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A Fuzzy Approach for Ranking Discrete Multi-Attribute Alternatives under Uncertainty

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Abstract: This paper presents a fuzzy approach for ranking discrete alternatives in multi-attribute decision-making under uncertainty. Linguistic variables approximated by fuzzy numbers were applied for facilitating the making of pairwise comparison by the decision maker in determining the alternative performance and attribute importance using fuzzy extent analysis. The resultant fuzzy assessments were aggregated using the simple additive utility method for calculating the fuzzy utility of each alternative across all the attributes. An ideal solution-based procedure was developed for comparing and ranking these fuzzy utilities, leading to the determination of the overall ranking of all the discrete multi-attribute alternatives. An example is provided that shows the proposed approach is effective and efficient in solving the multi-attribute decision making problem under uncertainty, due to the simplicity and comprehensibility of the underlying concept and the efficiency and effectiveness of the computation involved.

Keywords: fuzzy extent analysis; pairwise comparison; multi-attribute decision-making; ideal solutions; degree of dominance; fuzzy numbers

1. Introduction

Multi-attribute decision making problems are often present in real-world settings, in which discrete alternatives must be assessed by the decision maker to determine their overall rankings, with respect to multiple, usually conflicting, attributes under uncertainty [1–5]. To effectively solve this problem, subjective assessments usually have to be made to evaluate the alternative performance and attribute importance in a given situation [6,7]. These subjective assessments must then be aggregated in a computationally efficient manner to evaluate the overall performance of each alternative across all the attributes. This leads to the determination of the overall ranking of all the discrete multi-attribute alternatives under uncertainty [8–11].

Uncertainty is always present in human decision making in the real world [12–16]. This uncertainty does not only take the form of randomness that is usually dealt with using the probability theory [12,17]. Other forms of uncertainties such as inexactness, subjectiveness, vagueness, and imprecision are often present [12]. This is due to the existence of (a) incomplete information, (b) subjective assessments, (c) conflicting evidences, and (d) too much information in the decision-making process [12,18]. As a result, the process of human decision making often becomes complex and challenging [8,12,19]. To effectively deal with such uncertainties in human decision making, the fuzzy sets theory [20] has been widely used [8,12,21].

There exists a large amount of literature in multi-attribute decision making. Numerous approaches have been developed for solving various practical problems of different kinds in the real world [12,21–24], dating back to the fundamental work of Bellman and Zadeh [18]. Many successful applications of these approaches in solving specific real-world problems under different circumstances have been reported in literature [1,3,5,12]. There are, however, still several issues and concerns that are present on how multi-attribute decision making problems can be better solved, including (a) the challenge of using crisp numbers to express the subjective assessment of the decision maker in the decision making process [6,14], (b) the sophistication of using fuzzy numbers to represent the subjective and imprecise assessment of the decision maker [1], and (c) the computation effort required to efficiently calculate the performance index value of each alternative across all the attributes [1,16]. As a result, the development of effective approaches, capable of adequately solving the problem of multi-attribute decision making under uncertainty, is not only desirable but also necessary [4,12,17].

This paper presents a fuzzy approach for ranking discrete multi-attribute alternatives in multi-attribute decision-making under uncertainty. To better tackle the uncertainty inherent in the human decision-making process while effectively reducing the cognitive demand on the decision maker, linguistic variables approximated by fuzzy numbers were applied to facilitate the making of pairwise comparison [6] in determining the alternative performance and attribute importance using fuzzy extent analysis [25]. To appropriately aggregate the resultant fuzzy assessments, the simple additive utility method based on the utility theory [22,26] was adopted for calculating the fuzzy utility of each alternative across all the attributes. To avoid the unreliable and often complex process of comparing and ranking fuzzy utilities [6], an ideal solution-based procedure using the concept of the degree of dominance [27] was developed for comparing these fuzzy utilities, leading to determination of the overall ranking of the discrete multi-attribute alternatives across all the attributes. An example is provided to demonstrate the applicability of the fuzzy approach for solving the multi-attribute decision making problem under uncertainty in the real world. The result shows that the proposed approach is attractive in solving multi-attribute decision making problems under uncertainty, due to the simplicity and comprehensibility of the underlying concept and the efficiency and effectiveness of the computation involved.

The remaining sections in this article are organized as follows. Section 2 presents a brief review of the related literature, including multi-attribute decision-making and uncertainty modeling, to justify the need for this study. This paves the way for the development of a fuzzy approach to solve the multi-attribute decision making problem under uncertainty in Section 3. In Section 4, a real-life example is provided, followed by the conclusion in Section 5, in which the limitation of this study and the future research were discussed.

2. Multi-Attribute Decision Making under Uncertainty

A multi-attribute decision making problem usually consists of a set of available alternatives $\{A_i, i = 1, 2, \ldots, n\}$. These alternatives must be compared with respect to multiple, usually conflicting attributes $\{C_j, j = 1, 2, \ldots, m\}$ to determine their overall rankings with respect to all the attributes in a given situation [9,22]. Based on the classical utility theory [22,26], the alternative performance with respect to each attribute and the attribute importance against the overall objective of the problem must be subjectively assessed before aggregating these subjective assessments together to determine the overall performance of each alternative across all the attributes in a given situation [12]. This leads to the determination of the overall ranking of the discrete multi-attribute alternatives for facilitating the decision-making process in real situations.

Mathematically, the alternative performance with respect to each attribute can be represented as x_{ij} $(i = 1, 2, \ldots, n; j = 1, 2, \ldots, m)$ [12]. This leads to the determination of a decision matrix for the given multi-attribute decision making problem as follows:

$$\mathbf{X} = \begin{vmatrix} x_{11} & x_{12} & \ldots & x_{1m} \\ x_{21} & x_{22} & \ldots & x_{2m} \\ & & \ldots & \\ x_{n1} & x_{n2} & \ldots & x_{nm} \end{vmatrix}, \quad (1)$$

where x_{ij} is the performance assessment of alternative A_i, with respect to attribute C_j.

In a similar manner, the attribute importance, with respect to the overall objective of the multi-attribute decision making problem, can be assessed subjectively by the decision maker. This results in the determination of the attribute weighting, given as:

$$W = (w_1, w_2, \ldots, w_m), \tag{2}$$

where w_j ($j = 1, 2, \ldots, m$) is the weighting of the attribute C_j.

With the subjective assessments given above, a utility function is often implicitly or explicitly defined to aggregate these subjective assessments to produce an overall performance index value for each alternative across all the attributes [12,26]. This leads to the determination of the overall ranking of all the discrete alternatives across all the attributes in a given situation.

Subjectively assessing the performance of alternatives and the importance of attributes in real situations in a precise manner by the decision maker is difficult, if not impossible, in multi-attribute decision making [1,12,26,27]. This is due to the existence of uncertainties, in terms of the subjectiveness and imprecision inherent in human decision-making [2,27–29]. As a result, classical multi-attribute decision making approaches [22] are unsatisfactory in solving multi-attribute decision making problems under uncertainty [12]. This is due to their inability to effectively tackle such subjective and imprecise information in multi-attribute decision making [7,9,10,24]. To adequately resolve this difficulty, the fuzzy sets theory [20] has been widely used.

Bellman and Zadeh [18] are the first ones to apply the fuzzy sets theory [16] in their pioneering work to address the decision-making problem under uncertainty. This led to numerous approaches that were developed to solve multi-attribute decision making problems under uncertainty in the literature [2,12,13,15]. A comprehensive review of the related literature shows that four distinct types of approaches were developed in the literature for solving the multi-attribute decision making problem under uncertainty including (a) fuzzy ranking approaches [30], (b) fuzzy analytic hierarchy process approaches [4,31], (c) defuzzification-based approaches [6], and (d) fuzzy outranking approaches [12]. As discussed above, such existing approaches were not totally satisfactory for solving multi-attribute decision making problems under uncertainty, due to various issues and concerns present [1,6,12,17,20,32].

Multi-attribute decision making processes usually consist of three phases, including (a) assessing the alternative performance with respect to each attribute and evaluating the attribute importance with respect to the overall objective of the problem, (b) aggregating the assessments on the performance of alternatives and the importance of attributes for each alternative across all the attributes, and (c) selecting or ranking all the available alternatives based on the aggregated performance index value for each alternative across all the attributes [12,13]. It is common in real situations that the decision maker must subjectively assess the alternative performance and the attribute importance in multi-attribute decision making [17,23].

It is cognitively demanding on the decision maker to make subjective assessments on alternative performance and attribute importance in a precise manner in multi-attribute decision making [4,25]. To effectively reduce the cognitive burden on the decision maker in the subjective evaluation process, linguistic variables are often used to facilitate the making of subjective assessments on alternative performance and attribute importance in the decision-making process [6,10,12].

A linguistic variable is usually characterized by a quintuple (x, $T(x)$, X, G, M) [12]. X is the name of the variable. $T(x)$ denotes the term-set of x, that is, the set of terms of linguistic values of x, with each value being a fuzzy variable denoted generically by x and ranging over a universe of discourse X, which is associated with the base variable x. The variable G is a syntactic rule, usually in a grammatical form, for generating the term, X, of value x. M is a semantic rule for associating with each X its meaning, $M(x)$, which is a fuzzy subset of X. The base variable could also be vector-valued [9].

Linguistic variables are effective for approximating the subjective assessments of the decision maker in decision making in a fuzzy environment [3,12,14] and for describing complex phenomena which are hard to define precisely [4,12]. They form the basis for approximate reasoning in fuzzy decision making and fuzzy controlling [12]. The motivation for using linguistic variables rather than real numbers is that the linguistic characterizations of subjective assessments of the decision maker are, in general, less specific than numerical ones [12]. Traditionally, these linguistic variables are often represented in real numbers in classical multi-attribute decision making [22]. Such representation provides the decision maker with a convenient way to quantify the subjective assessments of the decision maker in decision making.

There are many criticisms about the appropriateness of real number representation in expressing the subjective assessment of the decision maker in multi-attribute decision making under uncertainty [6,18]. This is because real numbers cannot be realistically used to reflect the preference of the decision maker in evaluating the alternative performance and attribute importance. It is often difficult, if not impossible, to use such real numbers to express their subjective assessments, even if the decision maker knows the problem well [4].

To facilitate the making of subjective assessments in multi-attribute decision making under uncertainty, the technique of pairwise comparison [6,30] is adopted for assessing the alternative performance and attribute importance. To adequately tackle the uncertainty in human decision making, linguistic variables approximated by fuzzy numbers are applied to express the pairwise comparison assessment of the decision maker [4,25]. Table 1 presents the linguistic variables and their fuzzy number approximations for facilitating the making of pairwise comparison assessments in multi-attribute decision-making under uncertainty.

Table 1. Linguistic variables and their fuzzy number approximations.

Linguistic Variables	Fuzzy Numbers
Very Low (VL)	(1, 1, 3)
Low (L)	(1, 3, 5)
Medium (M)	(3, 5, 7)
High (H)	(5, 7. 9)
Very High (VH)	(7, 9, 9)

3. A Fuzzy Approach

Comparing and ranking discrete multi-attribute alternatives under uncertainty is complex and challenging [2,12]. Existing approaches are not totally satisfactory due to various issues and concerns, as discussed above [1,6,12]. To better address such issues and concerns in real world situations, this section presents a fuzzy approach for solving the multi-attribute decision-making problem under uncertainty. Several dominant concepts in fuzzy decision making, including pairwise comparison [6,30], fuzzy extent analysis [25], idea solutions [1,6,22,28], and degree of dominance [29,32], have been effectively combined in the development of the fuzzy approach for ranking the discrete multi-attribute alternatives under uncertainty.

To effectively reduce the cognitive demand on the decision maker in making the subjective assessments in human decision making, the fuzzy approach starts with assessing the alternative performance with respect to each attribute and the attribute importance against the overall objective of the problem, using the technique of pairwise comparison [6,25]. To better tackle the uncertainty in the pairwise comparison process, linguistic variables approximated by fuzzy numbers, defined as in Table 1, are used to express the subjective assessment of the decision maker in the evaluation process. As a result, $m + 1$ pairwise comparison matrices can be produced, in which m is the number of attributes, given as follows:

$$C_j \text{ or } W = \begin{bmatrix} \bar{a}_{11} & \bar{a}_{12} & \cdots & \bar{a}_{1k} \\ \bar{a}_{21} & \bar{a}_{22} & \cdots & \bar{a}_{2k} \\ \cdots & \cdots & \cdots & \cdots \\ \bar{a}_{k1} & \bar{a}_{k2} & \cdots & \bar{a}_{kk} \end{bmatrix}. \tag{3}$$

Fuzzy extent analysis combines the concept of extent analysis with the degree of possibility to calculate the weight from the fuzzy comparison matrices [25]. With the application of fuzzy extent analysis on (3) [4,25], the performance of each alternative (A_i) against each attribute C_j ($j = 1, 2, \ldots, m$), denoted as x_{ij}, and the attribute importance represented as w_j ($j = 1, 2, \ldots, m$) can be determined as

$$x_{ij} \text{ or } w_j = \frac{\sum_{s=1}^{k} \bar{a}_{ls}}{\sum_{l=1}^{k} \sum_{s=1}^{k} \bar{a}_{ls}}, \tag{4}$$

where $i = 1, 2, \ldots, n$; $j = 1, 2, \ldots, m$; and $k = m$ or n, depending on whether the reciprocal assessment matrix is for evaluating the alternative performance or attribute importance in a given situation.

Utility is a measure of desirability or satisfaction of the decision maker in a given situation [26]. It is related to the quantitative representation of the qualitative preference of the decision maker in subjectively assessing the performance of two alternatives in a given situation [12,26]. The use of such a concept in multi-attribute decision-making under uncertainty, provides the decision maker with a uniform scale to compare and rank discrete alternatives across all the attributes. This leads to the development of an additive utility function in multi-attribute decision-making for calculating the overall performance utility of each alternative across all the attributes [12,26].

Based on the simple additive utility function, commonly used in multi-attribute decision-making [16,17,22], the overall performance utility of each alternative across all the attributes can be calculated through aggregating the fuzzy assessments on alternative performance and attribute importance from (3) and (4), given as follows:

$$U(A_i) = \sum_{j=1}^{m} w_j x_{ij}. \tag{5}$$

$U(A_i)$ ($i = 1, 2, \ldots, n$) represents how well alternative A_i satisfies the decision maker's overall objective, with respect to the multiple, usually conflicting, attributes in the decision-making situation. The higher the utility of the alternative is, the more preferred the alternative [22].

When crisp numbers are used to represent the utilities of these alternatives, as in (5), comparing and ranking the discrete multi-attribute alternatives is simple and straightforward [18]. With the use of linguistic variables approximated by fuzzy numbers shown as in Table 1, the alternative utilities determined as in (5) are not crisp numbers but fuzzy utilities [22,24]. Due to the overlap between fuzzy utilities, commonly existent in real situations, comparing and ranking these fuzzy utilities to determine their overall rankings is not a trivial task [24,29,33].

Numerous procedures have been developed to compare and rank fuzzy utilities in the literature [12,24,29]. A comprehensive review of these ranking procedures shows that four groups of procedures can be identified in the literature, including (a) independent ranking, (b) reference-based ranking, (c) pairwise comparison-based ranking, and (d) linguistic approximation [22]. These ranking procedures have been developed from different perspectives, with numerous applications for solving various practical problems in real world settings [12,29,33].

The ranking procedures as discussed above are not totally satisfactory in comparing and ranking fuzzy utilities in multi-attribute decision-making under uncertainty [12,26,33,34]. There are various shortcomings in applying these procedures to compare and rank fuzzy utilities in decision making in a fuzzy environment. Some ranking procedures, for example, lack discrimination in differentiating similar fuzzy numbers [33,34]. Other ranking procedures are sophisticated and often computationally very demanding [31]. Furthermore, inconsistent and often counter-intuitive ranking outcomes are often produced under circumstances. This makes the decision-making process, through comparing and ranking fuzzy utilities, complex and challenging [6,12,26].

To adequately address the shortcomings of existing ranking procedures as above, a new ranking procedure is proposed in this section. Such a novel ranking procedure adopts the concept of ideal solutions [18,35], in terms of the fuzzy maximum and the fuzzy minimum [26] and the degree of dominance [29], for comparing and ranking these fuzzy utilities from (5). The use of this procedure can produce more consistent ranking outcomes in multi-attribute decision making under uncertainty due to the simplicity and comprehensibility of the underlying concept and the efficiency in computation involved in the decision-making process.

The proposed ranking procedure is based on the concept of idea solutions that are commonly used in multi-attribute decision making [18,22,28]. With the adoption of such a concept in multi-attribute decision making under uncertainty, the ideal solution, including the positive ideal solution and the negative ideal solution, must be determined first [35]. In this situation, the positive (negative) ideal solution represents the best (worst) possible value of each alternative, with respect to each attribute in the multi-attribute decision making problem [35].

Following this concept in developing the proposed procedure in ranking fuzzy utilities, the positive ideal solution and the negative ideal solution based on the available fuzzy utilities of the discrete alternatives from (5) can be determined as follows:

$$U^{+} = (u_{\min},\ u_{\max},\ u_{\max}) \tag{6}$$

$$U^{-} = (u_{\min},\ u_{\min},\ u_{\max}), \tag{7}$$

where u_{min} and u_{max} are the minimum and the maximum of the support of all the fuzzy utilities respectively in (5), represented as follows:

$$u_{\min} = \min \bigcup_{i=1}^{m} \sup(U(A_i)) \tag{8}$$

$$u_{\max} = \max \bigcup_{i=1}^{m} \sup(U(A_i)) \tag{9}$$

The concept of the degree of dominance is often used in traditional mathematics to compare two real numbers to determine how much bigger one real number is compared to the other with simple computation [28]. This concept is popular for developing various approaches and procedures in decision making and in fuzzy number ranking [12,28]. This is because the concept is simple to understand and efficient for computation [22].

Extending such a concept to the comparison of two fuzzy numbers, the degree of dominance that the positive ideal solution, defined as in (6), is on the fuzzy utility $U(A_i)$ of alternative A_i (i = 1, 2, ... , n) from (5) can then be calculated as:

$$d_i^+ = d(U^+ - U(A_i)) = \int_{u_{\min}}^{u_{\max}} D_{\max-i}(\alpha)\, d\alpha, \tag{10}$$

where

$$D_{\max-i} = U^+ - A_i = \left\{(z, \mu_{D_{\max-i}}(z)), z \in R\right\} \tag{11}$$

and the membership function of $D_{max\text{-}i}$ is defined using fuzzy mathematics [9] as

$$\mu_{D_{\max-i}}(z) = \sup_{z=x_i-x_j} (\min(\mu_{U^+}(x_i),\ \mu_{A_i}(x_j)), \quad x_i, x_j \in R). \tag{12}$$

Following the same process as above, the degree of dominance that each fuzzy utility $U(A_i)$ of alternative A_i (i = 1, 2, ... , n) from (5) has on the negative ideal solution, defined as in (7), can be determined as follows:

$$d_i^+ = d(U^+ - U(A_i)) = \int_{u_{\min}}^{u_{\max}} D_{\max-i}(\alpha)\, d\alpha, \tag{13}$$

where

$$D_{\max-i} = U^+ - A_i = \left\{(z, \mu_{D_{\max-i}}(z)), z \in R\right\}, \tag{14}$$

where the membership function of $D_{i\text{-}min}$ is determined using fuzzy mathematics [9] by

$$\mu_{D_{i-\min}}(z) = \sup_{z=x_i-x_j} (\min(\mu_{U^-}(x_i),\ \mu_{A_i}(x_j)), \quad x_i, x_j \in R). \tag{15}$$

The overall ranking of discrete alternatives across all the attributes can be determined by comparing and ranking the resulting fuzzy utilities, $U(A_i)$, from (5). Based on the ideal solution concept [28], the overall performance of each alternative across all the attributes can be calculated by simultaneously considering the degree of dominance between the fuzzy utility of alternatives, the positive ideal solution, and the negative ideal solution [12,22,28,36–38]. This leads to the calculation of an overall performance index value for each alternative A_i (i = 1, 2, ... , n) across all the attributes C_j (j = 1, 2, ... , m) as:

$$P_i = \frac{(d_i^-)^2}{(d_i^+)^2 + (d_i^-)^2}. \tag{16}$$

Based the overall performance index value as shown above, an alternative A_i (i = 1, 2, ... , n) is preferred if it has a larger index value [30,37]. This is consistent with the underlying principle of using the ideal solution concept, in which an alternative is preferred if it is closer to the positive ideal solution, while being farther away from the negative ideal solution at the same time [30,31,37]. This further shows that the underlying concept in the developed fuzzy approach is simple and comprehensible.

To better understand the proposed fuzzy approach above for ranking multi-attribute discrete alternatives, a flow diagram describing how the multi-attribute decision making process using the developed fuzzy approach works is depicted in Figure 1 as follows.

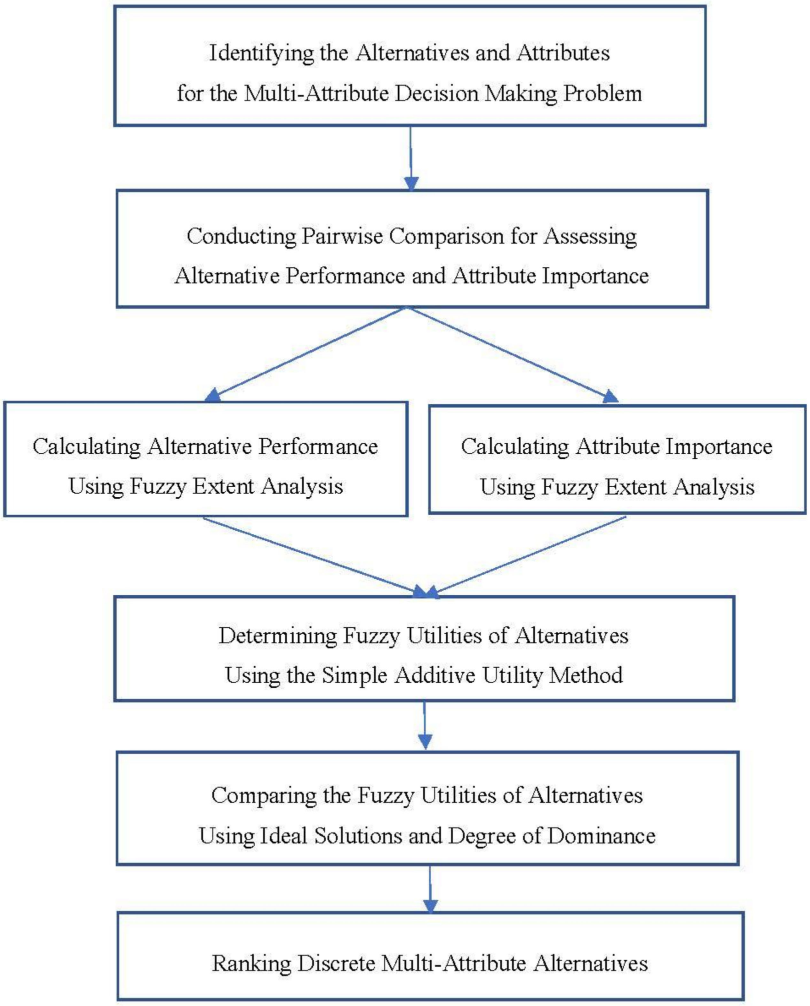

Figure 1. The flow chart for the proposed fuzzy approach.

4. An Example

Multi-attribute decision making under uncertainty is about selecting or ranking discrete alternatives against multiple, usually conflicting, attributes in specific decision-making situations [6,12]. It is widely used in various practical areas, including project selection [10,26], supplier chain evaluation [7], and supplier selection [16], just to name a few. To demonstrate the applicability of the proposed fuzzy approach for solving multi-attribute decision-making problems under uncertainty, an example of assessing three candidates for promotion in a university department is presented in this section. There are three candidates (A_1, A_2, and A_3) in a university department in the promotion round for consideration. After comprehensive consultation across the department, three attributes were identified, including research and scholarship (C_1), teaching and learning (C_2), and leadership and community services (C_3). The three candidates must be evaluated with respect to these three attributes to determine their overall ranking for promotion in this situation.

The situation described above is a typical problem of multi-attribute decision making under uncertainty. Based on the discussion in Section 3, with respect to the development of the fuzzy approach, linguistic variables defined as in Table 1 can be used for facilitating the making of the pairwise comparison by the decision maker to assess the candidate's performance and attribute importance in this situation. As a result, a pairwise comparison matrix for the attribute importance can be determined as follows:

$$w = \begin{pmatrix} M & L & VH \\ H & M & H \\ VL & L & M \end{pmatrix}.$$

Following the definition of these linguistic variables, as defined in Table 1, the pairwise comparison matrix for the attribute importance can be expressed as follows:

$$w = \begin{pmatrix} (3,\ 5,\ 7) & (1,\ 3,\ 5) & (7,\ 9,\ 9) \\ (5,\ 7,\ 9) & (3,\ 5,\ 7) & (5,\ 7,\ 9) \\ (1,\ 1,\ 3) & (1,\ 1,\ 3) & (3,\ 5,\ 7) \end{pmatrix}.$$

Applying the fuzzy extent analysis to the pairwise comparison matrix above, based on (4), the importance of the attributes with respect to the overall objective of the problem, presented as in (2), can be calculated as follows:

$$W = ((0.14, 0.29, 0.66), (0.08, 0.24, 0.59), (0.24, 0.47, 0.93)).$$

Following the same pairwise comparison procedure as above, the performance of each candidate with respect to each attribute can be assessed using the linguistic variable defined in Table 1, with their fuzzy number approximations. This leads to the determination of three fuzzy pairwise comparison matrices, as follows:

$$C_1 = \begin{pmatrix} (3,\ 5,\ 7) & (5,\ 7,\ 9) & (7,\ 9,\ 9) \\ (1,\ 3,\ 5) & (3,\ 5,\ 7) & (5,\ 7,\ 9) \\ (1,\ 1,\ 3) & (1,\ 3,\ 5) & (3,\ 5,\ 7) \end{pmatrix}$$

$$C_2 = \begin{pmatrix} (3,\ 5,\ 7) & (1,\ 3,\ 5) & (1,\ 1,\ 3) \\ (5,\ 7,\ 9) & (3,\ 5,\ 7) & (5,\ 7,\ 9) \\ (7,\ 9,\ 9) & (1,\ 3,\ 5) & (3,\ 5,\ 7) \end{pmatrix}$$

$$C_3 = \begin{pmatrix} (3,\ 5,\ 7) & (1,\ 1,\ 3) & (5,\ 7,\ 9) \\ (7,\ 9,\ 9) & (3,\ 5,\ 7) & (5,\ 7,\ 9) \\ (1,\ 3,\ 5) & (1,\ 3,\ 5) & (3,\ 5,\ 7) \end{pmatrix}$$

Applying the fuzzy extent analysis on these three pairwise matrices as in (4), the candidate performance with respect to each attribute can be calculated. This leads to the determination of the decision matrix expressed as in (1) for the multi-attribute decision making problem as follows:

$$X = \begin{pmatrix} (0.08,\ 0.20,\ 0.52) & (0.14,\ 0.33,\ 0.72) & (0.24,\ 0.47,\ 0.92) \\ (0.24,\ 0.47,\ 0.93) & (0.08,\ 0.24,\ 0.59) & (0.14,\ 0.29,\ 0.67) \\ (0.17,\ 0.38,\ 0.79) & (0.08,\ 0.20,\ 0.52) & (0.21,\ 0.42,\ 0.86) \end{pmatrix}$$

With the use of the additive utility function as in (5), the overall fuzzy utility of each candidate across all the attributes can be calculated, given as follows:

$$U(A_1) = (0.080, 0.358, 1.633), U(A_2) = (0.074, 0.330, 1.585), U(A_3) = (0.081, 0.356, 1.628)$$

Following the proposed fuzzy approach, these fuzzy utilities must be compared to determine the overall ranking of the three candidates for promotion. Using the proposed procedure for ranking these fuzzy utilities, the positive ideal solution and the negative ideal solution can be calculated with respect to (6) to (9) as follows:

$$U^+ = (0.074, 1.633, 1.633), U^- = (0.074, 0.074, 1.633)$$

Based on the positive ideal solution and the negative ideal solution shown above, the degree of dominance of the positive ideal solution on each fuzzy utility and the degree of dominance of each fuzzy utility on the negative ideal solution can be calculated by (10) to (15). Consequently, the overall performance index value for each candidate across all the attributes can be determined by (16). Table 2 shows a summary of the calculation result.

Table 2. An overview of the ranking of the candidates.

Candidates	d_i^+	d_i^-	P_i
A_1	0.423	0.097	0.050
A_2	0.450	0.069	0.023
A_3	0.425	0.094	0.047

Analysis of Table 2 above shows that the overall ranking of the three candidates is in the form of $A_1 > A_3 > A_2$. Obviously, candidate A_1 is the best choice in this situation, as it has the largest overall performance index value across all the attributes.

To demonstrate the advantages of the proposed fuzzy approach for solving the problem of multi-attribute decision making under uncertainty, a comparable approach based on the combination of the analytic hierarchy process [30] and the simple additive utility method [22,26] was adopted to solve the problem of ranking the three candidates with respect to the three attributes in this situation. This leads to the same ranking of the three candidates with respect to the three attributes in this situation. The use of the proposed fuzzy approach, however, shows several distinct merits for solving the problem of multi-attribute decision making under uncertainty. The proposed fuzzy approach, for example, is cognitively less demanding on the decision making on the subjective assessment in determining the candidate performance and the attribute importance. The adoption of linguistic variables is much easier than the use of crisp ratios for expressing the pairwise comparison assessment of the decision maker. The application of the ideal solution concept together with the degree of dominance in comparing the resultant fuzzy utilities avoided the shortcomings of existing ranking procedures in comparing and ranking fuzzy numbers [35]. Furthermore, the use of triangular fuzzy numbers in approximating linguistic variables makes the process of determining the overall performance of each alternative across all the attributes efficient due to the computation required. This shows that the proposed approach is attractive for solving the problem of multi-attribute decision making under uncertainty due to the advantages above.

With the development of the fuzzy approach for solving the problem of multi=attribute decision making under uncertainty, sensitivity analysis can be carried out by changing the assessments of the decision maker in the pairwise comparison process [12]. Such analysis can help explore the relationship between the overall linking of the discrete alternatives and the subjective assessments of the decision maker in specific situations. This is helpful when the decision maker does not understand the problem well at the beginning and solves the multi-attribute decision making problem under uncertainty. Through exploring the inter- and intra-relationships between the rankings of alternatives and the subjective assessments of the decision maker on alternative performance and attribute importance, the decision maker can learn to understand themselves and the problem. This can lead to better decisions being made in real situations.

5. Conclusions

Comparing and ranking discrete alternatives with respect to multiple, usually conflicting, attributes under uncertainty is often complex and challenging in real-world settings [6,22]. Existing approaches for adequately addressing such problems are not totally satisfactory in the literature. In this paper, a fuzzy approach is presented to effectively solve the problem of multi-attribute decision making under uncertainty. To reduce the cognitive demand on the decision maker in the decision-making process, the technique of pairwise comparison [6,30] was adopted to evaluate the alternative performance and attribute importance in a given situation. To facilitate the making of subjective assessments by the decision maker, linguistic variables approximated by fuzzy numbers were used to express the subjective assessment [6,12]. To avoid the complex and often unreliable process of comparing and ranking fuzzy utilities [33], a novel ranking procedure was developed based on the concept of ideal solutions [6] and the degree of dominance [27]. As a result, effective decisions can be made when comparing and ranking discrete alternatives against multiple, often conflicting, attributes under uncertainty. The applicability of the proposed fuzzy approach for solving the problem of multi-attribute decision making under uncertainty was demonstrated through the presentation of an example in the real-world setting.

There are several limitations in the development of this fuzzy approach for solving multi-attribute decision making problems under uncertainty in this study. The proposed fuzzy approach, for example, has not considered a decision-making situation in which multiple decision makers are present [7,14,36,38]. The uncertainty in terms of subjectiveness and imprecision in multi-attribute decision making can also be tackled using other types of fuzzy numbers, including the hesitant fuzzy numbers [2,15,18,39]. Furthermore, more real-world applications need to be identified to better show the applicability of the fuzzy approach in solving the general multi-attribute decision making problem under uncertainty [12]. All these limitations can be handled in future research in this area.

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