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Mathematical Model on Gravitational Electro-Magneto-Thermoelasticity with Two Temperature and Initial Stress in the Context of Three Theories

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Abstract: The main aim of this paper is to study two temperature thermoelasticity in a generalization form to solve the half-space problem of two dimensions under gravity, perturbed magnetic field, and initial stress. The fundamental equations are solved considering a new mathematical technique under Lord-Şhulman (LS), Green-Naghdi (GN type III) and three-phase-lag (3PHL) theories to investigate displacement, stress components, and temperature distribution. The results obtained by the three theories, i.e., (LS), (GN type III), and (3PHL), considering the absence and the presence of gravity, initial stress, and magnetic field have been compared. The results were numerically calculated and graphically displayed to exhibit the physical meaning of the phenomenon and the external parameters' effect. A comparison has been presented between the results obtained in the absence and the presence of the external considered parameters and with the previously obtained results by other researchers.

Keywords: thermoelasticity; magnetic field; initial stress; gravity; two temperature; Lord-Şhulman (LS); Green-Naghdi (GN type III); three-phase-lag (3PHL)

MSC: 73B30; 73C35; 73C60

1. Introduction

Waves propagation in a thermoelastic and thermoplastic homogeneous or non-homogeneous media has considerable importance in diverse topics, such as engineering, earthquakes, acoustics, and seismology, due to the non-homogeneities presence in the different layers of the earth's crust. A lot of materials, e.g., polymers, composites of solids, metals, and rocks, nearly have the same media properties as microstructures.

The strain and thermal fields coupling gave rise to the coupled thermoelasticity theory (CT theory). Duhamel [1,2] was the first postulated the originator of the thermal stresses theory who introduced the term of dilatation in equation of heat conduction. Biot [3] proposed the coupled thermoelasticity theory by inserting the modified Fourier heat conduction equation strain-rate term (diffusion equation) approaching the equation of heat conduction in a parabolic-type, which predicts a propagation with

finite speed for the waves in elastic media, although the thermal disturbance has infinite speed. This is physically unrealistic and conflicts with practical results. The theory of Lord-Shulman is the first generalization of the coupled theory. The model basis proposed by Lord and Shulman [4] modified the Fourier's law of the equation of heat conduction, presenting a new physical concept (acceleration needs a relaxation time for the heat flow) and denoted by (LS) theory. The heat equation that concerns the LS theory of these types of waves ensures finite speeds of propagation of elastic and heat waves. Lord and Shulman [4] and Green and Lindsay [5] presented generalized thermoelasticity theories advocating the finite wave speed due to the thermal signals in the solids. Chen and Gurtin [6] and Chen et al. [7] presented the theory of heat conduction in an elastic body depending on two different temperatures (conductivity temperature and thermodynamic temperature). Puri [8] pointed out the magneto-thermoelasticity effect on the plane waves' propagation.

Relaxation times during the process of generalized thermoelastic waves have been inserted to develop these theories made by modifying the conduction heat equation of Fourier's or reconsidering the Neuman-Duhamel relation (energy equation). Many researchers considered the electromagneto-thermoelastic wave propagation in a thermally conducting and electrically solid. The plane wave propagation in a solid considering the effect of the electro-magnetic field explained by Nayfeh and Nemat-Nasser [9]. Green and Naghdi [10–12] considered the concern (GN, types I, II, and III) theories; the linearized model-I version approaches to the model of thermoelastic in a classical form. Concerns (GN, type II) model, the production rate internal of entropy is identical to zero and implying no thermal energy dissipation. Also, this theory admits that the thermoelastic waves are undamped in a thermoelastic media and denoted by the thermoelasticity model with no energy dissipation. Finally, the GN-III model includes the two previous models as special cases admitting, in general, the dissipation of energy.

Chandrasekharaiah and Srinath [13] investigated the problem of thermoelastic waves without dissipation of energy due to an external point heat source. The waves' propagation phenomenon in solids, as a globe of elastic, was first discussed by Bromwich [14] who considered the gravity effect. Love [15] considered the effect of gravity parameter on Rayleigh waves and concluded that the Rayleigh waves velocity increased considerably from the gravity field for the large values of wavelengths. The gravity effect on propagation of waves in an elastic layer has been discussed [16]. Ezzat et al. [17] presented a magneto-thermoelasticity generalized model in a two-dimensional form. The problem of generalized thermoelasticity considering three theories and the state-space approach has been presented by Youssef and El-bary [18]. Youssef [19] formatted a new model of generalized thermoelasticity under two temperature dependence.

The heat conduction model of three-phase-lag has attracted considerable interest. Roy Choudhuri [20] considered the addition phase lag associated with the variable displacement of temperature to the two relaxation times presented by Tzou (please, see Ref. [21] and the therein references) that was related to the heat flux vector and the temperature gradient. The above-mentioned displacement variable of thermal is attributable to Green and Naghdi [10–12]. Youssef and Al-Lehaibi [22] pointed out some new problems with two temperature thermoelasticity considering relaxation time. The formulation of the state space vibration in the femtoseconds scale of nano-beam gold was studied by Elsibai and Youssef [23].

Abd-Alla and Abo-Dahab [24] pointed out the initial stress, diffusion, and rotation effect on a magneto-thermoelastic generalized problem with a spherical cavity. Abouelregal and Abo-Dahab [25] discussed the dual-phase-lag (DPL) model on non-homogeneous an infinite spherical cavity solid with a magnetic field. The initial stress and gravitational effect on generalized microstretch-magneto-thermoelastic solid medium for different models were presented by Othman et al. [26]. Lotfy [27] investigated interactions in an elastic generalized magneto-thermoelastic medium under two temperatures and three models. Lotfy and Hassan [28] explored thermal shock and two-temperature problems on the generalized thermoelasticity using normal mode techniques. The theory of two-temperature generalized thermoelasticity considering Youssef's theory for solving the boundary value

problems of two dimensional solid with different types of heating on its boundary was highlighted by Lotfy and Abo-Dahab [29]. The interaction between the magnetic field, thermal field, and elasticity considering heat transfer fractional derivative for a fibre-reinforced rotation thermoelastic has been investigated by Lotfy and Abo-Dahab [30]. Abo-Dahab et al. [31] discussed a generalized two dimensional thermoelasticity considering the magnetic field and rotation.

Recently, Lotfy et al. [32] have investigated the response of the thermomechanical model for semiconductor medium on a reflection diffusion photothermal waves. Abo-Dahab et al. [33] discussed the mechanical changes in a microstretch thermoelastic medium considering two-temperature due to pulse heating. Said and Othman [34] investigated the generalized electro–magneto-thermoelasticity considering three theories under the internal heat source and two temperatures in a finite conducting medium. Othman and Abd-Elaziz [35] applied three thermoelastic theories considering the micro-temperatures and gravity influence on the porous thermoelastic medium. Carini and Zampoli [36] studied in detail the three-phase-lag theory in porous matrices assuming three delay times in linearity of thermoelasticity. Marin et al. [37] discussed some results on the dipolar structure of bodies in the context of Green–Lindsay thermoelasticity. Kumar et al. [38] discussed the interactions of thermomechanical in magneto-thermoelastic and isotropic transversely considering GN type II theory under rotation.

The Earth’s electromagnetic impact through seismic propagation, the machine design of different elements, electromagnetic radiations emissions from plasma physics, nuclear devices, etc. has been discussed by more researchers. Marin and Craciun [39] investigated the uniqueness concerns the model composite materials boundary value problem in dipolar thermoelasticity.

In this paper, two-dimensional generalized thermoelasticity theory under two temperatures is considered for solve the boundary value problems half-space under the initial stress, gravity, and magnetic field. The governing equations have been solved using new mathematical methods considering Lord-Şhulman (LS), Green-Naghdi theory of type III (GN type III), and three-phase-lag (3PHL) theories to investigate displacement and stresses components and temperature distribution. Comparisons have been made with the predicted results by the three theories; (LS), (GN III), and (3PHL) with the absence and the presence of the magnetic field, initial stress, and gravity. The obtained results were numerically calculated and presented graphically to figure out the physical meaning of the phenomenon. A comparison has been made between the present results in the absence and the presence of the external considered parameters and with the previously obtained results by other researchers.

2. Formulation of the Problem

Considering an isotropic semi-infinite elastic solid, Oxyz is a Cartesian orthogonal coordinate system, any point O of the boundary of the plane, and Oy vertically downward to the medium, as displayed in Figure 1.

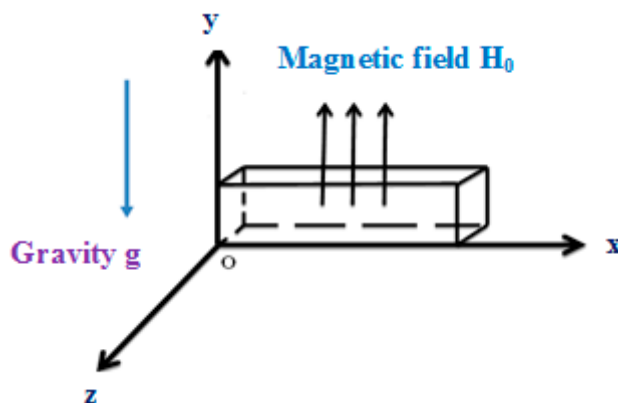


Figure 1. Shows the problem formulation.

We formulate the fundamental equation of the problem, as follows:

(i) The constitutive equation (stress–strain relation) considering initial stress takes the following form:

$$\sigma_{ij} = (\lambda\theta - \gamma TP)\delta_{ij} + 2\mu e_{ij} - P w_{ij}, \quad w_{ij} = \frac{1}{2}(u_{j,i} - u_{i,j}) \tag{1}$$

(ii) The equation of heat conduction assuming three thermoelastic theories forms is as follows (Kar and Kanoria [40]):

$$\left(K^* + \tau_v^* \frac{\partial}{\partial t} + K\tau_T \frac{\partial^2}{\partial t^2} \right) \nabla^2 \varphi = \left(1 + \tau_q \frac{\partial}{\partial t} + \frac{\tau_q^2}{2} \frac{\partial^2}{\partial t^2} \right) [\rho C_e \ddot{T} + \gamma T_0 \ddot{e}], \quad \tau_v^* = K + K^* \tau_v \tag{2}$$

(iii) The motion equation with body force and heat source absent takes the following form:

$$\sigma_{ji,j} + F_i = \rho \ddot{u}_i, \quad (i, j = 1, 2, 3) \tag{3}$$

(iv) The conductive and thermodynamic temperatures relation takes the form [19]

$$\varphi - T = a \nabla^2 \varphi \tag{4}$$

Consider that the displacement current is absent; the equations of linearized Maxwell’s that govern the magnetic field for a moving solid slowly having perfect electrical conductivity in the following form

$$\text{curl } \mathbf{h} = \mathbf{J} - \epsilon_0 \dot{\mathbf{E}}, \tag{5a}$$

$$\text{curl } \mathbf{E} = -\mu_e \frac{\partial \mathbf{h}}{\partial t}, \tag{5b}$$

$$\text{div } \mathbf{h} = 0, \tag{5c}$$

$$\text{div } \mathbf{E} = 0, \tag{5d}$$

$$\mathbf{E} = -\mu_e \left(\frac{\partial \mathbf{u}}{\partial t} \times \mathbf{H}_0 \right), \tag{5e}$$

$$\mathbf{E} = \text{curl}(\mathbf{u} \times \mathbf{H}_0), \tag{5f}$$

$$\mathbf{F}_i = \mu_e (\mathbf{J} \times \mathbf{H}_0)_i \tag{5g}$$

where we used

$$\mathbf{H} = \mathbf{H}_0 + \mathbf{h}(x, y, t), \quad \mathbf{H}_0 = (0, 0, H).$$

Using Equation (5), we obtain

$$\begin{aligned} F_x &= \mu_e H_0^2 \frac{\partial e}{\partial x} \\ F_z &= \mu_e H_0^2 \frac{\partial e}{\partial z} \\ F_y &= 0 \end{aligned} \tag{6}$$

The stress of Maxwell produced from the magnetic field can be formed as

$$\tau_{ij} = \mu_e \left[H_i h_j + H_j h_i - (\vec{H}_k \cdot \vec{h}_k) \delta_{ij} \right], \quad i, j = 1, 2, 3 \tag{7a}$$

which is reduced to

$$\tau_{xx} = \tau_{zz} = \mu_e H_0^2 \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial z} \right), \quad \tau_{xz} = 0. \tag{7b}$$

where all notations used have been defined in the “nomenclature” at the end of the paper.

Equation (2) is the generalized thermoelastic solid equation field, applicable to the following:

- i. (LS) theory: $K^* = \tau_v = \tau_T = \tau_q^2 = 0, \tau_q > 0$
- ii. (GN type II) theory: $\tau_v = \tau_T = \tau_q = 0$

iii. (3PHL) theory: $\tau_v < \tau_T < \tau_q > 0$

The non-dimensional variables take the following form

$$\begin{aligned} (x', z', u', v') &= C_0 \eta(x, z, u, v), (t', \tau'_T, \tau'_v, \tau'_q) = C_0^2 \eta(t, \tau_T, \tau_v, \tau_q), h' = \frac{h}{H_0} \\ (\theta', \varphi') &= \frac{(T, \varphi) - T_0}{T_0}, (\sigma'_{ij}, \tau'_{ij}) = \frac{(\sigma_{ij}, \tau_{ij})}{\rho C_0^2}, g' = \frac{g}{C_0^3 \eta} \end{aligned} \tag{8}$$

where $\eta = \frac{\rho C_e}{K}$, $C_2^2 = \frac{\mu}{\rho}$ and $C_0^2 = \frac{\lambda + 2\mu}{\rho}$.

When we substitute from Equation (8) into Equations (2)–(4), we get

$$\left(C_k + C_v \frac{\partial}{\partial t} + C_T \frac{\partial^2}{\partial t^2} \right) \nabla^2 \varphi - \left(1 + T_q \frac{\partial}{\partial t} + \frac{T_q^2}{2} \frac{\partial^2}{\partial t^2} \right) \left(\ddot{\theta} + \frac{\gamma}{\rho C_e} \ddot{e} \right) \tag{9}$$

$$\varphi - \theta = \beta \nabla^2 \varphi \tag{10}$$

where $C_k = \frac{K^*}{\rho C_e C_0^2}$, $C_v = \frac{\tau_v^*}{\rho C_e C_0^2}$, $C_T = \frac{K \tau_T \eta}{\rho C_e}$

The equations of motion approach

$$a_1^* \nabla^2 u + a_2 \frac{\partial e}{\partial x} - a_0 \frac{\partial \theta}{\partial x} + g \frac{\partial w}{\partial x} = \beta \ddot{u} \tag{11}$$

$$a_1^* \nabla^2 w + a_2 \frac{\partial e}{\partial z} - a_0 \frac{\partial \theta}{\partial z} - g \frac{\partial u}{\partial x} = \beta \ddot{w} \tag{12}$$

where $\varepsilon = \frac{\gamma}{\rho C_e}$, $a_1^* = \frac{2\mu - P}{2\rho C_0^2}$, $a_2 = \frac{2\lambda + 2\mu + P + 2\mu_e H_0^2}{2\rho C_0^2}$, $a_0 = \frac{\gamma T_0}{\rho C_0^2}$, $\beta = 1 + \frac{\varepsilon_0 \mu_e H_0^2}{\rho}$.

Assuming the scalar potential and vector potential functions π and ψ

$$u = \frac{\partial \Pi}{\partial x} - \frac{\partial \psi}{\partial z}, w = \frac{\partial \Pi}{\partial z} + \frac{\partial \psi}{\partial x} \tag{13}$$

Substituting from Equation (13) into Equations (11) and (12), we get

$$\left(\nabla^2 - \beta^* \frac{\partial^2}{\partial t^2} \right) \Pi - a_3^* \frac{\partial \psi}{\partial x} - a_0^* \theta = 0 \tag{14}$$

$$\left(\nabla^2 - \beta^{**} \frac{\partial^2}{\partial t^2} \right) \psi + a_4 \frac{\partial \Pi}{\partial x} = 0 \tag{15}$$

where

$$R_H^2 = \frac{\mu_e H_0^2}{\rho C_0^2}, \beta^* = \frac{\beta}{1 + R_H^2}, a_0^* = \frac{a_0}{1 + R_H^2}, a_3^* = \frac{g}{1 + R_H^2}, a_4 = \frac{g}{a_1^*}, \beta^{**} = \frac{\beta^*}{a_1^*}$$

The temperature Equation (9) approaches

$$\left(C_k + C_v \frac{\partial}{\partial t} + C_T \frac{\partial^2}{\partial t^2} \right) \nabla^2 \varphi - \left(1 + T_q \frac{\partial}{\partial t} + \frac{T_q^2}{2} \frac{\partial^2}{\partial t^2} \right) \frac{\partial^2}{\partial t^2} (\theta + \varepsilon \nabla^2 \pi) \tag{16}$$

3. Solution of the Problem

To solve Equations (10), and (11)–(13), we assume the following normal mode method:

$$[\Pi, \psi, \varphi, \theta, \sigma_{ij}] (x, z, t) = [\Pi^*, \psi^*, \varphi^*, \theta^*, \sigma_{ij}^*] (z) e^{i\omega(x-ct)} \tag{17}$$

Substituting from Equation (17) into Equations (14) and (15) using $D = \frac{d}{dz}$, we get:

$$[D^2 - A_1]\Pi^* - a_3^{**} \psi^* - a_0^* \theta^* = 0 \tag{18}$$

$$[D^2 - A_2]\psi^* + a_4^* \Pi^* = 0 \tag{19}$$

Equation (10) tends to

$$[D^2 - A_3] \varphi^* + \beta^{-1} \theta^* = 0 \tag{20}$$

Also, Equation (16) tends to

$$(D^2 - \omega^2)\varphi^* + B(D^2 - \omega^2)\Pi^* + A\theta^* = 0 \tag{21}$$

where $A_1 = \omega^2(1 - c^2\beta^*)$, $a_3^{**} = i\omega a_3^*$, $A_2 = \omega^2(1 - c^2\beta^{**})$, $a_4^* = i\omega a_4$, $A_3 = \frac{\beta\omega^2 + 1}{\beta}$.

where, $B = \varepsilon A$, $A = \omega^2 c^2 \left(\frac{1 - i\omega c(\tau_q - \frac{i\omega \tau_q^2}{2})}{C_k - i\omega(C_v - i\omega c C_T)} \right)$.

From Equations (18)–(21); eliminating Π^* , ψ^* , φ^* and θ^* , we obtain

$$\begin{vmatrix} D^2 - A_1 & -a_3^{**} & 0 & -a_0^* \\ a_4^* & D^2 - A_2 & 0 & 0 \\ 0 & 0 & D^2 - A_3 & \beta^{-1} \\ B(D^2 - \omega^2) & 0 & D^2 - \omega^2 & A \end{vmatrix} = 0$$

which tends to

$$[D^6 + AD^4 + BD^2 + C]\Pi^*(x) = 0 \tag{22}$$

where

$$\begin{aligned} A &= -\frac{(A - \beta^{-1})(A_1 + A_2) - \beta^{-1}\omega^2 + AA_3 + a_0^*B(\omega^2 + A_2 + A_3)}{(A - \beta^{-1} + a_0^*B)}, \\ B &= \frac{(A - \beta^{-1})[A_1A_2 + a_3^{**}a_4^*] - (A_1 + A_2)(\beta^{-1}\omega^2 - AA_3) + a_0^*B[\omega^2(A_2 + A_3) + A_2A_3]}{(A - \beta^{-1} + a_0^*B)}, \\ C &= \frac{(\beta^{-1}\omega^2 - AA_3)[A_1A_2 + a_3^{**}a_4^*] - a_0^*B\omega^2A_2A_3}{(A - \beta^{-1} + a_0^*B)}. \end{aligned} \tag{23}$$

In a similar behavior, we obtain

$$[D^6 + AD^4 + BD^2 + C](\psi^*, \varphi^*, \theta^*, \sigma_{ij}^*)(x) = 0 \tag{24}$$

which can be factorized to the following form

$$(D^2 - k_1^2)(D^2 - k_2^2)(D^2 - k_3^2)(\psi^*, \varphi^*, \theta^*, \sigma_{ij}^*)(x) = 0 \tag{25}$$

where $k_n^2 (n = 1, 2, 3)$ represent the following characteristic equation roots

$$K^6 + AK^4 + BK^2 + C = 0. \tag{26}$$

The Equation (25) solution, as $z \rightarrow \infty$, takes the form

$$\Pi^*(z) = \sum_{n=1}^3 M_n \exp(-k_n z), \tag{27}$$

$$\theta^*(z) = \sum_{n=1}^3 M'_n \exp(-k_n z), \tag{28}$$

$$\psi^*(z) = \sum_{n=1}^3 M_n'' \exp(-k_n z), \tag{29}$$

$$\varphi^*(z) = \sum_{n=1}^3 M_n''' \exp(-k_n z), \tag{30}$$

since

$$u^*(z) = i\omega \Pi^* - D \psi^*, \tag{31}$$

$$v^*(z) = D \Pi^* + i\omega \psi^*, \tag{32}$$

$$e^*(z) = i\omega u^* + D v^*. \tag{33}$$

Using Equations (31) and (32) to obtain the displacements amplitudes u and v taking into account that they are bounded as $x \rightarrow \infty$, we get

$$u^*(z) = i\omega \sum_{n=1}^3 (M_n + k_n M_n'' \exp(-k_n z)), \tag{34}$$

$$v^*(z) = -i\omega \sum_{n=1}^3 (k_n M_n + i\omega M_n'') \exp(-k_n z) \tag{35}$$

where the parameters M_n , M_n' , M_n'' , and M_n''' depend on c , β , and ω .

Using Equations (27)–(30) into Equations (18)–(21), we obtain

$$M' = \frac{(k_n^2 - A_1)(k_n^2 - A_2) + a_3^{**} a_4^*}{a_0^*(k_n^2 - A_2)} M_n = H_{1n} M_n, \tag{36}$$

$$M_n'' = -\frac{a_4^*}{K_n^2 - A_2} M_n = H_{2n} M_n, \tag{37}$$

$$M_n''' = -\beta^{-1} \left(\frac{(k_n^2 - A_1)(k_n^2 - A_2) + a_3^{**} a_4^*}{a_0^*(k_n^2 - A_2)(k_n^2 - A_3)} \right) M_n = H_{3n} M_n \tag{38}$$

where

$$H_{1n} = \frac{(k_n^2 - A_1)(k_n^2 - A_2) + a_3^{**} a_4^*}{a_0^*(k_n^2 - A_2)}, H_{2n} = -\frac{a_4^*}{K_n^2 - A_2},$$

$$H_{3n} = -\beta^{-1} \left(\frac{(k_n^2 - A_1)(k_n^2 - A_2) + a_3^{**} a_4^*}{a_0^*(k_n^2 - A_2)(k_n^2 - A_3)} \right)$$

where $n = 1, 2, 3$.

Thus, we have

$$\theta^*(z) = \sum_{n=1}^3 H_{1n} M_n \exp(-k_n z), \tag{39}$$

$$\psi^*(z) = \sum_{n=1}^3 H_{2n} M_n \exp(-k_n z), \tag{40}$$

$$\varphi^*(z) = \sum_{n=1}^3 H_{3n} M_n \exp(-k_n z). \tag{41}$$

From Equations (5), (14), and (33)–(35) into Equation (1), we can obtain

$$\sigma_{xx}^* = \sum_{n=1}^3 h_n M_n \exp(-k_n z) - \frac{P}{\lambda + 2\mu}, \tag{42}$$

$$\sigma_{zz}^* = \sum_{n=1}^3 h'_n M_n \exp(-k_n z) - \frac{P}{\lambda + 2\mu}, \tag{43}$$

$$\sigma_{xz}^* = \sum_{n=1}^3 h''_n M_n \exp(-k_n z), \tag{44}$$

$$\tau_{zz}^* = \frac{\mu_e H_0^2}{\lambda + 2\mu} [k_n(k_n - i\omega H_{2n}) - i\omega(i\omega + k_n H_{2n})], \tag{45}$$

$$h_n = i\omega(i\omega + k_n H_{2n}) + \frac{\lambda}{\lambda + 2\mu} k_n(k_n - i\omega H_{2n}) - \frac{\gamma T_0}{\lambda + 2\mu} H_{1n}, \tag{46}$$

$$h'_n = k_n(k_n - i\omega H_{2n}) + \frac{i\omega\lambda(i\omega + k_n H_{2n})}{\lambda + 2\mu} - \frac{\gamma T_0}{\lambda + 2\mu} H_{1n}, \tag{47}$$

$$h''_n = -\left(\frac{\mu + \frac{P}{2}}{\lambda + 2\mu}\right) k_n(i\omega + k_n H_{2n}) - \left(\frac{\mu - \frac{P}{2}}{\lambda + 2\mu}\right) i\omega(-k_n + i\omega H_{2n}). \tag{48}$$

4. Boundary Conditions (Application)

We will take the following application considering the thermal shock

(i) $\theta(x, 0, t) = f(x, 0, t)$

which tends to

$$\sum_{n=1}^3 H_{1n} M_n = f^* \tag{49}$$

(ii) $\sigma_{zz} + \tau_{zz} = \frac{-P}{\rho C_0^2}$

With the help of Equation (7b), it tends to the following form

$$\sum_{n=1}^3 \bar{h}'_n M_n = 0 \tag{50}$$

where

$$\bar{h}'_n = \left[k_n(k_n - i\omega H_{2n})(1 + \mu_e H_0^2) - i\omega(i\omega + k_n H_{2n}) \left(\frac{\lambda}{\lambda + 2\mu} + \mu_e H_0^2 \right) - \frac{\gamma T_0}{\lambda + 2\mu} H_{1n} \right]$$

Finally,

(iii) $\sigma_{xz} + \tau_{xz} = 0$

which tends to

$$\sum_{n=1}^3 h''_n M_n = 0 \tag{51}$$

Equations (49)–(51) can be rewritten in the form of matrices, as the form:

$$\begin{pmatrix} M_1 \\ M_2 \\ M_3 \end{pmatrix} = \begin{pmatrix} H_{11} & H_{12} & H_{13} \\ \bar{h}'_1 & \bar{h}'_2 & \bar{h}'_3 \\ h''_1 & h''_2 & h''_3 \end{pmatrix}^{-1} \begin{pmatrix} f^* \\ 0 \\ 0 \end{pmatrix}. \tag{52}$$

5. Numerical Results

To display the figure out for the physical meaning of the obtained thee earlier analytical procedure, we assume a numerical example for obtaining computational results in Table 1. The graphs show the influence of thermoelastic theories, initial stress, electro-magnetic field, and gravity with respect to

distance z on the displacement components u and w , components of stress σ_{xx} , σ_{zz} , σ_{xz} , τ_{zz} and $\sigma_{zz} + \tau_{zz}$, as well as temperature T .

Table 1. Physical constants for chosen material [41].

Parameter	Value	Parameter	Value
λ	$7.76 \times 10^9 \text{ N m}^{-2}$	K	$150 \text{ W m}^{-1} \text{ K}^{-1}$
μ	$3.86 \times 10^9 \text{ N m}^{-2}$	C_e	$383.1 \text{ J Kg}^{-1} \text{ K}^{-1}$
ρ	8954 Kg m^{-3}	α	$-1.28 \times 10^9 \text{ N m}^{-2}$
α_t	$1.78 \times 10^{-4} \text{ K}^{-1}$	K^*	$386 \text{ W m}^{-1} \text{ K}^{-1} \text{ s}^{-1}$
T_0	293 K	η	8886.73 m s^{-2}

Additionally, we used the constant values $\omega = \omega_0 + i\xi$, $\omega_0 = 2$, $\xi = 1$, $a = 1$, $f^* = 1$, $\tau_0 = 0.02 \text{ s}$, and $\varepsilon = 0.0168 \text{ F m}^{-1}$.

Considering these physical constants, the MATLAB package is used for calculating the numerical results.

From all the analytical results and numerical calculations via graphs obtained, we obvious that all physical quantities satisfy the boundary conditions at the origin point considered at the starting of the movement of the waves and approach to zero as the axial x approaches to infinity. This indicates that if these quantities vanish, the waves far from the origin point at the wall or the boundary agree with the practical results, and the previous results obtained by others.

First, from Figures 2–25, we can discuss the influence of initial stress P on the procedure calculated in the previous obtained results. Figures 2 and 3 show that the components of displacement components u and w have a strong effect on neglecting initial stress at the interval $(0, 1.15)$. However, they have a strong effect on the presence of initial stress at the interval $(1.15, 1.7)$. The decrease or increase approaches to zero as the distance z approaches to infinity. It is shown that the normal stresses displayed in Figures 4 and 5 σ_{xx} and σ_{zz} behave in the same manner during the intervals $(0, 0.3)$. While they have a strong impact with the initial stress, with the absence of P at $(0.3, 0.8)$ they have a strong effect. After that, the increase or decrease periodically approaches zero as the distance approaches to infinity. While the absence of P has a strong effect on the shear stress σ_{xz} at the interval $(0, 0.65)$ that schematics in Figure 6, it has a strong effect on $(0.65, 1.4)$ and periodically decreases or increases approach zero as z approaches infinity. Figure 7 illustrates that Maxwell’s normal stress τ_{zz} has a strong effect on the interval $z \in (0, 0.35)$ in the absence of initial stress, taking an inverse behavior for $(0.35, 1)$. After that, it periodically decreases or increases as large values of the distance z approaches zero. The total normal mechanical and Maxwell stresses have a strong behavior in the presence of P for $z \in (0, 0.3)$. In the absence of P , it decreases in $(0.3, 0.65)$ and it is interrupted periodically tending to zero as z approaches to infinity (see, Figure 8). Finally, Figure 9 displays the temperature distribution concerns to z considering the absence and presence of P .

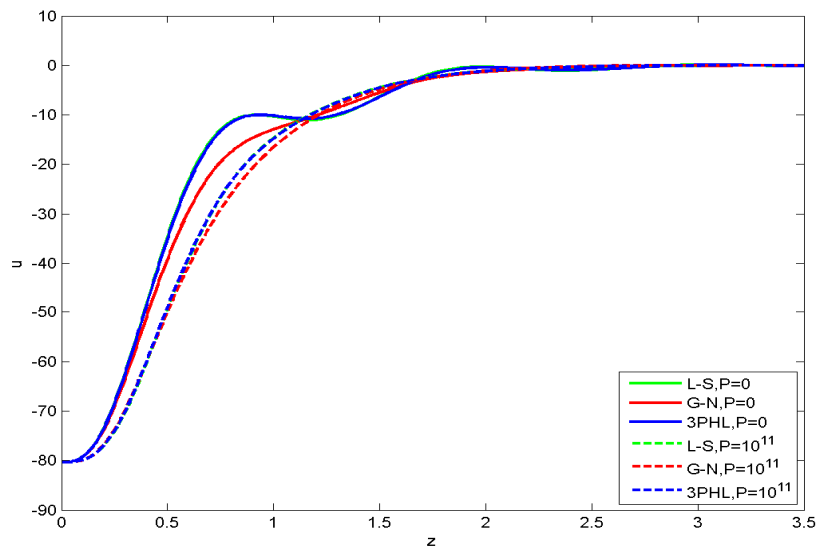


Figure 2. Displacement u concerning z under thermoelastic theories.

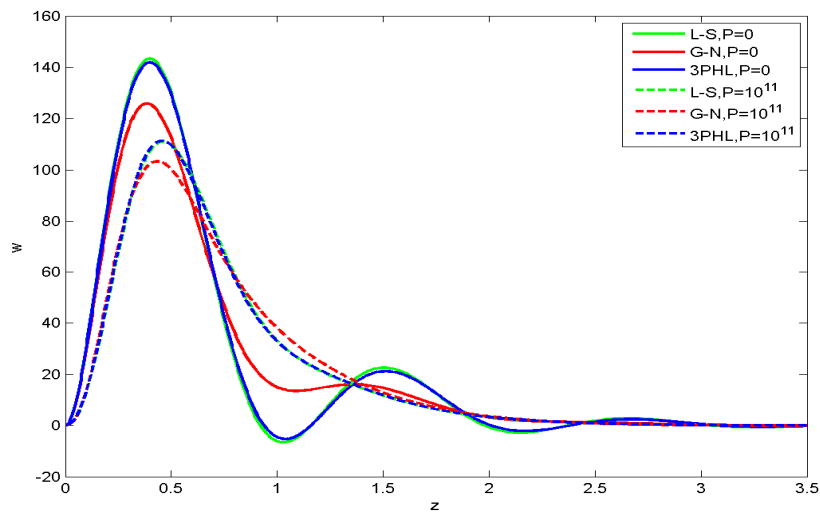


Figure 3. Displacement w concerning z under thermoelastic theories.

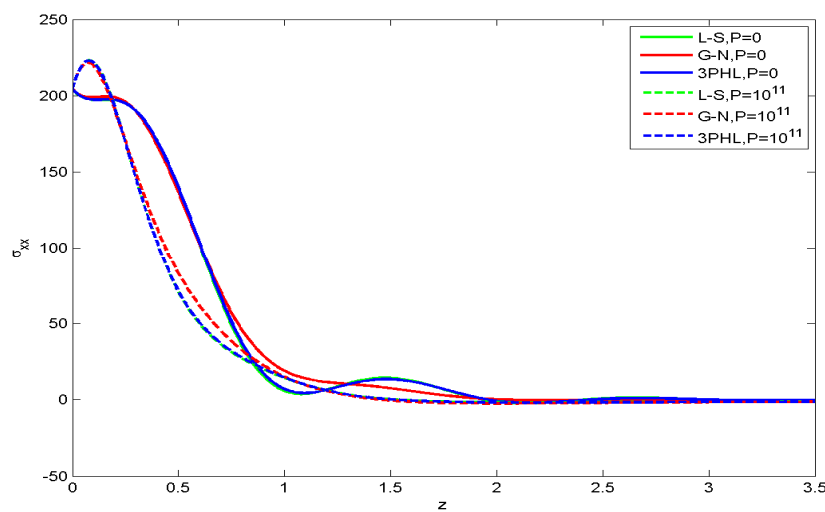


Figure 4. Normal stress σ_{xx} concerning z under thermoelastic theories.

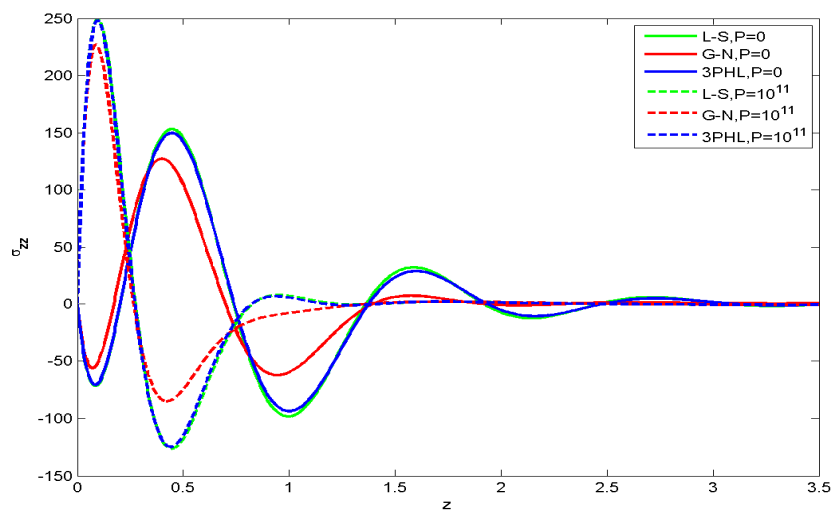


Figure 5. Normal stress σ_{zz} concerning z under thermoelastic theories.

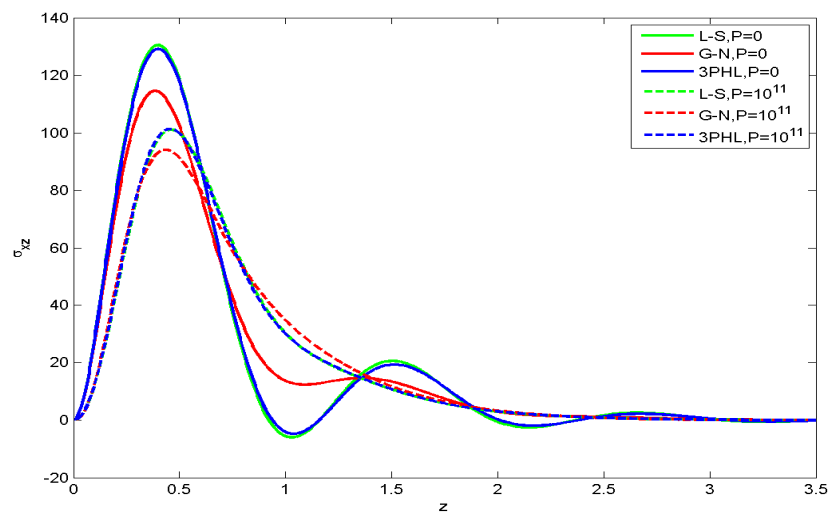


Figure 6. Shear stress σ_{xz} concerning z under thermoelastic theories.

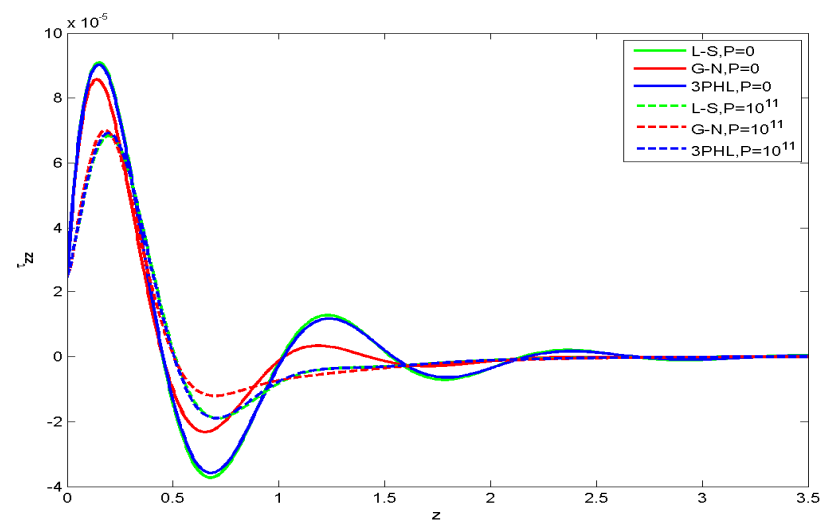


Figure 7. Maxwell's stress τ_{zz} concerning z under thermoelastic theories.

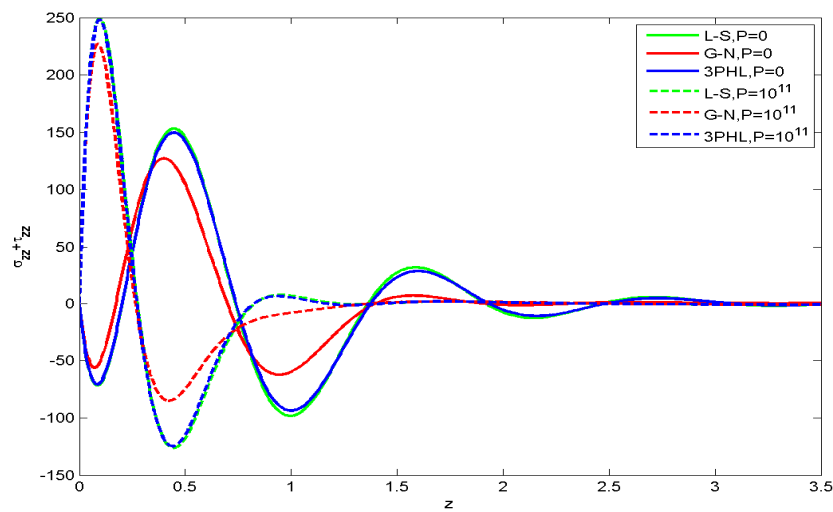


Figure 8. Total stresses $\tau_{zz} + \sigma_{zz}$ concerning z under thermoelastic theories.

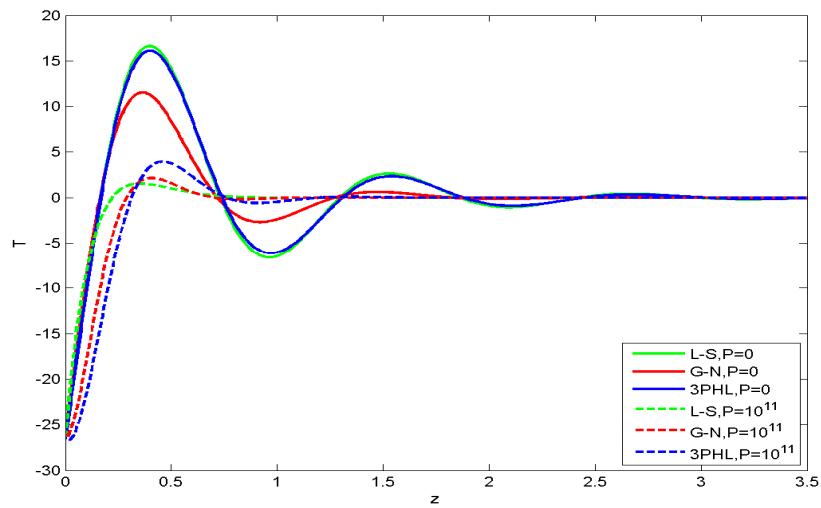


Figure 9. Temperature T concerning z under thermoelastic theories.

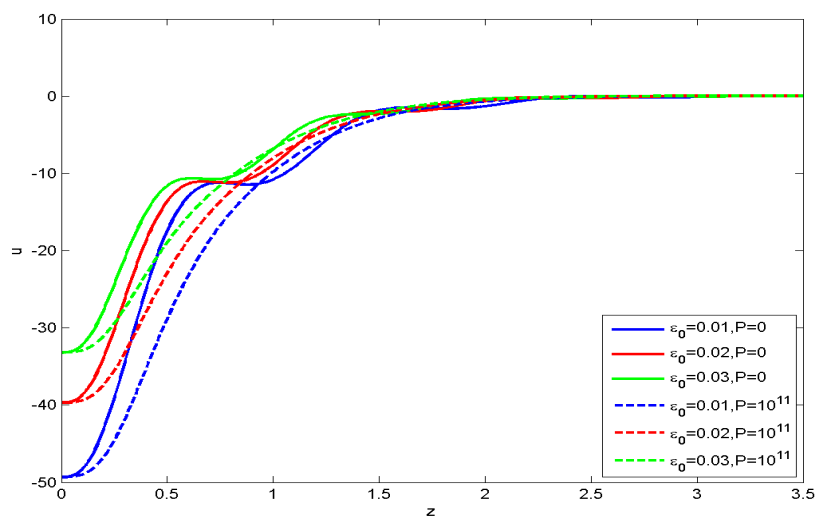


Figure 10. Horizontal displacement u concerning z with electric field ϵ_0 .

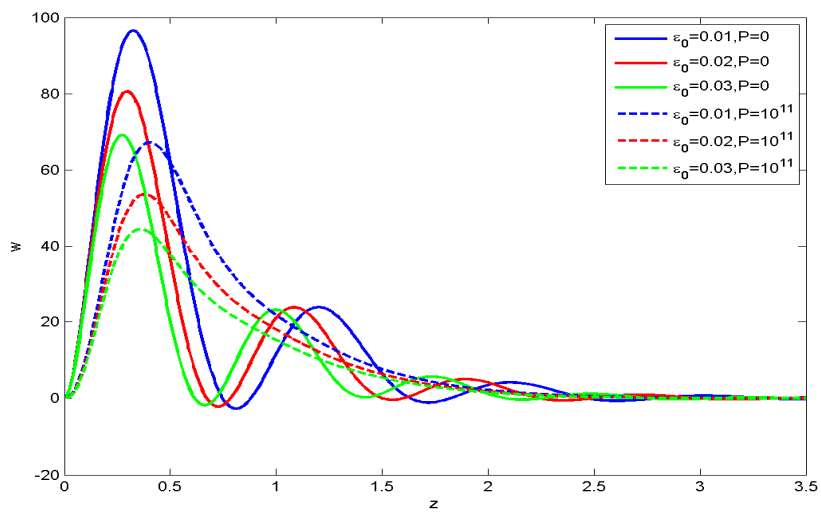


Figure 11. Vertical displacement w concerning z with electric field ϵ_0 .

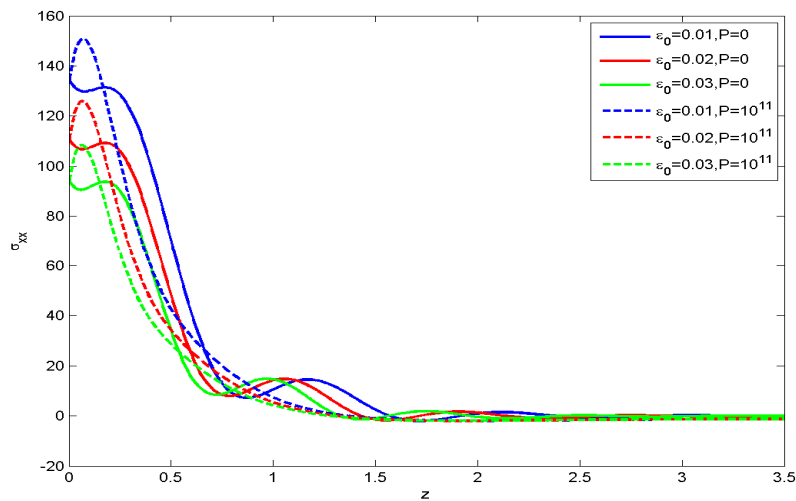


Figure 12. Normal stress σ_{xx} concerning z with electric field ϵ_0 .

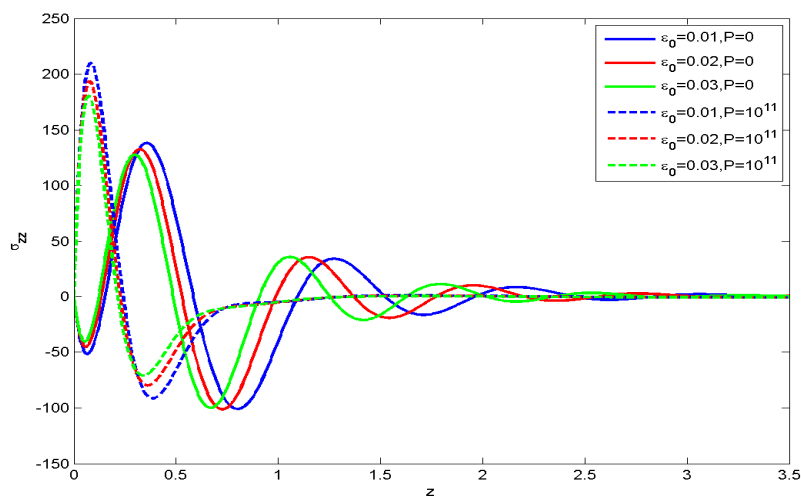


Figure 13. Normal stress σ_{zz} concerning z with electric field ϵ_0 .

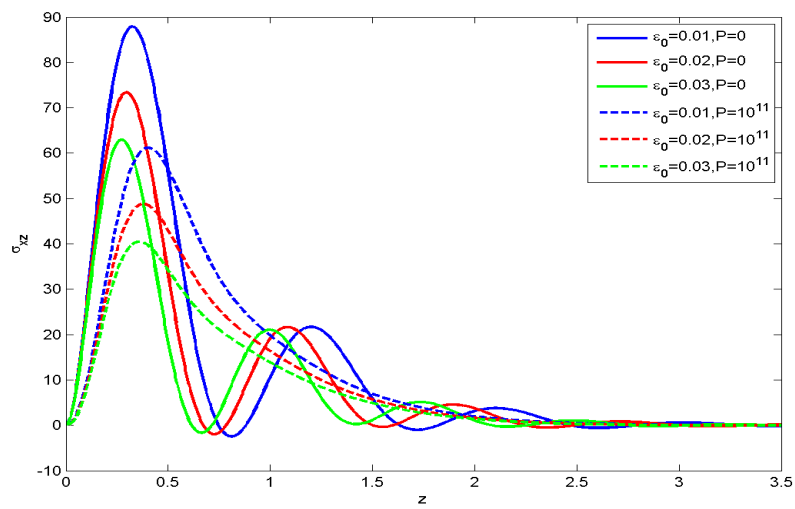


Figure 14. Normal stress σ_{xz} concerning z with electric field ϵ_0 .

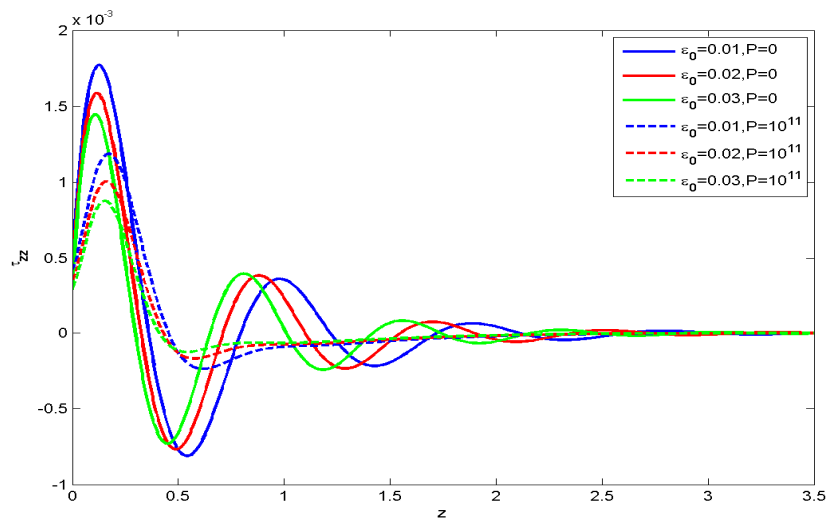


Figure 15. Maxwell stress τ_{zz} concerning z with electric field ϵ_0 .

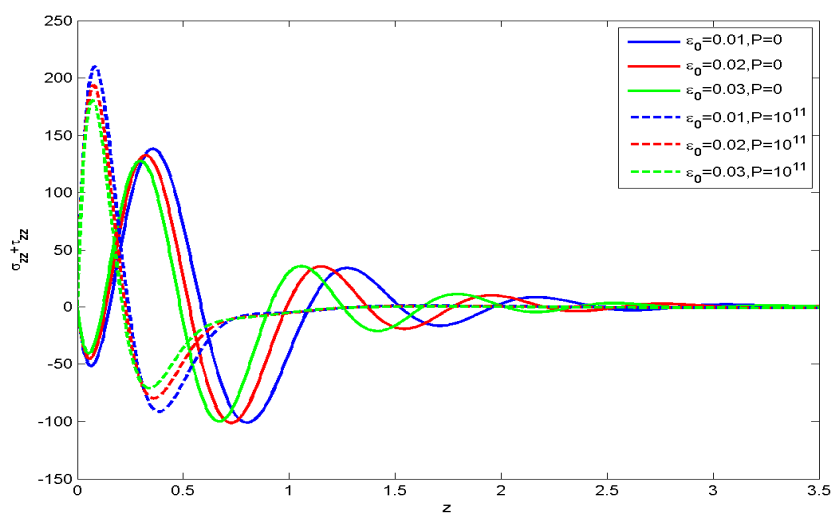


Figure 16. Total stresses $\tau_{zz} + \sigma_{zz}$ concerning z with electric field ϵ_0 .

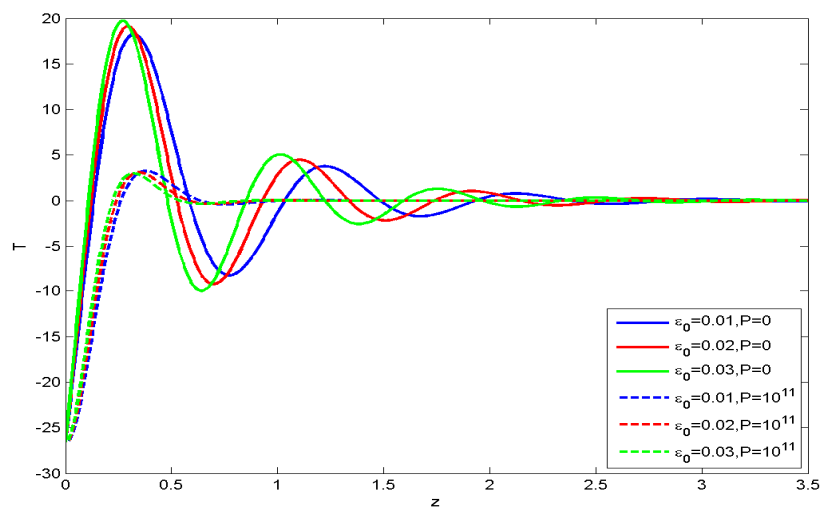


Figure 17. Temperature T concerning z with electric field ϵ_0 .

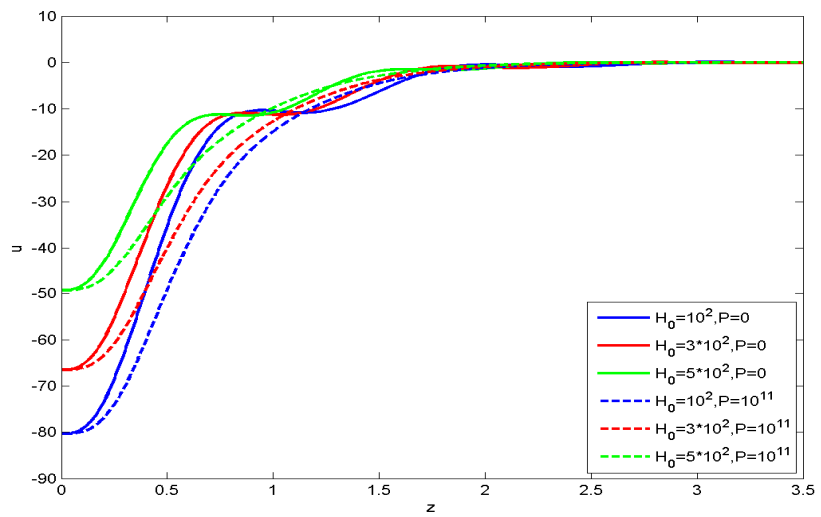


Figure 18. Horizontal displacement u concerning z with magnetic field H_0 .

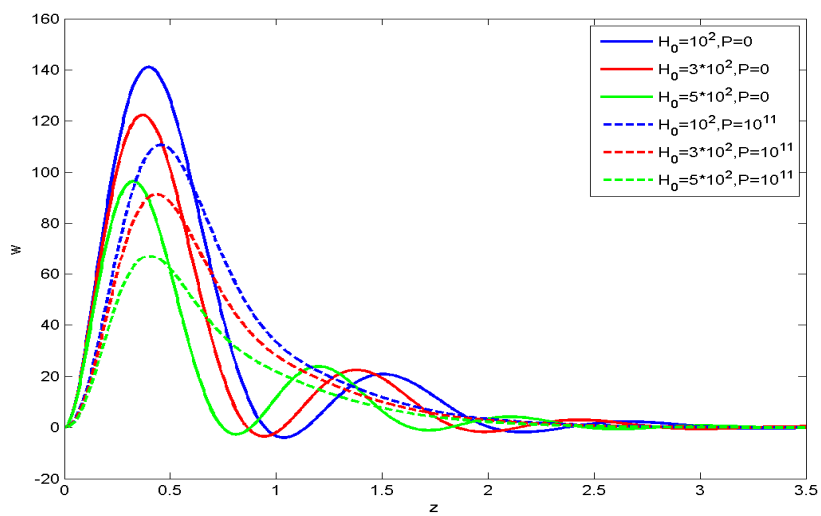


Figure 19. Vertical displacement distribution w concerning z with magnetic field H_0 .

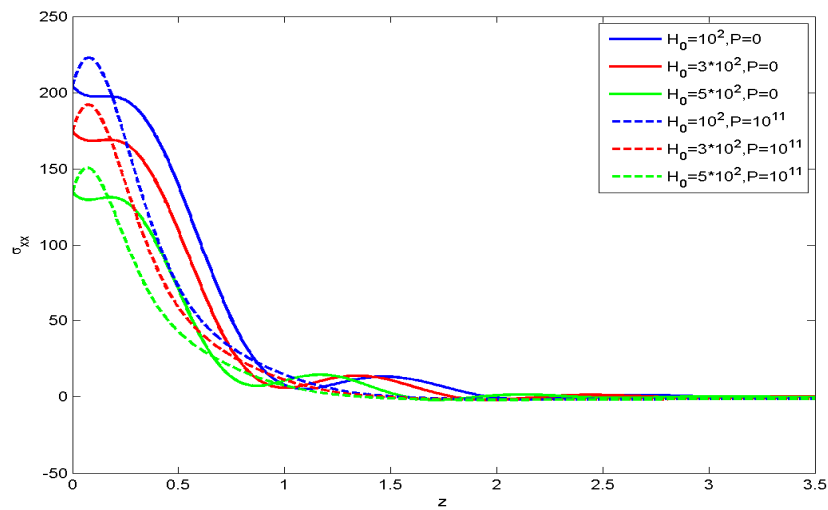


Figure 20. Normal stress σ_{xx} concerning z with magnetic field H_0 .

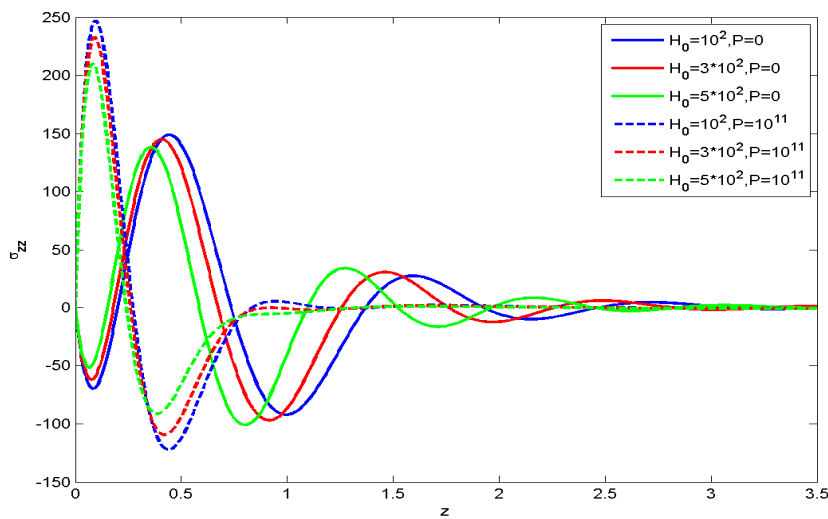


Figure 21. Normal stress σ_{zz} concerning z with magnetic field H_0 .

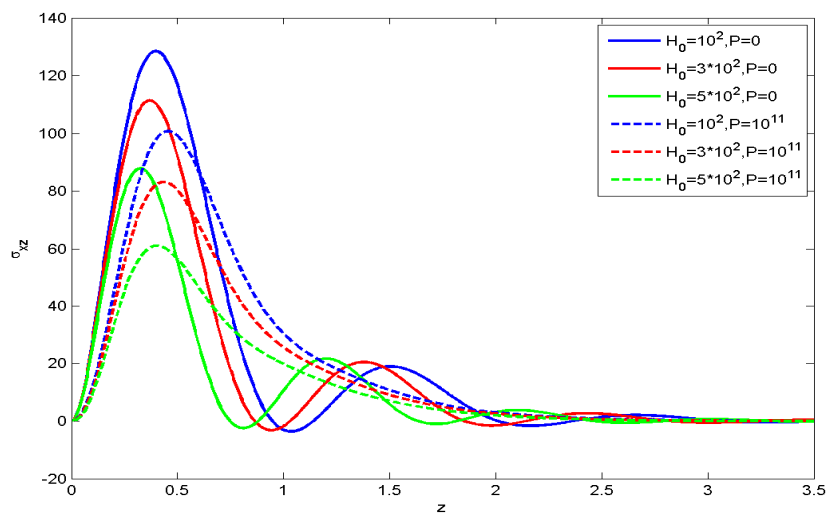


Figure 22. Shear stress σ_{xz} concerning z with magnetic field H_0 .

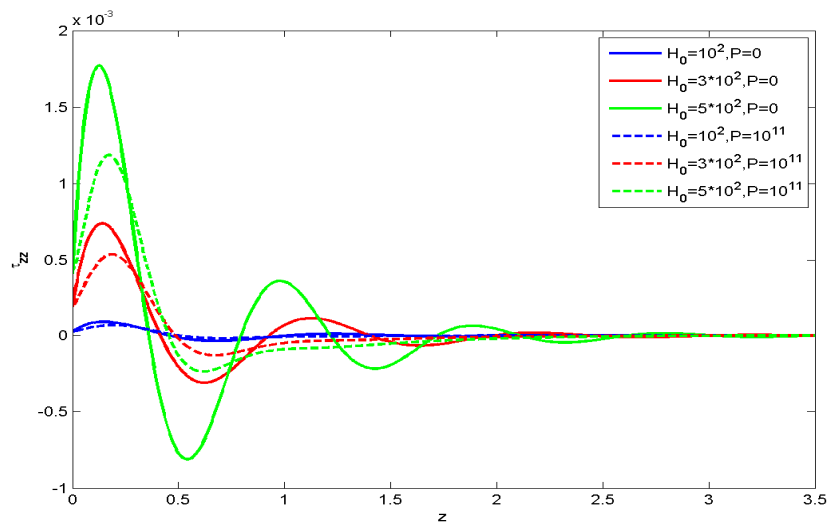


Figure 23. Maxwell stress τ_{zz} concerning z with magnetic field H_0 .

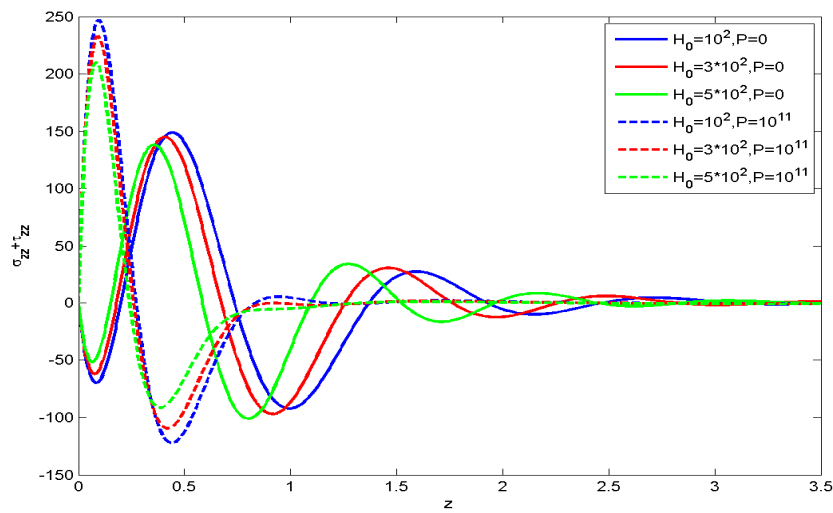


Figure 24. Total stress $\tau_{zz} + \sigma_{zz}$ concerning z with magnetic field H_0 .

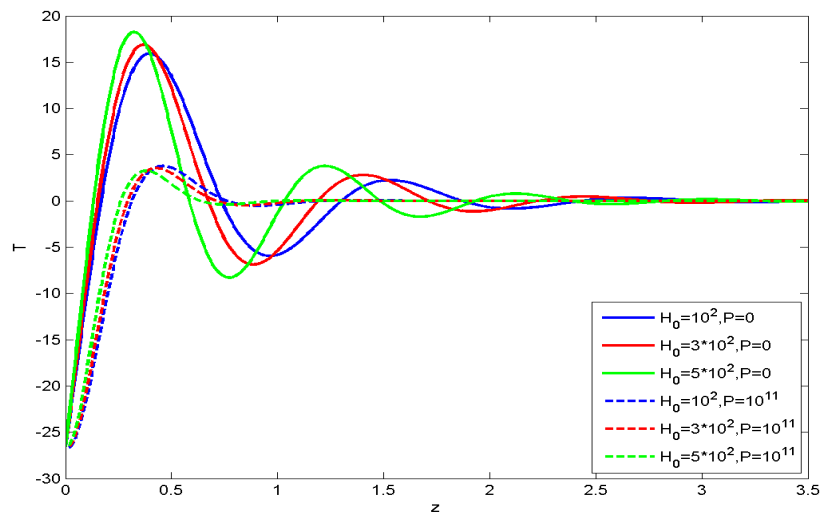


Figure 25. Temperature T concerning z with magnetic field H_0 .

Figures 2–9 illustrate the influence of theories with respect to distance z on the displacement components u and w ; the components of stress σ_{xx} , σ_{zz} , σ_{xz} , τ_{zz} and $\sigma_{zz} + \tau_{zz}$; as well as the temperature T . From these figures, we noticed the strong effect of Green–Naghdi theory (G–N) compared with the slight effect of Lord–Shulman (L–S), and the three-phase-lag model (3PHL) on u ; w ; σ_{xx} , σ_{zz} , σ_{xz} , τ_{zz} , $\sigma_{zz} + \tau_{zz}$; and T . We display this effect in detail. In Figure 2, for $z \in (0, 1.25)$, the displacement component u decreases for z in $(1.25, 1.7)$. Then, u periodically decreases or increases because the large values of z approach zero. The displacement component w behaves the same as u but in a different interval of the distance z , as shown in Figure 3. Figures 4 and 5 show that the components of stress σ_{xx} and σ_{zz} have the same manner concerning z , since they increase for $z \in (0, 0.3)$ and decrease for $z \in (0.3, 0.7)$. After that, they periodically increase or decrease because the large values of z approach zero. However, σ_{xz} , as shown in Figure 6, decreases when z increases in $(0, 0.65)$, increases in $(0.65, 1.4)$, and periodically decreases or increases approaching zero as z approaches infinity. Likewise, τ_{zz} decreases for $z \in (0, 0.3)$ and increases for $z \in (0.3, 0.8)$. Then, τ_{zz} decreases or increases periodically approaching zero as z approaches infinity (see, Figure 7). Figure 8 illustrates that $\sigma_{zz} + \tau_{zz}$ have an inverse behavior when compared with τ_{zz} . The temperature distribution T decreases for $z \in (0, 0.7)$ and increases for $z \in (0.7, 1.4)$. After that, T decreases or increases periodically approaching zero as z approaches infinity (see Figure 9).

The influence of the electric field on the displacement components u and w , the components of stress σ_{xx} , σ_{zz} , σ_{xz} , τ_{zz} and $\sigma_{zz} + \tau_{zz}$; and the temperature T concerning distance z appears in Figures 10–17. In Figure 10, the displacement component u increases with respect to z as the increased of electric field and approaches to zero as z approaches to infinity. On the contrary, w decreases with respect to z as the electric field increases and approaches to zero as z approaches to infinity, as shown in Figure 11. Moreover, Figure 12 illustrates that the stress component σ_{xx} decreases with respect to z as the electric field increases and approaches zero as z approaches infinity. However, σ_{zz} decreases for $z \in (0, 0.4)$ and increases for $z \in (0.4, 0.7)$ as the electric field increases and approaches zero as z approaches infinity, as seen in Figure 13. While the electric field increases in Figure 14, it decreases when σ_{xz} z takes the interval $(0, 0.65)$ and approaches zero as z approaches infinity. However, τ_{zz} decreases for $z \in (0, 0.5)$ and increases for $z \in (0.5, 1)$ as the electric field increases and approaches zero as z approaches infinity, as shown in Figure 15. Likewise, $\sigma_{zz} + \tau_{zz}$ decreases for $z \in (0, 0.4)$ and increases for $z \in (0.4, 0.7)$ as the electric field increases to zero as z approaches infinity, as shown in Figure 16. The temperature distribution T , on the contrary to $\sigma_{zz} + \tau_{zz}$, increases for $z \in (0, 0.4)$ and decreases for $z \in (0.4, 0.7)$ as the electric field increases and approaches zero as z approaches infinity as shown in Figure 17.

The Figures 18–25 show the impact of the magnetic field concerning distance z on the components u , w , σ_{xx} , σ_{zz} , σ_{xz} , τ_{zz} , $\sigma_{zz} + \tau_{zz}$, and T . In Figure 18, the component of displacement u increases with respect to z as the magnetic field increases and approaches zero as z approaches infinity. On the contrary, w decreases concerning z as the magnetic field increases and approaches zero as z approaches infinity, as shown in Figure 19. Figure 20 illustrates that the stress component σ_{xx} decreases with respect to z as the magnetic field increases and approaches zero as z approaches infinity. However, σ_{zz} decreases for $z \in (0, 0.45)$ and increases for $z \in (0.45, 0.7)$ as the magnetic field increases and approaches to zero as z approaches to infinity, see Figure 21. As the magnetic field increases in Figure 22, σ_{xz} decreases with respect to z and approaches to zero as z approaches infinity. However, τ_{zz} increases for $z \in (0, 0.5)$ and decreases for $z \in (0.5, 1.7)$ as the magnetic field increases and approaches zero as z approaches infinity (see, Figure 23). Figure 24 shows that $\sigma_{zz} + \tau_{zz}$ decreases for $z \in (0, 0.4)$ but increases for $z \in (0.4, 0.7)$ as the magnetic field increases and approaches zero as z approaches infinity. As shown in Figure 25, the temperature distribution T , on the contrary to $\sigma_{zz} + \tau_{zz}$, increases for $z \in (0, 0.45)$ and decreases for $z \in (0.45, 0.7)$ as the magnetic field increases and approaches to zero as z approaches infinity.

Figures 26–33 highlight the influence of the gravity with respect to distance z on the components of displacement u and w ; the components of stress σ_{xx} , σ_{zz} , σ_{xz} , τ_{zz} and $\sigma_{zz} + \tau_{zz}$; and the temperature T . In Figure 26, the displacement component u increases for z as the gravity increases and approaches

zero as z approaches infinity. Figure 27 shows that as the gravity increases, w decreases for $z \in (0, 0.45)$, increases for $z \in (0.45, 0.7)$, and approaches zero as z approaches infinity. Additionally, Figure 28 shows that the component of stress σ_{xx} increases concerning z as the gravity increases and approaches zero as z approaches to infinity. However, σ_{zz} increases for $z \in (0, 0.35)$ and decreases for $z \in (0.35, 1.7)$ because the gravity increases and approaches zero as z approaches infinity, see Figure 29. As gravity increases in Figure 30, σ_{xz} increases for $z \in (0, 0.3)$, decreases for $z \in (0.3, 0.7)$, and approaches zero as z approaches infinity. Likewise, τ_{zz} increases for $z \in (0, 0.6)$ and decreases for $z \in (0.6, 0.8)$ because the gravity increases and approaches zero as z approaches infinity, as displayed in Figure 31. As the gravity increases, $\sigma_{zz} + \tau_{zz}$ decrease for $z \in (0, 0.4)$ and increase for $z \in (0.4, 0.7)$ and approach zero as z approaches infinity, as shown in Figure 32. The gravity effect on the temperature distribution T concerning z is shown in Figure 33. Physically, it is obvious that all dependent components approach zero as the distance approaches infinity. This agrees with the physical meaning of the waves phenomenon if it is far from the origin point of propagation.

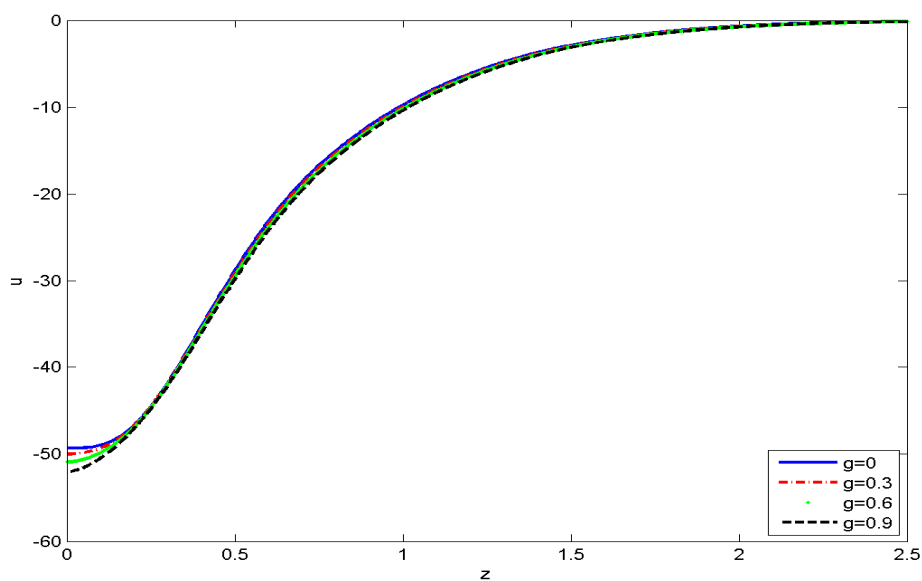


Figure 26. Horizontal displacement distribution u concerning z with gravity g .

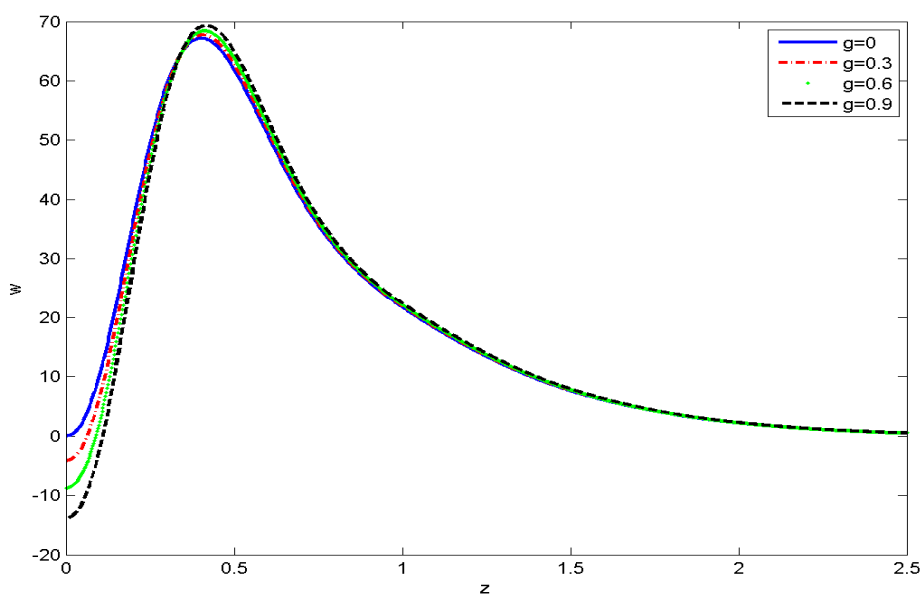


Figure 27. Vertical displacement w concerning z with gravity g .

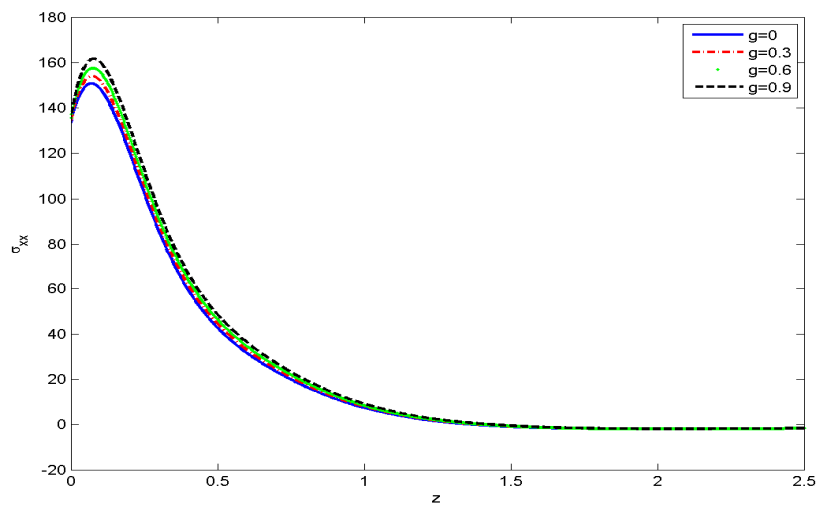


Figure 28. Normal stress σ_{xx} concerning z with gravity g .

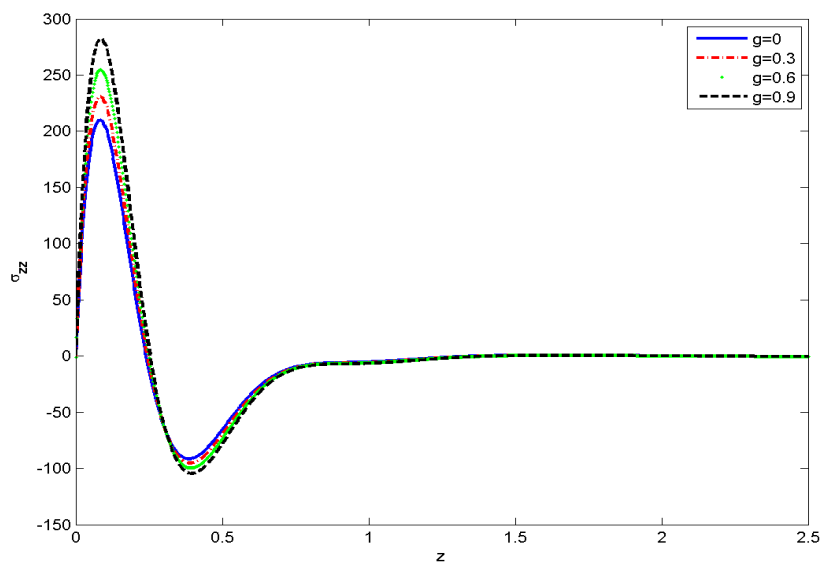


Figure 29. Normal stress σ_{zz} concerning z with gravity g .

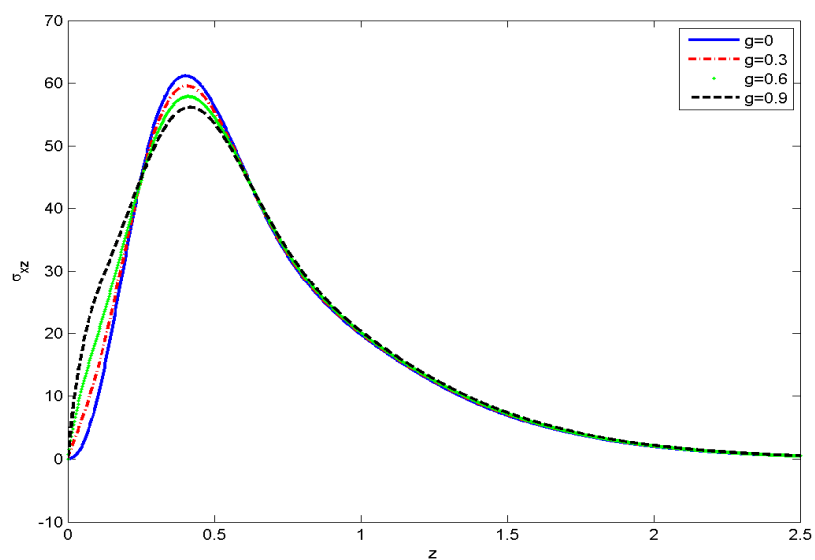


Figure 30. Shear stress σ_{xz} concerning z with gravity g .

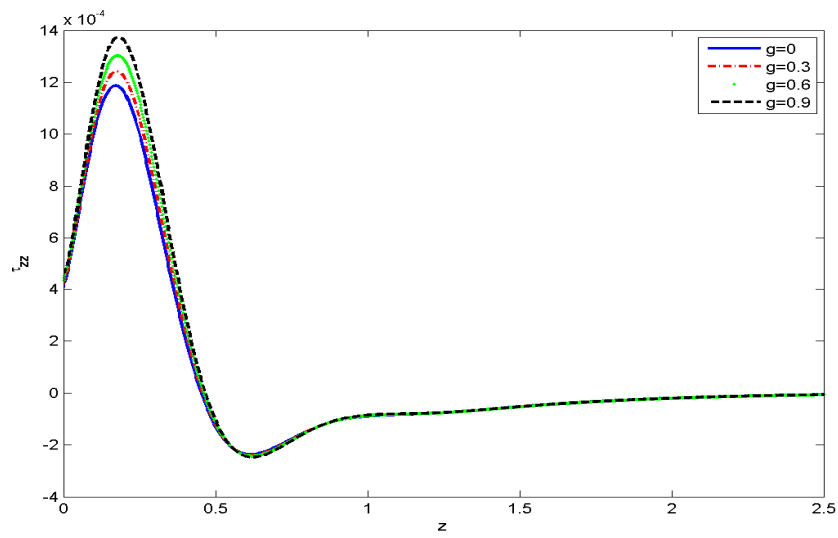


Figure 31. Maxwell stress τ_{zz} concerning z with gravity g .

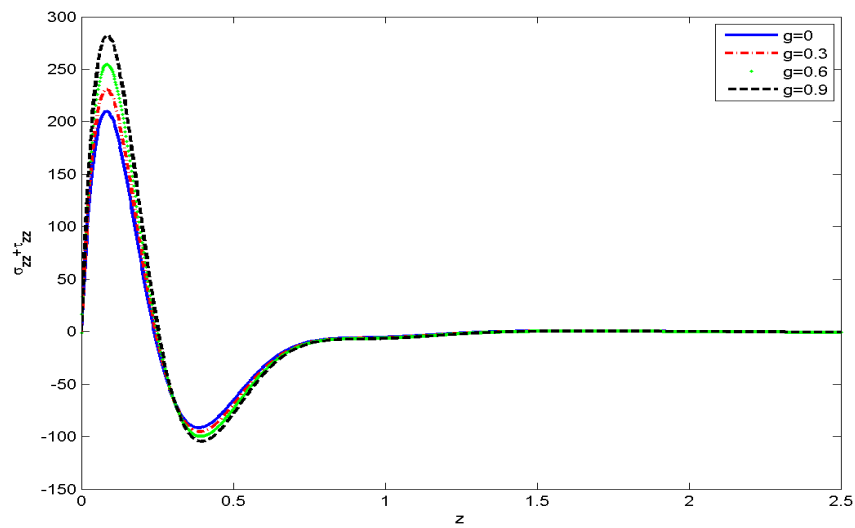


Figure 32. Total stress $\tau_{zz} + \sigma_{zz}$ concerning z with gravity g .

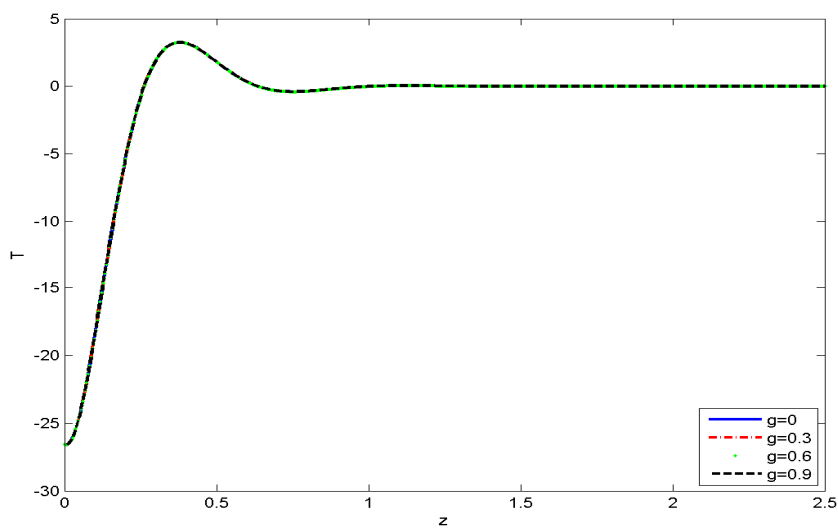


Figure 33. Temperature T concerning z with gravity g .

Finally, when the initial stress and gravity parameters are vanishes, the obtained results are deduced to the study of Said and Othman [34] which considered GN (type II) theory and internal heat source as special case from the present study.

6. Conclusions

In this work, we presented an analytical solution based upon the Lamé potentials and the normal mode technique for the problem of thermoelastic in a solid medium has been developed, utilized, and compared via graphs. The strong effect of Green-Naghdi theory (GN) compared with the slight effect between Lord–Shulman (LS) and three-phase-lag model (3PHL). The physical quantities converge to zero with an increasing of the distance z and continuously satisfying the assumed boundary conditions. The body deformation depends on the nature of external forces applied (electromagnetic field, two-temperature, initial stress, and gravity), also, the type of thermoelastic theories and boundary conditions. The time parameter, as well as relaxation time, gravity, and electromagnetic field have a strong effect and play a significant strong role in all the physical quantities obtained for the components of stresses, components of displacement, and temperature decreasingly or increasingly. Therefore, the presence of the field of electro-magnetic, gravity, two-temperature, initial stress, and relaxation times in the present model is of significance. The considered method is interesting and applicable to a wide range of phenomena in thermodynamics, thermoelasticity, and magneto-thermoelasticity. The transient behaviors of field variables are studied in detail, and the influences of variations in field variables on each other are discussed. Thus, they provide useful information for practical scientists/ technologists/ researchers/ seismologists/ engineers work in this experimental field on propagation of waves. This paper introduced the effect of gravity, initial stress, electromagnetic field, and two temperatures dependence on the components of displacement, temperature, and components of stress that indicate to their significant effects. Finally, the results provide a significant motivation to study the magneto-thermoelectric conducting materials as a new applicable class of electro-magneto-thermoelectric solids and should prove the useful for the researchers in the material science, designers of new materials, physicists, engineers, and those working on the electro-magneto-thermoelasticity development and in practical situations, especially in optics, geomagnetic, geophysics, acoustics, and oil prospecting.

Author Contributions: The idea of the paper was proposed and stimulated by S.M.A.-D. and Y.A.-H. They also participated in writing the main part of manuscript along with A.A.E.-B. and M.A., S.M.A.-D. made major contributions particularly in drawing the figures and the editorial part. S.M.A.-D. and A.A.E.-B. made a significant contribution at the revision stage. Y.A.-H. and M.A. prepared and formulated the references and were responsible in collaboration with S.M.A.-D. for manuscript revision. All authors contributed equally to the calculations and numerical results. All authors have read and agreed to the published version of the manuscript.

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Conflicts of Interest: The authors declare no conflict of interest.

Nomenclature

a	Two-temperature parameter
\mathbf{B}	The vector of induced magnetic field
C_e	Specific heat per unit mass
\mathbf{E}	Vector of induced electric field
e	Strain
e_{ij}	Tensor of strain
\mathbf{F}	Vector of body forces of Lorentz's
\mathbf{h}	Perturbed magnetic field
\mathbf{H}_0	Vector of primary magnetic field
\mathbf{J}	Vector of electric current density
K	Thermal conductivity characteristic the medium
K^*	Thermal conductivity
P	Initial stress
T	Absolute temperature
T_0	Medium natural temperature; $ (T - T_0)/T_0 < 1$
u_i	Vector of displacement
α_t	Linear thermal expansion coefficient
δ_{ij}	Kronecker delta
ϵ_0	Electric permittivity
$\eta = \frac{\rho C_e}{K}$	Entropy per unit mass
$\theta = T - T_0$	Thermodynamical temperature
λ, μ	Lame's parameters
μ_e	Magnetic permeability
ρ	Density
σ_{ij}	Stress tensor
τ_{ij}	Maxwell's stress tensor
τ_q, τ_T, τ_v	Relaxation times
$\varphi = \varphi_0 - T$	Conductive temperature

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