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Determining the Optimal Inventory and Number of Shipments for a Two-Resource Supply Chain with Correlated Demands and Remanufacturing Products Allowing Backorder

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Abstract: This study develops an integrated supplier–remanufacturer and customer (downstream manufacturer) inventory model that takes into account three-echelon system with correlated demands and remanufacturing products allowing a backorder goods condition. This paper improves the observable fact that the first model system customer might select two sources from remanufactured products or supplier products without defective items. The second model further considers the defective items during the screening duration. The results are examined analytically and numerically to show that the policy of single shipment in large lot sizes results in less total cost than a frequent shipments policy. We also explore the impact of recovery rate on the economic benefits of the inventory system. In addition, we also perform sensitivity analysis to study the impact of seven important parameters (transportation cost, recovery rate, screening rate, annual demand, defect rate, and backorder rate, holding cost,) on the optimal solution. Management insights were also discussed.

Keywords: inventory; logistics; lot-splitting shipments; defective products; backorder rate

1. Introduction

In view of the limited world resources being rapidly used by humankind, awareness is growing in terms of the importance of environmental protection. With this in mind, we hope to use circular economy or recycling to reduce the consumption of the earth's resources. The past decade has seen a wide range of literature on reverse logistics (Chan, Yin, & Chan, 2010 [1]; Alamri, 2011 [2]; Mona 2011 [3]; Lin, C-C., 2013 [4]; Li, Y., & Chen, Z., 2015 [5]; Wakhid, A.J., Rahmad, S., Pringgo, W. L., 2018 [6]) and sustainable product design/manufacturing/operations (Yuan-Shyi, 2005 [7]; El Saadany & Jaber, 2010 [8], 2011 [9]; Gungor & Gupta, 1999 [10]; Jaber & El Saadany, 2009 [11], 2011 [12]; Hodgson & Warburton, 2009 [13]). Some recent reviews (Akcali & Cetinkaya, 2011 [14]; Lin, T-Y., 2013 [15]) have shown that inventory and production planning in a closed-loop supply chain are quantitative models. The literature on manufacturing products and waste disposal models assumes that items can be recycled indefinitely, but this is usually inconsistent with the facts. For example, El Saadany, Jaber, and Bonney (in print) address this limitation. These subjects (Dyckhoff, Lackes, & Reese, 2003 [16]; Flapper, Van Nunen, & Van Wassenhove, 2005 [17]; Zhuang X., Yixiang T., 2018 [18]; Verhoeven, P. & Sinn, F., 2018 [19]; Y. Yi and J. Li 2018 [20]; Tanaka, R., Ishigaki, A., 2019 [21]) published in many edited books, although all these names are different, the basic idea is to integrate product returns with traditional supply chains, which may involve the acquisition of used products, reverse logistics, product disposition (sort, test, and grade), remanufacturing/repair,

and remarketing (Guide and Van Wassenhove, 2002 [22]). Many manufacturers in developed countries in North America and Europe have become obliged to prevent waste and pollution during the handling of end-of-use or end-of-life product returns. Therefore, the remanufacturer must design the function of recycling the defective products in order to obtain the maximum economic benefits from the product returns. However, the ability to resume proper operation depends on the quality of the return. Thierry, Salomon, Van Nunen, and Van Wassenhove (1995) conceptually describe five options: repair, refurbishment, remanufacturing, crushing, and recycling, depending on the quality of the return and the degree of disassembly [23]. Among these, the means of remanufacturing products is returning the quality of a product or product part to the level of a new product. Remanufacturing products are usually cheaper than new products, so it can save a lot of money. Currently, remanufacturable items include plastic products, machinery, automotive parts, office furniture, and so on.

Typically, we see that demand exceeds the rate of return of products. As a result, the correlation between demand and recycling will increase order and inventory or system complexity. Thus, we seek to find reasonable correlation between optimal demand and remanufacturing from products with a short product life (such as tires and copying paper); however, between demand and recycling for longevity products (such as durable products, and electrical and electronic equipment) the replacement rate is relatively low. In the literature (Lin, C.C., Lin, C.-W., 2011), it is generally assumed that demand and return are independent, and extending a single-stage closed-loop supply chain to a multi-tier inventory level makes the supply chain system more complex [24]. Recently, Yuan and Gao (2010) developed an inventory control model consisting of retailers, manufacturers, suppliers, and collectors to determine demand and return rates without shortages [25]. Although these papers make meaningful and valuable contributions to the literature, they make many assumptions, including the independence between demand and benefits and the irrelevance of certain costs (settings, inventory holdings, and shortages). Mitra (2009, 2012) solves the above cost problem; however, this article assumes independence and dependence between demand and return [26,27].

Lin (2014) [28] mainly discusses the remanufacturing rate in the supply chain system for suppliers and remanufacturers, and the minimization of the total cost of shipping batches. According the above literature, the assumption is that backorder and out-of-stock costs are not allowed. However, in practice, backorder is possible. In the case of backorder, the dual supply (supplier–remanufacturer) will give a discount on the price. The longer the backorder period, the greater the discount, so this paper considers the situation of allowing backorder for further study.

In this study, we consider the credit period as a mechanism for improving the coordination among a single supplier, a single remanufacturer and a single customer (downstream manufacturer) for a single product which grade is based on different ratios by a new item and a remanufactured item. Our model aims to view the three-echelon closed-loop supply chain model as an integrated whole and determine the optimum remanufacturing level and supply level and suitable delivery times that would minimize the total system cost. Our model reveals the correlation between demands and returns, and the interrelationship among the opportunity costs of the supplier, the remanufacturer and the customer (downstream manufacturer) as one of the boundaries for the delivery period. Among the supplier, the remanufacturer and customer save on costs because of the coordination, making the model a win-win proposition.

The rest of this paper is organized as follows: Section 2 defines the inventory management system conditions and symbols. Section 3 discusses the integrated model formulations, both without defective items and with defective items under the backorder condition, and analyzes the effects of the problem parameters on the optimal solutions, respectively. Section 4 presents a numerical analysis. Section 5 provides our conclusions.

2. System Conditions and Symbols

This paper considers a three-echelon inventory system including remanufactured products. The outside return products are remanufactured, which have a 100% recovery rate for goods as new

products. Besides, the other outside returns are new products from suppliers. Both outside returns are perfectly complementary and meet customer demand. We assume that remanufactured products and new products have the same value and the same holding cost for inventory. We also assume that external suppliers of remanufactured products and new products are immediately replenishing. We also simulate the replenishment of remanufactured products and supply new products with the same supply chain cycle time. By considering the backorder condition in the model, customer orders may be be underpaid, which means that some orders can be provided in the current period.

We suppose that set-up cost, holding cost of inventory and allowed stock-out cost in all levels and transportation cost for suppliers and remanufactured, and customer (downstream manufacturer) stock stage contain defective items and without defective items in the system. There may be many costs in a closed-loop supply chain, such as collection, sorting, recycling, disposal and remarketing. However, in this article, from the perspective of inventory management, we simply take the basic costs of the system as constituent elements, such as set-up costs, inventory holding costs, stock-out cost and transportation costs. The objective is to determine that minimize the (expected) total costs of the system at all the stages. In the second model, it is further assumed that while supplier and remanufacturer products with defective items are inspected by the customers. The numerical conditions we assume are as follows:

1. The three-echelon inventory system consists of a single supplier, a single remanufacturer and a single customer (downstream manufacturer).
2. A single product is considered.
3. The remanufacturing rate and demand rate are known.
4. The screening rate is higher than the remanufacturing rate and demand rate.
5. The lead-time and defect rate are fixed.
6. There is an unlimited planning period.
7. Allow supply backorder and out-of-stock inventory.
8. Not consider quantity discounts.
9. Not consider inventory space constraints.

The notations used in this paper are shown in Table 1.

Table 1. Notations.

For the Remanufacturer		For the Supplier		For the Customer	
S_m	Set-up cost for remanufacturer	S_S	Set-up cost for supplier	S_b	Ordering cost for customer
H_m	Inventory holding cost per unit per period for remanufacturer	H_S	Inventory holding cost per unit per period for supplier	H_b	Inventory holding cost per unit per period for customer
M	Remanufacturing rate	D	Demand rate	q	Batch size/order quantity
r	Recovery rate ($0 < r < 1$)	n	Integer multiple	p	Defect rate
T	Cycle time length	x	Screening rate	C_b	Unit screening cost
F_m	Transportation cost for remanufacturer	F_S	Transportation cost for supplier	C_S	Shortage cost
t1	No out-of-stock period	t2	Out-of-stock period	s	Backorder quantity
L	Lead time for replenishment	π_0	Profit margin per unit of goods	π_x	Backorder discount per unit for goods
β	Backorder ratio	R_q	Stock quantity in reorder of point (ROP)	Q	Total quantity ($Q = n*q$)

3. Model Formulation

In this section, we will develop a three-echelon inventory system (Figure 1). The downstream stage is that the customer belongs to the demand side, and the upstream stage is that the remanufacturer and supplier belong to the dual supply side. This is the total cost of the integrated supplier–remanufacturer

(that is dual supply) and customer in the first model. In addition, we extend the above inventory system, adding that the downstream stage customers will completely screen out defective products, and the dual supply side remains unchanged in the second model. Symbols and diagrams for model operation contents are as follows.
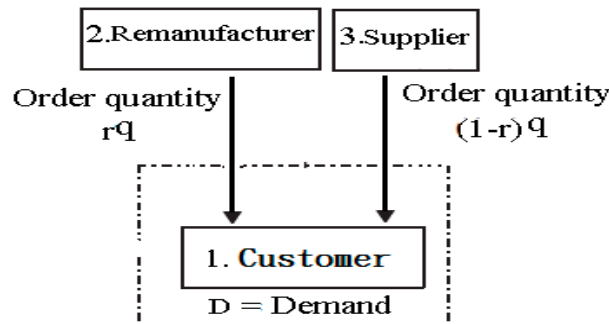

Figure 1. The framework of the three-echelon inventory system.

The system of a customer's acquirement and supplement customer demand from both external suppliers and remanufacturers can be shown in Figure 1. It is clear from Figure 1 that if q is the customer's demand quantity, the cycle length is $\frac{q}{D}$. The recycle ordering amount in the remanufacturer is rq. Therefore, the order quantity with the supplier is $(1 - r)q$. The number of orders for a supplier is n, where n $(1 - r)q$ is an integer.

For backorder condition model, out-of-stock and under-allocation means that the customer is allowed to make up for out-of-stock products within a certain period of time. At this time, a discounted loss of material prices may occur, or it may be to meet the allowable period by increasing the cost of overtime to rush to work. The cost of transportation or outsourcing costs can be summed up as the cost of stock-outs. Because the cost of stock-outs is estimated, customers and suppliers often have different views, so it is difficult to accurately estimate the cost.

The backorder model is shown in Figure 2. Cs is the unit cost of stock, t1 is the no out-of-stock period, and t2 is the out-of-stock period. We show derivation process of out of stock model. Average inventory level is $(q - s)/2$ and average lack of inventory level $s/2$. The geometric figure $\triangle ABD$, $\triangle ACE$ and $\triangle EHD$ are similar triangles, so the following proportional relationships exist.

$$\frac{t_1}{q-s} = \frac{(t_1+t_2)}{q} = \frac{t_2}{s}, \text{ then } \frac{t_1}{t} = \frac{q-s}{q}, \frac{t_2}{t} = \frac{s}{q} \tag{1}$$

The total cost of the allowance model for backorders includes the purchase cost of the materials themselves, order costs per year, storage costs per year and backorder costs per year.

$$\text{TC} = P \times D + C_0 \times \frac{D}{q} + C_h \times \frac{q-s}{2} \times \frac{t_1}{t} + C_s \times \frac{s}{2} \times \frac{t_2}{t} = P \times D + C_0 \times \frac{D}{q} + C_h \times \frac{(q-s)^2}{2q} + C_s \times \frac{s^2}{2q} \tag{2}$$

With the above formula differential, we get:

$$q^* = \sqrt{\frac{2C_0 \times D}{C_h}} \times \sqrt{\frac{C_S + C_h}{C_S}}, \quad S = q \times \frac{C_h}{C_s + C_h}, \quad q - S = q \times \frac{C_S}{C_S + C_h} \tag{3}$$
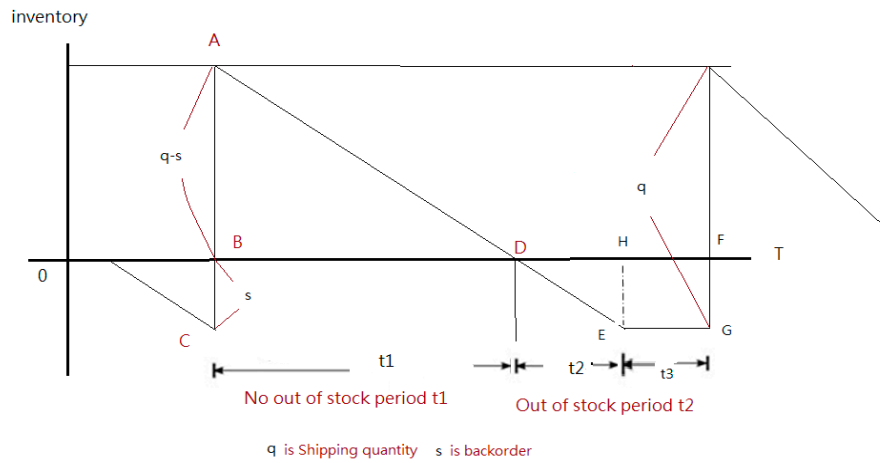
Figure 2. Allowed backorder stock model inventory level chart.

3.1. Establishing Model 1: The Supplier and Remanufacturer Splits a Shipment into Several Small Lot Sizes for Customer (Downstream Manufacturer) without Dective Items for a Backorder Model

Based on the optimal delivery strategy, Lin (2014) [28] and Kim & Ha (2003) [29] proposed a model considering the total relevant costs for the supplier and buyer, and determined the optimal order quantity, number of deliveries/set-ups, and shipping quantities in a simple JIT single-supplier, single-remanufacturer–single-customer structure.

Figure 3 shows the three-echelon inventory system diagrams for the backorder model. It can be seen from Figure 2 that customers, suppliers, and remanufacturers have the same length of supply and demand cycle. Based on customer environmental protection considerations, there are ordered both remanufactured products and new products from remanufacturers and suppliers to meet customer demand. Simultaneously we add the out-of-stock model by using heuristic derivation, importing concept of out-of-stock costs to modify Mona's 2011 formula as follows. When an out-of-stock period causes customers to wait for a long time, some customers with lower loyalty will switch to other suppliers.
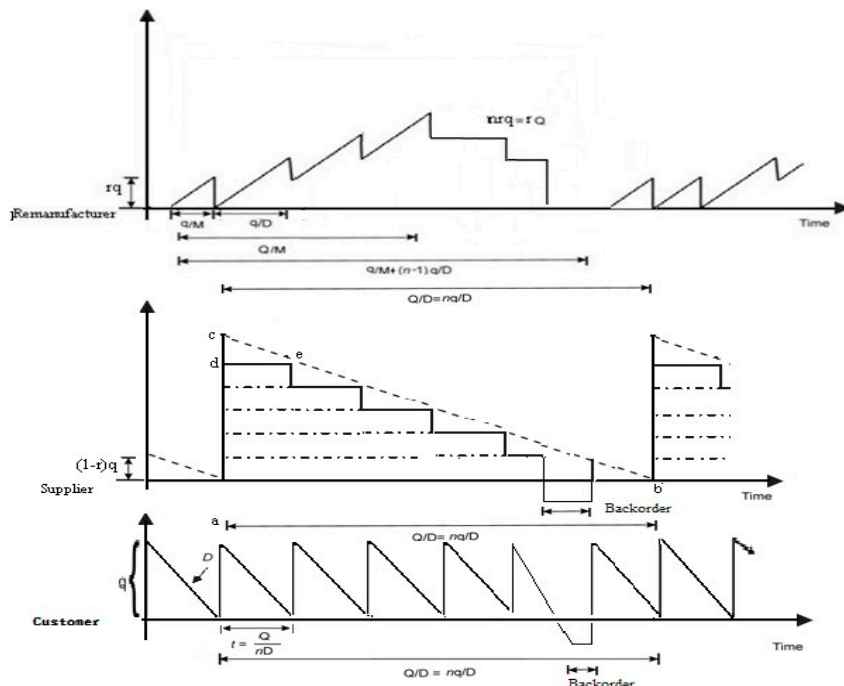

Figure 3. On-hand inventories at three echelons for the backorder model without defective items.

Ouyang et al. [30,31] believe that in the event of a shortage, some customers are still willing to wait out the shortage due to trust and loyalty toward the supplier. At this time, the supplier often provides backorder discounts to compensate for losses due to waiting or increase in production costs. Therefore, how to find the best relationship between the discounts of the amount owed and the period of backorder owing for minimizing the total cost of inventory is explored in a future study.

From Figure 3, it is apparent that the number of set-ups at customer, supplier and remanufacturer will be $\frac{D}{q}$, $\frac{D}{nq}$ and $\frac{D}{nq}$, here $nq = Q$, The expression for the average on-hand inventory at remanufacturer, supplier and customer can be derived as follows:

The expression for the average inventory holding cost at the remanufacturer can be derived as follows: (refer to Mona 2011) [3] and Lin's research [28]:

$$H_{\mathrm{m}}\frac{nq^2}{2D}[r(2-n)\frac{D}{M} + r(n-1)] \tag{4}$$

The average inventory holding cost at **supplier** can be derived as follows:

$$H_S\frac{nq^2}{2D}(n-1)(1-r) \tag{5}$$

From the **customer's** viewpoint, the average inventory holding cost is:

$$H_b\frac{1}{2}\frac{nq^2}{D} \tag{6}$$

According Ouyang et al. (1996) [30], we establish the allow out-of-stock mode in Equation (7); D is demand quantity for the customer and follows normal distribution with the probability density function f(D). The mean value is uL, the standard deviation is $\sigma\sqrt{L}$ the ROP is $R_q = uL + k\sigma\sqrt{L}$; the expected quantity out-of-stock each period is the Equation (7). L is the lead-time for replenishment.

$$\mathrm{B(d)} = \int (D - R_q)f(D)dD = \sigma\sqrt{L}\psi(k), \psi(k) = \varphi(k) - k[1 - \Phi(k)] \tag{7}$$

The Φ and $\Phi(K)$ is the representative probability density function (pdf) and cumulative distribution function (cdf), respectively. The backorder per period is $(1 - \beta) \times B(r)$, such as β ($0 \leqq \beta \leqq 1$) is the proportion of stocks understocked in stock-out period. In the out-of-stock cost function, there is a functional relationship between the out of stock discount (β) and the out-of-stock period (L). In other words, the longer the out-of-stock period, the more out-of-stock discounts may be given. We assume that the out-of-stock pattern in this case conforms to the normal distribution. Then, the cost of stock out can be expressed as follows: β is backorder ratio, π_0 is profit margin per unit of goods, π_x is backorder discount per unit of goods according Ouyang et al. (1996) [30].

$$\mathrm{TC(Cs)} = \frac{D}{q}[\pi_x\beta + \pi_0(1 - \beta)] \tag{8}$$

To combine remanufacturer cost, TC(M) consists of a set-up cost (S_m), transportation cost and on-hand inventory cost (nF_m). Supplier cost TC(S) consists of a set-up cost (Ss), transportation cost (nFs) and on-hand inventory cost and customer cost. TC(B) consists of a ordering cost (S_b) and on-hand inventory cost and the backorder cost TC(Cs) becomes the supply total costs (TC(M-S-B-Cs) as follows: (refer to Lin 2014) [28]

$$\text{Total cost (M-S-B-Cs)}_{\text{model1}} = \text{TC(MSBCs)}_{\text{model1}} = \text{TC(M)} + \text{TC(S)} + \text{TC(B)} + \text{TC(Cs)} \tag{9}$$

$$= S_{\mathrm{m}} + nF_{\mathrm{m}} + H_{\mathrm{m}}\frac{Q^2}{D2n}[r(2-n)\frac{D}{M} + r(n-1)] \qquad \text{the formula means TC(M)}$$

$$+ S_S + nF_S + H_S\frac{1}{2}n(n-1)(1-r)\frac{q^2}{D} \qquad \text{the formula means TC(S)}$$

$$+ S_b + H_b\frac{1}{2}\frac{nq^2}{D} \qquad \text{the formula means TC(B)}$$

$$+ \frac{D}{q}[\pi_x\beta + \pi_0(1-\beta)] \qquad \text{the formula means TC(Cs)}$$

There are $\frac{1}{T} = \frac{D}{Q}$ cycles in one time period. Hence, the model average total cost is as follows:

$$\begin{aligned} ETC(Q,n) = {} & \frac{D[S_{\mathrm{m}}+S_S+S_b+n(F_m+F_S)]}{Q} + H_{\mathrm{m}}\frac{Q}{2n}\left[r(2-n)\frac{D}{M} + r(n-1)\right] + \\ & H_S\frac{Q}{2n}(n-1)(1-r) + H_b\frac{Q}{2n} + \frac{D^2}{nq^2}[\pi_x\beta + \pi_0(1-\beta)] \end{aligned} \tag{10}$$

At first, we let n be fixed for finding the unique solution. Taking the Equation (10) derivative with Q will be:

$$Q^*(n) = \sqrt{\frac{2D(S+nF)}{H(n)}} \times \sqrt{\frac{\pi_x + \pi_0}{\pi_x}} \tag{11}$$

Let n be fixed; $ETC(Q,n)$ is decreasing on $(0; Q^*(n)]$ and $ETC(Q,n)$ is increasing on $[Q^*(n), \infty]$. Therefore, we can obtain at one of $Q^*(n), ETC(Q,n)$ has the optimal solution. Plug Equation (11) into Equation (9), and rearranging the results will be:

$$ETC(Q^*,n) = \sqrt{2D(S+nF)/H(n)\pi_x(t,d)} \tag{12}$$

Since n is an integer, the optimal number of shipments from the supplier and remanufacturer to the customer must be satisfied as follows: [28]

$$n^*(n^*-1) \le \frac{S\theta}{F\varphi} \le n^*(n^*+1) \tag{13}$$

For finding the overall optimal solution, to help the operations managers plan and make the decisions quickly and correctly, an algorithm is developed as follows.

3.2. Model 2: The Supplier and Remanufacturer Splits a Shipment into Several Small Lot Sizes for Demand (Downstream Manufacturer) with Defective Items

In this model, we extend the previous model, and we assume that the customer 100% checks and screens out defects that are immediately removed from the original inventory. Figure 4 displays the behavior of the inventory level for this model. Suppliers and remanufacturers arrive at customers with replenishment; the cycle length remains the same, but the cycles in one-time period at customer, supplier, and remanufacturer will be $\frac{D}{(1-p)q}$, $\frac{D}{n(1-p)q}$ and $\frac{D}{n(1-p)q}$, respectively.

From the remanufacturer's viewpoint, the total cost of a product should include the set-up cost, holding cost, product delivery cost, and screening cost. S_{m} is the set-up cost (including maintenance); $H_{\mathrm{m}}\frac{nq^2}{2D}[r(2-n)\frac{D}{M} + r(1-p)(n-1)]$ is the remanufacturer's holding costs for transporting batches; nF_{m} is the shipping freight cost.

The remanufacturer's holding cost is derived as follows (refer to Mona 2011 [3] and Lin's research [28]):

$$H_{\mathrm{m}}\frac{nq^2}{2D}[r(2-n)\frac{D}{M} + r(1-p)(n-1)] \tag{14}$$

The total cost for the remanufacturer is:

$$TCM = S_{\mathrm{m}} + nF_{\mathrm{m}} + H_{\mathrm{m}}\frac{Q^2}{D2n}\left[r(2-n)\frac{D}{M} + r(1-p)(n-1)\right]$$
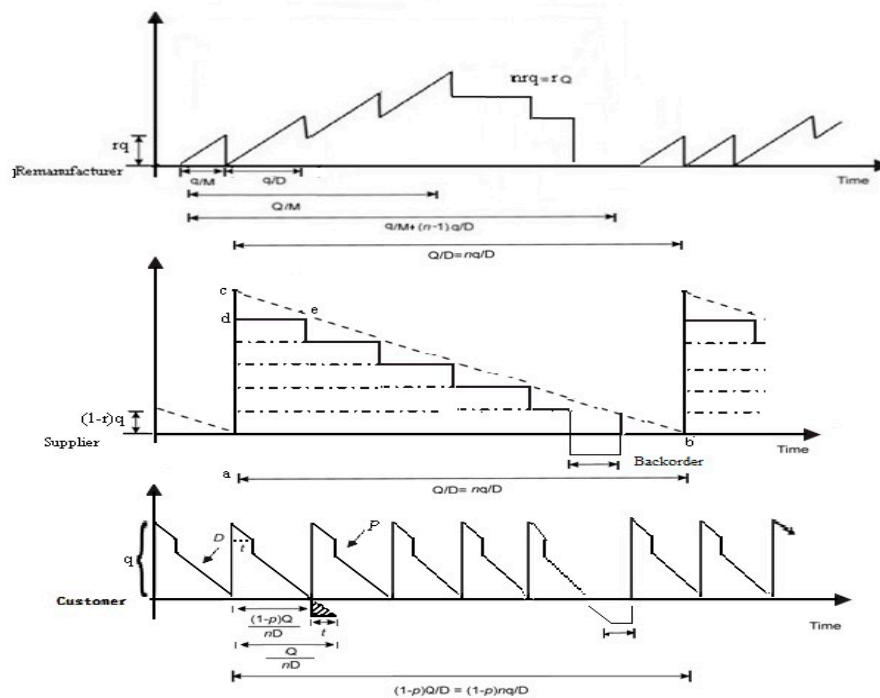
Figure 4. On-hand inventories at three echelons **for backorder model** with defective items.

The supplier's holding cost is derived as follows:

$$H_S \frac{1}{2} n(n-1)(1-r) \frac{(1-p)q^2}{D} \tag{15}$$

The total cost for the supplier is:

$$\text{TCS} = S_S + nF_S + H_S \frac{1}{2} n(n-1)(1-r) \frac{(1-p)q^2}{D}$$

From the customer's viewpoint, the average on-hand inventory cost is:

$$H_b \frac{1}{2} \frac{n(1-p)q^2}{D} \tag{16}$$

The total cost for the customer is:

$$\text{TCB} = S_b + C_b Q + H_b \frac{Q^2}{2n} \left(\frac{(1-p)^2}{D} + \frac{2p}{x} \right)$$

The cost of stock out is:

$$\frac{D}{q} [\pi_x \beta + \pi_0 (1-\beta)]$$

The combined remanufacturer, supplier and customer and backorder total costs is

Total cost$(\text{M-S-B-Cs})_{\text{model2}} = \text{TC(MSBCs)}_{\text{model2}} = \text{TC(M)} + \text{TC(S)} + \text{TC(B)} + \text{TC(Cs)}$(Lin 2014) [28]

$$\begin{aligned}
&= S_m + nF_m + H_m \frac{Q^2}{D2n} \left[r(2-n)\frac{D}{M} + r(1-p)(n-1) \right] + S_S + nF_S + \\
&\quad H_S \frac{1}{2} n(n-1)(1-r) \frac{(1-p)q^2}{D} + S_b + C_b Q + \\
&\quad H_b \frac{Q^2}{2n} \left(\frac{(1-p)^2}{D} + \frac{2p}{x} \right) + \frac{D}{q} [\pi_x \beta + \pi_0 (1-\beta)]
\end{aligned} \tag{17}$$

There are $\dfrac{1}{T} = \dfrac{D}{(1-p)Q}$ cycles in one time period. Hence, the model average total cost, as follows:

$$
\begin{aligned}
ETC(Q,n) =\ & \frac{[S_m + S_S + S_b + n(F_m + F_S)]D}{Q(1-E[p])} + \frac{C_b D}{(1-E[p])} + H_m \frac{Q}{2n(1-E[p])}[\\
& r(2-n)\frac{D}{M} + r(1-E[p])(n-1)] + H_S[\frac{Q}{2n(1-E[p])}(n-1)(1-r)(1-E[p])] \\
& + H_b \frac{QD}{2n(1-E[p])}(\frac{E[(1-p)^2]}{D} + \frac{2E[p]}{x}) + \frac{D^2}{nq^2}[\pi_x \beta + \pi_0(1-\beta)]
\end{aligned}
\tag{18}
$$

$$
Q^* = \sqrt{\frac{2nD(S+nF)}{H(n)}} \times \sqrt{\frac{\pi_x + \pi_0}{\pi_x}}
\tag{19}
$$

$$
ETC(Q^*,n) = \sqrt{2nD(S+nF)H(n)\pi_x(t,d)}
$$

Since n is an integer, the optimal number of shipments from the vendor to the customer must be satisfied as follows:

$$
n^*(n^*-1) \leq \frac{S\theta}{F\varphi} \leq n^*(n^*+1)
\tag{20}
$$

4. Numerical Analysis

This analysis considers two kinds of materials, the resources of the supplier and the remanufacturer. The customer (downstream manufacturer) will consider two kinds models of inventory without defective items when considering the backorder condition with defective items. We further calculate the total relevant customer, remanufacturer and supplier cost by determining the optimal order quantity cycle lot size, a number of deliveries, and dual supplier shipment quantity, and Table 2 is a numerical example using Kim and Ha (2003) as the parameters needed for the analysis model in this paper [29].

Some values in Table 2 are based on Salameh & Jaber's (2000) [32] values for fixed costs, screening rate, expected defect rate, $E[p] = 0.02$, and another definition of demand, the remanufacturing rate. Considering the cost structure and using the algorithm in Section 3, we can get the optimal order quantity, the best shipment times and the lowest annual total cost as follows:

According to the Figure 5 as shown below, assume supplier and remanufacturer have same condition (such as quality, cost, etc.). Then, we integrate the supplier and the remanufacturer, and the recovery rate dropped slightly by 50.3%; therefore, we can obtain the quantity of remanufactured $r^*q = 274$, the quantity of suppliers $(1-r^*)q = 252$, $Q^{**} = 2088$ units $ETC(2088, 4) = \$8391$ per year.

However, if the single delivery policy ($n = 1$) were used, we also compare Model 1 in terms of the effects on the optimal strategies and the performance of a shipment split into several small lot sizes (compared with a single shipment). The values for M, D, S_m, Ss, S_b, H_m, H_s, H_b, x, and [P] are the same as those given in Table 2. We assume $F_s = 25$, and $F_m = 10, 25, 100$. Also, we assume r takes the six values 0.1, 0.2, 0.3, 0.4, 0.5, 0.6. and $\beta = 0.05$. Substituting these values into the derived formulas, we obtain the results summarized in Table 3. By comparing a shipment of several lot sizes with a single shipment strategy, we will obtain cost savings (CS), as follows: [28]

$$
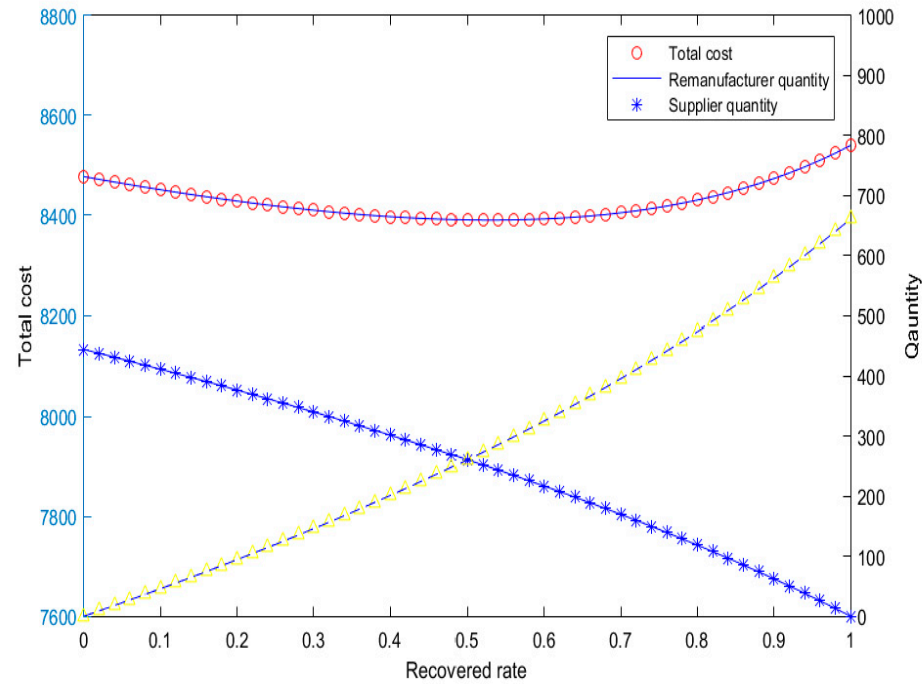CS = \frac{ETC_{n=1}^{**} - ETC^{**}}{ETC^{**}} \times 100\%.
\tag{21}
$$

Figure 5. Remanufacturer and supplier-ordering system for the customer economic curve.

Table 2. Parameter values.

For the Remanufacturer		For the Supplier		For the Customer	
Setup cost for remanufacturer	S_m = $300/cycle	Setup cost for Supplier	S_S = $600/cycle,	Ordering cost for buyer	S_d = $25/cycle
Remanufacturer's inventory holding cost	H_m = $3/per unit/per year	Supplier's inventory holding cost	H_S = $3/per unit/per year	Customer's inventory holding cost	H_d = $5/per unit/per year
Remanufacturing rate	M = 19,200 units/year,	Demand rate	D = 4800 units/year,	Screening rate	x = 152,000 units/year
Transportation cost for remanufacturer	F_m = $25/delivery,	Transportation cost for supplier	F_S = $25/delivery,	Except defect rate	$E[p]$ = 0.02.
Profit margin per unit	π_0 = 5%	Backorder discount	π_x = 5%	Backorder ratio	β = 0.05

Table 3. Comparison of performance of a shipment split into several small lot sizes and a single shipment under backorder condition.

	r	With a Shipment Split into Several Small Lot Sizes(nq = Q)						With a Single Shipment (n = 1)				Model 1	Model 2
		Model 1			Model 2			Model 1		Model 2			
		n^{**}	Q^{**}	ETC^{**}	n^{**}	Q^{**}	ETC^{**}	$Q^{**}_{n=1}$	$ETC^{**}_{n=1}$	$Q^{**}_{n=1}$	$ETC^{**}_{n=1}$	CS	CS
$F_m = 10$	0.1	4	1933	6368* (5841 + 527)	4	1960	8869 (8270 + 599)	1375	**7113** (6830 + 283)	1388	**9567** (9103 + 464)	16.46%	12.11%
β=0.05	0.2	5	2067	6334 (5810 + 524)	5	2097	8841 (8265 + 579)	1365	7165 (6883 + 282)	1377	9620 (9130 + 490)	18.06%	13.17%
	0.3	5	2142	6302 (5780 + 522)	5	2174	8817 (8234 + 583)	1355	7217 (6894 + 323)	1368	9672 (9320 + 352)	19.64%	14.18%
	0.4	5	2225	6278 (5759 + 519)	5	2261	8800 (8210 + 590)	1345	7269 (6919 + 350)	1358	9725 (9423 + 302)	21.09%	15.11%
	0.5	5	2421	6279 (5758 + 521)	5	2358	**8792** (8215 + 577)	1335	7320 (6934 + 386)	1347	9776 (9502 + 274)	21.99%	15.88%
	0.6	6	2547	6274 (**5754** + 520)	6	2593	8818 (8321 + 497)	1326	7370 (6993 + 377)	1338	9828 (9632 + 196)	23.01%	16.18%
	0.7	6	2551	**6270** (5760 + 510)	6	2597	8825 (8341 + 484)	1327	7372 (7003 + 369)	1336	9856 (9679 + 177)	23.03%	16.22%
$F_m = 25$	0.1	4	1987	6545 (6010 + 535)	4	2014	9043 (8437 + 606)	1385	**7168** (6880 + 288)	1398	**9622** (9242 + 380)	13.98%	10.46%
β = 0.05	0.2	4	2045	6514 (5906 + 608)	4	2075	9019 (8419 + 600)	1375	7221 (6889 + 332)	1388	9675 (9272 + 403)	15.49%	11.44%
	0.3	4	2109	6488 (5868 + 620)	4	2142	8999 (8405 + 594)	1365	7273 (6895 + 378)	1378	9728 (9302 + 426)	16.90%	12.38%
	0.4	4	2180	6467 (**5826** + 639)	4	2215	8985 (8397 + 6368)	1356	7325 (6932 + 393)	1368	9780 (9340 + 440)	18.23%	13.22%
	0.5	4	2258	**6453** (5864 + 589)	4	2297	**8978** (8412 + 566)	1346	7377 (6950 + 427)	1359	9833 (9410 + 423)	19.42%	13.99%
	0.6	5	2508	6465 (5873 + 592)	5	2388	8981 (8459 + 522)	1337	7428 (6973 + 455)	1348	9884 (9519 + 365)	20.09%	14.60%

Table 3. *Cont.*

	r	With a Shipment Split into Several Small Lot Sizes($nq = Q$)						With a Single Shipment ($n = 1$)				Model 1	Model 2
		Model 1			Model 2			Model 1		Model 2			
		n^{**}	Q^{**}	ETC^{**}	n^{**}	Q^{**}	ETC^{**}	$Q^{**}_{n=1}$	$ETC^{**}_{n=1}$	$Q^{**}_{n=1}$	$ETC^{**}_{n=1}$	CS	CS
$F_m = 100$	0.1	2	1871	7188 (6632 + 556)	2	1902	9674 (9049 + 625)	1437	**7438** (7104 + 334)	1451	**9890** (9488 + 402)	7.13%	5.72%
$\beta = 0.05$	0.2	2	1895	7189 (6596 + 593)	2	1927	9680 (9036 + 644)	1427	7493 (7132 + 361)	1440	9946 (9520 + 426)	7.97%	6.29%
	0.3	3	2175	7176 (6570 + 606)	3	2210	9674 (9028 + 646)	1417	7547 (7286 + 261)	1430	10,001 (9584 + 417)	9.04%	7.02%
	0.4	3	2231	7163 (6536 + 627)	3	2268	9666 (8995 + 6368)	1407	7601 (7365 + 236)	1420	10,055 (9610 + 445)	10.14%	7.74%
	0.5	3	2291	7152 (**6522** + 630)	3	2332	**9664** (9012 + 652)	1396	7655 (7487 + 171)	1410	10,109 (9639 + 470)	11.17%	8.40%
	0.6	3	2357	**7148** (6556 + 592)	3	2400	9667 (9032 + 635)	1387	7708 (7584 + 124)	1399	10,163 (9689 + 474)	12.09%	9.00%

Note1: The Bottom line bold represents the minimum value in the block. Note2: The Expected cost ETC** is composition by the sum of cost (Equation (4), Equation (5), Equation (6)) and the out of stock cost (Equation (8)) such as 6368* = 5841 + 527.

The results is shown in Table 3 and the sensitivity analysis is shown in Table 4.

1. Under the condition of a shipment split into several small lot sizes, it shows that when F_m is constant, r (rq is order remanufacturing quantity of upstream level), n** and Q** increases, ETC** decreases then increase at reversal point. Regardless of whether there is a defective item, in the case of multiple shipments, under the same backorder ratio (β) and number of shipments(n**), different recovery rates(r) have different cost-saving rates (CS). The higher the recovery rate (r), the higher the cost-saving rate (CS). Since the replacement cost of the recovered product is lower, so that it can be provided to the customer at a cheaper price, the customer's acquisition cost is lower, which represents a higher cost savings. Thus, managers should pay attention to the economic value of transportation and recycling strategies to increase the company's overall operating profit.

2. In contrast, in the single-shipment (n = 1) transportation supply chain, when r increases, Q ** decreases, ETC ** increases, and CS increases. The total cost is higher than the multi-batch method. Therefore, the best strategy is to integrate upstream and downstream supply chain, in order to reduce the overall cost compared to stand-alone strategies

3. When *r* is constant, F_m and Q^{**} increase, β and n^{**} decreases (because $n^{**} = [n^*]$ is an integer and leads to some Q^{**} increases). ETC^{**} and $ETC^{**}_{n=1}$ increase, but CS decreases. The results same as the traditional EOQ model and the transportation model.

4. In Table 3, if the shortage is not allowed, the total expected cost is obtained by shipping 6 times with $n^{**} = 6$, r = 0.6. The ETC = 5754 is the lowest. If the shortage is considered, ß = 0.05, $n^{**} = 6$, r = 0.7, the cost is 6270 as the lowest, so in this article, the replacement rate r and the shortage rate ß will affect the total expected cost. In the follow-up research, we can discuss the interference between the two variable recovery rate (r) and backorder rate (β).

We also perform sensitivity analysis to study the impact of five important parameters (screening rate, annual demand, defect rate, backorder rate and holding cost) on the optimal solution in Table 4.

Table 4. The sensitivity analysis of combination of X, D, [P], β, H_m corresponding to the values of Q^{**}, n^{**} and ETC^{**}, respectively.

x	D	$[p]$	β	H_m	n^{**}	Q^{**}	ETC^{**}
152,000	4800	0.02	0.01	3	4.0	1960	8952
				4.5	4.0	2056	9221
			0.05	3	3.0	2265	10,192
				4.5	4.0	2190	9645
		0.03	0.01	3	4.0	2065	9094
				4.5	4.0	2072	9188
			0.05	3	4.0	2199	9508
				4.5	4.0	2205	9608
	7200	0.02	0.01	3	4.0	2519	11,885
				4.5	4.0	2530	11,989
			0.05	3	4.0	2681	12,396
				4.5	4.0	2694	12,506
		0.03	0.01	3	4.0	2537	11,952
				4.5	4.0	2548	11,956
			0.05	3	4.0	2700	12,357
				4.5	4.0	2713	12,469
228,000	4800	0.02	0.01	3	4.0	2051	9127
				4.5	4.0	2057	9220
			0.05	3	4.0	2184	9545
				4.5	4.0	2190	9644
		0.03	0.01	3	4.0	2065	9293
				4.5	4.0	2072	9186

Table 4. *Cont.*

x	D	[p]	β	H_{m}	n^{**}	Q^{**}	ETC^{**}
			0.05	3	4.0	2199	9507
				4.5	4.0	2205	9607
	7200	0.02	0.01	3	4.0	2519	11,884
				4.5	4.0	2530	11,988
			0.05	3	4.0	2681	12,395
				4.5	4.0	2694	12,505
		0.03	0.01	3	4.0	2537	11,850
				4.5	4.0	2548	11,955
			0.05	3	4.0	2701	12,356
				4.5	4.0	2713	12,468

1. When the screening rate (x), demand rate (D) and defect rate (p) increase, then Q ** and ETC ** also increase. The screening rate (x) increases, that is, the more defective products are selected, the more customers will increase the number of orders and reduce the cost of holding, thus Q ** and ETC ** increase. This result corresponds to the works of Maddah and Jaber (2008)[33], Maddah et al. (2010) [34] and Lin, T-Y. (2013) [15].

2. As Demand rate (D) increases, Q ** and ETC ** increase. Because customers will generate a large number of orders in order to meet the demand. When the defect rate(p) increases, Q ** and ETC ** increases. Since the higher the defect rate, the higher the number of orders and the higher the number of deliveries, the total annual cost will increase as well.

3. When backorder rate (β) is higher, Q ** and ETC ** increase. Because the higher the out-of-stock rate and the higher the number of deliveries, the total cost will increase

5. Summary and Conclusions

As inventory management issues increase, businesses face a greater need to improve their financial performance by cutting (shipping) on inventory holding cost, and integrating the supply chain to allow all members to share the minimum joint total cost. Although Mitra (2012) [27] can solve the problem of defective returned products, in real life, defective products are returned to upstream manufacturers. In addition, environmental awareness is rising. To ensure an image of corporate social responsibility, this study specifically proposes to purchase a certain proportion of remanufacturing products and mixed new products. We developed an algorithm with the ability to make the decisions quickly and correctly to find the overall best solution. In order to better match the inventory situation in the real world, this article considers an inventory model with two states—nondefective and defective items—and incorporates different recovery rates for each model. From these models, we derive closed-form formulas and derive the optimal ordering and shipping strategies.

In reality, shortage cost refers to the cost incurred when inventory is in short supply, which can be further divided into backorder cost or the cost of a loss in sales. Losses caused by stock shortages often have different cost valuation methods due to different positions of buyers and sellers. Therefore shortage costs are the hardest to estimate inventory costs. The level of this cost and the number of stock-outs are related to the unit of shortage cost. When performing inventory model analysis, it can choose different inventory costs according to different needs. It must be specifically stated that the total cost include the ordering cost, holding cost and backorder cost.

From the parameters analysis and the illustrative numerical example, we assume that storage is neglected. Thus, we find that:

1. In the two-stage remanufacturer and supplier supportable stocks, it shows the best ordering recovered rate r* (r* q) for the customer. Therefore, the small batch delivery method can save costs more than a single shipment, so it is better to adopt the upstream and downstream integration strategy of the supply chain than the single strategy.

2. The replacement rate and the backorder rate will increase expected total cost. After considering the backorder rate, under a specific number of shipments, replacement rate, and screening rate, the expected total cost will increase as the backorder rate increases. In the sensitivity analysis, it is also found that the increase of the screening rate, demand rate, defect rate and backorder rate will increase the total cost. Therefore, the ordering strategy adopted by the management personnel needs to decentralize the source of supply. Procurement should avoid concentrating on a few resettlement or supplier sources to ensure the company's best inventory policy.

It is suggested that future research should consider more realistic conditions and more complex inventory models to coordinate conflicts in the supply chain system and achieve a win-win policy for all parties, such as the time value of money, customers willing to acquire defective products at low prices, and multiple suppliers, including multi-customer cooperation and restrictions between each other.

Lin (2014) [28] does not allow the backorder in the supply chain system. However, in real case, there are often unexpected situations (sudden increase in demand, mechanical equipment factors such as repairs required for damage or careless operation of personnel) lead to backorder. Therefore, this study considers allowing backorders for shortages, and finds the optimal number of deliveries (n), the backorder rate (β) and the recovery rate (r), which will affect the total cost. Subsequent research can explore the interaction effects of these three variables to achieve the lowest total cost model.

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