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The LR-Type Fuzzy Multi-Objective Vendor Selection Problem in Supply Chain Management

Irfan Ali ¹, Armin Fügenschuh ^{2,*}, Srikant Gupta ³ and Umar Muhammad Modibbo ^{1,4}

¹ Department of Statistics & Operations Research, Aligarh Muslim University, Aligarh 202002, India; irfii.st@amu.ac.in (I.A.); umarmodibbo@mautech.edu.ng (U.M.M.)

² Brandenburg University of Technology Cottbus-Senftenberg, Platz der Deutschen Einheit 1, 03046 Cottbus, Germany; fuegenschuh@b-tu.de

³ Jaipuria Institute of Management, Jaipur 302033, India; srikant.gupta@jaipuria.ac.in

⁴ Department of Statistics & Operations Research, Modibbo Adama University of Technology, Yola, PMB 2076, Nigeria

* Correspondence: fuegenschuh@b-tu.de; Tel.: +49-355-69-3127

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Abstract: Vendor selection is an established problem in supply chain management. It is regarded as a strategic resource by manufacturers, which must be managed efficiently. Any inappropriate selection of the vendors may lead to severe issues in the supply chain network. Hence, the desire to develop a model that minimizes the combination of transportation, deliveries, and ordering costs under uncertainty situation. In this paper, a multi-objective vendor selection problem under fuzzy environment is solved using a fuzzy goal programming approach. The vendor selection problem was modeled as a multi-objective problem, including three primary objectives of minimizing the transportation cost; the late deliveries; and the net ordering cost subject to constraints related to aggregate demand; vendor capacity; budget allocation; purchasing value; vendors' quota; and quantity rejected. The proposed model input parameters are considered to be LR fuzzy numbers. The effectiveness of the model is illustrated with simulated data using R statistical package based on a real-life case study which was analyzed using LINGO 16.0 optimization software. The decision on the vendor's quota allocation and selection under different degree of vagueness in the information was provided. The proposed model can address realistic vendor selection problem in the fuzzy environment and can serve as a useful tool for multi-criteria decision-making in supply chain management.

Keywords: multi-objective optimization; vendor selection problem; weighted criterion; LR fuzzy numbers; fuzzy goal programming

1. Introduction

One of the most vexing problems facing purchasing managers in organizational business decision-making is a vendor selection problem (VSP). A supply chain is a complex network in both manufacturing and service industries with interconnected components such as the suppliers of raw materials, enterprise, manufacturers, retailers, distributors, storage facilities and transporters of goods and services to desired customers in its simplest form. Thus, decision-making in a supply chain encompasses all the involved partners and activities in satisfying the demand of customers. Vendor selection plays a vital role in supply chain management; this is because of the pressing need to incorporate alliance strategies with the vendors. The types of equipment and materials supplied from vendors play a significant role in the effective management of the supply network. A location of a vendor, for instance, has a substantial impact on the firm's logistic decision for planning transportation

and distribution. Hence, selecting potential vendors is an integral part of achieving the overall objectives of the supply chain. The fewer the vendors, the more they are reliable similarly, the more substantial the number, the lesser the purchasing risk but increase the associated cost. There are many criteria for evaluating any selected vendor(s), which can vary from individual to another based on their performance characteristics. For example, 23 selection criteria are identified by Dickson [1] and 18 by Dempsey [2]. Also, constraints such as system policies, ordering quantities, vendors quota allocation, time to delivery, and the like exist in the process. Hence, VSP is a multi-objective decision-making problem. According to Xia and Wu [3] supplier selection problem can be either single or multiple sourcing. In the former, the buyer decides the best among available suppliers. In contrast, in the later, the buyer's requirement cannot be satisfied by a single supplier due to certain constraints listed above and hence, he must decide accordingly. Moreover, uncertainty is another property of VSP because of the imperfect information inherently in the chain, as such fuzzy set theory has to be incorporated to handle the vagueness and imperfections in the decision parameters. The goal programming developed by Charnes et al. [4] emerged as a powerful and strong technique for solving multi-criteria decision-making problems. Many researchers such as Lee [5], Ignizio [6] and many more improved the goal programming technique since its inception. Undoubtedly, the method is regarded as one of the breakthroughs in handling multi-criteria decision-making problems. On the other hand, the concept of fuzzy sets is seeing as one possible way of improving the modeling and formulation of vague parameters Zadeh [7]. A fuzzy programming concept was developed by Zimmermann [8] for solving multi-criteria decision-making problems. However, one of the significant issues which bedeviling the decision-makers is the problems where the coefficients are imprecise and vague or modeling of an ill-conditioned optimization problem. Such situations cannot be optimized using classical methods. According to Bellman and Zadeh [9], the constraints and goals of such may be viewed as fuzzy. In this study, imprecise input decision parameters are considered to be LR fuzzy numbers, and multi-objective vendor selection problem (MOVSP) is formulated using the concept of fuzzy goal programming with relative weight. We also proposed the aggregated weighted criterion method which is attached to each fuzzy number itself. We compared the result from using the aggregated weighted criterion with those results obtained using the standard weighted criterion, which is a link to each objective function in the form of priority. We demonstrate this approach using a numerical example. Section 2 of the paper reviewed various literature related to mathematical programming in VSP and classified them according to the solution techniques. Section 3 discusses the fuzzy set theory to make it self-contained. Section 4 presents the formulation of MOVSP, conversion from fuzzy to crisp MOVSP under different cases also discussed, and fuzzy goal programming and its computational procedures in brief. Section 5 outline a case study of the MOVSP considering two scenarios of fixed and varying demand. Section 6 analyses and present results from different techniques employed in the study, while Section 7 concludes the paper by drawing the effectiveness regarding the developed solution procedure and suggest an area for further investigations.

2. Literature Review

Research on supply chain and supplier selection has a long tradition since the 1960s, the criteria for vendor selection and vendor rating is a main central area of research in supply chain management (SCM). Studies related to this include but not limited to Ravindran [10], Wind et al. [11], Stewart [12], Hinkle et al. [13], Miller [14], McMillan [15], Lucas and Moore [16], Wieters and Ostrom [17], Manzer et al. [18]. More recently, fuzzy set methods were introduced in the study of supply chains, in order to cope with robustness, uncertainty and vagueness, see for instance [19–22]. Three quantitative techniques used for decision-making in supplier selection are mathematical programming, multiple attribute, and intelligent approaches. Of the three methods, mathematical programming models are used extensively in the vendor selection problem (VSP) [see Tables 1 and 2]. The nature of the vendor selection in the supply chain was discussed as a multi-criterion decision-making problem by Kumar et al. [23]. Weber and Current [24] gave the concept of multi-objective programming technique in selecting

multiple conflicting criteria of vendors alongside their order quantities. Several authors such as Xia and Wu [3], Dahel [25], Pokharel [26], Tsai and Wang [27], Rezaei and Davoodi [28] worked on the multi-objective vendor selection problem (MOVSP). Kumar et al. [23,29] formulated a fuzzy mixed integer goal programming model for a multiple sourcing supplier selection problems (MSSP) with cost, quality, and delivery as fuzzy goals, subject to the constraints of buyer's demand, the suppliers' capacity, and others. The max-min technique developed by Zimmermann [8] employed in solving the multi-objective problem in their study. A systematic vendor selection process was developed by Lamberson et al. [30] to identify and prioritize relevant criteria, evaluate the trade-offs between the economic, technical, and performance criteria. Kumar et al. [31] used a lexicographic goal programming approach for solving a piecewise linear VSP of quantity discounts.

In real-life situations, the criteria for purchasing department may have various weights related to their strategies. To cope with these problems, Amid et al. [32,33] formulated a fuzzy-based model for MSSP, including three fuzzy goals: cost, quality and delivery, which are subject to capacity restriction and market demand. They used the additive model developed by Tiwari et al. [34] in solving their multi-criterion model. A weighted max-min (WMM) model was subsequently proposed by Lin [35] for solving fuzzy multi-criterion model of supplier selection. The approach was applied later by Amid et al. [36] to a fuzzy multi-objective supplier selection problem (MOSSP). Liao and Kao [37] combined the Taguchi loss function, AHP, and multi-choice goal programming model to solve the supplier selection problem. Liao and Kao [38] also gave a two-stage model for selecting suppliers in a company which engaged in the watch manufacturing sector by using a fuzzy technique for order preference by similarity to an ideal solution (TOPSIS) with fuzzy triangular numbers and multi-choice goal programming (MCGP) to optimize the problem. Wu et al. [39] used a trapezoidal membership function and solved the probabilistic multi-criteria vendor selection model by using sequential programming, considering risk factors. Ozkok and Tiryaki [40] established a compensatory fuzzy approach for solving a multi-objective linear supplier selection problem with multiple items by using cost, service, and quality as objectives. Some latest research work in the vendor selection problem using fuzzy goal programming includes: Pandey et al. [41], Mirzaee et al. [42], Abbas [43], Sari [44], Wadhwa [45], Kamal et al. [46], Shaw [47], Aggarwal et al. [48], Islam and Deb [49], Ho [50], Torres-Ruiz and Ravindran [51], Mahmudul Hassan et al. [52], Alizadeh and Yousefi [53], Ozkan and Aydin [54], Jia et al. [55], Krishankumar et al. [56]. Arikani [57] formulated a multi-objective VSP based on price, quality, customer service and delivery criteria, by using Lai and Hwang [58,59], gave fuzzy augmented approach to solve the fuzzy MOVSP. Shirkouhi et al. [60] developed an interactive two-phase fuzzy multi-objective linear programming model for the supplier selection under multi-price level and multi-products. Kilic [61] developed an integrated approach, including a fuzzy technique for selecting the best supplier in a multi-item/multi-supplier environment. Rouyendegh and Saputro [62] described an optimum decision-making method for selecting a supplier and allocating order by applying fuzzy TOPSIS and MCGP. Jadidi et al. [63] developed a normalized goal programming model with predetermined goals and predetermined weights for solving a multi-objective supplier selection problem. Chang et al. [64] considered multiple aspiration levels and vague goal relations to help the decision-makers for choosing better suppliers by using multi-choice goal programming (MCGP) with the fuzzy approach. Karimi and Rezaeinia [65] adopted a revised multi-segment goal programming model for selecting the suppliers. Sivrikaya et al. [66] adopted a fuzzy-AHP goal programming approach with linguistic variables that are express in trapezoidal fuzzy numbers, and applied to assess weights and ratings of supplier selection criteria.

The single objective programming problem can be easily solved. However, the “multi-criteria decision-making” (MCDM) problem cannot be directly solved using any algorithm, and therefore some other algorithms are used to address this issue. The algorithms which are frequently use for solving the MCDM problems are “fuzzy goal programming, goal programming and interactive fuzzy-goal programming respectively.” These algorithms are used to obtain the compromise solution of the MCDM problems. Presently, several methods are available to solve MCDM problems; one of the simplest methods is weighted criterion method. In weighting method we can use the following criterion: point allocation, pairwise comparison, ranking or rating methods, and trade-off analysis. Each criterion differs according to its accuracy, ease of use, complexity for users, and theoretical foundations and produces different sets of weight criteria. Selecting a proper method of weighting is a crucial step for solving an MCDM problem. The main purpose of any criteria weighting method is to attach ordinal values to different standards indicating their relative importance in an MCDM problem. Weighted criterion works well if the objectives behaved well, and trade-offs between the objectives allow the weights to be certainly determined. The weights can be changed both during and after the optimization. This change allows the decision maker to optimize only for one objective, change the weights and optimize to another objective. It can accomplish by setting one weight to 1 and all the others to 0 (or some small positive number). Also, the decision maker can make his priority level for the objective functions and optimize the problem accordingly to his preference. The choices of weights assigned to the objective functions depend on the choices of the decision maker. The flexibility in the weights allows to the decision maker to generate new solutions accordingly to his special ordering of the objective functions. Therefore, there is always an opportunity to solve the problem for setting the better-improved solutions.

Motivated from the literature, we extend a non-fuzzy MOVSP model from Kumar et al. [23] to imprecise parameters which are considered to be fuzzy numbers. This extended model is approximated using the α -cut approach by a series of classical (non-fuzzy) linear programs, which are solved by standard linear programming techniques and numerical solvers.

Table 1. Summary of Literature Review.

Authors	Model Objectives	Techniques Used	Converted to Single Objective By
Jia et al. [55]	Multiple	Preemptive GP	Expected & join chance constraints
Tirkolaee et al. [67]	Multiple	FANP,FTOPSIS & FDEMATEL	WGP
Maragatham et al. [68]	Multiple	FILP	Modified Zimmermann’s approach
Sumrit [69]	Multiple	Fuzzy Delphi, Fuzzy SWARA	MCDM framework proposed
Shen et al. [70]	Single	Fuzzy TOPSIS	MCGP
Mahmudul Hassan et al. [52]	Multi-Attribute	FTOPSIS	MCGP
Alizadeh and Yousefi [53]	Multiple	MCGP	Utility Function
Ozkan and Aydin [54]	Multiple	IFAHP	FGP
Islam and Deb [49]	Multiple	Neutrosophic AHP & GP	Compared with FGP
Ho [50]	Multiple	WGP, FG	WGP, MINMAX MCGP
Torres-Ruiz and Ravindran [51]	Multiple	Preemptive, Non-preemptive & FGP	DEA
Mirzaee et al. [42]	Multiple	MILP	Preemptive & WFP, Max-min & Classical GP
Abbas [43]	Multiple	Intractive Fuzzy GP	Alpha-cut, FGP
Sari [44]	Multiple	FAHP, FGP	FGP
Kamal et al. [46]	Multiple	WFGP	Weighted root power mean, linear, exponential, & hyperbolic
Charles et al. [71]	Multiple	Additive, Weighted & Preemptive GP	FGP
Aggarwal et al. [48]	Multiple	Non-preemptive GP	Weighted Sum AOF
Shaw [47]	Multiple	IGP,WGP, IFG	Interactive fuzzy ϵ -constraint
Wadhwa [45]	Multiple Criteria	FGP, Tcebycheff Min-max	Preemptive & non-preemptive GP
Pandey et al. [41]	Multiple	WFGP	Hyperbolic membership function
Razmi et al. [72]	Multiple	IFGP	Two-step GP
Sivrikaya et al. [66]	Multiple	FAHP, GP	Trapezoidal Fuzzy, geometric mean Method
Jadidi et al. [63]	Multiple	WGP, TOPSIS, Min-Max GP, WMin-Max,	Compromise Programming, Normalized GP developed
Karimi and Rezaeinia [65]	Multiple	multi-segment goal programming	multi-segment goal programming revised
Rouyendegh and Saputro [62]	Multi-Criteria	FTOPSIS, MCGP	Triangular fuzzy numbers
Arikan [57]	Multiple	FGP	Fuzzy additive, augmented max-min model
Shirkouhi et al. [60]	Multiple	FMOLP	Piecewise linear membership
Kilic [61]	Multiple	FTOPSIS & MILP	Many MCDM proposed
Ozkok and Tiryaki [40]	Multi-item	MLSSP-MI	Fuzzy operator
Ozkok and Tiryaki [40]	Multiple	Weners’ fuzzy and μ_{and} operator	compensatory fuzzy approach developed
Rezaei and Davoodi [28]	Multiple	Genetic Algorithm	Two Multiobjective Mixed-Integer Non-linear models developed

Table 2. Summary of Literature Review.(Cont'd).

Authors	Model Objectives	Techniques Used	Converted to Single Objective by
Chang et al. [64]	Multiple	MCGP	FMCGP
Liao and Kao [37,38]	Multi-Criteria	AHP, Taguschi loss function, FTOPSIS	MCGP
Wu et al. [39]	Multiple	FMOP	Simulation
Amid et al. [32,33,36]	Multiple	Fuzzy-assymmetric	weighted additive, alpha-cute approach
Pokharel [26]	Two Objectives	STEP Method	MOP
Xia and Wu [3]	Multi-Criteria	AHP	MILP
Yong [73]	Multi-Criteria	Fuzzy TOPSIS	new FTOPSIS proposed
Kumar et al. [23,29]	Multiple	Fuzzy Integer	Linear membership
Lin [35]	Multiple	FGP	Weighted Max-min
Dulmin and Mininno [74]	Multiple criteria	promethee/ gaia techniques	MCDM investigated
Tam and Tummala [75]	Multi-Criteria	AHP	AHP-based model developed
Muralidharan et al. [76]	Multiple-Criteria	Confidence interval approach	Ranking methodology proposed
Petroni and Braglia [77]	Multi-Attribute	PCA	Multi-attribute approach discussed
Yahya and Kingsman [78]	Multiple criteria	AHP	Vendor rating and comparison
Mandal and Deshmukh [79]	Qualitative research	Interpretive Structural Modeling	Vendor selection framework
Lai and Hwang [58]	Single	Probalistic LP	Augmented max-min proposed
Weber and Current [24]	Multi-criteria	ILP	IMB XT
Tiwari et al. [34]	Multiple	aditive & weighted GP	additive FGP formulated

3. Fuzzy Sets Preliminaries

This section is devoted to an introduction to our fuzzy set notations and definitions, which will be used in the next section for the problem formulation. Starting with several basic definitions involving fuzzy sets, fuzzy numbers and its types, especially LR fuzzy numbers, are outlined. A brief discussion of α -cuts for Gaussian and exponential fuzzy numbers is also given. For more details, we refer to [7,9,80,81].

Let X be a set, called *universe of discourse*. A mapping $\mu : X \rightarrow [0, 1]$ is a *membership function*, if $\mu(x) \in [0, 1]$. A *fuzzy set* \tilde{A} is the pair (X, μ) .

Let $x \in X$ and $\tilde{A} := (X, \mu)$. If $\mu(x) = 0$, then x is *not included* in \tilde{A} . If $\mu(x) = 1$, then x is *fully included* in \tilde{A} . If $\mu(x) \in (0, 1)$, then x is *partially included* in \tilde{A} .

For $\tilde{A} := (X, \mu)$ being a fuzzy set and $\alpha \in [0, 1]$ the following (non-fuzzy, classical, or crisp) sets are defined: The α -cut or α -level set is $A \geq \alpha := A_\alpha := \{x \in X : \mu(x) \geq \alpha\}$ and the *strong α -cut* or *strong α -level set* is $A > \alpha := A'_\alpha := \{x \in X : \mu(x) > \alpha\}$. Based on these two, we define the *support* $S(A) = Supp(A)$ as the strong 0-cut and the *core* or *kernel* $C(A) = Core(A)$ as the 1-cut of \tilde{A} .

The *height* of a fuzzy set \tilde{A} is defined by $h(\tilde{A}) = Hgt(\tilde{A}) := \sup\{\mu(x) : x \in X\}$. If $h(\tilde{A}) = 1$, then \tilde{A} is said to be *normal*, otherwise it is called *subnormal*.

Let $X \subseteq \mathbb{R}^n$. Then the fuzzy set $\tilde{A} := (X, \mu)$ is called *convex*, if for all $x, y \in X$ and all $\lambda \in [0, 1]$ it holds that $\mu(\lambda x + (1 - \lambda)y) \geq \min\{\mu(x), \mu(y)\}$.

A fuzzy set $\tilde{A} = (X, \mu)$ for $X \subseteq \mathbb{R}$ is called a *fuzzy number* if \tilde{A} is convex and normal, μ is piecewise continuous, A_α is a closed interval for all $\alpha \in (0, 1]$, and $S(A)$ is bounded.

Some special cases of fuzzy numbers with their α -cuts are:

- A *trapezoidal fuzzy number* $\tilde{A} = (\mathbb{R}, \mu)$ is a fuzzy number whose membership function μ is defined as:

$$\mu(x) = \begin{cases} 0, & x < a, \\ \frac{x-a}{b-c}, & a \leq x \leq b, \\ 1, & b \leq x \leq c, \\ \frac{d-x}{d-c}, & c \leq x \leq d, \\ 0, & x > d, \end{cases}$$

where $a, b, c, d \in \mathbb{R}$ with $a \leq b \leq c \leq d$. Please note that the membership function is piecewise continuous and \tilde{A} is normal, hence it is a well-defined fuzzy number. Further, note that the quadruple (a, b, c, d) is sufficient to describe \tilde{A} , so we write $\tilde{A} = (a, b, c, d)$ as abbreviation for a trapezoidal fuzzy number. In case of $b = c$, the trapezoidal fuzzy number is called a *triangular fuzzy number*, and one can write $\tilde{A} = (a, b, d)$. The α -cut of the trapezoidal fuzzy number $\tilde{A} = (a, b, c, d)$ is the closed interval

$$\tilde{A}_\alpha = [\tilde{A}_\alpha^L, \tilde{A}_\alpha^R] = [a + (b - a)\alpha, d - (d - c)\alpha].$$

The α -cut of a triangular fuzzy number $\tilde{A} = (a, b, d)$ is the closed interval

$$\tilde{A}_\alpha = [\tilde{A}_\alpha^L, \tilde{A}_\alpha^R] = [a + (b - a)\alpha, d - (d - b)\alpha].$$

- A trapezoidal fuzzy number is a special case of an *LR fuzzy number* $\tilde{A} = (\mathbb{R}, \mu)$, where in addition to (a, b, c, d) , two continuous, strictly monotone function $L, R : [0, 1] \rightarrow [0, 1]$ with $L(0) = R(0) = 0$ and $L(1) = R(1) = 1$ are specified, and μ is defined as

$$\mu(x) = \begin{cases} 0, & x < a, \\ L\left(\frac{x-a}{b-c}\right), & a \leq x \leq b, \\ 1, & b \leq x \leq c, \\ R\left(\frac{d-x}{d-c}\right), & c \leq x \leq d, \\ 0, & x > d, \end{cases}$$

Please note that the core of \tilde{A} is the interval $[b, c]$. The *left spread* and the *right spread* are defined as $\beta := b - a$ and $\gamma := d - c$, respectively. The α -cut is

$$\tilde{A}_\alpha = [\tilde{A}_\alpha^L, \tilde{A}_\alpha^R] = [a + L^{-1}(\alpha)\beta, d - R^{-1}(\alpha)\gamma].$$

- A fuzzy number $\tilde{A} = (\mathbb{R}, \mu)$ is called a *Gaussian fuzzy number*, if its membership function μ is defined as

$$\mu(x) = \begin{cases} 0, & x < m - \sigma_L, \\ \exp\left(-\frac{(m-x)^2}{2\sigma_L^2}\right), & m - \sigma_L \leq x < m, \\ \exp\left(-\frac{(x-m)^2}{2\sigma_R^2}\right), & m \leq x < m + \sigma_R, \\ 0, & x \geq m + \sigma_R, \end{cases}$$

where $m \in \mathbb{R}$. Here σ_L, σ_R are called the *left spread* and *right spread*, respectively. As abbreviation, we write $\tilde{A} = (m, \sigma_L, \sigma_R)$, and if $\sigma_L = \sigma_R$, we write as abbreviation $\tilde{A} = (m, \sigma)$. Please note that $\{m\}$ is the core of the Gaussian fuzzy number. Please note that the Gaussian fuzzy number is a special case of the LR fuzzy number. The α -cut of a Gaussian fuzzy number $\tilde{A} = (m, \sigma)$ is

$$\tilde{A}_\alpha = [\tilde{A}_\alpha^L, \tilde{A}_\alpha^R] = [m - \sigma\sqrt{-2\ln\alpha}, m + \sigma\sqrt{-2\ln\alpha}].$$

- An *exponential fuzzy number* is a fuzzy number which has a membership function μ given by

$$\mu(x) = \begin{cases} 0, & x < m - \sigma_L, \\ \exp\left(-\frac{m-x}{\sigma_L}\right), & m - \sigma_L \leq x < m, \\ \exp\left(-\frac{x-m}{\sigma_R}\right), & m \leq x < m + \sigma_R, \\ 0, & x \geq m + \sigma_R, \end{cases}$$

where $\{m\}$ is the core, for $m \in \mathbb{R}$. Here σ_L, σ_R are called the *left spread* and *right spread*, respectively. As abbreviation, we write $\tilde{A} = (m, \sigma_L, \sigma_R)$, and if $\sigma_L = \sigma_R$, we write as abbreviation $\tilde{A} = (m, \sigma)$. Please note that the exponential fuzzy number is a special case. The α -cut of an exponential fuzzy number $\tilde{A} = (m, \sigma)$ is

$$\tilde{A}_\alpha = [\tilde{A}_\alpha^L, \tilde{A}_\alpha^R] = [m - \sigma \ln(1/\alpha), m + \sigma \ln(1/\alpha)].$$

4. Multi-Objective Vendor Selection Problem (MOVSP)

The MOVSP is an extension of the problem discussed by [23], where a system of n vendors with a deterministic parameter is considered. The list of symbols used in the model formulation is given in Table 3.

Table 3. Nomenclature.

Indices	
j	vendor's index, $\forall i = 1, 2, \dots, n$
k	objectives index, $\forall k = 1, 2, 3, \dots, K$
Parameters	
n	vendors' number who are competing for selection
D	aggregate item demand for a fixed planning period
p_j	per unit item price for the quantity ordered q_j to the j^{th} vendor
t_j	unit item transportation cost from the j^{th} vendor
l_j	percentage for the late delivered units by the j^{th} vendor
U_j	upper limit for the j^{th} vendor's quantity available
B_j	each vendor's budget constraint allocation
s_j	percentage for the j^{th} vendor rejected units
r_j	rating value for the j^{th} vendor
P	vendor's minimum total purchasing value
f_j	j^{th} vendor quota flexibility
F	vendor's minimum value flexible in supply quota
R	maximum affordable rejection by a purchaser
Decision Variable	
q_j	order quantity is given to the vendor j

The MOVSP is formulated as follows:

$$\text{Optimize } \left\{ \begin{array}{l} Z_1 = \sum_{j=1}^n p_j q_j, \text{ related to net - ordering - costs} \\ Z_2(X) = \sum_{j=1}^n t_j q_j, \text{ related to net - transportation costs} \\ Z_3(X) = \sum_{j=1}^n l_j q_j, \text{ related to late delivered items} \\ \text{Subject to set of constraints :} \\ \sum_{j=1}^n q_j = D, \text{ constraint related to aggregate demand} \\ q_j \leq U_j, \quad \text{for all } j = 1, 2, \dots, n., \text{ constraint related to vendor capacity} \\ p_j q_j \leq B_j \quad \text{for all } j = 1, 2, \dots, n., \text{ constraint related to budget allocation} \\ \sum_{j=1}^n r_j q_j \geq P. \text{ constraint related to Purchasing value} \\ \sum_{j=1}^n f_j q_j \leq F. \text{ constraint related to vendor's quota} \\ \sum_{j=1}^n s_j q_j \leq R \text{ constraint related to rejected items} \\ q_j \geq 0, \quad \forall j = 1, 2, \dots, n. \text{ constraint related to non - negative vendor} \end{array} \right. \quad (1)$$

The assumptions contained in [23] is hold, we therefore, formulate the MOVSP with n decision variables and m constraints under fuzzy logic set theory. The fuzzy mathematical formulation is stated as follows:

$$\begin{array}{l} \text{Minimize } \left(Z_1 = \sum_{j=1}^n \tilde{p}_j q_j, Z_2 = \sum_{j=1}^n \tilde{t}_j q_j, Z_3 = \sum_{j=1}^n \tilde{l}_j q_j \right) \\ \text{subject to } \sum_{j=1}^n q_j = \tilde{D}, \\ q_j \leq \tilde{U}_j, \quad \forall j = 1, 2, \dots, n \\ \tilde{p}_j q_j \leq \tilde{B}_j, \quad \forall j = 1, 2, \dots, n \\ \sum_{j=1}^n \tilde{s}_j q_j \leq \tilde{R}, \\ \sum_{j=1}^n \tilde{r}_j q_j \geq \tilde{P}, \\ \sum_{j=1}^n \tilde{f}_j q_j \leq \tilde{F}, \\ q_j \geq 0, \quad \forall j = 1, 2, \dots, n \end{array} \quad (2)$$

In problem (2), we assume that all the vague parameters are LR type fuzzy number.

4.1. The Conversion Procedure of Fuzzy MOVSP into Crisp MOVSP

In this section, the MOVSP formulated under three different types of LR fuzzy numbers, enlightening their applications in real-world problems.

Case 1. The vague input parameters of the problem (2) is defined by the generalized exponential LR normalize fuzzy number $\tilde{A} = (a, b, \sigma_L, \sigma_R)$, where a, b are left and right extreme (lower and upper values) endpoints, and σ_L, σ_R are left and right spread. Let $L(\alpha), R(\alpha)$ are two monotonic functions respectively.

$$L(\alpha) = \exp \left\{ - \left(\frac{a - x}{\sigma_L} \right) \right\} \Rightarrow L^{-1}(\alpha) = a - \sigma_L \left(\ln \frac{1}{\alpha} \right)$$

and

$$R(\alpha) = \exp \left\{ - \left(\frac{x - b}{\sigma_R} \right) \right\} \Rightarrow R^{-1}(\alpha) = b + \sigma_R \left(\ln \frac{1}{\alpha} \right).$$

We now define the ranking function value as

$$\begin{aligned} R(\tilde{A}) &= \lambda \int_0^1 \left[a - \sigma_L \left(\ln \frac{1}{\alpha} \right) \right] d\alpha + (1 - \lambda) \int_0^1 \left[b + \sigma_R \left(\ln \frac{1}{\alpha} \right) \right] d\alpha \\ &= \lambda [a - \sigma_L] + (1 - \lambda) [b + \sigma_R] \end{aligned} \tag{3}$$

Problem (2) is transformed into the equivalent crisp form using Equation (3):

$$\begin{aligned} \text{Minimize } [(Z_1(Q))_\lambda] &= \sum_{j=1}^n \left(\lambda (a_{pj} - \sigma_{pR}) + (1 - \lambda) (b_{pj} + \sigma_{pL}) \right) q_j \\ \text{Minimize } [(Z_2(Q))_\lambda] &= \sum_{j=1}^n \left(\lambda (a_{tj} - \sigma_{tR}) + (1 - \lambda) (b_{tj} + \sigma_{tL}) \right) q_j \\ \text{Minimize } [(Z_3(Q))_\lambda] &= \sum_{j=1}^n \left(\lambda (a_{lj} - \sigma_{lR}) + (1 - \lambda) (b_{lj} + \sigma_{lR}) \right) q_j \\ \text{subject to} \\ \sum_{j=1}^n q_j &= \lambda (D - \sigma_{DR}) + (1 - \lambda) (D - \sigma_{DL}), \\ q_j &\leq \lambda (a_{Uj} - \sigma_{UR}) + (1 - \lambda) (b_{Uj} + \sigma_{UL}), \\ \left(\lambda (a_{pj} - \sigma_{pR}) + (1 - \lambda) (b_{pj} + \sigma_{pL}) \right) q_j &\leq \lambda (a_{Bj} - \sigma_{BR}) + (1 - \lambda) (b_{Bj} + \sigma_{BL}), \\ \sum_{i=1}^n \left(\lambda (a_{sj} - \sigma_{sR}) + (1 - \lambda) (b_{sj} + \sigma_{sL}) \right) q_j &\leq \lambda (a_R - \sigma_{RR}) + (1 - \lambda) (b_R + \sigma_{RL}), \\ \sum_{i=1}^n \left(\lambda (a_{rj} - \sigma_{rR}) + (1 - \lambda) (b_{rj} + \sigma_{rL}) \right) q_j &\geq \lambda (a_P - \sigma_{PR}) + (1 - \lambda) (b_P + \sigma_{PL}), \\ \sum_{i=1}^n \left(\lambda (a_{fj} - \sigma_{fR}) + (1 - \lambda) (b_{fj} + \sigma_{fL}) \right) q_j &\leq \lambda (a_F - \sigma_{FR}) + (1 - \lambda) (b_F + \sigma_{FL}), \\ q_j &\geq 0, \quad \forall j = 1, 2, \dots, n, \end{aligned} \tag{4}$$

where $\lambda \in [0, 1]$ is an arbitrary chosen fixed value.

Case 2. The vague input parameters of the problem (2) is defined by the Gaussian LR normalize fuzzy number $\tilde{A} = (a, b, \sigma_L, \sigma_R)$.

$$L(\alpha) = \exp \left\{ - \left(\frac{a - x}{\sigma_L} \right)^2 \right\} \Rightarrow L^{-1}(\alpha) = a - \sigma_L \sqrt{-\ln \alpha}.$$

and

$$R(\alpha) = \exp \left\{ - \left(\frac{x - b}{\sigma_R} \right)^2 \right\} \Rightarrow R^{-1}(\alpha) = b + \sigma_R \sqrt{-\ln \alpha}.$$

We now define the ranking function value as

$$R(\tilde{A}) = \lambda \int_0^1 \left[a - \sigma_L \sqrt{-\ln \alpha} \right] d\alpha + (1 - \lambda) \int_0^1 \left[b + \sigma_R \sqrt{-\ln \alpha} \right] d\alpha$$

$$= \lambda \left[a - \sqrt{\frac{\pi}{2}} \sigma_L \right] + (1 - \lambda) \left[b + \sqrt{\frac{\pi}{2}} \sigma_R \right] \tag{5}$$

Problem (2) is transformed into the equivalent crisp form using Equation (5) as

$$\begin{aligned} \text{Minimize } [(Z_1(Q))_\lambda] &= \sum_{j=1}^n \left(\lambda \left(a_{p_j} - \sqrt{\frac{\pi}{2}} \sigma_{pR} \right) + (1 - \lambda) \left(b_{p_j} + \sqrt{\frac{\pi}{2}} \sigma_{pL} \right) \right) q_j \\ \text{Minimize } [(Z_2(Q))_\lambda] &= \sum_{j=1}^n \left(\lambda \left(a_{t_j} - \sqrt{\frac{\pi}{2}} \sigma_{tR} \right) + (1 - \lambda) \left(b_{t_j} + \sqrt{\frac{\pi}{2}} \sigma_{tL} \right) \right) q_j \\ \text{Minimize } [(Z_3(Q))_\lambda] &= \sum_{j=1}^n \left(\lambda \left(a_{l_j} - \sqrt{\frac{\pi}{2}} \sigma_{lR} \right) + (1 - \lambda) \left(b_{l_j} + \sqrt{\frac{\pi}{2}} \sigma_{lR} \right) \right) q_j \\ \text{subject to} \\ \sum_{j=1}^n q_j &= \lambda \left(D - \sqrt{\frac{\pi}{2}} \sigma_{DR} \right) + (1 - \lambda) \left(D - \sqrt{\frac{\pi}{2}} \sigma_{DL} \right), \\ q_j &\leq \lambda \left(a_{u_j} - \sqrt{\frac{\pi}{2}} \sigma_{uR} \right) + (1 - \lambda) \left(b_{u_j} + \sqrt{\frac{\pi}{2}} \sigma_{uL} \right), \\ \left(\lambda \left(a_{p_j} - \sqrt{\frac{\pi}{2}} \sigma_{pR} \right) + (1 - \lambda) \left(b_{p_j} + \sqrt{\frac{\pi}{2}} \sigma_{pL} \right) \right) q_j &\leq \lambda \left(a_{B_j} - \sqrt{\frac{\pi}{2}} \sigma_{BR} \right) + (1 - \lambda) \left(b_{B_j} + \sqrt{\frac{\pi}{2}} \sigma_{BL} \right), \\ \sum_{i=1}^n \left(\lambda \left(a_{s_j} - \sqrt{\frac{\pi}{2}} \sigma_{sR} \right) + (1 - \lambda) \left(b_{s_j} + \sqrt{\frac{\pi}{2}} \sigma_{sL} \right) \right) q_j &\leq \lambda \left(a_R - \sqrt{\frac{\pi}{2}} \sigma_{RR} \right) + (1 - \lambda) \left(b_R + \sqrt{\frac{\pi}{2}} \sigma_{RL} \right), \\ \sum_{i=1}^n \left(\lambda \left(a_{r_j} - \sqrt{\frac{\pi}{2}} \sigma_{rL} \right) + (1 - \lambda) \left(b_{r_j} + \sqrt{\frac{\pi}{2}} \sigma_{rL} \right) \right) q_j &\geq \lambda \left(a_P - \sqrt{\frac{\pi}{2}} \sigma_{PR} \right) + (1 - \lambda) \left(b_P + \sqrt{\frac{\pi}{2}} \sigma_{PL} \right), \\ \sum_{i=1}^n \left(\lambda \left(a_{f_j} - \sqrt{\frac{\pi}{2}} \sigma_{fR} \right) + (1 - \lambda) \left(b_{f_j} + \sqrt{\frac{\pi}{2}} \sigma_{fL} \right) \right) q_j &\leq \lambda \left(a_F - \sqrt{\frac{\pi}{2}} \sigma_{FR} \right) + (1 - \lambda) \left(b_F + \sqrt{\frac{\pi}{2}} \sigma_{FL} \right), \\ q_j &\geq 0, \quad \forall j = 1, 2, \dots, n, \end{aligned} \tag{6}$$

where $\lambda \in [0, 1]$ is an arbitrary chosen fixed value.

Case 3. The vague input parameters of the problem (2) is defined by the Gaussian LR normalize fuzzy number $\tilde{A} = (a, b, \sigma_L, \sigma_R)$.

$$L(\alpha) = \exp \left\{ -\frac{1}{2} \left(\frac{a - \alpha}{\sigma_L} \right)^2 \right\} \Rightarrow L^{-1}(\alpha) = a - \sigma_L \sqrt{-2 \ln \alpha}$$

and

$$R(\alpha) = \exp \left\{ -\frac{1}{2} \left(\frac{\alpha - b}{\sigma_R} \right)^2 \right\} \Rightarrow R^{-1}(\alpha) = b + \sigma_R \sqrt{-2 \ln \alpha}$$

We now define the ranking function value as

$$\begin{aligned} R(\tilde{A}) &= \lambda \int_0^1 \left[a - \sigma_L \sqrt{-2 \ln \alpha} \right] d\alpha + (1 - \lambda) \int_0^1 \left[b + \sigma_R \sqrt{-2 \ln \alpha} \right] d\alpha \\ &= \lambda \left[a - \sqrt{\pi} \sigma_L \right] + (1 - \lambda) \left[b + \sqrt{\pi} \sigma_R \right] \end{aligned} \tag{7}$$

Problem (2) is transformed into the equivalent crisp form using Equation (5) as

$$\begin{aligned} \text{Minimize } [(Z_1(Q))_\lambda] &= \sum_{j=1}^n \left(\lambda \left(a_{p_j} - \sqrt{\pi} \sigma_{pR} \right) + (1 - \lambda) \left(b_{p_j} + \sqrt{\pi} \sigma_{pL} \right) \right) q_j \\ \text{Minimize } [(Z_2(Q))_\lambda] &= \sum_{j=1}^n \left(\lambda \left(a_{t_j} - \sqrt{\pi} \sigma_{tR} \right) + (1 - \lambda) \left(b_{t_j} + \sqrt{\pi} \sigma_{tL} \right) \right) q_j \\ \text{Minimize } [(Z_3(Q))_\lambda] &= \sum_{j=1}^n \left(\lambda \left(a_{l_j} - \sqrt{\pi} \sigma_{lR} \right) + (1 - \lambda) \left(b_{l_j} + \sqrt{\pi} \sigma_{lR} \right) \right) q_j \\ \text{subject to} \\ \sum_{j=1}^n q_j &= \lambda \left(D - \sqrt{\pi} \sigma_{DR} \right) + (1 - \lambda) \left(D - \sqrt{\pi} \sigma_{DL} \right), \\ q_j &\leq \lambda \left(a_{u_j} - \sqrt{\pi} \sigma_{uR} \right) + (1 - \lambda) \left(b_{u_j} + \sqrt{\pi} \sigma_{uL} \right), \\ \left(\lambda \left(a_{p_j} - \sqrt{\pi} \sigma_{pR} \right) + (1 - \lambda) \left(b_{p_j} + \sqrt{\pi} \sigma_{pL} \right) \right) q_j &\leq \lambda \left(a_{B_j} - \sqrt{\pi} \sigma_{BR} \right) + (1 - \lambda) \left(b_{B_j} + \sqrt{\pi} \sigma_{BL} \right), \\ \sum_{i=1}^n \left(\lambda \left(a_{s_j} - \sqrt{\pi} \sigma_{sR} \right) + (1 - \lambda) \left(b_{s_j} + \sqrt{\pi} \sigma_{sL} \right) \right) q_j &\leq \lambda \left(a_R - \sqrt{\pi} \sigma_{RR} \right) + (1 - \lambda) \left(b_R + \sqrt{\pi} \sigma_{RL} \right), \\ \sum_{i=1}^n \left(\lambda \left(a_{r_j} - \sqrt{\pi} \sigma_{rL} \right) + (1 - \lambda) \left(b_{r_j} + \sqrt{\pi} \sigma_{rL} \right) \right) q_j &\geq \lambda \left(a_P - \sqrt{\pi} \sigma_{PR} \right) + (1 - \lambda) \left(b_P + \sqrt{\pi} \sigma_{PL} \right), \\ \sum_{i=1}^n \left(\lambda \left(a_{f_j} - \sqrt{\pi} \sigma_{fR} \right) + (1 - \lambda) \left(b_{f_j} + \sqrt{\pi} \sigma_{fL} \right) \right) q_j &\leq \lambda \left(a_F - \sqrt{\pi} \sigma_{FR} \right) + (1 - \lambda) \left(b_F + \sqrt{\pi} \sigma_{FL} \right), \\ q_j &\geq 0, \quad \forall j = 1, 2, \dots, n, \end{aligned} \tag{8}$$

where $\lambda \in [0, 1]$ is an arbitrary chosen fixed value.

4.2. Fuzzy Goal Programming Procedure

To solve the formulated problems (4), (6) and (8), we use Zimmermann’s fuzzy programming approach, see [8,82]. This approach was applied to solve various multi-objective optimization problems (such as linear programming, non-linear programming, or stochastic programming). In this section, we present a brief fuzzy programming method for solving the crisp MOVSP (4), (6) and (8), which in an abstract way can be written as:

$$\begin{aligned} & \text{Minimize } [(Z_1(Q))_\lambda, (Z_2(Q))_\lambda, \dots, (Z_k(Q))_\lambda], \quad k = 1, 2, \dots, K \\ & \text{subject to } x \in X \end{aligned}$$

where $x \in X$ is used to represent all set of the feasible constraint of the MOVSP (4), (6) and (8), respectively. We assume that the problem is feasible and that an optimal compromise solution exists.

Step 1. Firstly, we solve the problem to obtain the ideal solutions (g_1, g_2, \dots, g_k) for the respective objective functions. Using these ideal solutions, we formulate a pay-off matrix. Then lower and upper bound of each of the objective functions is estimated from the pay-off matrix as: $g_k \leq [(Z_k(Q))_\lambda] \leq u_k, \quad k = 1, 2, \dots, K$ Where g_k are imprecise aspiration levels of fuzzy goals $(Z_k(Q))_\lambda$ for which we define linear membership functions in the next step.

Step 2. In this step, we define a fuzzy membership function for the k -th objective function $(Z_k(Q))_\lambda$:

1. For first minimize type objective function, the membership function is constructed as:

$$\mu_1([(Z_1(Q))_\lambda]) = \begin{cases} 1, & [(Z_1(Q))_\lambda] \leq g_1 \\ \frac{u_1 - [(Z_1(Q))_\lambda]}{u_1 - g_1}, & g_1 \leq [(Z_1(Q))_\lambda] \leq u_1 \\ 0, & [(Z_1(Q))_\lambda] \geq u_1 \end{cases}$$

2. For second minimize type objective function, the membership function is constructed as:

$$\mu_2([(Z_2(Q))_\lambda]) = \begin{cases} 1, & [(Z_2(Q))_\lambda] \leq g_2 \\ \frac{u_2 - [(Z_2(Q))_\lambda]}{u_2 - g_2}, & g_2 \leq [(Z_2(Q))_\lambda] \leq u_2 \\ 0, & [(Z_2(Q))_\lambda] \geq u_2 \end{cases}$$

For third minimize type objective function, the membership function is constructed as:

$$\mu_3([(Z_3(Q))_\lambda]) = \begin{cases} 1, & [(Z_3(Q))_\lambda] \leq g_3 \\ \frac{u_3 - [(Z_3(Q))_\lambda]}{u_3 - g_3}, & g_3 \leq [(Z_3(Q))_\lambda] \leq u_3 \\ 0, & [(Z_3(Q))_\lambda] \geq u_3 \end{cases}$$

Here u_1, u_2 and u_3 are upper tolerance limit for the fuzzy goal $(Z_1(Q))_\lambda, (Z_2(Q))_\lambda$ and $(Z_3(Q))_\lambda$, respectively.

Step 3. Next, we consider the conversion of the objective functions into fuzzy goals using aspiration level to the corresponding objective function. Thus, problems (4), (6) and (8) are approximated as a fuzzy goal program by taking certain aspiration levels and introducing an auxiliary variable for the deviation from below to the objective function.

The minimization type objective function is approximated as

$$\frac{u_k - [(Z_k(Q))_\lambda]}{u_k - g_k} + d_k^- \geq 1, \quad k = 1, 2, \dots, K$$

The above membership function is used to formulate the multi-objective optimization problem as a single-objective optimization problem. Now we can use the FGP approach for attaining the highest degree of membership and thus the above problem is transformed into a goal programming problem as

$$\begin{aligned} \text{Min } Z &= \sum_{k=1}^3 w_k^- d_k^- \\ \text{subject to } &x \in X \\ &\frac{u_k - [(Z_k(Q))_\lambda]}{u_k - g_k} + d_k^- \geq 1, \quad k = 1, 2, \dots, K, \end{aligned}$$

Here, Z represents the achievement function of the problems (4), (6) and (8), and $w_k^- = \frac{1}{u_k - g_k} \forall k = 1, 2, \dots, K$ attached to each of the objective functions measures the relative importance of a deviation from their respective target.

4.3. Computational Procedure

The conversion method presented in Section 3 can be organized into the following steps as:

- Step 1.** Formulate the real world problem as a mathematical programming problem with fuzzy parameter. In our case, we formulated the MOVSP with LR fuzzy parameters.
- Step 2.** Define the membership function for fuzzy parameters in the mathematical formulation of the stated problem.
- Step 3.** Compute the ranking function for these fuzzy parameters to convert them into an equivalent crisp parameter.
- Step 4.** Determine the solution of multi-objective programming problem with objectives $(Z_1(Q))_\lambda$, $(Z_2(Q))_\lambda$ and $(Z_3(Q))_\lambda$ respectively with crisp parameter as obtained in Step 3 for an individual optimal solution by any optimization software (Lingo, R, Matlab and any other appropriate software).
- Step 5.** Set the optimal solution obtained in Step 4 as a goal, and compute the appropriate aspiration level for each goal.
- Step 6.** Repeat the Steps 4 and 5 for the various choices of values of lambda set by the decision maker.
- Step 7.** Compute $w_k^- = \frac{1}{u_k - g_k} \forall k = 1, 2, \dots, K$.
- Step 8.** Finally, formulate the problem into a deterministic goal programming problem.
- Step 9.** Solve the deterministic goal programming problem for various values of alpha-cut set by decision maker by any optimization software.
- Step 10.** Choose the optimal solution from the obtained solution set and the corresponding decision variables.

5. Case Study

In order to illustrate the proposed work, we considered a simulated case study for the vendor Selection Problem in continuation of [31] with some extensions. The instance consists of four vendors. Their profiles with uncertainty are giving in Table 4. All the parameters of the problem are considered to be randomly distributed and are simulated using uniform distribution through the R software package. If the purchasers follow a 95% (2σ limits) for the accepted policy, then the rejection maximum limit should not be more than 5% of the demand. Hence, the tolerance for the maximum rejection for a purchaser should be $25,000 \times 0.05 = 1250$. The minimum value for the vendors' quota flexibility and that of the total purchase for supplied items are the policy decisions, which depend on the demand. The minimum flexible value in the quota of the suppliers is $F = f \cdot D$, and that of total purchase value for the items supplied is given as $P = r \cdot D$. The other information is fuzzy. We assumed that fuzzy numbers $\tilde{A} = (a, b, \sigma_L, \sigma_R)$ are LR type fuzzy numbers. The instance data for the vendors with LR type fuzzy numbers are given in Table 4. Furthermore, we studied the following pattern of the fuzzy number, i.e., Exponential, Normal, and Gaussian-Normal. Parameters in the form of LR-fuzzy numbers $(a, b, \sigma_L, \sigma_R)$ are given below:

Table 4. Instance data for the Vendor Selection Problem with LR fuzzy numbers.

Vendors	1	2	3	4
\tilde{p}_j	(110, 130, 10, 15)	(305, 325, 15, 20)	(250, 265, 13, 18)	(355, 370, 12, 20)
\tilde{t}_j	(15, 17, 2, 3)	(11, 12, 1, 2)	(5, 7, 0.9, 1.8)	(21, 24, 4, 6)
\tilde{l}_j	(2, 2.5, 0.3, 0.7)	(3, 4.5, 0.6, 0.9)	(9, 11, 0.8, 1.1)	(4, 6, 0.5, 0.7)
\tilde{U}_j	(5600, 5800, 200, 400)	(16,500, 16,900, 600, 750)	(7000, 7900, 250, 450)	(5500, 5800, 310, 420)
\tilde{B}_j	(1,250,000, 1,300,000, 50,000, 60,000)	(5,000,000, 5,500,000, 65,000, 70,000)	(1,750,000, 1,800,000, 40,000, 45,000)	(300,000, 325,000, 10,000, 15,000)
\tilde{s}_j	(4, 6, 0.8, 1.2)	(4, 5.5, 0.6, 0.8)	(1, 2, 0.1, 0.2)	(7.5, 8.5, 1.5, 1.8)
\tilde{f}_j	(0.05, 0.06, 0.001, 0.002)	(0.02, 0.03, 0.002, 0.003)	(0.07, 0.09, 0.001, 0.003)	(0.03, 0.04, 0.001, 0.002)
\tilde{r}_j	(0.87, 0.88, 0.01, 0.02)	(0.90, 0.94, 0.04, 0.06)	(0.91, 0.93, 0.03, 0.05)	(0.89, 0.90, 0.08, 0.09)

Using the ranking function approaches (3), (5), and (7) as defined above in the preliminary section, the LR generalized exponential, the LR normalized, and the LR Gaussian-Normal fuzzy number, respectively, are transformed into an equivalent deterministic form. Tables 5–7 provide the parameters equivalent to the deterministic or crisp value for cases 1, 2, 3 at the distinct value of $\lambda = 0, 0.5$ and 1, respectively.

Table 5. Optimistic, pessimistic and most likely value of the fuzzy parameter for case 1.

Vendors	λ	1	2	3	4
p_j	0	145.00	345.00	283.00	390.00
	0.5	122.50	317.50	260.00	366.50
	1	100.00	290.00	237.00	343.00
t_j	0	20.00	14.00	8.80	30.00
	0.5	16.50	12.00	6.45	23.50
	1	13.00	10.00	4.10	17.00
l_j	0	3.2	5.4	12.10	6.70
	0.5	2.45	3.9	10.15	5.10
	1	1.7	2.4	8.20	3.50
U_j	0	6200	17,650	8350	6220
	0.5	5800	16,775	7550	5705
	1	5400	15,900	6750	5190
B_j	0	1,360,000	5,570,000	1,845,000	340,000
	0.5	1,280,000	5,252,500	1,777,500	315,000
	1	1,200,000	4,935,000	1,710,000	290,000
s_j	0	7.20	6.30	2.20	10.30
	0.5	5.20	4.85	1.55	8.15
	1	3.20	3.40	0.90	6.00
f_j	0	0.0620	0.0330	0.0930	0.0420
	0.5	0.0555	0.0255	0.0810	0.0355
	1	0.0490	0.0180	0.0690	0.0290
r_j	0	0.90	1.00	0.98	0.99
	0.5	0.88	0.93	0.93	0.90
	1	0.86	0.86	0.88	0.81

Table 6. Optimistic, pessimistic and most likely value of the parameter for case 2.

Vendors	λ	1	2	3	4
p_j	0	148.79	350.00	287.55	395.06
	0.5	123.13	318.13	260.63	367.52
	1	97.46	286.20	233.71	339.96
t_j	0	20.76	14.51	9.25	31.52
	0.5	16.62	12.13	6.56	23.75
	1	12.49	9.74	3.87	15.98
l_j	0	3.37	5.63	12.38	6.87
	0.5	2.50	3.94	10.18	5.13
	1	1.63	2.25	7.99	3.37
U_j	0	6301	17,840	8464	6326
	0.5	5825	16,794	7575	5719
	1	5349	15,748	6687	5111
B_j	0	1,375,198	5,587,731	1,856,398	343,799
	0.5	1,281,266	5,253,133	1,778,133	315,633
	1	1,187,335	4,918,536	1,699,868	287,467
s_j	0	7.51	6.51	2.25	10.75
	0.5	5.25	4.87	1.56	8.18
	1	2.99	3.25	0.87	5.62
f_j	0	0.0625	0.0337	0.0937	0.0425
	0.5	0.0556	0.0256	0.0813	0.0356
	1	0.0487	0.0174	0.0687	0.0287
r_j	0	0.91	1.02	0.99	1.02
	0.5	0.88	0.93	0.93	0.91
	1	0.86	0.85	0.87	0.79

Table 7. Optimistic, pessimistic and most likely value of the parameter for case 3.

Vendors	λ	1	2	3	4
p_j	0	135.43	338.22	269.79	380.63
	0.5	123.64	322.73	257.18	368.69
	1	111.85	307.24	244.57	356.75
t_j	0	17.82	13.15	7.52	27.04
	0.5	16.39	11.88	6.18	23.06
	1	14.96	10.61	4.84	19.08
l_j	0	2.93	4.88	10.84	5.99
	0.5	2.48	3.87	9.61	4.98
	1	2.03	2.86	8.38	3.97
U_j	0	5932	17,230	7885	6043
	0.5	5700	16,817	7505	5689
	1	5468	16,403	7124	5335
B_j	0	1,331,185	5,394,682	1,777,112	323,239
	0.5	1,280,245	5,231,522	1,764,536	308,615
	1	1,229,305	5,068,362	1,751,960	293,990
s_j	0	6.44	5.61	1.87	8.67
	0.5	5.51	4.63	1.49	7.54
	1	4.58	3.65	1.11	6.41
f_j	0	0.0605	0.0286	0.0875	0.0395
	0.5	0.0547	0.0246	0.0794	0.0357
	1	0.0488	0.0206	0.0714	0.0319
r_j	0	0.88	0.95	0.96	0.91
	0.5	0.87	0.92	0.94	0.88
	1	0.85	0.88	0.91	0.84

When demand is fixed to $D = 25,000$ units. Using values of Table 5, problem (4) of case 1 was solved using the FGP approach as defined in Section 4.2. The results obtained at the distinct value of $\lambda = 0, 0.5$ and 1 are given in Table 8.

Table 8. Compromise objective value and allocation for case 1 with fixed demand.

λ	Objective Value	Vendor's Allocation
0	(7,232,987, 385,099.2, 1431.551)	(6200, 14,845, 6084, 871)
0.5	(6,455,521, 298,038.7, 1328.458)	(5800, 11,505, 6836, 859)
1	(5,911,035, 232,290, 962.99)	(5400, 122,005, 6750, 845)

Similarly, problems (6) and (8) of case 2 and 3 respectively were solved for the distinct value of $\lambda = 0, 0.5$ and 1 using the FGP approach, and the result is summarized below in Tables 9 and 10.

Table 9. Compromise objective value and allocation for case 2 with fixed demand.

λ	Objective Value	Vendor's Allocation
0	(7,118,268, 382,976.7, 1674.064)	(6301, 11,374, 6455, 870)
0.5	(6,467,487, 301,375.7, 1337.023)	(5825, 11,495, 6822, 858)
1	(5,839,909, 224,235.7, 922.5766)	(5349, 12,120, 6686, 845)

Table 10. Compromise objective value and allocation for case 3 with fixed demand.

λ	Objective Value	Vendor's Allocation
0	(6,837,807, 331,160.2, 1506.335)	(5932, 11,632, 6587, 849)
0.5	(6,522,167, 292,957, 1055.71)	(5700, 11,602, 6861, 837)
1	(6,206,943, 254,909.6, 1072.01)	(5468, 11,584, 7124, 824)

The optimal result for various discrete choices for λ is presented in Tables 8–10. They show that vendor 4 has lost most of his quota due to inferior performance on the criterion set up by the manufacturer, i.e., percentage of flexibility, purchase rating, percentage of rejection, purchasing cost, and budget allocation. However, vendor 2 received more quota allocation as he performed best among the other vendors on the different performance criteria. The quota for vendor 3 is higher than the vendor 1 due to their inferior capacity. The result also indicates that if the value of λ increases or decreases, then the quota allocation to different vendors also starts changing consistently.

When the demand is not fixed ($D = 22,000$ or $23,000$ or 2000 or 4000) units. Since the parameters are in uncertain form; we generated the optimistic, pessimistic and most likely value of the fuzzy parameters, and their corresponding solutions. The compromise solution values for Z_1, Z_2 and Z_3 and optimum allocation of order quantities for the vendors for the same discrete values for λ are summarized below in Tables 8–10.

By using the values in Table 5, problem (4) of case 1 was solved using the FGP approach as defined in Section 4.2. The results obtained at the distinct value of $\lambda = 0, 0.5$ and 1 are given in Table 11.

Table 11. Compromise objective value and allocation for case 1 without fixed demand.

λ	Objective Value	Vendor's Allocation
0	(7,710,017, 395,237.2, 1769.69)	(6200, 13,410, 6519, 871)
0.5	(5,979,271, 280,038.7, 1269.95)	(5800, 10,005, 6836, 859)
1	(4,461,035, 182,290, 842.99)	(5400, 7005, 6750, 845)

Similarly, problem (6) and (8) of case 2 and 3 respectively were solved for the distinct value of $\lambda = 0, 0.5$ and 1 using the FGP approach, and the result is summarized in Tables 12 and 13.

Table 12. Compromise objective value and allocation for case 2 without fixed demand.

λ	Objective Value	Vendor's Allocation
0	(8,172,818, 426,695.3, 1833.757)	(6301, 14,387, 6455, 870)
0.5	(6,070,779, 286,249.6, 1287.89)	(5825, 10,248, 6822, 858)
1	(4,263,805, 170,597.5, 798.66)	(5349, 6613, 6686, 845)

Table 13. Compromise objective value and allocation for case 3 without fixed demand.

λ	Objective Value	Vendor's Allocation
0	(6,866,894, 332,291.1, 1510.53)	(5932, 11,718, 6587, 849)
0.5	(5,788,602, 265,953.7, 990.71)	(5700, 9329, 6861, 837)
1	(4,784,421, 205,785.3, 939.58)	(5468, 6954, 7124, 824)

The optimal result for various discrete choices λ is presented in Tables 11–13. They show that vendor 4 lost most of their quota due to inferior performance on the criterion set up by the manufacturer, i.e., percentage of flexibility, purchase rating, percentage of rejection, purchasing cost, and budget allocation. However, the second vendor received more quota allocation as he performed best among the other vendors on the different performance criteria. The quota for vendor 3 is higher than the vendor 1 due to their inferior capacity. The result also indicates that if the value of λ increases or decreases, then the quota allocation to different vendors also starts changing consistently.

6. Result Analysis and Discussion

The efficiency of the solution at a different value λ was compared within the different methods by calculating the Relative Effectiveness (R.E.).

Here $T_{Approach_1}$ represents the trace value obtained when demand is fixed and $T_{Approach_2}$ represents the trace value when demand is not fixed. Approach 1, and Approach 2 are based on the use of different types of ranking function and are evaluated on the different scales of λ , i.e., $0, 0.5$, and 1 , because of this value of the objective function and the quota allocation to the vendor's also changes. However, among all the methods used, case 3 provides the best optimum result under the given restrictions, and the solution found, is very close to the aspiration level set by the decision-maker.

Pessimistic Result ($\lambda = 0$). From the different solutions tables and their obtained trace value, we can see that the results attained from Approach 2 are very close to the aspiration goal set by the decision-maker. Approach 2 provides a better objective value in comparison to Approach 1 for an aggregate demand of 25,000 units that yields a minimum net ordering cost of 6,837,807 rupees, minimum transportation cost of 331,160.2 rupees and a minimum late delivered units to be around 1506 (aggregated) during the process. Based on the optimization technique and the results generated from it show that the manufacturer decides to purchase 11,632 units form vendor 2, 6587 units form vendor 3, 5932 units from vendor 1 and purchase only 849 units from vendor 4 due to their most inferior performance on the criteria set (viz. the highest percentage rejections, high percentage late deliveries, less vendor rating value, less quota flexibility value, etc.).

Most Likely Result ($\lambda = 0.5$). From the different solutions tables and their obtained trace value, we can see that the results attained from Approach 1 are very close to the aspiration goal set by the decision-maker. Approach 1 provides a better objective value in comparison to Approach 2 for an aggregate demand of 25,000 units that yields a minimum net ordering cost of 6,455,521

rupees, minimum transportation cost of 298,038.7 rupees and a minimum late delivered units to be around 1328 (aggregated) during the process. Based on the optimization technique and the results generated from it, show that the manufacturer decides to purchase 11,505 units form vendor 2, 6836 units form vendor 3, 5800 units from vendor 1 and purchase only 859 units from vendor 4 due to their most inferior performance on the criteria set (viz. the highest percentage rejections, high percentage late deliveries, less vendor rating value, less quota flexibility value, etc.). (5,839,909, 224,235.7, 922.5766)

Optimistic Result ($\lambda = 1.0$). From the different solutions tables and their obtained trace value, we can see that the results attained from Approach 2 is very close to the aspiration goal set by the decision-maker. Approach 1 provides a better objective value in comparison to Approach 2 for an aggregate demand of 25,000 units that yields a minimum net ordering cost of 5,839,909 rupees, minimum transportation cost of 224,235.7 rupees and a minimum late delivered units to be around 923 (aggregated) during the process. Based on the optimization technique and the results generated from it show that the manufacturer decides to purchase 12,120 units form vendor 2, 6686 units form vendor 3, 5349 units from vendor 1 and purchase only 845 units from vendor 4 due to their most inferior performance on the criteria set (viz. the highest percentage rejections, high percentage late deliveries, less vendor rating value, less quota flexibility value, etc.). A similar conclusion can be drawn for the uncertain demand. Notable, the objective values and order quantities associated with different λ values do not need to be the same.

The solution to the problem instance given above was solved by LINGO 16.0. The following results were generated, which indicate that most of the goals are attainable either with some minor and significant improvement in the set of targets. The result analysis of vendor selection for discrete choices of λ is given in Figures 1 and 2.

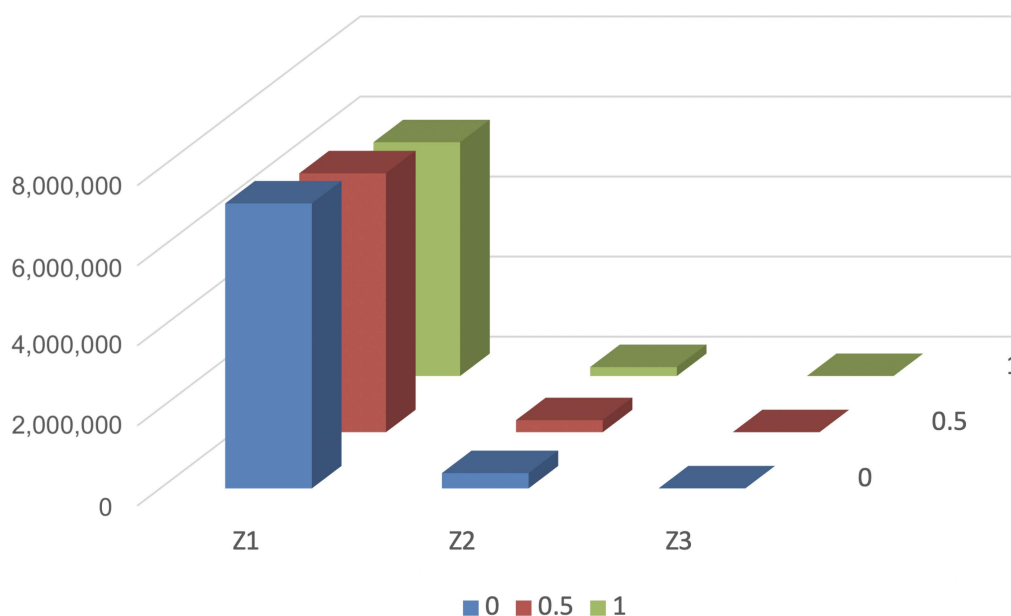


Figure 1. Value of objective function.

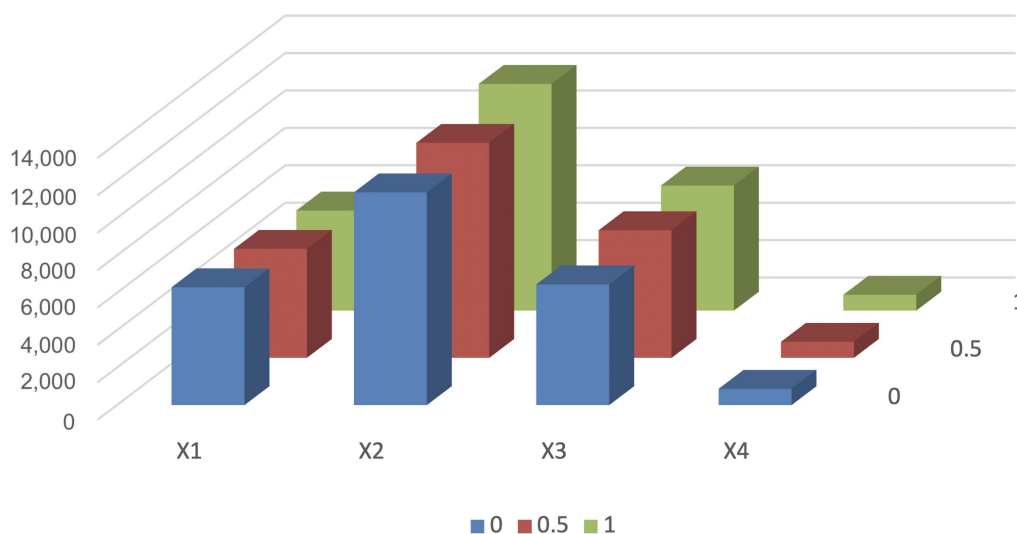


Figure 2. Vendor’s quota.

Mean value Method. Since Table 6 is in the case of vagueness, where the input information is present in the form of LR fuzzy numbers. The suitable fuzzy set theory concept was applied to obtain the equivalent crisp form. Moreover, we can consider the mean method of extreme values to handle the vagueness in the input parameters. Therefore, Table 4 is revised accordingly to the mean values method, and resultant Table 14 is given below:

Table 14. Averaging data of Vendor Selection Problem.

Vendors	1	2	3	4
p_i	122.50	316.30	258.80	364.50
t_i	16.30	11.80	6.20	23.00
l_i	2.35	3.825	10.075	5.05
U_i	5750	16,737	7500	5678
B_i	1,277,500	5,251,250	1,776,250	313,750
q_i	5.10	4.80	1.525	8.075
f_i	0.055	0.025	0.081	0.035
r_i	0.878	0.925	0.925	0.898

Also, we used some well-known existing multi-objective approaches, i.e., goal programming, Chebyshev goal programming, and also the two different types of fuzzy goal programming for solving the proposed MOVSP model and found that the solution obtained by these approaches are ranging within the solution range provided by the proposed approach. The different approaches for solving the proposed MOVSP model are shortly defined below:

The Goal Programming. The data given in Table 15 was used to solve the MOVSP by the goal programming approach.

$$\begin{aligned}
 &Min \sum_{k=1}^K \lambda_k \\
 &s.t. \\
 &Z_k(x) - \lambda_k \leq Z_k^*(x), \quad k = 1, 2, \dots, K \\
 &g_i(x) \{ \leq, =, \geq \} b_i, \quad i = 1, 2, \dots, m \\
 &x \geq 0
 \end{aligned}$$

After using the goal programming approach, the optimal compromise solution is obtained as (6,398,528, 282,442.2, 1301.87) with the order quantities (5750,12,387,6863,0).

The Chebyshev Goal Programming. The data given in Table 15 was used to solve the MOVSP by the Chebyshev goal programming approach.

$$\begin{aligned}
 & \text{Min } \lambda \\
 & \text{s.t.} \\
 & Z_k(x) + \lambda \geq Z_k^*(x), \quad k = 1, 2, \dots, K \\
 & g_i(x) \{ \leq, =, \geq \} b_i, \quad i = 1, 2, \dots, m \\
 & \underline{x} \geq \underline{0}
 \end{aligned}$$

After using the Chebyshev goal programming approach, the optimal compromise solution is obtained as (6,439,980, 292,074.2, 1313.05) with order quantities (5750, 11,527, 6863, 860).

The Fuzzy Goal Programming. The data given in Table 15 was used to solve the MOVSP by the fuzzy goal programming approach.

$$\begin{aligned}
 & \text{Max } \lambda \\
 & \text{s.t.} \\
 & \lambda \leq \frac{U_k - Z_k(x)}{U_k - L_k}, \quad k = 1, 2, \dots, K \\
 & g_i(x) \{ \leq, =, \geq \} b_i; \quad i = 1, 2, \dots, m \\
 & \underline{x} \geq \underline{0} \\
 & 0 \leq \lambda \leq 1
 \end{aligned}$$

After using the fuzzy goal programming approach, the optimal compromise solution is obtained as (6,925,348, 297,672, 1140.43) with order quantities (4063, 16,602, 3819, 516). Also, by introducing the auxiliary variables for each objective function as λ_k , the above model is reformulated as:

$$\begin{aligned}
 & \text{Max } \sum_{k=1}^K \lambda_k \\
 & \text{s.t.} \\
 & \lambda_k \leq \frac{U_k - Z_k(x)}{U_k - L_k}, \quad k = 1, 2, \dots, K \\
 & g_i(x) \{ \leq, =, \geq \} b_i; \quad i = 1, 2, \dots, m \\
 & \underline{x} \geq \underline{0} \\
 & 0 \leq \lambda_k \leq 1
 \end{aligned}$$

After using the fuzzy goal programming approach with auxiliary variables, the optimal compromise solution is obtained as (6,731,794, 299,191.8, 1088.83) with order quantities (5078, 16,590, 3332, 0).

Table 15. Relative efficiency of the methods.

Problem 1(a) of case 1			
λ	$T_{Approach1}$	$T_{Approach2}$	$(R.E. = T_{Approach1}/T_{Approach2})$
0	7,619,517.75	7,502,918.76	1.0155
0.5	6,754,888.16	6,770,199.72	0.9977
1	6,144,287.99	6,065,067.28	1.0131
Problem 1(a) of case 2			
λ	$T_{Approach1}$	$T_{Approach2}$	$(R.E. = T_{Approach1}/T_{Approach2})$
0	7,619,517.75	7,170,473.54	1.0626
0.5	6,754,888.16	6,816,179.71	0.9910
1	6,144,287.99	6,462,924.61	0.9506
Problem 1(a) of case 3			
λ	$T_{Approach1}$	$T_{Approach2}$	$(R.E. = T_{Approach1}/T_{Approach2})$
0	7,502,918.76	7,170,473.54	1.0464
0.5	6,770,199.72	6,816,179.71	0.9934
1	6,065,067.28	6,462,924.61	0.9384

7. Conclusions

A Multi-Objective Vendor Selection Problem (MOVSP) incorporating vagueness in the parameters was presented in this paper, which is useful to select vendors' quota allocation in supply chain management. Two approaches, certainty and uncertainty in demand along with different types of defuzzification methods of fuzzy numbers are considered. Various techniques such as mean value method, goal programming with some of its variants are incorporated in this work Under both cases of fixed and uncertainty in demand considered, three values of the fuzzy parameters are generated. That is, the optimistic, the pessimistic, and the most likely as shown in Tables 5–7. The trace values of each case are represented with an approach. Different ranking functions were used in evaluating the approaches at varying values of λ . The relative effectiveness of these approaches is compared to ascertain the efficiency of the solution (Table 15). The optimal compromise multi-objective solution for various λ values is presented in Tables 8–13. It can be seen that the fourth vendor lost most of their quota as a result of inferior performance on the criteria such as flexibility percentage, purchase rating, rejection percentage, purchasing cost, and budget allocation set by the decision-maker. However, the second vendor received more quota allocation due to his best performance among the vendors on the different criteria. The quota for the third vendor is higher than the first vendor due to their low capacity. The result also indicates quota allocation to vendors changes with the value of λ consistently. Fuzzy multi-objective programming is used to solve the entire MOVSP. Since in multiple objective optimization problems, a single satisfying solution for all the objectives is not possible, the concept of Pareto multi-objective solution is considered. In this regard, we solved the formulated MOVSP to obtain the compromise multi-objective solution of the problem to enable manufacturers to make a viable and profitable decision in their manufacturing process. The Pareto solution for the various vendors is analyzed above. The technique can handle the complexity of quota allocation problems of vendors in the supply chain even if their capacities are vague, which may be due to the information gap between suppliers and buyers. The proposed MOVSP under fuzzy environment has advantages over the deterministic techniques in the sense that precise information is not readily available in supply chain management. The inherent uncertainty in the input information such as transportation cost; the demand, the price, late delivery cost of an item as well as budget and vendors capacity are considered to be fuzzy. In addition, this will enable decision-makers to address VSPs under uncertainty. Another advantage of this proposed model is that, any available commercial

packages such as LINDO/LINGO, GAMS and the like could be useful in solving such problems. A case study with simulated data concerning vendor selection is used to demonstrate the effectiveness of the proposed MOVSP model. This research opens to researchers a new methodology which can be applied to solve other similar supplier selection problems with slight modifications. For instance, in the future, this study can be further examined under an extended fuzzy environment, or where an astronomical data on the real-life situation is available, a probabilistic approach to solving the problem can be employed.

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