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Editorial

Graph-Theoretic Problems and Their New Applications

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Received: 15 March 2020; Accepted: 16 March 2020; Published: 19 March 2020

Graph theory is an important area of Applied Mathematics with a broad spectrum of applications in many fields. In the Call for Papers for this issue, I asked for submissions presenting new and inoovative approaches for traditional graph-theoretic problems as well as for new applications of graph theory in emerging fields, such as network security, computer science and data analysis, bioinformatics, operations research, engineering and manufacturing, physics and chemistry, linguistics, or social sciences.

In response to the Call for Papers, we had an enormous resonance, and altogether 151 submissions have been received among which finally 20 papers have been accepted for this special issue, all of which are of high quality, reflecting the great interest in the area of Graph Theory. This corresponds to an acceptance rate of 13.2%. The authors of these accepted publications come from 13 different countries: USA, China, Pakistan, India, Iran, Marocco, Slovenia, United Arab Emirates, Oman, Spain, Mexico, Serbia, and Belarus, where most authors are from the first two countries.

All submissions have been reviewed, as a rule, by at least three experts in the field of Graph Theory. Subsequently, all published papers in this special issue are briefly surveyed in increasing order of their publication dates. We hope that the readers will find interesting theoretical ideas in this special issue and that researchers will find new inspirations for future works. Since also for my previous special issue 'Discrete Optimization: Theory, Algorithms, and Applications' a lot of graph-theoretic works have been submitted, I would like to remind the readership on the follow-up issue 'Novel Approaches for Discrete Optimization Problems' with a submission deadline of May 31, 2020, where also works related to graphs and networks are mentioned in the Call for Papers.

The first accepted paper by Tilley [1] is related to the 4-color theorem which has been proven by showing that a minimal counterexample does not exist. Here the author proves that a minimum counterexample must also satisfy a particular coloring property which he denotes as Kempe–Locking. However, the main intention of this paper is not an alternative proof of the 4-color theorem but an exploratory paper aimed at gaining a deeper understanding of why the 4-color theorem is true and a new approach to understand why planar graphs are 4-colorable by investigating whether the connectivity and coloring properties are compatible.

Zhang et al. [2] consider so-called generalized hypergraphs H denoted as r-uniform if all the hyperedges have the same cardinality r. Such a graph is called a generalized hypertree GHT, if after removing any hyperedge E, $GHT - E$ has exactly k components with $2 \le k \le r$. Focusing first on the case $k = 2$, they determine bounds on the number of edges. In particular, the authors show that an r-uniform generalized GHT on n vertices has at least $2n/(r+1)$ edges and at most $n - r + 1$ edges if $r \ge 3, n \ge 3$ and that the lower and upper bounds on the edge number are tight. Finally, the case of a fixed value $k \le r - 1$ is also discussed.

Wang et al. [3] deal with the matrix $A_\alpha(G) = \alpha D(G) + (1-\alpha)A(G)$ with $\alpha \in [0,1]$, introduced by Nikiforov [4], where $A(G)$ is the adjacent matrix and $D(G)$ is the resulting diagonal matrix of the degrees of a graph G. They determine the graphs with largest $A_\alpha(G)$-spectral radius with fixed vertex or edge connectivity. In addition, the corresponding extremal graphs are given and equations satisfying the $A_\alpha(G)$-spectral radius are derived.

Jan et al. [5] deal with fuzzy graphs. The goal of this paper is to show that there are some serious flaws in the existing definitions of several root-level generalized fuzzy graph structures with the help of some counterexamples. To achieve this, first, we aim to improve the existing definition for interval-valued fuzzy graphs, interval-valued intuitionistic fuzzy graphs and their complements. The authors also point out that a single-valued neutrosophic graph is not well defined in the literature by illustrative examples and present then a new definition and an application of such graphs in decision making.

Wang et al. [6] deal with the k-rainbow domination number which is the minimum weight of a k-rainbow dominating function. In particular, they determine this domination number of the grid graph $P_3 \square P_n$ for $k \in \{2, 3, 4\}$ and all n, where P_m is a path of order m.

Ye et al. [7] consider the Cartesian product of directed circles C_m and C_n of length m and n, respectively $(n \geq m \geq 3)$. In this paper, the authors extend the known results from the literature for m up to 21. They also give the exact values of the domination numbers for n up to 31.

Xu et al. [8] deal with a particular topological index, namely with the distance degree index introduced by Dobrynin and Kochetova [9]. Topological indices can be used, e.g., for predicting physical, chemical, or pharmaceutical properties of organic molecules and chemical compounds. The authors derive expressions for the distance degree index for a variety of graphs, namely for a line graph, a subdivision graph, a vertex-semitotal graph, an edge-semitotal graph, a total graph, and a paraline graph.

Liu et al. [10] consider the normalized Laplacian which plays an important role when studying the structural properties of non-regular networks. They determine the normalized Laplacian spectrum of a linear heptagonal network by a decomposition theorem for the normalized Laplacian matrix and elementary operations. In addition, the authors derive explicit formulas for the degree-Kirchhoff index and the number of spanning trees with respect to a linear heptagonal network. Here the authors use the relationships between the roots and coefficients.

Wang et al. [11] consider another distance-based topological index, namely the Padmakar-Ivan (PI) index. They obtain results for this index from trees to recursively clustered trees, the so-called k-trees. Moreover, tight upper bounds of such indices for k-trees are obtained by recursive relationships, and also the corresponding extremal graphs are given. In addition, the PI values of some classes of k-trees are derived and compared.

Liu et al. [12] deal with several topological indices. In particular, they derive expressions for reformulated Zagreb indices of some derived graphs, such as the complement graph, the line graph, the subdivision graph, the edge-semitotal graph, the vertex-semitotal graph, the total graph and the paraline graph of a graph.

Liu et al. [13] use the edge isoperimetric problem to determine the exact wirelengths of embedding an enhanced hypercube into windmill and necklace graphs by partitioning the edge set of the host graph. The results obtained in this paper may have a great impact on parallel computing systems.

Yang et al. [14] consider the subtree problem of so-called fan graphs, wheel graphs and also graphs obtained from partitioning wheel graphs under dynamic evolution. The enumeration of these subtree numbers is done through so-called subtree generation functions of graphs. In particular, they study extremal graphs, subtree fitting problems and subtree densitiy behaviors of the graphs under consideration.

Huang et al. [15] deal with the idea of regularity in neutrosophic graph theory. They describe the utility of a regular neutrosophic graph and a bipartite neutrosophic graph to model an assignment problem, a road transport network, and a social network. Neutrosophic graphs are a useful concept to cope with uncertainty resulting from the inconsistent or indeterminate information in real-world problems. In particular, a regular neutrosophic graph, a star neutrosophic graph, a regular complete neutrosophic graph, a complete bipartite neutrosophic graph and a regular strong neutrosophic graph are introduced. The authors prove some properties of these graphs. Moreover, the concept of an m-highly irregular neutrosophic graph on cycle and path graphs is introduced. The definition of busy and free nodes in a regular neutrosophic graph is also presented. In addition, some properties of complement and isomorphic regular neutrosophic graphs are also given.

Hui et al. [16] derive necessary and sufficient conditions for the graph join of a cycle with m vertices and a path with n vertices to be induced matching-extendable and bipartite-matching extendable, respectively. A graph G is called induced matching extendable, if every induced matching in this graph is included in a perfect matching of G. Similarly, a graph is bipartite matching extendable if every bipartite matching is included in a perfect matching. The paper finishes with some suggestions for future work, e.g., to investigate the relationships between k-extendable and forbidden subgraphs of a graph.

Falcon et al. [17] derive some results on graph theory in the context of molecular processes occurring during the S-phase of a mitotic cell cycle. After presenting some basic concepts on genetics, genetic algebras, evolution algebras, graph theory, and isotopisms of algebras, they introduce a total-colored graph that can be associated with any given evolution algebra over a finite field. Finally, the existence of a faithful functor between both considered categories of evolution algebras and their total-colored graphs is shown.

Carbollosa et al. [18] introduce the f-index and the f-polynomial of a graph. Using this polynomial of several topological indices, they study relations, e.g., of the inverse degree index, the generalized first Zagreb index, and sum lordeg indices. They obtain inequalities involving the f-polynomial of many graph operations including the corona product graph, the join graph, and line graph and the Mycielskian graph. This leads to new inequalities for the topological indices considered.

De la Sen et al. [19] consider so-called $(s-q)$-graphic contraction mappings in b-metric like spaces. Their approach is used to show that a Picard sequence is Cauchy in the context of a b-metric like space which generalizes known results from the literature. The obtained results are illustrated by some examples.

Marappan et al. [20] deal with the asymptotic analysis of several evolutionary operators (mutations and crossovers) for finding the chromatic number of a graph which is the minimum number of colors necessary to color the vertices of a graph such that no adjacent vertices have the same color. The selection of an appropriate operator has a great influence on finding good bounds for the chromatic number as well as on the achievement of a faster convergence with a smaller population size. In addition, necessary and sufficient conditions for the global convergence of evolutionary algorithms have been derived. Finally, the stochastic convergence of some recently suggested evolutionary operators is investigated.

Worawannotai et al. [21] consider particular domination games. Such a game is played by two players, namely the dominator and the staller, which alternatively choose vertices until all vertices are dominated. They study a version of a domination game, where the set of chosen vertices is always independent. This game turns out to be a competition-independence game, which is played by a Diminisher and a Sweller, who want to construct a maximal independent set M: however, while the Diminisher tries to minimize $|M|$, the Sweller wishes to maximize $|M|$. In this paper, the authors check whether some well-known results for domination games also hold for such competition-independence games and describe a family of graphs for which many parameters are equal.

Sotskov [22] gives a detailed review about mixed graph colorings in relation to scheduling problems with minimizing the makespan. Such a mixed graph contains both directed arcs and undirected edges. He presents known results for two types of vertex colorings, referring to the chromatic number and the strict chromatic number of a graph, respectively, and he also reviews the complexity of these problems. Then he discusses in detail how the results for mixed graph colorings can be used for job shop scheduling problems with unit processing times as well as general shop scheduling problems. Further separate sections deal with colorings of arcs and edges of a mixed graph as well as with non-strict colorings of a mixed graph.

Acknowledgments: As the guest editor, I would like to thank all authors for submitting their work to this special issue and also all referees for their support by giving timely and insightful reports. My special thanks go to the staff of the journal Mathematics for the good and pleasant cooperation during the preparation of this issue.

Conflicts of Interest: The author declares no conflict of interest.

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