)	Delivery Cost (4)	% (4)/(1)	Customer Holding Cost (5)	% (5)/(1)	Other In-House Cost (6)	% (6)/(1)
0.00	3	0.5638	$2,239,231	$0	0%	$140,680	6.28%	$71,818	3.20%	$123,738	5.52%	$2,028,712	90.52%
0.05	3	0.5684	$2,286,723	$144,275	6.31%	$130,833	5.72%	$71,272	3.12%	$123,358	5.39%	$1,816,985	79.46%
0.10	3	0.5730	$2,301,276	$257,184	11.18%	$121,294	5.27%	$70,745	3.07%	$122,941	5.34%	$1,729,111	75.14%
0.15	3	0.5775	$2,315,912	$369,772	15.97%	$112,062	4.84%	$70,237	3.03%	$122,486	5.29%	$1,641,354	70.87%
0.20	3	0.5819	$2,330,633	$482,042	20.68%	$103,134	4.43%	$69,749	2.99%	$121,992	5.23%	$1,553,716	66.66%
0.25	3	0.5861	$2,345,440	$593,995	25.33%	$94,508	4.03%	$69,280	2.95%	$121,458	5.18%	$1,466,199	62.51%
0.30	3	0.5903	$2,360,334	$705,634	29.90%	$86,182	3.65%	$68,831	2.92%	$120,884	5.12%	$1,378,803	58.42%
0.35	3	0.5943	$2,375,317	$816,961	34.39%	$78,154	3.29%	$68,402	2.88%	$120,268	5.06%	$1,291,532	54.37%
0.40	**3**	**0.5982**	**$2,390,389**	$927,977	38.82%	**$70,423**	**2.95%**	$67,992	2.84%	$119,611	5.00%	$1,204,385	50.38%
0.45	3	0.6019	$2,405,551	$1,038,685	43.18%	$62,985	2.62%	$67,603	2.81%	$118,912	4.94%	$1,117,365	46.45%
0.50	3	0.6055	$2,420,805	$1,149,087	47.47%	$55,840	2.31%	$67,235	2.78%	$118,170	4.88%	$1,030,473	42.57%
0.55	3	0.6089	$2,436,150	$1,259,185	51.69%	$48,984	2.01%	$66,886	2.75%	$117,386	4.82%	$943,708	38.74%
0.60	3	0.6122	$2,451,588	$1,368,982	55.84%	$42,416	1.73%	$66,558	2.71%	$116,558	4.75%	$857,074	34.96%
0.65	3	0.6152	$2,467,120	$1,478,478	59.93%	$36,135	1.46%	$66,251	2.69%	$115,688	4.69%	$770,569	31.23%
0.70	3	0.6182	**$2,482,746**	$1,587,676	63.95%	$30,137	1.21%	$65,964	2.66%	$114,775	4.62%	$684,194	27.56%
0.75	3	0.6209	$2,498,466	$1,696,577	67.90%	$24,421	0.98%	$65,698	2.63%	$113,819	4.56%	$597,950	23.93%
0.80	3	0.6234	$2,514,280	$1,805,185	71.80%	$18,985	0.76%	$65,453	2.60%	$112,820	4.49%	$511,837	20.36%
0.85	3	0.6257	$2,530,190	$1,913,500	75.63%	$13,827	0.55%	$65,228	2.58%	$111,780	4.42%	$425,854	16.83%
0.90	3	0.6279	$2,546,195	$2,021,525	79.39%	$8,945	0.35%	$65,024	2.55%	$110,698	4.35%	$340,002	13.35%
0.95	3	0.6298	$2,562,294	$2,129,261	83.10%	$4,336	0.17%	$64,841	2.53%	$109,576	4.28%	$254,279	9.92%
1.00	3	0.6315	**$2,483,483**	$2,236,710	90.06%	$0	0%	$64,679	2.60%	$108,415	4.37%	$73,680	2.97%

Table A2. Impacts of changes in $\overline{\pi}$ on sum of uptime, rework time, and utilization.

$\overline{\pi}$	n^*	$T_\pi{}^*$	Sum of Manufacture–ing Uptime (in Year)	Sum of Rework Time (in Year)	Machine Idle Time Per Cycle (in Year)	Utilization (Uptime) (A)	Utilization (Rework Time) (B)	Total Utilization (A) + (B)
0.00	3	0.5638	0.1638	0.2070	0.1930	0.291	0.367	**0.658**
0.05	3	0.5684	0.1567	0.1980	0.2137	0.276	0.348	0.624
0.10	3	0.5730	0.1494	0.1887	0.2349	0.261	0.329	0.590
0.15	3	0.5775	0.1420	0.1793	0.2562	0.246	0.310	0.556
0.20	3	0.5819	0.1345	0.1698	0.2776	0.231	0.292	0.523
0.25	3	0.5861	0.1269	0.1600	0.2992	0.217	0.273	0.490
0.30	3	0.5903	0.1191	0.1502	0.3210	0.202	0.254	0.456
0.35	3	0.5943	0.1112	0.1401	0.3430	0.187	0.236	0.423
0.40	**3**	**0.5982**	0.1032	0.1300	0.3650	0.173	0.217	**0.390**
0.45	3	0.6019	0.0950	0.1197	0.3872	0.158	0.199	0.357
0.50	3	0.6055	0.0868	0.1093	0.4094	0.143	0.181	0.324
0.55	3	0.6089	0.0784	0.0987	0.4318	0.129	0.162	0.291
0.60	3	0.6122	0.0700	0.0881	0.4541	0.114	0.144	0.258
0.65	3	0.6152	0.0615	0.0773	0.4764	0.100	0.126	0.226
0.70	3	0.6182	0.0529	0.0665	0.4988	0.086	0.108	0.193
0.75	3	0.6209	0.0442	0.0556	0.5211	0.071	0.090	0.161
0.80	3	0.6234	0.0355	0.0446	0.5433	0.057	0.072	0.128
0.85	3	0.6257	0.0267	0.0335	0.5655	0.043	0.054	0.096
0.90	3	0.6279	0.0178	0.0224	0.5877	0.028	0.036	0.064
0.95	3	0.6298	0.0089	0.0112	0.6097	0.014	0.018	0.032
1.00	3	0.6315	0.0000	0.0000	0.6315	0.000	0.000	0.000

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